Linearized waveform inversion (LWI) of multicomponent blended data with polarization filters

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Outline

1. Artifacts in imaging blended data
2. Multicomponent (MC) data
3. New LWI objective function
4. Polarization filtering of MC blended data
   - SVD-based approach
   - Prediction-error filter-based
5. Road ahead

Some nomenclature

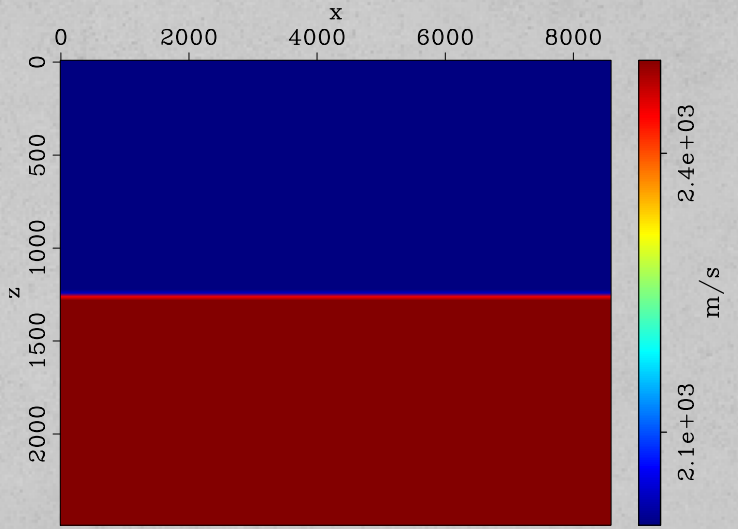
- Blending - done in the field (determined by the acquisition)
- Encoding - performed in the computer (can change from iteration to iteration)

Direct imaging of blended data

- A well-studied problem (Xue et al., 2014; Dai et al., 2011; Jiang et al., 2010; Tang et al., 2009, …)

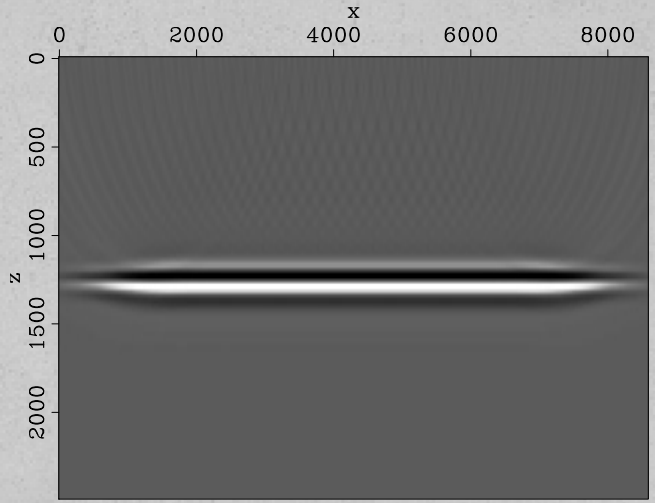
- The challenge is with the artifacts (cross terms)

Single interface model



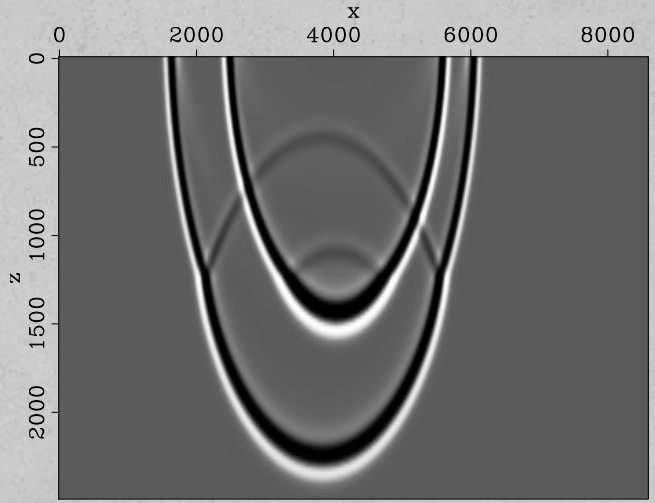
m

Conventional migrated image



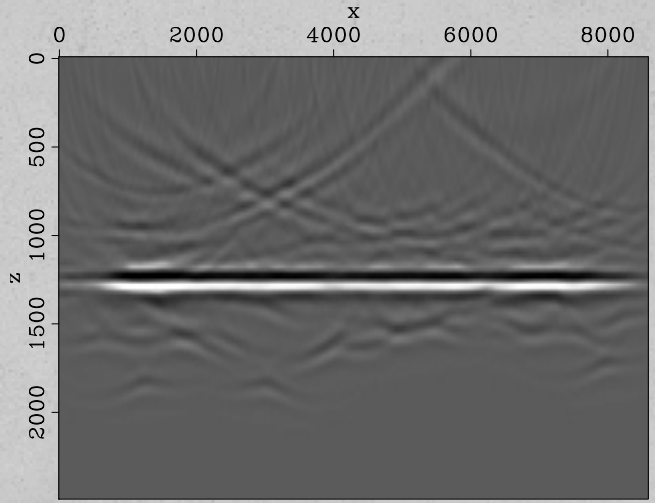
$L'd$

Blended modeling



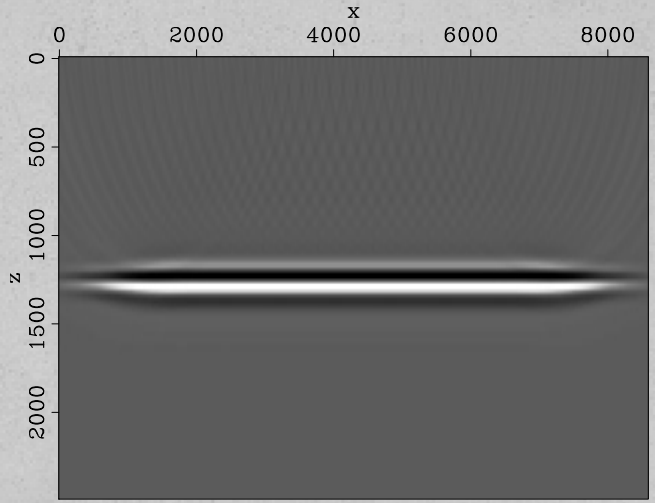
$\widetilde{\mathbf{L}}\mathbf{m}$

Blended migrated image



$\widetilde{L}'\widetilde{d}$

Conventional migrated image



$L'd$

Cross terms

- Conventional modeling and imaging

$$\mathbf{L}_1\mathbf{m} = \mathbf{d}_1$$

$$\mathbf{L}_2\mathbf{m} = \mathbf{d}_2$$

$$\widehat{\mathbf{m}} = \mathbf{L'}_1\mathbf{d}_1 + \mathbf{L'}_2\mathbf{d}_2$$

Cross terms

- Conventional modeling and imaging

$$\mathbf{L}_1\mathbf{m} = \mathbf{d}_1$$

$$\mathbf{L}_2\mathbf{m} = \mathbf{d}_2$$

$$\widehat{\mathbf{m}} = \mathbf{L'}_1\mathbf{d}_1 + \mathbf{L'}_2\mathbf{d}_2$$

- Blended modeling and imaging

$$(\mathbf{L}_1 + \mathbf{L}_2)\mathbf{m} = \mathbf{d}_1 + \mathbf{d}_2$$

$$\begin{aligned}\widehat{\mathbf{m}} &= (\mathbf{L'}_1 + \mathbf{L'}_2)(\mathbf{d}_1 + \mathbf{d}_2) \\ &= \mathbf{L'}_1\mathbf{d}_1 + \mathbf{L'}_2\mathbf{d}_2 + \mathbf{L'}_1\mathbf{d}_2 + \mathbf{L'}_2\mathbf{d}_1\end{aligned}$$

Cross terms

- Conventional modeling and imaging

$$\mathbf{L}_1\mathbf{m} = \mathbf{d}_1$$

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- Blended modeling and imaging

$$(\mathbf{L}_1 + \mathbf{L}_2)\mathbf{m} = \mathbf{d}_1 + \mathbf{d}_2$$

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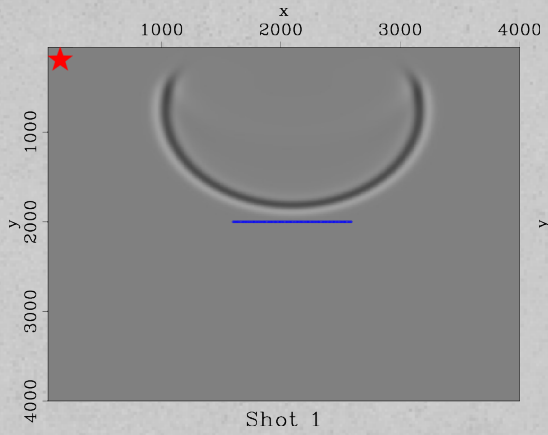
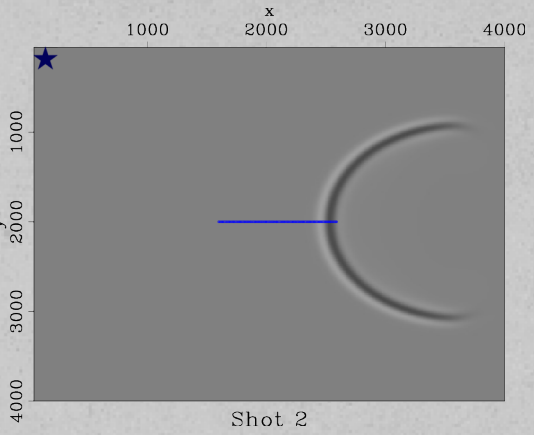
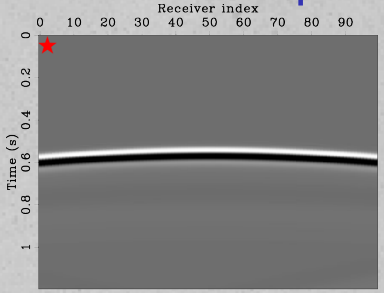
Two shots - different azimuths

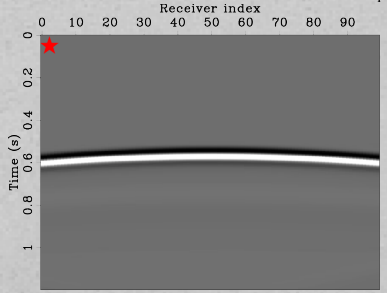




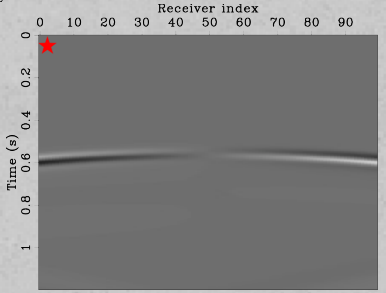
Shot one - multicomponent data



Pressure

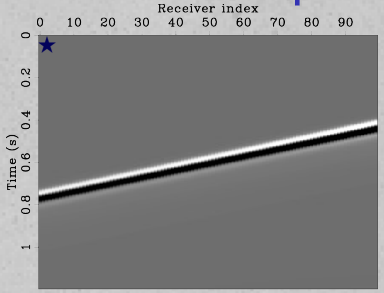


Y component

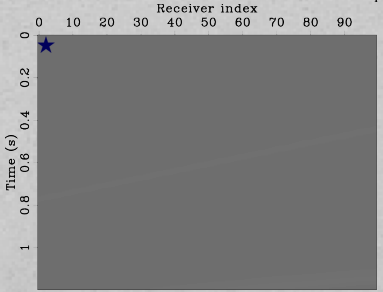


X component

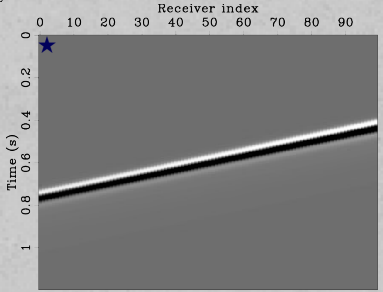
Shot two - multicomponent data



Pressure

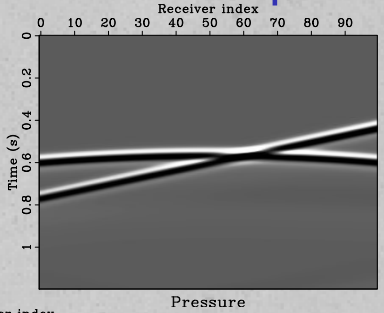


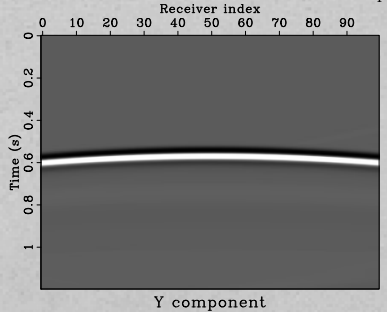
Y component

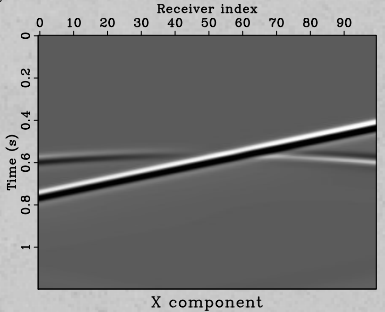


X component

Blended multicomponent data







Can the directional information measured on multicomponent data help mitigate the cross terms?

LWI of blended data

$$J(\mathbf{m}) = \frac{1}{2}||\widetilde{\mathbf{L}}\mathbf{m} - \widetilde{\mathbf{d}}||_2^2$$

- $\mathbf{m}$ - reflectivity
- $\widetilde{\mathbf{d}}$ - blended data
- $\widetilde{\mathbf{L}}$ - linearized blended wave equation modeling operator

LWI of blended data

$$J(\mathbf{m}) = \frac{1}{2}||\widetilde{\mathbf{L}}\mathbf{m} - \widetilde{\mathbf{d}}||_2^2$$

LWI of blended data with polarization filters

$$J(\mathbf{m}) = \frac{1}{2}||\widetilde{\mathbf{L}}\mathbf{m} - \widetilde{\mathbf{d}}||_2^2$$

$$J(\mathbf{m}) = \frac{1}{2}||\mathbf{L}\mathbf{m} - \mathbf{P}(\widetilde{\mathbf{d}})||_2^2$$

- $\mathbf{m}$ - reflectivity
- $\widetilde{\mathbf{d}}$ - blended multicomponent data
- $\mathbf{L}$ - independently models multicomponent data
- $\mathbf{P}$ - polarization filters; $\mathbf{P}(\widetilde{\mathbf{d}}) \approx \mathbf{d}$

Workflow

$$J(\mathbf{m}) = \frac{1}{2}||\mathbf{Lm} - \mathbf{P}(\widetilde{\mathbf{d}})||_2^2$$

$$\widetilde{\mathbf{d}} = \mathbf{d}_1 + \mathbf{d}_2$$

At each iteration of the inversion …

Workflow

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At each iteration of the inversion …

1. Independently model $\mathbf{d}_{1p} \approx \mathbf{d}_1$ and $\mathbf{d}_{2p} \approx \mathbf{d}_2$

Workflow

$$J(\mathbf{m}) = \frac{1}{2}||\mathbf{Lm} - \mathbf{P}(\widetilde{\mathbf{d}})||_2^2$$

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At each iteration of the inversion …

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3. Use $\mathbf{P}_1$ and $\mathbf{P}_2$ to give $\mathbf{d}$

Workflow

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At each iteration of the inversion …

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3. Use $\mathbf{P}_1$ and $\mathbf{P}_2$ to give $\mathbf{d}$
   - i.e. $\mathbf{P}_1(\tilde{\mathbf{d}}) \approx \mathbf{d}_1$ and $\mathbf{P}_2(\tilde{\mathbf{d}}) \approx \mathbf{d}_2$

Polarization filters

1. Singular-value decomposition (SVD)-based approach
2. Prediction error-filter (PEF)-based apprach

Polarization analysis via the SVD

- $\mathbf{y}$: Y-component trace, length nt
- $\mathbf{x}$: X-component trace, length nt

Polarization analysis via the SVD

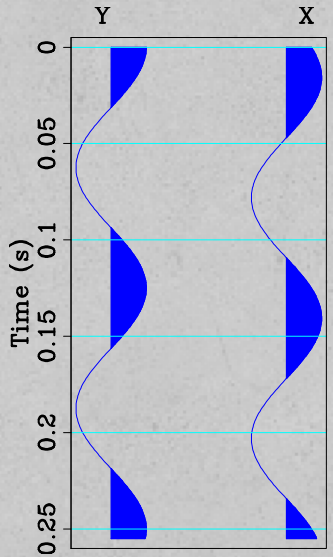
- $\mathbf{y}$: Y-component trace, length nt
- $\mathbf{x}$: X-component trace, length nt

$$\mathbf{D} = \left[\mathbf{y} \;\; \mathbf{x} \right]_{nt \times 2} \rightarrow \text{data matrix}$$

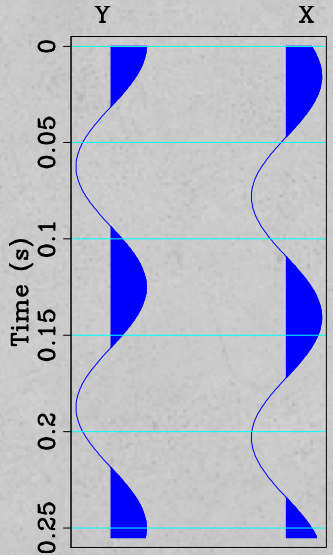
$$\mathbf{D} = \mathbf{U}_{nt \times 2} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \end{bmatrix}$$

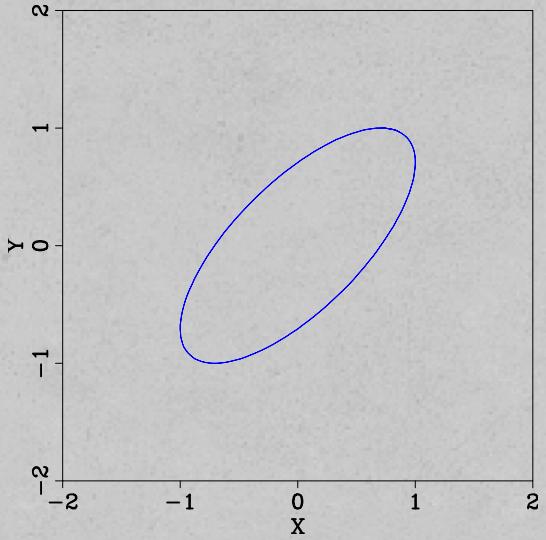
- $\mathbf{v}_1$ and $\mathbf{v}_2$ are "polarization vectors"

Simple MC data

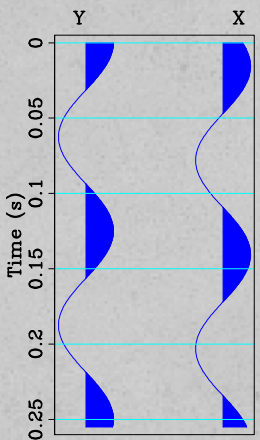


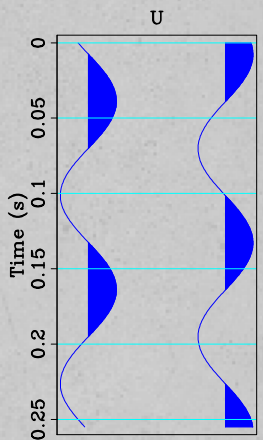
Simple MC data

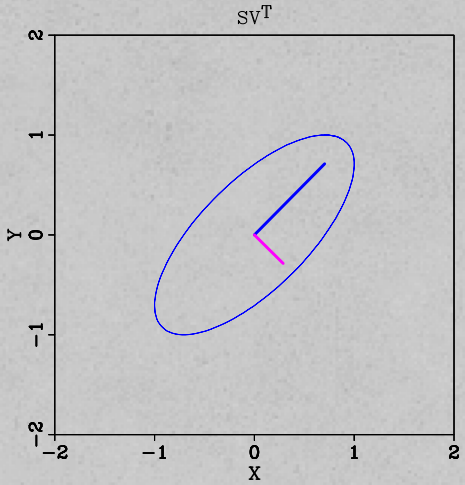




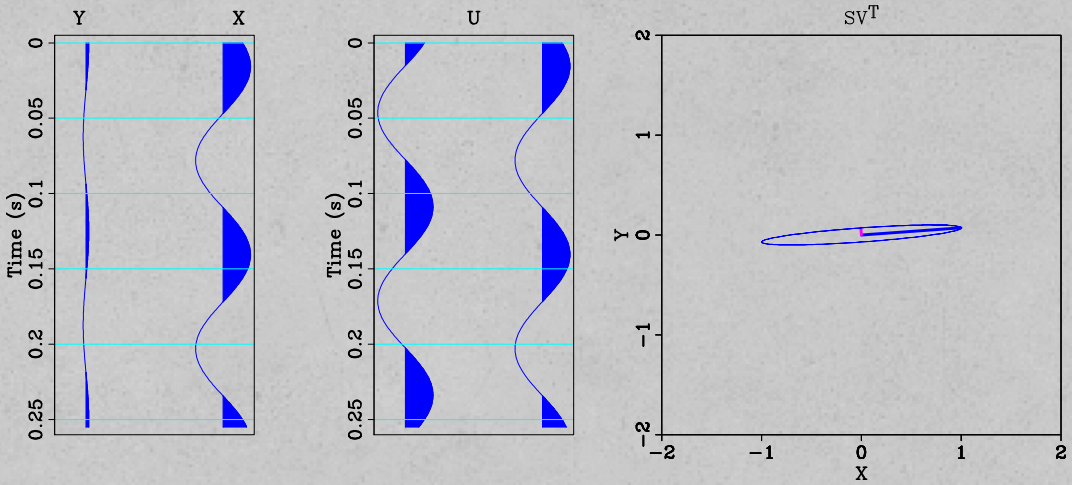
SVD of simple MC data



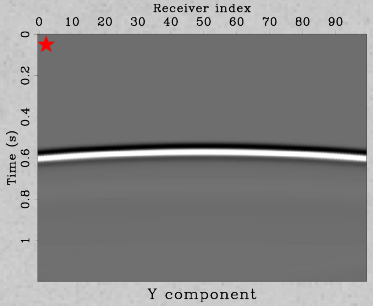




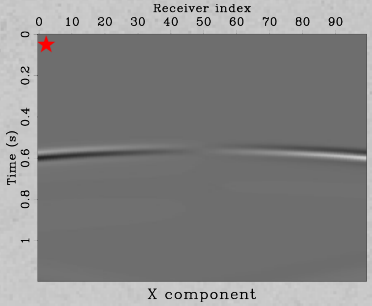
Polarized in X



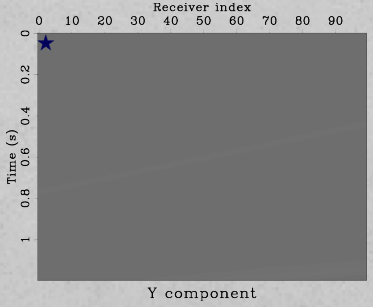
Independently modeled "proxy" data



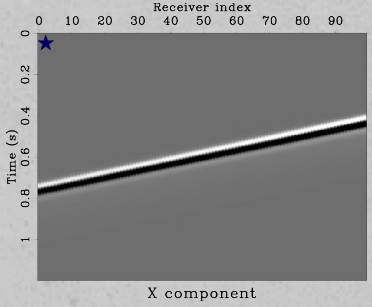
Y component



X component

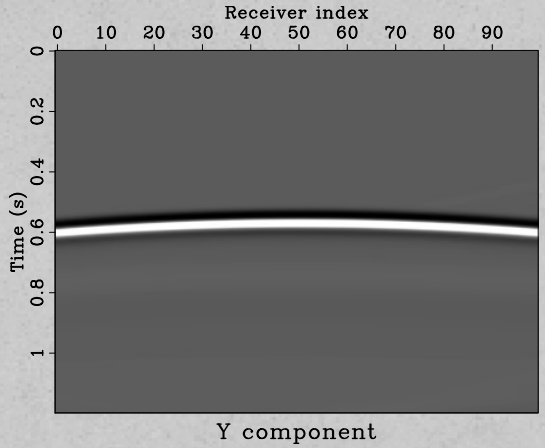


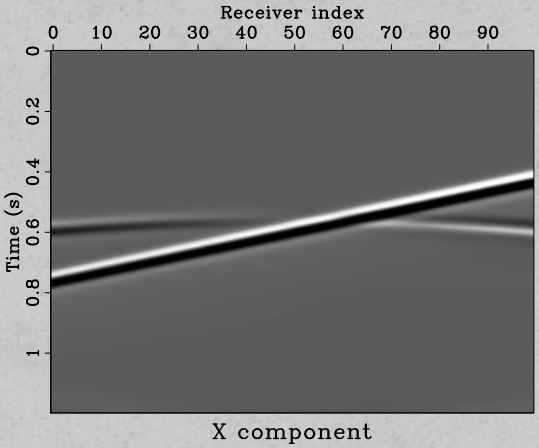
Y component



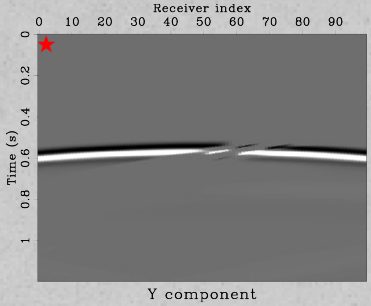
X component

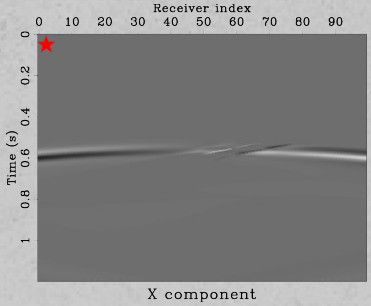
Blended multicomponent data

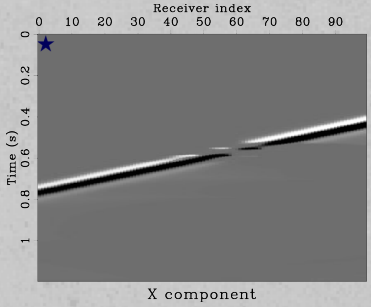




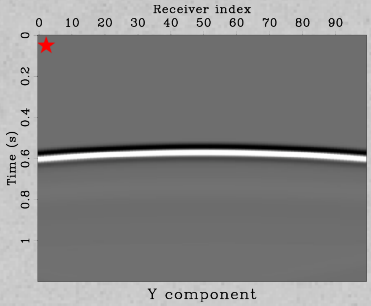
SVD-based polarization filtering: results

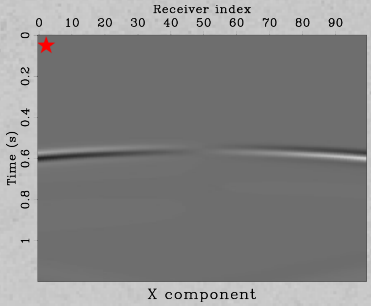


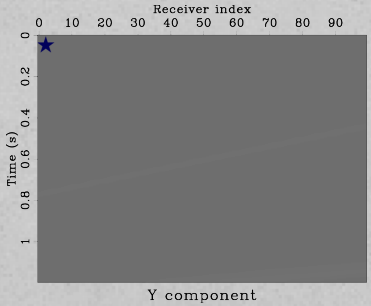


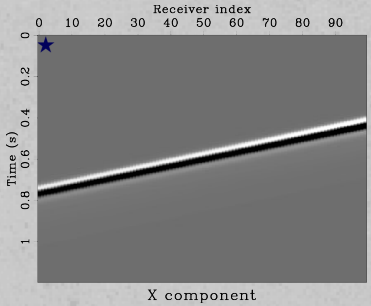


Unblended data









SVD approach: pros and cons

Pros:

- Filters based on direction (polarization)
- Computationally cheap

Cons:

- Does not make use of receiver axis
- Filters only over the time window
- Nonlinear: $\text{svd}(\mathbf{A} + \mathbf{B}) \neq \text{svd}(\mathbf{A}) + \text{svd}(\mathbf{B})$

Prediction-error filters (PEFs)

- Filters globally (not just in time windows)
- Extended to multiple spatial dimensions
- Can handle non-stationarity
- Linear
- Annihilation filters (good for signal and noise separation)

PEFs in LWI of blended data

$$J(\mathbf{m}) = \frac{1}{2}||\mathbf{Lm} - \mathbf{P}\widetilde{\mathbf{d}}||_2^2$$

$$\widetilde{\mathbf{d}} = \mathbf{d}_1 + \mathbf{d}_2$$

PEFs in LWI of blended data

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$$\widetilde{\mathbf{d}} = \mathbf{d}_1 + \mathbf{d}_2$$

1. Estimate PEFs ($\overline{\mathbf{A}}_1$ and $\overline{\mathbf{A}}_2$) on the proxy data $\mathbf{d}_{1p} \approx \mathbf{d}_1$ and $\mathbf{d}_{2p} \approx \mathbf{d}_2$

PEFs in LWI of blended data

$$J(\mathbf{m}) = \frac{1}{2}||\mathbf{Lm} - \mathbf{P}\widetilde{\mathbf{d}}||_2^2$$

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2. Use the PEFs to approximately deblend the data

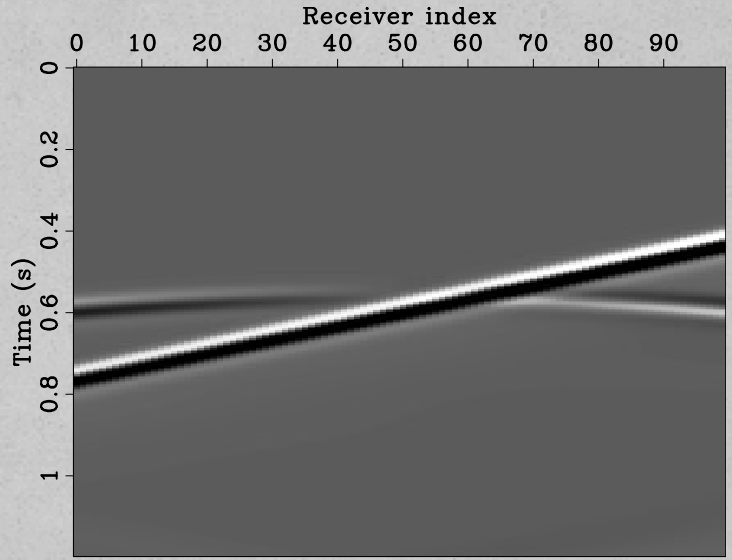
PEFs in LWI of blended data

$$J(\mathbf{m}) = \frac{1}{2}||\mathbf{L}\mathbf{m} - \mathbf{P}\widetilde{\mathbf{d}}||_2^2$$

$$\widetilde{\mathbf{d}} = \mathbf{d}_1 + \mathbf{d}_2$$

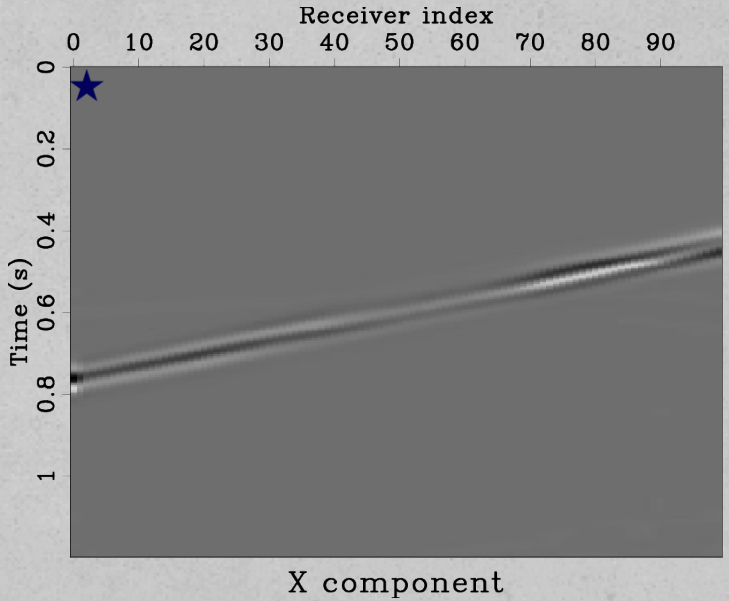
1. Estimate PEFs ($\overline{\mathbf{A}}_1$ and $\overline{\mathbf{A}}_2$) on the proxy data $\mathbf{d}_{1p} \approx \mathbf{d}_1$ and $\mathbf{d}_{2p} \approx \mathbf{d}_2$
2. Use the PEFs to approximately deblend the data
   - $\overline{\mathbf{A}}_1(\mathbf{d}_1 + \mathbf{d}_2) \approx \overline{\mathbf{A}}_1\mathbf{d}_2$
   - $\overline{\mathbf{A}}_2(\mathbf{d}_1 + \mathbf{d}_2) \approx \overline{\mathbf{A}}_2\mathbf{d}_1$

Blended X-component



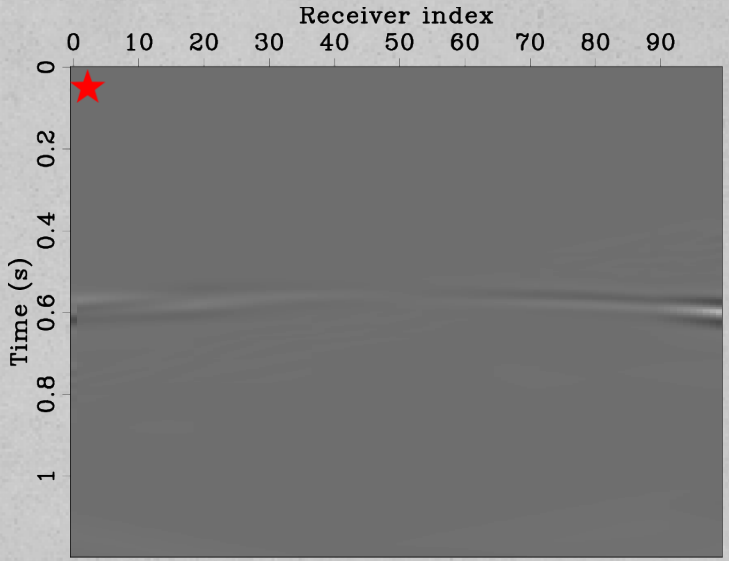
$\tilde{\mathbf{d}}$

X-component: $\overline{\mathbf{A}}_1\widetilde{\mathbf{d}}$



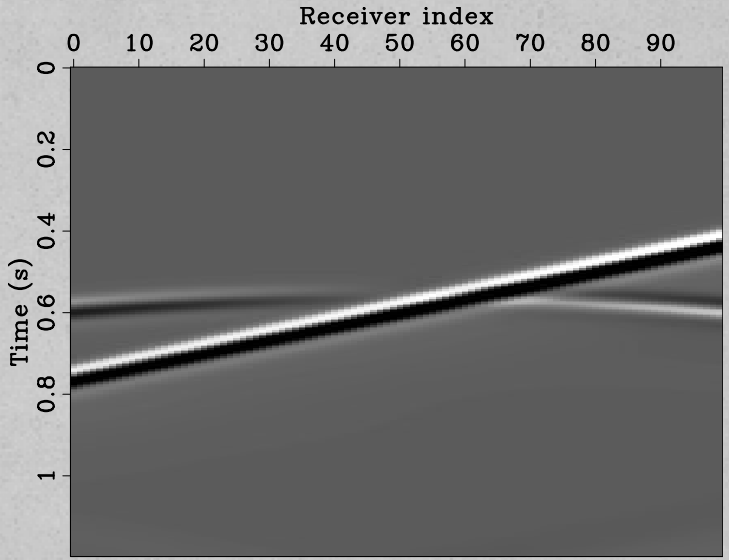
$\overline{\mathbf{A}}_1\widetilde{\mathbf{d}}$

X-component: $\overline{\mathbf{A}}_2\widetilde{\mathbf{d}}$



$\overline{\mathbf{A}}_2\widetilde{\mathbf{d}}$

Blended X-component



$\tilde{\mathbf{d}}$

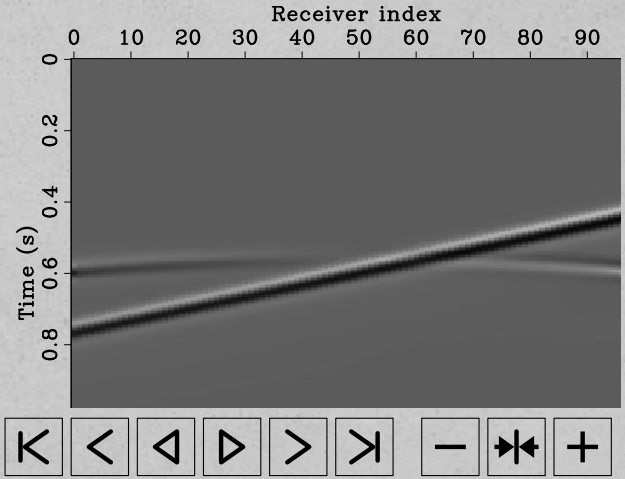
Signal and noise inversion using PEFs

$$J(\mathbf{d}_1, \mathbf{d}_2) = \frac{1}{2}||\mathbf{d}_1 + \mathbf{d}_2 - \tilde{\mathbf{d}}||_2^2 + \frac{1}{2}||\overline{\mathbf{A_1}}(\mathbf{d}_2 - \tilde{\mathbf{d}})||_2^2 + \frac{1}{2}||\overline{\mathbf{A_2}}(\mathbf{d}_1 - \tilde{\mathbf{d}})||_2^2$$

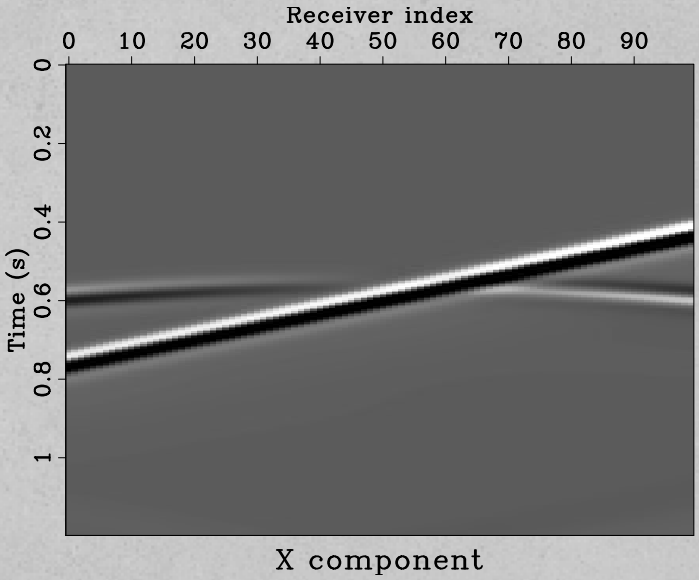
Signal and noise inversion using PEFs

$$J(\mathbf{d}_1, \mathbf{d}_2) = \underbrace{\frac{1}{2}||\mathbf{d}_1 + \mathbf{d}_2 - \tilde{\mathbf{d}}||_2^2}_{\text{deblending}} + \underbrace{\frac{1}{2}||\overline{\mathbf{A}}_{\mathbf{1}}(\mathbf{d}_2 - \tilde{\mathbf{d}})||_2^2 + \frac{1}{2}||\overline{\mathbf{A}}_{\mathbf{2}}(\mathbf{d}_1 - \tilde{\mathbf{d}})||_2^2}_{\text{signal and noise separation}}$$

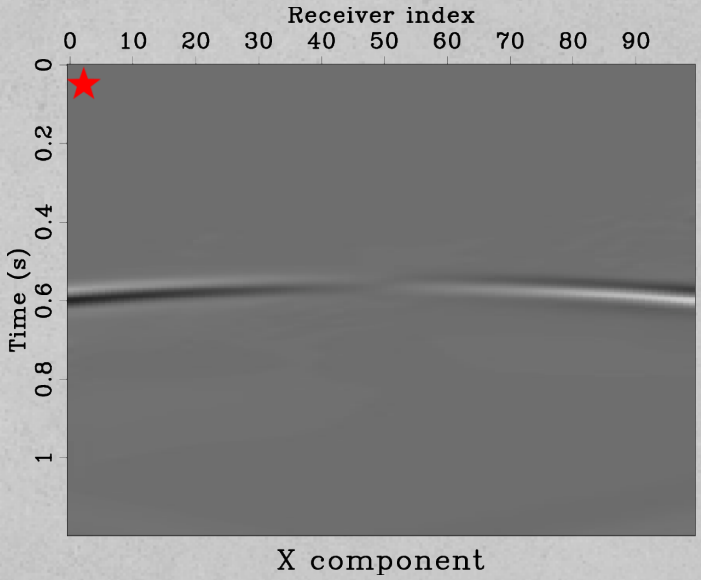
X-component PEF inversion results



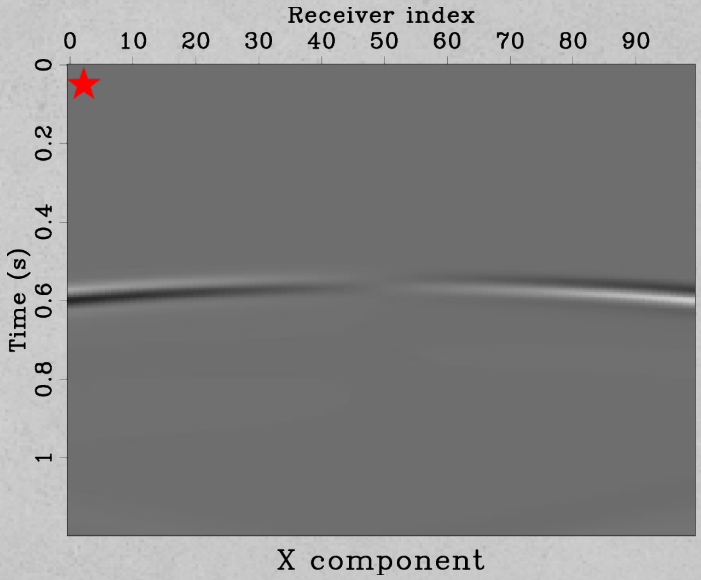
Blended X-component



Deblended X-component

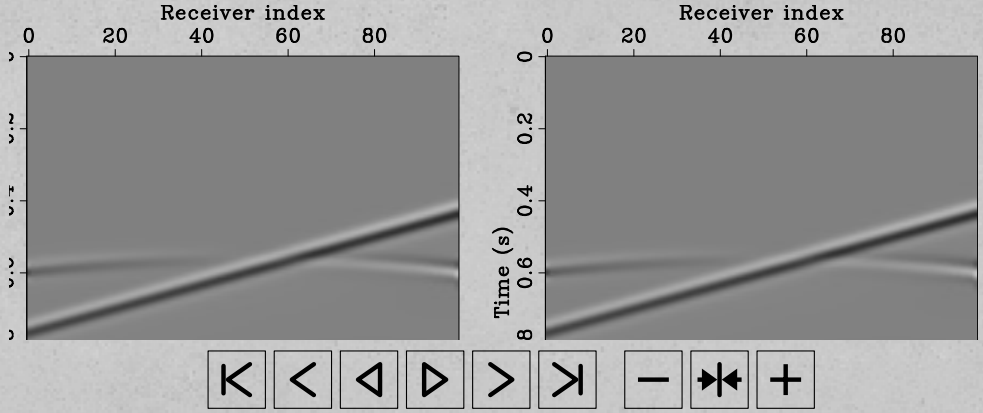


Unblended X-component

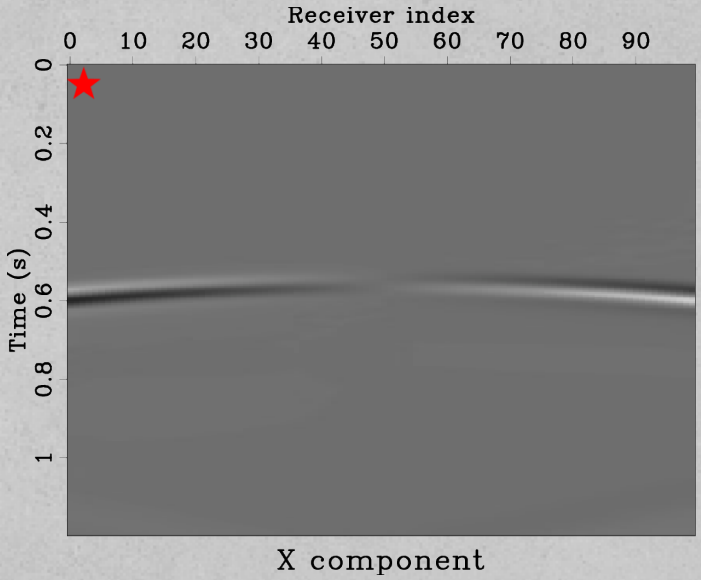


Can PEFs be used as polarization filters?

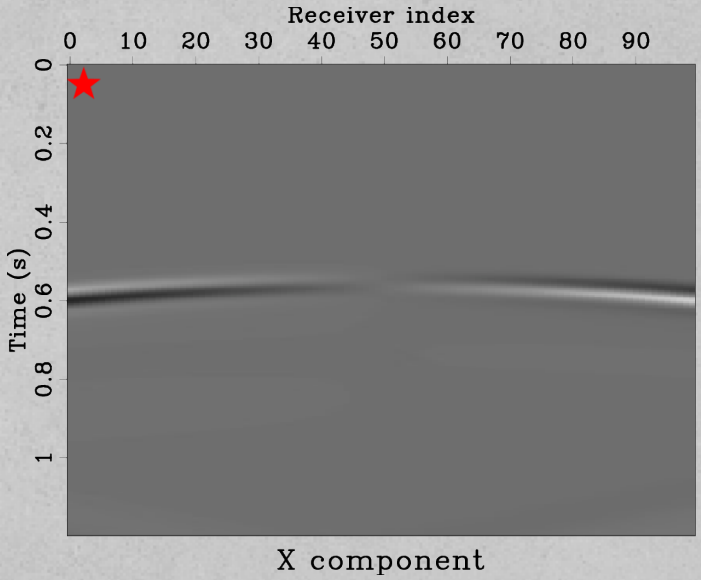
Single vs. multicomponent PEF inversion



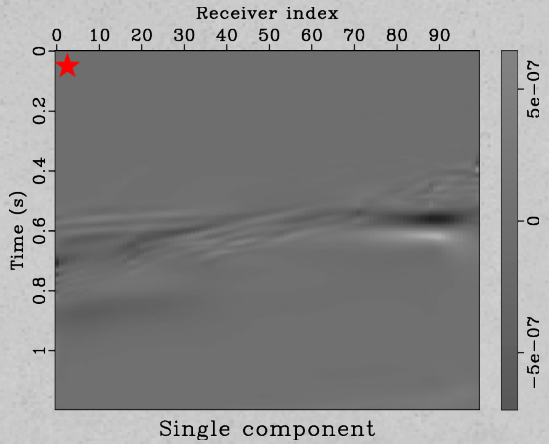
Deblended X-component using MC PEF

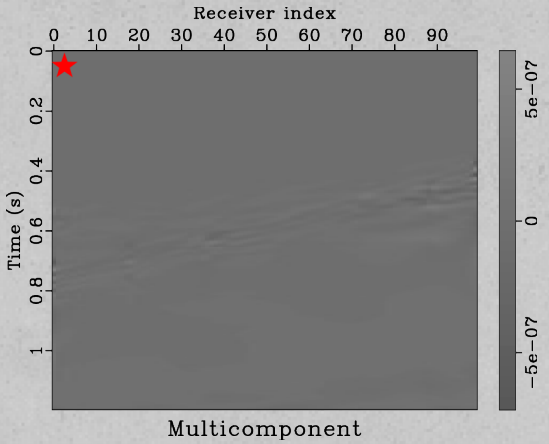


Unblended X-component

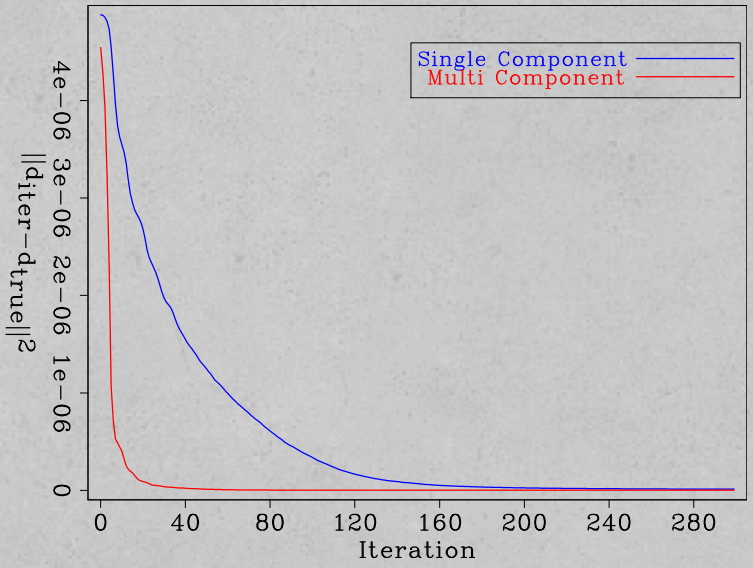


Unblended and Deblended difference

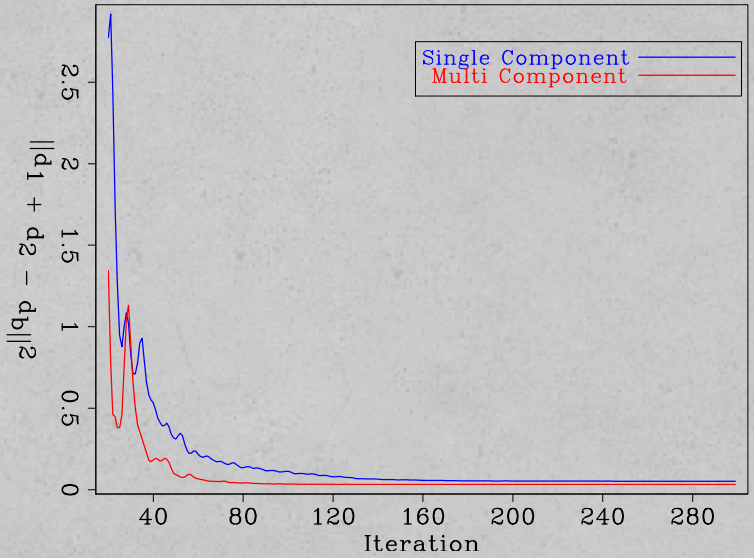




Model residual comparison



Data residual comparison



Conclusions

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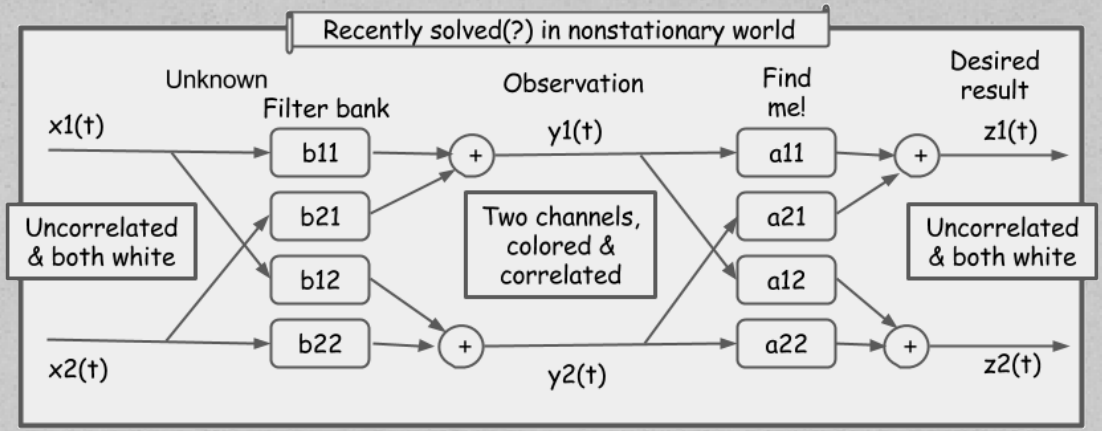
- Our PEF-based approach outperformed the SVD-based approach
- Multicomponent PEFs appear to give better signal and noise separation results

Road ahead


Recently solved(?) in nonstationary world
Unknown
Filter bank
Observation
Find me!
Desired result
x1(t)
b11
b21
b12
b22
x2(t)
y1(t)
y2(t)
a11
a21
a12
a22
z1(t)
z2(t)
Uncorrelated & both white
Two channels, colored & correlated
Uncorrelated & both white

Multicomponent acoustic data

1. Thanks to Kietta, we have multicomponent acoustic data

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2. Four component data, pressure sensor and 3 geophones (particle velocity)
   - Receivers are suspended in the water via cable

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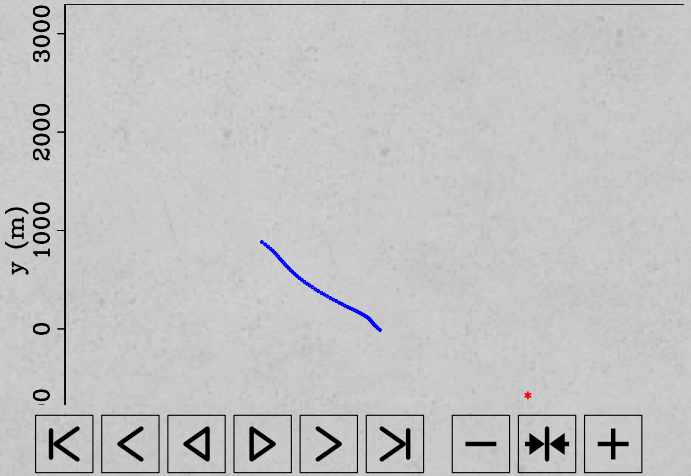
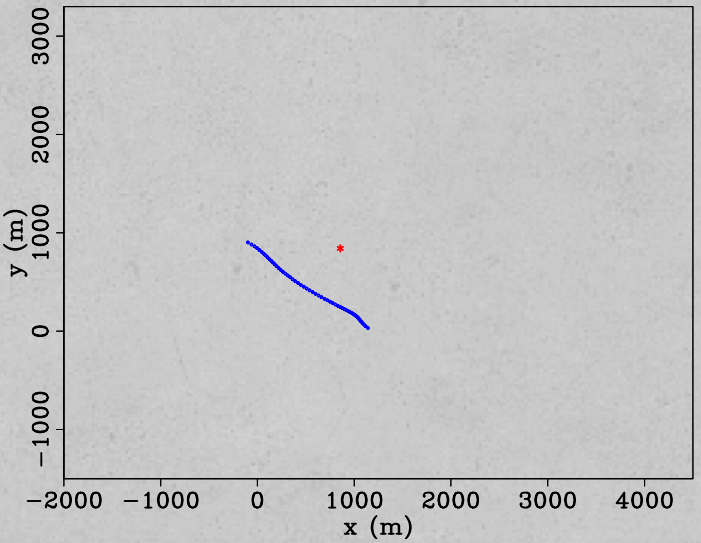
2. Four component data, pressure sensor and 3 geophones (particle velocity)
   - Receivers are suspended in the water via cable
3. Acquired in Mediterranean sea
   - 2.5 km water depth

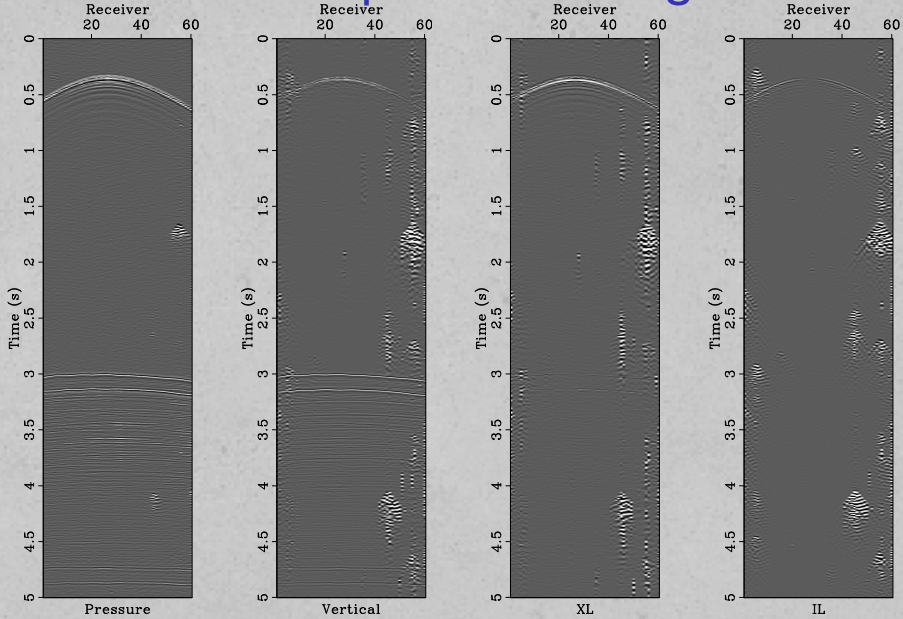
Acquisition geometry



One shot



Multicomponent shot gathers



Acknowledgements

- Kietta for allowing me to show the data
- SEP sponsors for your support

Questions?

References

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Xue, Z., Y. Chen, S. Fomel, J. Sun, et al., 2014, Imaging incomplete data and simultaneous-source data using least-squares reverse-time migration with shaping regularization: Presented at the 2014 SEG Annual Meeting, Society of Exploration Geophysicists.