STANFORD UNVERSITY

SEP meeting 2018

Target-oriented elastic full-waveform inversion through extended-migration redatuming

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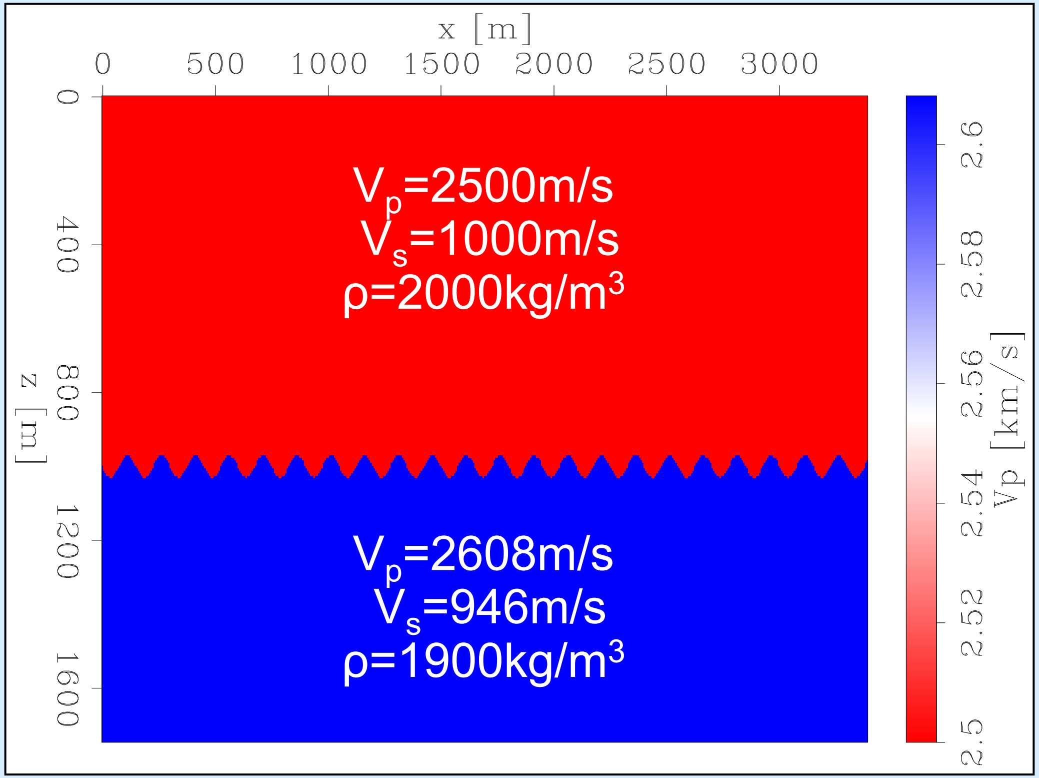
Agenda

- Why do we need an elastic FWI approach?
- Why a target-oriented approach?
- Redatuming through an extended least-squares migration
- Synthetic examples
- Conclusions

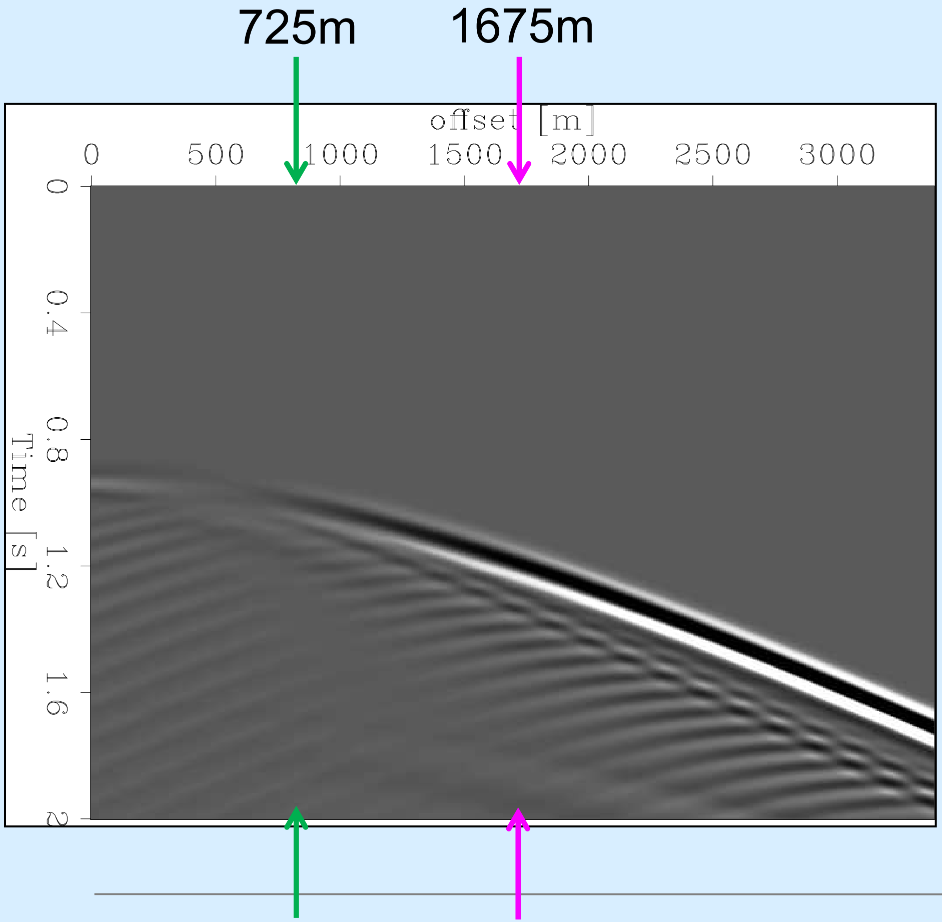
Amplitude vs angle: Zoeppritz

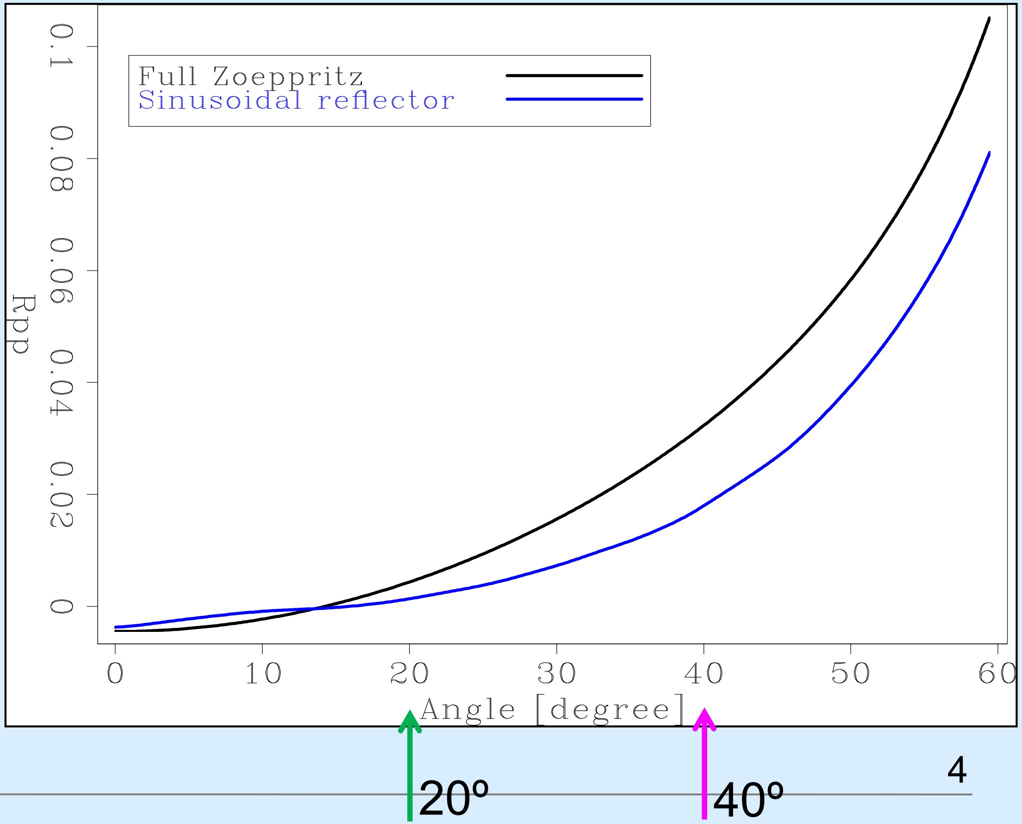
Sinusoidal reflector: planar reflector assumption

$\Delta V_p>0;\ \Delta V_s<0\ \Delta\rho<0$



Amplitude vs angle: Zoeppritz

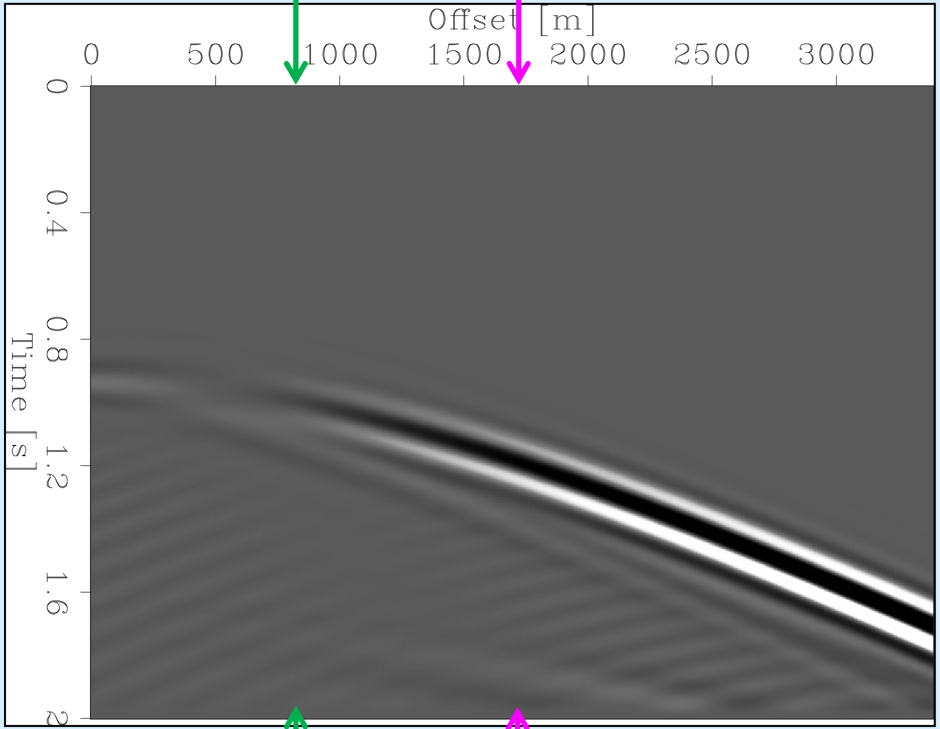


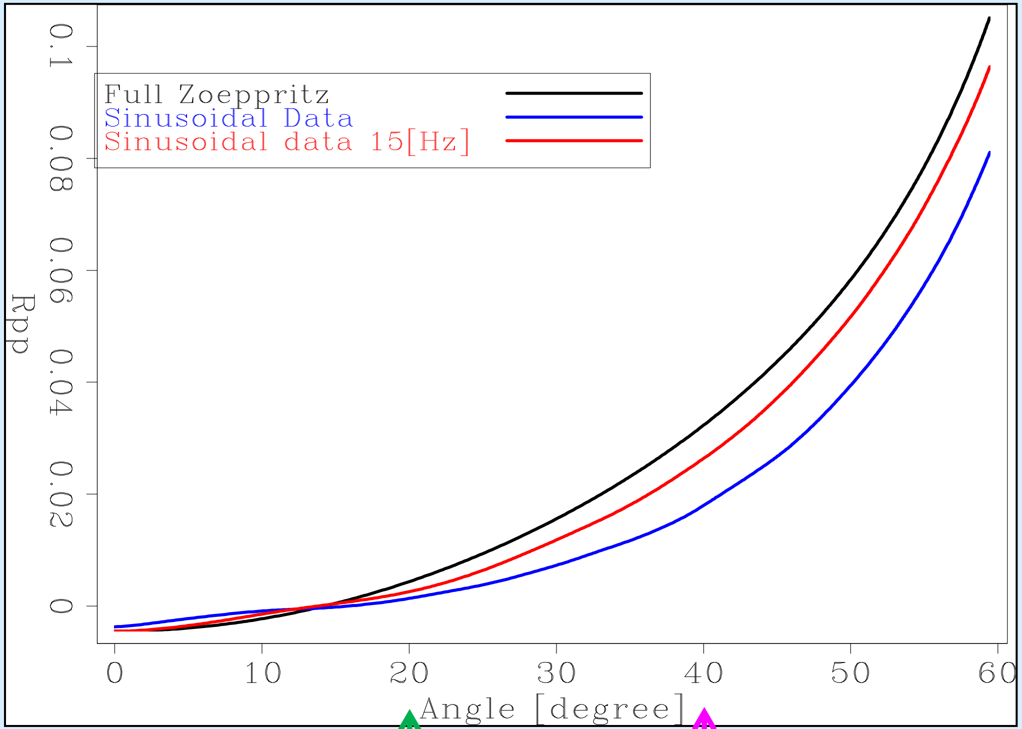


Amplitude vs angle: Zoeppritz

After low pass up to 15Hz

725m 1675m





20º 40º

Why elastic FWI?

Full waveform inversion uses all the frequencies in the data!

$$\boldsymbol{V}_p^*, \boldsymbol{V}_s^*, \boldsymbol{\rho}^* = \underset{\boldsymbol{V}_p, \boldsymbol{V}_s, \boldsymbol{\rho}}{\operatorname{argmin}} \boldsymbol{\phi}\left(\boldsymbol{V}_p, \boldsymbol{V}_s, \boldsymbol{\rho}\right) = \underset{\boldsymbol{V}_p, \boldsymbol{V}_s, \boldsymbol{\rho}}{\operatorname{argmin}} \frac{\mathbf{1}}{\mathbf{2}} \left\| \boldsymbol{f}\left(\boldsymbol{V}_p, \boldsymbol{V}_s, \boldsymbol{\rho}\right) - \boldsymbol{d} \right\|_2^2$$

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Pros:

- **Non-linear effects properly taken into account (e.g., multiples)**
- **Bandlimited nature of wavelet correctly considered**

Full waveform inversion uses all the frequencies in the data!

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Pros:

- **Non-linear effects properly taken into account (e.g., multiples)**
- **Bandlimited nature of wavelet correctly considered**

Cons:

- **Computationally expensive! (especially in 3D)**

Let’s compare acoustic and elastic finite-difference modeling

The computational cost of elastic modeling

Let's compare acoustic and elastic finite-difference modeling

2D acoustic isotropic:

$$\frac{\partial^2 p}{\partial t^2} = \nabla^2 p + s$$

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2D elastic isotropic:

$$\frac{\partial^2 u_x}{\partial t^2} = \frac{\partial}{\partial x}\left[(\lambda + 2\mu)\frac{\partial u_x}{\partial x}\right] + \frac{\partial}{\partial z}\left[\lambda \frac{\partial u_z}{\partial z}\right] + s_x$$

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The computational cost of elastic modeling

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1. Computational cost of ∇^2 approximately twice compared to $\frac{\partial}{\partial x}$ or $\frac{\partial}{\partial z}$
2. $1.6V_s \approx V_p$ in most cases

The computational cost of elastic modeling

Let's compare acoustic and elastic finite-difference modeling

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In 2D:

Elastic propagations are approximately 12 times more intensive than acoustic ones

The computational cost of elastic modeling

Let's compare acoustic and elastic finite-difference modeling

2D acoustic isotropic:

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2D elastic isotropic:

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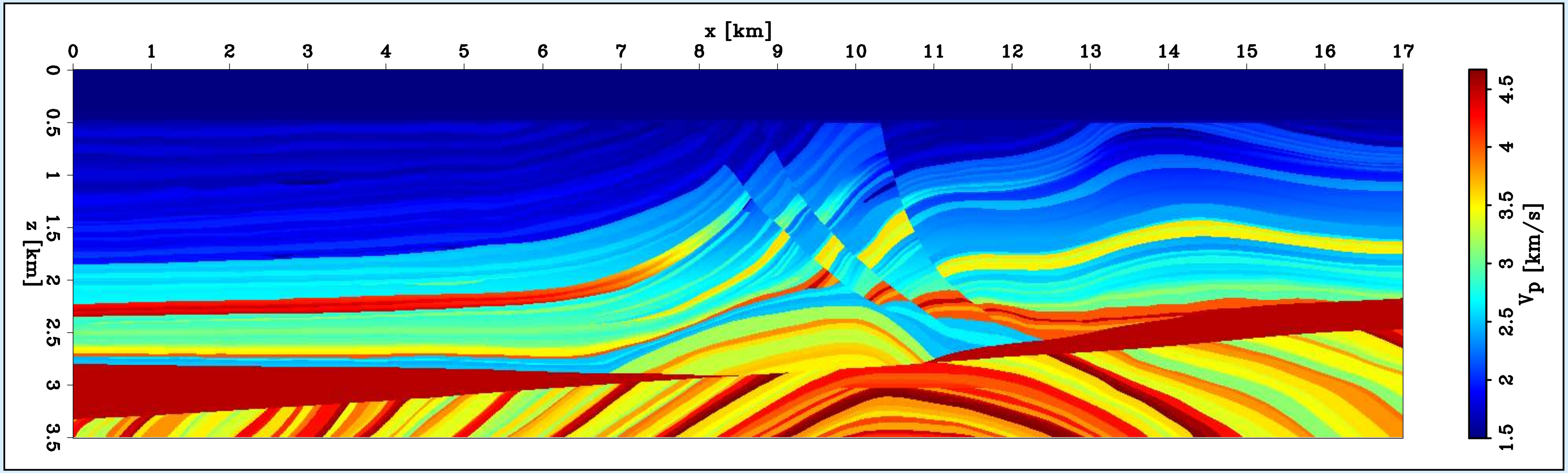
In 3D:

Elastic propagations are almost 30 times more intensive than acoustic ones

Why a target-oriented FWI?

In most field applications, high-resolution elastic properties are needed only within target areas



Martin et al. 2006

Why a target-oriented FWI?

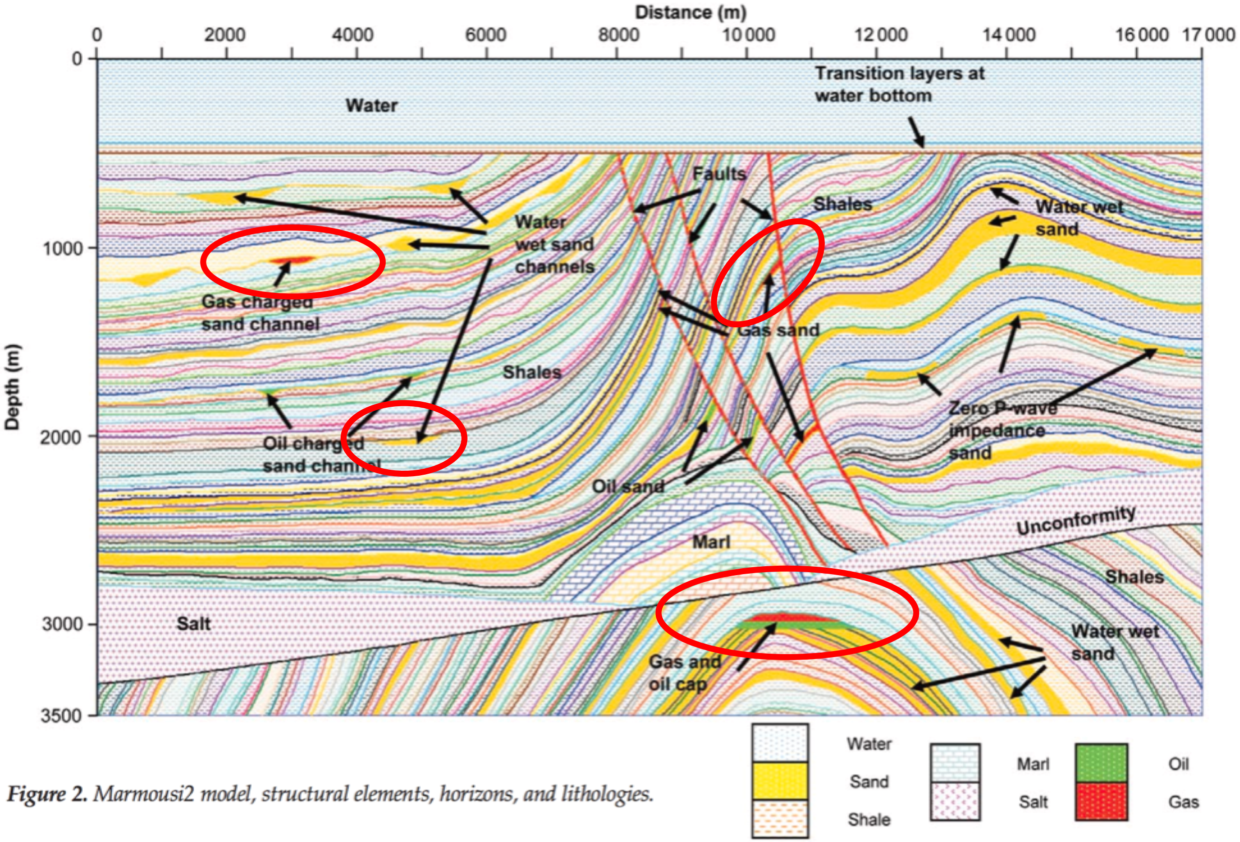


Figure 2. Marmousi2 model, structural elements, horizons, and lithologies.

Martin et al. 2006

Why a target-oriented FWI?

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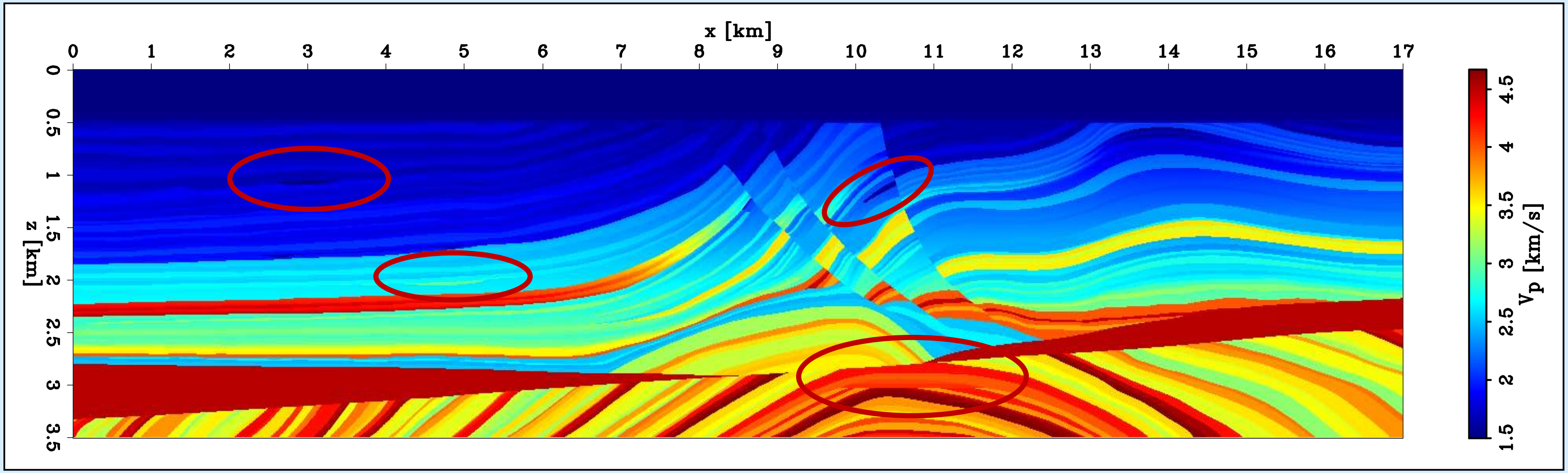


Martin et al. 2006

How can we perform a target-oriented elastic FWI?

Reconstruct the data coming from a target area (i.e., redatuming)

How to perform a target-oriented FWI?

How can we perform a target-oriented elastic FWI?

Reconstruct the data coming from a target area (i.e., redatuming)

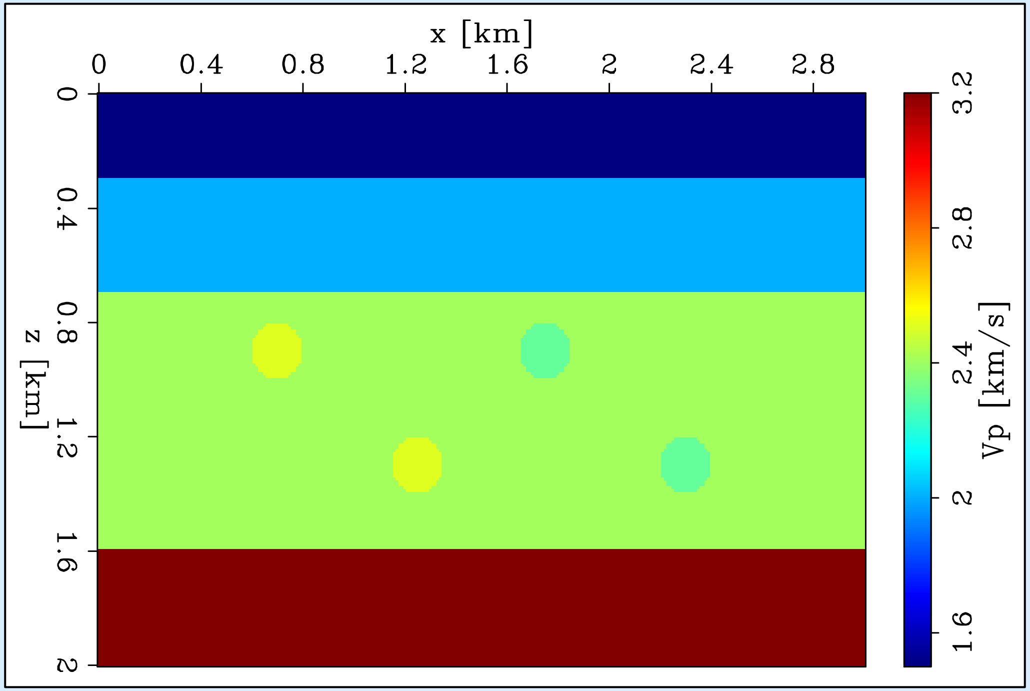
Redatuming could be performed by downward continuation. However, it is limited because:

Dense source-receiver sampling!

How can we use an extended migration to reconstruct data at a different subsurface level?

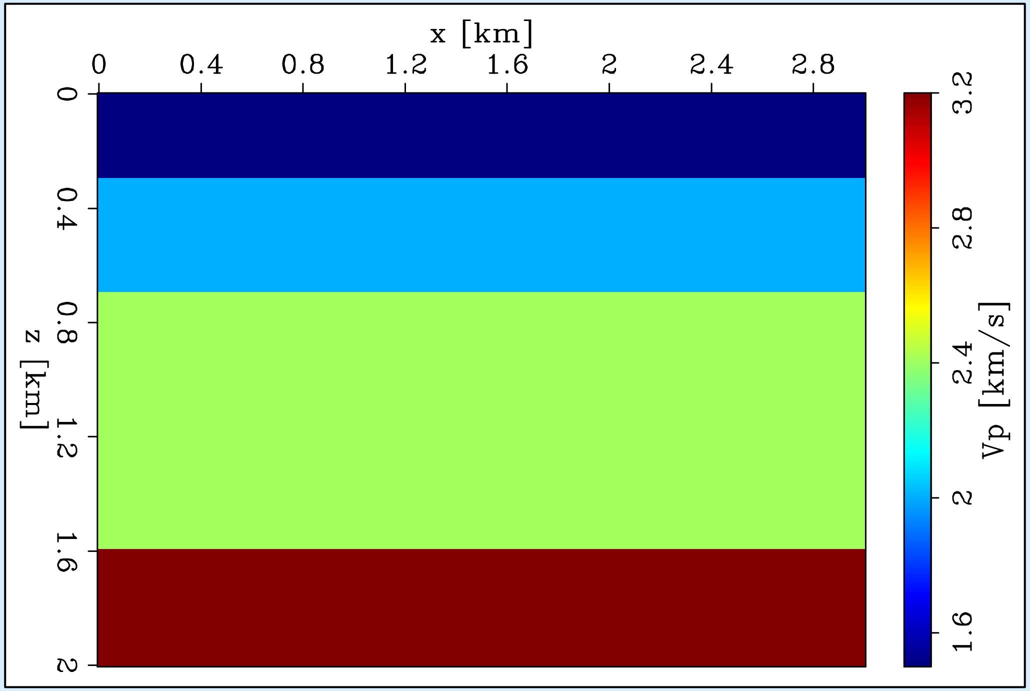
True velocity model:

$$m_{true}$$



Initial velocity model:

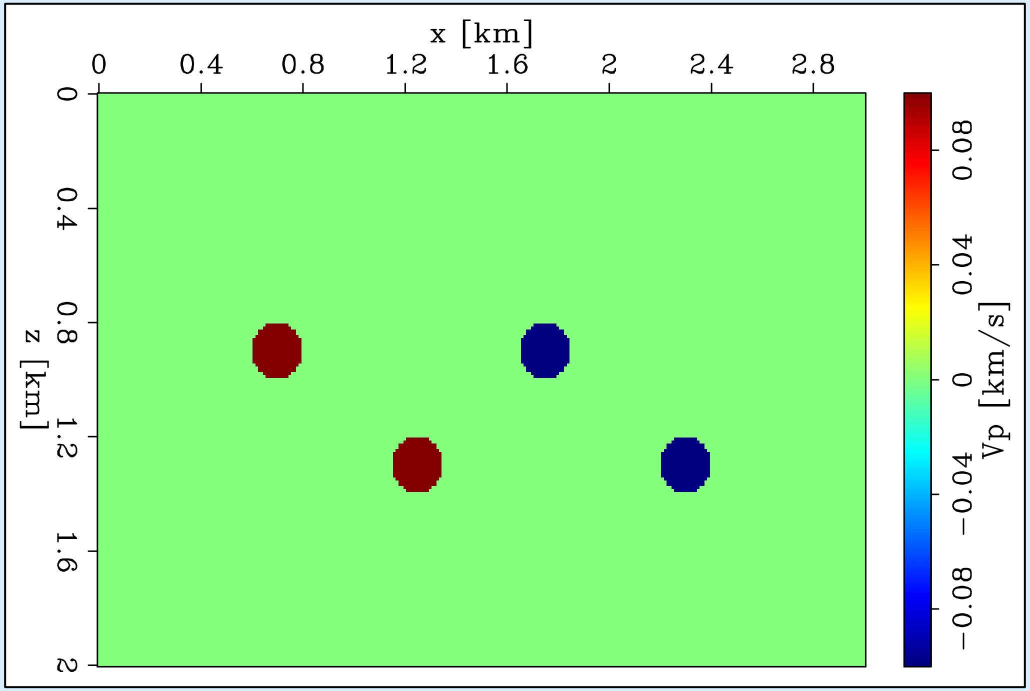
$$\boldsymbol{m_0}$$



Redatuming acoustic example

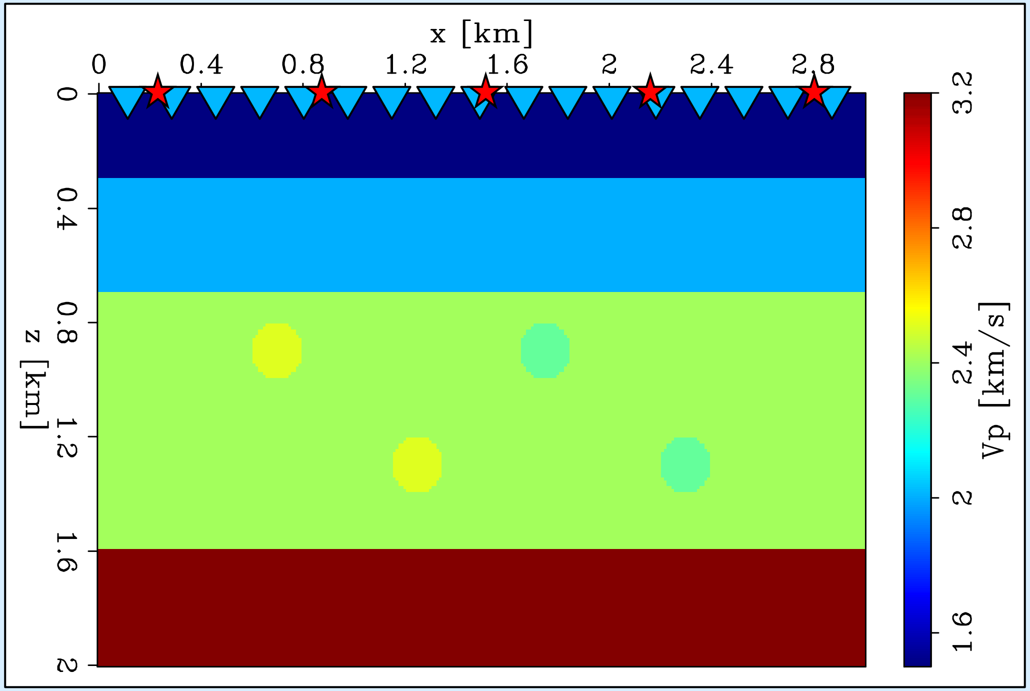
Velocity perturbation:

$$\Delta m$$



We would like to move the surface data:

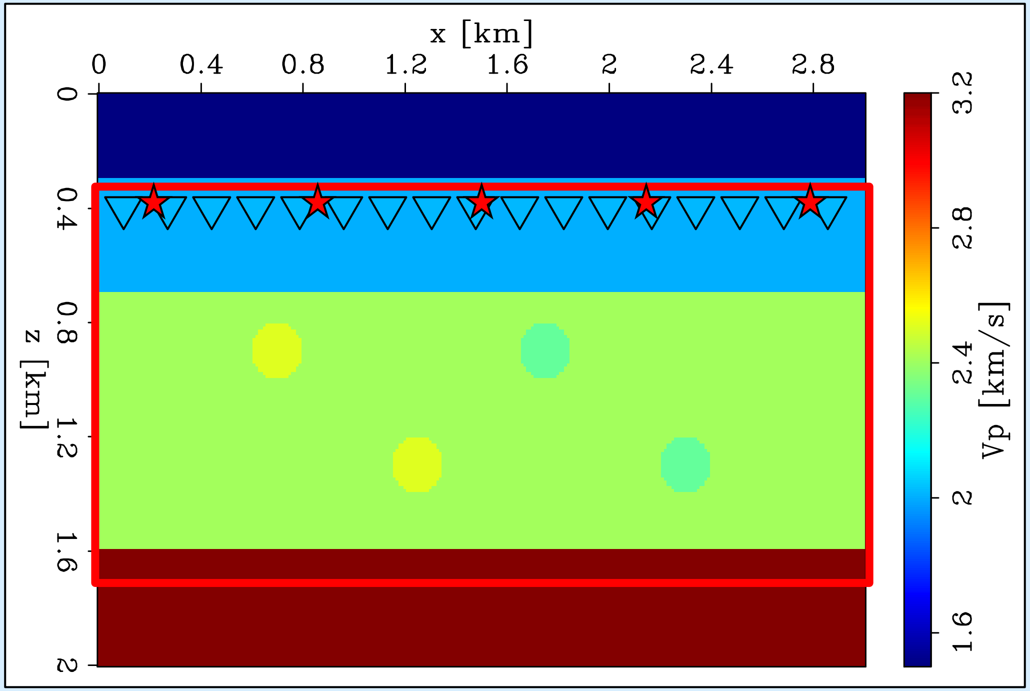
$$f(m_{true})$$



To a target area only:

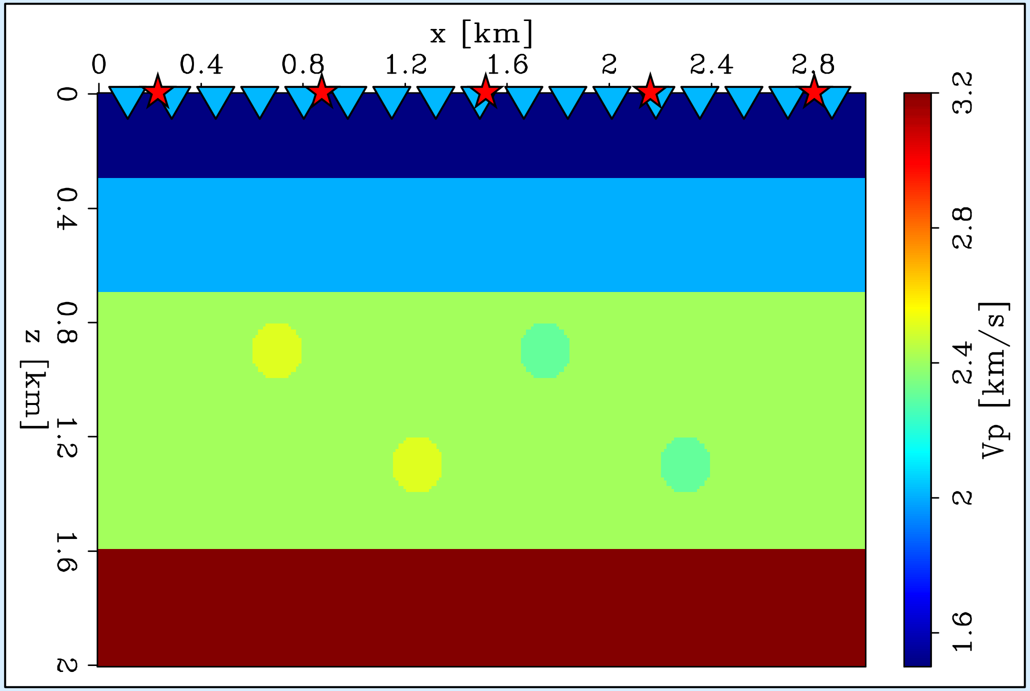
$$f'(Km_{true})$$

K = truncation to target area



Redatuming acoustic example

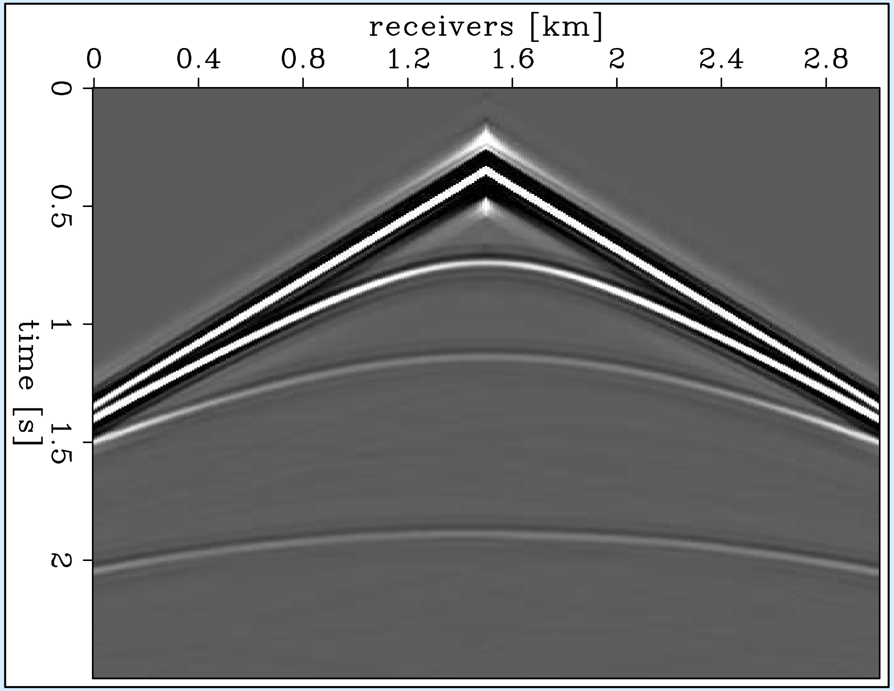
31 shots with 301 receivers
2-30Hz broadband wavelet
Acoustic isotropic modeling



Redatuming acoustic example

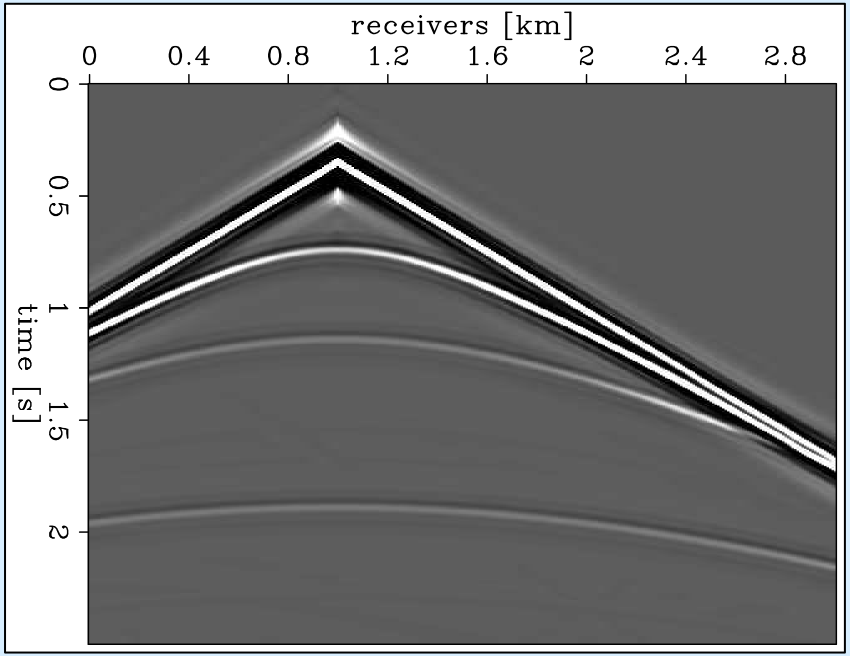
$f(m_{true})$

Observed data



$f(m_0)$

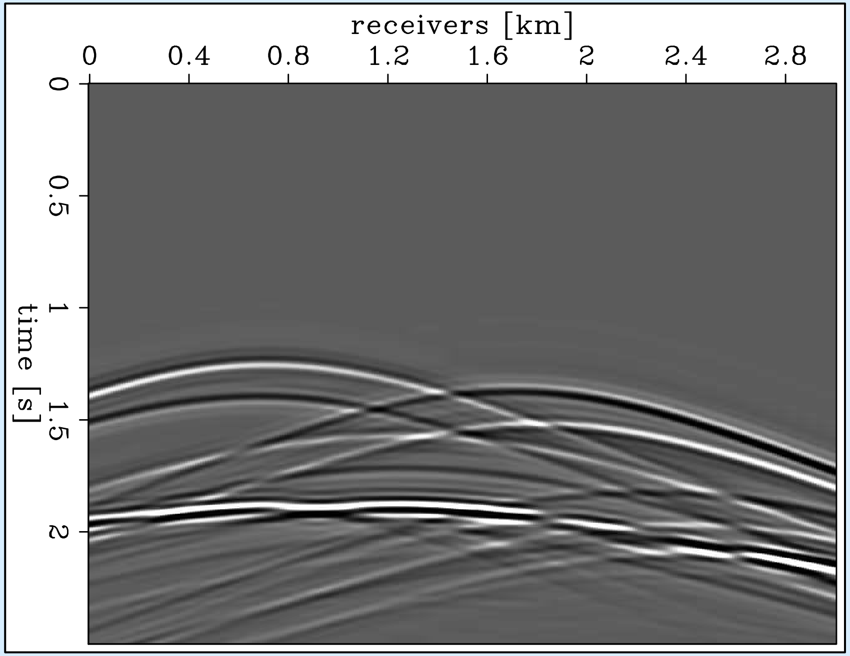
Initial modeled data



Redatuming acoustic example

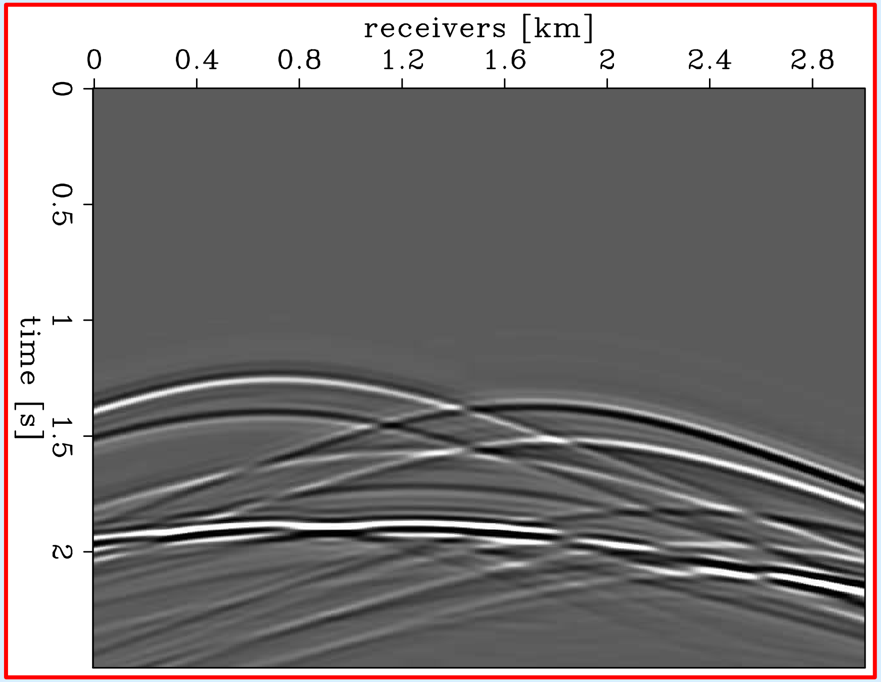
$$\Delta d = f(m_{true}) - f(m_0)$$

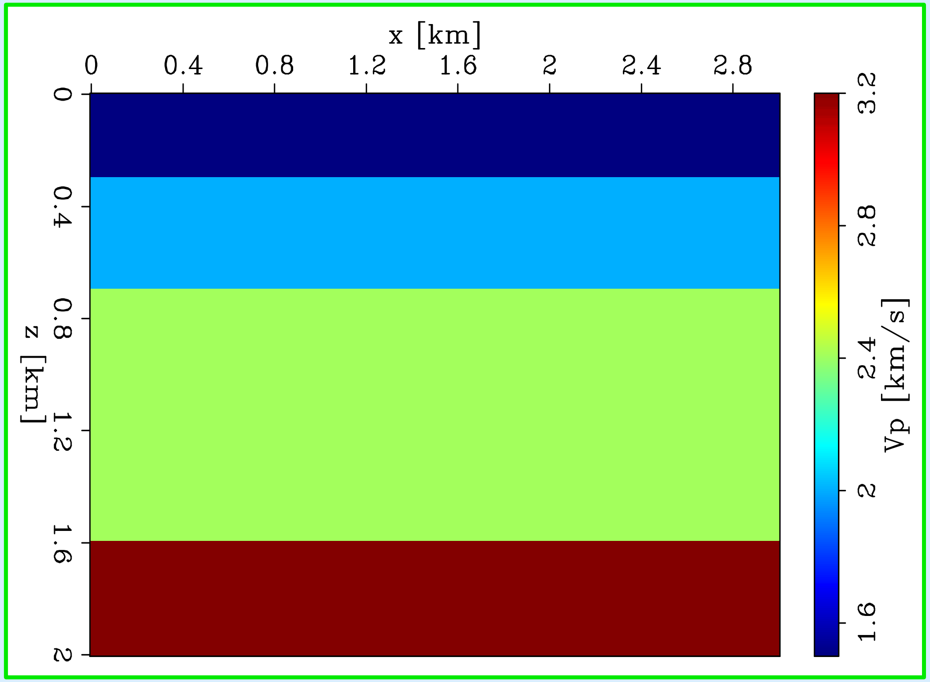
Surface data difference



Redatuming acoustic example

$$\Delta\widetilde{\boldsymbol{m}}^* = \underset{\Delta\widetilde{\boldsymbol{m}}}{\operatorname{argmin}} \left\| \widetilde{\boldsymbol{B}}(\boldsymbol{m}_0)\Delta\widetilde{\boldsymbol{m}} - \Delta\boldsymbol{d} \right\|_2^2$$





Redatuming acoustic example

$$\Delta\tilde{\boldsymbol{m}}^* = \underset{\Delta\tilde{\boldsymbol{m}}}{\operatorname{argmin}} \left\| \tilde{\boldsymbol{B}}(\boldsymbol{m}_0)\Delta\tilde{\boldsymbol{m}} - \Delta\boldsymbol{d} \right\|_2^2$$

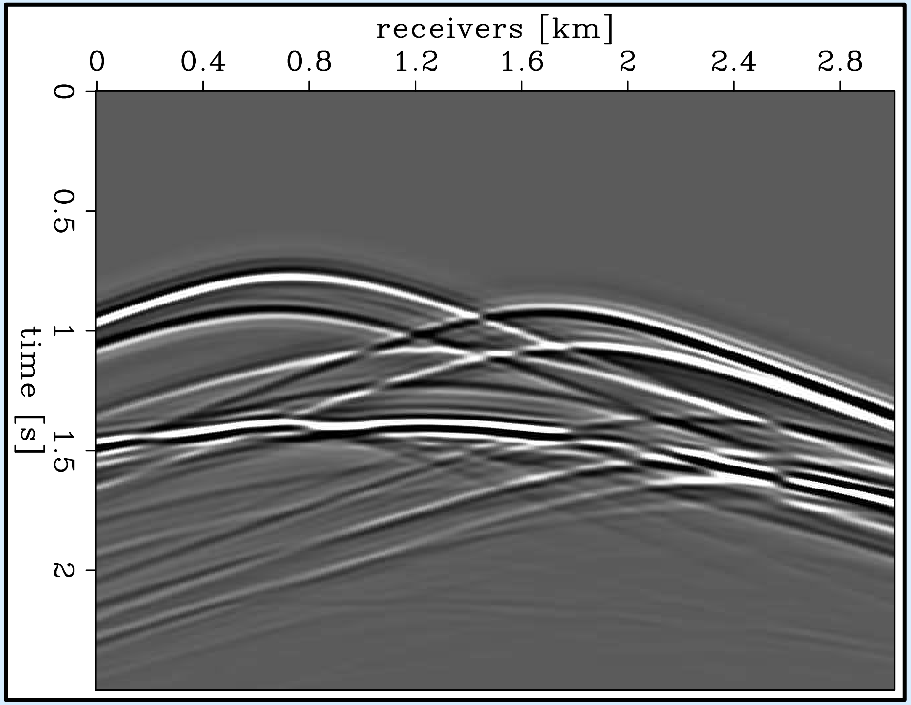
Can use the extended image to reconstruct the data difference at a new subsurface level?

$$\mathbf{\Delta d}' = \boldsymbol{f}'(\boldsymbol{K}\boldsymbol{m}_{true}) - \boldsymbol{f}'(\boldsymbol{K}\boldsymbol{m}_0) \overset{?}{=} \tilde{\boldsymbol{B}}'\boldsymbol{K}\Delta\tilde{\boldsymbol{m}}^*$$

Redatuming acoustic example: results

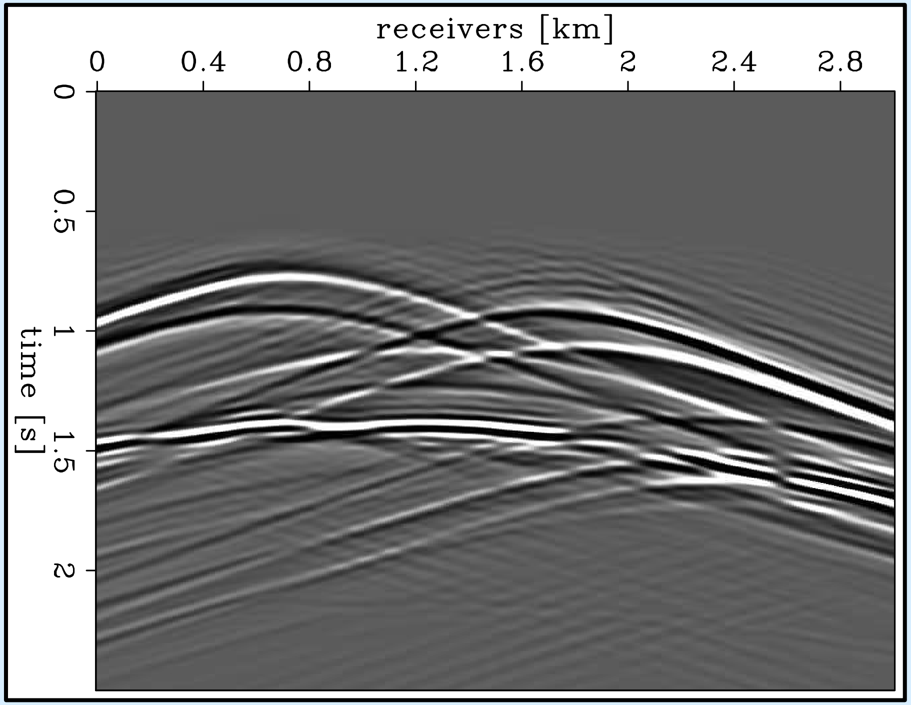
$$\Delta \mathbf{d}' = f'(Km_{true}) - f'(Km_0)$$

True subsurface data difference



$$\widetilde{\boldsymbol{B}}'(\boldsymbol{m_0})\boldsymbol{K}\Delta\widetilde{\boldsymbol{m}}^*$$

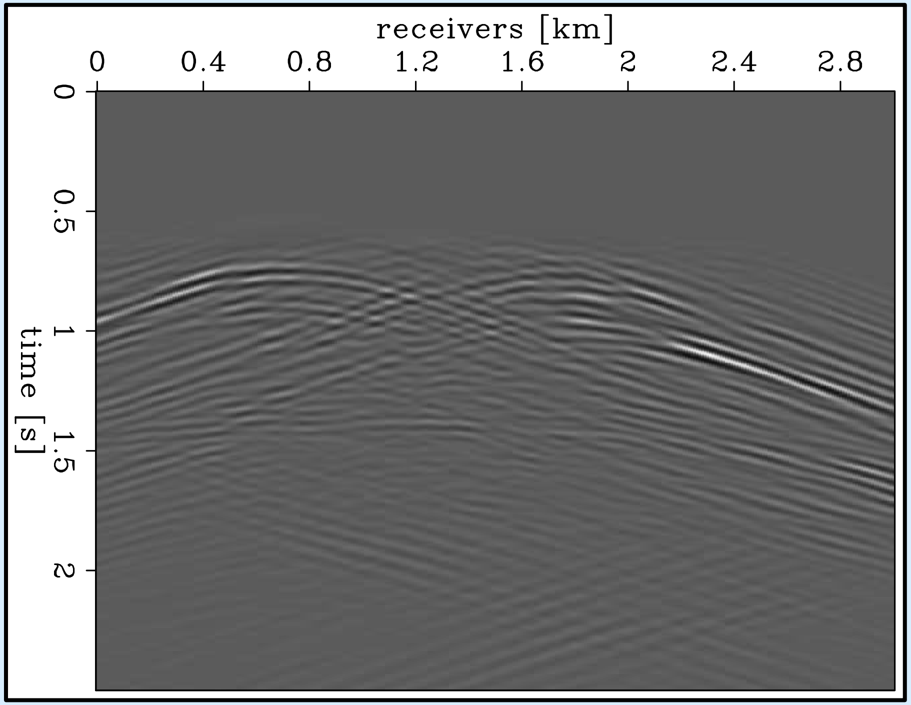
Reconstructed subsurface data difference



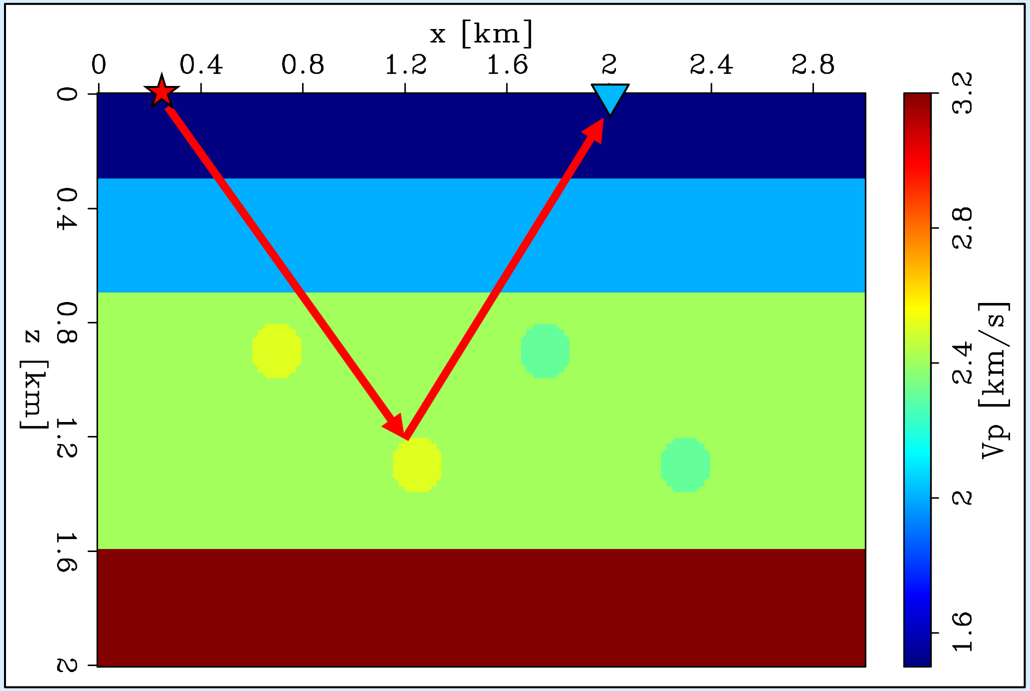
Redatuming acoustic example: results

$$\Delta \mathbf{d}' - \widetilde{B}'(m_0)K\Delta\widetilde{m}^*$$

Reconstruction error



Why does it work?



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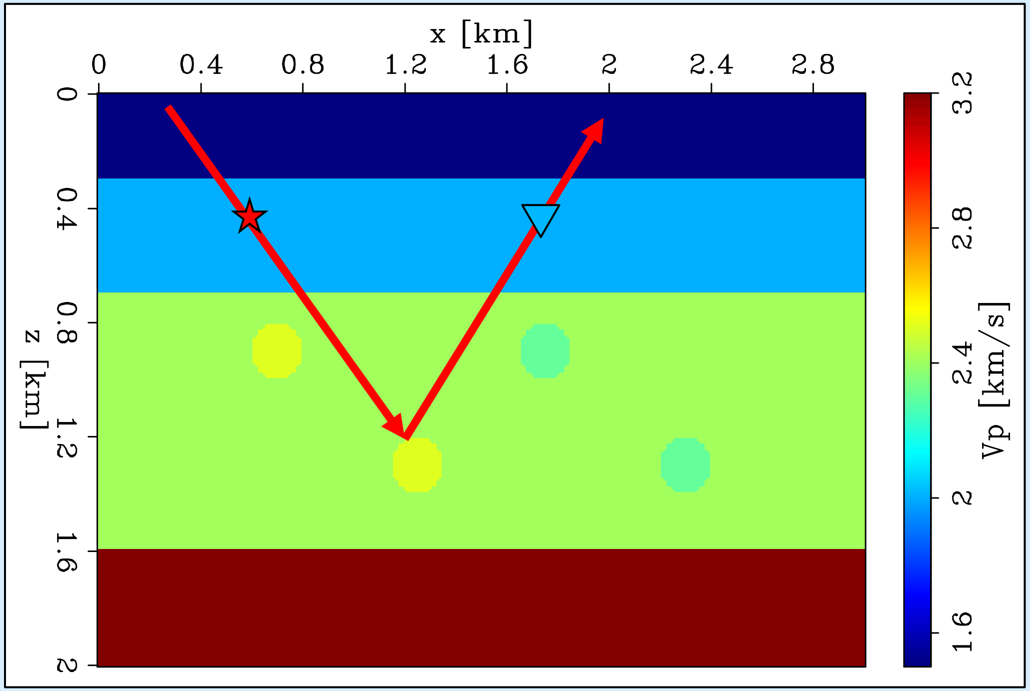


Why does it work?

$$\Delta\boldsymbol{d} = \boldsymbol{f}(\boldsymbol{m}_{\boldsymbol{true}}) - \boldsymbol{f}(\boldsymbol{m}_0) \xrightarrow{\underset{\Delta\widetilde{\boldsymbol{m}}}{\operatorname{argmin}} \left\| \widetilde{\boldsymbol{B}}(\boldsymbol{m}_0)\Delta\widetilde{\boldsymbol{m}} - \Delta\boldsymbol{d} \right\|_2^2} \color{red}{\Delta\widetilde{\boldsymbol{m}}^*}$$

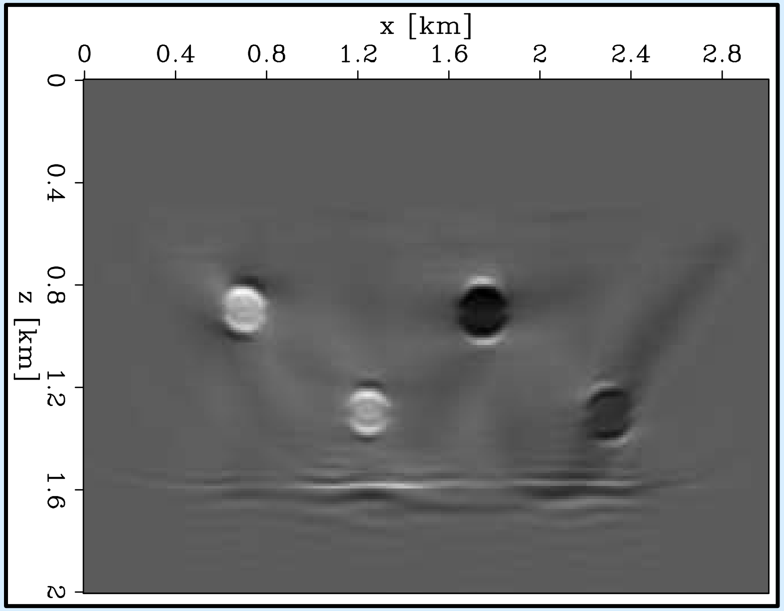
$$\Delta\boldsymbol{d}' = \boldsymbol{f}'(\boldsymbol{K}\boldsymbol{m}_{\boldsymbol{true}}) - \boldsymbol{f}'(\boldsymbol{K}\boldsymbol{m}_0) \xrightarrow{\underset{\Delta\widetilde{\boldsymbol{m}}}{\operatorname{argmin}} \left\| \widetilde{\boldsymbol{B}'}(\boldsymbol{K}\boldsymbol{m}_0)\Delta\widetilde{\boldsymbol{m}} - \Delta\boldsymbol{d}' \right\|_2^2} \Delta\widetilde{\boldsymbol{m}}'^* \approx \boldsymbol{K}\color{red}{\Delta\widetilde{\boldsymbol{m}}^*}$$

$$\Delta\tilde{\boldsymbol{m}}^* = \underset{\Delta\tilde{\boldsymbol{m}}}{\operatorname{argmin}} \left\| \tilde{\boldsymbol{B}}(\boldsymbol{m}_0)\Delta\tilde{\boldsymbol{m}} - \Delta\boldsymbol{d} \right\|_2^2$$

Surface acquisition

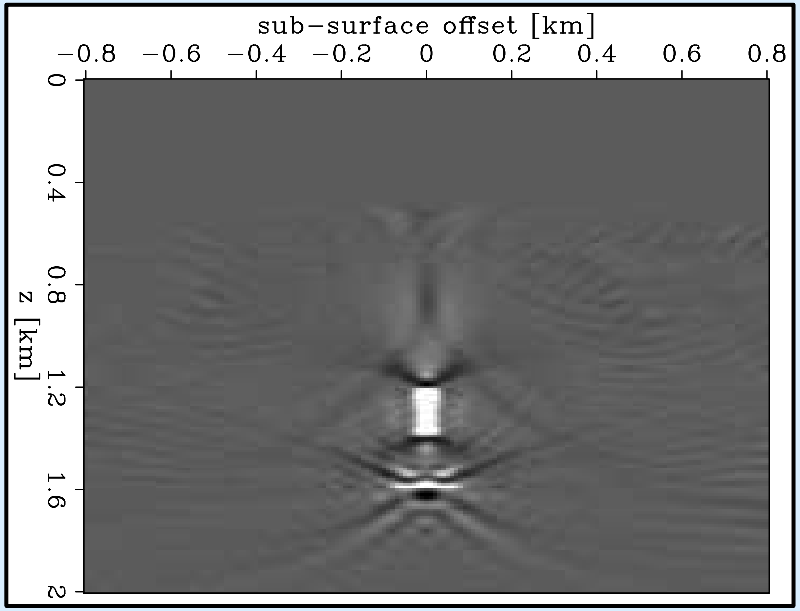
Zero-subsurface offset section

$\Delta\tilde{\boldsymbol{m}}^*(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{h_x} = \boldsymbol{0})$



CIG

$\Delta\tilde{\boldsymbol{m}}^*(\boldsymbol{x} = \boldsymbol{1.25km}, \boldsymbol{z}, \boldsymbol{h_x})$

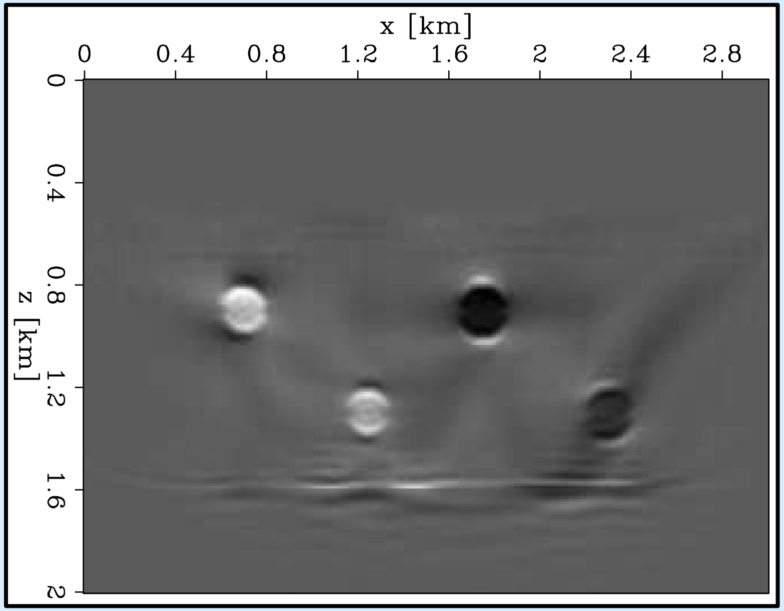


$$\Delta\tilde{\boldsymbol{m}}'^{*} = \underset{\Delta\tilde{\boldsymbol{m}}}{\operatorname{argmin}} \left\| \widetilde{\boldsymbol{B}'}(\boldsymbol{K}\boldsymbol{m}_0)\Delta\tilde{\boldsymbol{m}} - \Delta\boldsymbol{d}' \right\|_2^2$$

Sunk acquisition (0.4 Km)

Zero-subsurface offset section

$\Delta\tilde{\boldsymbol{m}}^{*}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{h_x} = \boldsymbol{0})$



CIG

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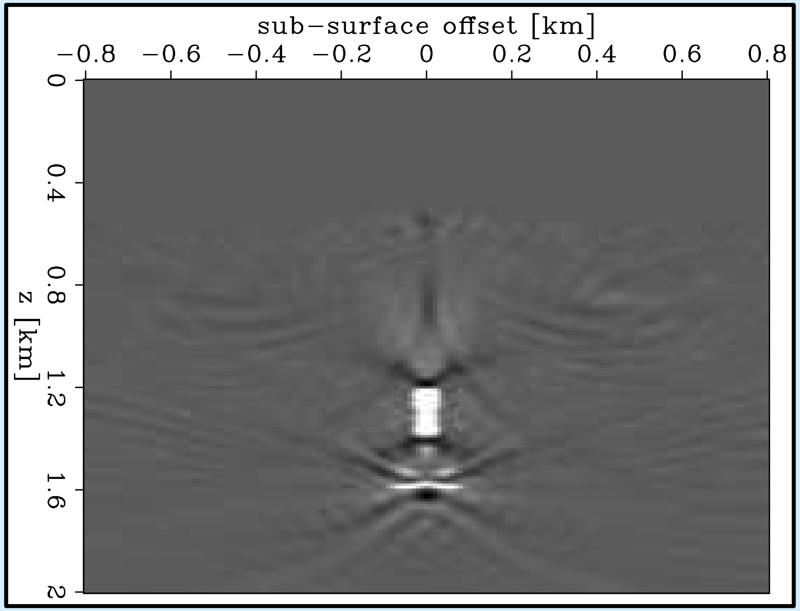
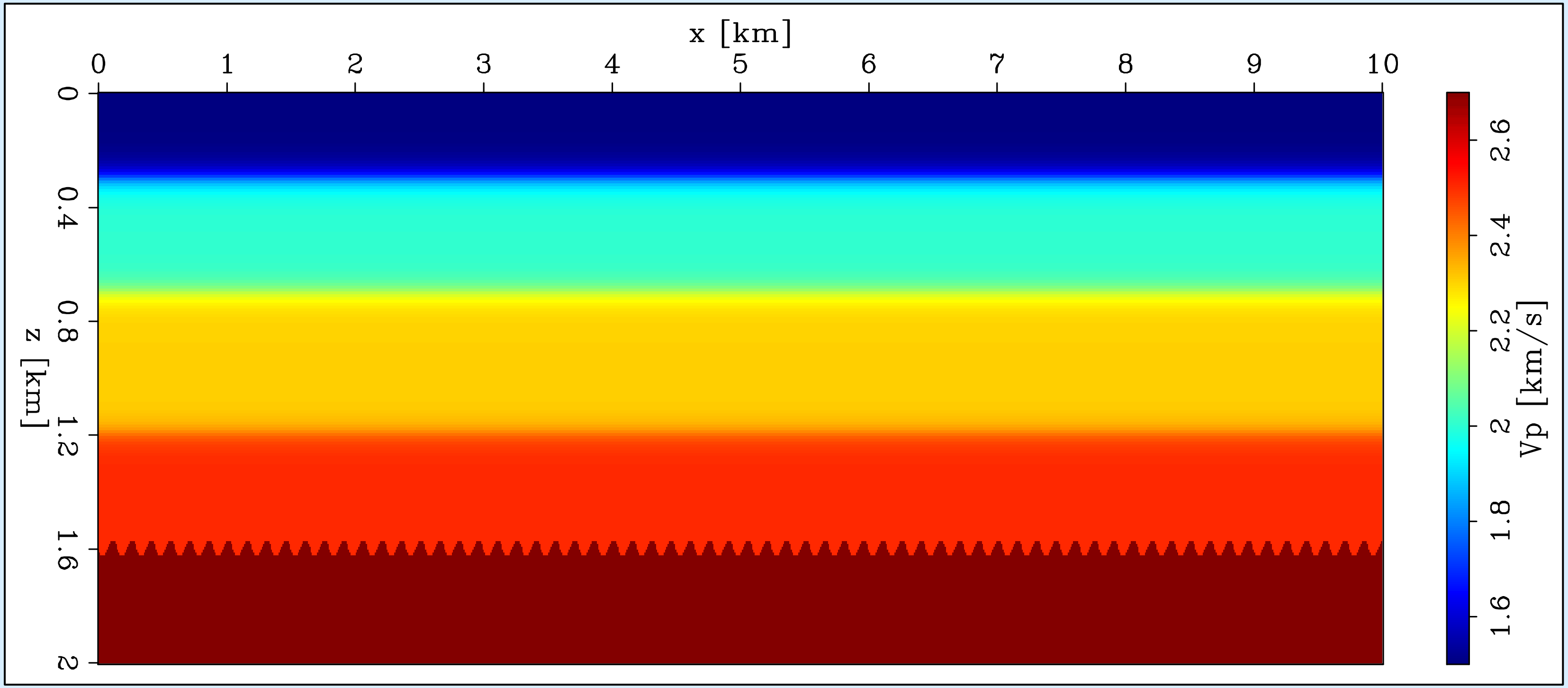


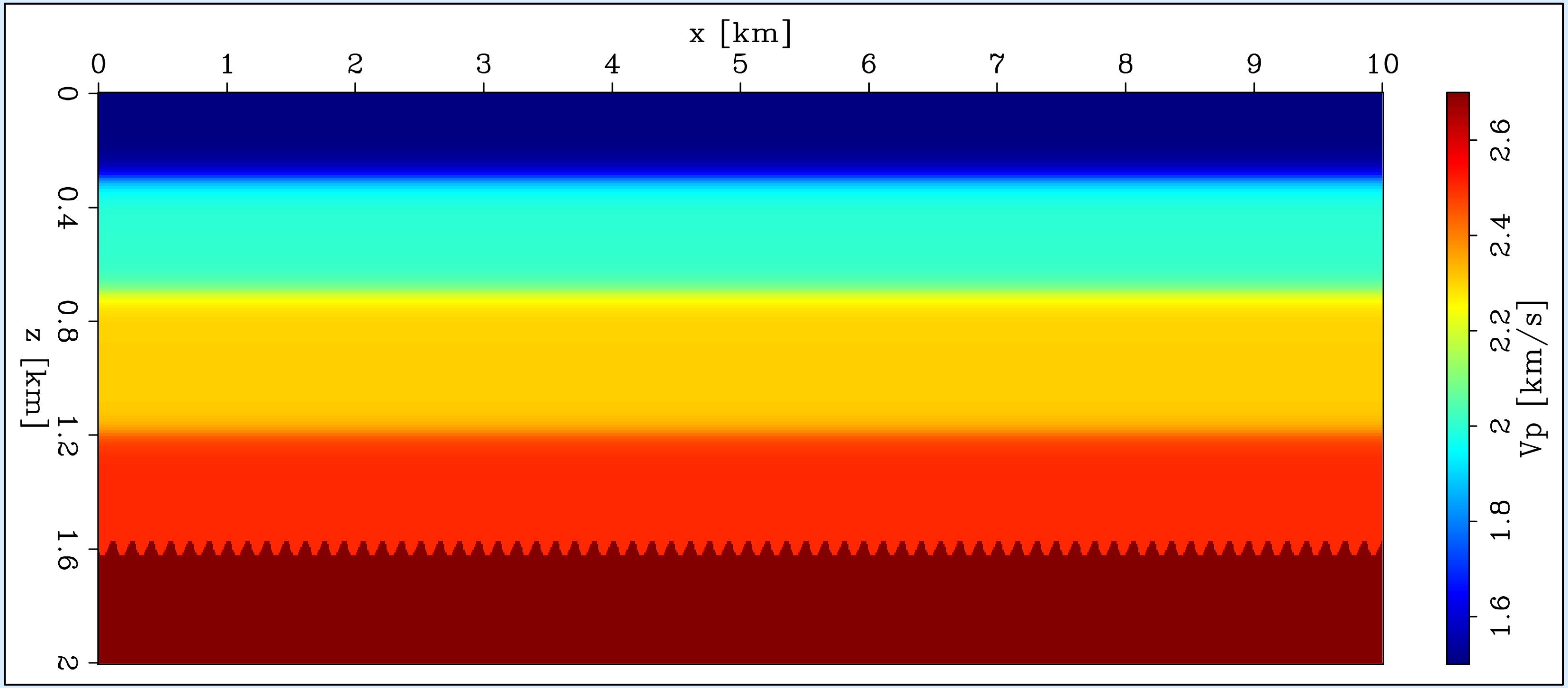
Target-oriented elastic FWI: layered model

Let's apply the same process to an *elastic isotropic* case

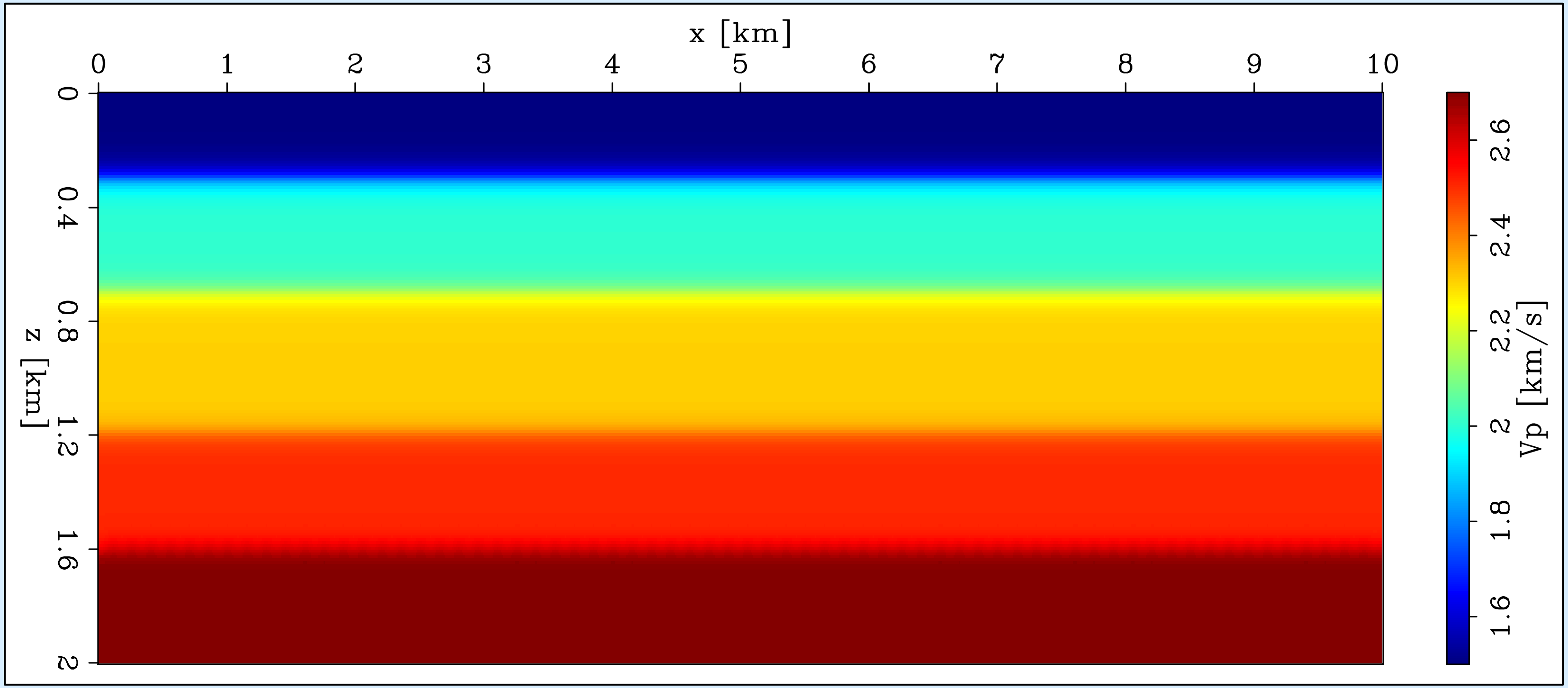


Target-oriented elastic FWI: layered model

100 shots and 1000 receivers at $z = 0$ km
Explosive source with energy between 2-30 Hz

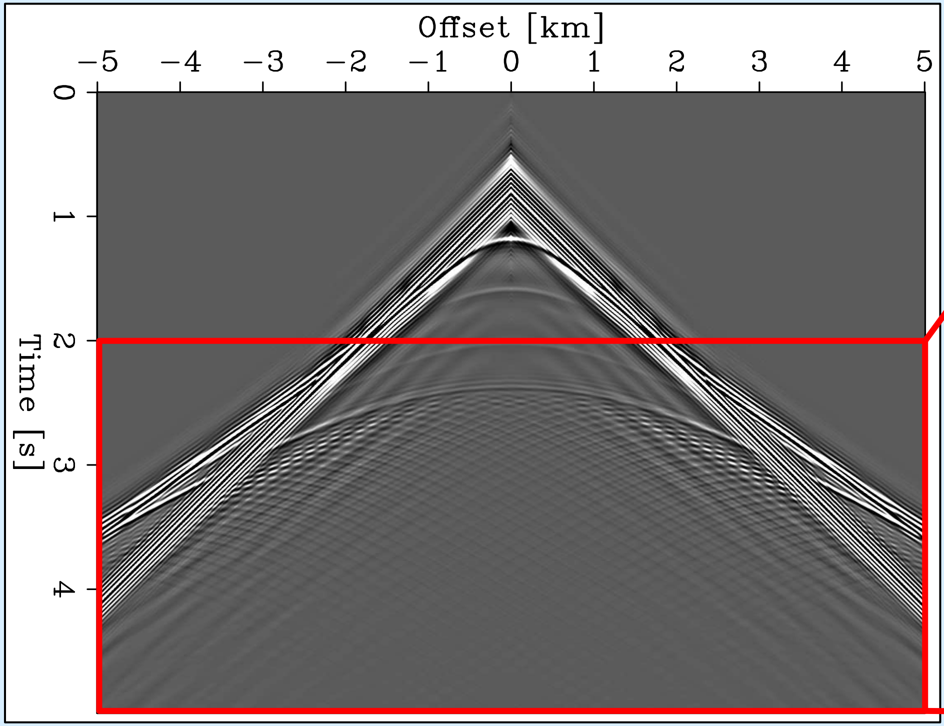


We start by smoothing the last target reflector

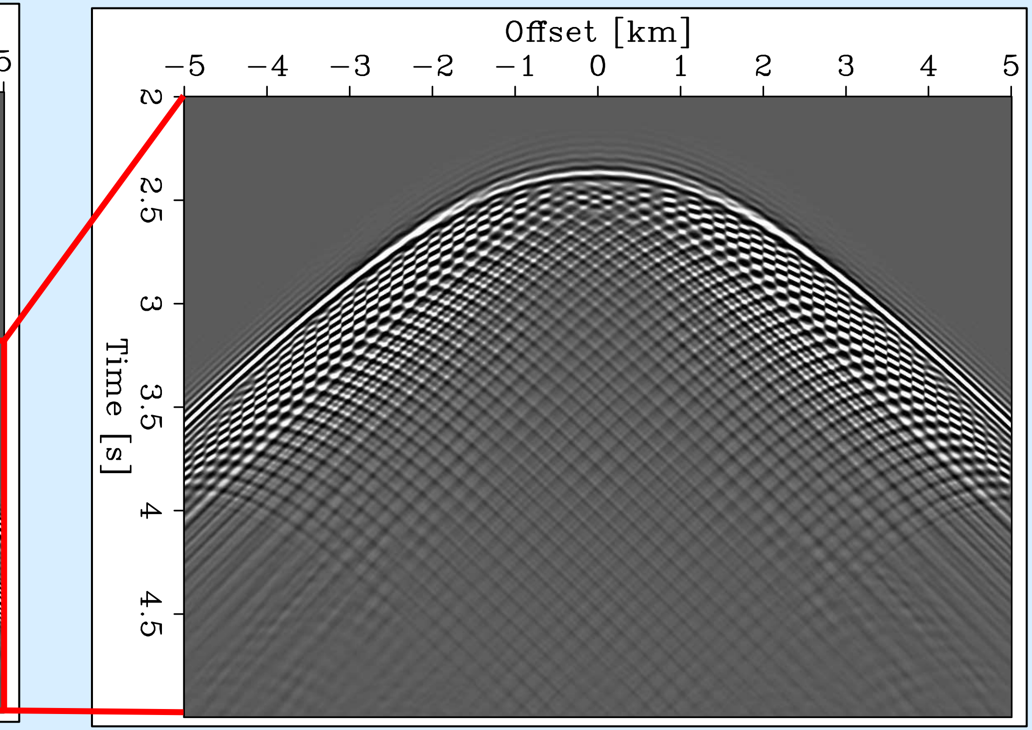


Target-oriented elastic FWI: data

Surface pressure data
$f(m_{true})$



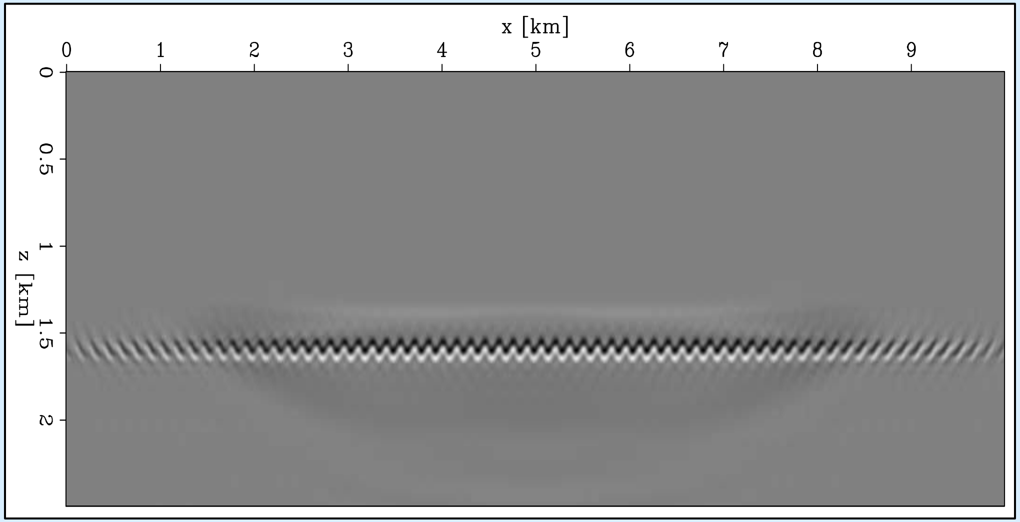
Surface data residuals
$\Delta d = f(m_{true})$ - $f(m_0)$



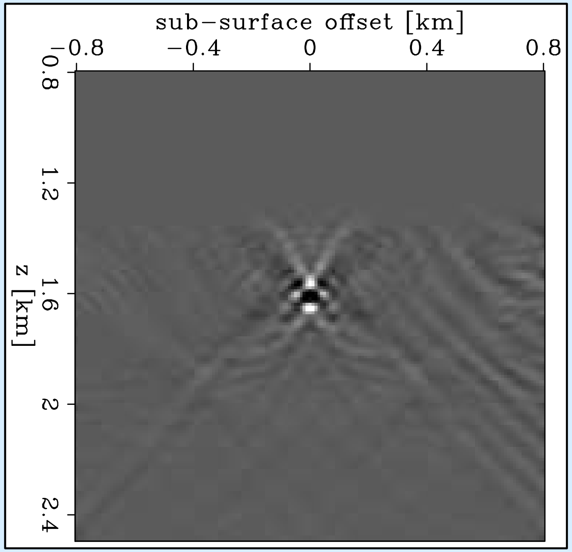
$$\Delta\tilde{\boldsymbol{m}}^* = \underset{\Delta\tilde{\boldsymbol{m}}}{\operatorname{argmin}} \left\| \tilde{\boldsymbol{B}}(\boldsymbol{m_0})\Delta\tilde{\boldsymbol{m}} - \Delta\boldsymbol{d} \right\|_{\boldsymbol{2}}^{\boldsymbol{2}}$$

Surface acquisition

Zero-subsurface offset section
$\Delta\tilde{\boldsymbol{m}}^*(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{h_x} = \boldsymbol{0})$



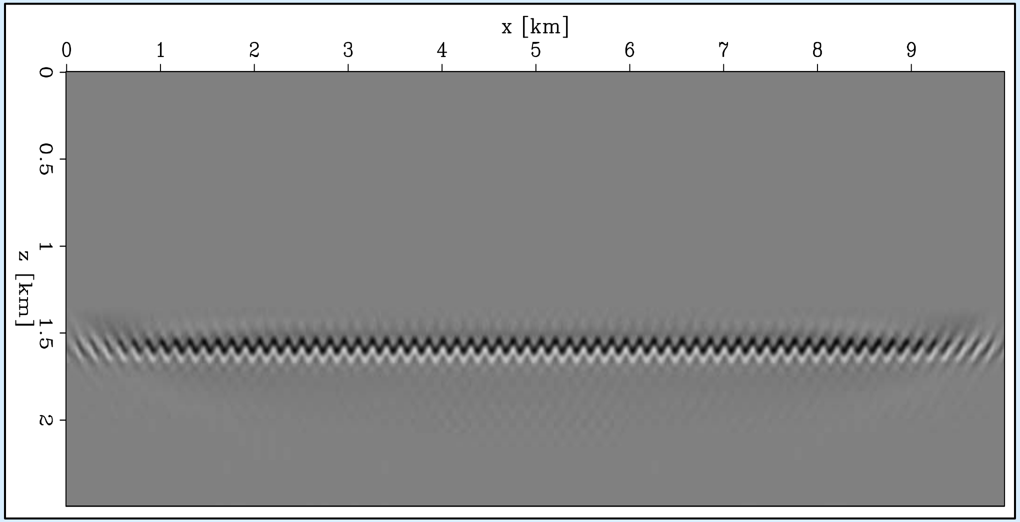
CIG
$\Delta\tilde{\boldsymbol{m}}^*(\boldsymbol{x} = \boldsymbol{5km}, \boldsymbol{z}, \boldsymbol{h_x})$



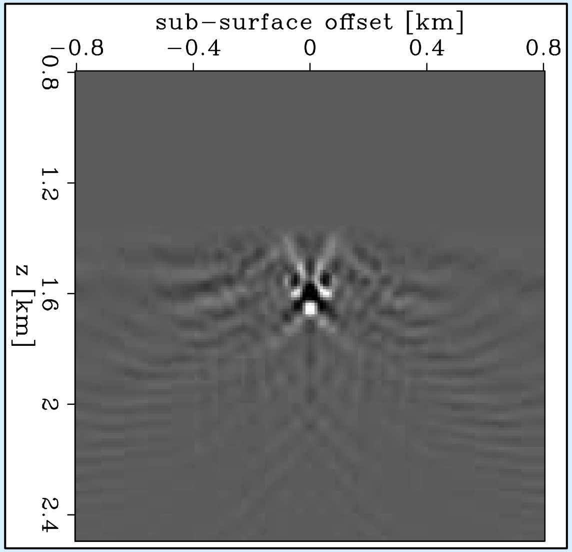
$$\Delta\tilde{\boldsymbol{m}}'^{*} = \underset{\Delta\tilde{\boldsymbol{m}}}{\operatorname{argmin}} \left\| \widetilde{\boldsymbol{B}'}(\boldsymbol{m}_0)\Delta\tilde{\boldsymbol{m}} - \Delta\boldsymbol{d}' \right\|_2^2$$

Sunk acquisition (1.2 km)

Zero-subsurface offset section
$\Delta\tilde{\boldsymbol{m}}^{*}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{h_x} = \boldsymbol{0})$



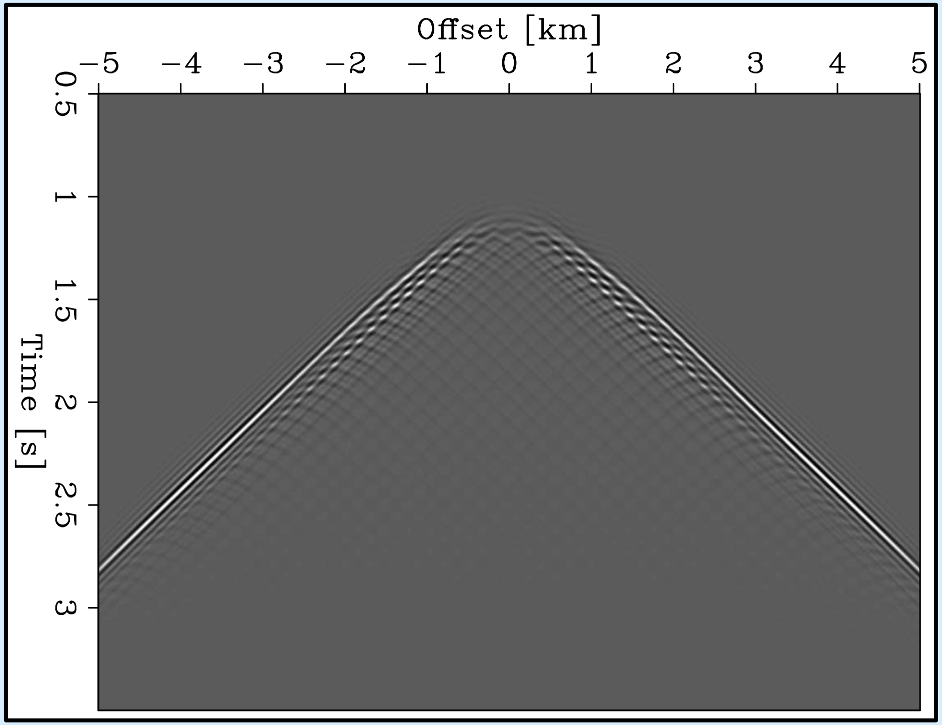
CIG
$\Delta\tilde{\boldsymbol{m}}^{*}(\boldsymbol{x} = \boldsymbol{5km}, \boldsymbol{z}, \boldsymbol{h_x})$



Target-oriented elastic FWI: redatuming

$$\Delta \mathbf{d}' = f'(Km_{true}) - f'(Km_0)$$

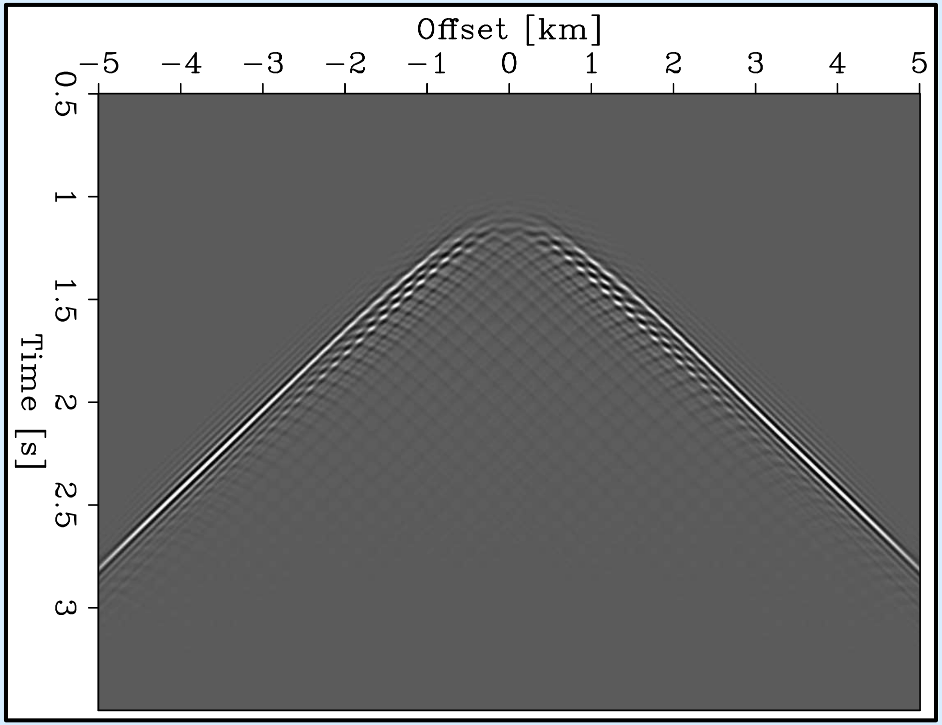
True subsurface data difference



Target-oriented elastic FWI: redatuming

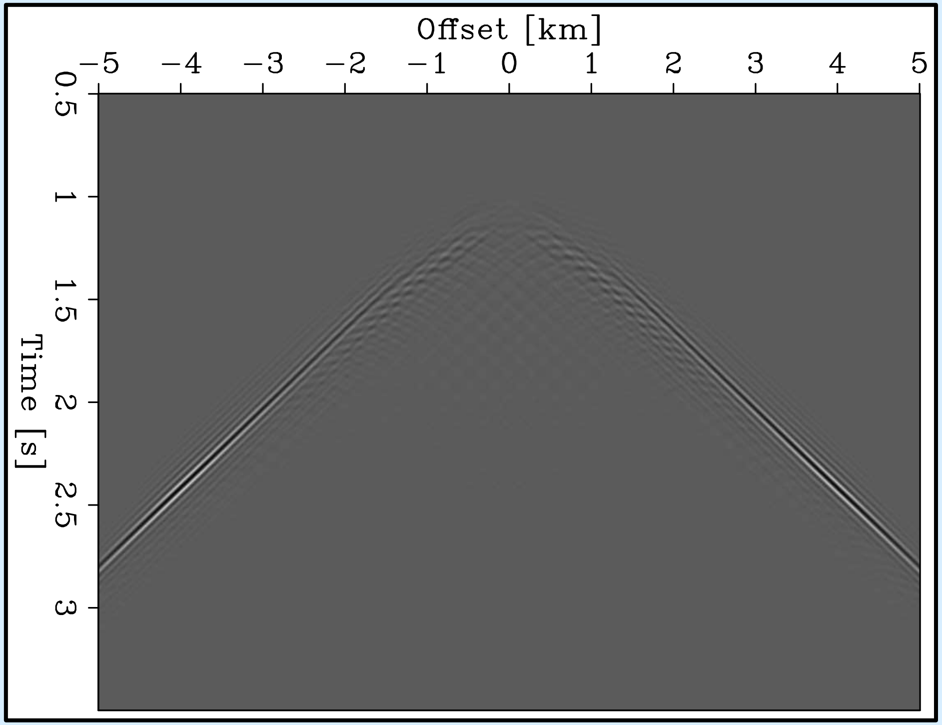
$$\widetilde{\boldsymbol{B}}'(\boldsymbol{m_0})\boldsymbol{K}\Delta\widetilde{\boldsymbol{m}}^*$$

Reconstructed subsurface data difference



$$\Delta\mathbf{d}' - \widetilde{\boldsymbol{B}}'(\boldsymbol{m}_0)\boldsymbol{K}\Delta\widetilde{\boldsymbol{m}}^*$$

Reconstruction error



Target-oriented elastic FWI: results

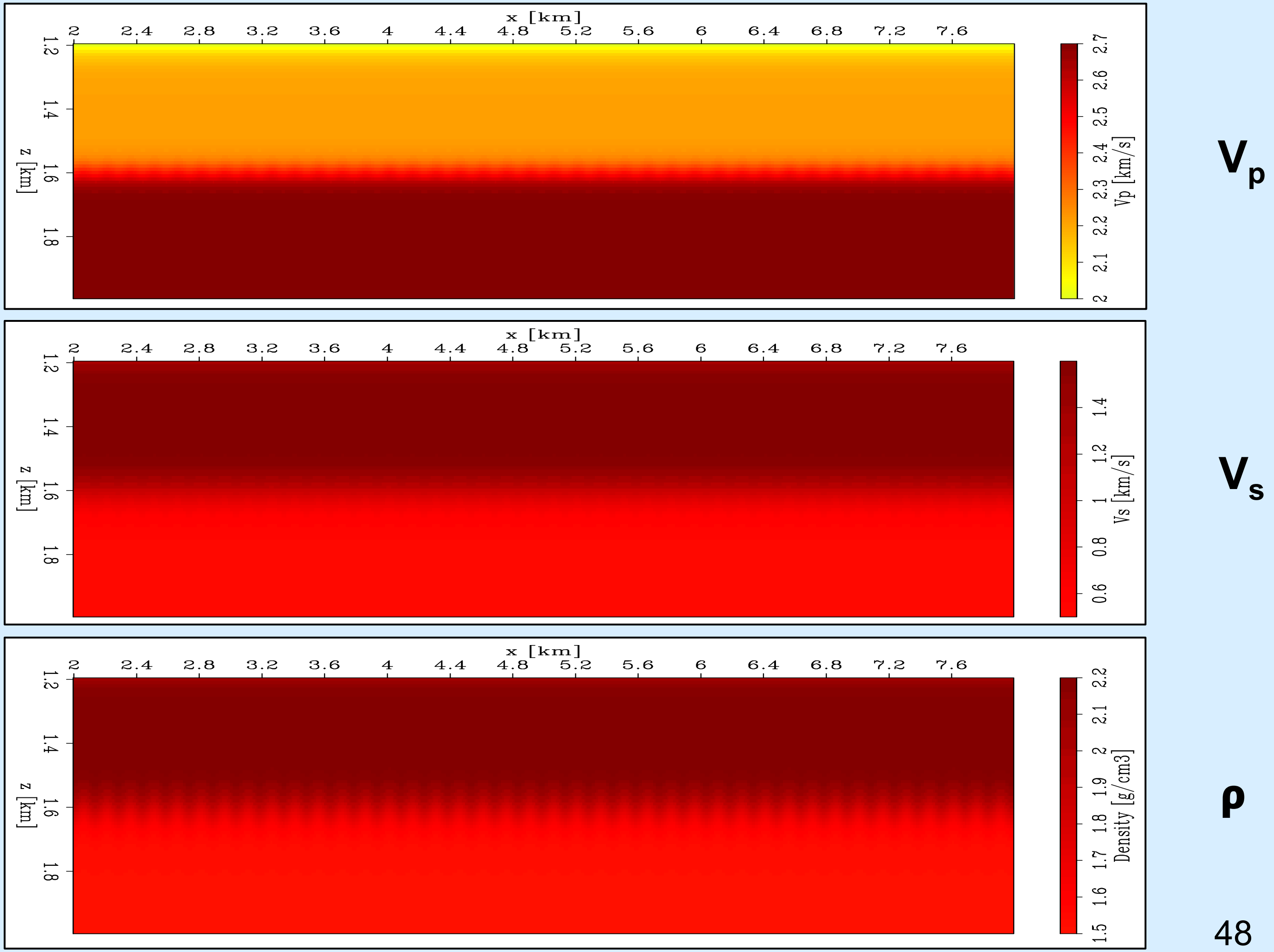
Elastic FWI:

- 100 iterations of multiscale approach 2-10/2-20/2-30 Hz

- 60 shots with 600 receivers

- Vp, Vs, Density parameterization

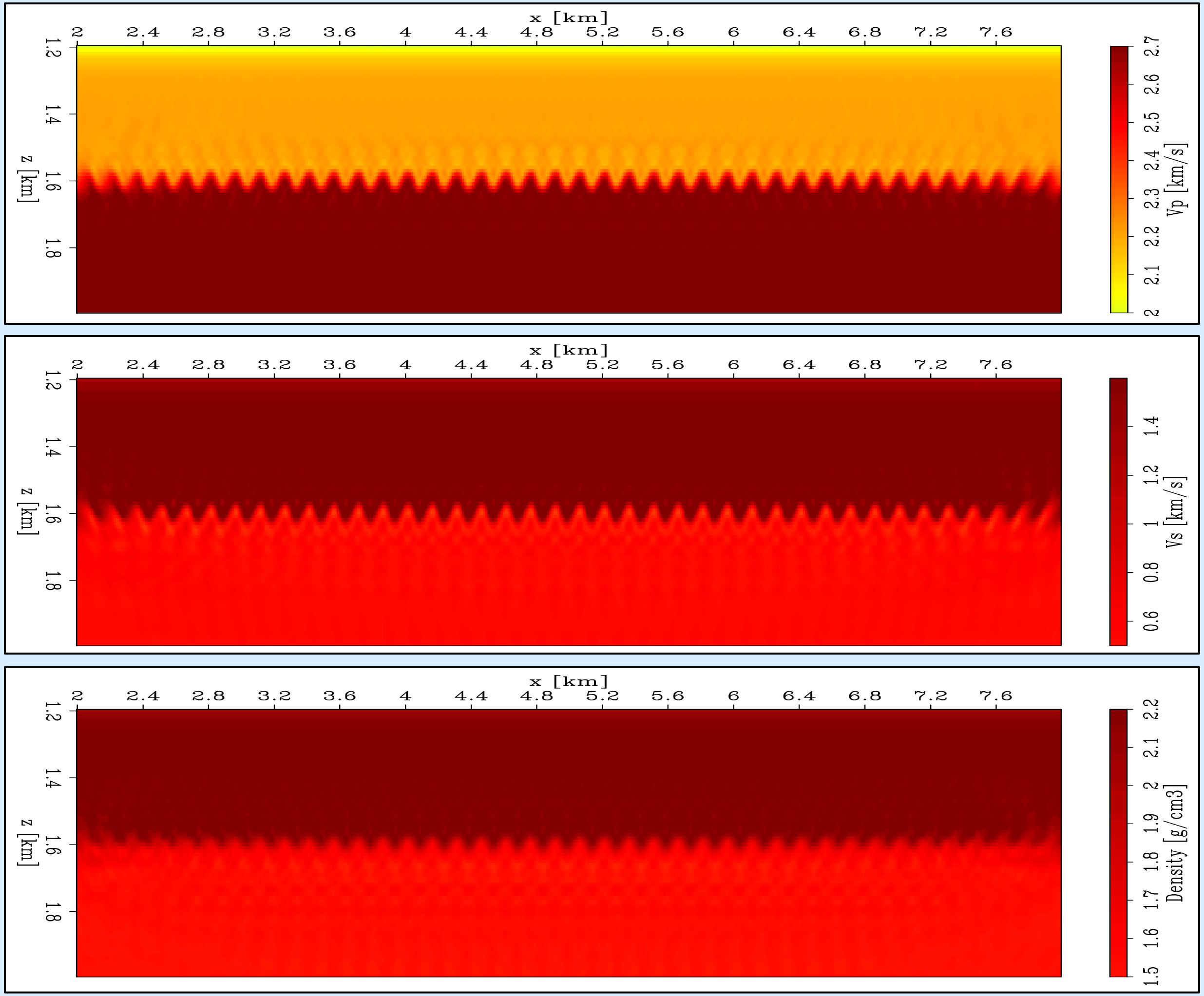
- Pressure only data



Target-oriented elastic FWI: results

Elastic FWI:

Inverted model using the true subsurface data



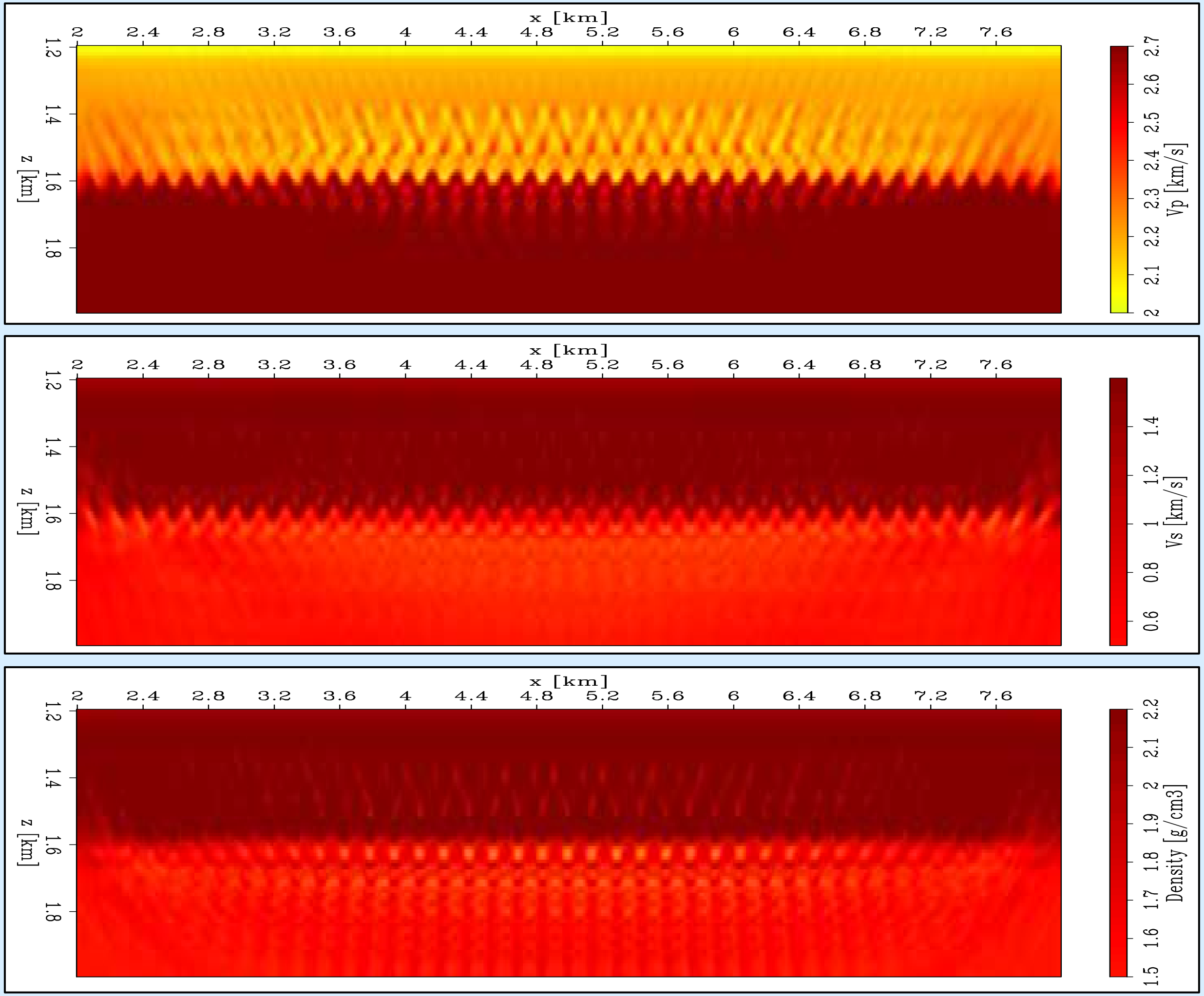
V_p

V_s

ρ

Elastic FWI:

Inverted model using the reconstructed subsurface data

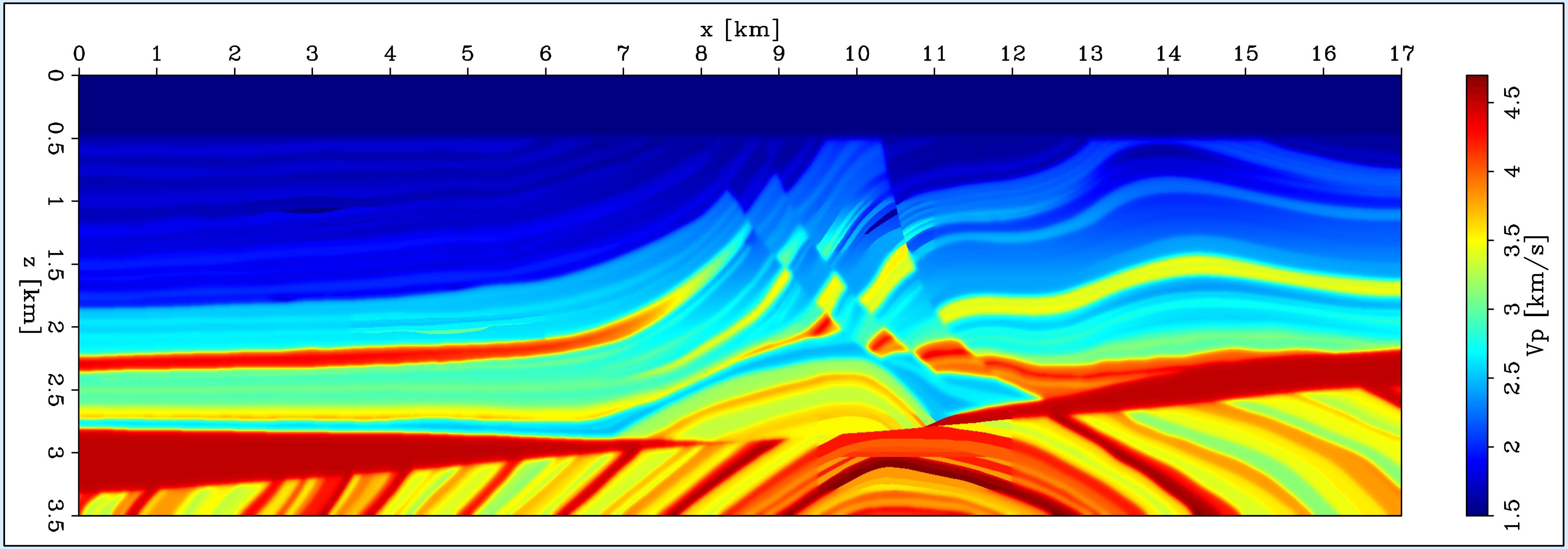


V_p

V_s

ρ

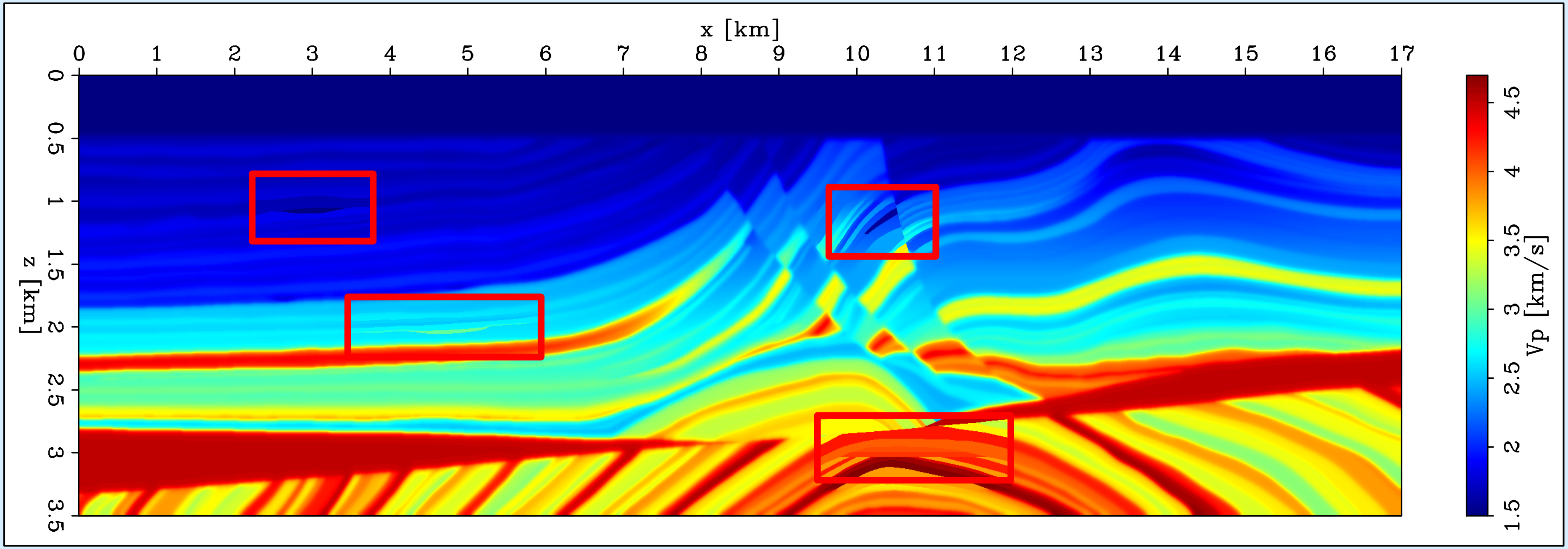
Let's try the same process on the Marmousi2 model (*elastic isotropic*)



Martin et al. 2006

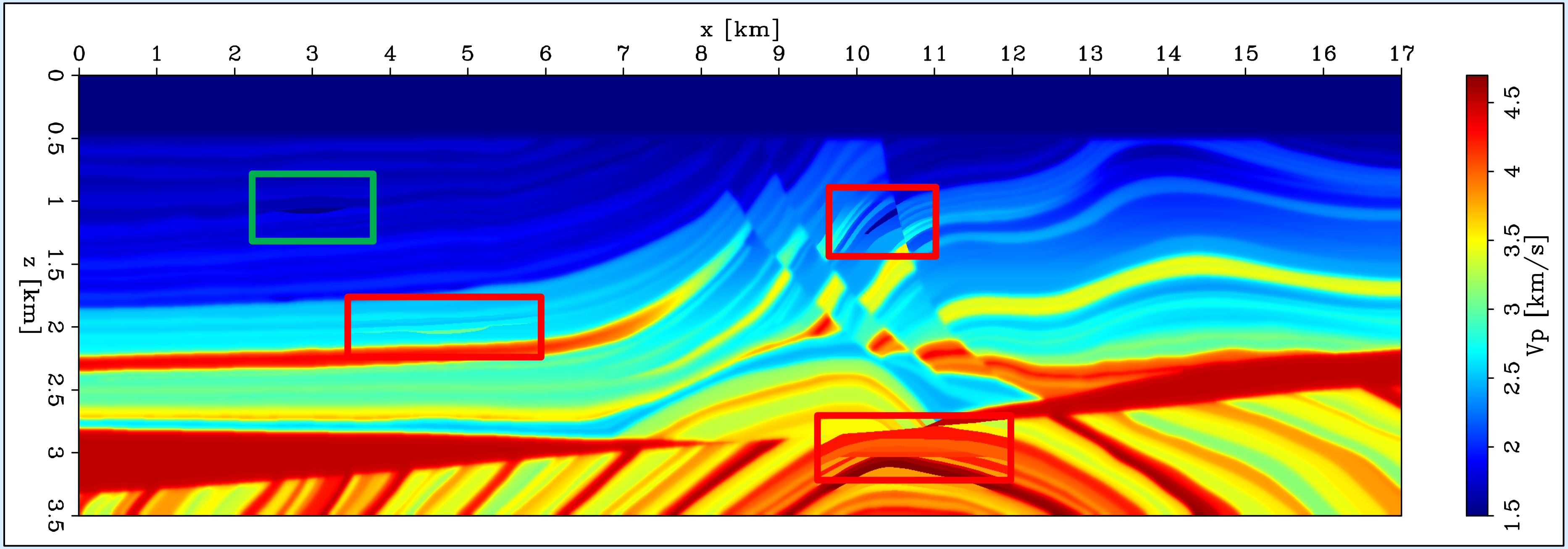
Target-oriented elastic FWI: Marmousi2

We are going to look at one of these four target areas



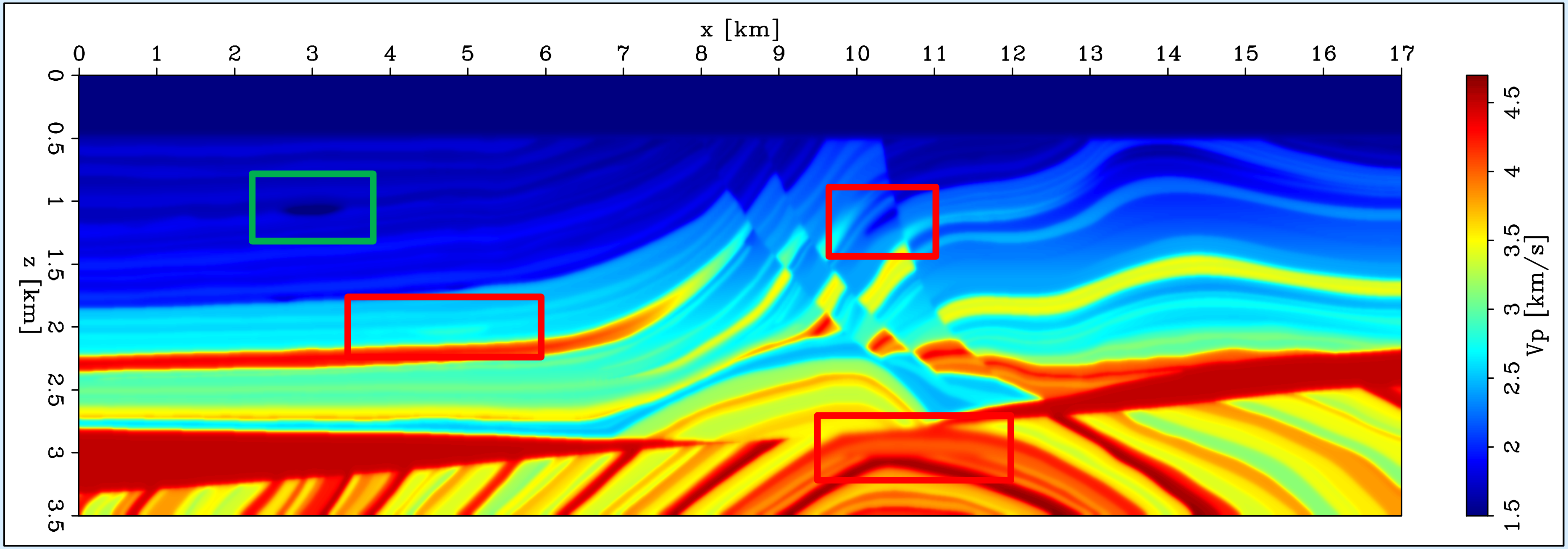
We are going to look at one of these four target areas

True Vp model



We are going to look at one of these four target areas

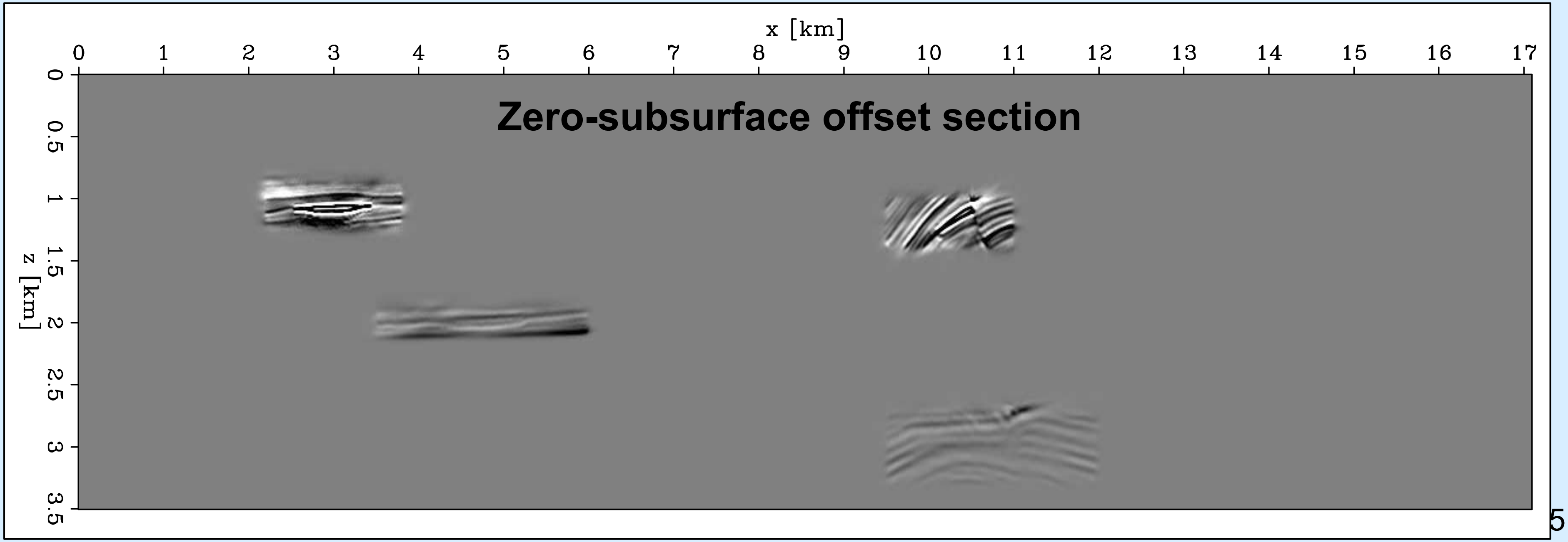
Intial Vp model



Target-oriented elastic FWI: redatuming

- 170 shots with 1701 receivers at the surface
- Pressure data only between 2-30 Hz content

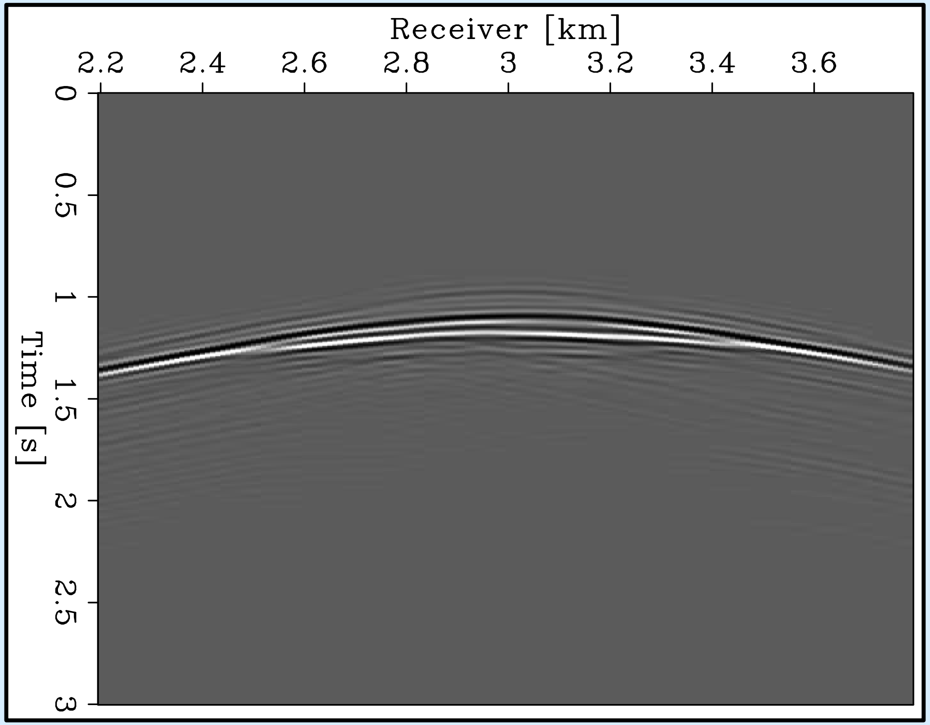
$$\Delta\widetilde{\boldsymbol{m}}^{*} = \underset{\Delta\widetilde{\boldsymbol{m}}}{\operatorname{argmin}} \left\| \widetilde{\boldsymbol{B}}(\boldsymbol{m_0})\Delta\widetilde{\boldsymbol{m}} - \Delta\boldsymbol{d} \right\|_2^2$$



Target-oriented elastic FWI: redatuming

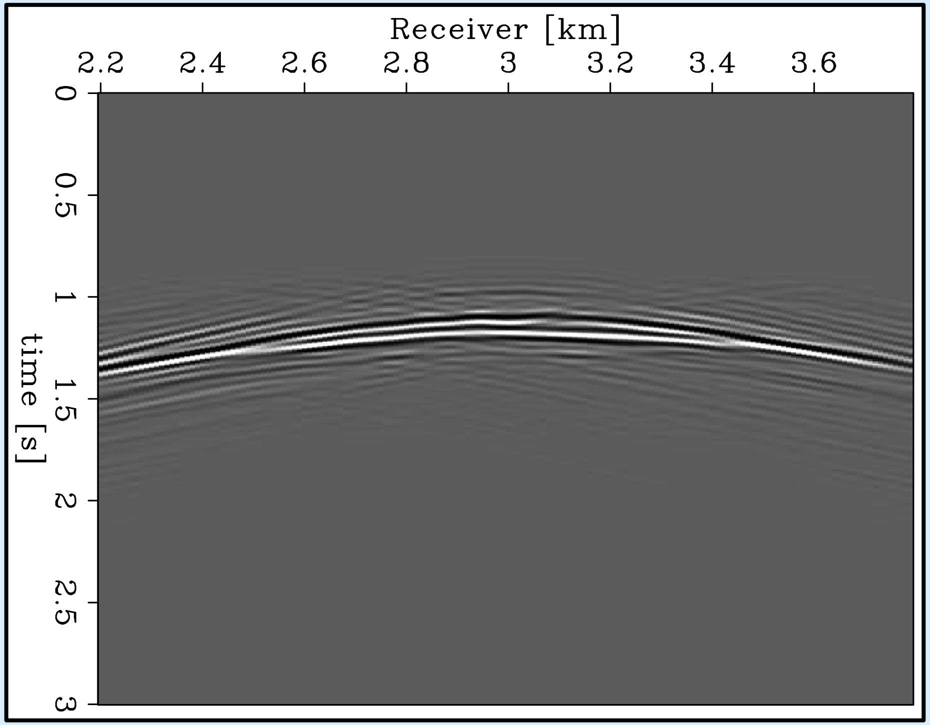
$$\Delta \mathbf{d}' = f'(Km_{true}) - f'(Km_0)$$

True subsurface data difference



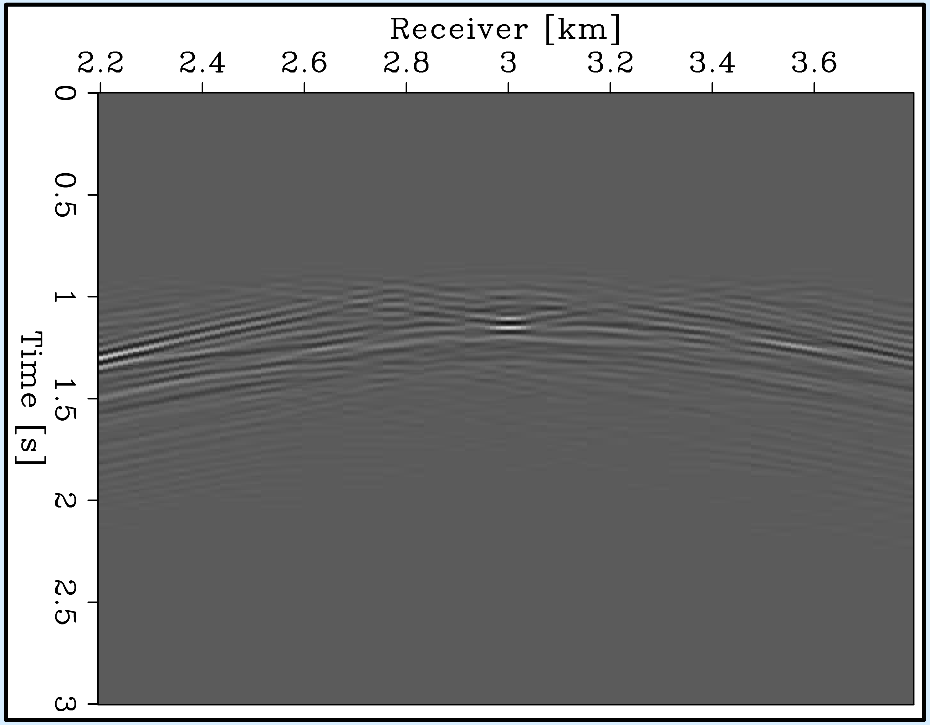
$$\widetilde{\boldsymbol{B}}'(\boldsymbol{m_0})\boldsymbol{K}\Delta\widetilde{\boldsymbol{m}}^*$$

Reconstructed subsurface data difference



$$\mathbf{\Delta d'} - \widetilde{\boldsymbol{B}}'(\boldsymbol{m_0})\boldsymbol{K}\Delta\widetilde{\boldsymbol{m}}^*$$

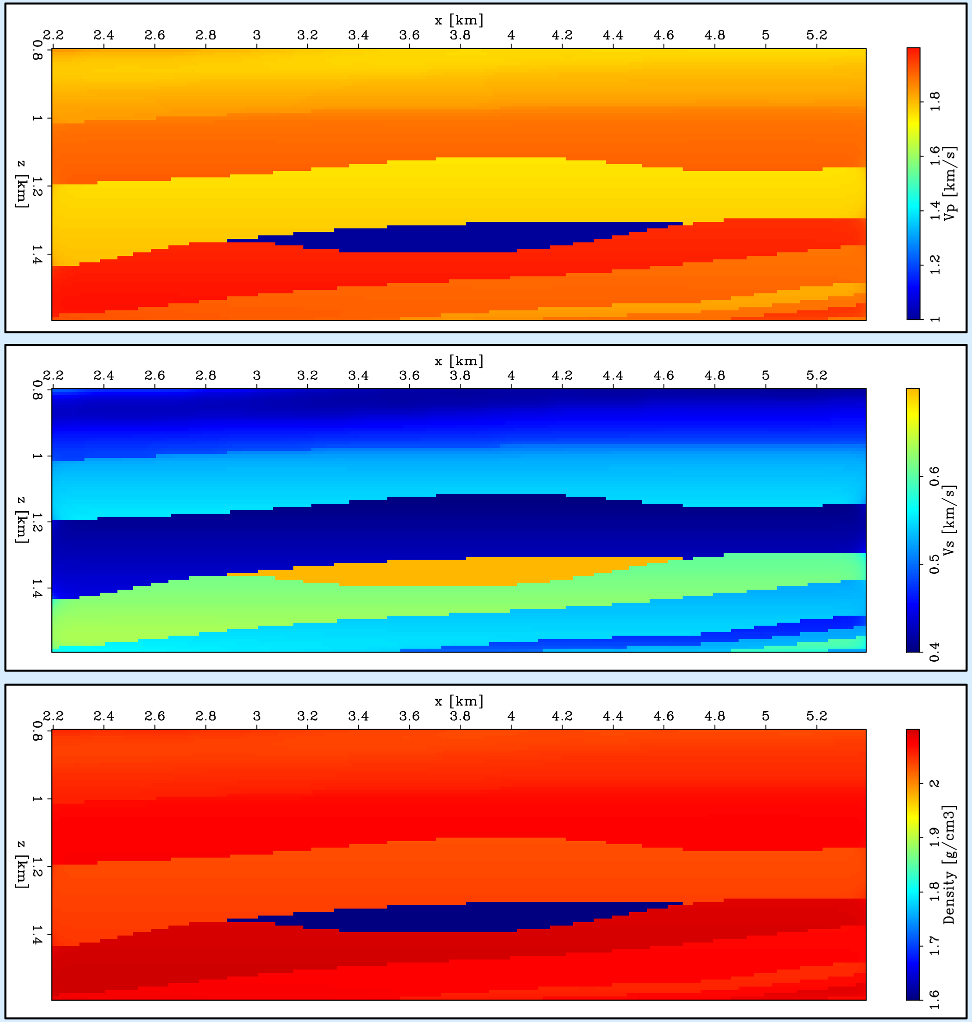
Reconstruction error



Target-oriented elastic FWI: results

Elastic FWI:

- **100 iterations of NLCG**
- **16 shots with 160 receivers**
- **Vp, Vs, Density parameterization**
- **Pressure only data**



V_p

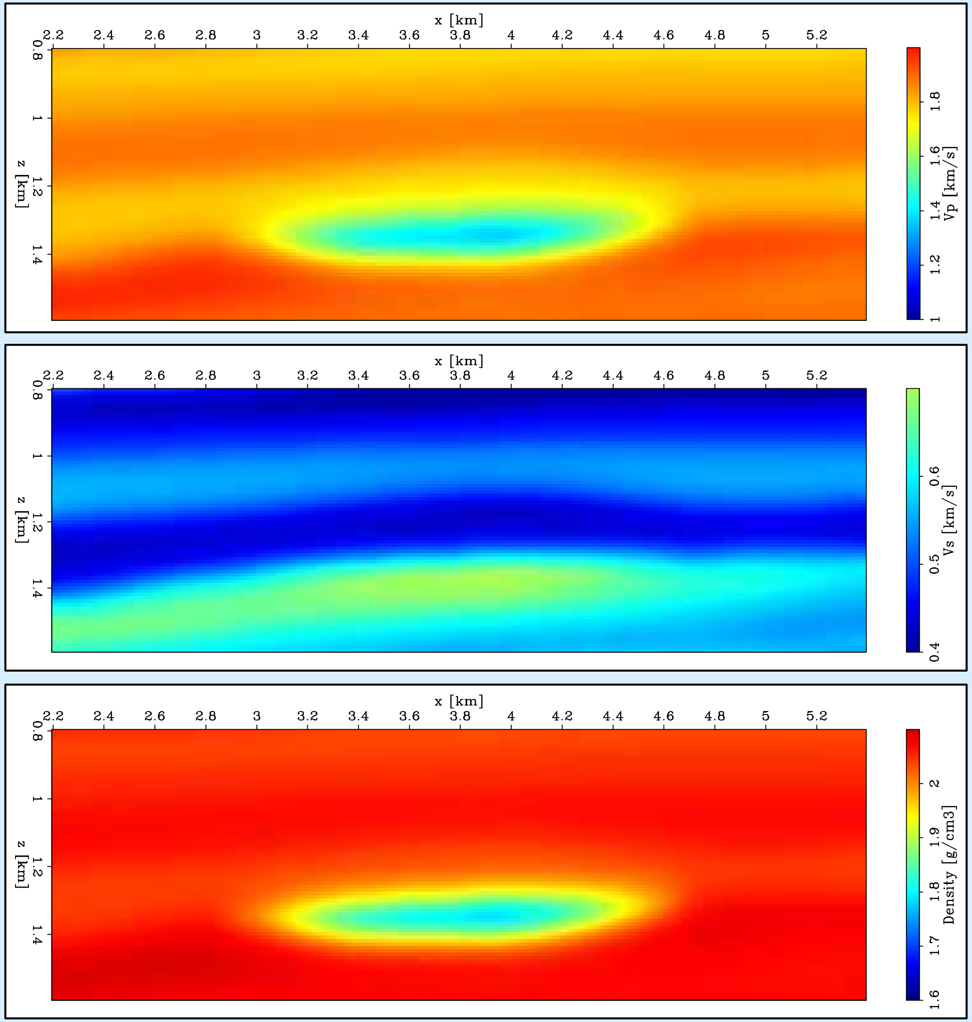
V_s

ρ

Target-oriented elastic FWI: results

Elastic FWI:

Starting model



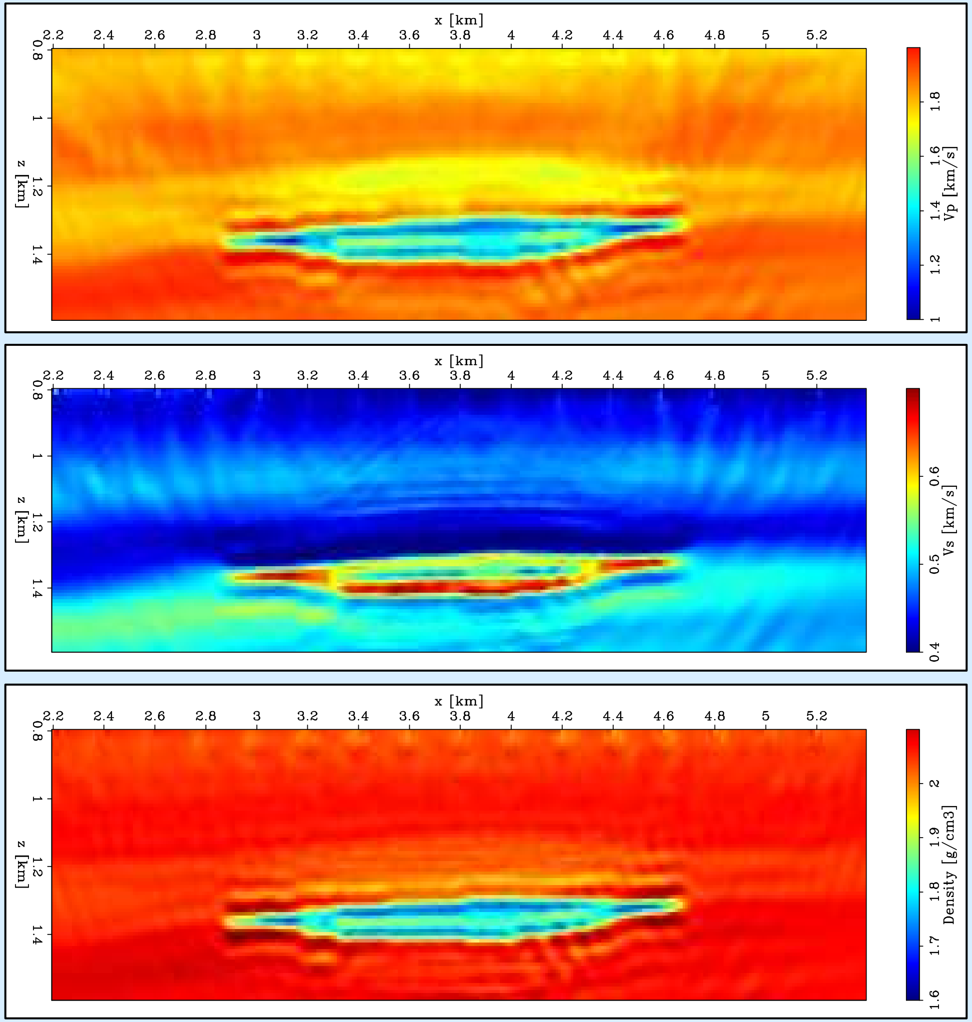
V_p

V_s

ρ

Elastic FWI:

Inverted model using the true subsurface data



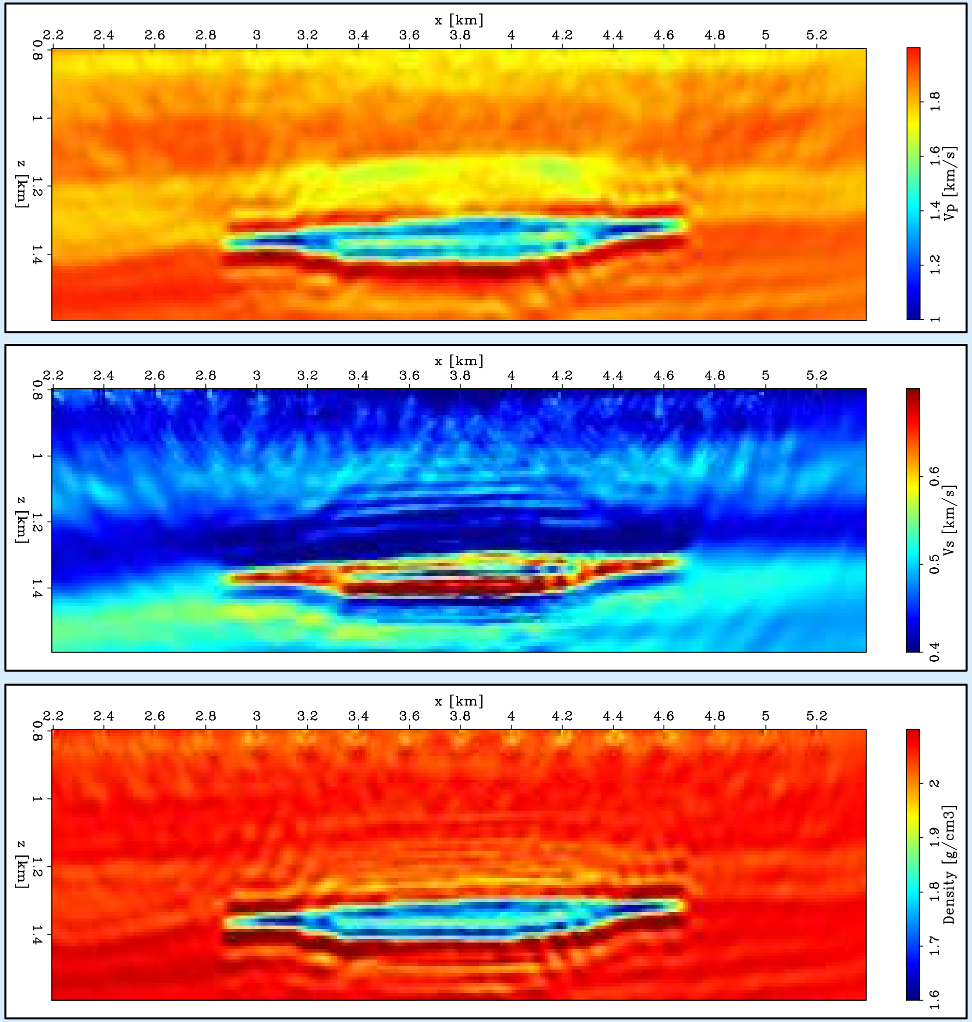
V_p

V_s

ρ

Elastic FWI:

Inverted model using the reconstructed subsurface data

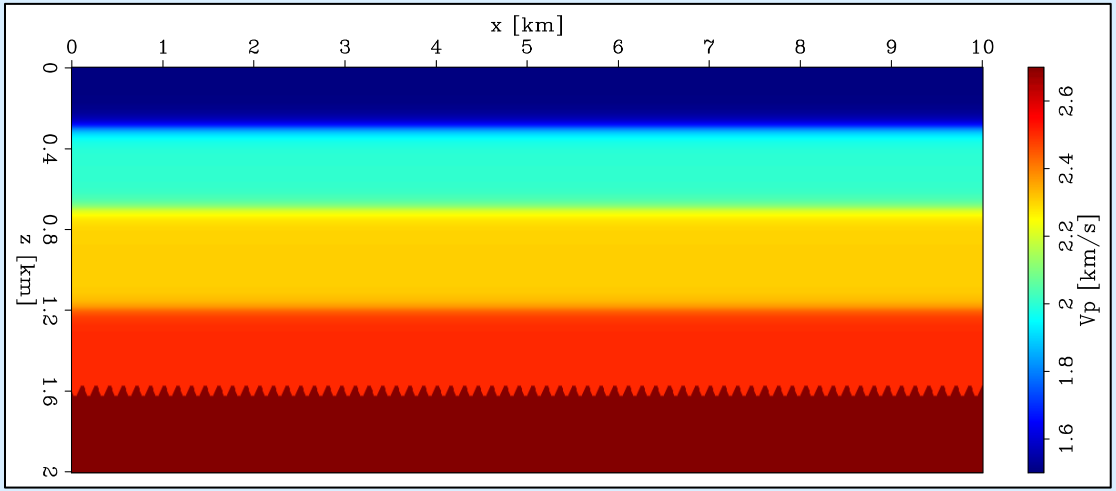


V_p

V_s

ρ

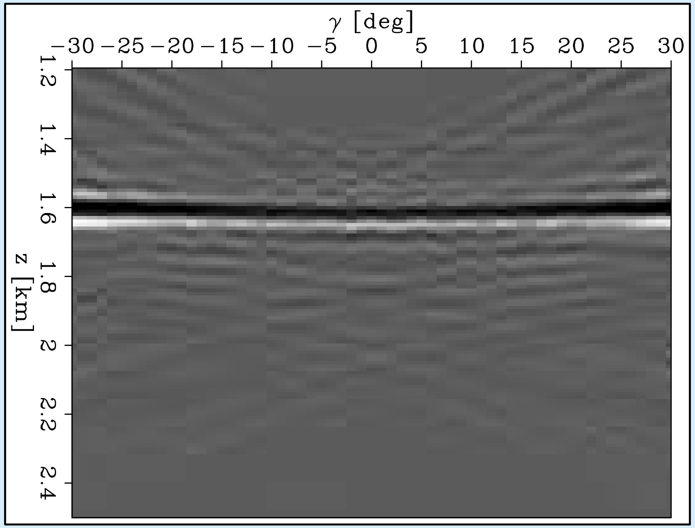
What is the cause of the reconstruction artifacts?



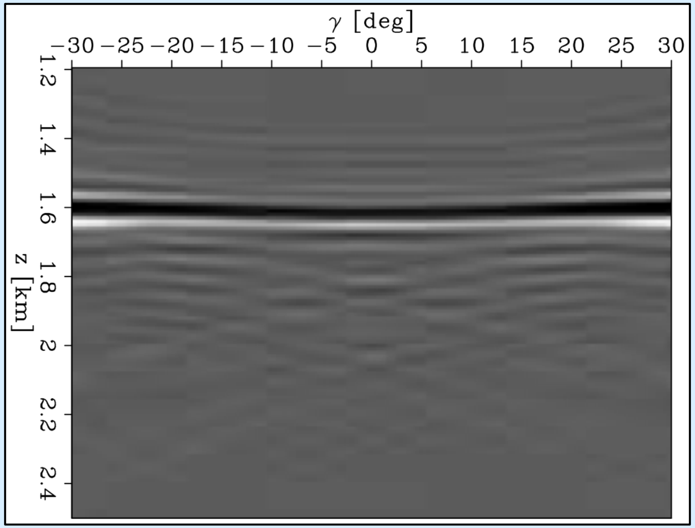
What is the cause of the reconstruction artifacts?

Probably the illumination

Angle-domain CIG
100m source sampling



Angle-domain CIG
20m source sampling

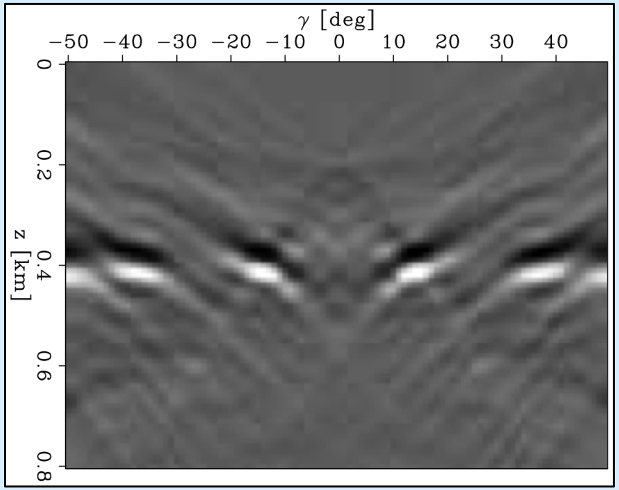


Redatuming: illumination and regularization

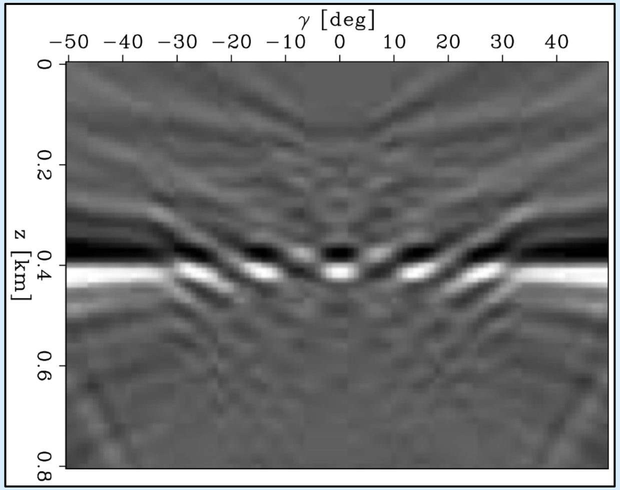
Image-space regularization is a viable solution (Prucha, 2005)

For a simple flat elastic reflector

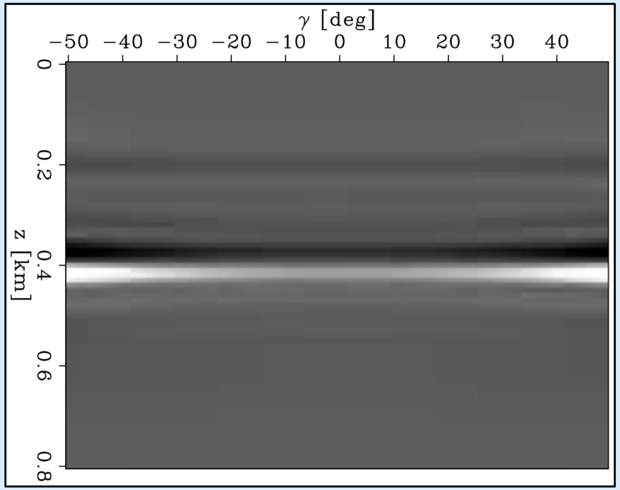
**Angle-domain CIG
200m source sampling**



**Angle-domain CIG
100m source sampling**



**Angle-domain CIG
Regularized 100m sampling**



Conclusions

- Diminishing the computational cost of elastic FWI is fundamental to make this process feasible for field exploration datasets

- Redatuming process performed by extended least-squares migration is able to reconstruct the data coming from a target area

- The reconstructed data can be used to performed a target-oriented FWI

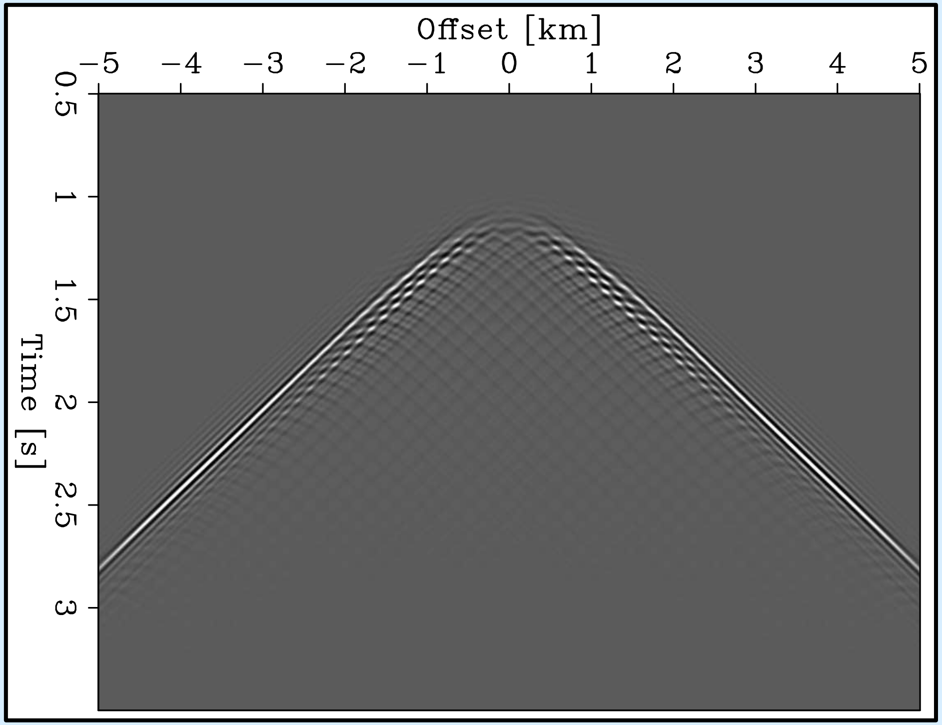
- Future work will involve: removal of reconstruction artifacts (regularization), analytical proof of image equivalence, 4D studies

Thank you for your attention
Questions?

Target-oriented elastic FWI: redatuming

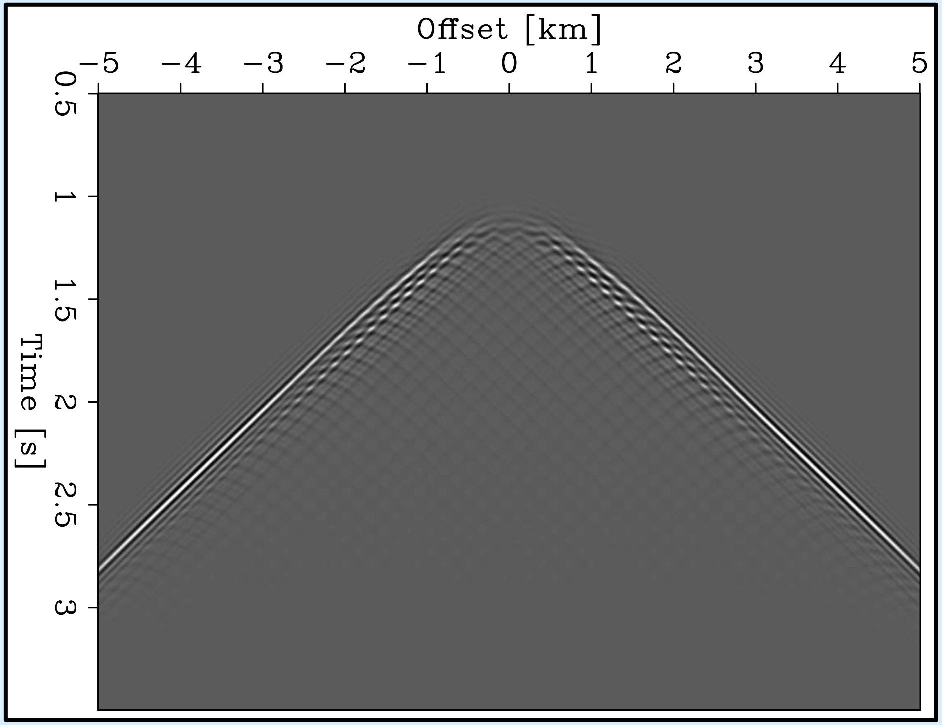
$$\widetilde{\boldsymbol{B}}'(\boldsymbol{m_0})\boldsymbol{K}\Delta\widetilde{\boldsymbol{m}}^*$$

Reconstructed subsurface data difference (extended)



$$\Delta\mathbf{d}' = f'(Km_{true}) - f'(Km_0)$$

True subsurface data difference



Target-oriented elastic FWI: redatuming

$$B'(m_0)K\Delta m^*$$

Reconstructed subsurface data difference (non-extended)