Shape Optimization Using Randomized Radial Basis Functions and Interpreter Guidance

Taylor Dahlke, SEP-172

5/2018

Overview

Motivation

Level set basics

Radial basis functions

Interpreter guidance

Demonstration on Sigsbee model

Conclusions

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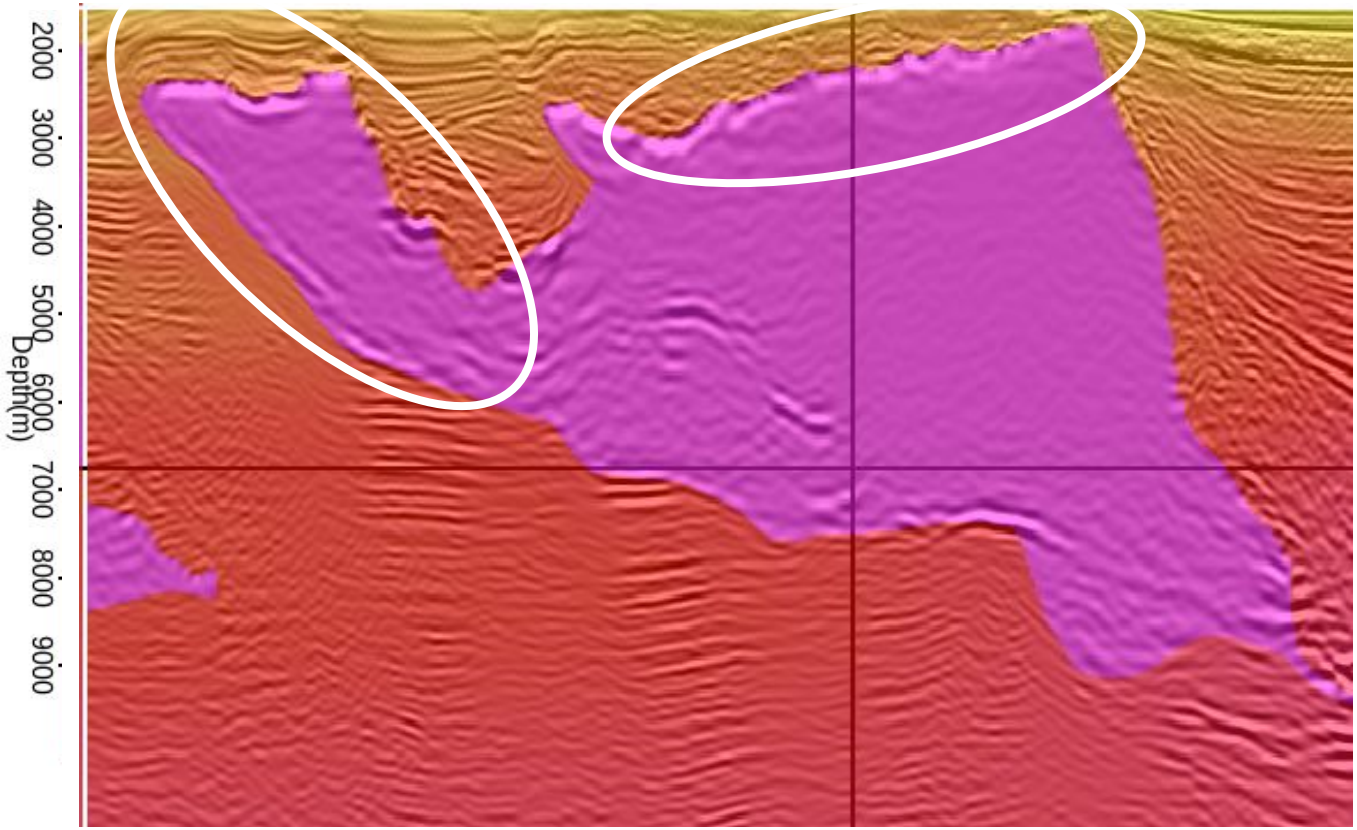
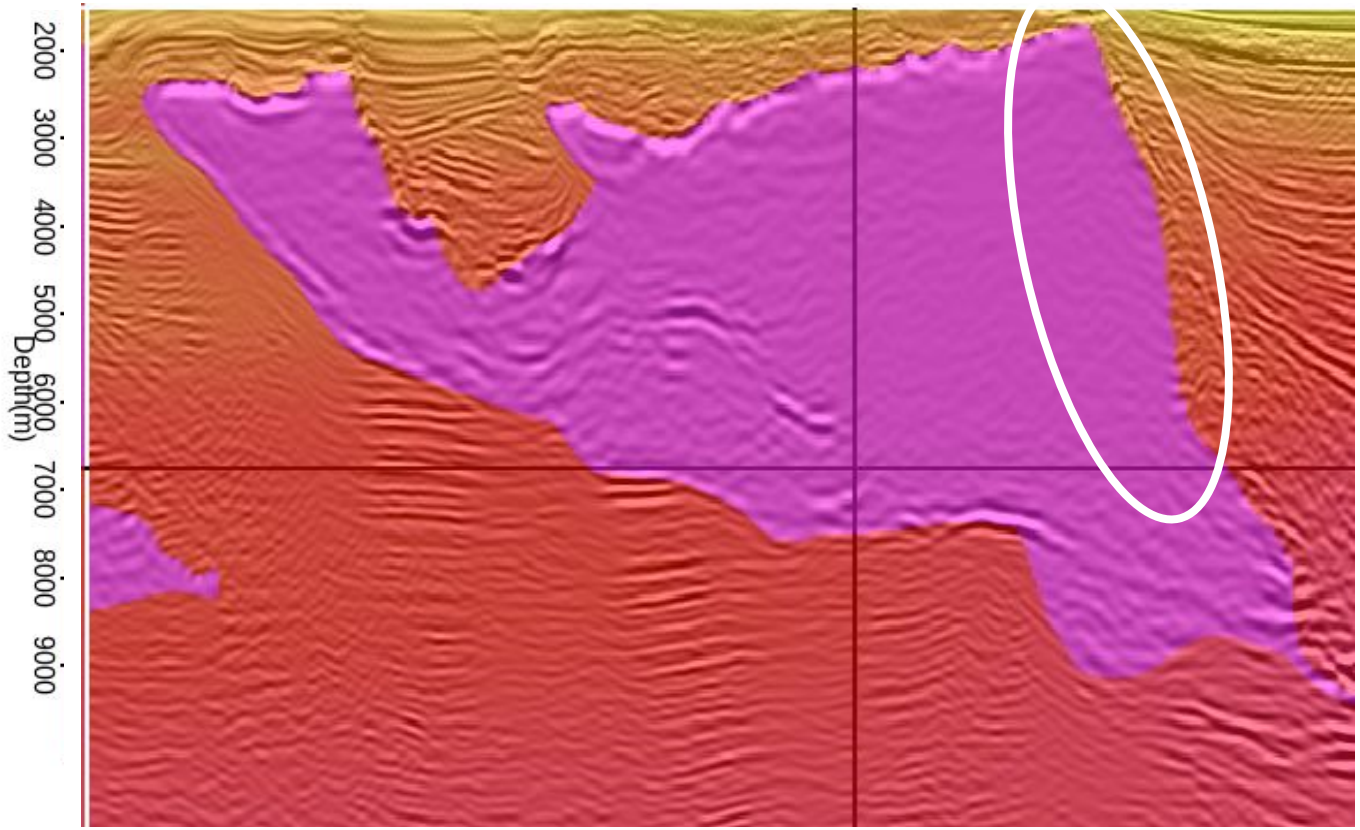
Motivation: Why use level sets?



Figure, Adam Halpert (Thesis) 2014

Motivation: Why use level sets?



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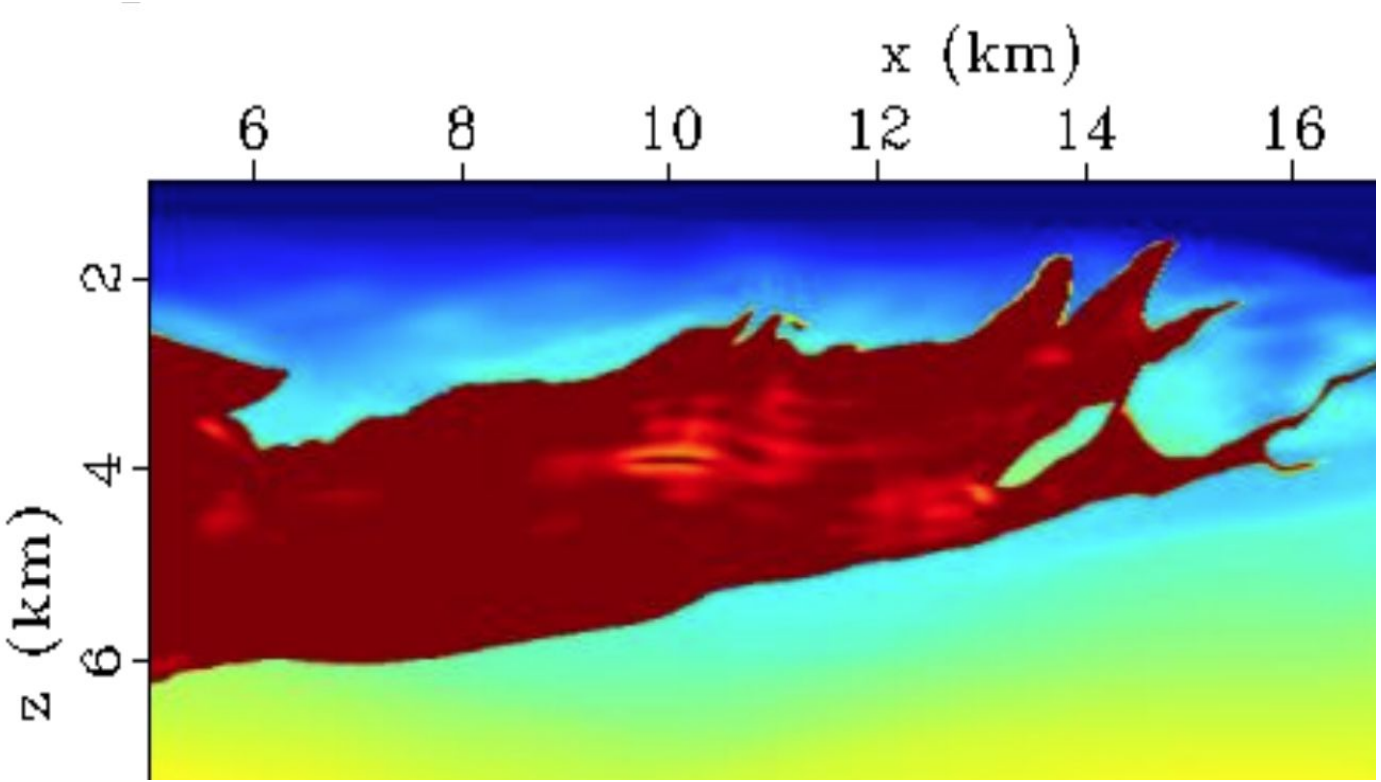

BEFORE FWI

Figure: Xukai Shen, “Salt model building at Atlantis with Full Waveform Inversion“, SEG 2017

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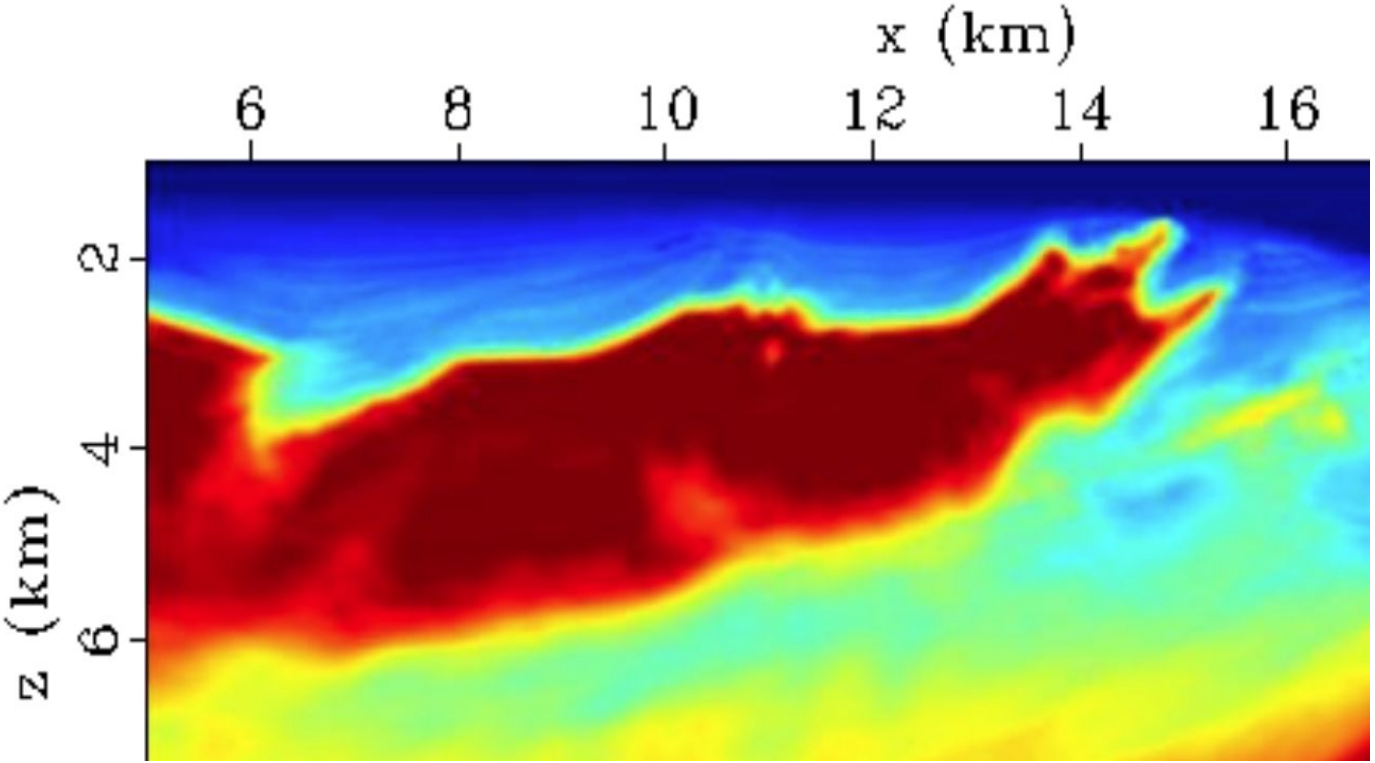

AFTER FWI

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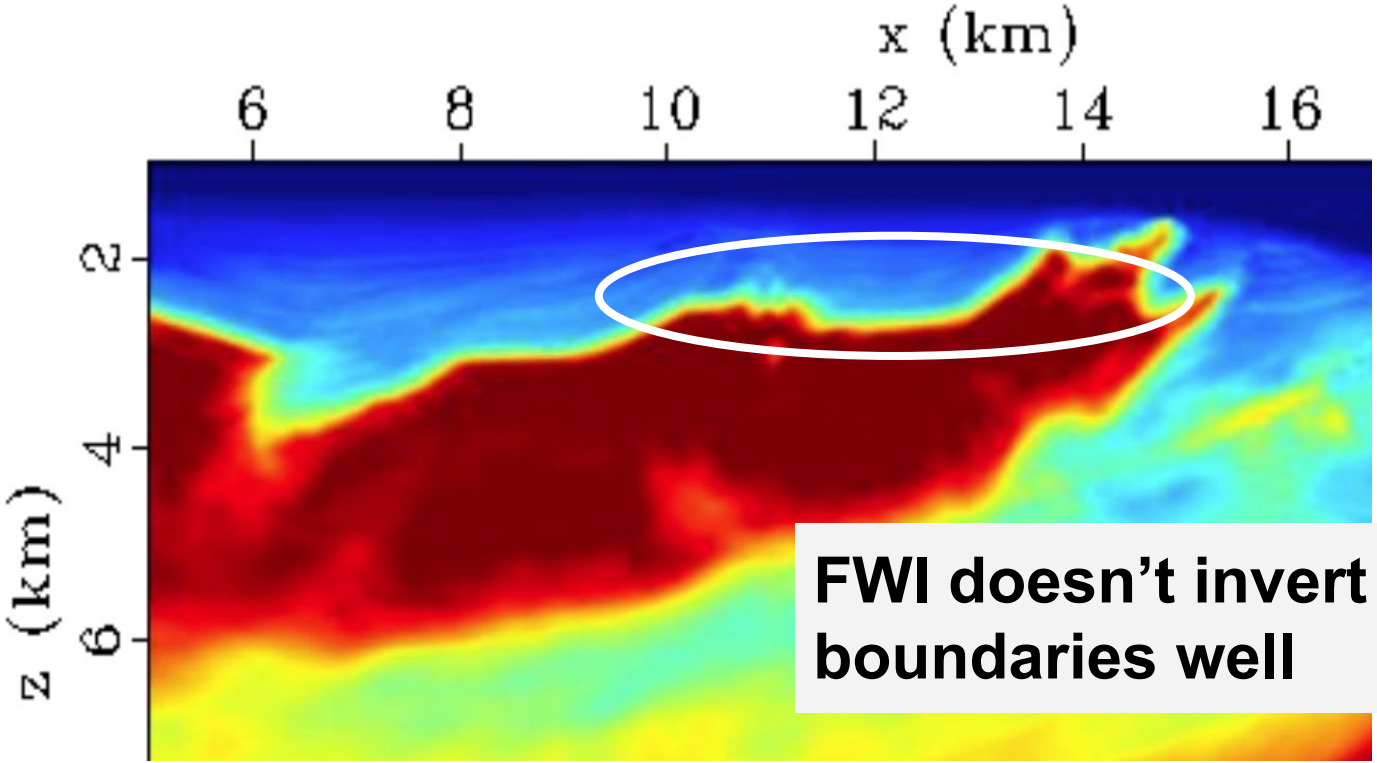

AFTER FWI

FWI doesn’t invert sharp boundaries well

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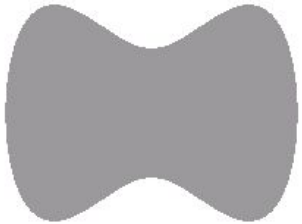
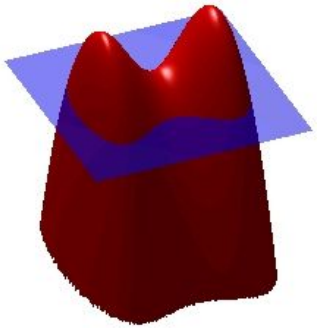
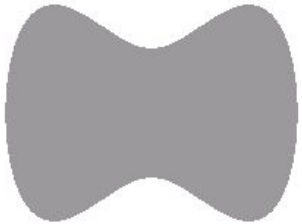


Level set



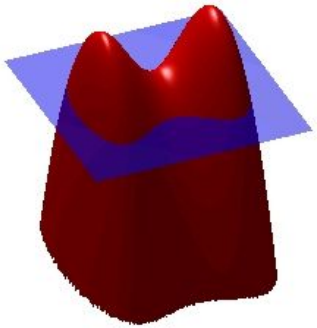
Implicit surface

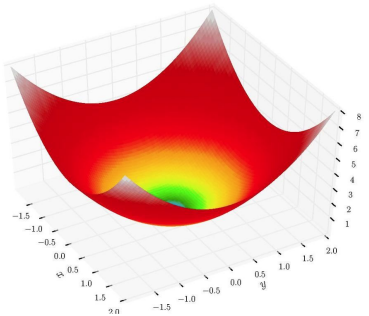
Motivation: Why use level sets?

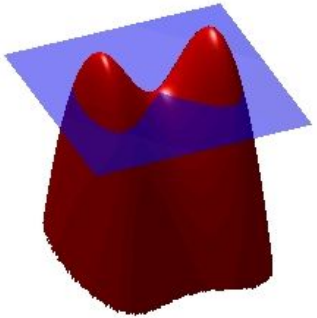


Level set

Low frequency update







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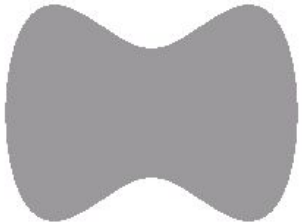
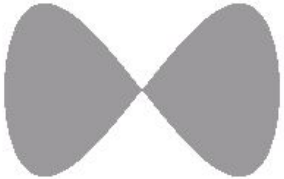


High frequency change



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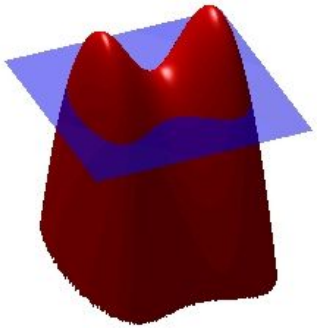

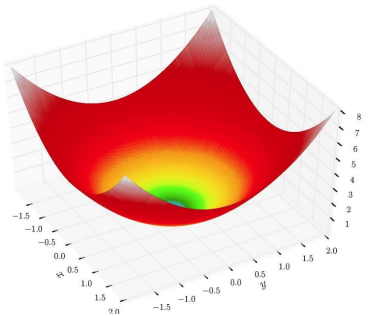

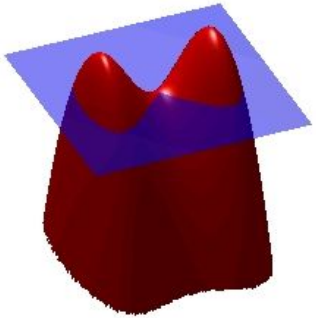

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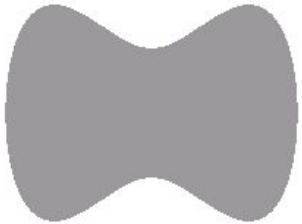

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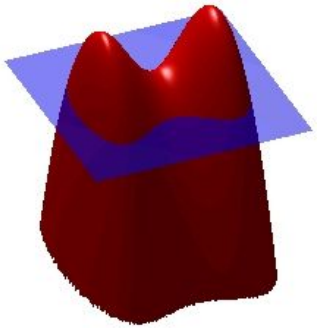

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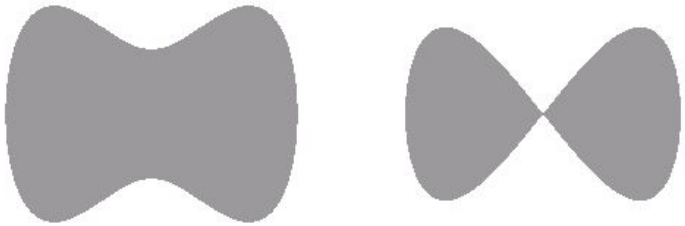

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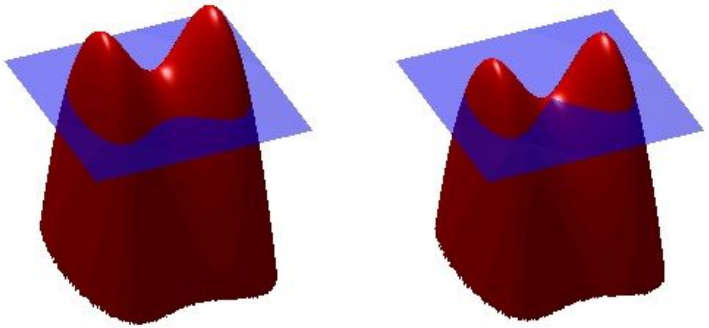

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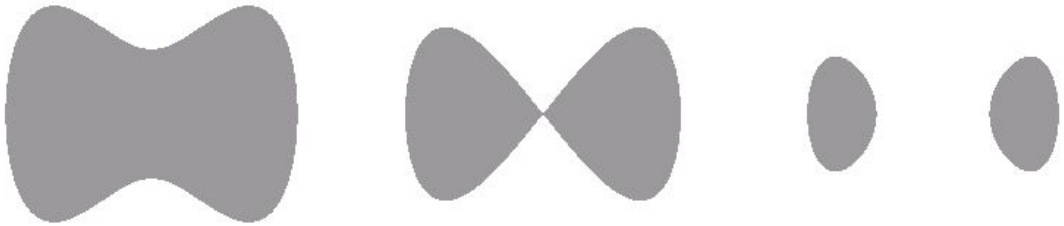

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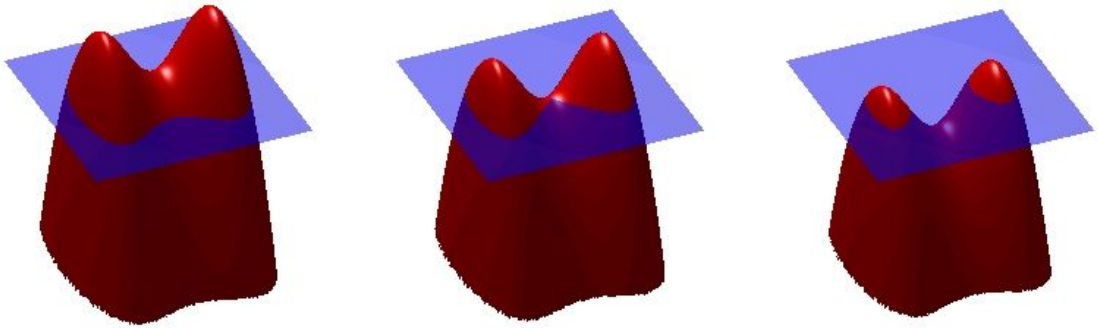
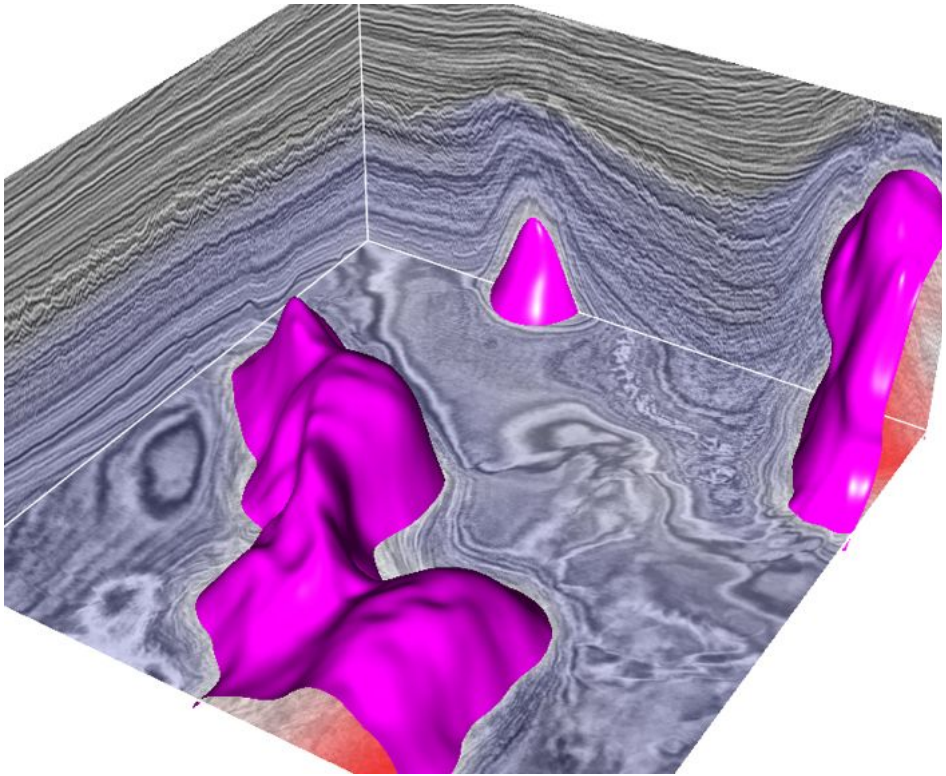


Implicit surface

Motivation: Why use level sets?

- Salt bodies can often be approximated as homogeneous.
- Level sets naturally define boundaries of homogeneous bodies
- This makes them a useful tool for salt modeling



Wu, X., 2016, Methods to compute salt likelihoods and extract salt boundaries from 3D seismic images. **Geophysics**, 81(6)

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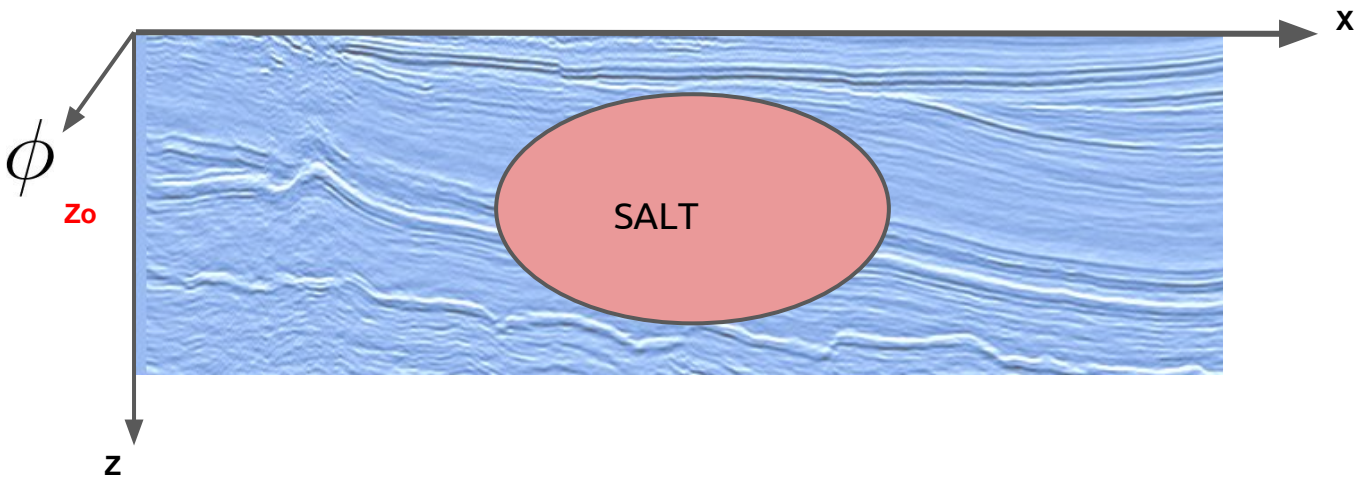
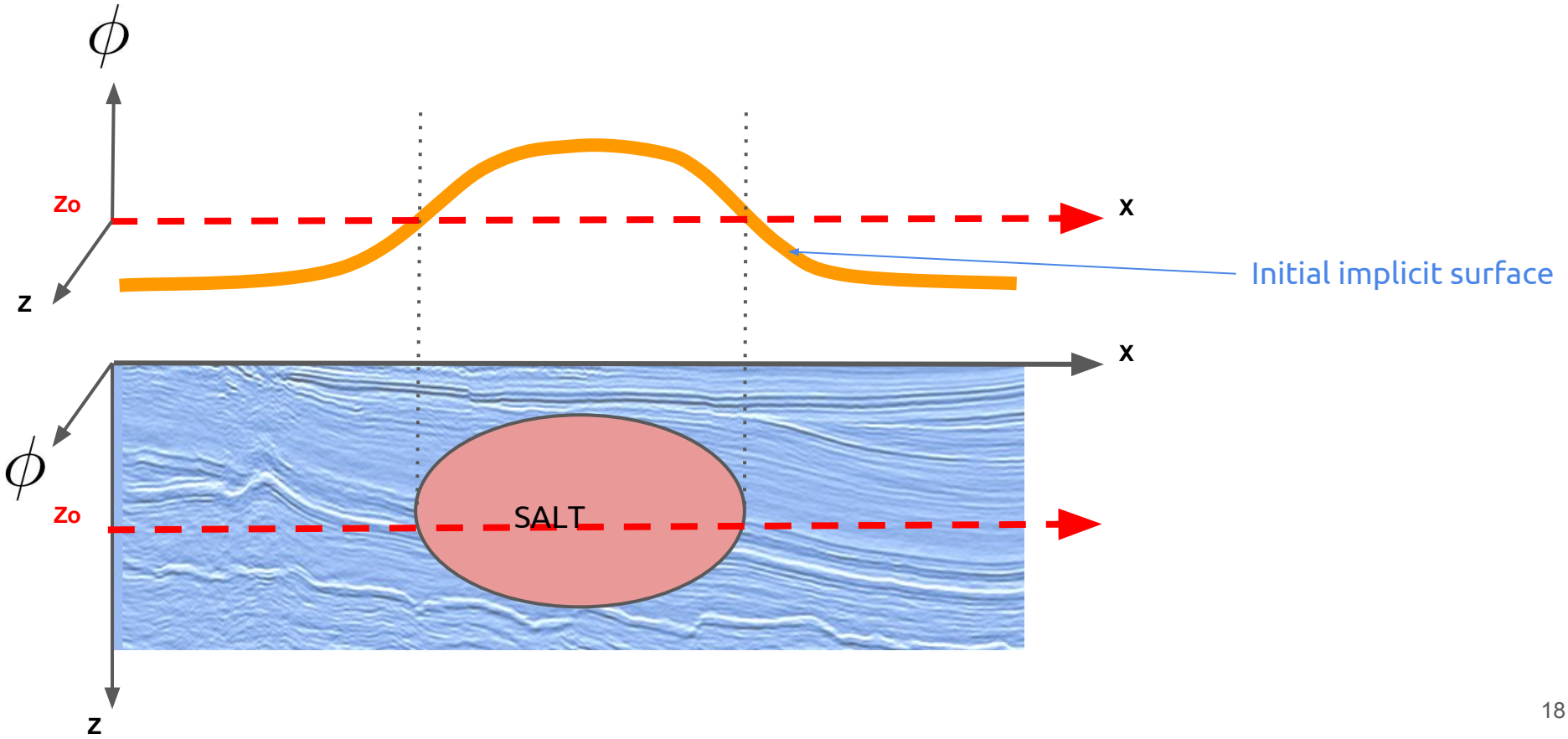
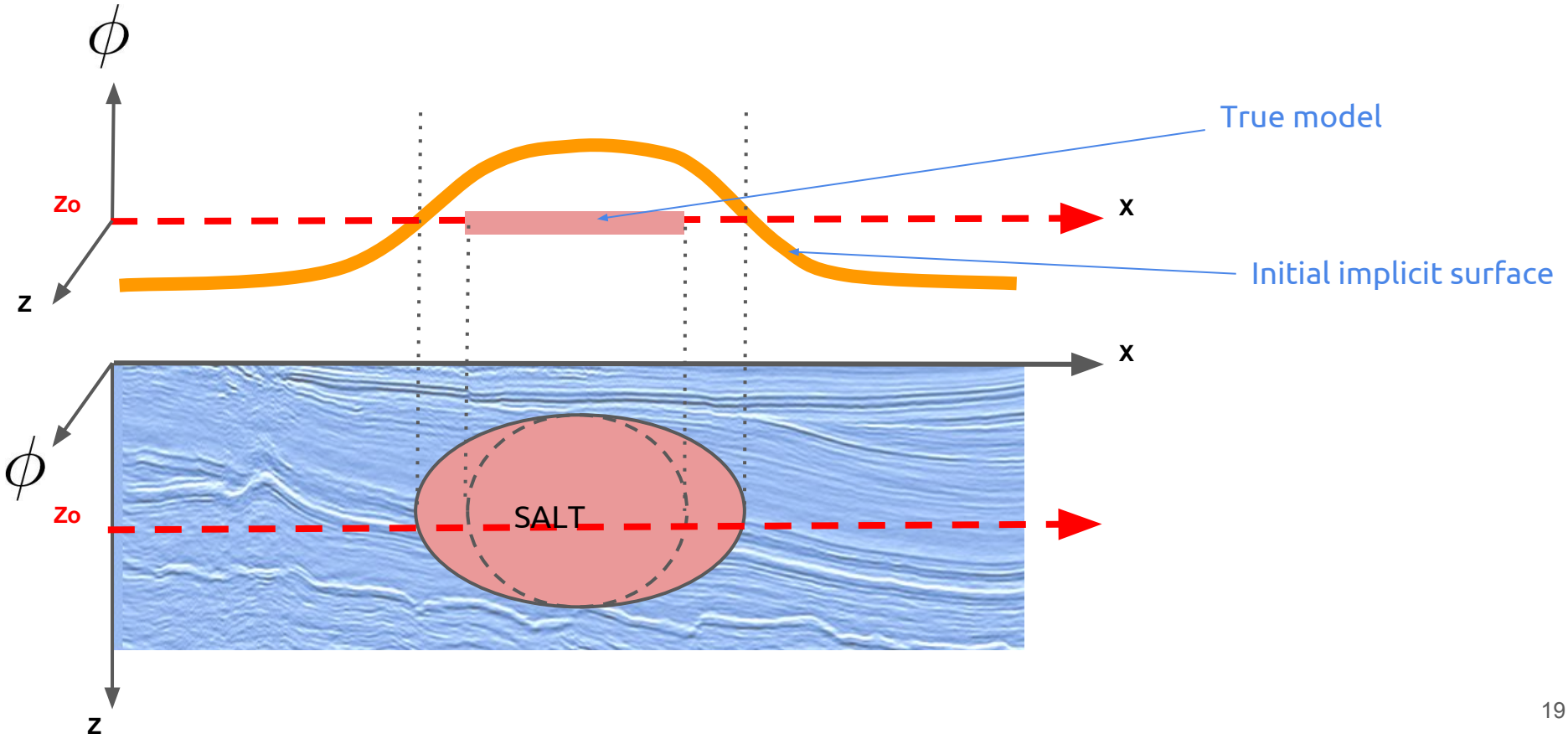
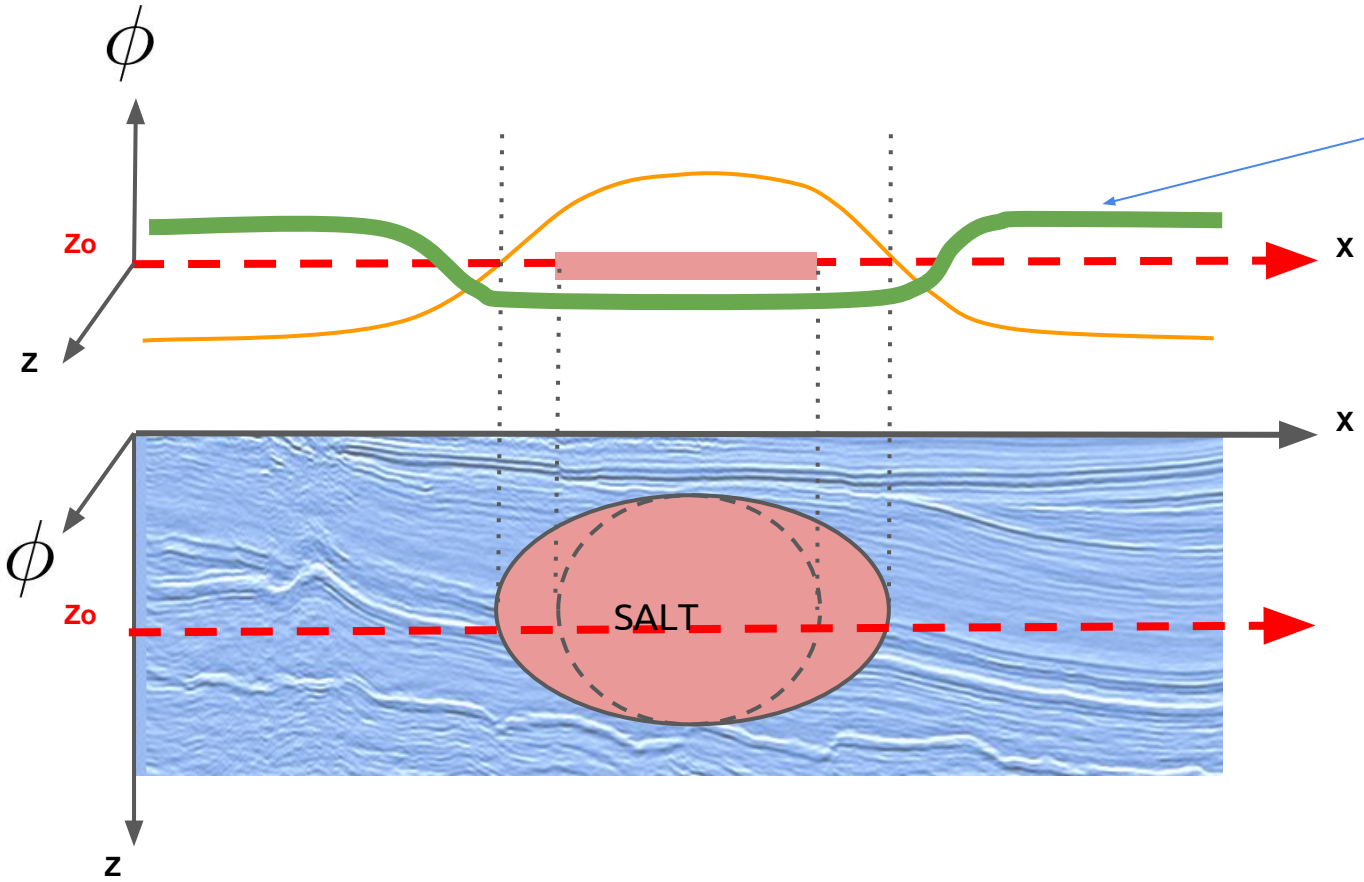
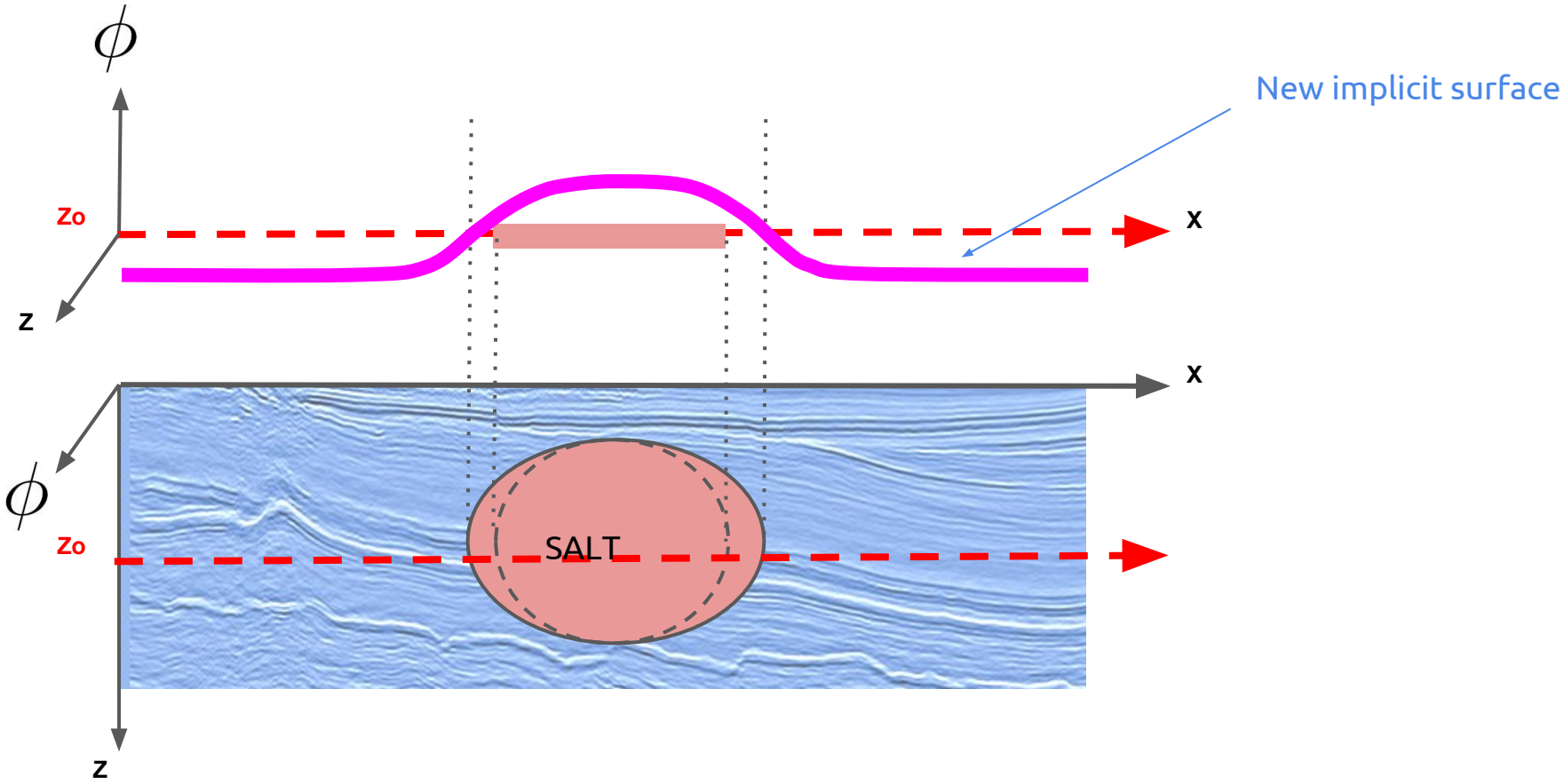

x
ϕ
Zo
SALT
z


ϕ
Zo
z
x
Initial implicit surface
x
ϕ
Zo
SALT
z


ϕ
Zo
z
x
True model
Initial implicit surface
x
ϕ
Zo
SALT
z


ϕ
Zo
z
x
Implicit surface update
x
ϕ
Zo
SALT
z


ϕ
Zo
z
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New implicit surface
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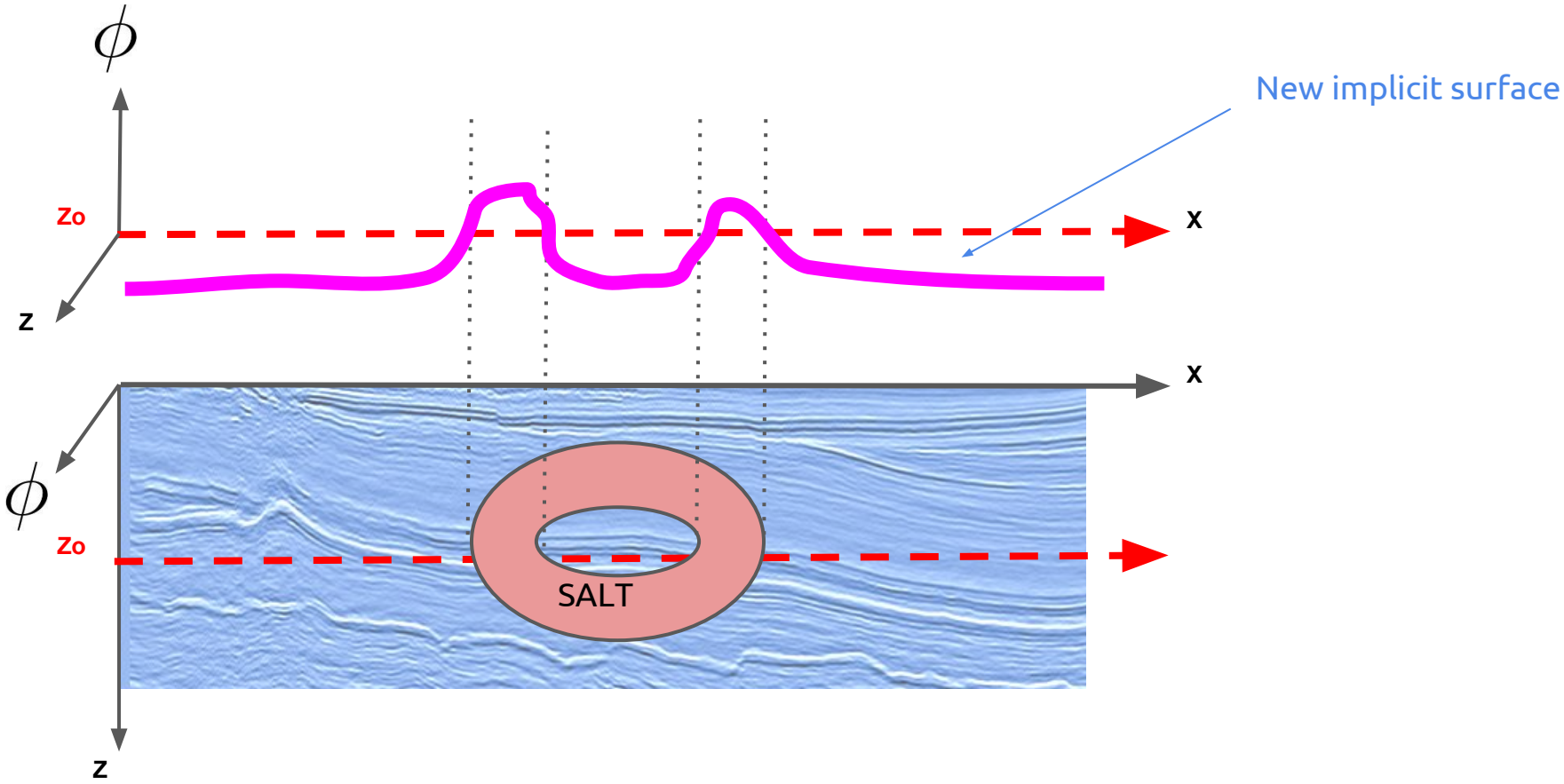
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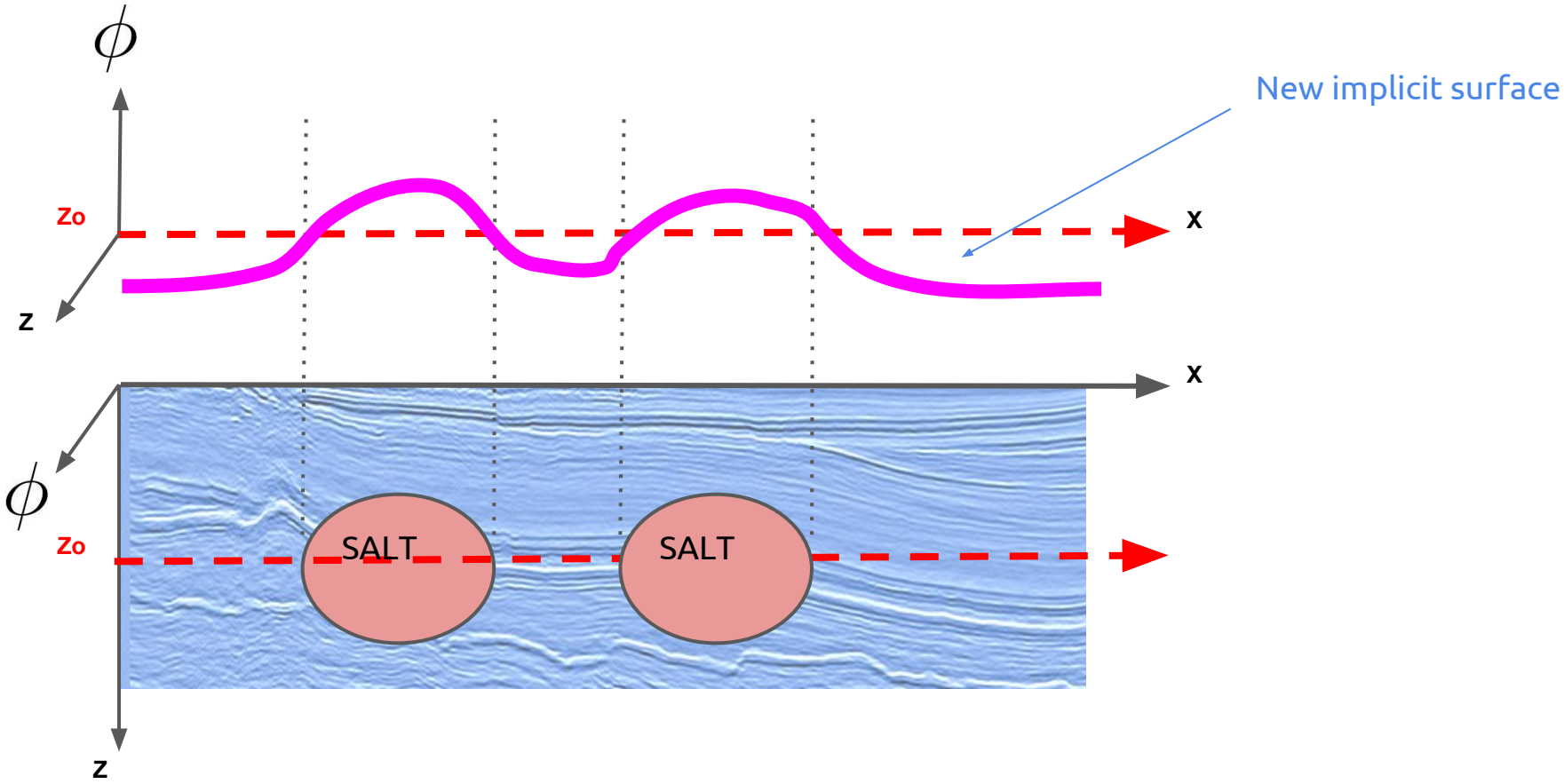
ϕ
Zo
z
x
New implicit surface
x
ϕ
Zo
SALT
SALT
z

Derivation: Objective function

Typical FWI:

$$\| F(m) - d_{obs} \|_2^2$$

Derivation: Objective function

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$$\left\| F(m) - d_{obs} \right\|_2^2$$

L2 Norm

Non-linear forward modeling

Observed pressure data (acoustic)

Derivation: Objective function

Level set FWI:

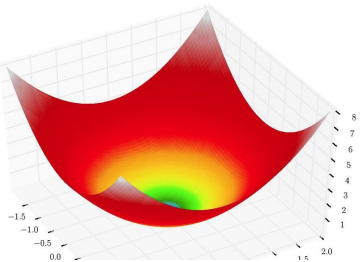
$$\left\| F(m(\phi, b)) - d_{obs} \right\|_2^2$$

Derivation: New model space

$$m(\phi,b)=H(\phi)(c_s-b)+b$$

Derivation: New model space

$$m(\phi, b) = H(\phi)(c_s - b) + b$$



Implicit surface

Derivation: New model space

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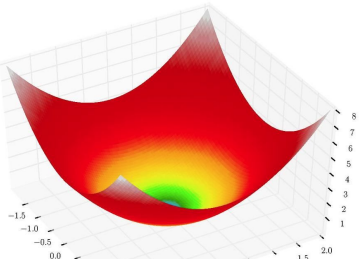

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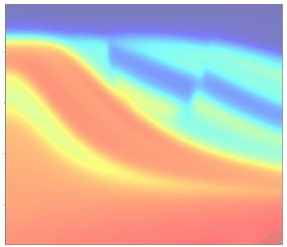
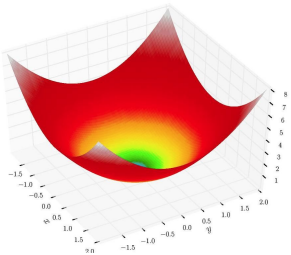
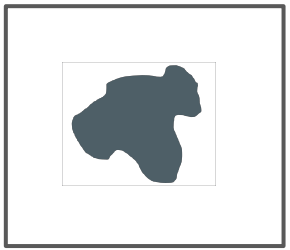


Background velocity

Derivation: New model space

$$m(\phi, b) = H(\phi)(c_s - b) + b$$

H() =



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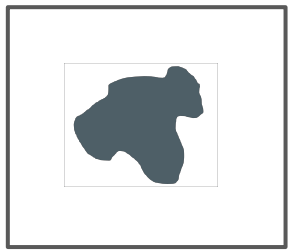

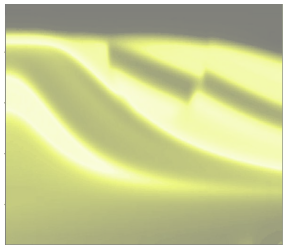

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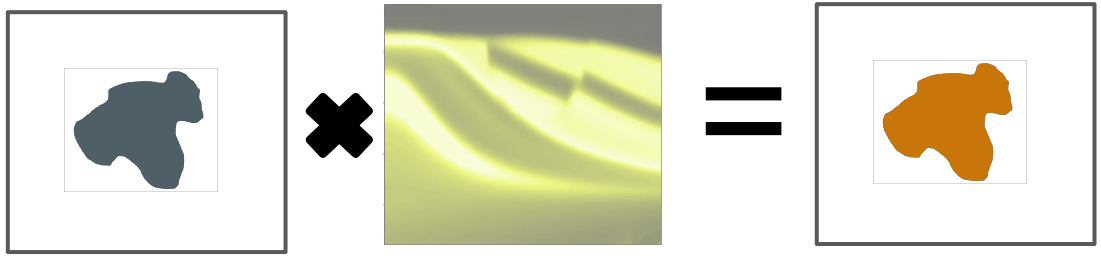
$$m(\phi, b) = H(\phi)(c_s - b) + b$$



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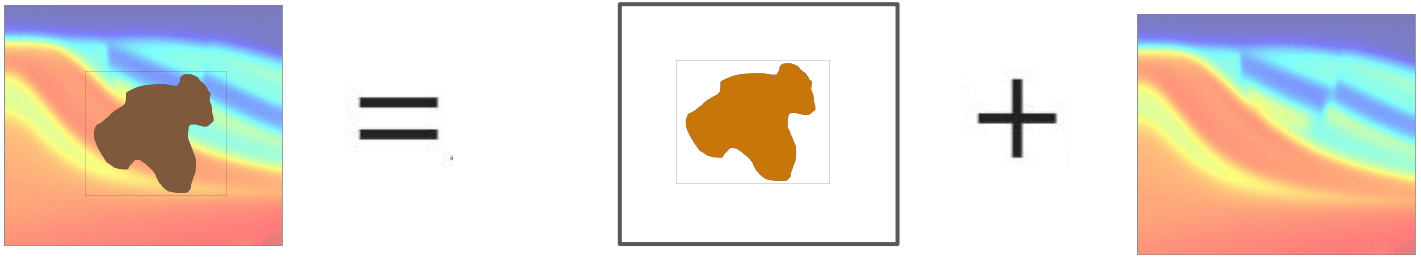


Derivation: Implicit surface gradient

Level set FWI:

$$\| F(m(\phi, b)) - d_{obs} \|_2^2$$

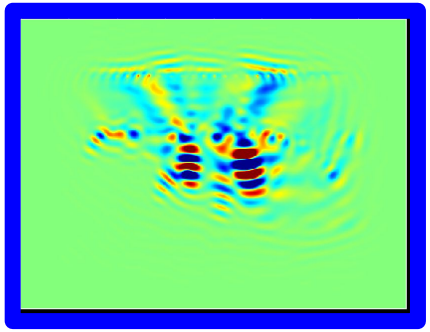
Derivation: Gradient

Level set FWI:

$$\| F(m(\phi, b)) - d_{obs} \|_2^2$$

$$g_\phi = \frac{\delta m}{\delta \phi}^T \frac{\delta F(m)}{\delta m}^T \Big|_{m_o} \triangle d$$

Derivation: Gradient



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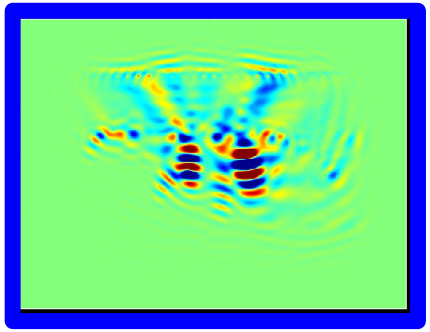
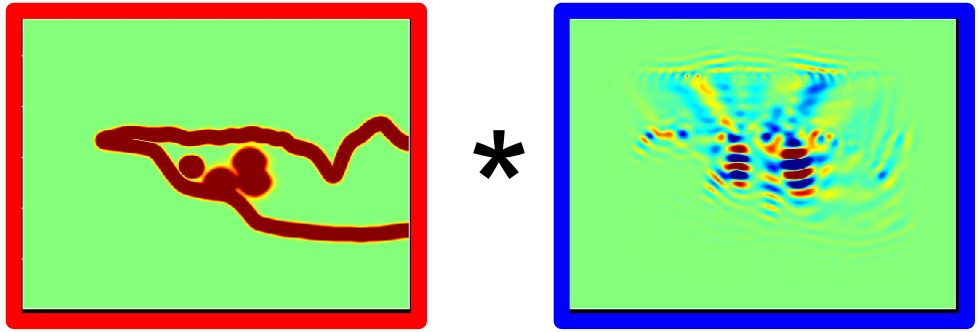


$$g_\phi = \tilde{\delta}(\phi)(c_s - b)\left.\frac{\delta F(m)}{\delta m}\right|_{m_o}^{T} \triangle d$$

Derivation: Gradient



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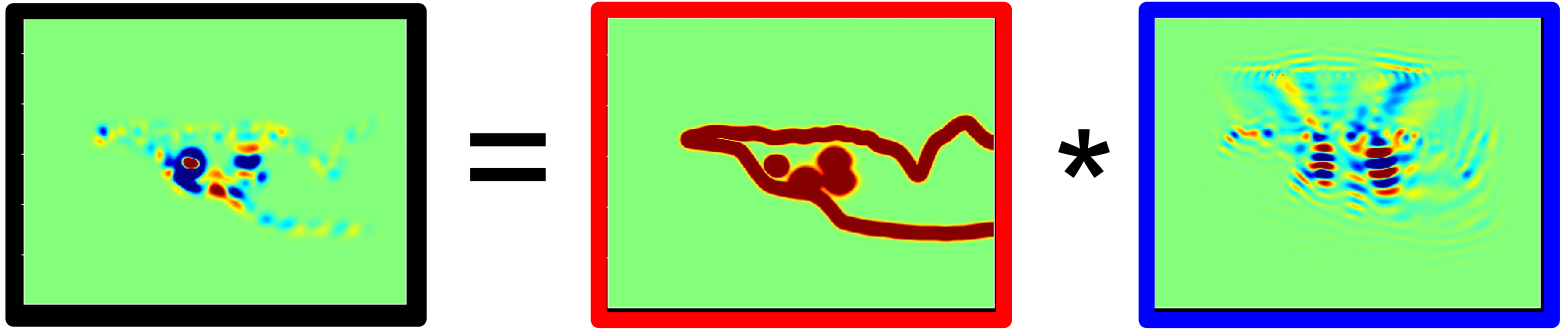

$$g_\phi = \delta(\phi)(c_s - b)\left.\frac{\delta F(m)}{\delta m}^T\right|_{m_o} \triangle d$$

Derivation: Gauss-Newton step

- Expensive to perform GN Hessian inversion
- Cost depends on model size
- Can we intelligently reduce the size of our model?

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Build implicit surface with RBFs

$$\phi(\alpha) = \sum_{i=1}^{n} \alpha_i e^{-\epsilon(\delta x^2 + \delta y^2)}$$

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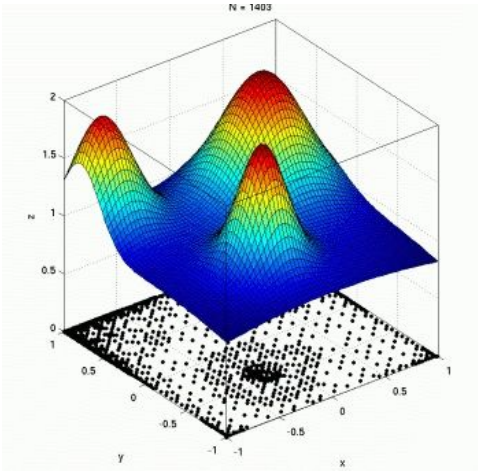


$$= \sum_{i=1}^{3} \alpha_i$$

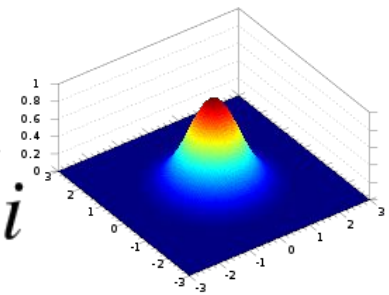
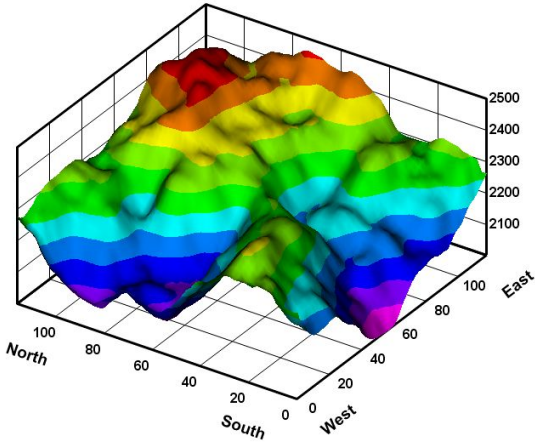


Image credit: http://www.it.uu.se/research/scientific_computing/project/rbf/rbfpde

Build implicit surface with RBFs

$$\phi(\alpha) = \sum_{i=1}^{n} \alpha_i e^{-\epsilon(\delta x^2 + \delta y^2)}$$



$$= \sum_{i=1}^{100} \alpha_i$$

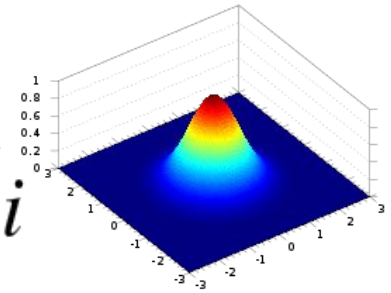


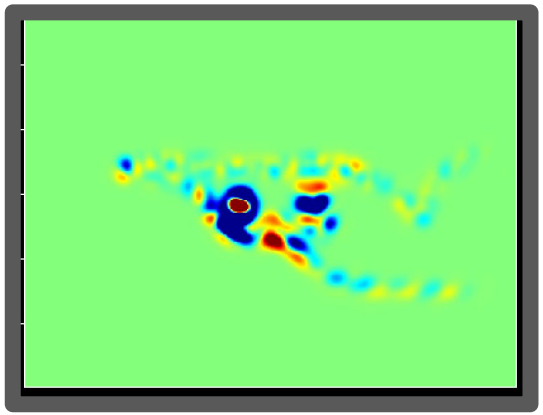
Image credit: http://www.dplot.com/features.htm

Model reduction

$$g_\alpha = \frac{\delta\phi}{\delta\alpha}^T \frac{\delta m}{\delta\phi}^T \frac{\delta F(m)}{\delta m}^T \Big|_{m_o} \triangle d$$

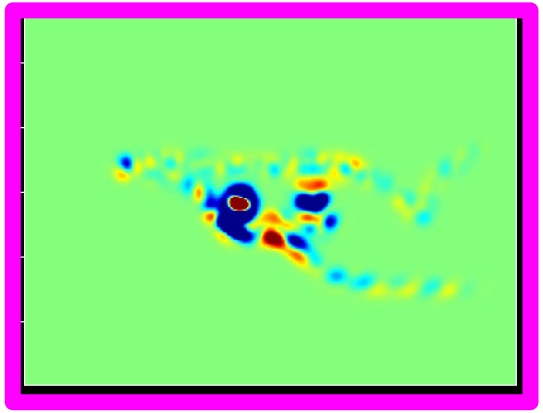
Model reduction

$$g_\alpha = \frac{\delta \phi}{\delta \alpha}^T$$



Model reduction

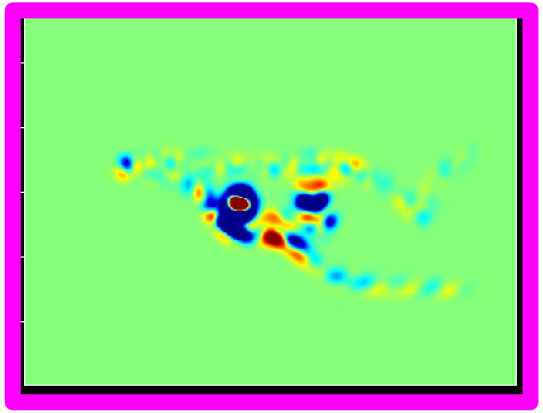
$$g_\alpha = \frac{\delta \phi}{\delta \alpha}^T$$



~ 100,000,000

Model reduction

$$g_\alpha = \frac{\delta \phi}{\delta \alpha}^T$$



Roughly 100 times fewer parameters!

~ 100,000

~ 10,000,000

Initial salt model example

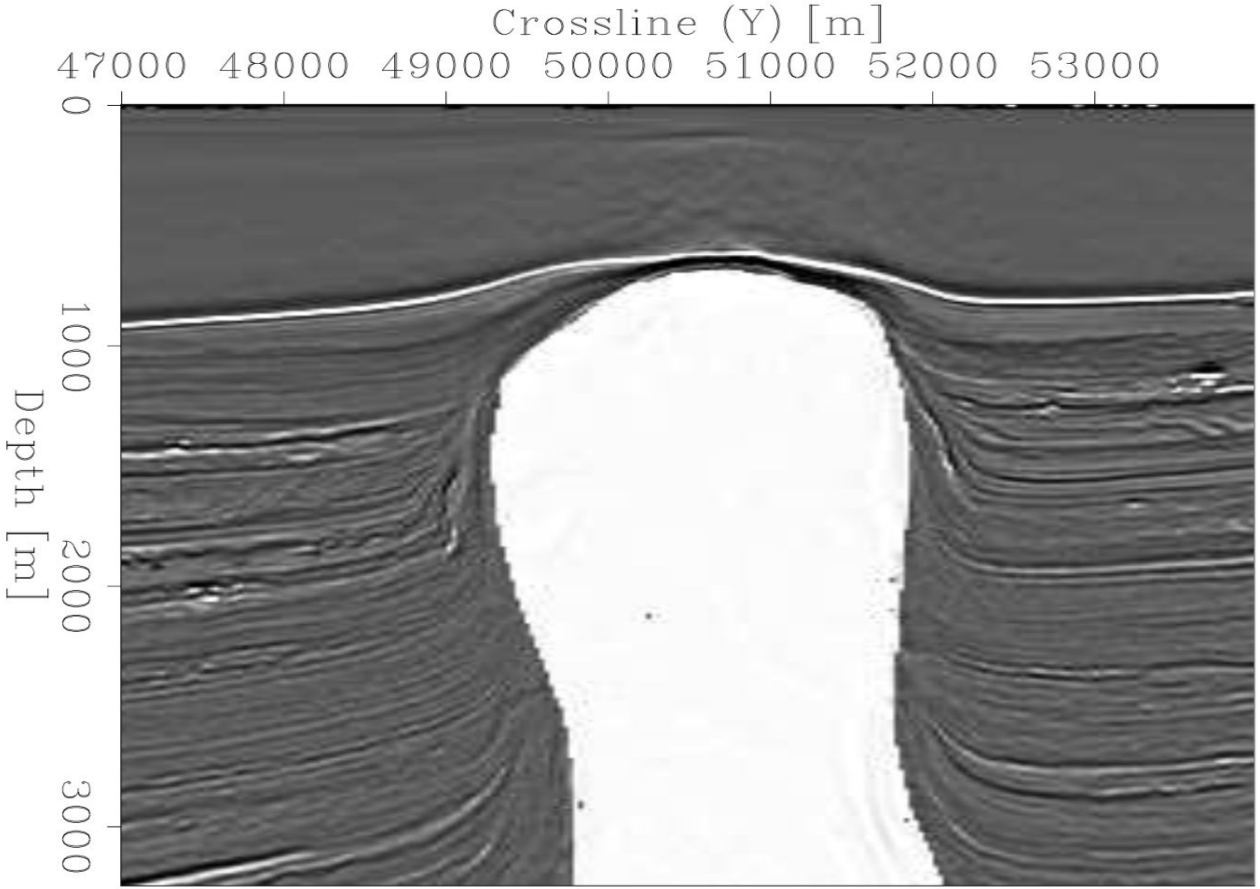
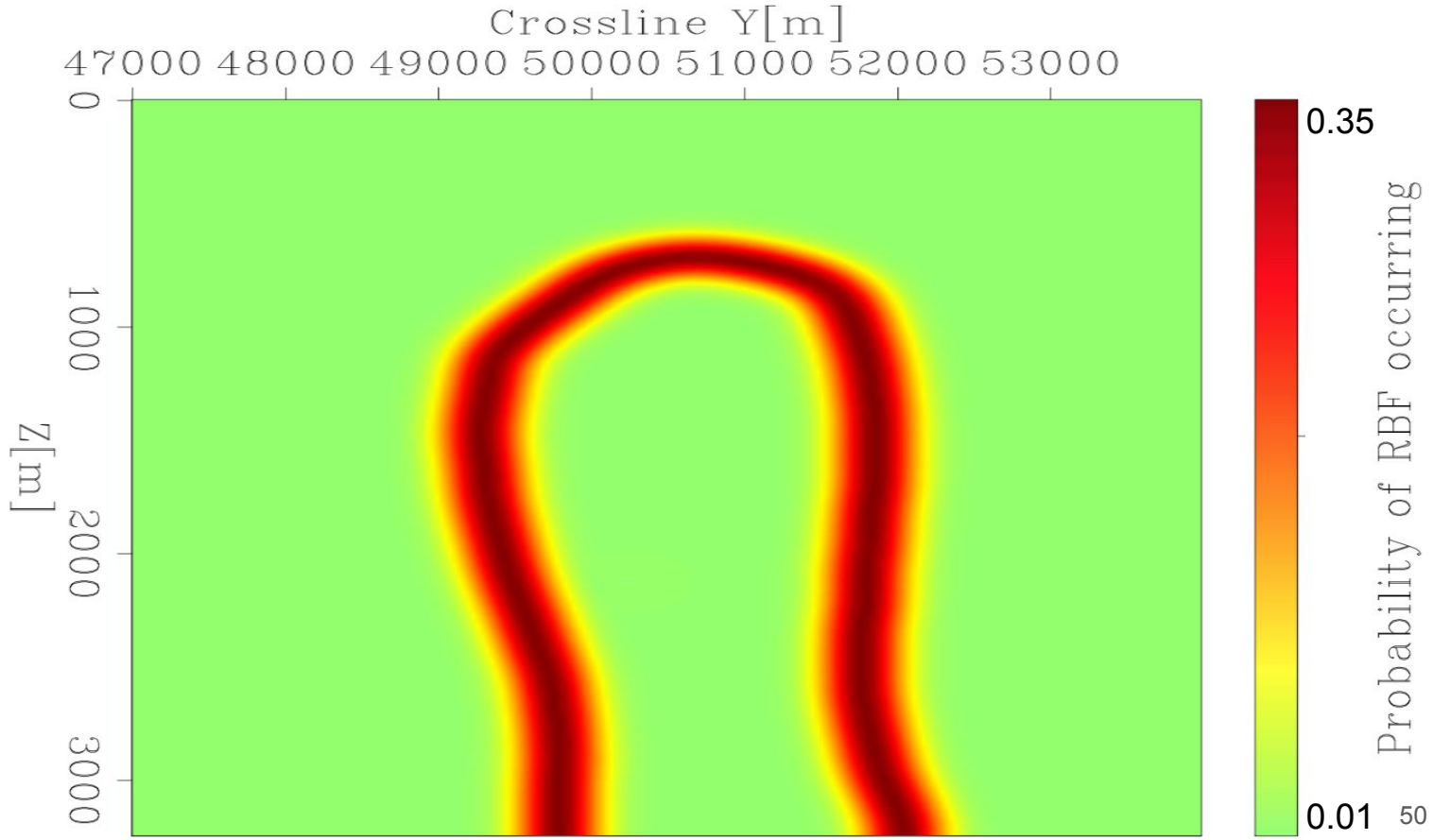
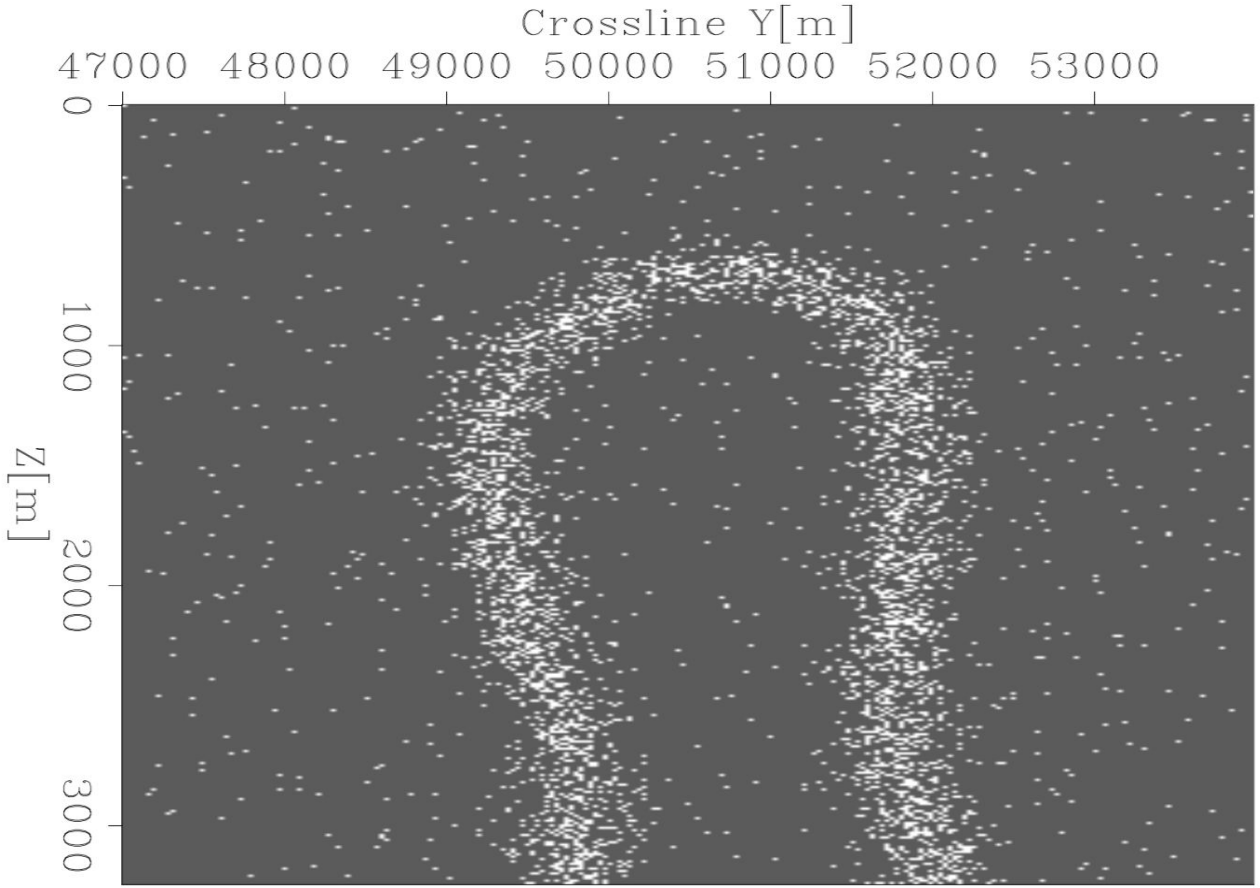


Image courtesy of Shell Exploration & Production company.

Probability function for boundary position



Randomly chosen RBF center positions



Overview

Motivation

Level set basics

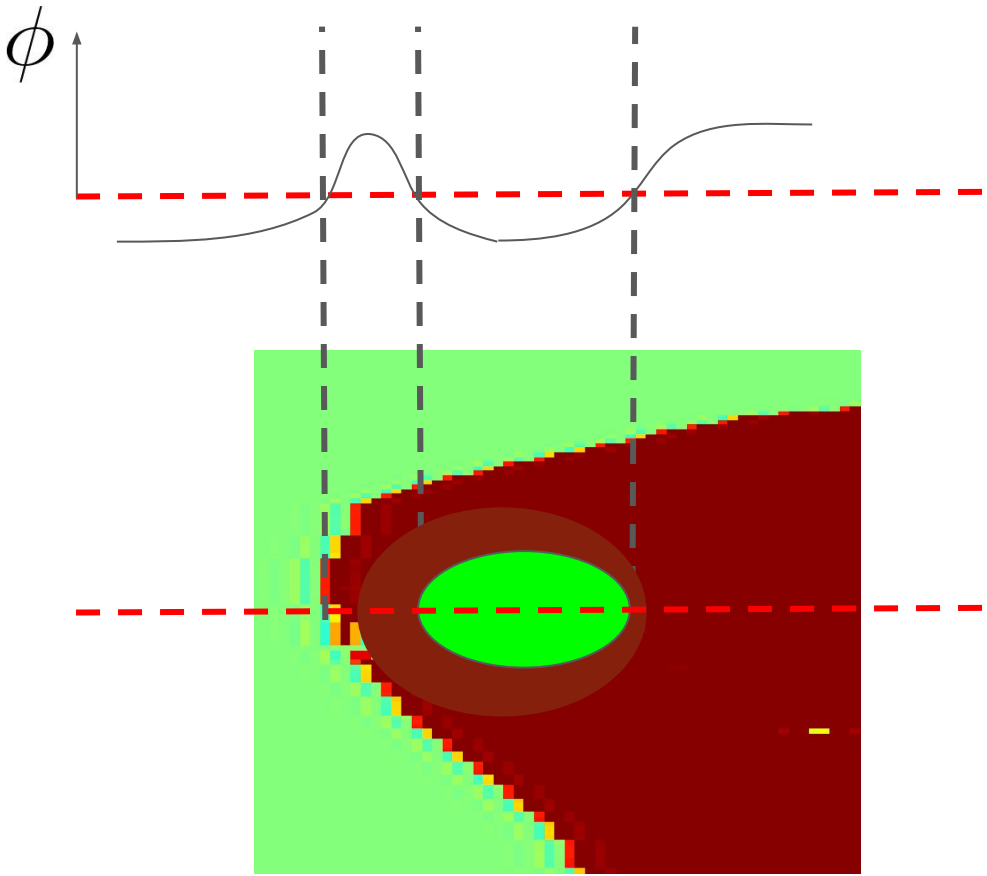
Radial basis functions

Interpreter guidance

Demonstration on Sigsbee model

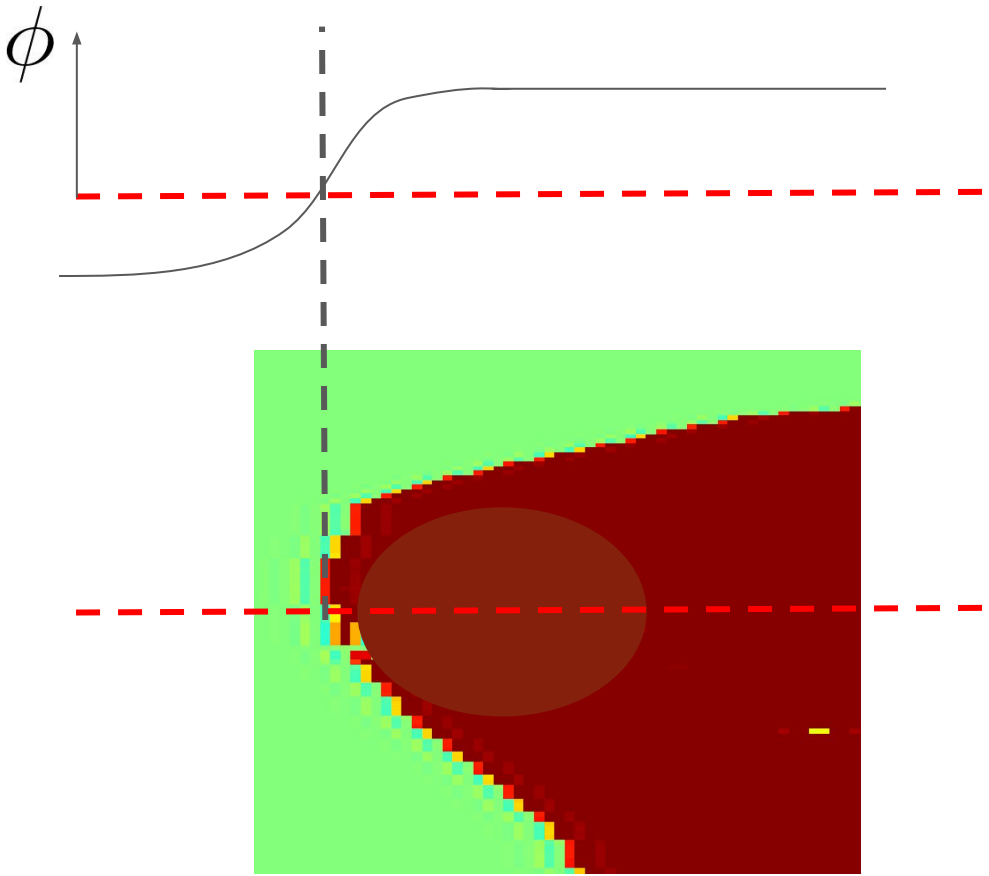
Conclusions

Problem: Current method can’t readily find inclusions



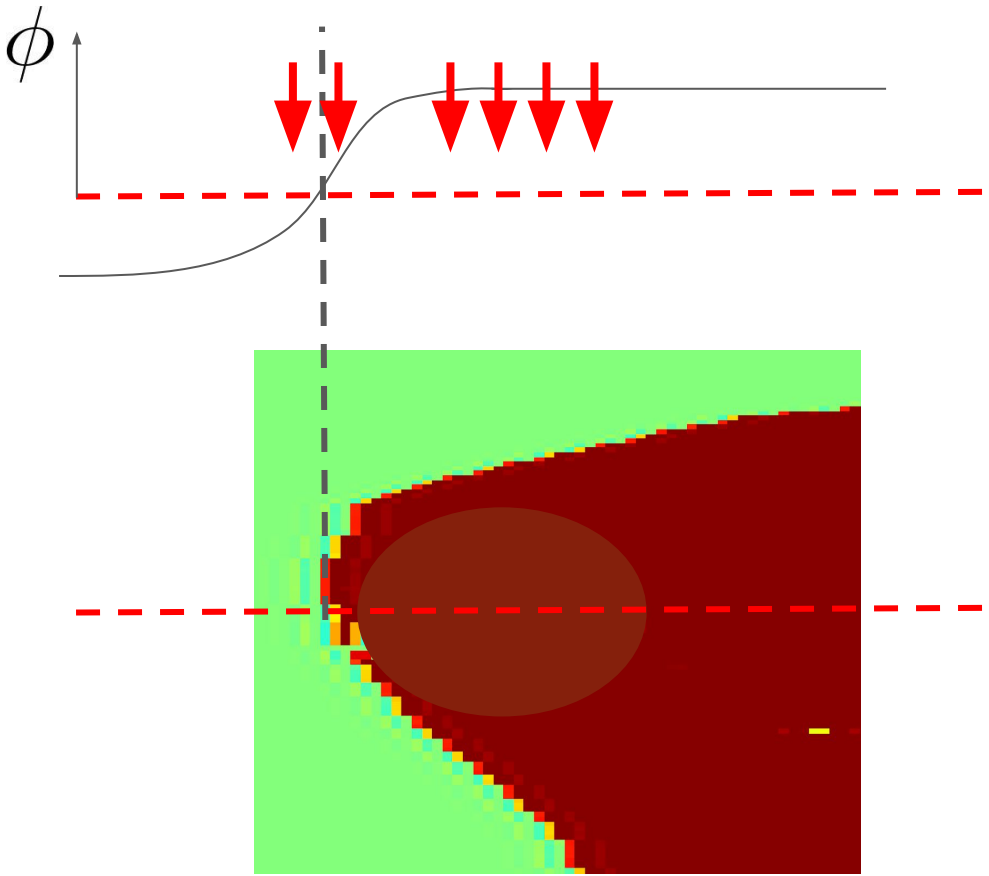
True model

Problem: Current method can’t readily find inclusions



Initial model

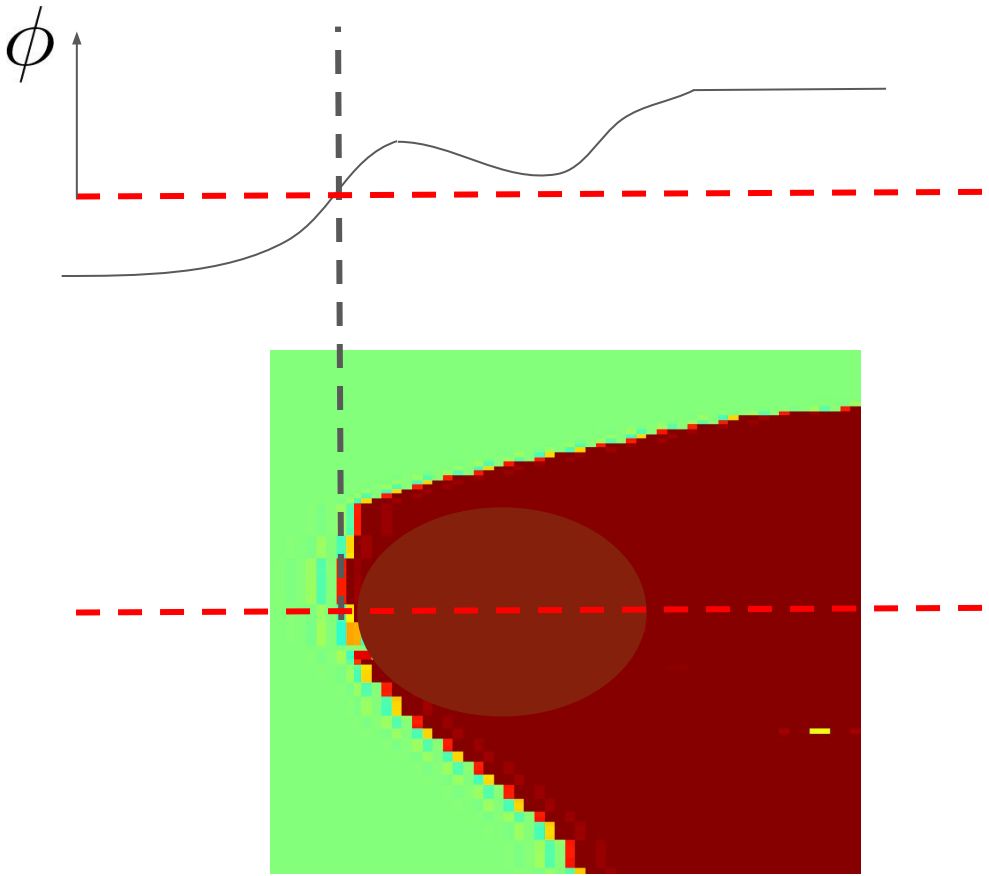
Problem: Current method can’t readily find inclusions



Search direction

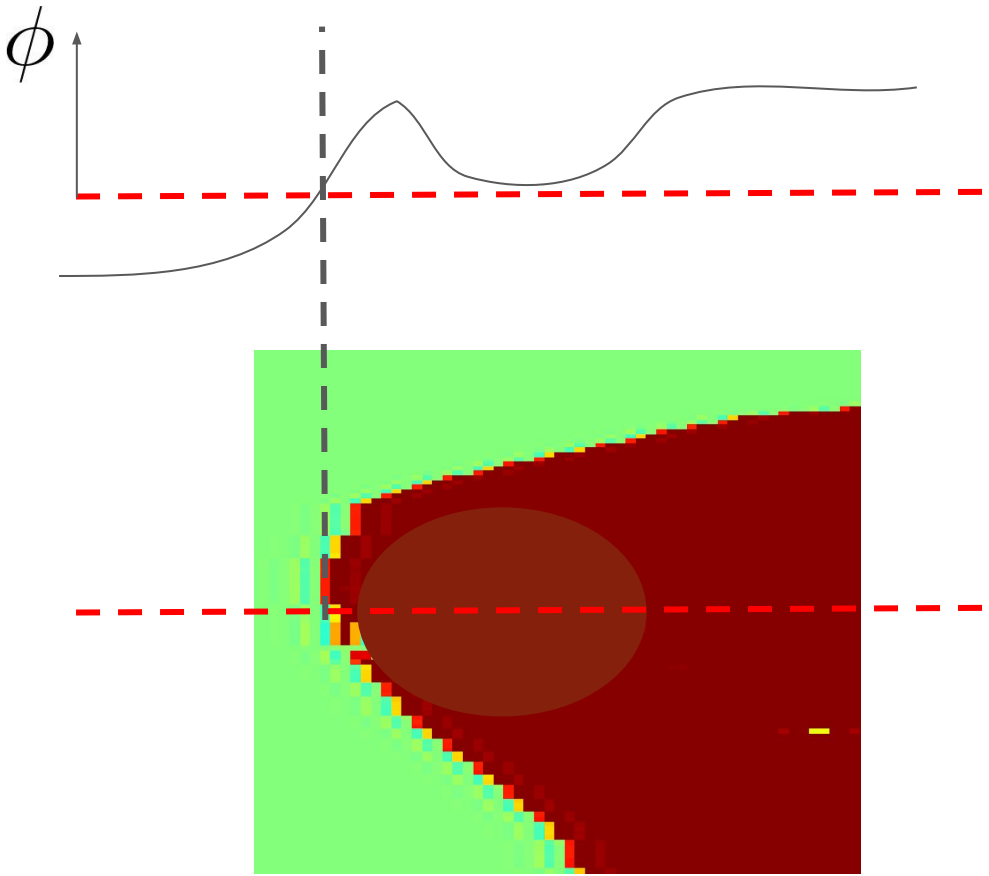
Initial model

Problem: Current method can’t readily find inclusions



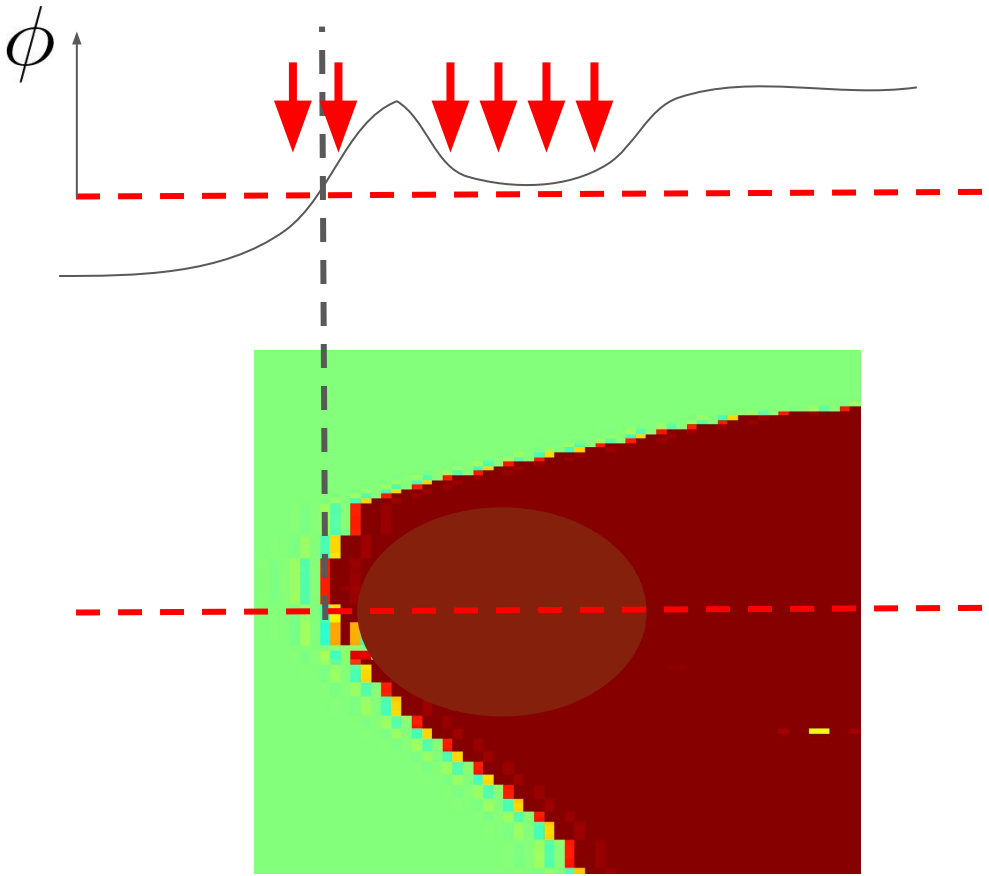
Updated model
(Not impressive)

Solution: Lower surface where update is probable



Initial model

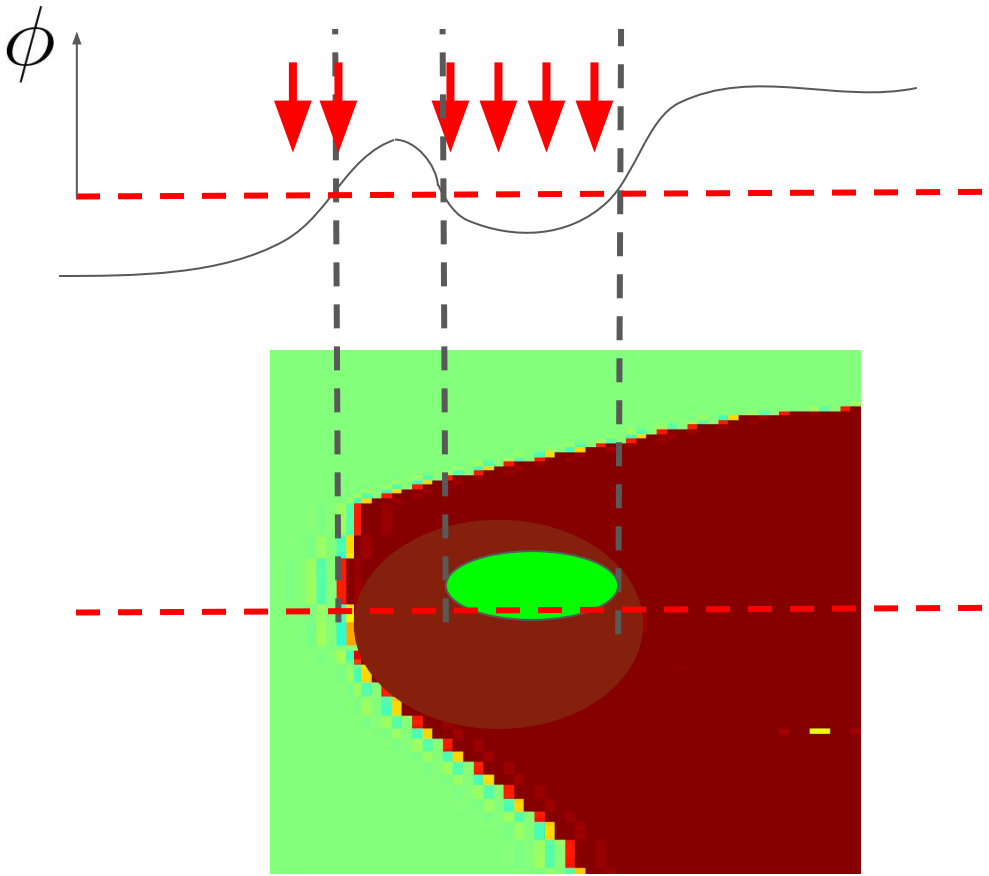
Solution: Lower surface where update is probable



Search direction

Initial model

Solution: Lower surface where update is probable



Updated model
(Better this time)

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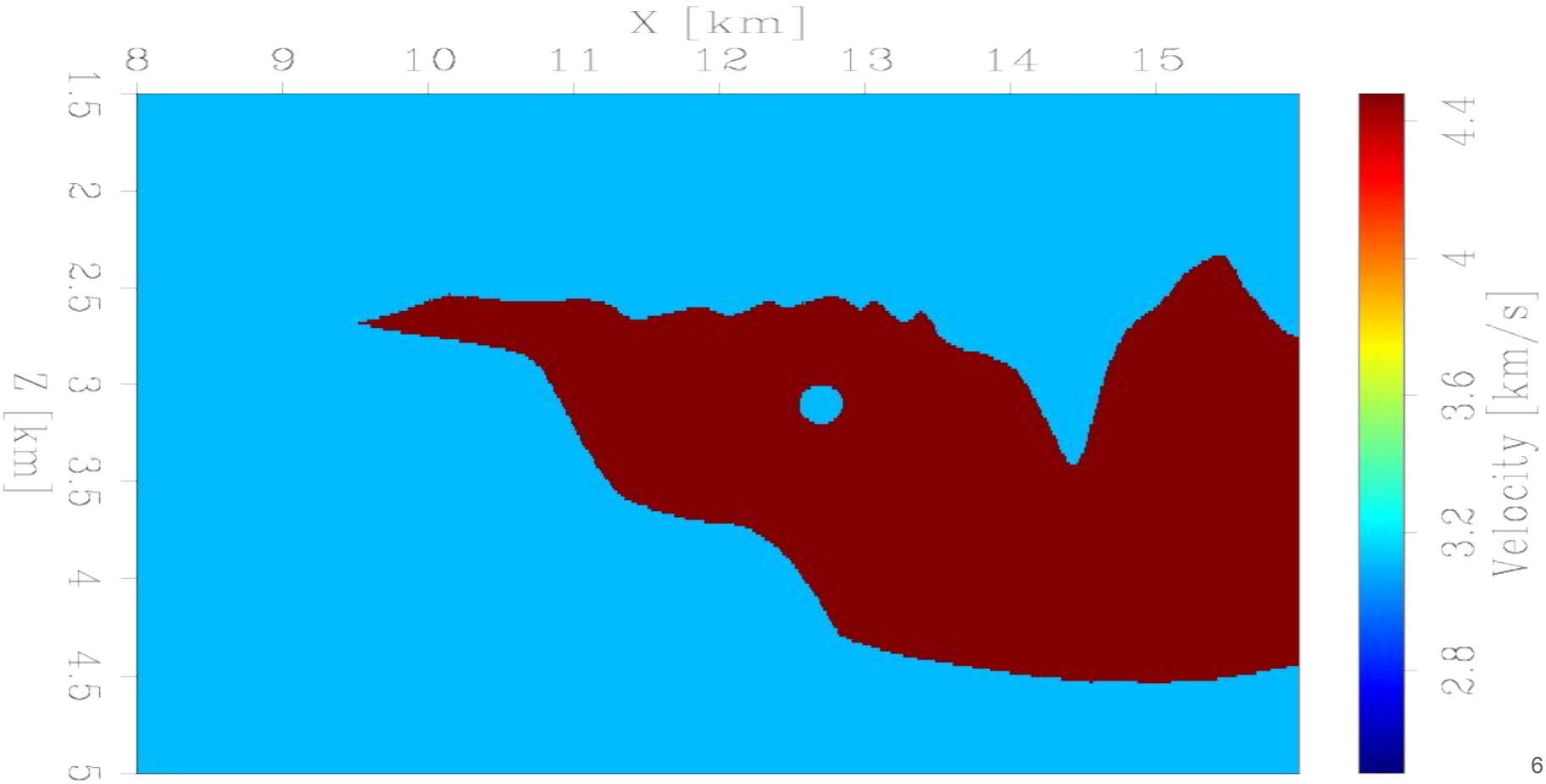
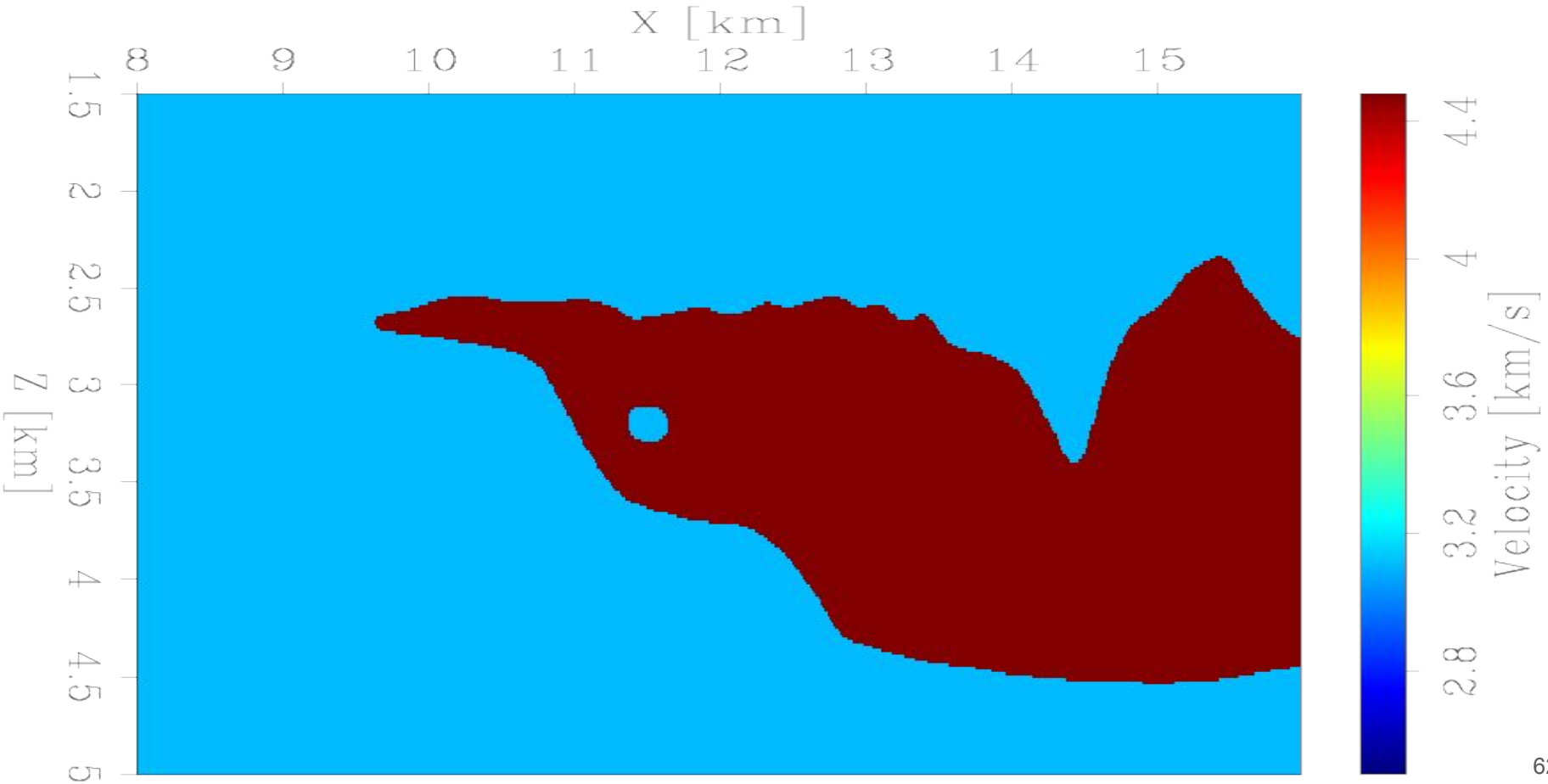
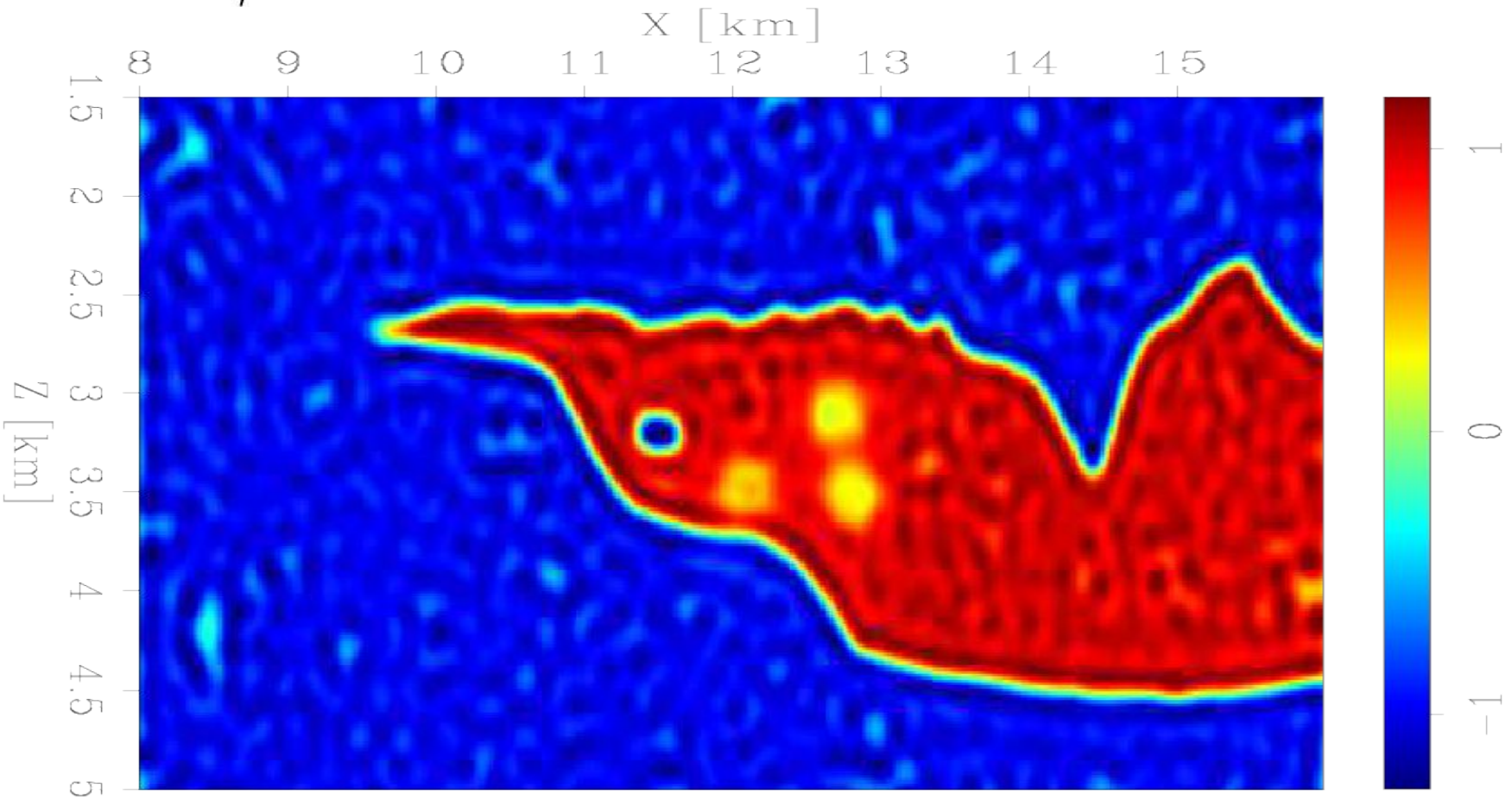

X [km]
8
9
10
11
12
13
14
15
Z [km]
1.5
2
2.5
3
3.5
4
4.5
5
Velocity [km/s]
2.8
3.2
3.6
4
4.4

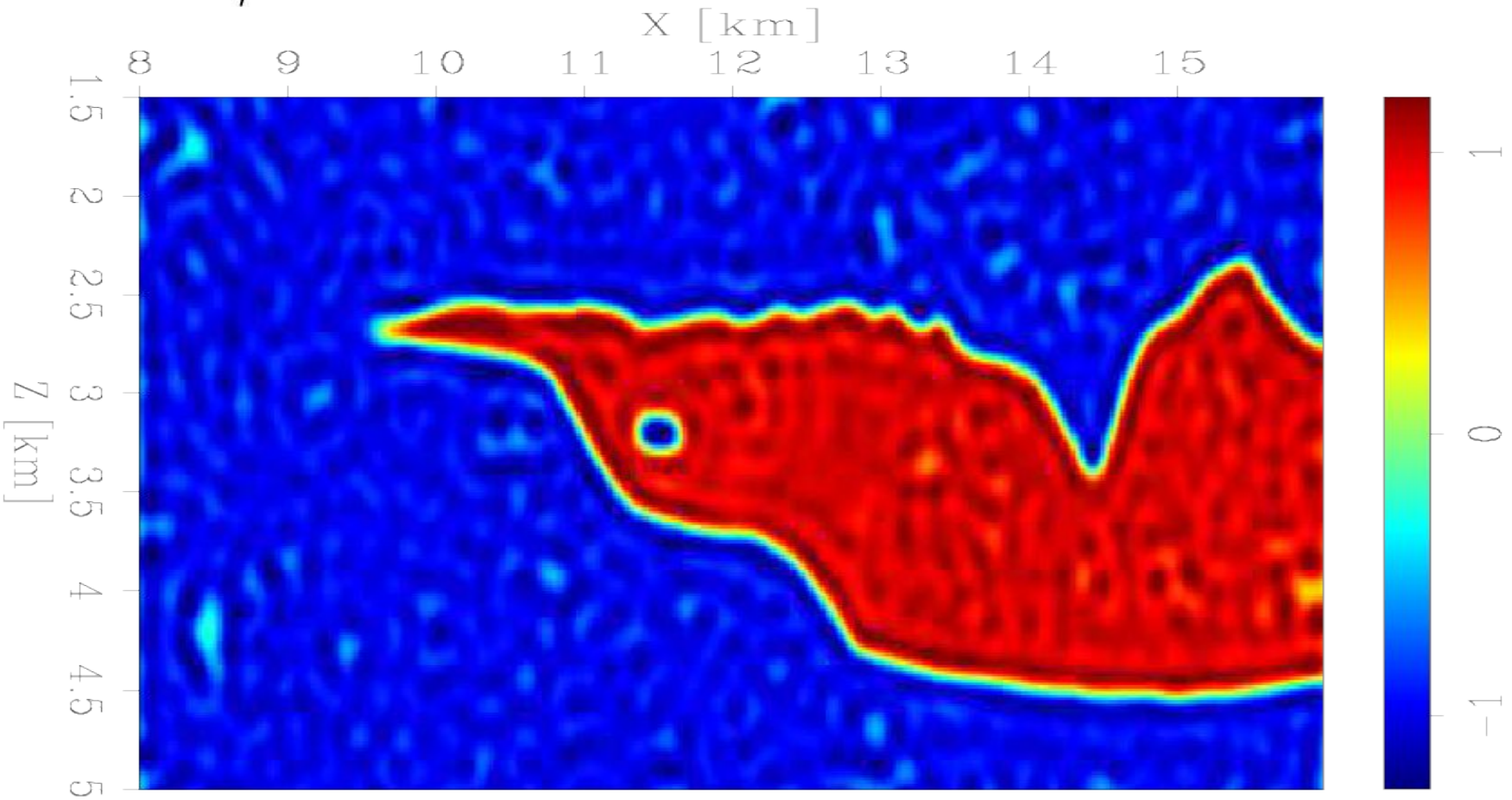
Initial model



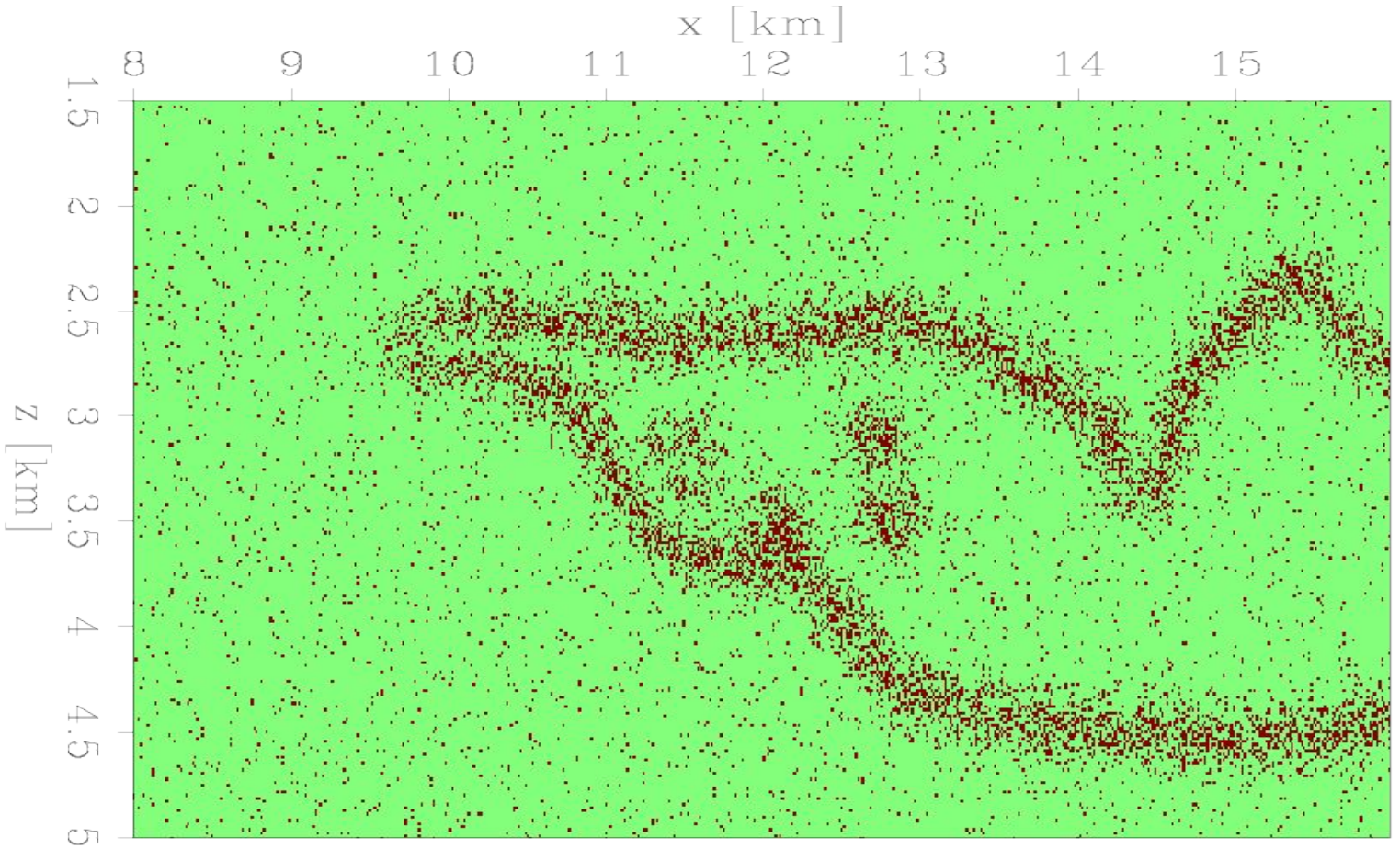
Initial ϕ (guided)



Initial ϕ (unguided)

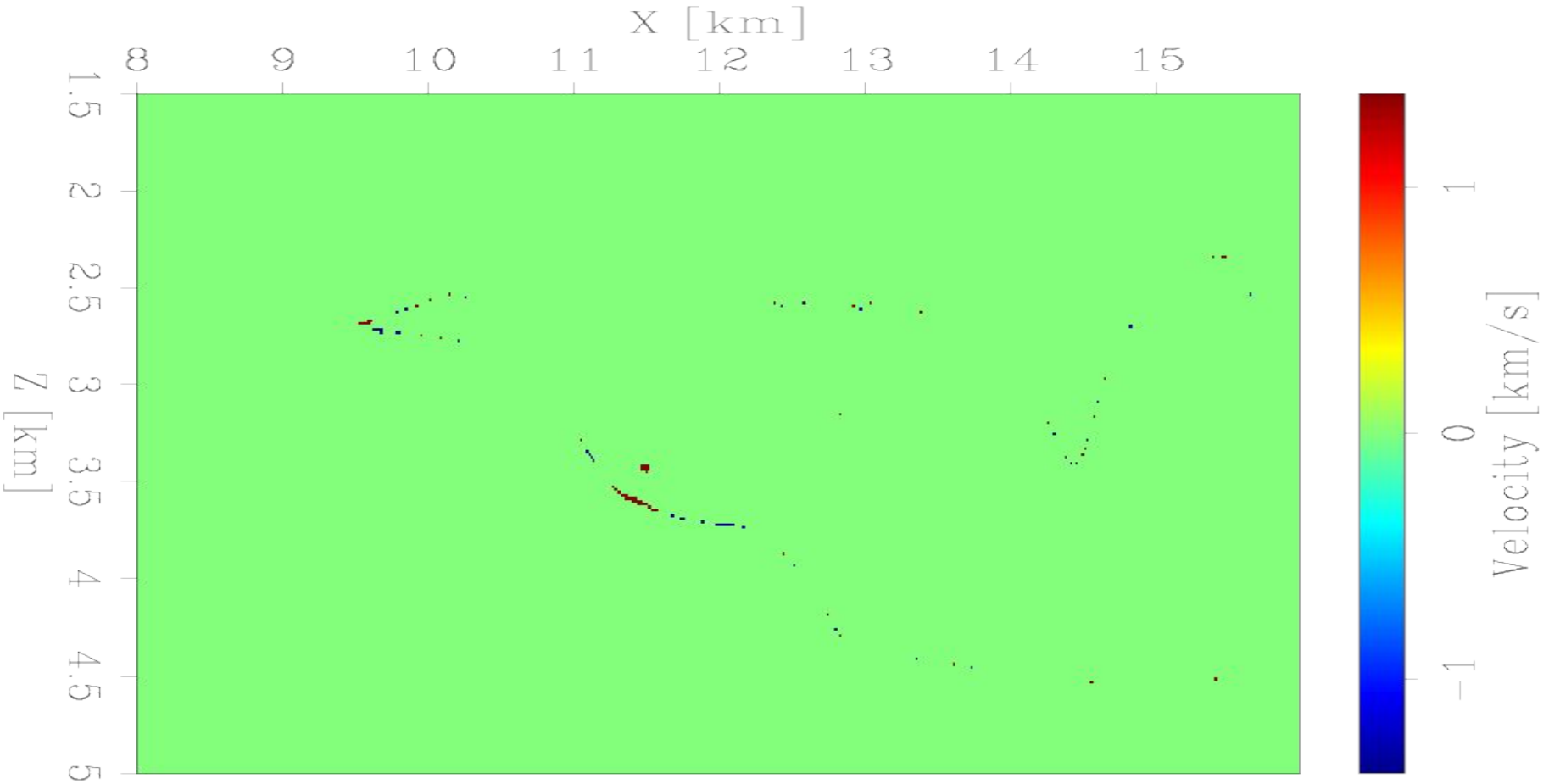


RBF center locations

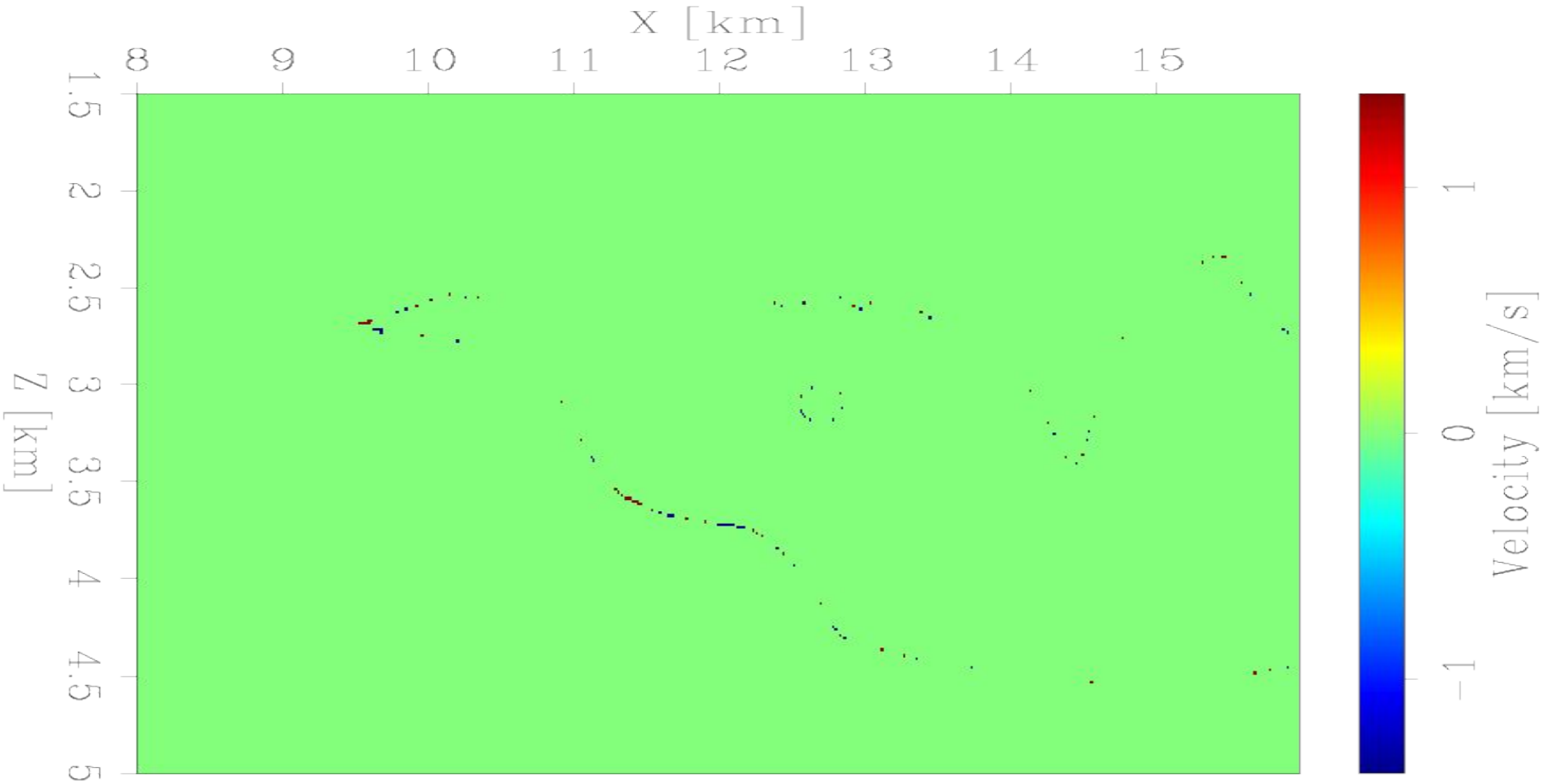


Guided inversion

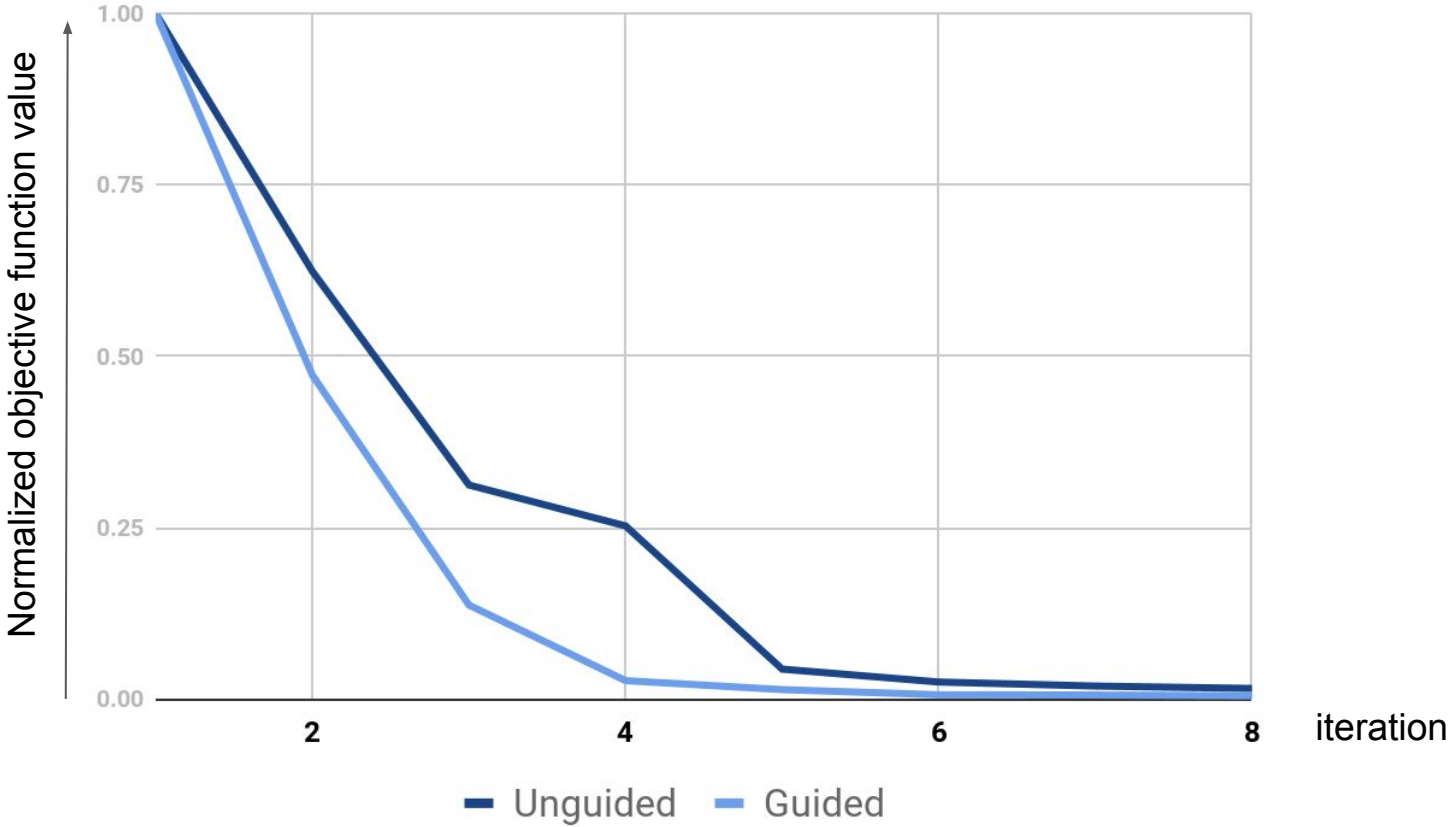
$(m_{true} - m_{it=8})$



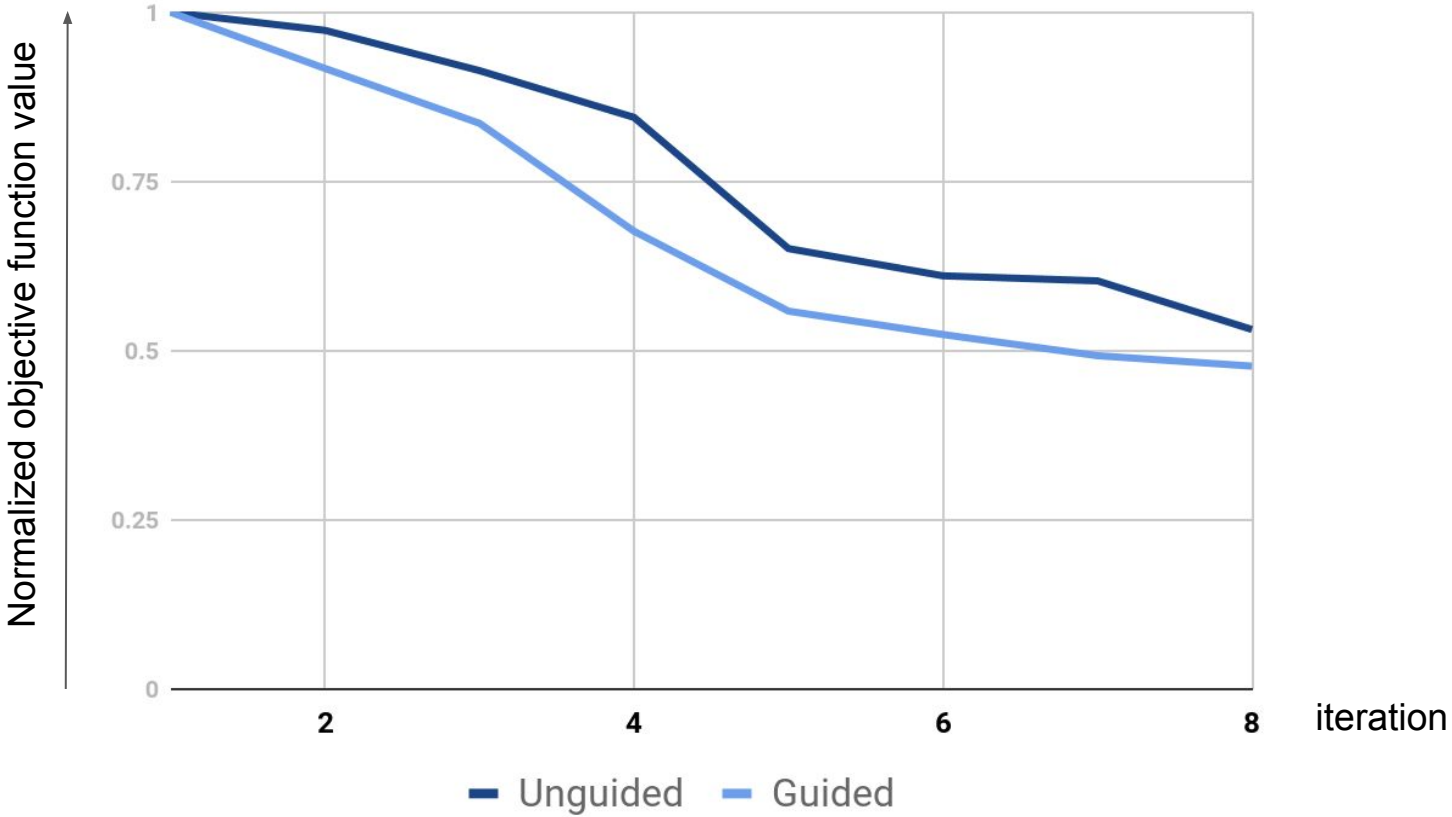
Unguided inversion $(m_{true} - m_{it=8})$



Data residual norm



Velocity model residual norm



Conclusions

- Radial basis functions allow for us to reduce the model space significantly and still perform effective inversion.
- Using interpreter guidance on the implicit surface allows us to elegantly include interpreter guidance in the inversion.
- Despite having “bad” interpreter guidance, the data still guided our inversion to the correct answer more quickly than in the unguided case.

Acknowledgments

Shell Exploration & Production Company, Biondo Biondi & Robert Clapp