

# **Texture Based Classification Of Seismic Image Patches Using Topological Data Analysis**

Rahul Sarkar

Institute for Computational and Mathematical Engineering (ICME)  
Stanford University

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This is joint work with **Bradley J. Nelson**  
(5<sup>th</sup> year Ph.D. student, ICME)

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**Oral Presentation Details**

Abstract Number 640

Deep Learning and Data Analytics – Methods and Applications I

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9:20 AM - 9:45 AM

Room 11

# Abbreviations

The following abbreviations will appear in this talk in various places.

**TDA:** Topological Data Analysis

**PH:** Persistent Homology

I will explain them in this talk.

**ML:** Machine Learning

**SVM:** Support Vector Machines

**RF:** Random Forest

**NN:** Neural Network

**CNN:** Convolutional Neural Network

These are machine learning specific terminologies. I'll assume working knowledge of these methods.

# TDA applications in geosciences / oil & gas

- “Topological data analysis and machine learning for recognizing atmospheric river patterns in large climate datasets”, Muszynski et al., 2019, Geoscientific Model Development
  - “Topological Data Analysis of Oil and Gas Petrophysical Data”, Andrea Cortis, 2015, Ayasdi
  - “Topological Data Analysis to Solve Big Data Problem in Reservoir Engineering: Application to Inverted 4D Seismic Data”, Alfaleh et al., 2015, SPE Annual Technical Conference and Exhibition
- 
- The first first paper uses the *same topological technique as us (persistent homology)*, but in a very different way.
  - The last two papers use a *different topological technique (mapper)* for exploratory data analysis.

# Our contribution & modest claim

- This is quite possibly the first application of TDA based methods that use **persistent homology** for a seismic imaging application.

## More generally...

- This is quite possibly the first application of TDA based methods that use **persistent homology** for a problem relevant to the oil and gas industry.

# Seismic textures

- In a seismic image, different lithologies often have very different “visual appearances”.
- For example, a salt bodies appear different from sedimentary sections.
- The trained human eye of seismic interpreters can easily detect these differences.

## **Seismic interpreter’s job (simplistic viewpoint)**

Segment seismic images based on a combination of

- Seismic texture
- Historical memory
- Geological knowledge

# Related work

## Pre-ML age

- **Seismic attributes:** Developing attributes for easier seismic interpretation.

## Post-ML age

- **Seismic texture based ML:** Semi to full automation of seismic interpretation, some directly using textural attributes.

## Most relevant

- **Image segmentation based on seismic patch classification:** Chevitarese et al. (2018)<sup>‡</sup> perform seismic image segmentation using 2D patches, and CNNs.

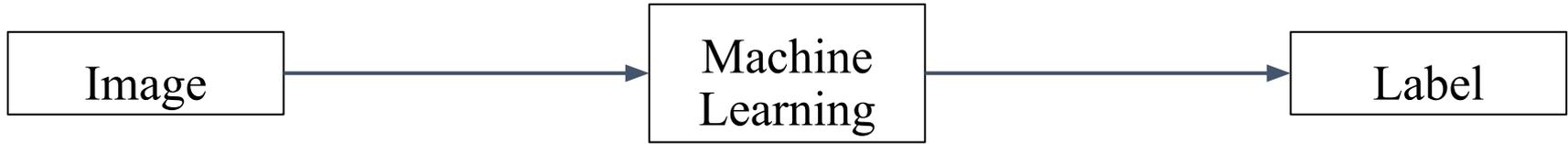
<sup>‡</sup> Chevitarese, D.S., Szwarcman, D., Brazil, E.V. and Zadrozny, B, “Efficient Classification of Seismic Textures”, 2018, International Joint Conference on Neural Networks, IEEE, 1–8.

# Seismic texture classification

## What we want



## A popular strategy



## Our roadmap



# ML challenges — texture classification

## Machine learning with images

- Treating each pixel as a feature is not well-suited for classification tasks.
- Relationship between neighboring pixels matter.
- Most existing methods use derived features (either hand-crafted or learned).

## Challenges of texture classification

- Areas with similar “look and feel”. This can be hard to quantify.  
(Think: I know it when I see it, but can’t describe exactly what I’m seeing.)
- Repetitive / recurrent (but not necessarily periodic).
- Scale, translation, rotation, deformation invariant.
- What kind of features can capture these properties?

# Why topology?

## Features of “algebraic topology”

- Study of topological spaces up to homotopy equivalence (continuous deformation).
- Identifies quantities that are **scale**, **translation**, **rotation**, and **deformation** invariant.

## Topological data analysis

- Tools to understand topology in data.
- Turns topological information into features (real numbers), that computers can process.
- Adapts tools from algebraic topology to study discrete point cloud data.



**Continuous deformation of a coffee mug to a donut**

# Simplicial Complex

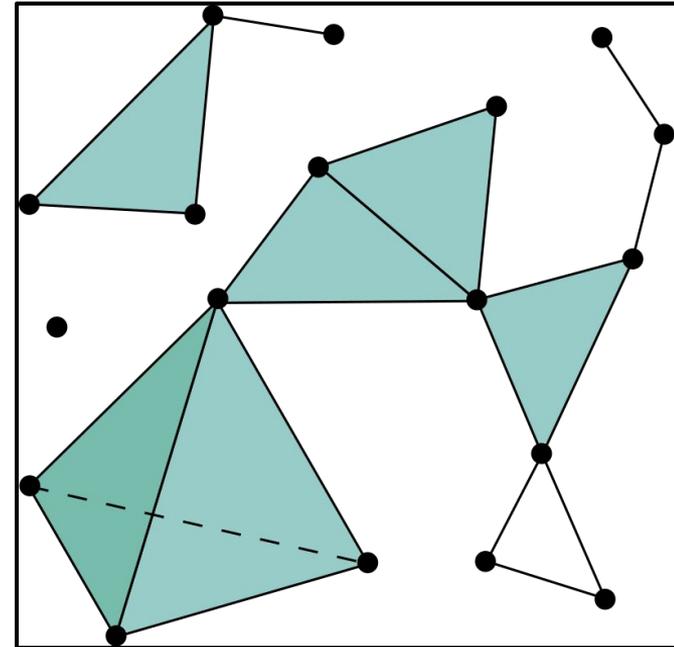
The key topological object (relevant to our work) is a **simplicial complex**. Abstractly this is a triangulation of a topological space.

## Definition of a simplicial complex

A set of simplices\* (points, lines, triangles, and higher dimensional objects) that satisfy the following two properties:

- Every face of a simplex is also a simplex.
- Intersection of any two simplices is a face of each simplex.

**A simplicial complex**



Source: Wikipedia

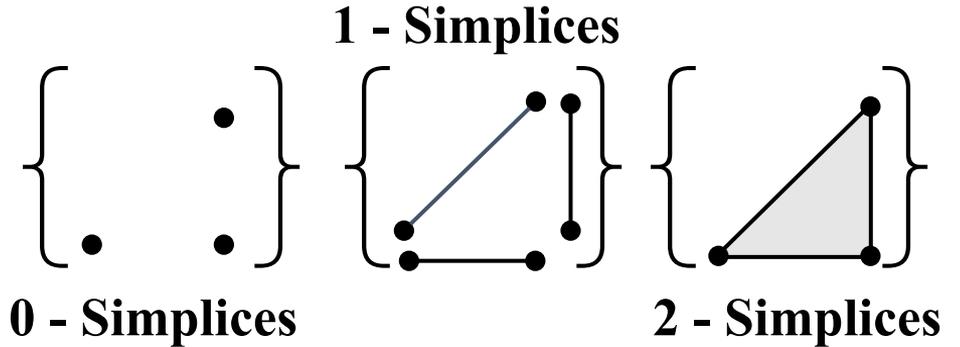
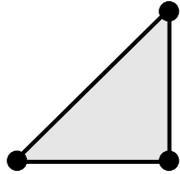
\* “Simplices” is the plural of the word “simplex”.

# Simplices of a simplicial complex

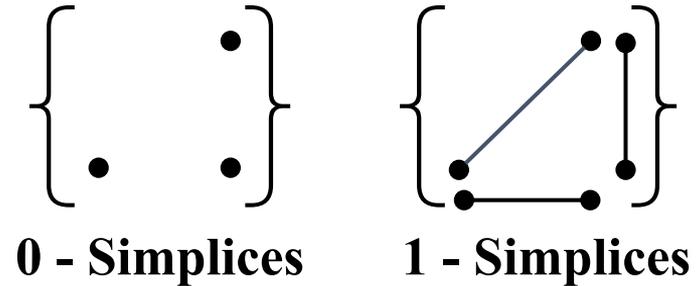
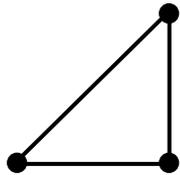
**Topological space**

**Simplicial complex**

**Filled triangle**



**Triangle with a hole**



# Homology of a simplicial complex

Consider formal linear combinations of vertices / edges / triangles in a simplicial complex  $X$  of dimension 2. This produces a set of vector spaces  $C_k(X)$  ( $k = 0$  for vertices,  $k = 1$  for edges...). There are linear boundary maps  $\partial_k : C_k(X) \rightarrow C_{k-1}(X)$

$$\partial(\bullet \longrightarrow \circ) = (\circ) - (\bullet)$$

$$\partial\left(\begin{array}{c} \bullet \\ \diagdown \quad \diagup \\ \bullet \quad \circ \end{array}\right) = \bullet \longrightarrow \circ + \begin{array}{c} \bullet \\ \diagdown \\ \circ \end{array} + \begin{array}{c} \bullet \\ \diagup \\ \circ \end{array} = \begin{array}{c} \bullet \\ \diagdown \quad \diagup \\ \bullet \quad \circ \end{array}$$

with the property that  $\partial \circ \partial = 0$ .

The  $k^{\text{th}}$  **homology group**, and the  $k^{\text{th}}$  **Betti number** are defined as

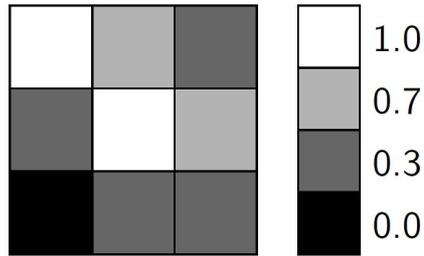
$$H_k(X) = \ker(\partial_k) / \text{img}(\partial_{k+1}), \quad \beta_k = \dim H_k(X).$$

- $\beta_0$  counts clusters that are not connected (called **connected components**).
- $\beta_1$  counts cycles that are not boundaries (called **holes**).

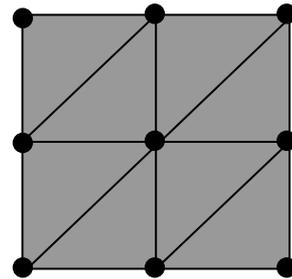
# Turning an image into a topological space

One way to do this is to form a simplicial complex as follows:

- Pixels become points in the space
- Adjacent pixels are connected by an edge
- Diagonal edges added by **Freudenthal triangulation**
- 3 adjacent pixels are spanned by a triangle



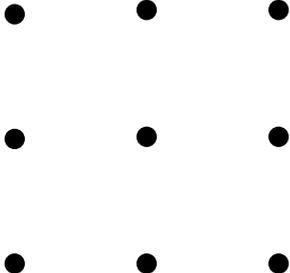
**3 x 3 image**



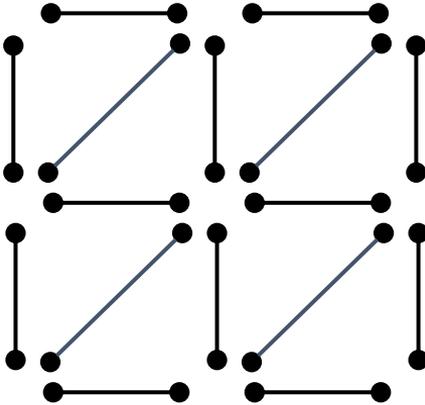
**Freudenthal triangulation**

# Resulting simplicial complex

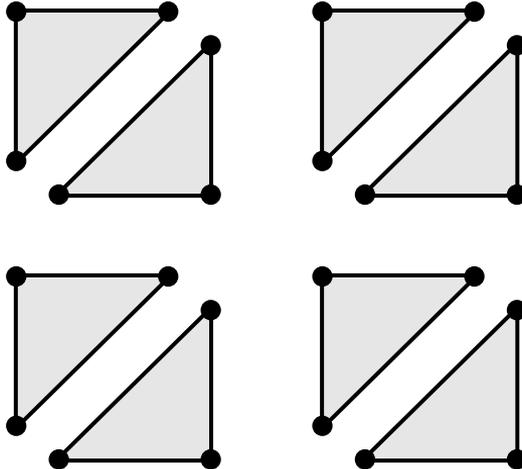
**0 - Simplices**



**1 - Simplices**

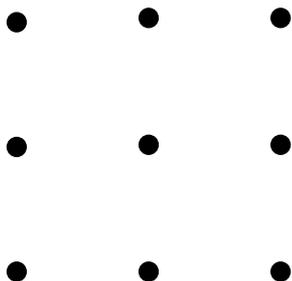


**2 - Simplices**

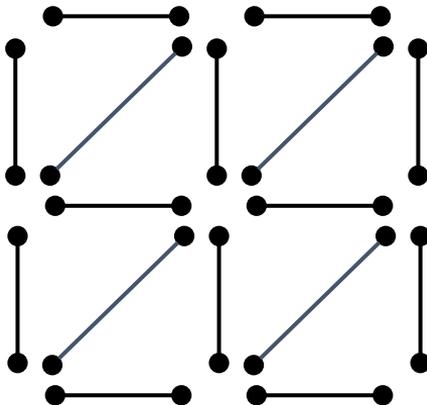


# Need for filtered topological spaces

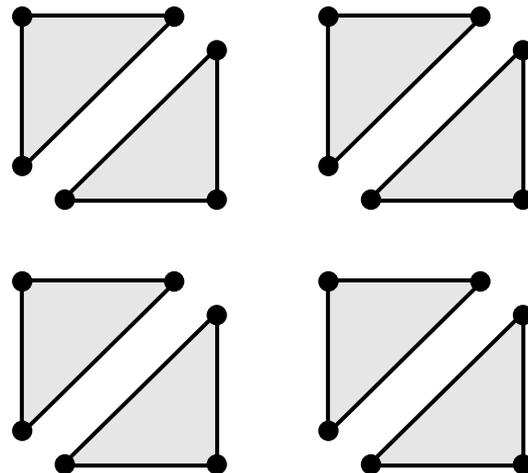
## 0 - Simplices



## 1 - Simplices



## 2 - Simplices

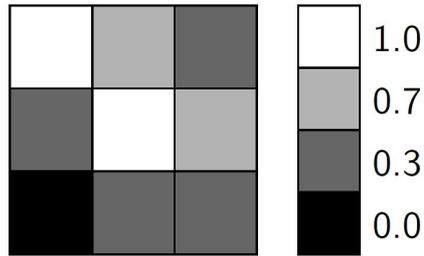


**Problem:** Topological spaces created from all pixels in the image always generate exactly the same simplicial complex — useless for classification.

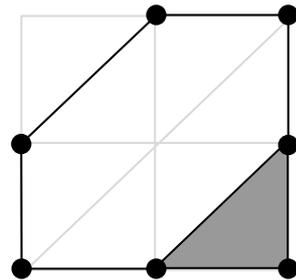
# Filtered topological spaces

A more interesting topological space:

- Choose some pixel value  $w$ .
- Only points with pixel values  $\leq w$  are used.
- Only edges with both endpoints are included.
- Only triangles with boundary edges are included.



**3 x 3 image**



**Topological space at  $w = 0.7$**

# Filtration and persistence

## Key ideas

- Create a sequence of nested topological spaces.
- Track homology changes across the topological spaces.
- Turn this information into quantifiable numbers.

## Nested topological spaces or Filtration

We use a *sublevel set filtration*.

- Vary pixel value  $w$  from minimum to maximum pixel value.
- For each  $w$ , we construct a filtered topological space  $X_w$ .
- Property:  $u \leq w \Rightarrow X_u \subseteq X_w$ .

# Persistent homology

**Persistent homology** is the tool that quantifies how homology changes across a filtration.

**Input:** A filtration  $\{X_w\}_w$ .

**Output:** A collection of pairs of real numbers for each homology dimension  $k$ , calculated as

$$PH_k(\{X_w\}_w) = \{(b_j, d_j)\}_j.$$

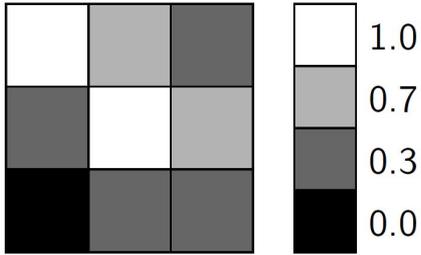
These are called **birth-death pairs**, and track how homology changes over the filtration.

## Properties:

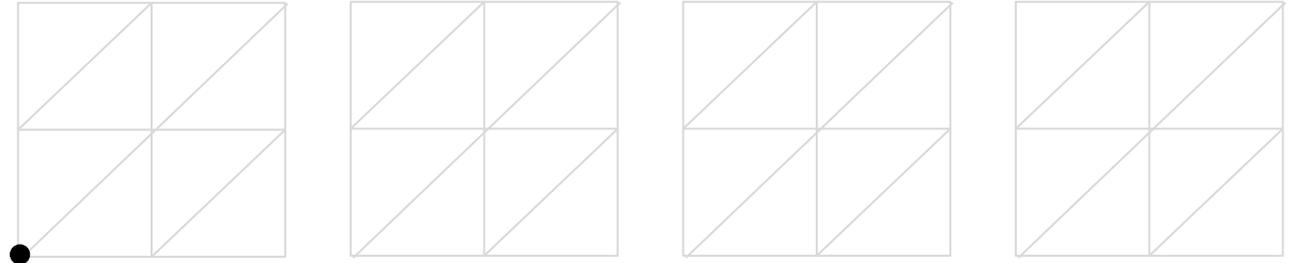
- Homotopy invariant (deformation, rotation, translation).
- Stable to perturbations of pixel values.

# Example of how a filtration is built

## Example Image



## Corresponding Filtration

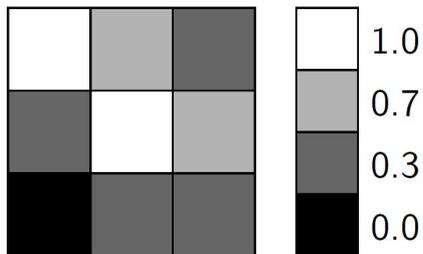


$$w = 0$$
$$\beta_0 = 1, \beta_1 = 0$$

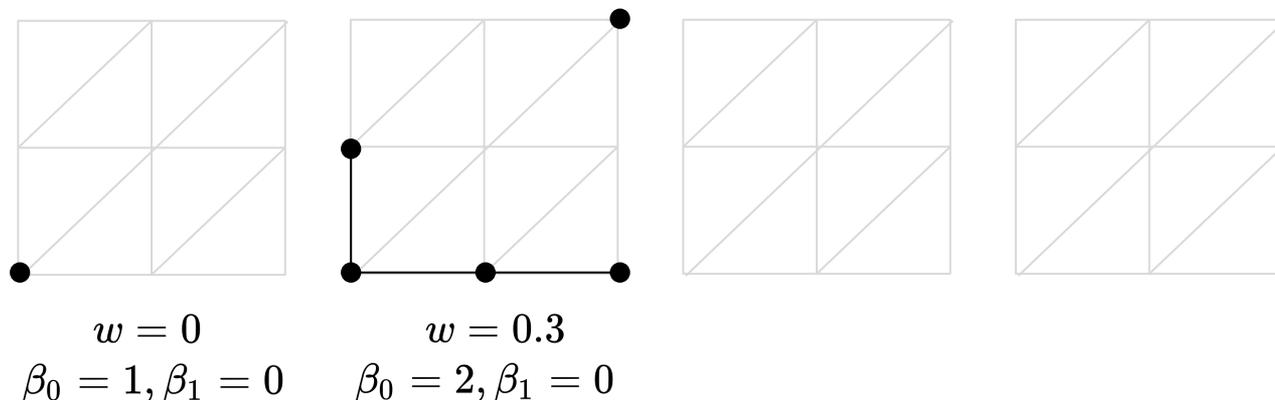
At  $w = 0$ , a single point appears, and  $H_0$  homology is born.

# Example of how a filtration is built

## Example Image



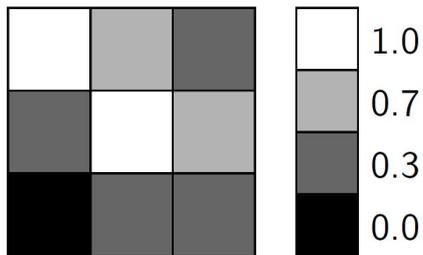
## Corresponding Filtration



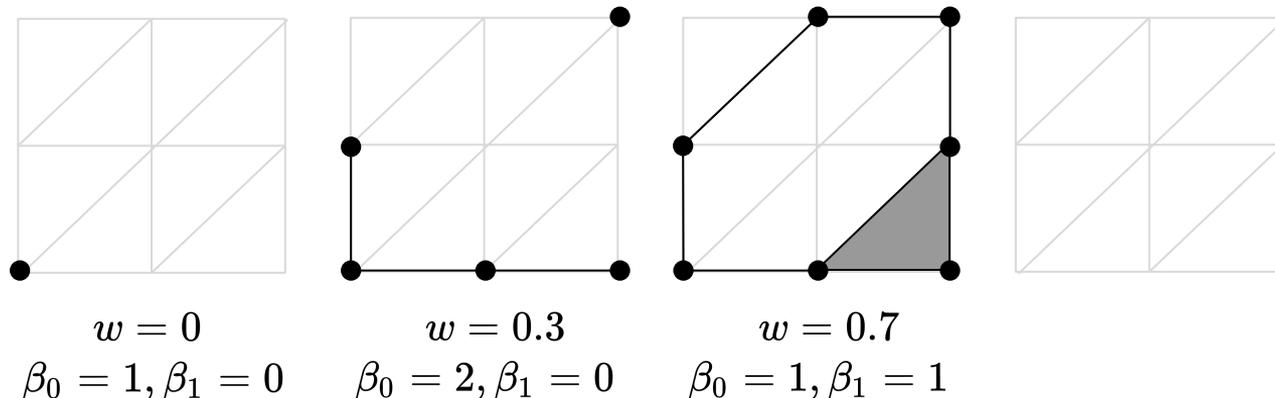
At  $w = 0.3$ , several points connect to the first point, and a new component emerges.  $H_0$  homology is born one more time.

# Example of how a filtration is built

## Example Image



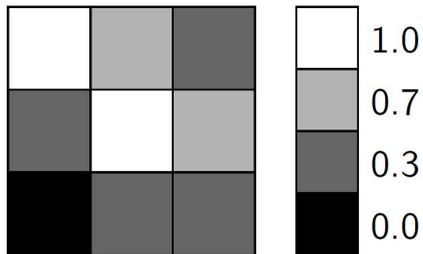
## Corresponding Filtration



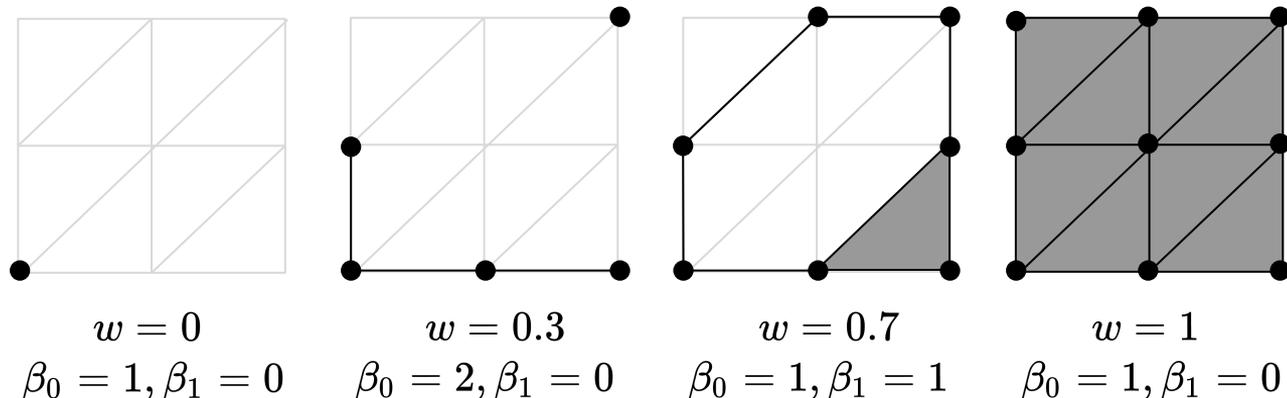
At  $w = 0.7$ , the two components join, and a hole appears. We also see our first triangle. So  $H_0$  homology has died, while  $H_1$  homology is born.

# Example of how a filtration is built

## Example Image



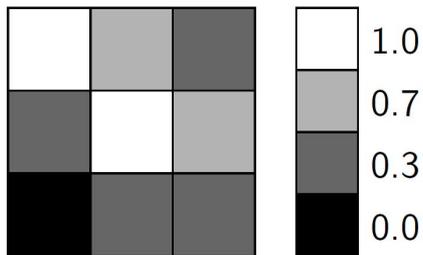
## Corresponding Filtration



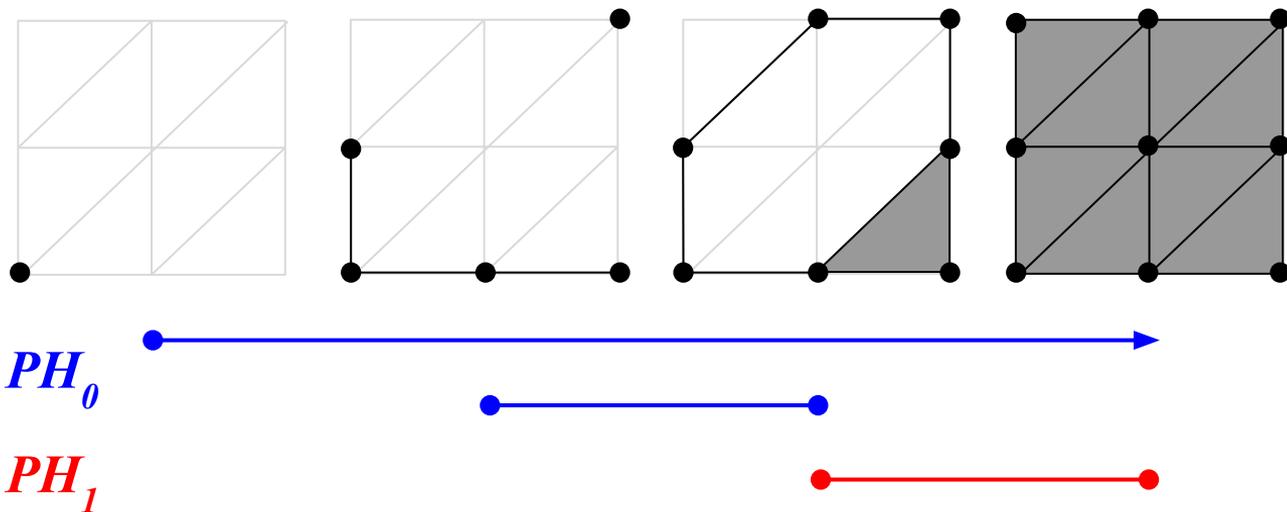
At  $w = 1$ , all points are now present, and all edges and triangles fill in the space. The hole has now disappeared, and so  $H_1$  homology has died.

# Example of how a filtration is built

## Example Image



## Corresponding Filtration



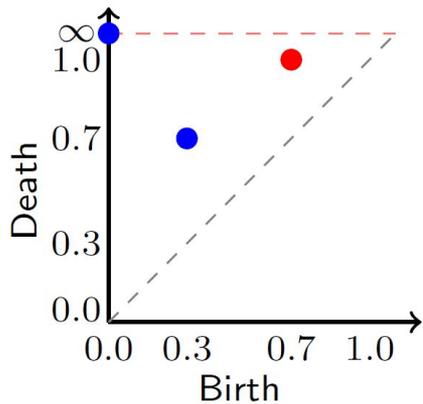
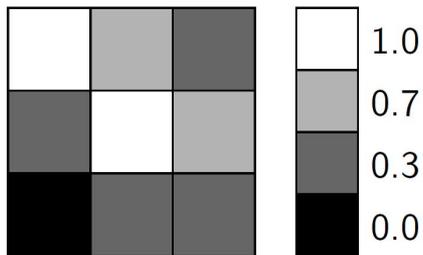
### Persistence Barcode:

Information about how components appear and merge is encoded in  $PH_0$ .

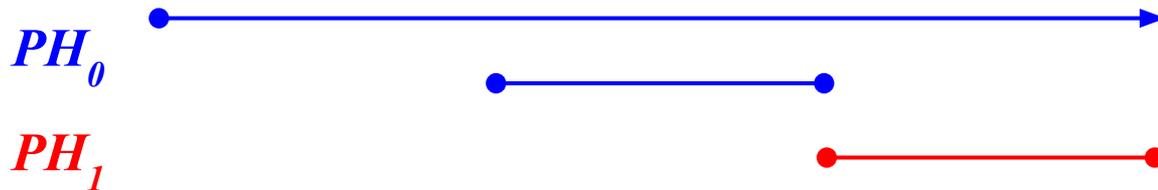
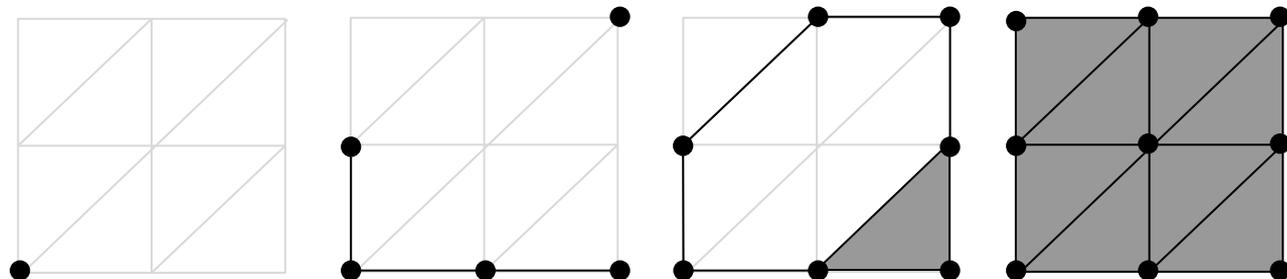
Information about how 1D holes appear and fill is encoded in  $PH_1$ .

# Example of how a filtration is built

## Example Image



## Corresponding Filtration



### Persistence Diagram:

The start and endpoints of the barcode are plotted in the plane.

Each point is referred to as a **birth-death** pair.

# Applications on a real 2D dataset

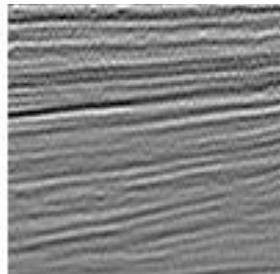
For the rest of this talk we will use the LANDMASS<sup>†</sup> dataset to demonstrate the workflow and our results. This is a publicly available dataset of **two sets** of labeled 2D seismic image patches, each with **4 classes**.

	LANDMASS-1	LANDMASS-2
Image Size (pixels)	99 x 99	150 x 300
<b>Class Names</b>	<b>Number of Images</b>	<b>Number of Images</b>
1. Horizons	9385	1000
2. Chaotic Horizons	5140	1000
3. Fault Patches	1251	1000
4. Salt Domes	1891	1000

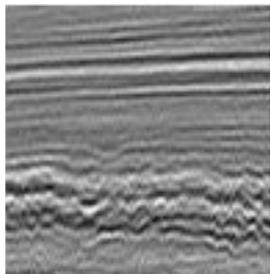
<sup>†</sup>[Alaudah, Y., Wang, Z., Long, Z. and AlRegib, G. \[2015\] LANDMASS Seismic Dataset.](#)

# Sample images (images not to scale)

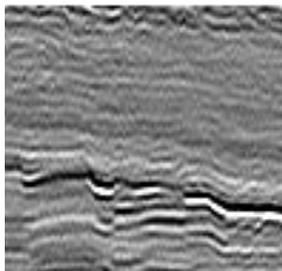
## LANDMASS-1



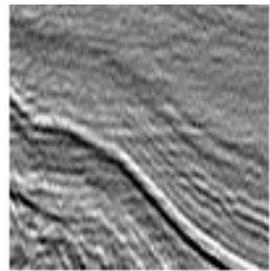
Horizons



Chaotic Horizons

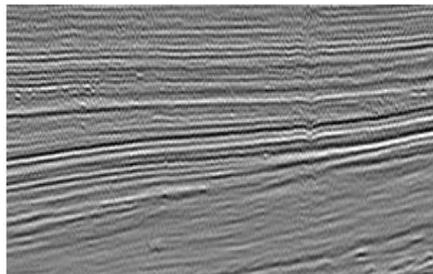


Fault Patches

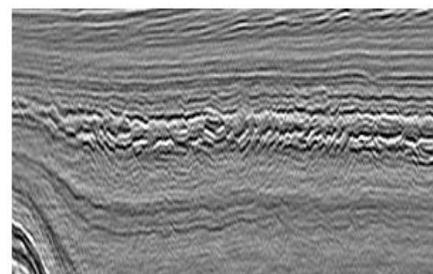


Salt Domes

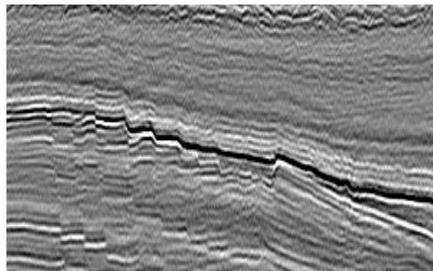
## LANDMASS-2



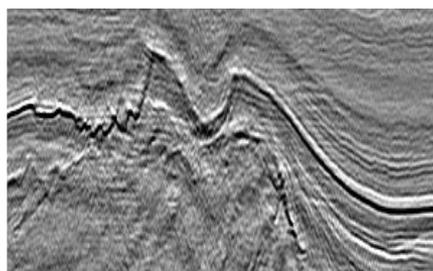
Horizons



Chaotic Horizons



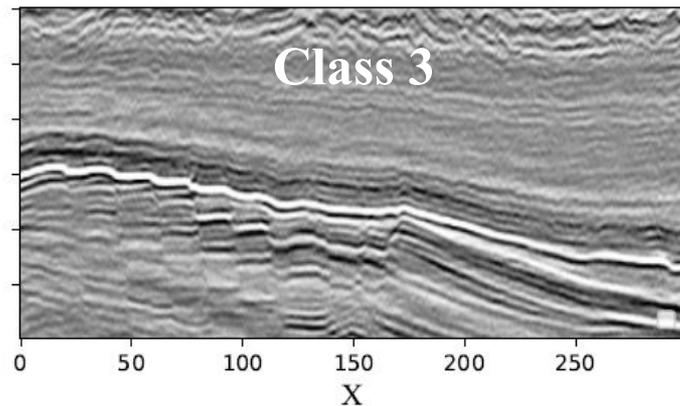
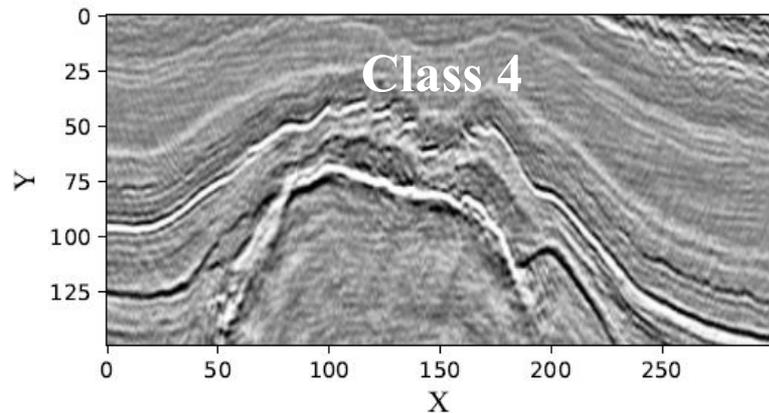
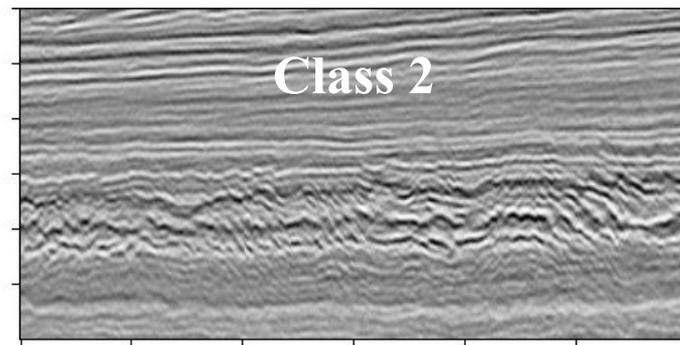
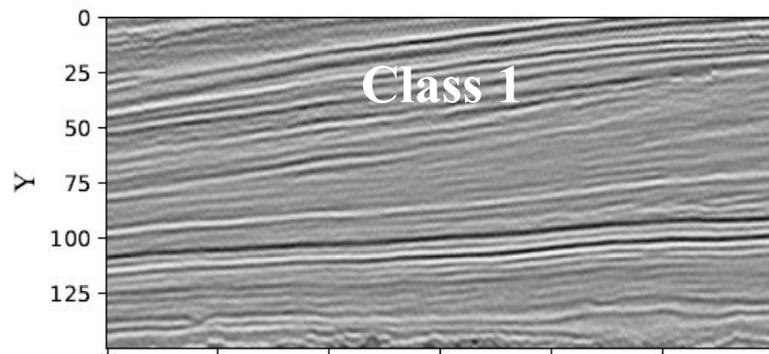
Fault Patches



Salt Domes

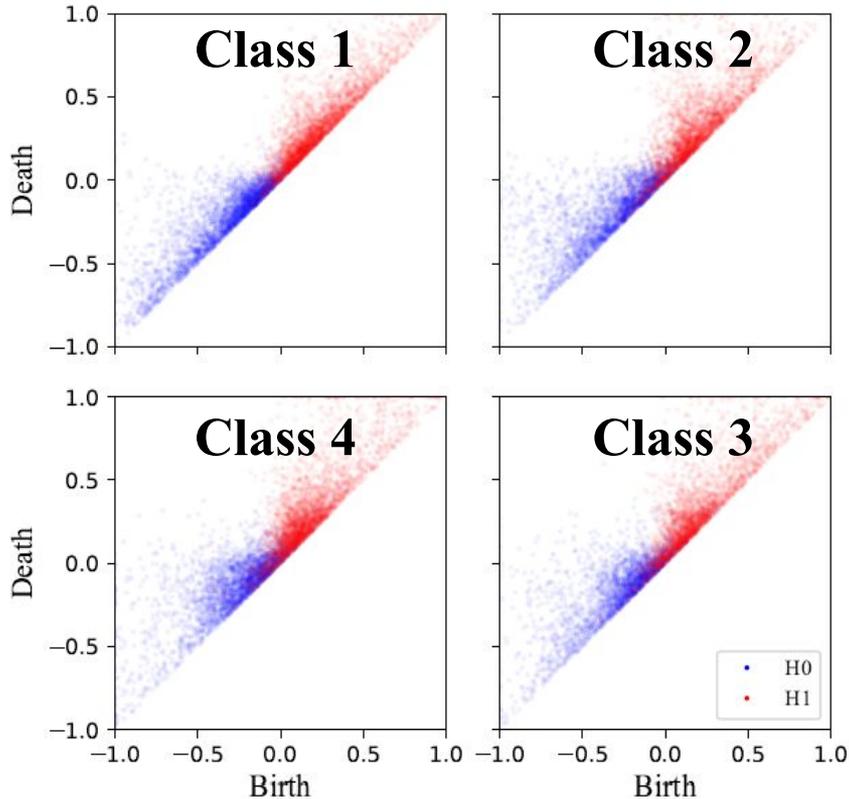
# Persistence diagram results (LANDMASS-2)

## Sample Images



# Persistence diagram results (LANDMASS-2)

## Persistence Diagrams



Subtle differences between the persistence diagrams.

To train a classifier we need:

- Statistically significant **intra-class similarity**.
- Statistically significant **inter-class dissimilarity**.

Currently working on how to make this more precise, and generate metrics.

# Need for featurization of persistence diagrams

We want to use a machine learning (ML) approach for training a classifier based on the persistence diagrams.

So far:



## Key points about the persistence diagrams:

- Every image produces a different number of birth-death pairs.
- We want a *standard number of features* for a ML workflow.

# Polynomial featurization

One approach is based on polynomial functions<sup>†</sup>, which we adopt in our work:

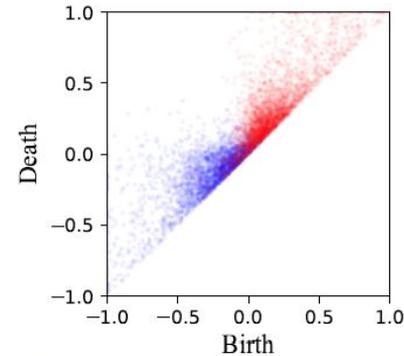
$$p(\alpha; \{b_i, d_i\}_{i \in J}) = \frac{1}{|J|} \sum_{i \in J} \sum_{j,k} \alpha_{j,k} (d_i - b_i)^j (d_i + b_i)^k.$$

For both homology dimensions 0 and 1 we choose:

$$\alpha_{j,k} = \delta_{j=j_0, k=k_0}$$

where  $(j_0, k_0) \in \{0, 1, 2, 3\}^2 - \{(0, 0)\}$ .

This gives us a total of  $15 \times 2 = 30$  features per persistence diagram.



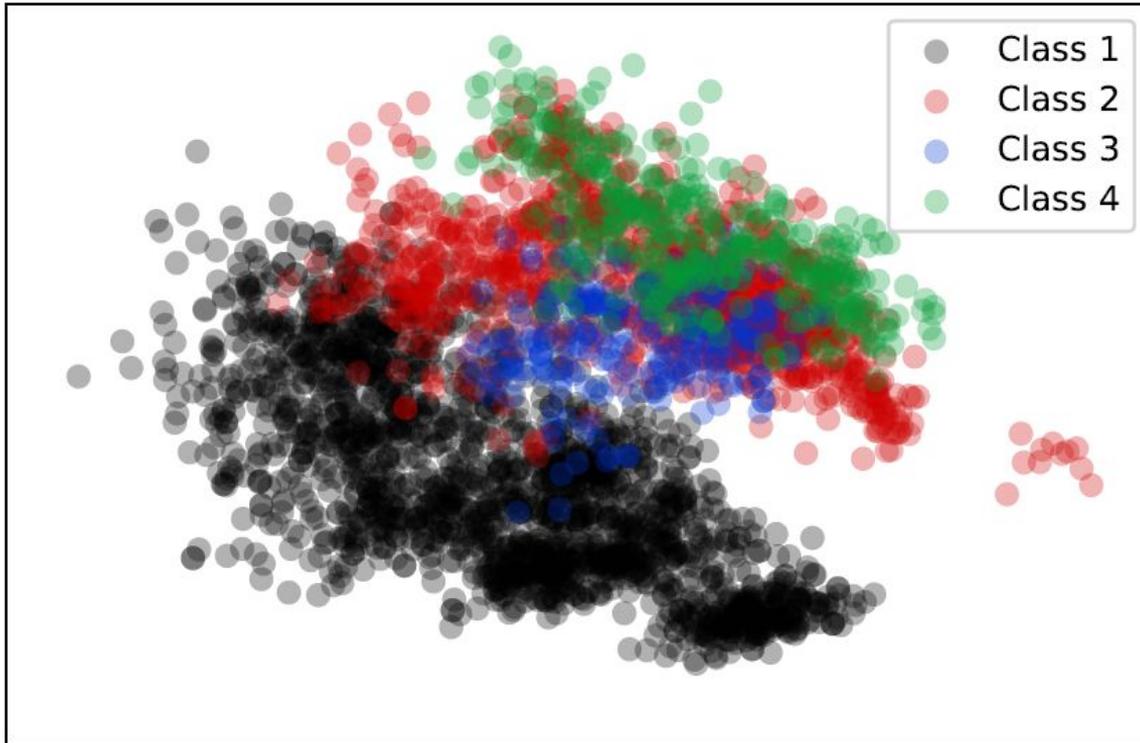
**Featurization**



<sup>†</sup> A. Adcock, E. Carlsson, G. Carlsson. The ring of algebraic functions on persistence barcodes. *Homology, Homotopy and Applications*. 18(1) 2016.

# LANDMASS-1 features

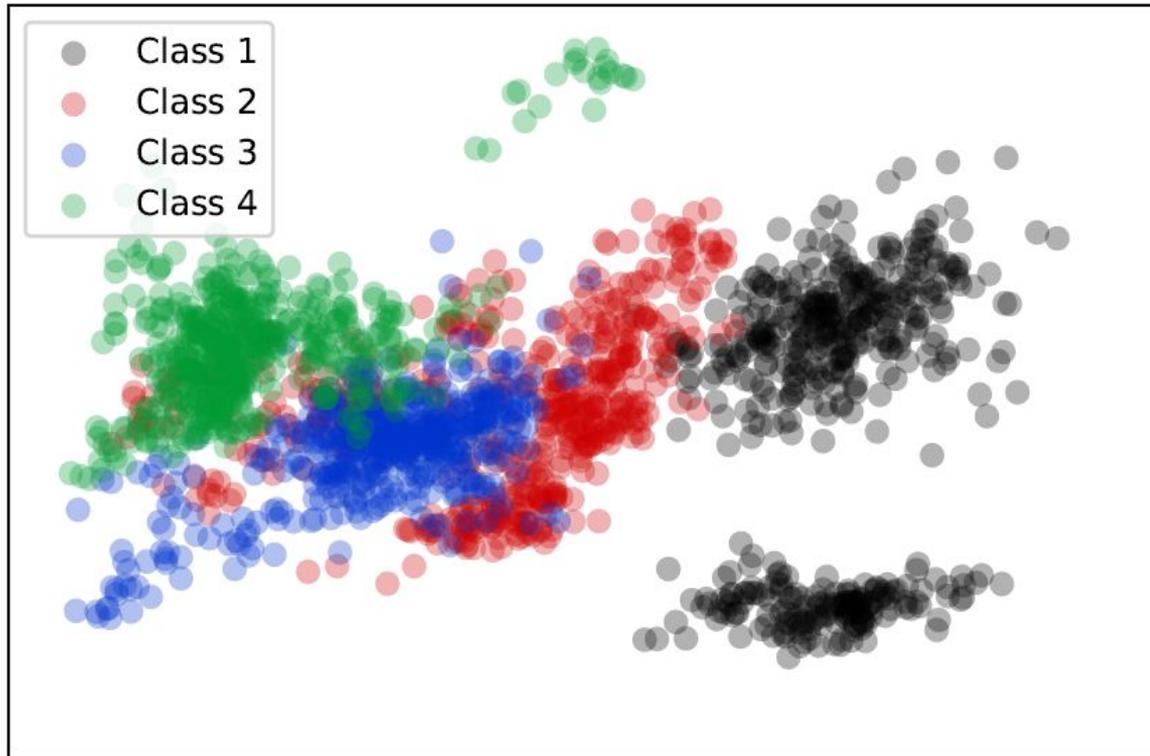
Projection of polynomial features into top two principal components. Each point is an image in the LANDMASS-1 dataset.



- Class 1 separates nicely from the other classes.
- With 2 principal components, classes are not well separated.
- More components are needed.

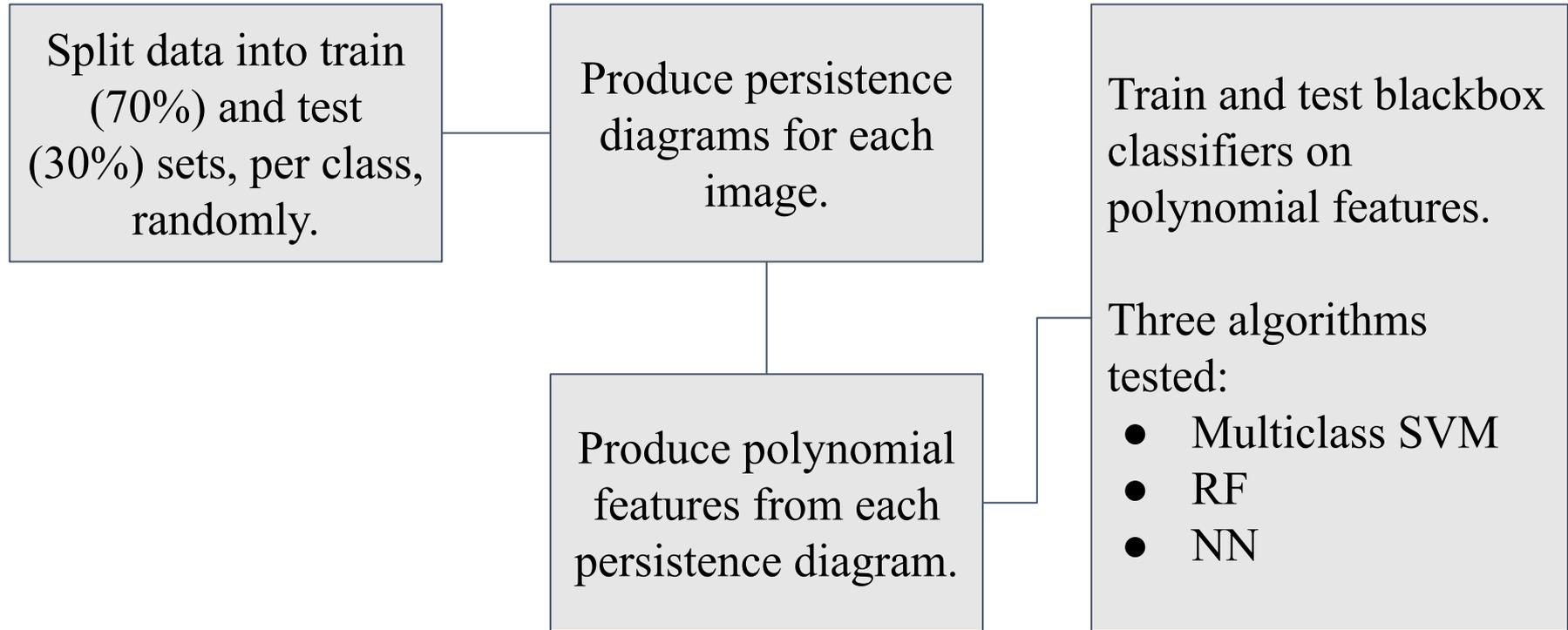
# LANDMASS-2 features

Projection of polynomial features into top two principal components. Each point is an image in the LANDMASS-2 dataset.

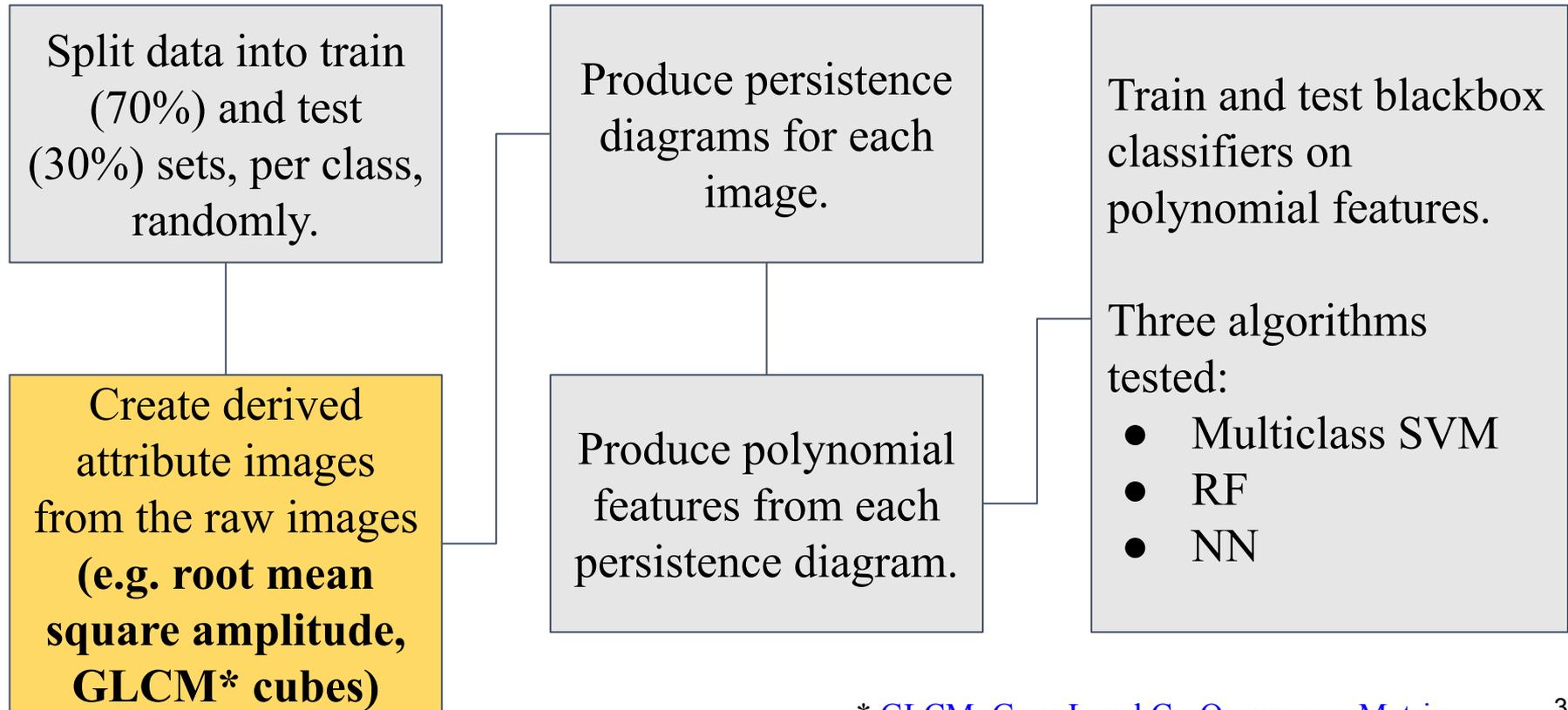


- Classes reasonably well separated with just top 2 principal components.
- Equal class sizes help classification.

# Machine learning workflow



# Derived attribute image based ML workflow



\* GLCM: Gray-Level Co-Occurrence Matrix

# Classification results: RF classifier

Attribute	RF Accuracy
Raw	99.9 / 98.6 / 95.2 / 93.3
Image	100.0 / 98.0 / 100.0 / 96.3
GLCM	99.9 / 97.9 / 82.1 / 93.3
Mean	100.0 / 97.0 / 97.3 / 91.7
RMS	99.3 / 96.1 / 88.0 / 82.0
Amplitude	99.7 / 96.0 / 96.0 / 91.7
GLCM	99.3 / 94.9 / 80.8 / 91.2
Correlation	99.7 / 93.7 / 92.0 / 97.0
GLCM	98.5 / 95.7 / 96.3 / 74.0
Variance	99.0 / 95.3 / 96.7 / 89.7

Class 1 / Class 2 / Class 3 / Class 4

Top Row: **LANDMASS-1**

Bottom Row: **LANDMASS-2**

Classification accuracy of **raw image, and best 4 attributes** with respect to RF classifier.

➤ Nonlinear classifiers do much better.

# Classification results: NN classifier

Attribute	NN Accuracy
Raw	100.0 / 99.6 / 99.7 / 98.4
Image	100.0 / 100.0 / 99.0 / 95.0
GLCM	100.0 / 97.8 / 92.8 / 97.0
Mean	100.0 / 96.0 / 95.7 / 96.3
RMS	99.5 / 99.1 / 96.3 / 91.5
Amplitude	99.7 / 99.0 / 93.7 / 91.3
GLCM	99.8 / 93.6 / 87.7 / 96.7
Correlation	100.0 / 95.7 / 93.7 / 98.3
GLCM	99.3 / 98.3 / 98.1 / 87.3
Variance	99.7 / 99.0 / 99.3 / 95.0

Class 1 / Class 2 / Class 3 / Class 4

Top Row: **LANDMASS-1**

Bottom Row: **LANDMASS-2**

Classification accuracy of **raw image, and best 4 attributes** with respect to RF classifier.

➤ Nonlinear classifiers do much better.

# Classification results: Multiclass SVM classifier

Attribute	SVM Accuracy			
Raw	99.8	75.2	0.0	0.0
Image	100.0	55.0	88.3	74.3
GLCM	100.0	18.6	34.1	29.3
Mean	62.7	19.0	4.0	100.0
RMS	100.0	1.0	0.0	0.0
Amplitude	74.7	85.7	71.3	61.7
GLCM	100.0	0.0	0.0	0.0
Correlation	64.7	32.0	89.3	32.3
GLCM	96.6	94.1	92.8	67.7
Variance	97.3	93.3	91.7	87.0

Class 1 / Class 2 / Class 3 / Class 4

Top Row: **LANDMASS-1**

Bottom Row: **LANDMASS-2**

Classification accuracy of **raw image, and best 4 attributes** with respect to RF classifier.

- Linear classifiers like SVM perform poorly.
- Need nonlinear decision boundaries.

# Conclusions

- TDA derived features perform well for texture classification in seismic images.
- Nonlinear decision boundary classifiers are necessary for good classification accuracy.
- These features could augment existing ML workflows for similar tasks.

## **Current and future work:**

- Develop metrics to better quantify differences in persistence diagrams.
- How many polynomial features do we need?
- Extend the workflow to 3D labeled data.

# Software used in this study

- **GUDHI**<sup>[1]</sup> in Python — persistent homology calculations.
- **Scikit-learn**<sup>[2]</sup> in Python — SVM and RF classifiers.
- **Tensorflow**<sup>[3]</sup> in Python — NN classifier.

[1] C. Maria, “Filtered Complexes, GUDHI User and Reference Manual”, [http://gudhi.gforge.inria.fr/doc/latest/group\\_simplex\\_tree.html](http://gudhi.gforge.inria.fr/doc/latest/group_simplex_tree.html), 2015.

[2] F. Pedregosa et al., “Scikit-learn: Machine Learning in Python”, *Journal of Machine Learning Research* 12, 2011.

[3] M. Abadi et al., “TensorFlow: Large-Scale Machine Learning on Heterogeneous Systems”, Whitepaper, <https://www.tensorflow.org/>, 2015.

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‡ Institute for Computational and Mathematical Engineering, Stanford University

‡ Department of Geophysics, Stanford University

‡ Department of Mathematics, Stanford University

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# Questions

Thank you for listening!

Questions?

If you need more information contact us by email at:

[rsarkar@stanford.edu](mailto:rsarkar@stanford.edu), [bjnelson@stanford.edu](mailto:bjnelson@stanford.edu)