Multiple attenuation using a $t-x$ pattern-based subtraction method

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SUMMARY

We present a new pattern-based method that separates multiples from primaries. This method finds its mathematical foundation in the work conducted by Nemeth (1996) on coherent noise attenuation by least-squares migration. We show that a similar inverse problem can be formulated to attenuate coherent noise in seismic data. In this paper, we use deconvolution with prediction error filters to model the signal and noise vectors in a least-squares sense. This new formulation of the noise separation problem has been tested on 2-D real data and achieves similar results to the Wiener approach. However, we show that the main strength of this new method is its ability to incorporate regularization in the inverse problem in order to decrease the correlation effects between noise and signal.

INTRODUCTION

Pattern-based noise attenuation techniques are known for their ability to remove multiple reflections in the most complex geology. These methods exploit the spatial predictability of the noise and signal with prediction error filters (PEF's). The pattern-based methods contrast with adaptive subtraction techniques, which assume that the primary wavefield has minimum energy (Weglein et al., 1997). Thus, the adaptive subtraction techniques are hampered by the requirement that a source wavelet be calculated (Verschuur et al., 1992). Both pattern-based and adaptive subtraction techniques assume that a multiple model is known in advance. The most popular way to derive the multiple model is using the "Delft approach" (Verschuur et al., 1992) in which the multiple model is calculated via autoconvolution of the recorded wavefield (Figure 1). A well-known problem of the single convolution approach to modeling is that it models the kinematics for every multiple correctly, but does not preserve the relative amplitude of high-order multiples. Brown and Clapp (2000) show that a pattern-based approach can compensate for amplitude errors in the noise modeling.

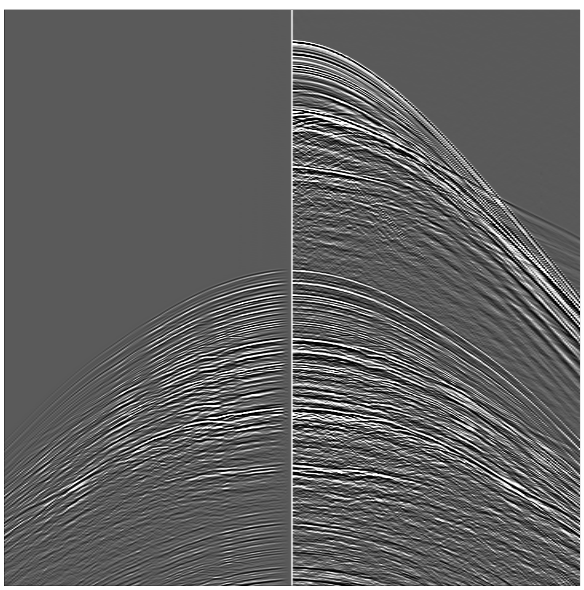

Figure 1: Left: multiple model at one shot location using Delft approach. Right: Shot record at the same location. The kinematics of all multiples are correctly modeled. The relative amplitude of high-order multiples is not preserved.

The recent pattern-based techniques in the literature are approximately equivalent to Wiener optimal estimation (Castleman, 1996) since they utilize the PEF to approximate the signal and noise power spectra. For instance, Spitz (2000) uses $f-x$ domain PEF while Brown and Clapp (2000) and Clapp and Brown (2000) use $t-x$ domain PEF.

We present a new method that is not based on the Wiener reconstruction of the signal. Following Nemeth (1996), we use the PEF's as prediction operators as opposed to filtering operators as in the Wiener approach. Nonetheless, this method belongs to the pattern-based type since PEF's are still estimated for the noise separation. Our goal is to show that this new methodology leads to a proper extraction of the multiples and has the potential to overtake the classical Wiener formulation. In the first part of this paper, we review the theoretical developments of both the Wiener-like scheme and the new proposed technique, discussing their differences and similarities. Then, we compare the two strategies on a surface-related multiple attenuation problem in complex geology.

THEORY REVIEW

We present the theoretical basis for both the Wiener method and the new proposed scheme. We show that our new method offers the opportunity to better separate noise from signal using inverse theory.

Wiener method

A constrained least-squares problem using PEF's gives a similar expression for the noise estimation than does the Wiener method. To see this, consider the recorded data to be the simple superposition of "signal", i.e., primary reflections and "noise", i.e., surface-related multiples: $\mathbf{d}=\mathbf{s}+\mathbf{n}$. For the special case of uncorrelated signal and noise, the so-called *Wiener estimator* is a filter, which when applied to the data, yields an optimal (least-squares sense) estimate of the embedded signal (Castleman, 1996). The frequency response of this filter is

$$\mathbf{H}=\frac{\mathbf{P_s}}{\mathbf{P_n}+\mathbf{P_s}}, \tag{1}$$

where $\mathbf{P_s}$ and $\mathbf{P_n}$ are the signal and noise power spectra, respectively.

Abma (1995) solved a constrained least squares problem to separate signal from spatially uncorrelated noise:

$$\begin{array}{rcl} \mathbf{Nn} & \approx & 0 \\ \epsilon\mathbf{Ss} & \approx & 0 \\ \text{subject to} & \leftrightarrow & \mathbf{d}=\mathbf{s}+\mathbf{n} \end{array} \tag{2}$$

where the operators $\mathbf{N}$ and $\mathbf{S}$ represent $t-x$ domain convolution with nonstationary Prediction Error filters (PEF's) which whiten the unknown noise $\mathbf{n}$ and signal $\mathbf{s}$, respectively. ϵ is a Lagrange multiplier. Minimizing the quadratic objective function suggested by equation (2) with respect to $\mathbf{s}$ leads to the following expression for the estimated signal:

$$\hat{\mathbf{s}}=\left(\mathbf{N^TN}+\epsilon^2\mathbf{S^TS}\right)^{-1}\mathbf{N^TN\ d} \tag{3}$$

Multiple attenuation

By construction, the frequency response of a PEF approximates the inverse power spectrum of the data from which it was estimated. Thus we see that the approach of equation (2) is similar to the Wiener reconstruction process. We refer to this approach as a "Wiener-like" method. It has been successfully used by Brown and Clapp (2000) for ground-roll attenuation and by Clapp and Brown (2000) for multiple separation.

Now we give a method that computes the signal PEF needed in equation (2). Spitz (1999) showed that for uncorrelated signal and noise, the signal PEF can be expressed in terms of a PEF, $\mathbf{D}$, estimated from the data $\mathbf{d}$, and a PEF, $\mathbf{N}$, estimated from the noise model:

$$\mathbf{S} = \mathbf{D}\mathbf{N}^{-1}. \tag{4}$$

Equation (4) states that the signal PEF equal the data PEF deconvolved by the noise PEF. Spitz' result applies to one-dimensional PEF's in the $f-x$ domain, but our use of the helix transform (Claerbout, 1998) permits stable inverse filtering with multidimensional $t-x$ domain filters.

Proposed method

Here, we show that the formalism used by Nemeth (1996) can help to better separate correlated noise and signal. But first, we detail the analogies and differences between the Wiener-like and proposed method.

In equation (2), the noise and signal PEF's filter the data components. Now, following Nemeth (1996), the noise and signal nonstationary PEF's predict the data components via a deconvolution as follows:

$$\mathbf{d} = \mathbf{N}^{-\mathbf{1}}\mathbf{m_n} + \mathbf{S}^{-\mathbf{1}}\mathbf{m_s}. \tag{5}$$

We call $\mathbf{m_s}$ the signal model and $\mathbf{m_n}$ the noise model (not to be confused with the multiple model that we use to compute the noise PEF). Clearly, $\mathbf{N}^{-\mathbf{1}}\mathbf{m_n}$ models the noise vector $\mathbf{n}$ and $\mathbf{S}^{-\mathbf{1}}\mathbf{m_s}$ the signal vector $\mathbf{s}$. Because we use PEF's in equation (5), this approach is pattern-based in essence. But now, the PEF's are not used to approximate the power spectra for the Wiener estimation, but rather used to model the data components. Now, we show the opportunities for improved results offered by such a formulation for the signal/noise separation.

Equation 5 can be written in an easier form. With $\mathbf{L_n} = \mathbf{N}^{-\mathbf{1}}$ and $\mathbf{L_s} = \mathbf{S}^{-\mathbf{1}}$, we can define $\mathbf{L} = (\mathbf{L_n}\ \ \mathbf{L_s})$ and $\mathbf{m}' = (\mathbf{m_n}\ \ \mathbf{m_s})$. Hence, the fitting goal becomes

$$\mathbf{0} = \mathbf{L}\mathbf{m} - \mathbf{d}, \tag{6}$$

leading to the familiar normal equations

$$\mathbf{L}'\mathbf{L}\mathbf{m} = \mathbf{L}'\mathbf{d}. \tag{7}$$

Using linear algebra, we can prove that the least-squares solution of $\mathbf{m}$ is

$$\begin{pmatrix} \hat{\mathbf{m_n}} \\ \hat{\mathbf{m_s}} \end{pmatrix} = \begin{pmatrix} (\mathbf{L_n}'\overline{\mathbf{R_s}}\mathbf{L_n})^{-1}\mathbf{L_n}'\overline{\mathbf{R_s}} \\ (\mathbf{L_s}'\overline{\mathbf{R_n}}\mathbf{L_s})^{-1}\mathbf{L_s}'\overline{\mathbf{R_n}} \end{pmatrix}\mathbf{d}, \tag{8}$$

with

$$\begin{aligned} \overline{\mathbf{R_s}} &= \mathbf{I} - \mathbf{L_s}(\mathbf{L_s}'\mathbf{L_s})^{-1}\mathbf{L_s}', \\ \overline{\mathbf{R_n}} &= \mathbf{I} - \mathbf{L_n}(\mathbf{L_n}'\mathbf{L_n})^{-1}\mathbf{L_n}'. \end{aligned} \tag{9}$$

$\overline{\mathbf{R_s}}$ and $\overline{\mathbf{R_n}}$ can be seen as signal and noise filters respectively since $\mathbf{L_s}(\mathbf{L_s}'\mathbf{L_s})^{-1}\mathbf{L_s}'$ and $\mathbf{L_n}(\mathbf{L_n}'\mathbf{L_n})^{-1}\mathbf{L_n}'$ are the data resolution operators for the signal and the noise.

The orthogonality between the noise operator $\mathbf{L_n}$ and the signal operator $\mathbf{L_s}$ condition the existence of $\mathbf{m}$ as derived in equation (8). If the two operators overlap completely, then the Hessians $\mathbf{L_n}'\overline{\mathbf{R_s}}\mathbf{L_n}$ and $\mathbf{L_s}'\overline{\mathbf{R_n}}\mathbf{L_s}$ are not invertible. If the two operators overlap only partially, then Nemeth (1996) proves that the separability of the signal and noise can be improved if we introduce a regularization term. If we use a model space regularization (Fomel, 1997), we have then

$$\begin{pmatrix} \hat{\mathbf{m_n}} \\ \hat{\mathbf{m_s}} \end{pmatrix} = \begin{pmatrix} (\mathbf{L_n}'\overline{\mathbf{R_s}}\mathbf{L_n} + \epsilon^2\mathbf{C_n}'\mathbf{C_n})^{-1}\mathbf{L_n}'\overline{\mathbf{R_s}} \\ (\mathbf{L_s}'\overline{\mathbf{R_n}}\mathbf{L_s} + \epsilon^2\mathbf{C_s}'\mathbf{C_s})^{-1}\mathbf{L_s}'\overline{\mathbf{R_n}} \end{pmatrix}\mathbf{d}, \tag{10}$$

with $\mathbf{C_n}$ and $\mathbf{C_s}$ the regularization operators for the noise model $\mathbf{m_n}$ and the signal model $\mathbf{m_s}$. A data space regularization can also improve the separation.

Again, the signal PEF's are approximated using Spitz' choice, i.e., $\mathbf{S} = \mathbf{D}\mathbf{N}^{-\mathbf{1}}$. In equation (5), the outcome of the inversion is $\mathbf{m_n}$ and $\mathbf{m_s}$. The estimated signal $\tilde{\mathbf{s}}$ is then easily derived as follows:

$$\tilde{\mathbf{s}} = \mathbf{d} - \mathbf{N}^{-\mathbf{1}}\mathbf{m_n}. \tag{11}$$

We call this new method the subtraction method. We now compare the Wiener-like approach and the subtraction method for multiple attenuation.

MULTIPLE ATTENUATION RESULTS

We use the very popular Gulf of Mexico line provided by WesternGeco (Figure 2). In the middle of the seismic section, the data contain a shallow salt body which generates strong reverberations and diffractions. This dataset is generally utilized to benchmark multiple attenuation techniques. We show that the subtraction method compares favorably with the Wiener-like approach. Notice that the subtraction method did not incorporate a regularization term as suggested above.

Figure 3 shows a CMP gather extracted from the Gulf of Mexico dataset and infested with multiples. The main patterns are accurately modeled (right). This CMP gather is taken outside the salt boundaries. The multiple attenuation starts by the PEF estimation for the data and noise model. Then, the noise attenuation begins using either the Wiener-like method or the subtraction scheme. We show the the result of the multiple attenuation using both methods in Figure 4. The multiples have been correctly attenuated in the two cases.

We now extract a CMP gather from above the salt. Multiple attenuation becomes more challenging because the salt play generates strong internal multiples, diffractions and shadow zones that are difficult to incorporate in the noise model using a simple convolutional technique. Figure 5 shows the selected CMP gather inside the salt boundaries with the corresponding multiple model. Despite the inherent difficulty of modeling subsalt multiples, the kinematics look correct. Figure 6 shows the estimated signal. As expected, the remaining signal is less coherent inside the salt boundaries than outside. Nonetheless, the two schemes reveal hidden information in a similar way.

CONCLUSION

We applied a new $t-x$ domain, pattern-based signal/noise separation technique to a 2-D line contaminated with multiples. This technique differs from the previous Wiener-like method because the data components are not filtered but predicted. The goal of this work was to show (1) that the noise attenuation can be formulated in a totally different way using a new fitting goal, (2) that this new formulation leads to a proper subtraction of the noise components and (3) that so far, this method is comparable in efficiency to the Wiener-like method. The first results are very encouraging. Yet, we have not explored the possibilities offered by the regularization to improve the signal-noise separation.

ACKNOWLEDGMENTS

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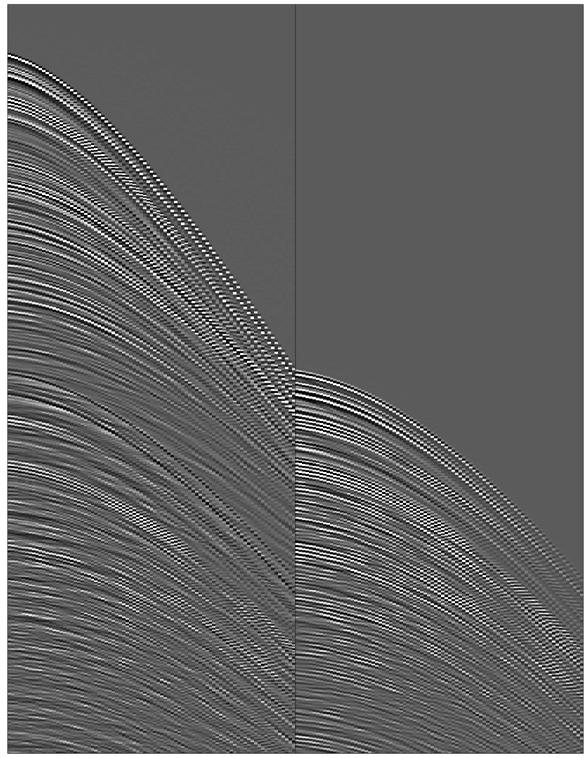

Figure 3: Left, CMP gather infested with multiples, outside the salt boundaries. Right, multiple model at the same location.

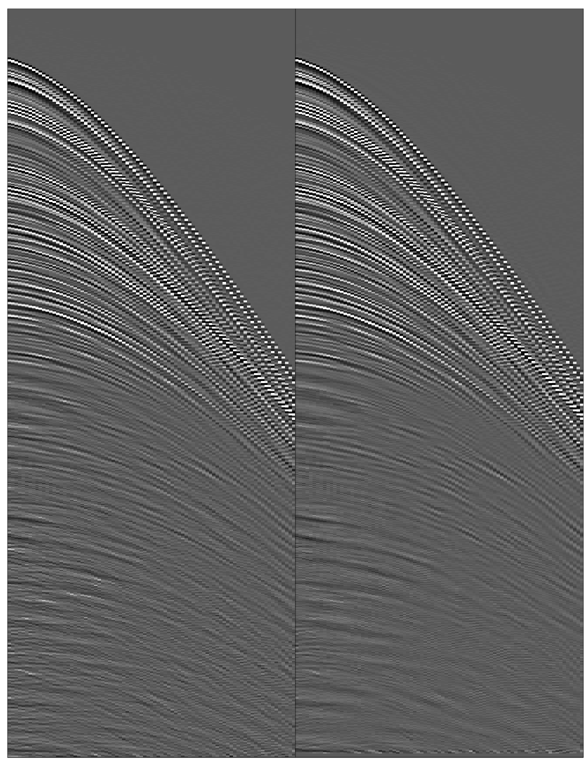

Figure 4: Left, estimated signal using the subtraction method. Right, estimated signal using the Wiener-like method.

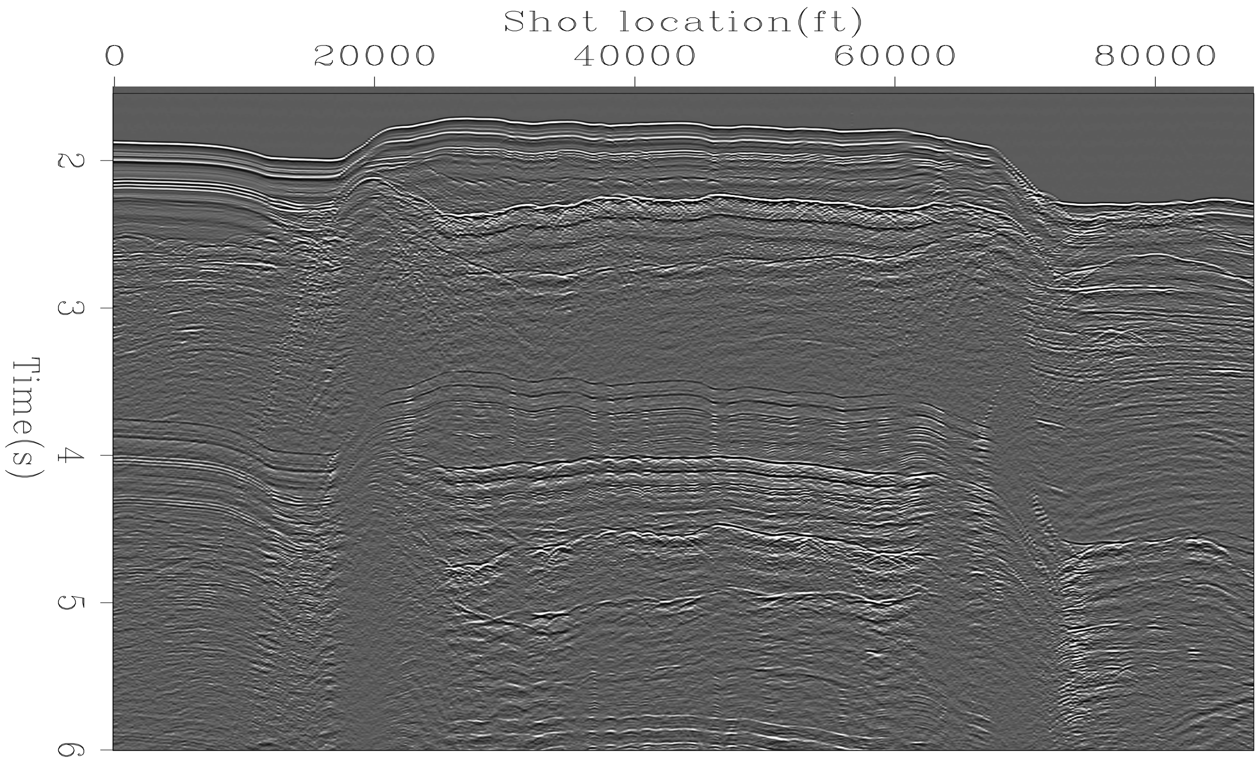

Figure 2: Zero offset section of the Gulf of Mexico data. Notice the strong water-column multiples after 3.5 seconds.

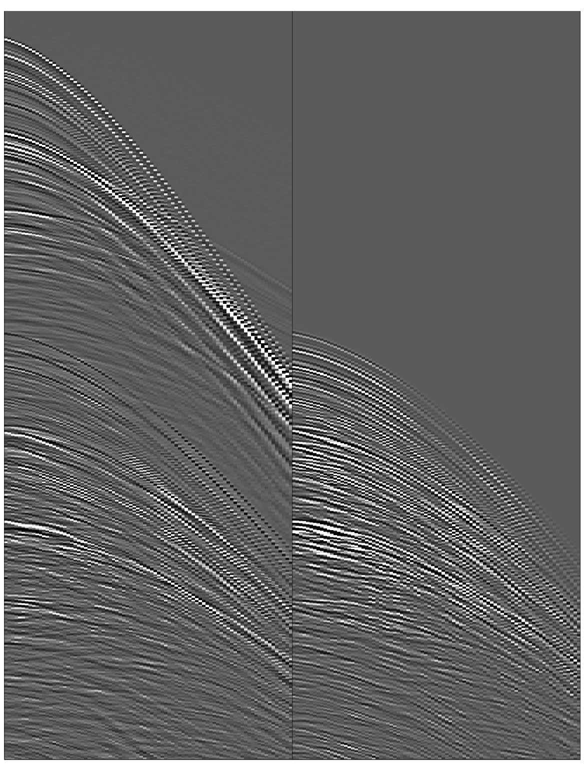

Figure 5: Left, CMP gather infested with multiples, inside the salt boundaries. Right, multiple model at the same location.

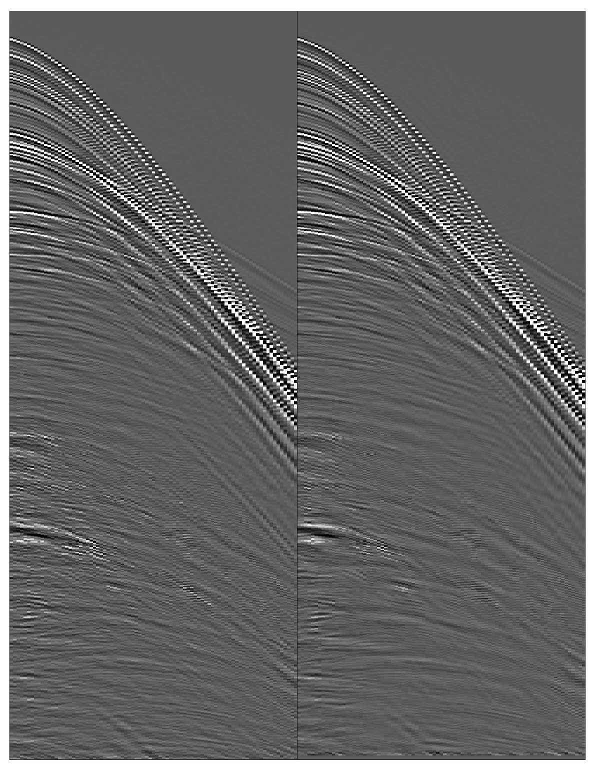

Figure 6: Left, estimated signal using the subtraction method. Right, estimated signal using the Wiener-like method.