

Outline

- ▶ **Motivation**
- ▶ **Theory**
- ▶ Inversion examples:
 - ▶ Synthetic: VTI Marmousi
 - ▶ Field: MC3 I I from ExxonMobil
- ▶ **Conclusions and future work**

Motivation

- ▶ **Anisotropic WEMVA**
 - ▶ Anisotropy VS. isotropy
 - ▶ Wavefield VS. ray
 - ▶ Image space VS. data space

- ▶ **Examples**
 - ▶ Propagator: Two-way VS. one-way
 - ▶ Objective function test
 - ▶ Preconditioning

Theory

▶ Objective function

$$J = \frac{1}{2} \sum_{\mathbf{h}} \langle \mathbf{h}I_{\mathbf{h}}, \mathbf{h}I_{\mathbf{h}} \rangle$$

L: modeling operator

f : source wavelet

f': receiver data

I_h: image at subsurface offset h

S_{+h}: shifting operator by +h

▶ Constraints (State equations)

$$\mathbf{L}(s_n, \eta)\mathbf{p} = \mathbf{f}$$

Source wavefield

$$\mathbf{L}^*(s_n, \eta)\mathbf{q} = \mathbf{f}'$$

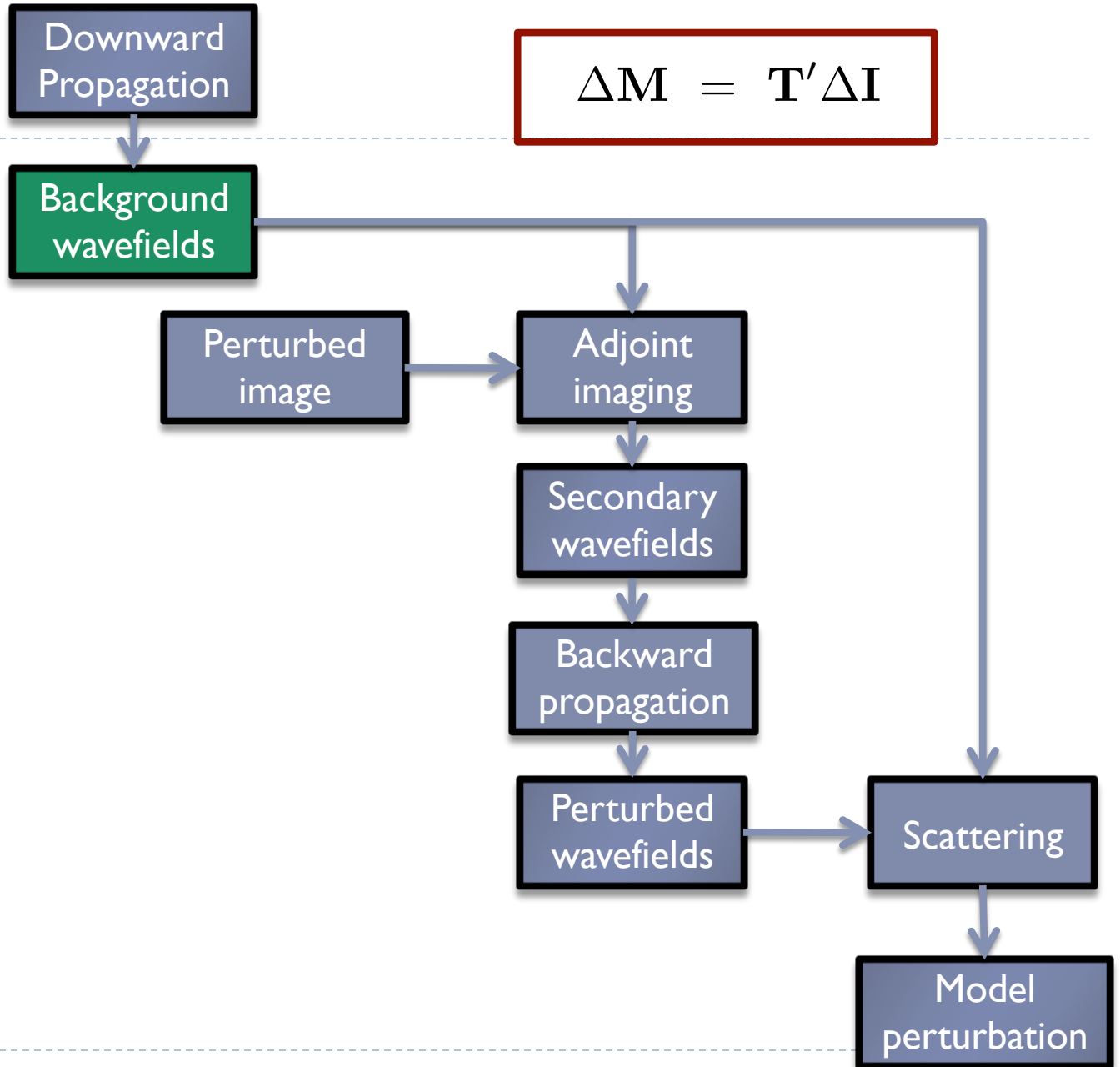
Receiver wavefield

$$I_{\mathbf{h}} = (S_{+\mathbf{h}}\mathbf{p})^*(S_{-\mathbf{h}}\mathbf{q})$$

Imaging condition

Theory

$$\mathbf{L}(s_n, \eta)\mathbf{p} = \mathbf{f}$$
$$\mathbf{L}^*(s_n, \eta)\mathbf{q} = \mathbf{f}'$$



Theory

Downward Propagation

$$\Delta \mathbf{M} = \mathbf{T}' \Delta \mathbf{I}$$

\mathbf{p}, \mathbf{q}

$$\frac{\partial \mathcal{L}}{\partial I_{\mathbf{h}}} = -\gamma_{\mathbf{h}} + \mathbf{h}^2 I_{\mathbf{h}} = 0, \quad \forall \mathbf{h}$$

Perturbed image

Adjoint imaging

Secondary wavefields

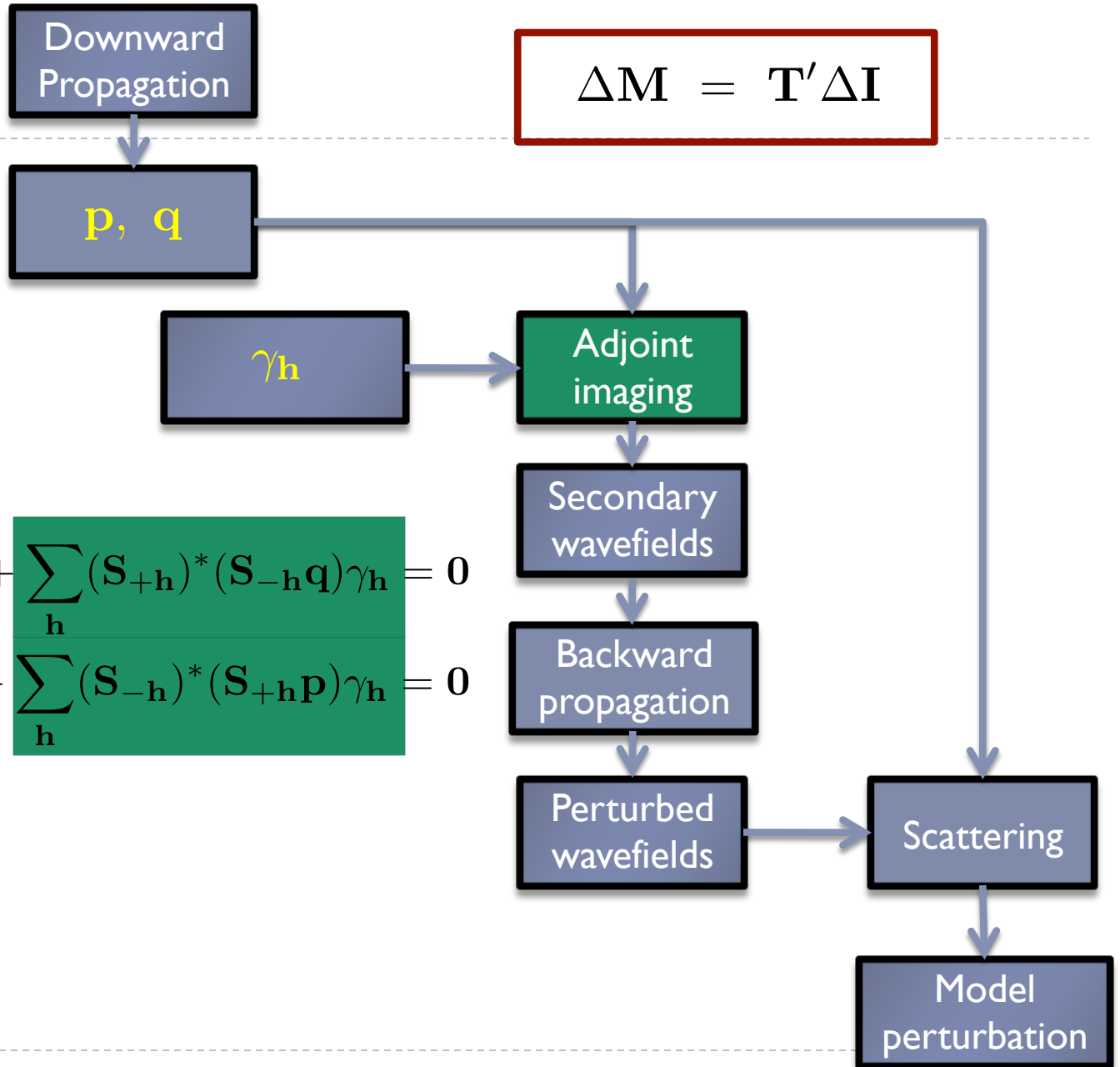
Backward propagation

Perturbed wavefields

Scattering

Model perturbation

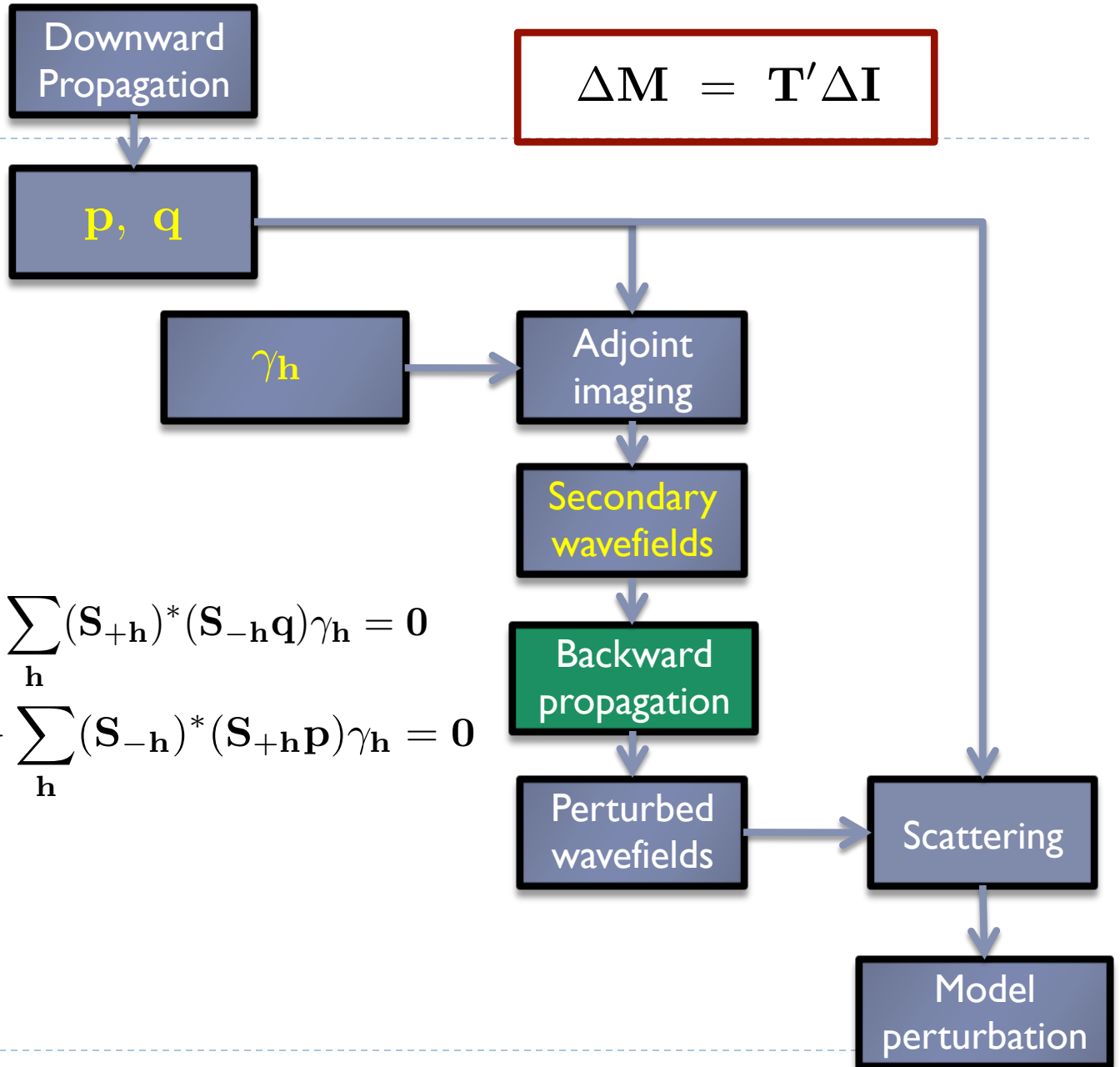
Theory



$$\frac{\partial \mathcal{L}}{\partial \mathbf{p}} = -\mathbf{L}^*(s_n, \eta) \lambda + \sum_{\mathbf{h}} (\mathbf{S}_{+\mathbf{h}})^* (\mathbf{S}_{-\mathbf{h}} \mathbf{q}) \gamma_{\mathbf{h}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{q}} = -\mathbf{L}(s_n, \eta) \mu + \sum_{\mathbf{h}} (\mathbf{S}_{-\mathbf{h}})^* (\mathbf{S}_{+\mathbf{h}} \mathbf{p}) \gamma_{\mathbf{h}} = 0$$

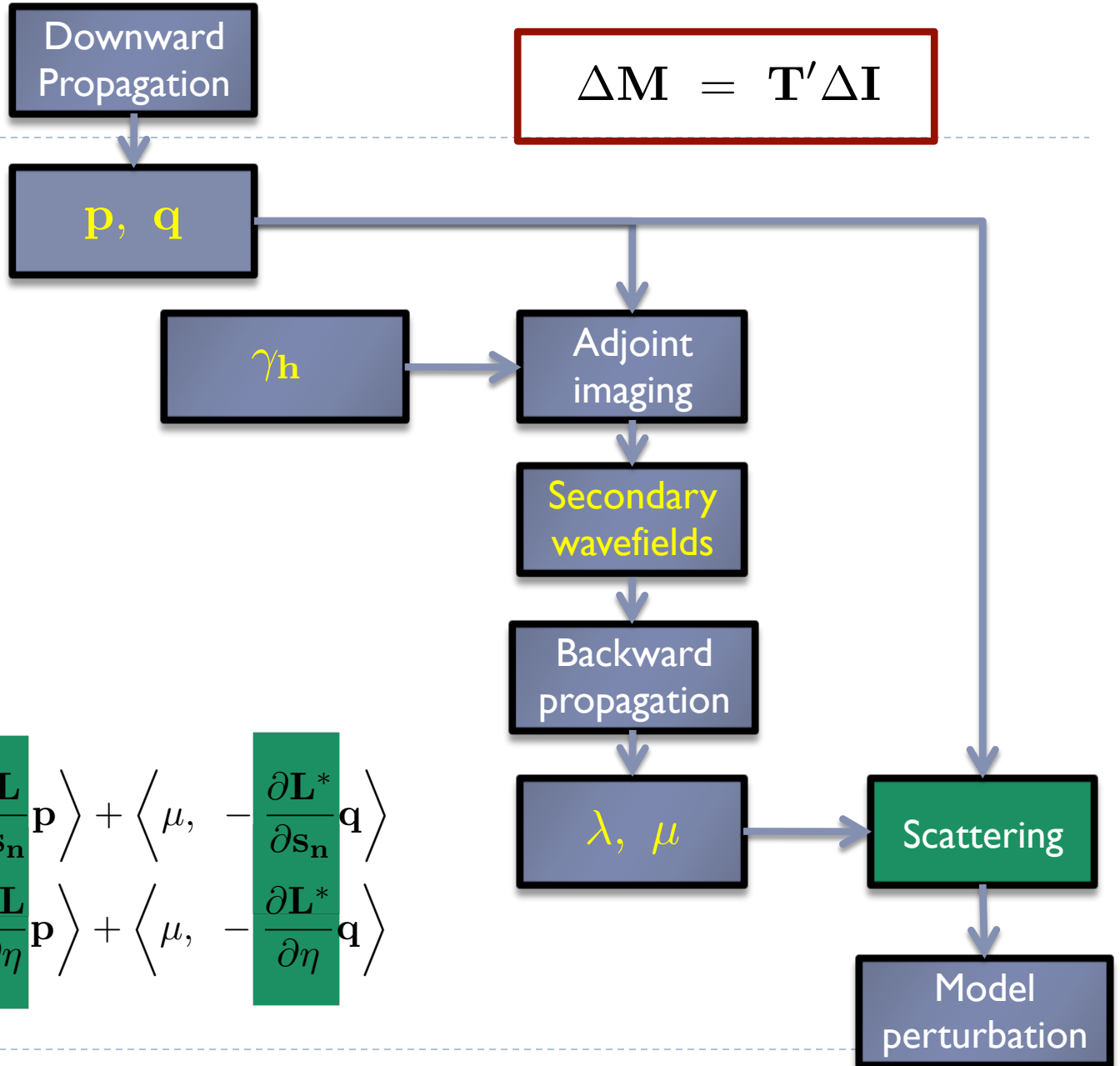
Theory



$$\frac{\partial \mathcal{L}}{\partial \mathbf{p}} = -\mathbf{L}^*(s_n, \eta) \lambda + \sum_{\mathbf{h}} (\mathbf{S}_{+\mathbf{h}})^* (\mathbf{S}_{-\mathbf{h}} \mathbf{q}) \gamma_{\mathbf{h}} = \mathbf{0}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{q}} = -\mathbf{L}(s_n, \eta) \mu + \sum_{\mathbf{h}} (\mathbf{S}_{-\mathbf{h}})^* (\mathbf{S}_{+\mathbf{h}} \mathbf{p}) \gamma_{\mathbf{h}} = \mathbf{0}$$

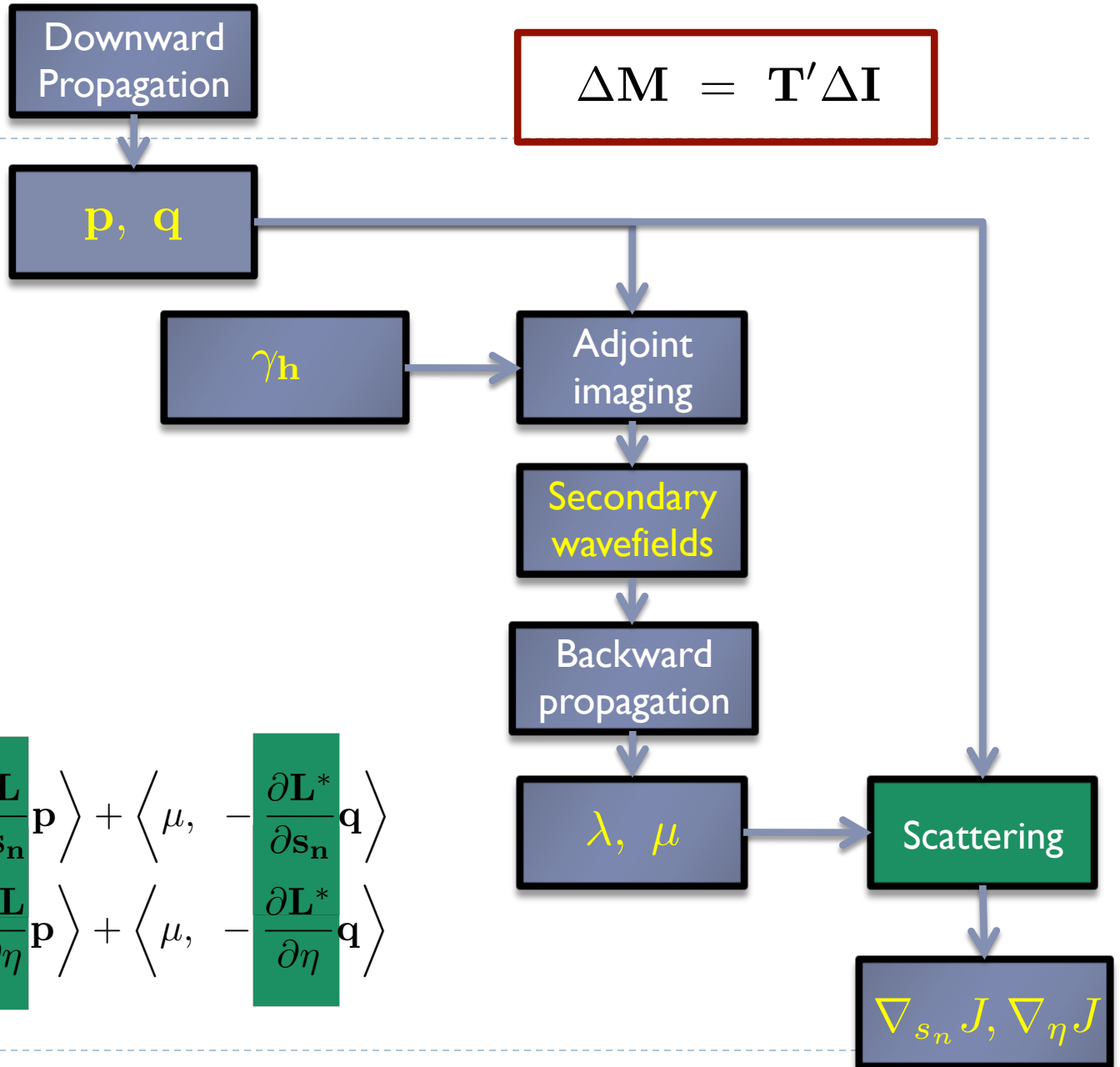
Theory



$$\nabla_{s_n} J = \left\langle \lambda, -\frac{\partial \mathbf{L}}{\partial \mathbf{s}_n} \mathbf{p} \right\rangle + \left\langle \mu, -\frac{\partial \mathbf{L}^*}{\partial \mathbf{s}_n} \mathbf{q} \right\rangle$$

$$\nabla_{\eta} J = \left\langle \lambda, -\frac{\partial \mathbf{L}}{\partial \eta} \mathbf{p} \right\rangle + \left\langle \mu, -\frac{\partial \mathbf{L}^*}{\partial \eta} \mathbf{q} \right\rangle$$

Theory



$$\nabla_{s_n} J = \left\langle \lambda, -\frac{\partial \mathbf{L}}{\partial \mathbf{s}_n} \mathbf{p} \right\rangle + \left\langle \mu, -\frac{\partial \mathbf{L}^*}{\partial \mathbf{s}_n} \mathbf{q} \right\rangle$$

$$\nabla_{\eta} J = \left\langle \lambda, -\frac{\partial \mathbf{L}}{\partial \eta} \mathbf{p} \right\rangle + \left\langle \mu, -\frac{\partial \mathbf{L}^*}{\partial \eta} \mathbf{q} \right\rangle$$

Preconditioning

$$d\mathbf{m} = \mathbf{B}\Sigma d\mathbf{n}$$

$$\nabla_{\mathbf{n}}J = \left(\frac{\partial\mathbf{m}}{\partial\mathbf{n}}\right)^* \nabla_{\mathbf{m}}J = \Sigma^* \mathbf{B}^* \nabla_{\mathbf{m}}J$$

$$\mathbf{m}_{i+1} = \mathbf{m}_i + \alpha_i \mathbf{B}\Sigma\Sigma^* \mathbf{B}^* \nabla_{\mathbf{m}}J$$

$\mathbf{m} = [s_n, \eta]'$ model parameter

$\mathbf{B} = \begin{vmatrix} \mathbf{B}_s & 0 \\ 0 & \mathbf{B}_\eta \end{vmatrix}$ Spatial standard deviation matrix

$\mathbf{n} = [\tilde{s}_n, \tilde{\eta}]'$ Preconditioning parameter

$\Sigma = \begin{vmatrix} \sigma_{ss} & \sigma_{s\eta} \\ \sigma_{\eta s} & \sigma_{\eta\eta} \end{vmatrix}$ X-parameter standard deviation matrix

Preconditioning

$$d\mathbf{m} = \mathbf{B}\Sigma d\mathbf{n}$$

$$\nabla_{\mathbf{n}}J = \left(\frac{\partial\mathbf{m}}{\partial\mathbf{n}}\right)^* \nabla_{\mathbf{m}}J = \Sigma^* \mathbf{B}^* \nabla_{\mathbf{m}}J$$

Semi-positive definite matrix

$$\mathbf{m}_{i+1} = \mathbf{m}_i + \alpha_i \mathbf{B}\Sigma\Sigma^* \mathbf{B}^* \nabla_{\mathbf{m}}J$$

$$\mathbf{m} = [s_n, \eta]' \quad \text{model parameter}$$

$$\mathbf{B} = \begin{vmatrix} \mathbf{B}_s & 0 \\ 0 & \mathbf{B}_\eta \end{vmatrix} \quad \text{Spatial standard deviation matrix}$$

$$\mathbf{n} = [\tilde{s}_n, \tilde{\eta}]' \quad \text{Preconditioning parameter}$$

$$\Sigma = \begin{vmatrix} \sigma_{ss} & \sigma_{s\eta} \\ \sigma_{\eta s} & \sigma_{\eta\eta} \end{vmatrix} \quad \text{X-parameter standard deviation matrix}$$

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- ▶ Motivation
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- ▶ **Inversion examples:**
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Example: VTI Marmousi

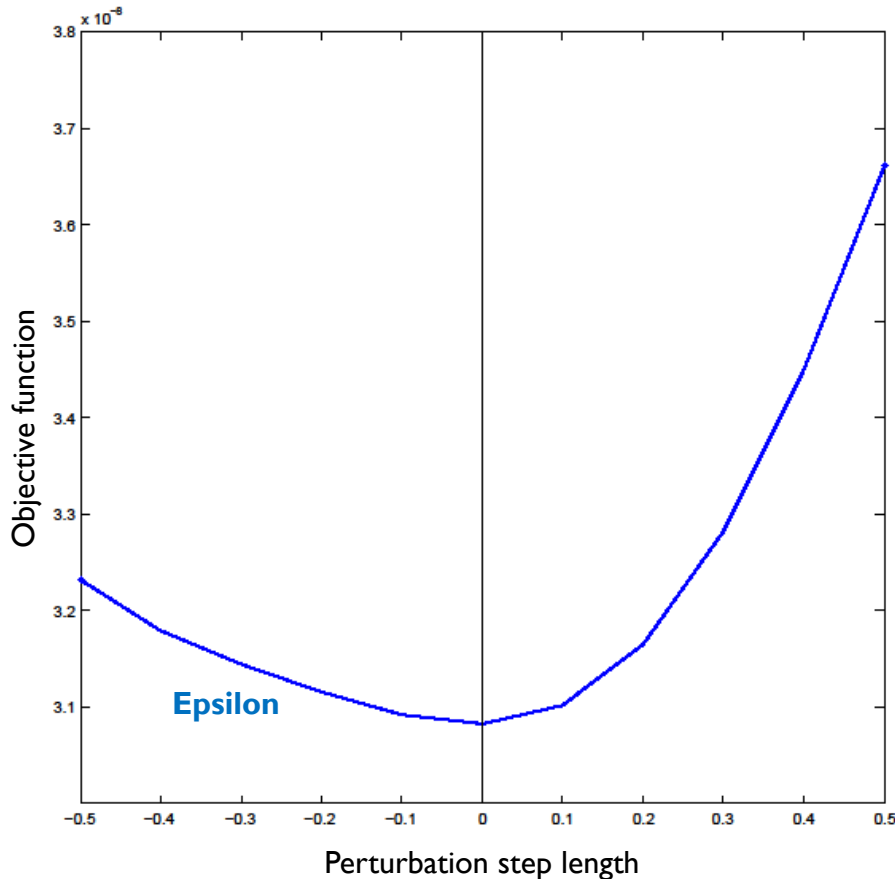
- ▶ Model grid: 300*901, 10m spacing in x and z
- ▶ Source: 180 shots * 50m
- ▶ Born modeled data with maximum offset = 6km * 10m
- ▶ Propagator: first-order two-way VTI wave-equation (Duveneck et al., 2008)
- ▶ Objective function: $J = \frac{1}{2} \langle \mathbf{DI}_a, \mathbf{DI}_a \rangle$

$$\mathbf{B} = \begin{vmatrix} \mathbf{B}_s & 0 \\ 0 & \mathbf{B}_\eta \end{vmatrix} \quad \text{B-spline interpolator}$$

$$\mathbf{\Sigma} = \begin{vmatrix} \sigma_{ss} & 0 \\ 0 & \sigma_{\eta\eta} \end{vmatrix} \quad \text{Scaling without X-terms}$$

Example: VTI Marmousi

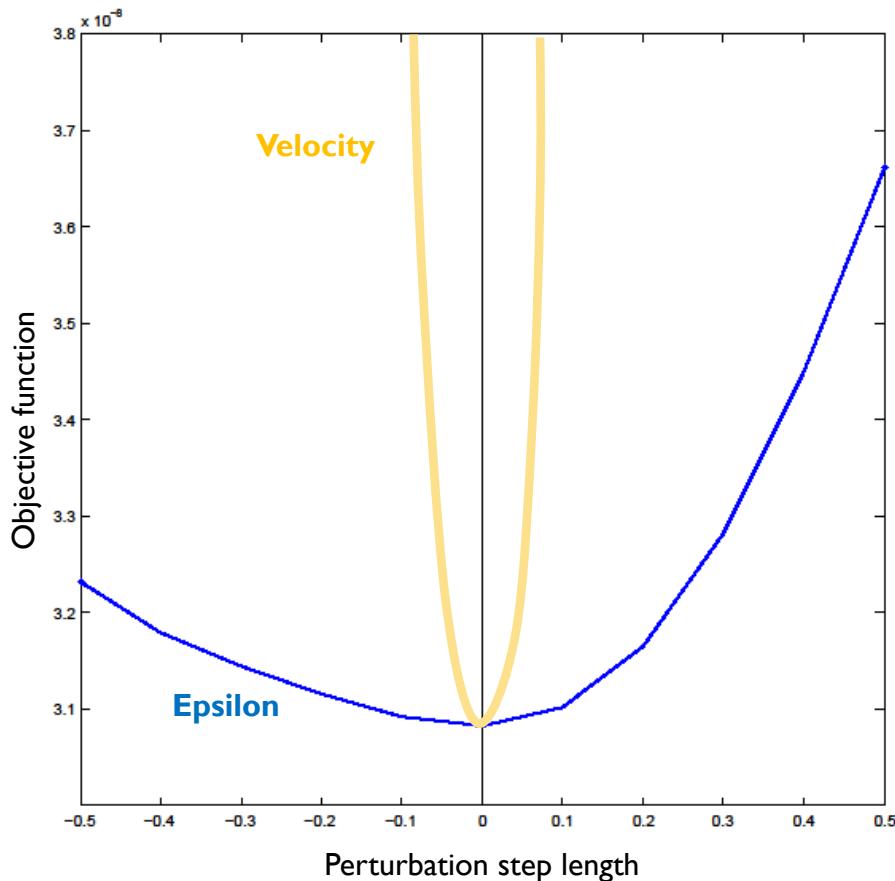
$$\text{Objective function: } J = \frac{1}{2} \langle \mathbf{DI}_a, \mathbf{DI}_a \rangle$$



- Objective function can very well characterize the epsilon error.

Example: VTI Marmousi

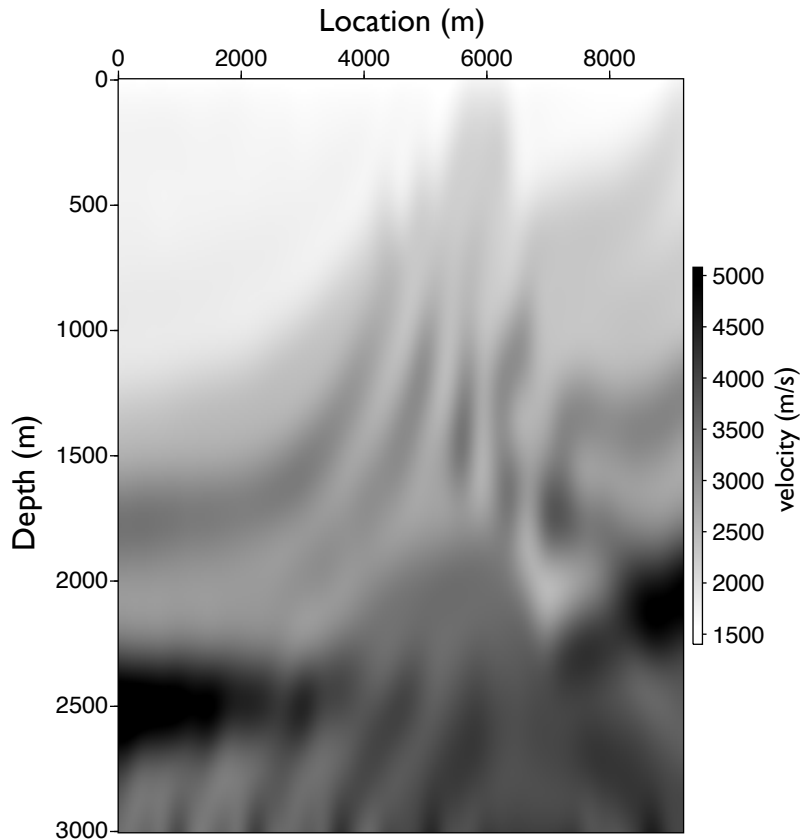
$$\text{Objective function: } J = \frac{1}{2} \langle \mathbf{DI}_a, \mathbf{DI}_a \rangle$$



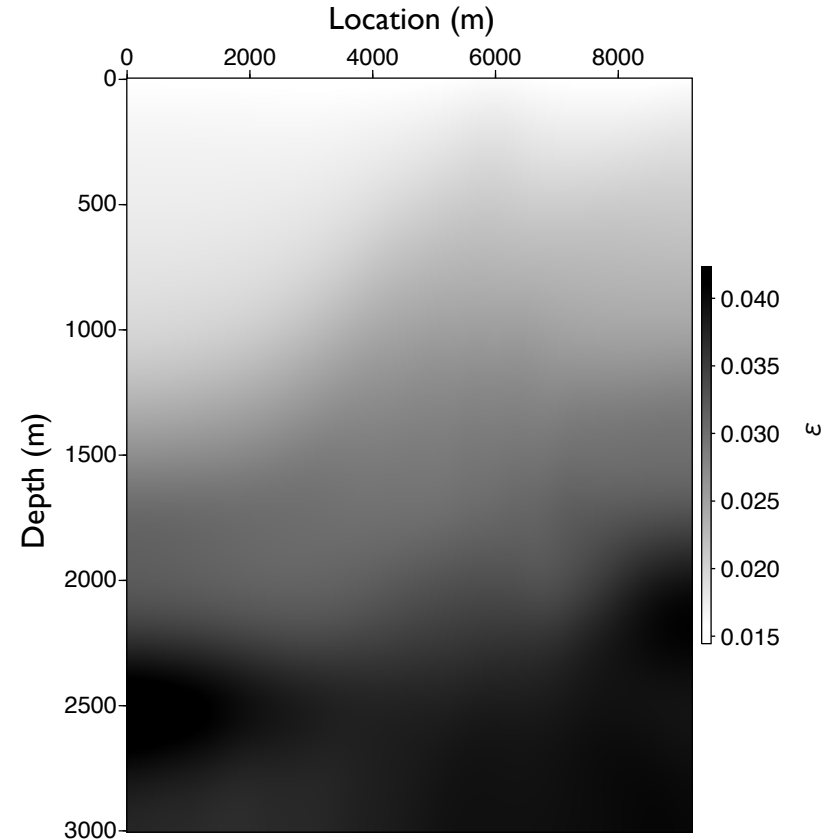
- ▶ Objective function can very well characterize the epsilon error.
- ▶ Flat bottom:
 - ▶ low resolution
 - ▶ High tolerance for the purpose of imaging

Example: VTI Marmousi

True v model

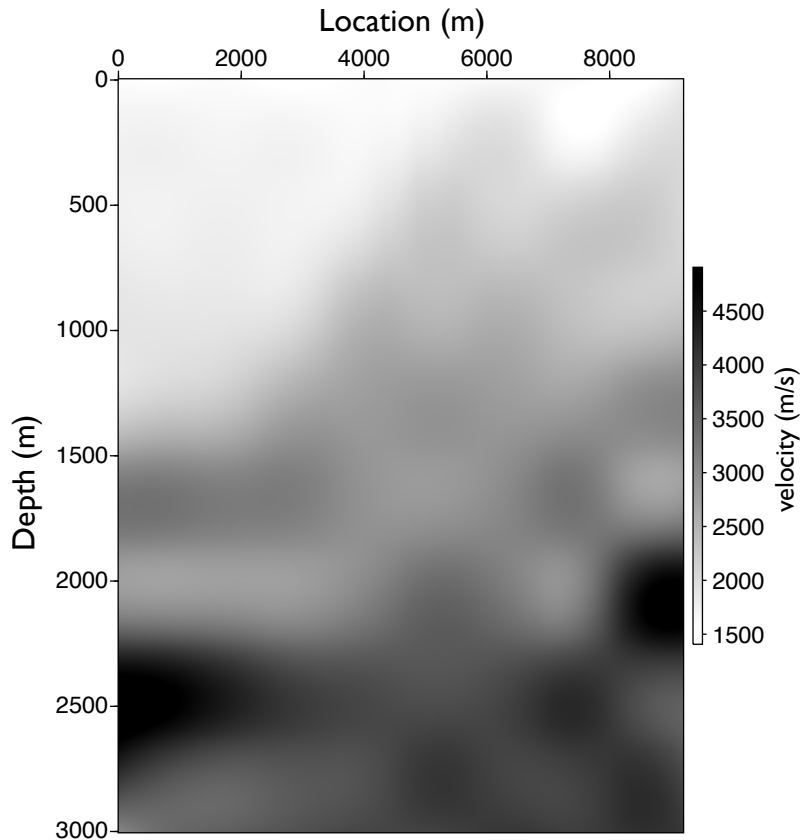


True ε model

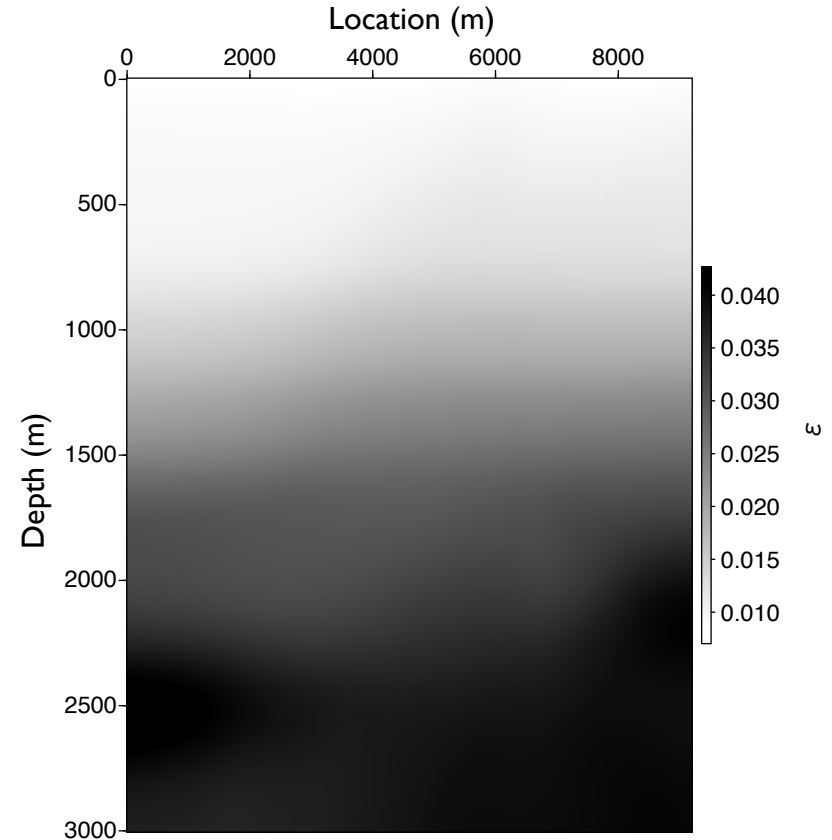


Example: VTI Marmousi

Initial v model

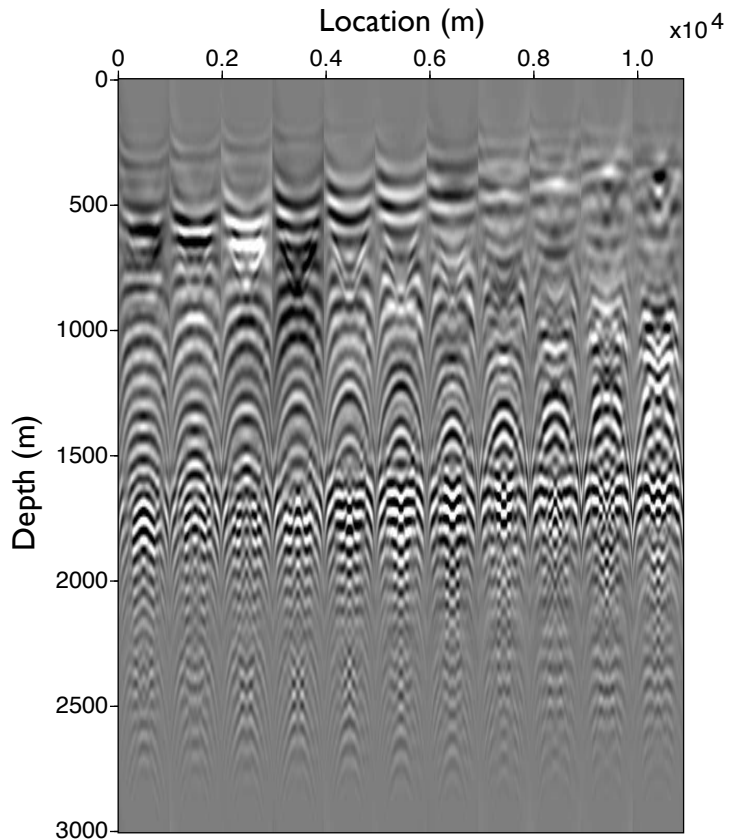


Initial ϵ model

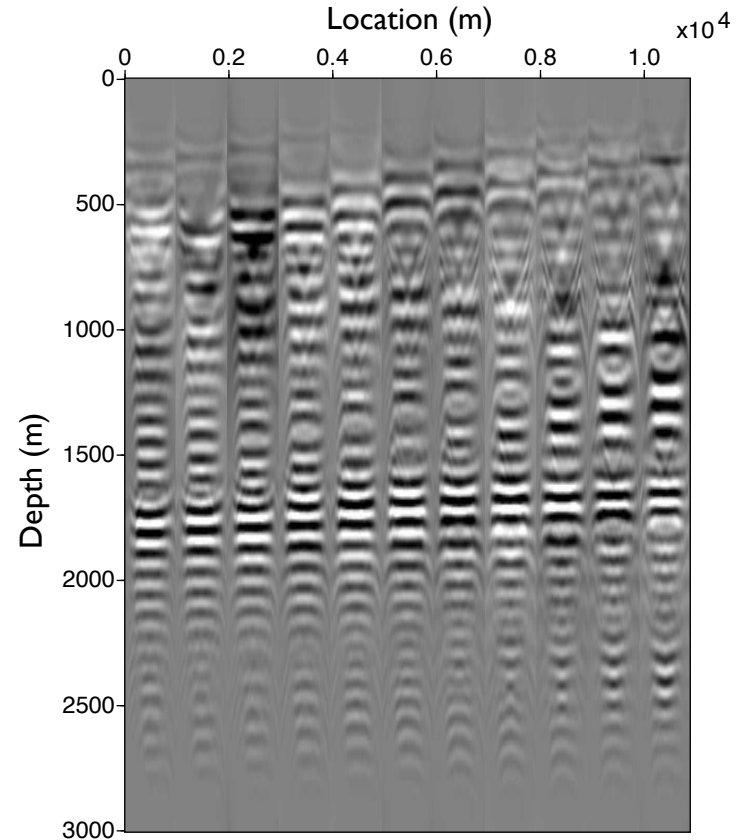


Example: VTI Marmousi

Initial angle gathers

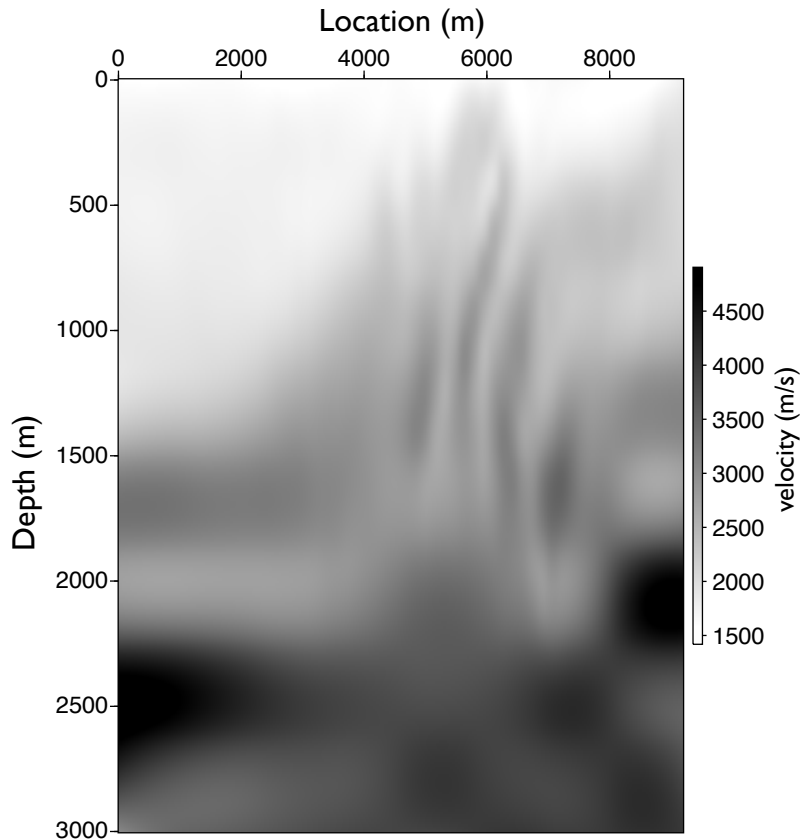


Final angle gathers

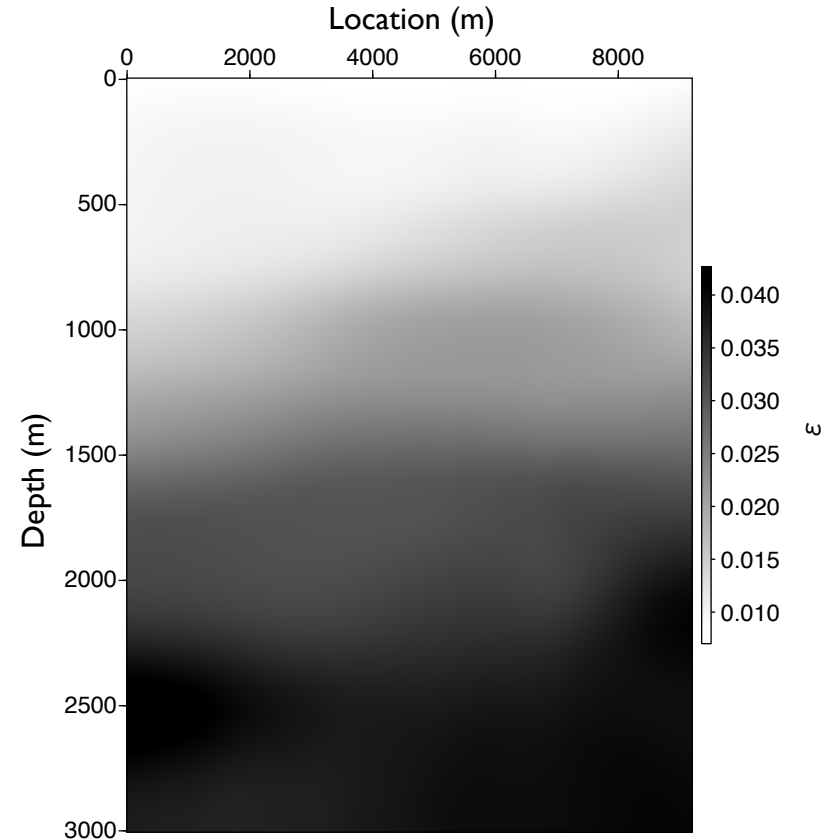


Example: VTI Marmousi

Inverted v model

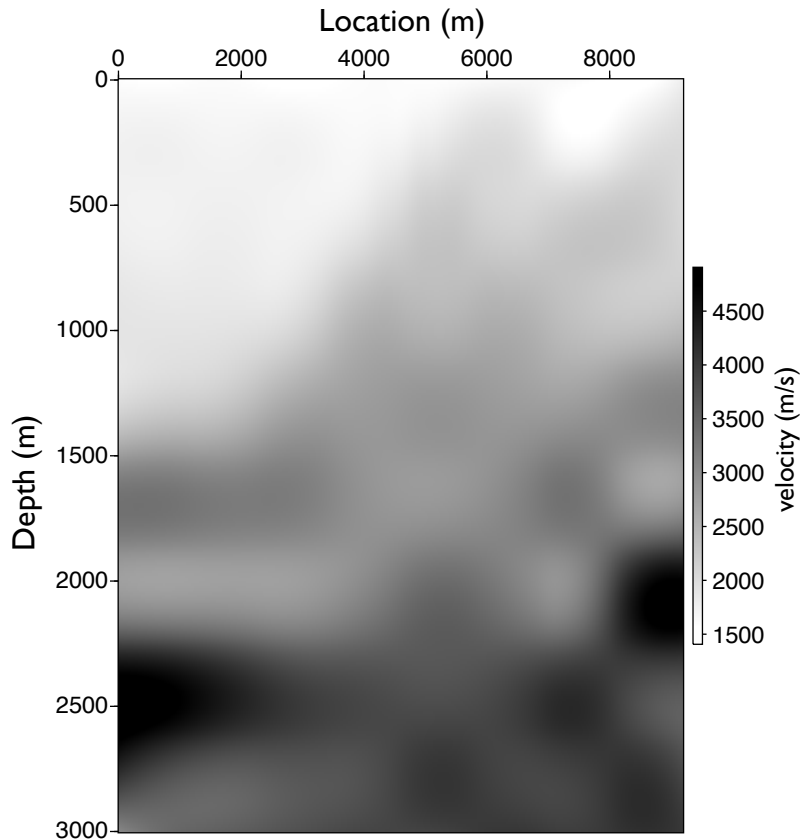


Inverted ϵ model

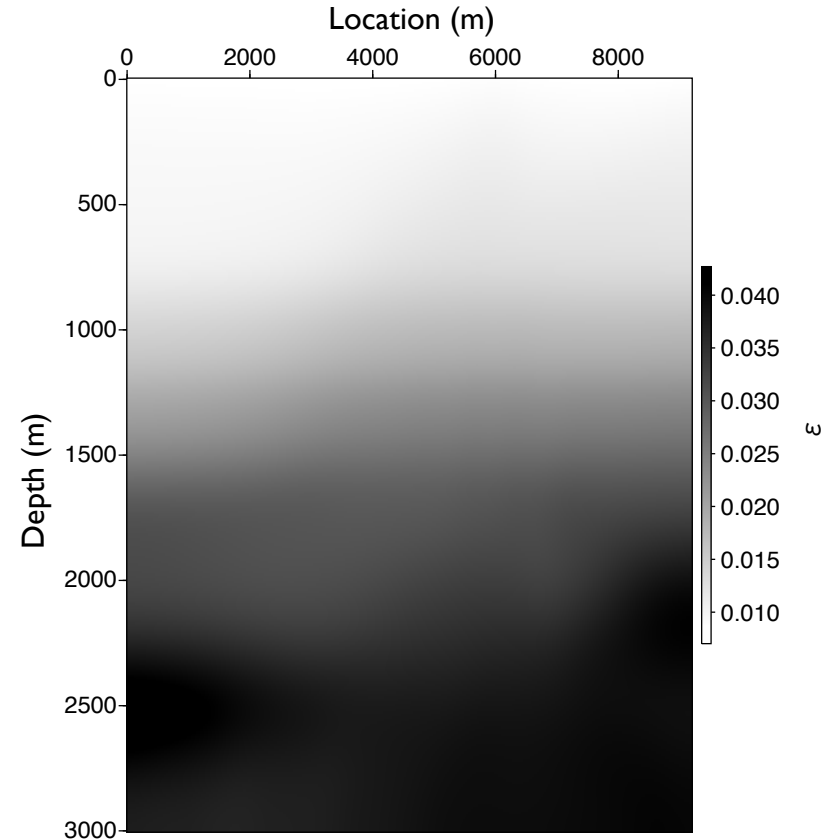


Example: VTI Marmousi

Initial v model

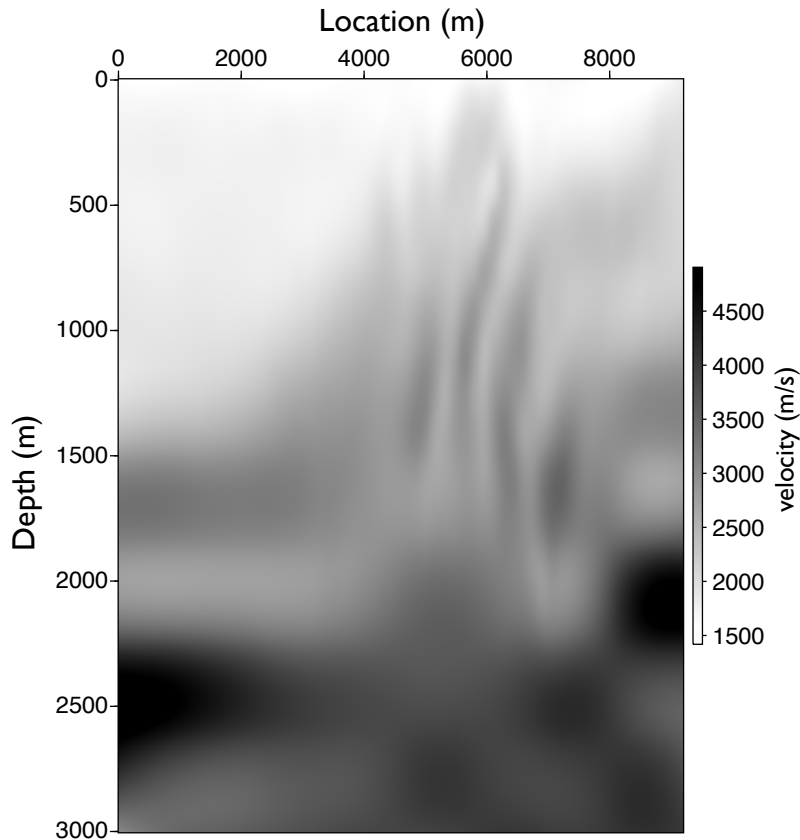


Initial ϵ model

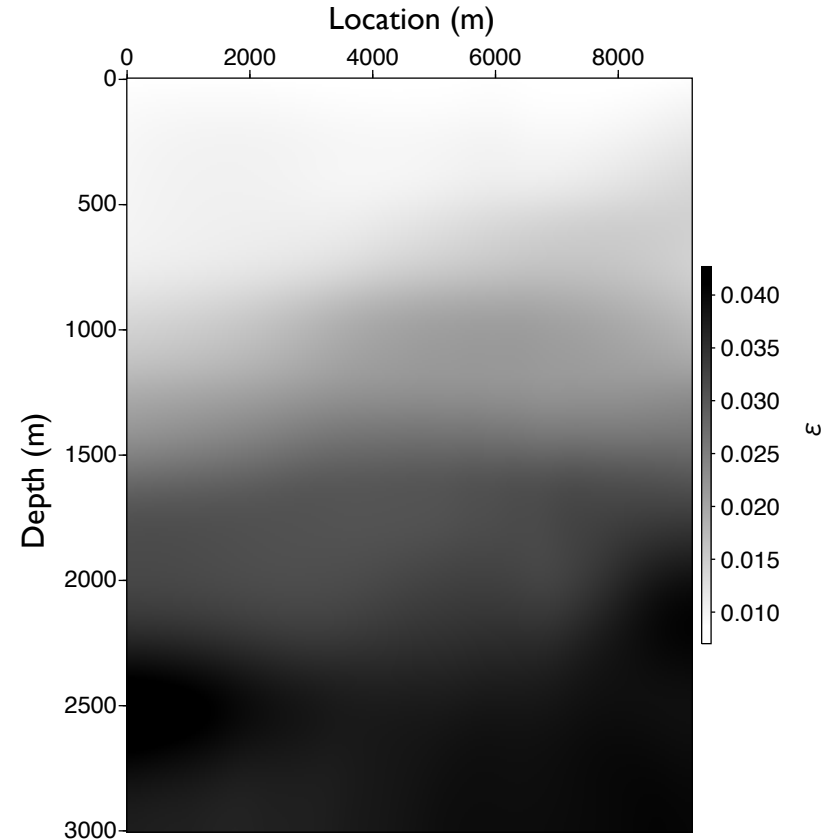


Example: VTI Marmousi

Inverted v model

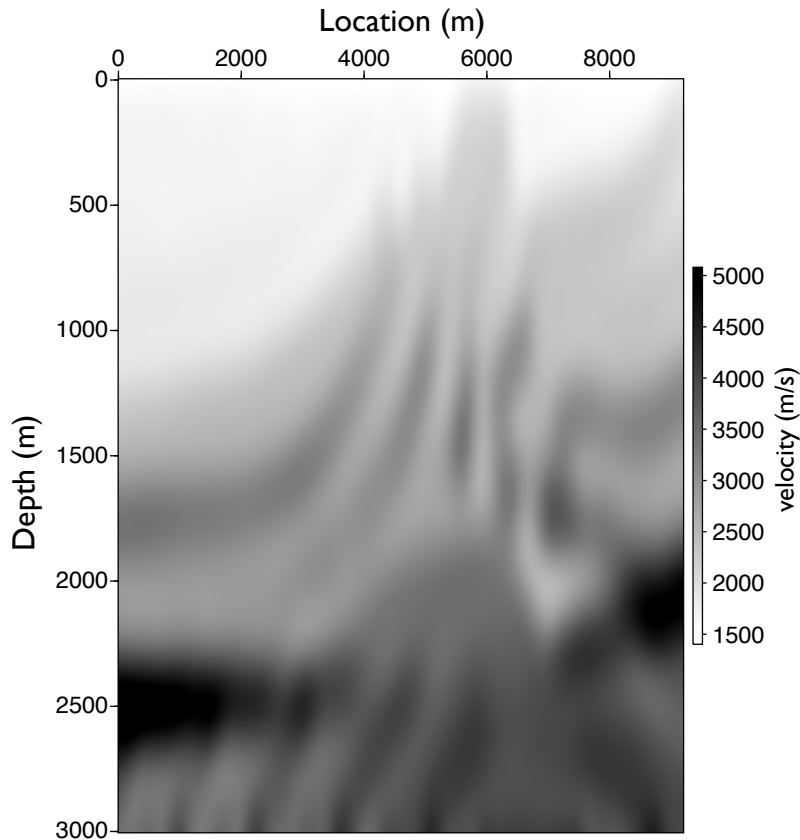


Inverted ϵ model

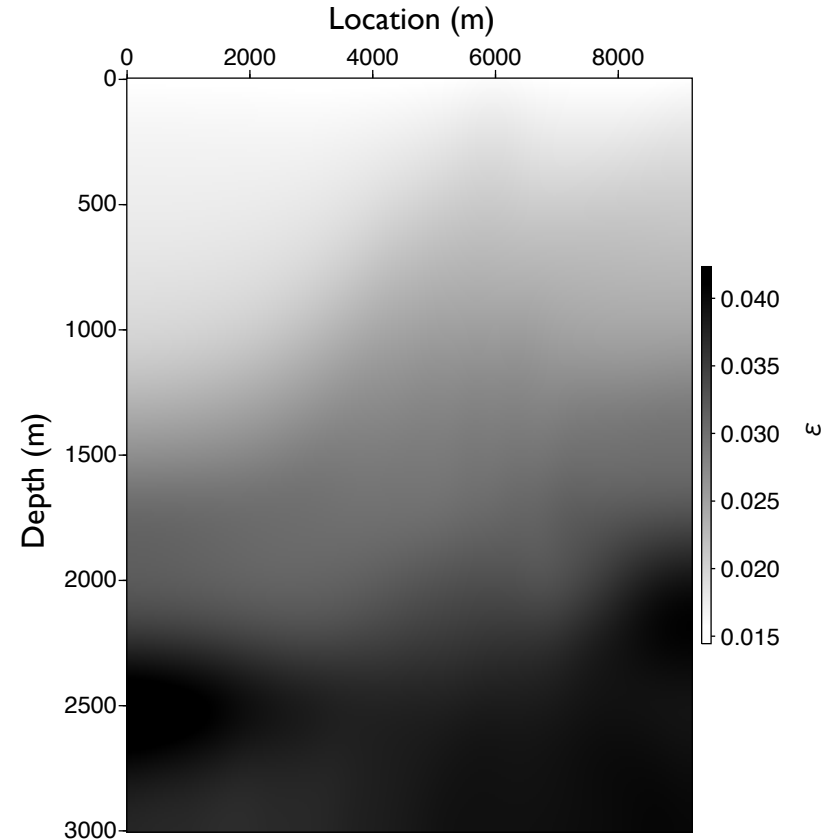


Example: VTI Marmousi

True v model



True ϵ model



Example: VTI Marmousi

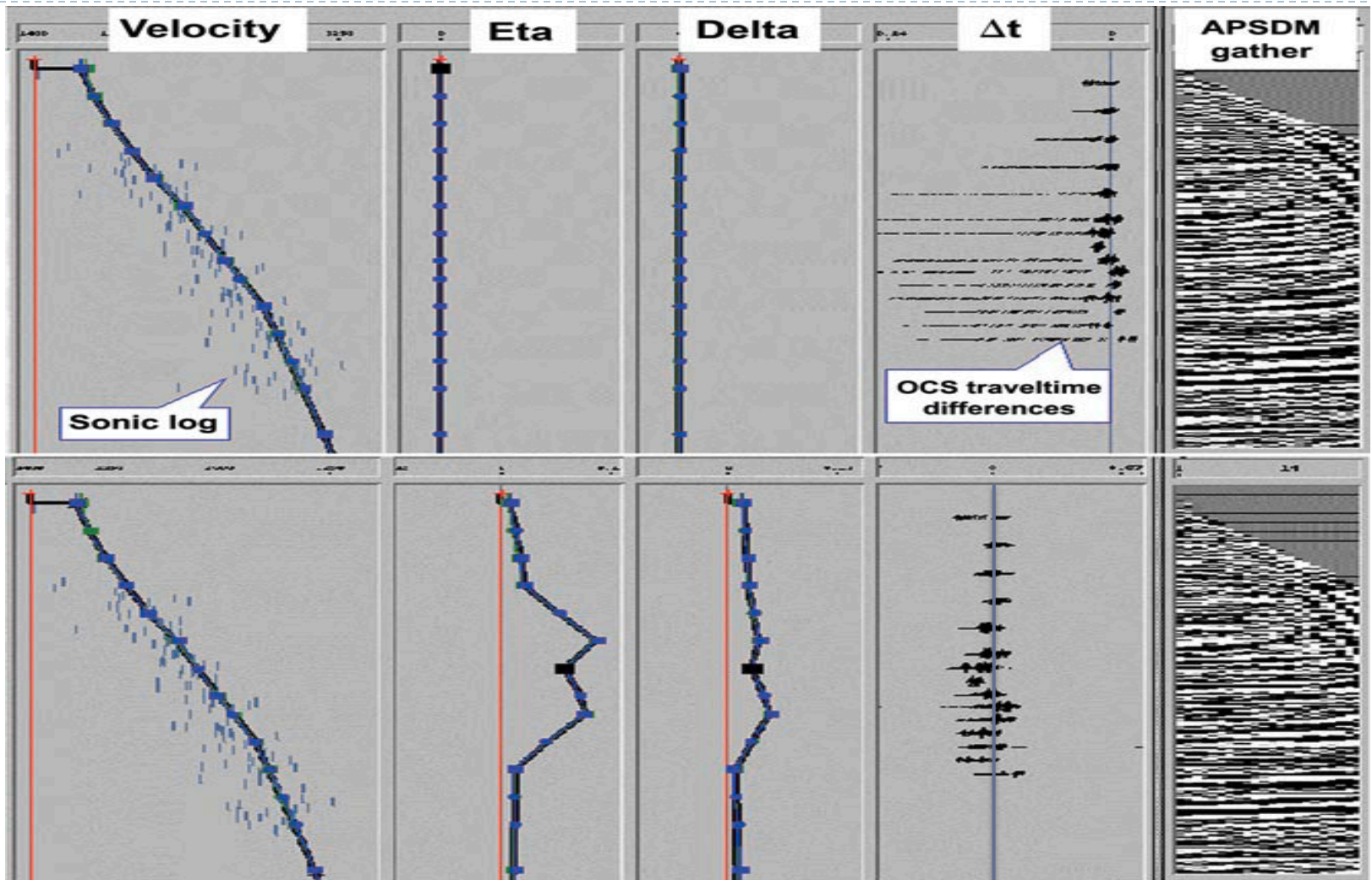
▶ Summary

- ▶ Able to resolve high wavenumber velocity anomalies
- ▶ Able to better define the anisotropic model
- ▶ First-order two-way VTI wave-equation
 - + Simplicity in code
 - + Accuracy for high angle waves
 - 5 fields to propagate
 - Large I/O requirements
 - High computational cost

Example: ExxonMobil field data

- ▶ Streamer geometry: 270 shots * 50 m
- ▶ Offset: maximum 4 km; minimum 150 m; 25 m spacing
- ▶ Initial model built using an interactive visualization method (Bear et al., 2005)

Example: ExxonMobil field data



Courtesy of Bear et al. (2005)



Example: ExxonMobil field data

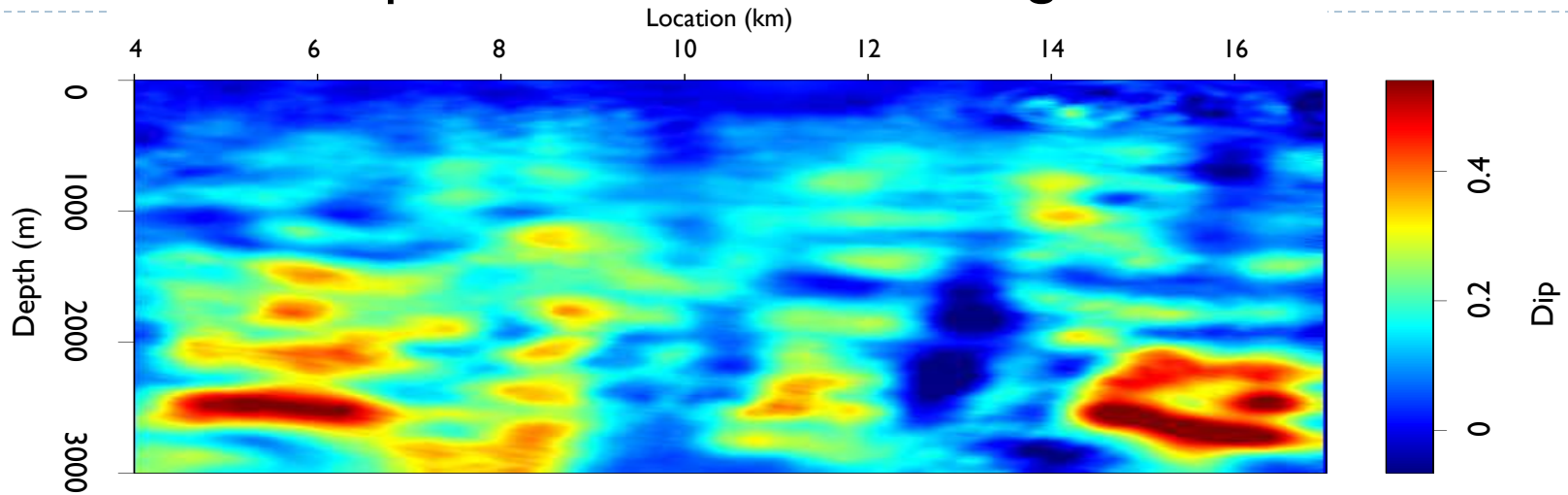
- ▶ Streamer geometry: 270 shots * 50 m spacing
- ▶ Offset: maximum 4 km; minimum 150 m; 25 m spacing
- ▶ Propagator: PSPI VTI one-way wave-equation (Tang and Clapp, 2006)
- ▶ Objective function: DSO + stack power

$$\mathbf{B} = \begin{vmatrix} \mathbf{B}_s & 0 \\ 0 & \mathbf{B}_\eta \end{vmatrix} \quad \text{Steering operator}$$

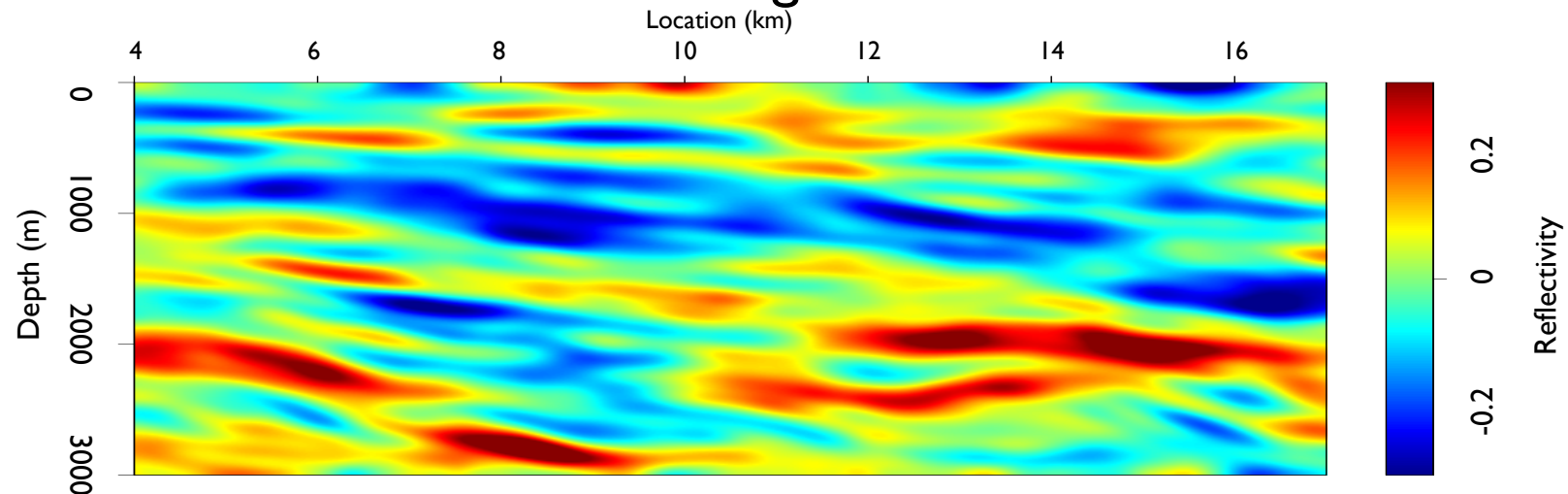
$$\Sigma = \begin{vmatrix} \sigma_{ss} & 0 \\ 0 & \sigma_{\eta\eta} \end{vmatrix} \quad \text{Scaling without X-terms}$$

Example: ExxonMobil field data

Dip field from the initial image

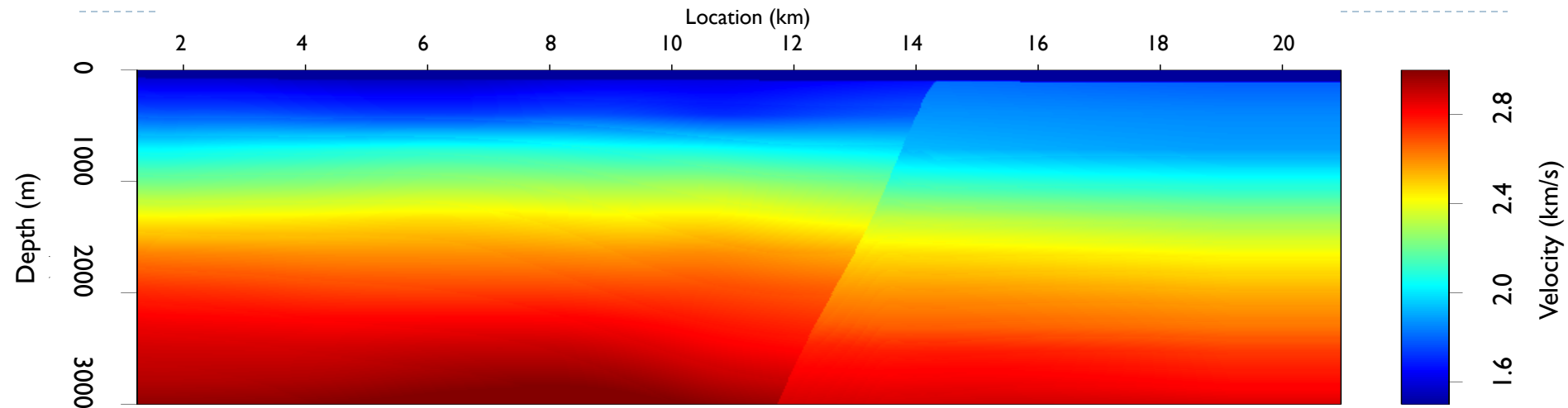


Reflector created using random number

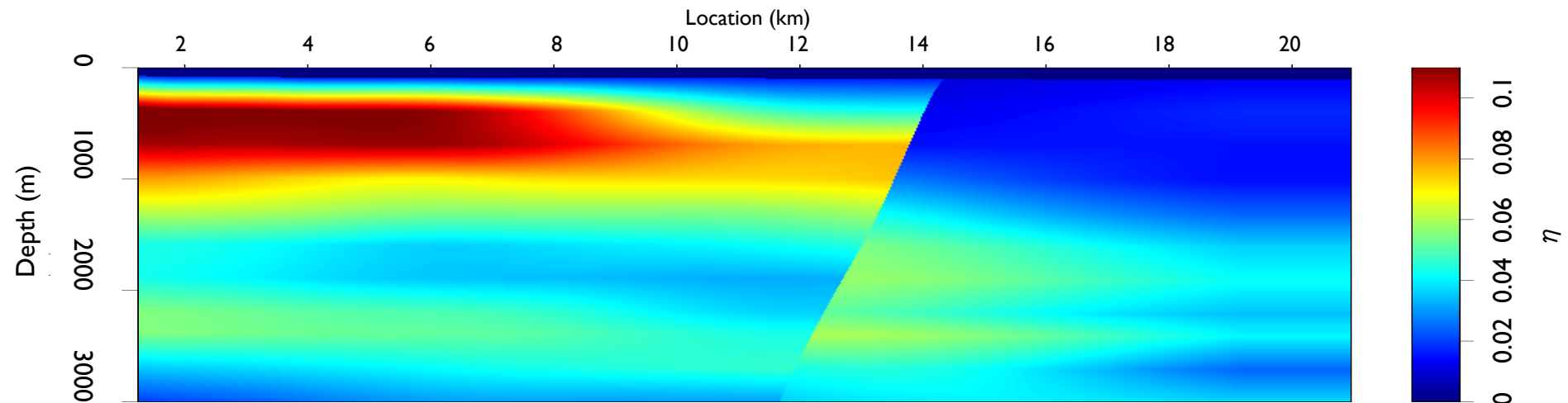


Example: ExxonMobil field data

Initial v model

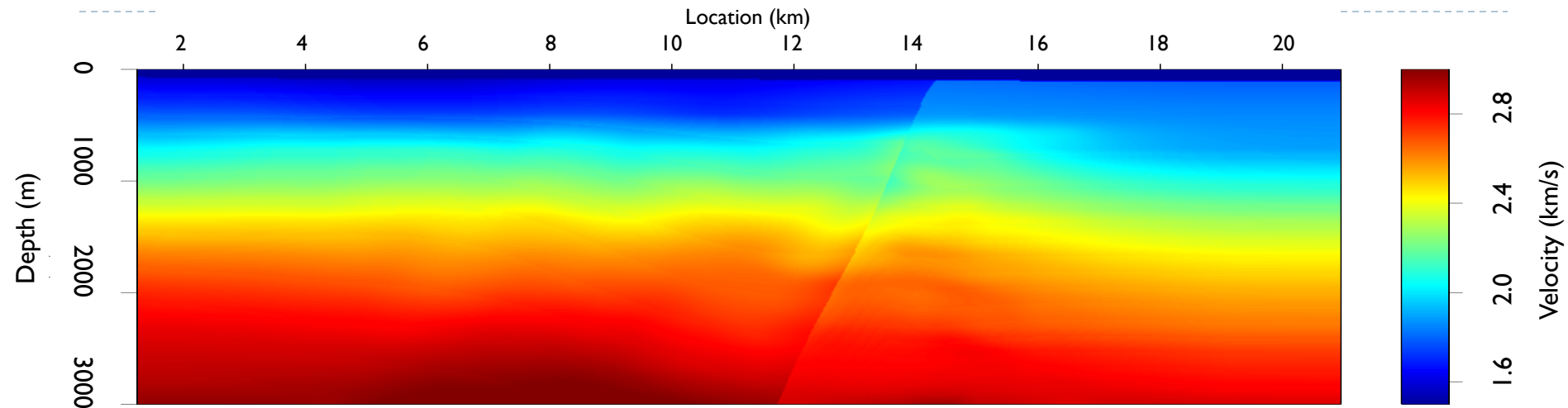


Initial η model

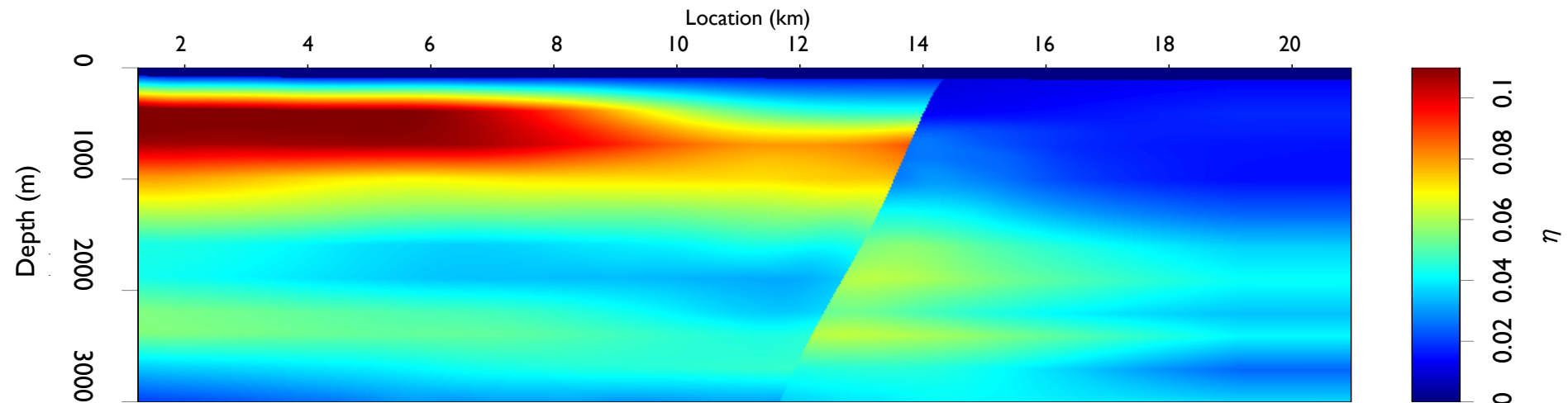


Example: ExxonMobil field data

Inverted v model

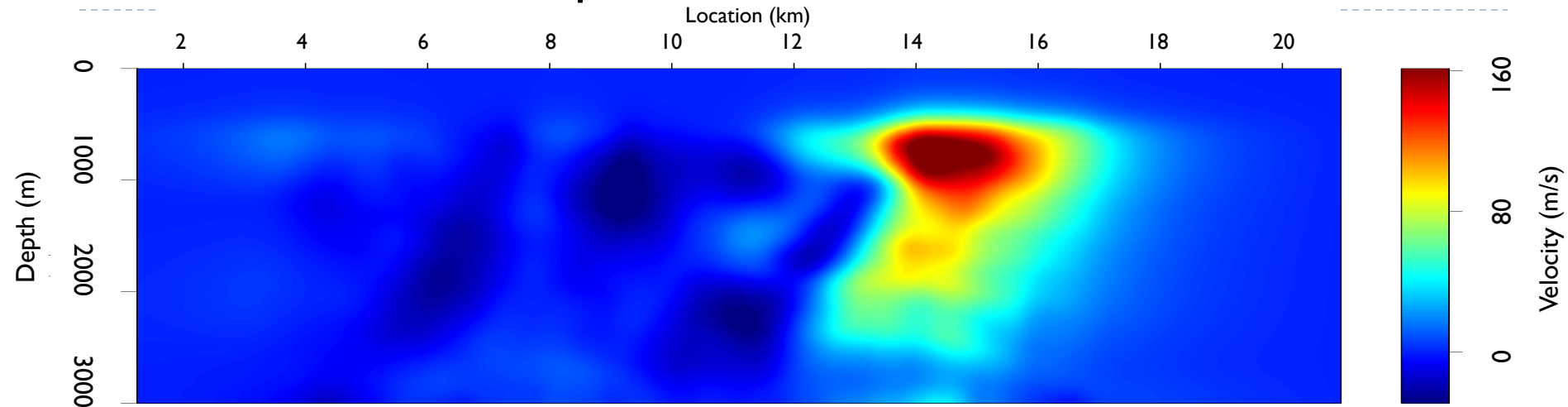


Inverted η model

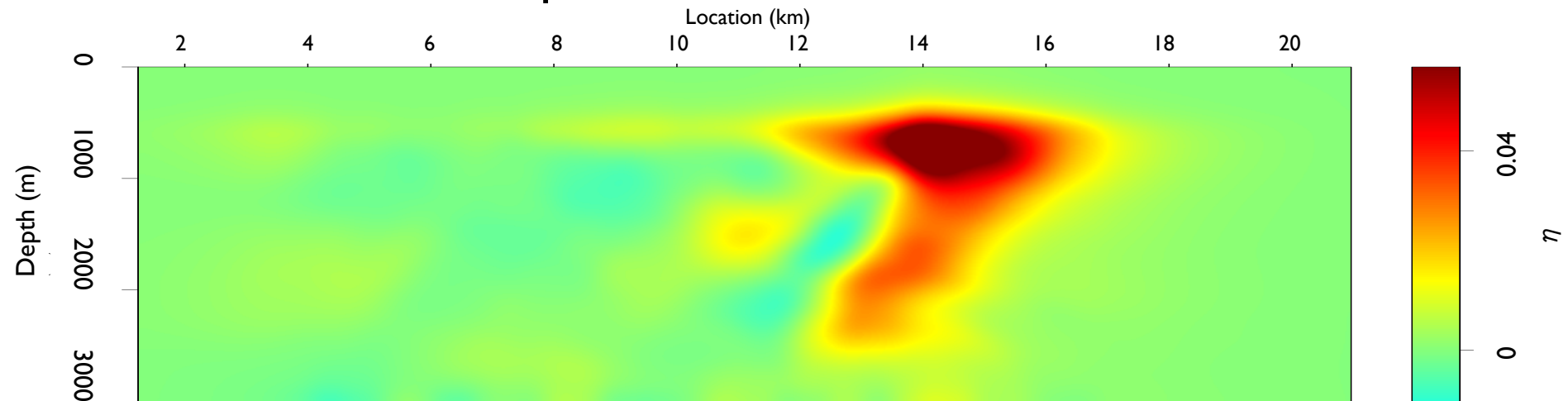


Example: ExxonMobil field data

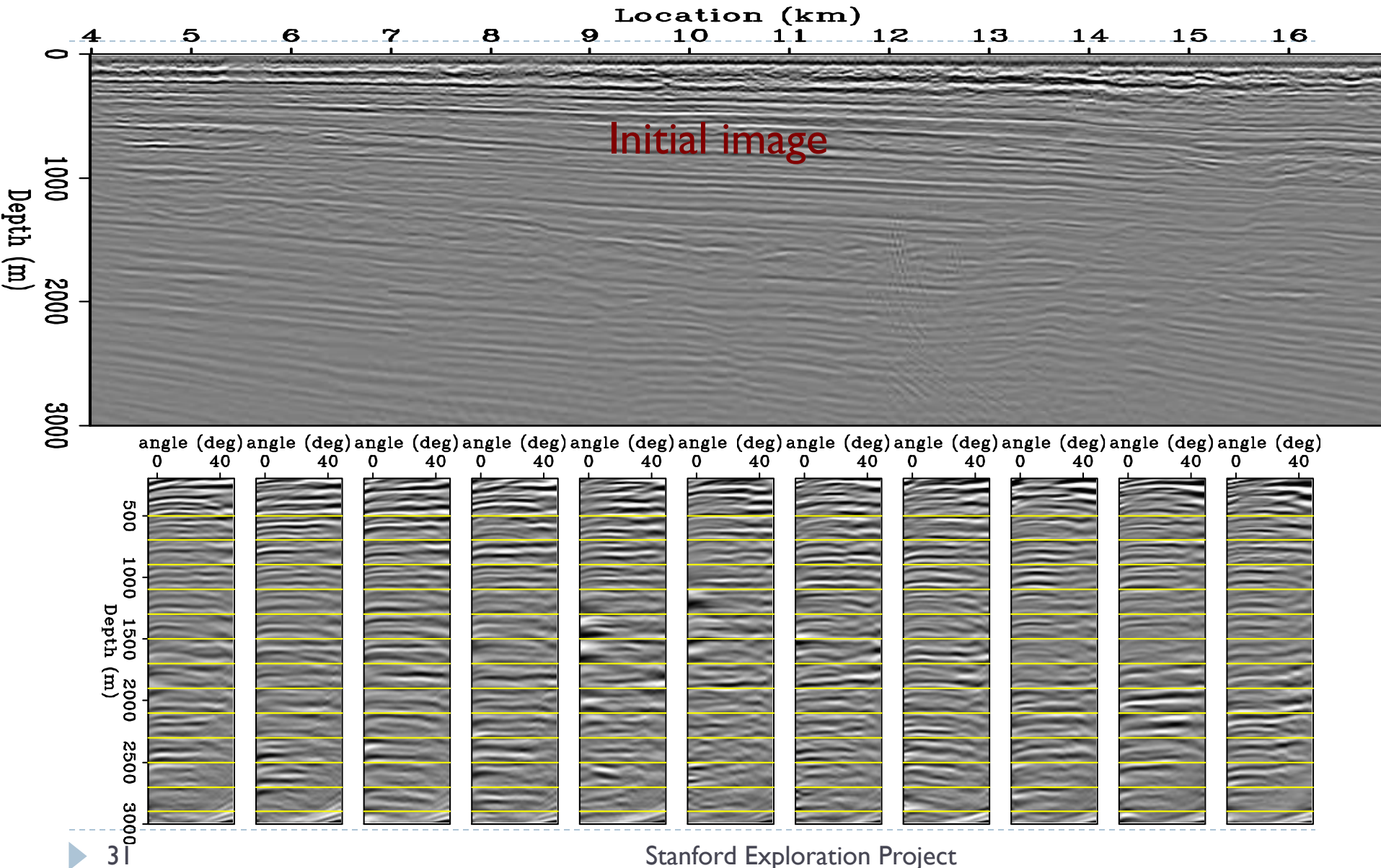
Updates in v model



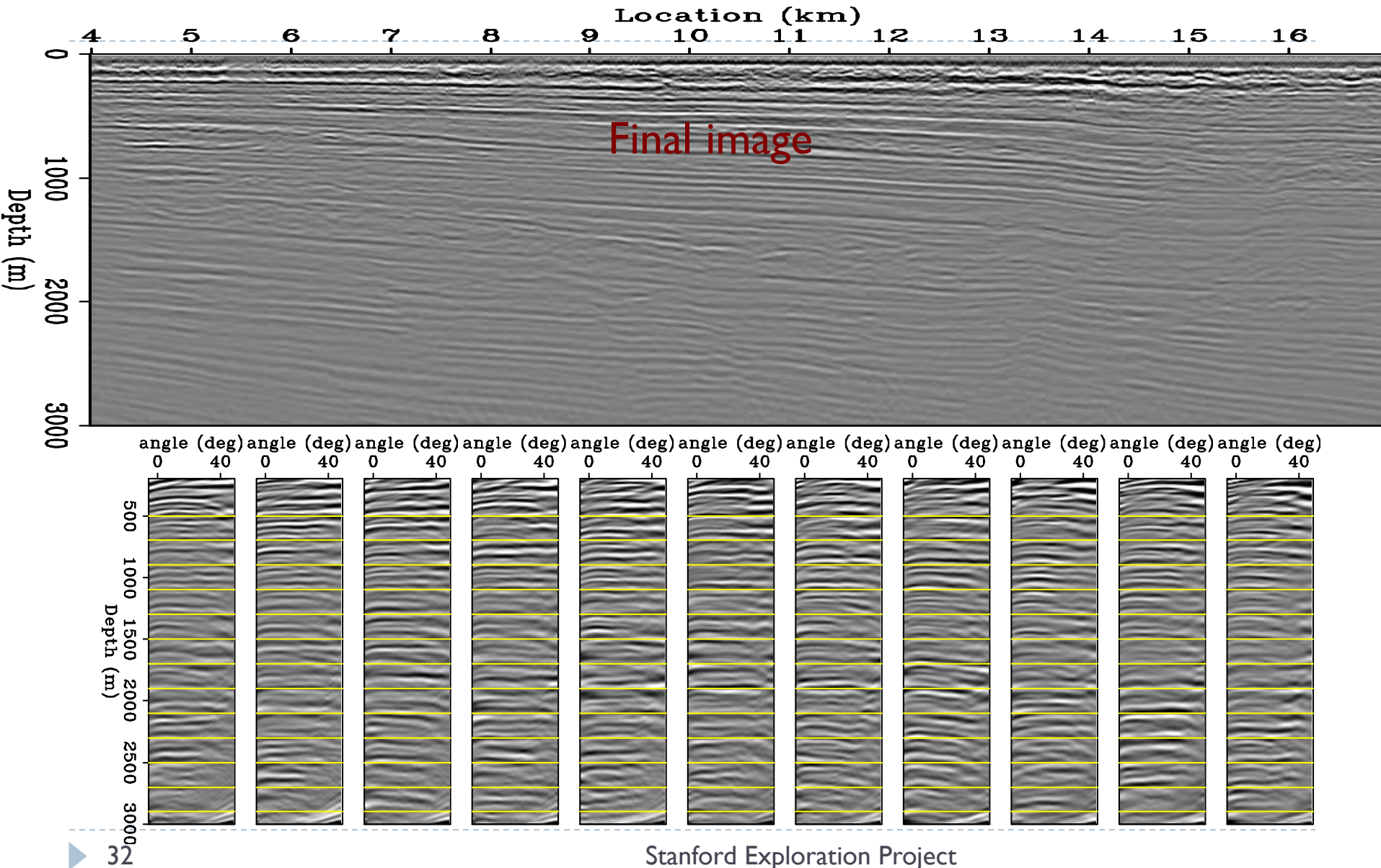
Updates in η model



Example: ExxonMobil field data



Example: ExxonMobil field data



Example: ExxonMobil field data

▶ Summary

- ▶ Able to identify a shallow velocity and η anomalies
- ▶ Improve the stacked image: higher resolution and better defined fault
- ▶ Improve the flatness of the angle domain common image gathers
- ▶ PSPI one-way VTI wave-equation
 - + Low (relative) computational cost
 - + Works for gently dipping reflectors
 - Inaccurate for high angle waves

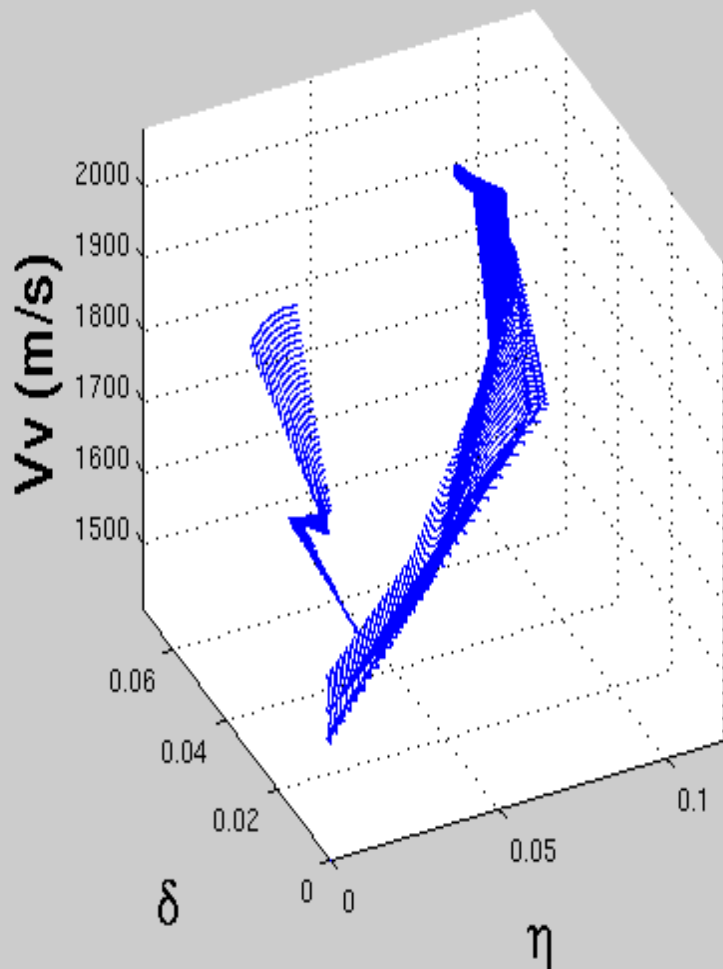
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Conclusions and discussions

- ▶ Anisotropic WEMVA can improve the anisotropic earth model and produce better focused images.
- ▶ An optimal propagator is needed to achieve accuracy and efficiency at the same time.
- ▶ Ambiguity between velocity and other anisotropic parameters.

MC3 I I – 3D scatter of parameters



▶ Prior information

- ▶ Distinct trends across the fault
- ▶ Information need to be spread according to geology
- ▶ Cross-parameter standard deviation matrix (Li et al., 2011)

Acknowledgement

- ▶ We thank Shell International Exploration & Production Company for the permission to publish the work.
- ▶ Thanks to ExxonMobil for the permission to publish the field data.
- ▶ Thank you all for your attention!

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