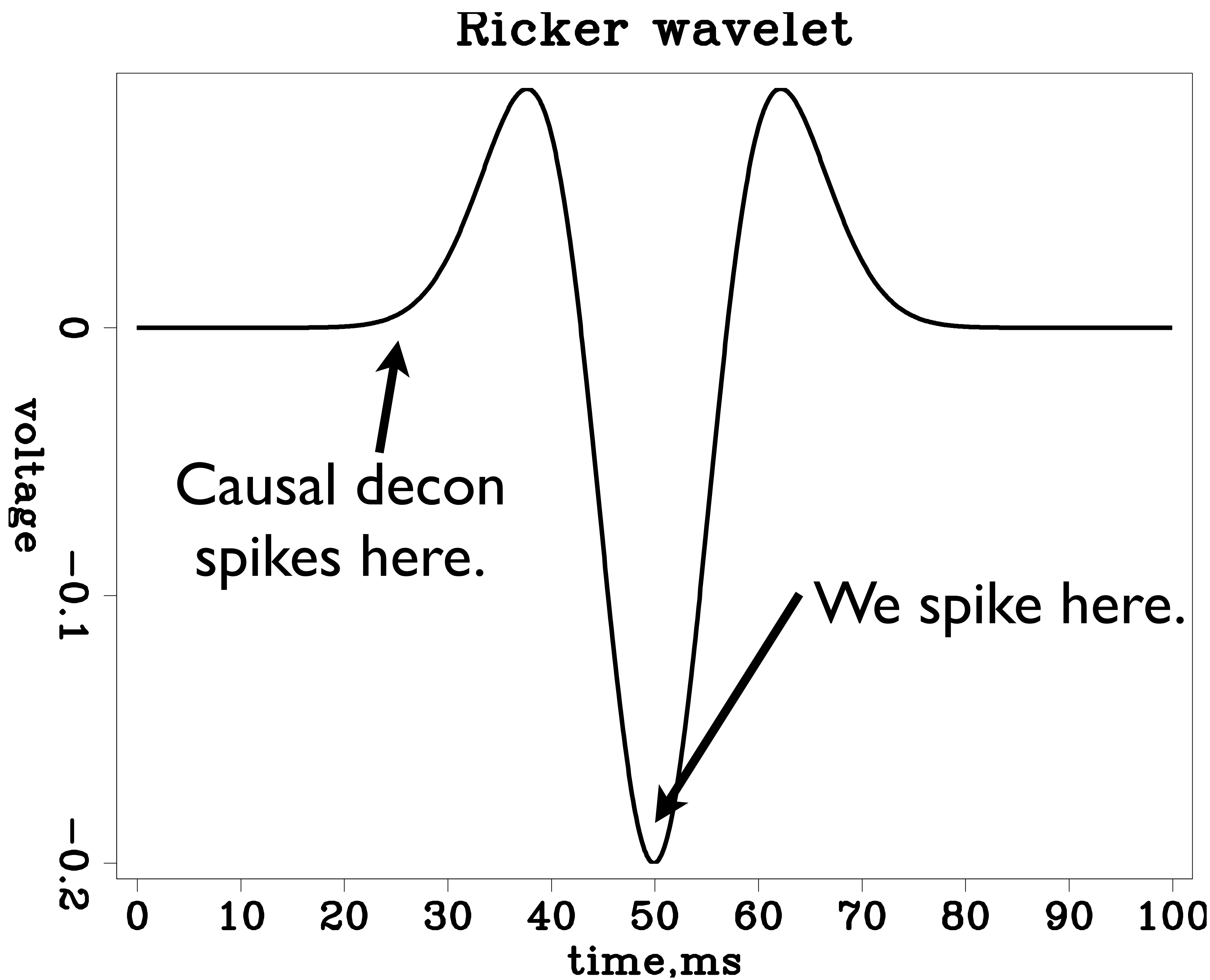
Mostly causal decon clarifies marine seismogram polarity.

by Jon Claerbout

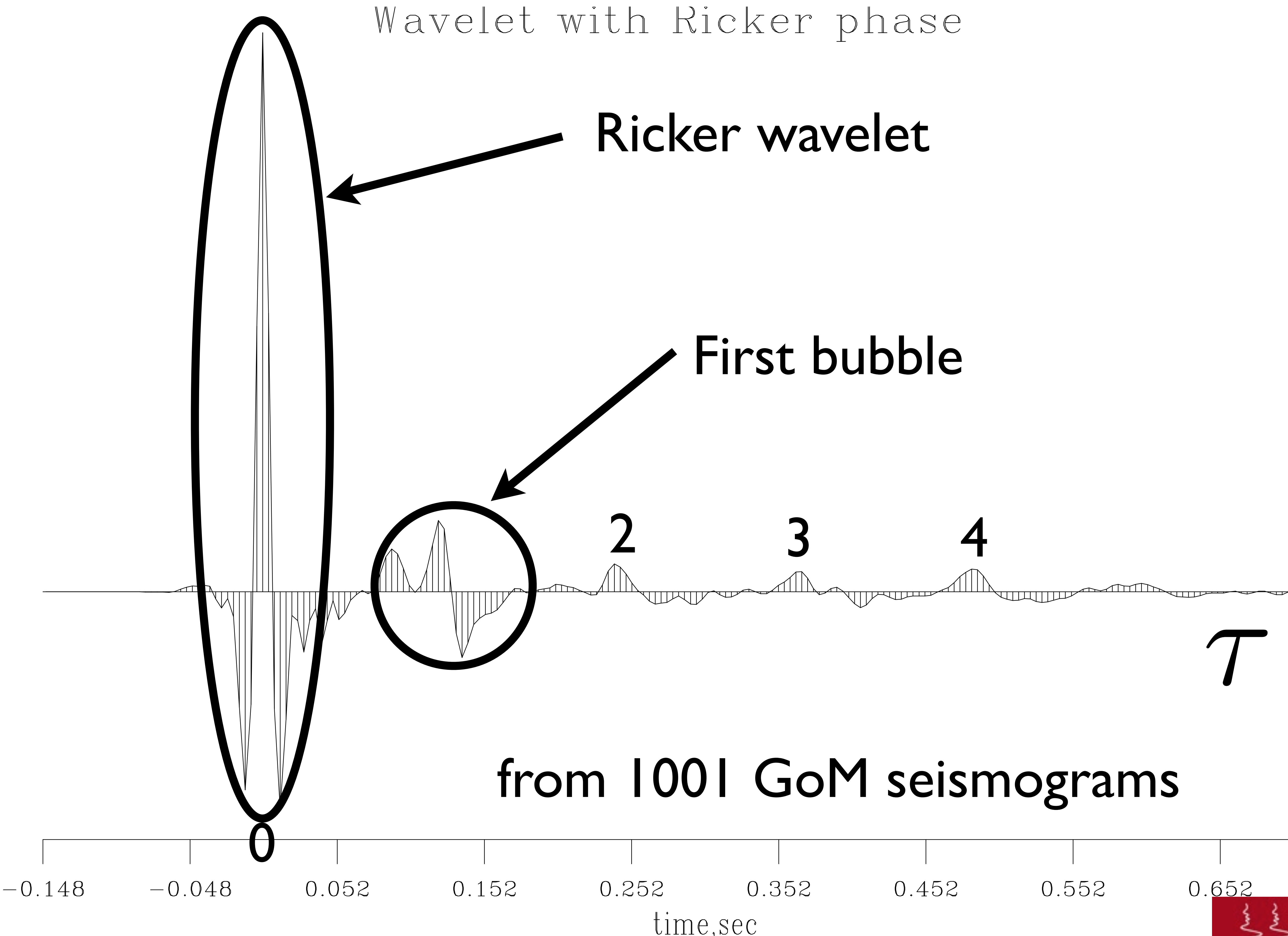
SEP meeting
May 2012

Report page 147, page 305

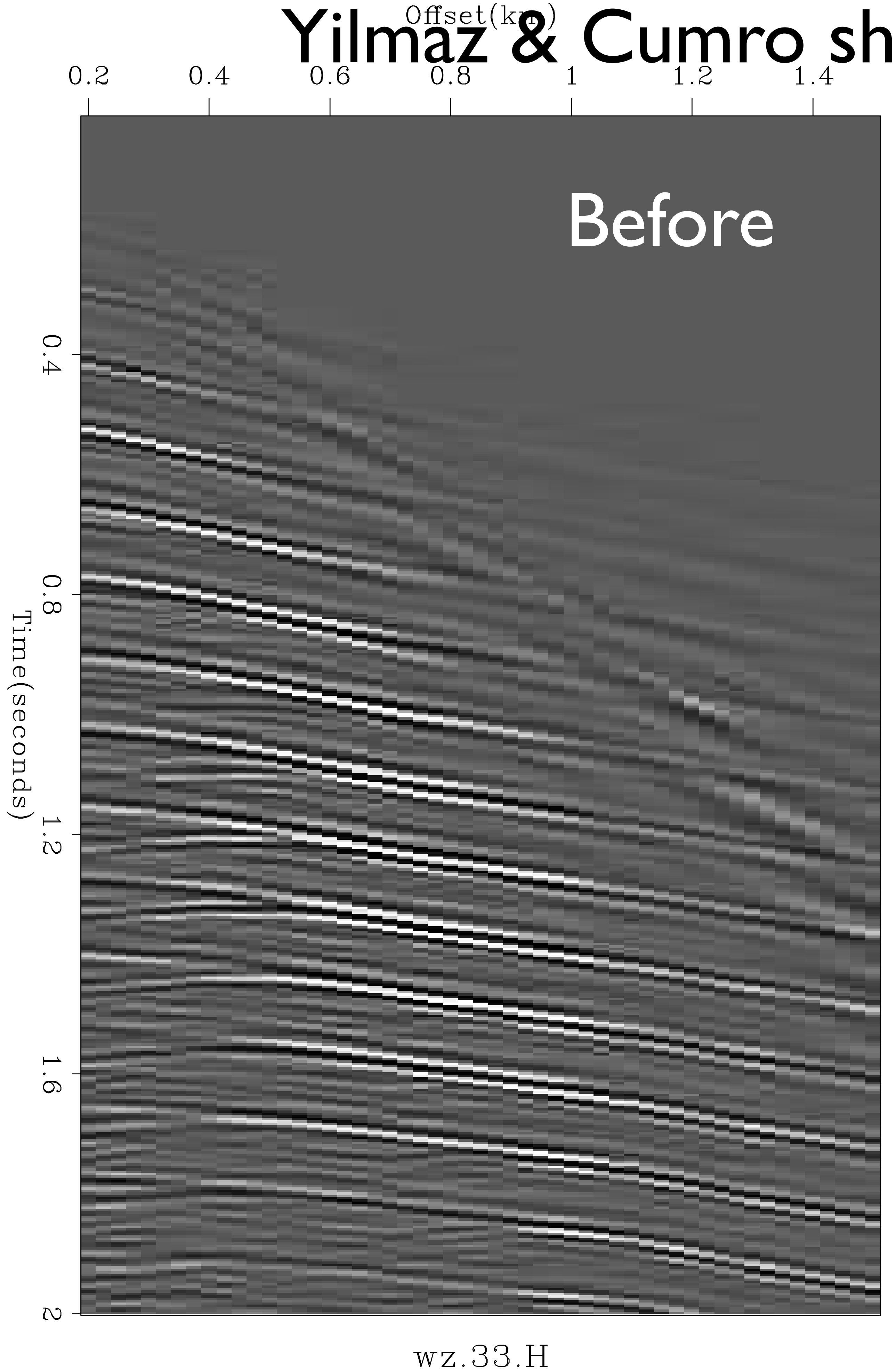

Ricker wavelet
voltage
0
−0.1
−0.2
0
10
20
30
40
50
60
70
80
90
100
time,ms
Causal decon spikes here.
We spike here.

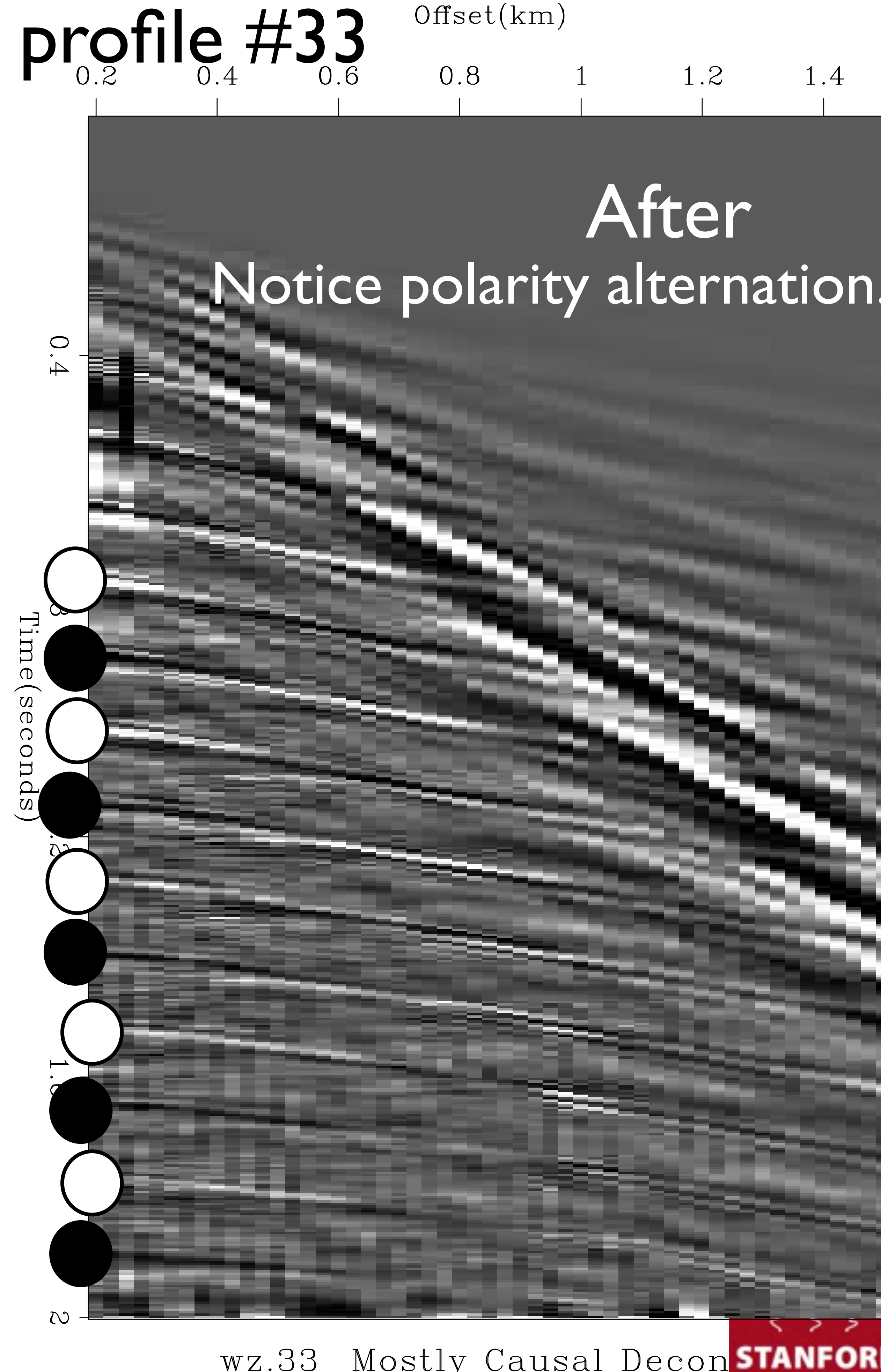
See vplot movie

Shot waveforms varying with the amount of pre-causal time.


Wavelet with Ricker phase
Ricker wavelet
First bubble
2
3
4
τ
from 1001 GoM seismograms
0
−0.148
−0.048
0.052
0.152
0.252
0.352
0.452
0.552
0.652
time,sec

Yilmaz & Cumro shot profile #33





STANFORD EXPLORATION PROJECT

Why does this work?

Deconvolve with the right wavelet.

Then seismogram polarity becomes clear.

Why does this work?

Deconvolve with the right wavelet.

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Why does conventional decon fail?

Ricker wavelets have no causal inverse.

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Generally equivalent terms and concepts

Blind decon
Predictive decon
Causal decon

$t,\ N^2$ ↕

Autoregression, Yule&Walker 1927
Minimum-phase decon, MIT GAG 1954
Wiener-Levinson-Burg decon, Toeplitz

$\omega,\ N \log N$ ↕

Kolmogoroff decon (1939)
(in my textbook FGDP 1974)
(the code is in my book PVI 1992)

STANFORD EXPLORATION PROJECT

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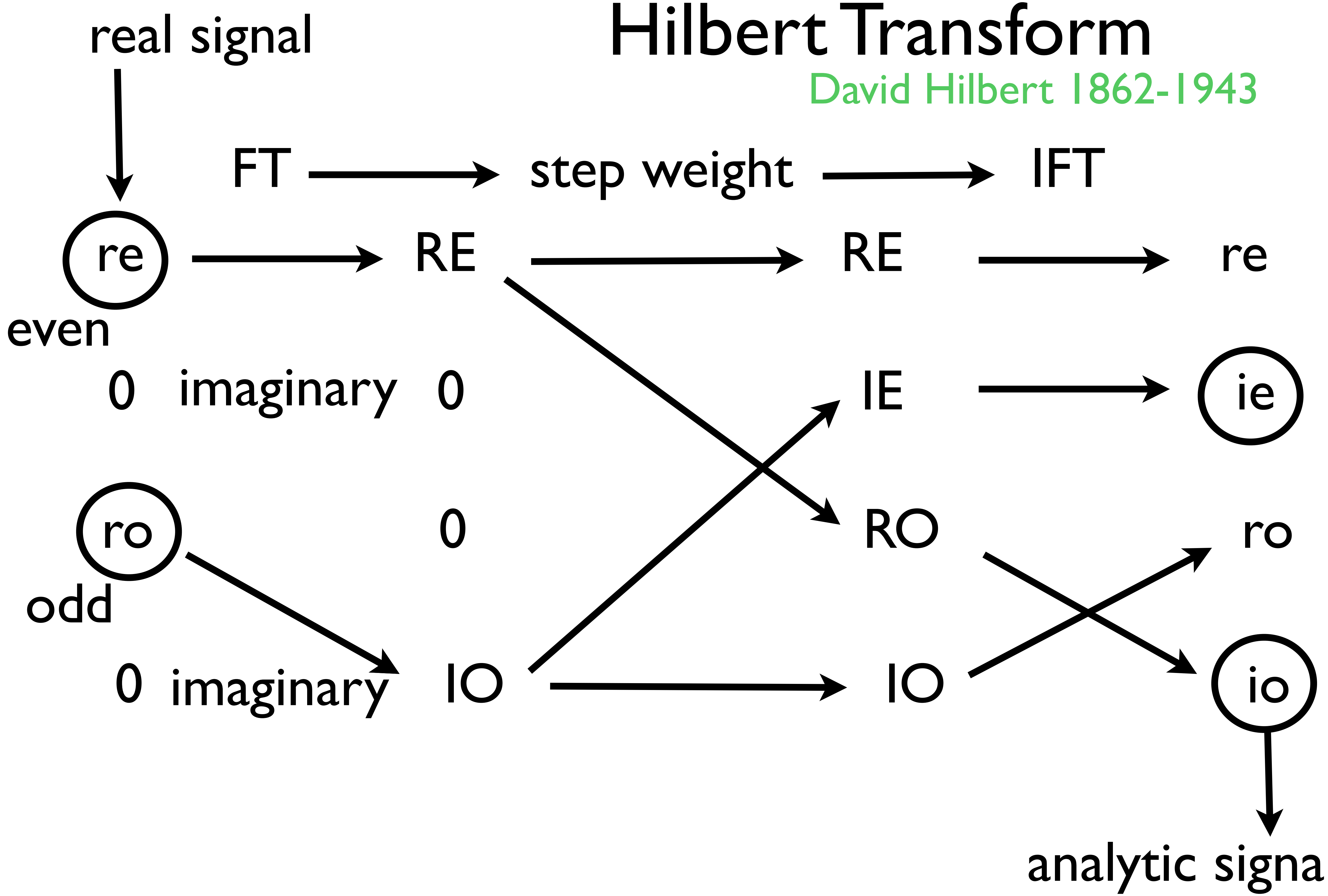
$t,\ N^2$ ↕ (Autoregression through Wiener-Levinson-Burg decon)

$\omega,\ N \log N$ ↕ (Kolmogoroff decon)

I adapt Kolmogoroff to "mostly causal" inverse.

Hilbert Transform

David Hilbert 1862-1943



Imaginary part is 90 degree phase shifted.

$$\bar{S} \quad = \quad e^{\log \bar{S}} \quad = \quad e^{\bar{U}}$$

40 years later
Kolmogorov

Hilbert $$S \quad = \quad e^{\log \bar{S} + i\Phi} \quad = \quad e^{\bar{U} + i\Phi} \quad = \quad e^{U}$$

$\bar{S}$ is a given amplitude spectrum.
Φ is an unknown phase.
$s_\tau = \mathrm{FT}^{-1}[S]$ is the shot waveform.
$u_\tau = \mathrm{FT}^{-1}[U]$ is "lag-log" parameter space.

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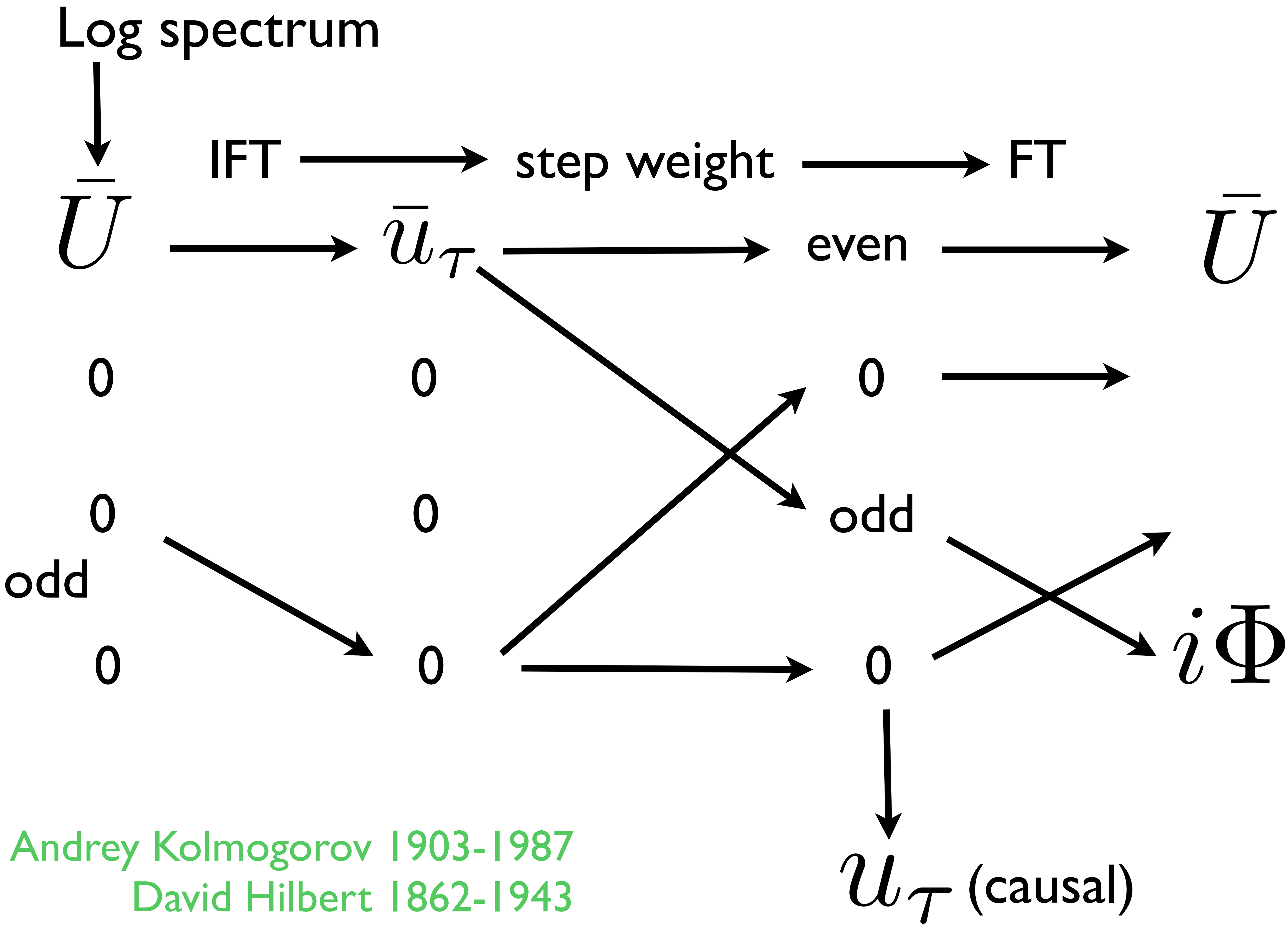

Andrey Kolmogorov 1903-1987
David Hilbert 1862-1943

Kolmogoroff-Wiener theorem (about 1940):

“If lag-log space u_τ is causal, then shot s_τ is too.”

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With this theorem you can do deconvolution by spectral factorization (and more!)

So let's prove it.

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Kolmogoroff-Wiener theorem (about 1940): "If lag-log space is causal then shot is too."

Given a causal time function $(1, u_1, u_2, u_3, \cdots)$ with $Z = e^{i\omega\Delta t}$, the Z-transform $U(Z) = 1 + u_1 Z + u_2 Z^2 + u_3 Z^3 + \cdots$ is secretly a Fourier series. Exponentiate $U(Z)$ by writing $e^{U(Z(\omega))}$ for all ω then Fourier transforming.

Another exponential is $e^U = 1 + U + U^2/2! + U^3/3! + \cdots$. Inserting U into e^U gives us a new polynomial (infinite series) with no powers of $1/Z$. It always converges because of the powerful influence of the denominator factorials. Thus we have shown that the "exponential of a causal is a causal".

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In Fourier space,

the wavelet is e^{U};

its inverse is e^{-U}.

Main facts about lag-log u(t) space

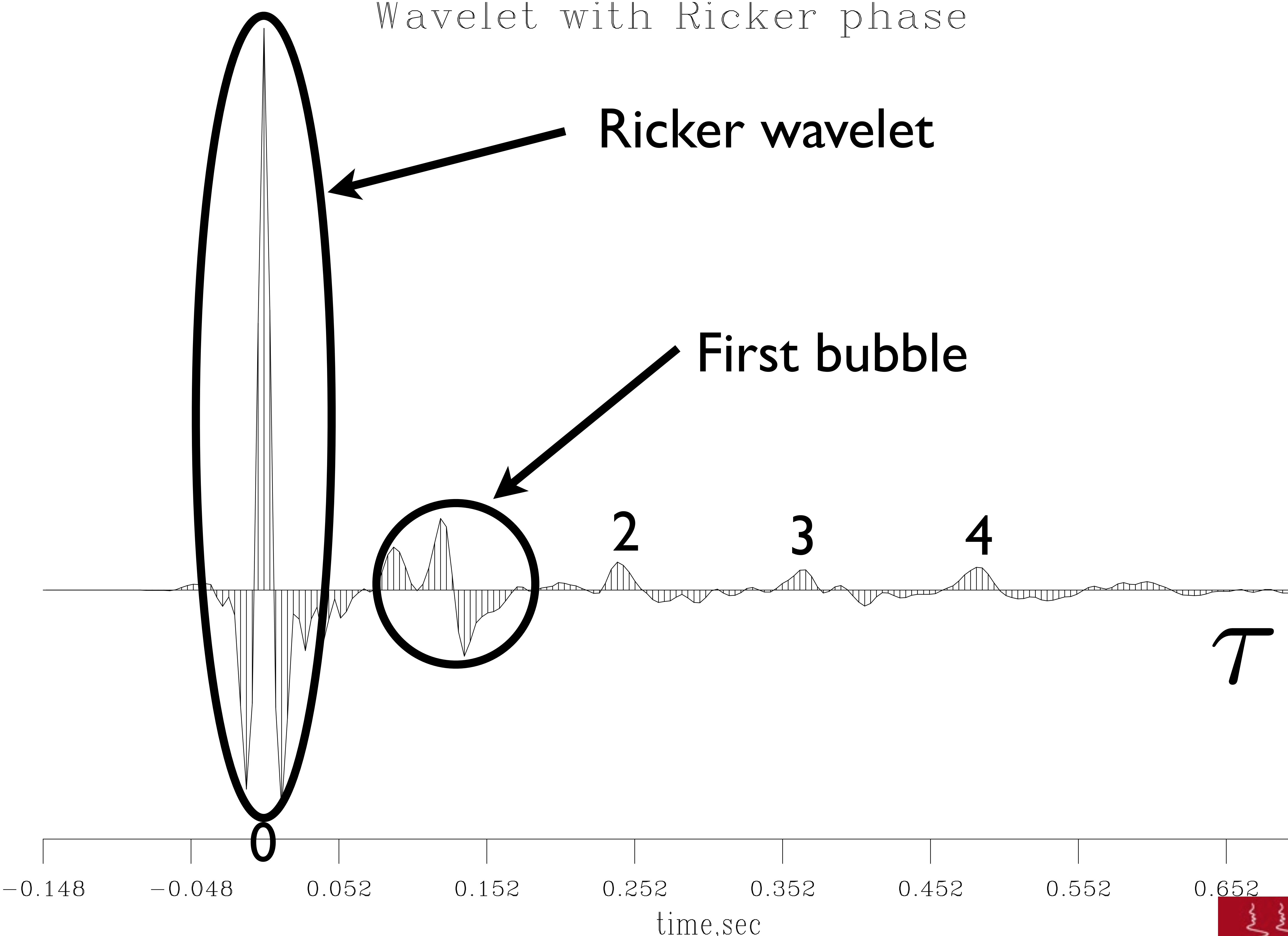
1. Large valued lags in u_τ affect *only large lags* in the wavelet $\mathrm{IFT}(\mathrm{e}^{\mathrm{U}})$ or the decon filter $\mathrm{IFT}(\mathrm{e}^{-\mathrm{U}})$. Why? Put $U = 1 + Z^{10}$ into e^U.

Main facts about lag-log u(t) space

1. Large valued lags in u_τ affect *only large lags* in the wavelet IFT(e^{U}) or the decon filter IFT($\mathrm{e}^{-\mathrm{U}}$). Why? Put $U = 1 + Z^{10}$ into e^U.

2. Small valued lags in u_τ affect *mainly the small lags* in the wavelet and the decon filter.

Wavelet with Ricker phase



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3. The bubble is at the large lags; the Ricker wavelet is at the small.

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Here comes the innovation.

4. We are going to mess with the small lags. Ricker has no phase. No odd part, no phase.

The innovation

Identify the odd part of the lag-log space.

Weight it down at small lags.

That gives even response (Ricker-like) at small lags.

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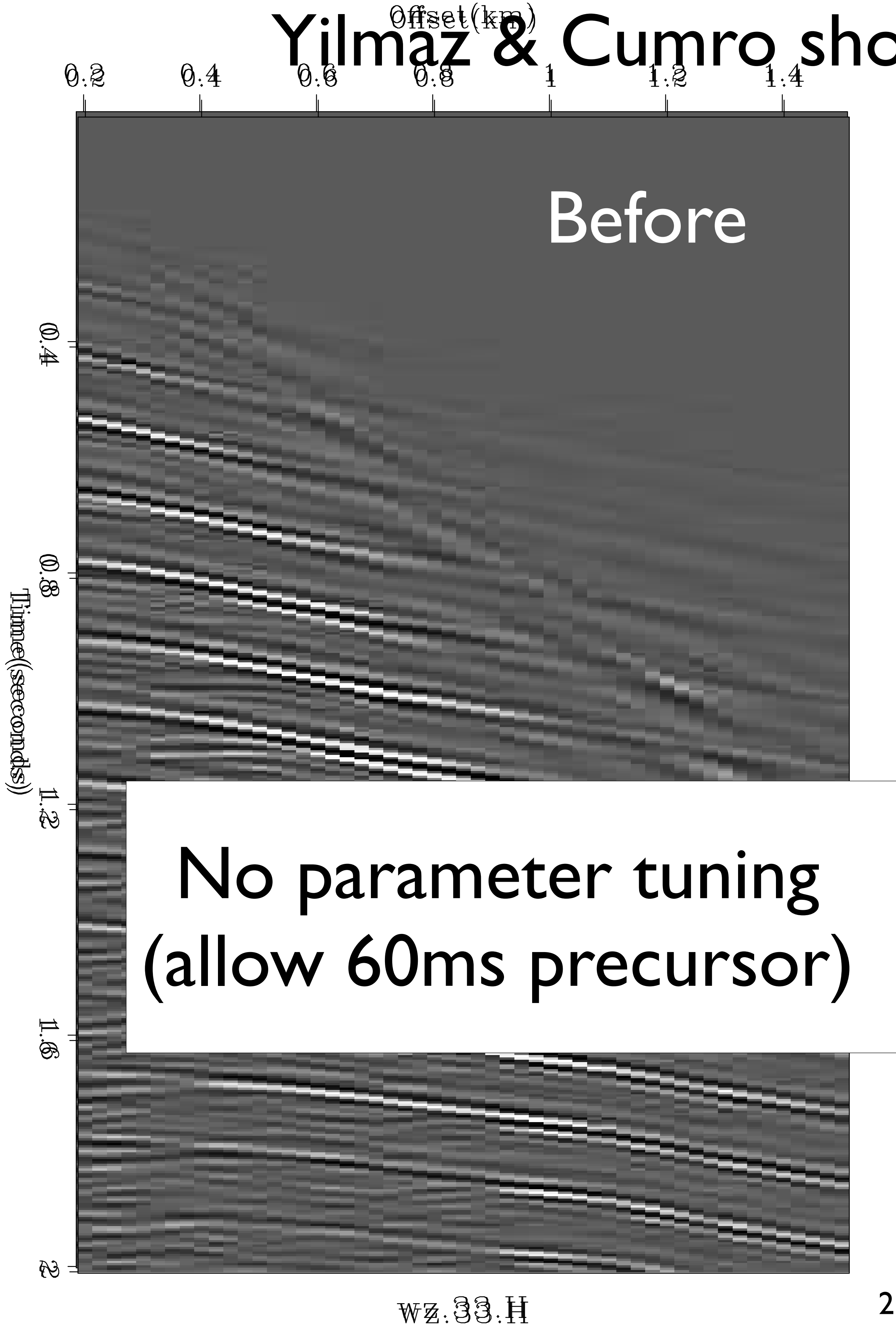

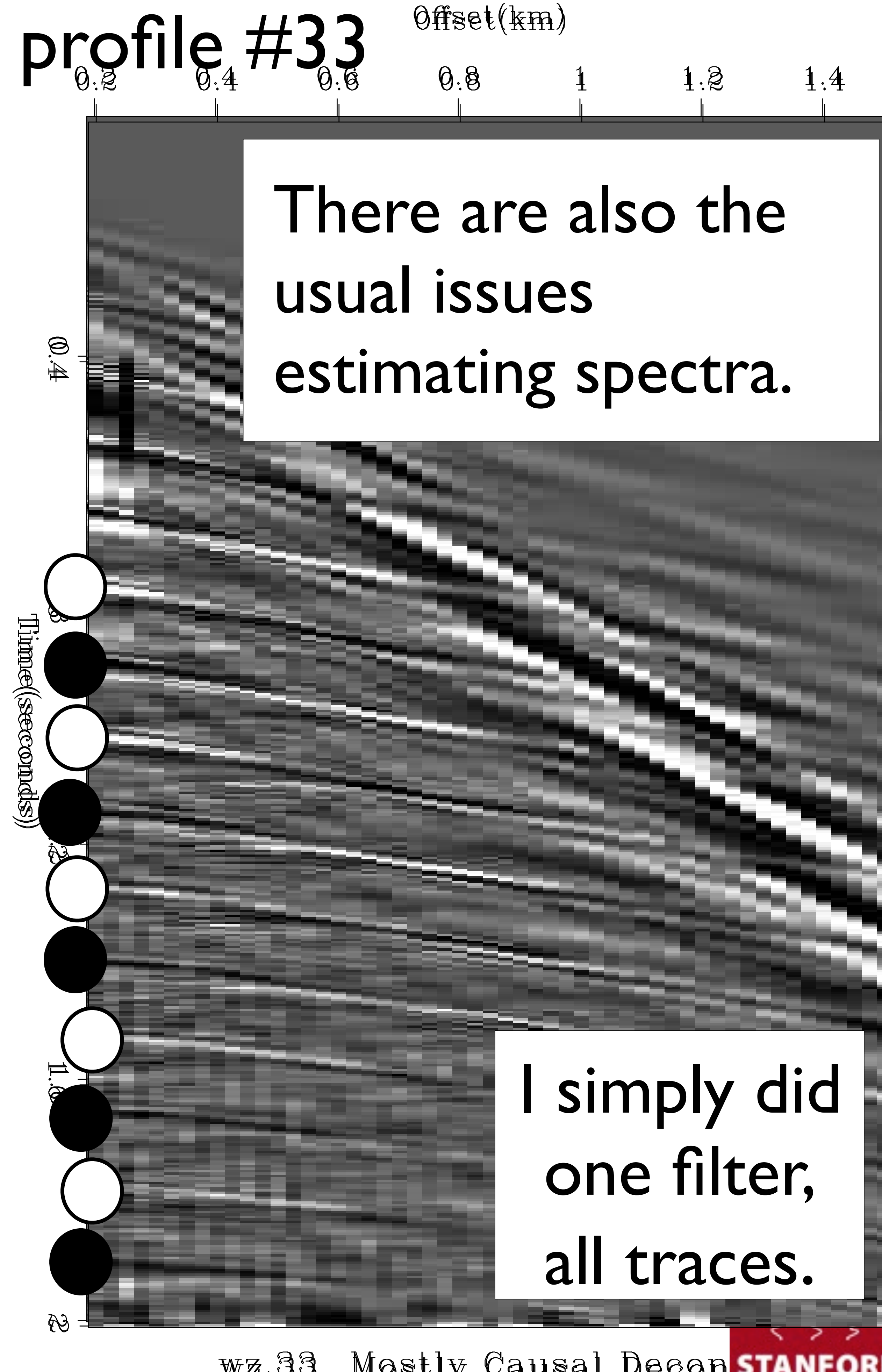

What's good about this

Predictive deconvolution makes the assumption that
the inverse source wavelet is causal,
which is untrue for Ricker wavelets.

Thus marine seismology is ripe for a revolution,
after which polarity should be routinely observable.

It's a starting solution and
a regularization for inverse theory.

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What's bad about this

It makes the false assumption that
a white output is desirable.

It ignores sparseness
as a characteristic of much real data.

It makes the false assumption that
echo data may be gained before filtering.

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That's all there is to it!

The code is listed in the article.
(six lines added to the textbook code)

Enjoy!

p.s. If you make any examples, I'd love to see them.

The end...

The end...

The last practice talk for this talk
is available at youtube.com

http://sep.stanford.edu/sep/jon/