Imaging with multiples using linearized full-wave inversion

May 22nd, 2012
SEP 147 p. 249-262

Mandy Wong*
Biondo Biondi
Shuki Ronen

Overview

- Introduction
- Theory
  - Linearized full-wave inversion
- Synthetic examples
  - One reflector model
  - Sigbee2B model
- Discussion and Conclusion

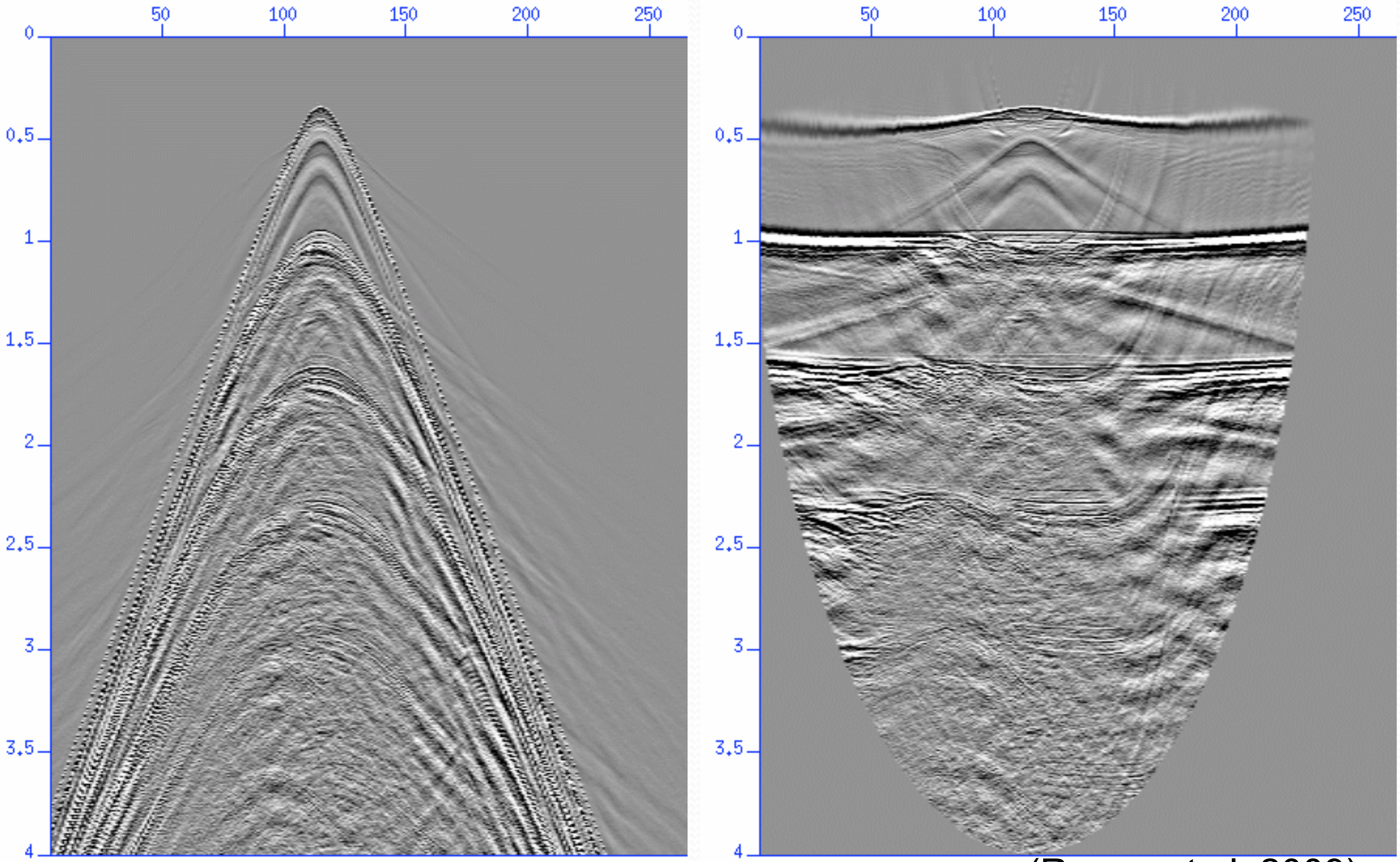
Imaging primary after demultiple

- Traditional multiple removal tools
  - Deconvolution
  - radon-transform demultiple
  - f-k demultiple
- Convolution-based techniques
  - Surface-related multiple removal (SRME)
  - Generalized surface multiple prediction (GSMP)
- Model based techniques
  - Modeling and subtractions

Imaging with multiples

- Motivations for imaging with multiples
  - multiples can provide sub-surface information not found in primary
  - incomplete removal of all multiples from the primaries
    - Migrating multiples as primary results in crosstalk artifacts in the image
  - Multiples are sensitive to velocity information

Imaging with multiples



(Ronen et al. 2009)

Imaging with multiples

- Reiter et al. (1991) developed a prestack Kirchhoff time-migration method first-order water-layer reverberation.
- Using surface-related multiples as source
  - Shot-profile migration (Guitton 2002)
  - Source-receiver migration (Shan 2003)
  - Reverse time migration (Liu et al. 2011)
- Muijs et al. (2007) image primary and free-surface multiples for OBS data with up-going and down-going data.
- Brown (2004) uses linearized inversion with a NMO based operator

Overview

- Introduction
- Theory
  - Linearized full-wave inversion
- Synthetic examples
  - One reflector model
  - Sigbee2B model
- Discussion and Conclusion

Linearized full-wave inversion (LFWI)

Linearizing the wave equation with respect to model m

$$s(\mathbf{x}) = s_o(\mathbf{x}) + \Delta s(\mathbf{x})$$

$s(\mathbf{x})$ true slowness

$s_o(\mathbf{x})$ migration slowness

$\Delta s(\mathbf{x})$ slowness perturbation

Linearized full-wave inversion (LFWI)

Linearizing the wave equation with respect to model m

$$s(\mathbf{x}) = s_o(\mathbf{x}) + \Delta s(\mathbf{x})$$

$$m(\mathbf{x}) = \Delta s(\mathbf{x}) s_o(\mathbf{x})$$

$s(\mathbf{x})$ **true slowness**

$s_o(\mathbf{x})$ **migration slowness**

$m(\mathbf{x})$ **model**

$\Delta s(\mathbf{x})$ **slowness perturbation**

Linearized full-wave inversion (LFWI)

LFWI forward modeling equation

$$d^{\,\mathrm{mod}}(\mathbf{x}_r, \mathbf{x}_s.\omega) = \sum_{\mathbf{x}} \omega^2 f_s(\omega) G_o(\mathbf{x}_s, \mathbf{x}, \omega) m(\mathbf{x}) G_o(\mathbf{x}, \mathbf{x}_r, \omega)$$

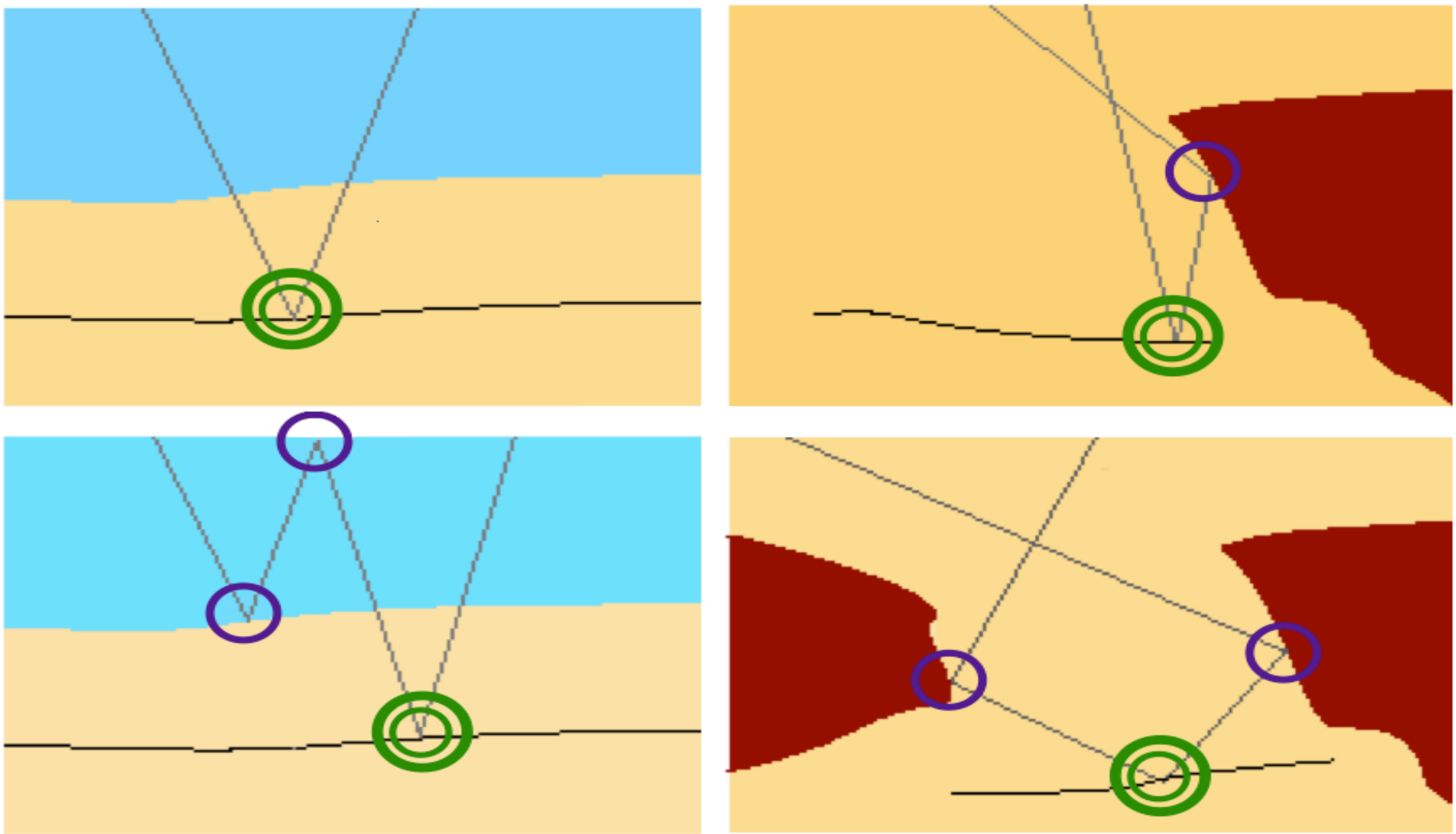
$d^{\,\mathrm{mod}}(\mathbf{x}_r, \mathbf{x}_s.\omega)$ **forwarded modeled data**

$f_s(\omega)$ **source waveform**

$G_o(\mathbf{x}_1, \mathbf{x}_2, \omega)$ **Green's function of the two-way acoustic constant density wave-equation over the migration slowness**

Linearized full-wave inversion (LFWI)

$$d^{\text{mod}}(\mathbf{x}_r, \mathbf{x}_s . \omega) = \sum_{\mathbf{x}} \omega^2 f_s(\omega) G_o(\mathbf{x}_s, \mathbf{x}, \omega) m(\mathbf{x}) G_o(\mathbf{x}, \mathbf{x}_r, \omega)$$



Linearized full-wave inversion (LFWI)

LFWI forward modeling equation

$$d^{\,\mathrm{mod}}(\mathbf{x}_r, \mathbf{x}_s.\omega) = \sum_{\mathbf{x}} \omega^2 f_s(\omega) G_o(\mathbf{x}_s, \mathbf{x}, \omega) m(\mathbf{x}) G_o(\mathbf{x}, \mathbf{x}_r, \omega)$$

Adjoint of LFWI forward modeling

$$\hat{m}(\mathbf{x}) = \sum_{\mathbf{x}_r, \mathbf{x}_s, \omega} \omega^2 f_s^*(\omega) G_o^*(\mathbf{x}_s, \mathbf{x}, \omega) G_o^*(\mathbf{x}, \mathbf{x}_r, \omega) d(\mathbf{x}_r, \mathbf{x}_s.\omega)$$

Linearized full-wave inversion (LFWI)

LFWI forward modeling equation

$$\mathbf{d}^{mod} = \mathbf{F}\mathbf{m}$$

Adjoint of LFWI forward modeling

$$\hat{\mathbf{m}} = \mathbf{F}^T \mathbf{d}$$

Linearized full-wave inversion (LFWI)

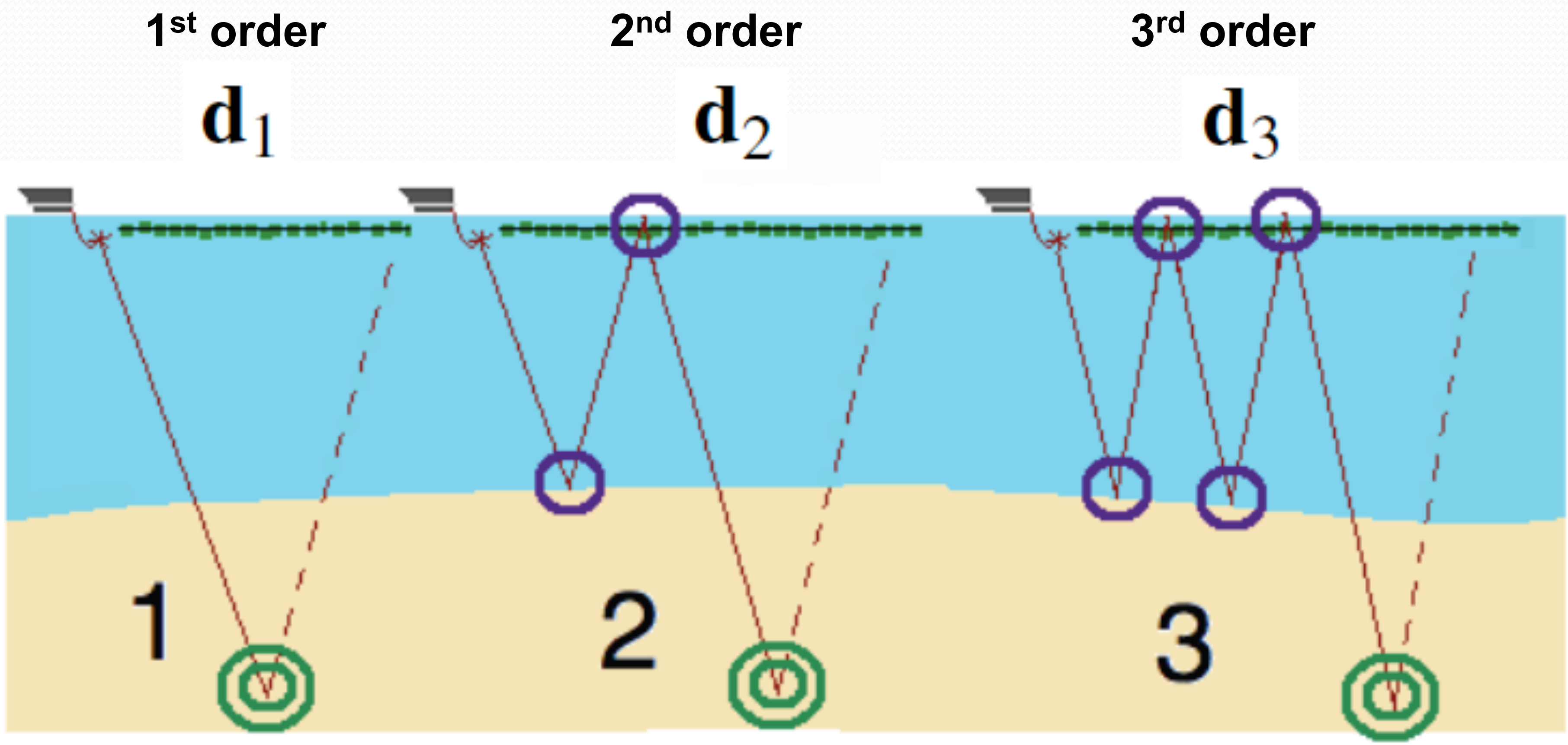
Minimizing the objective function

$$S(\mathbf{m}) = \|\mathbf{Fm} - \mathbf{d}\|^2$$

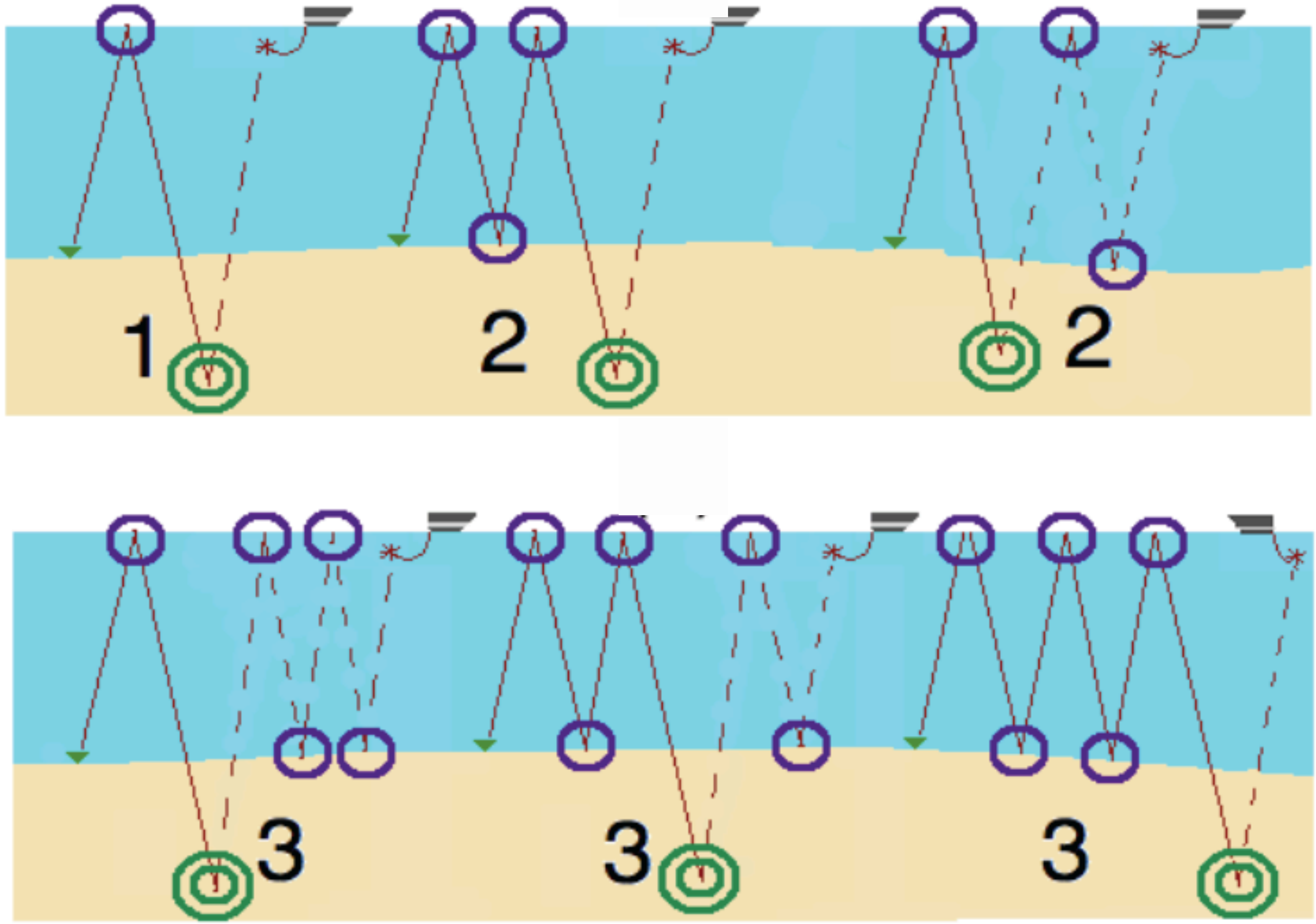
$\mathbf{F}$ LFWI forward modeling operator

$\mathbf{d}$ recorded data

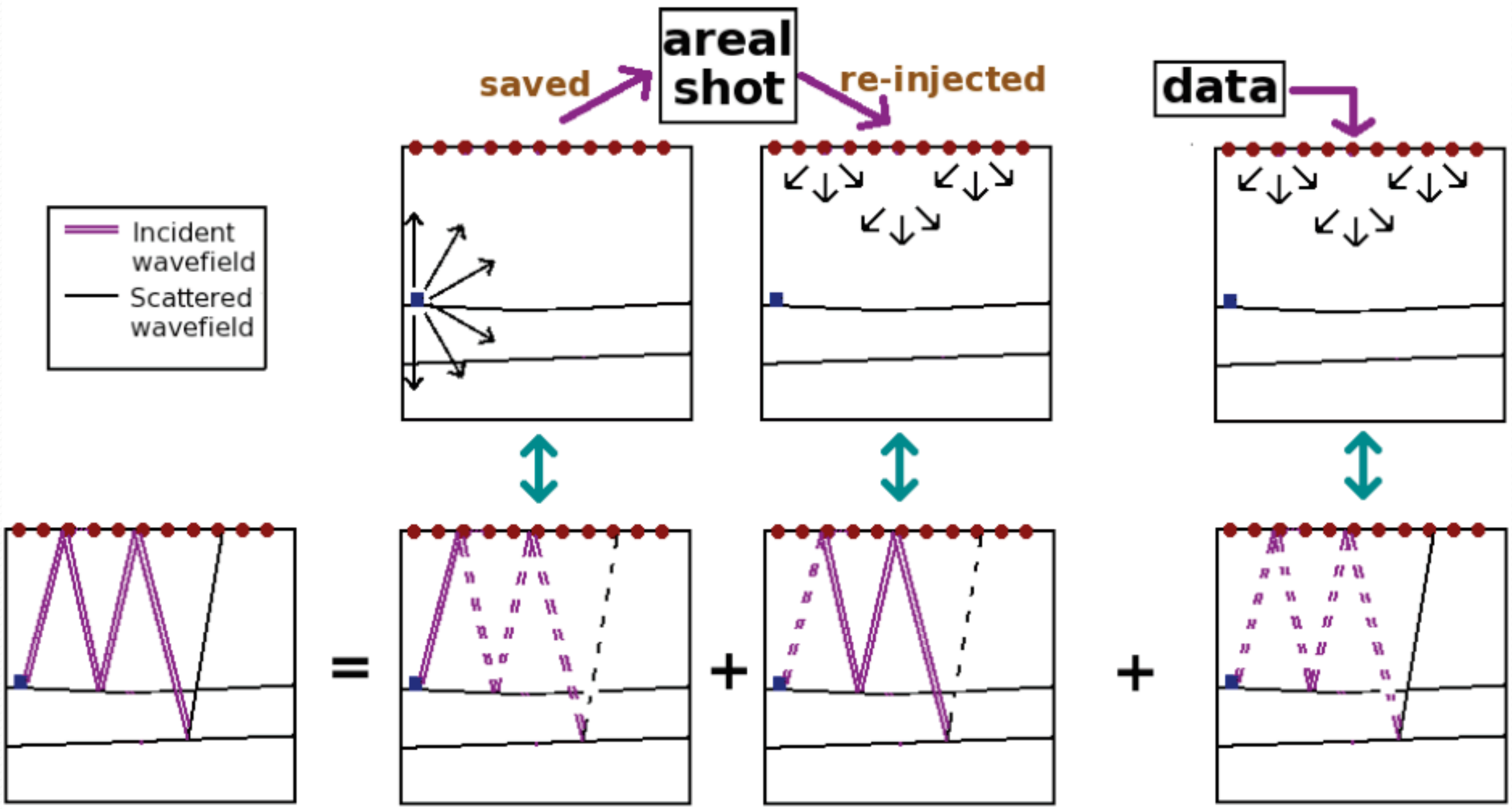
LFWI with towed-streamer data



LFWI with OBN data



LFWI with OBN data



LFWI vs. primary-only

LFWI forward modeling operator

$$\mathbf{F} = \mathbf{L}_1 + \mathbf{L}_2 + \mathbf{L}_3 + \dots$$

Primary only forward modeling operator

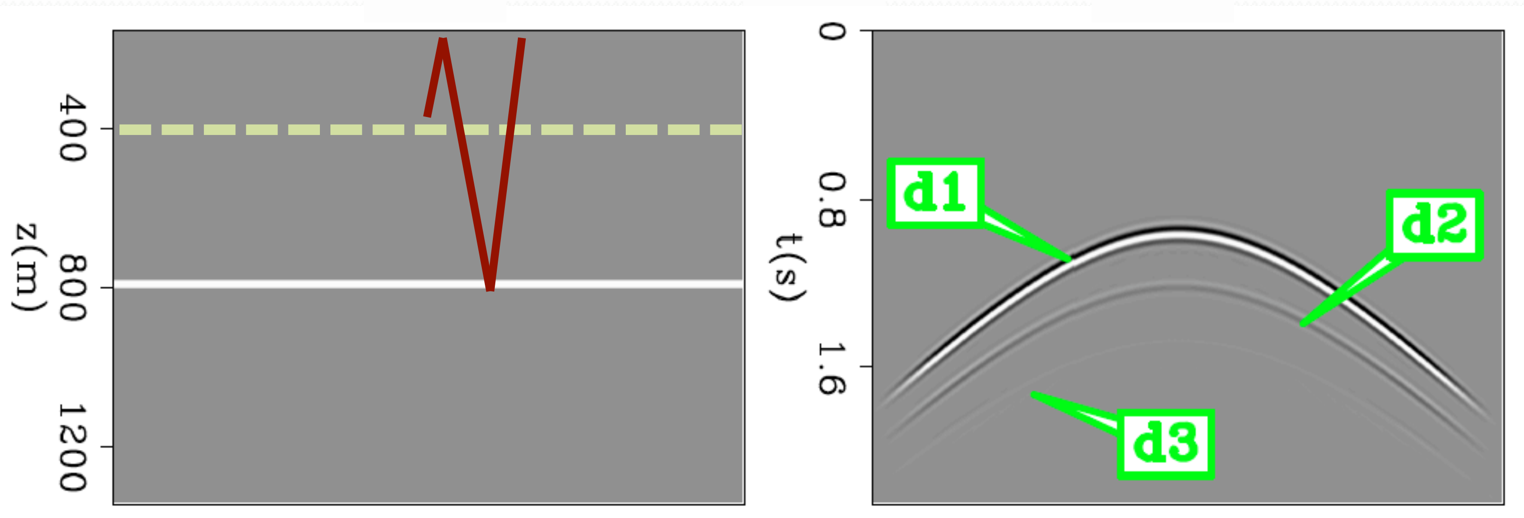
$$\mathbf{L}_p = \mathbf{L}_1$$

Overview

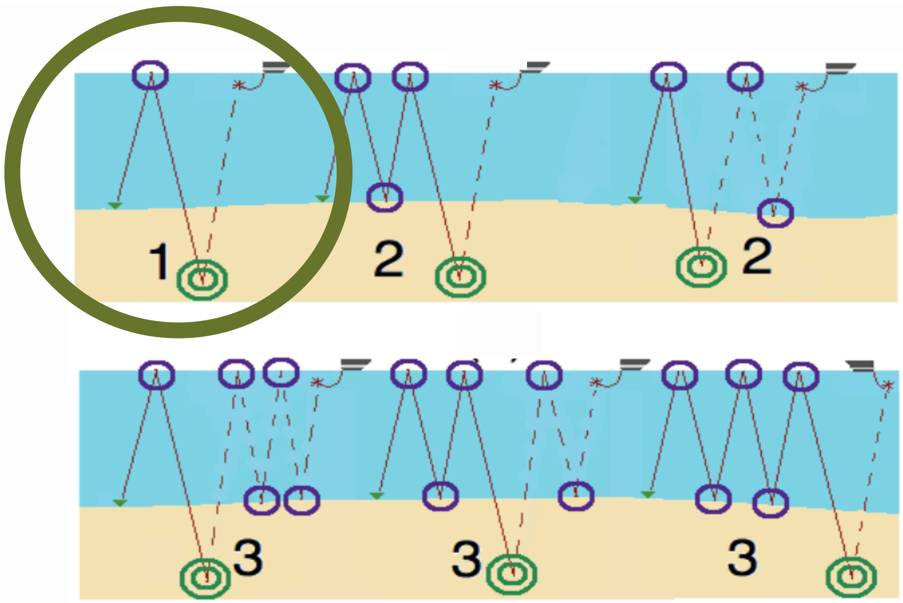
- Introduction
- Theory
  - Linearized full-wave inversion
- Synthetic examples
  - One reflector model
  - Sigbee2B model
- Discussion and Conclusion

One reflector below seabed with OBN geometry

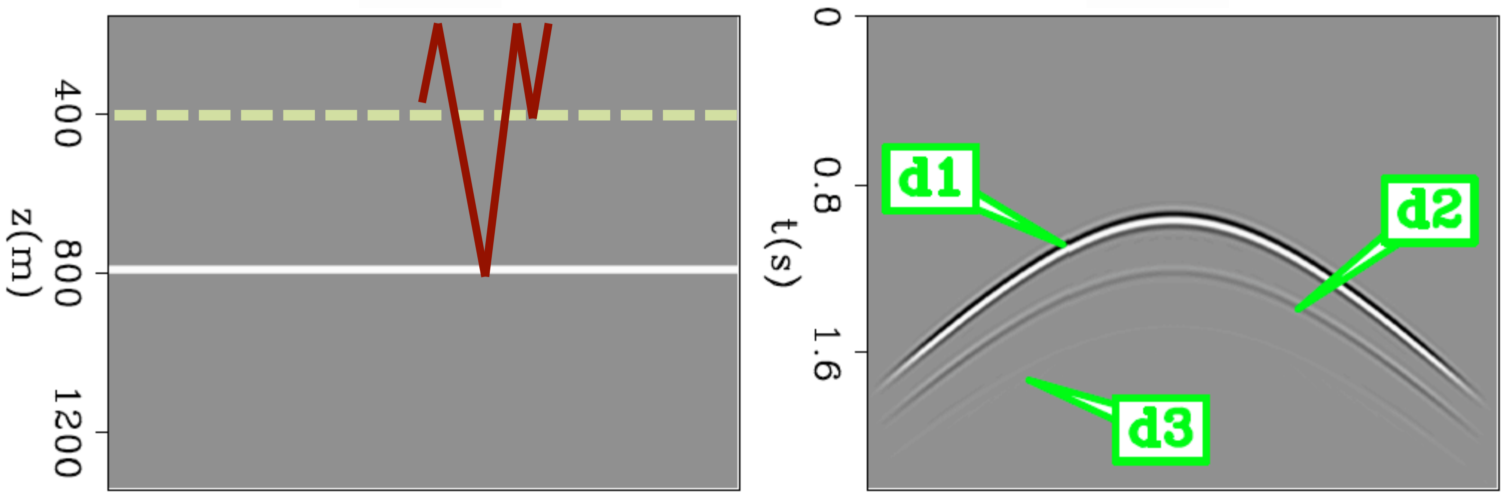
OBN synthetic data – first order

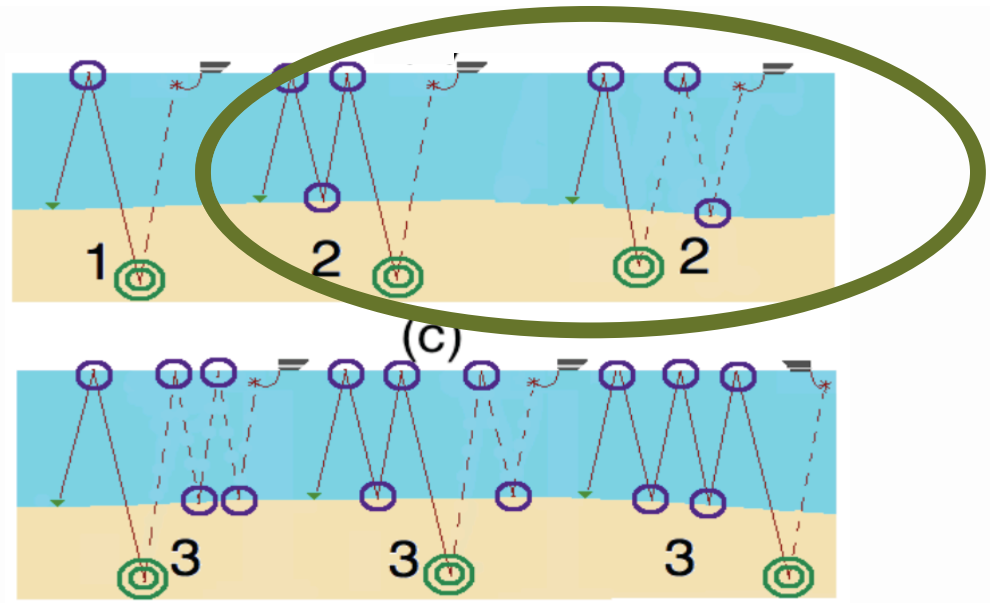


$$\mathbf{d}_1 = \mathbf{L}_1 \mathbf{m}$$



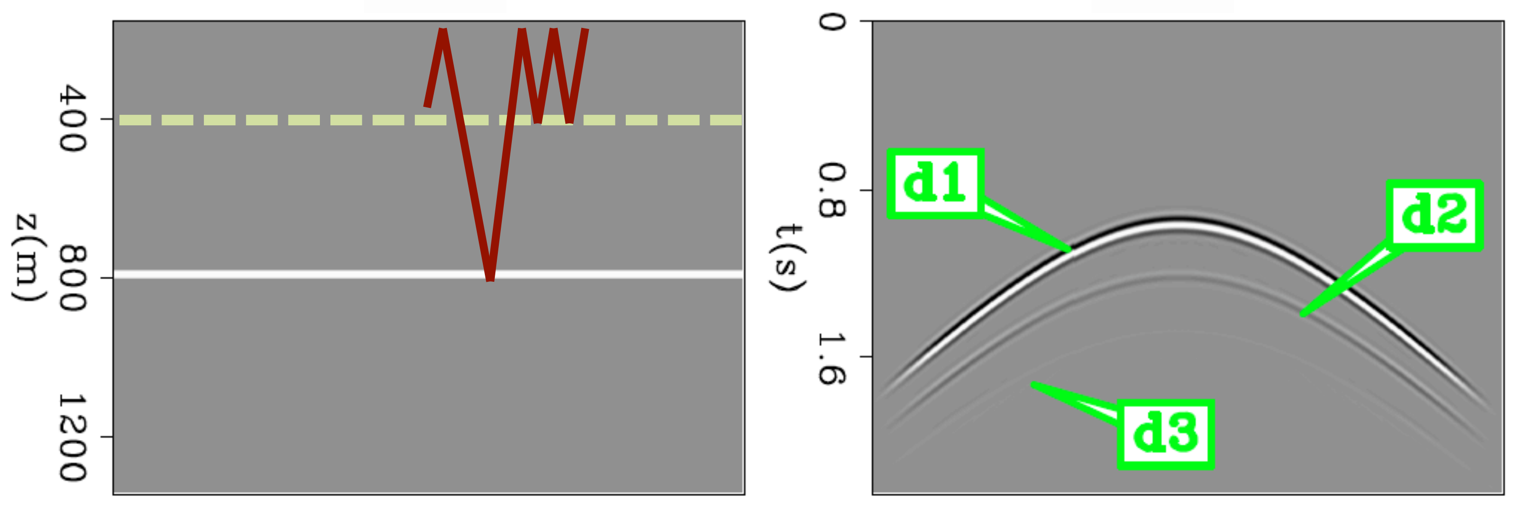
OBN synthetic data – second order



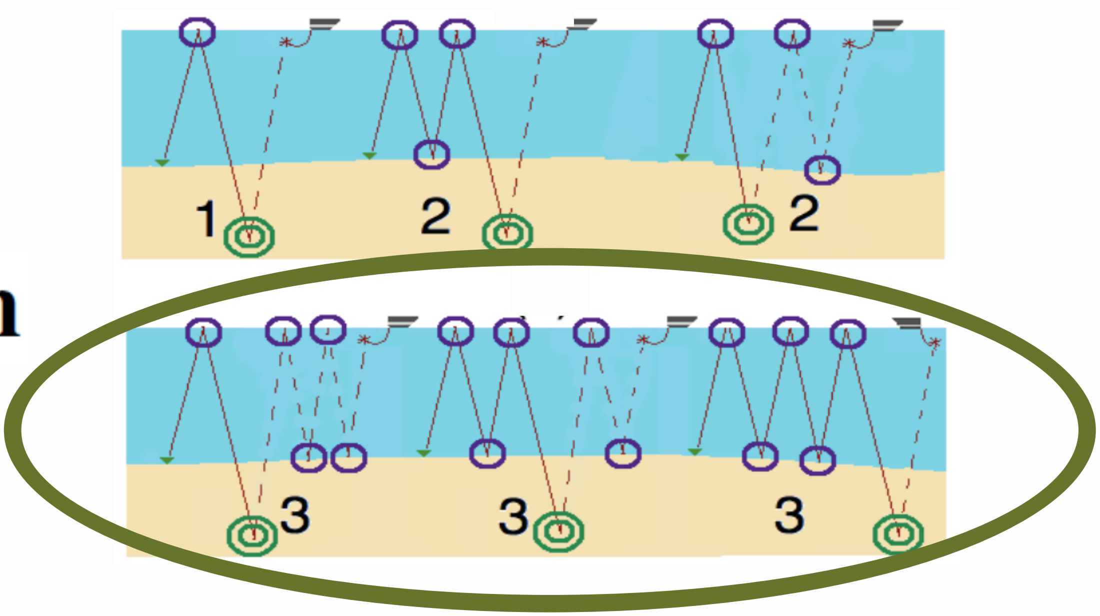


$$\mathbf{d}_2 = \mathbf{L}_2\mathbf{m}$$

OBN synthetic data – third order



$$\mathbf{d}_3 = \mathbf{L}_3 \mathbf{m}$$



Signal and crosstalks

$$\mathbf{F}^T\mathbf{d} = (\mathbf{L}_1 + \mathbf{L}_2 + \mathbf{L}_3 + \ldots)^T(\mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 + \ldots)$$

$$= (\mathbf{L}_1^T + \mathbf{L}_2^T + \mathbf{L}_3^T + \ldots)(\mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 + \ldots)$$

$$\hat{\mathbf{m}}_{\mathrm{signal}}$$

Signal and crosstalks

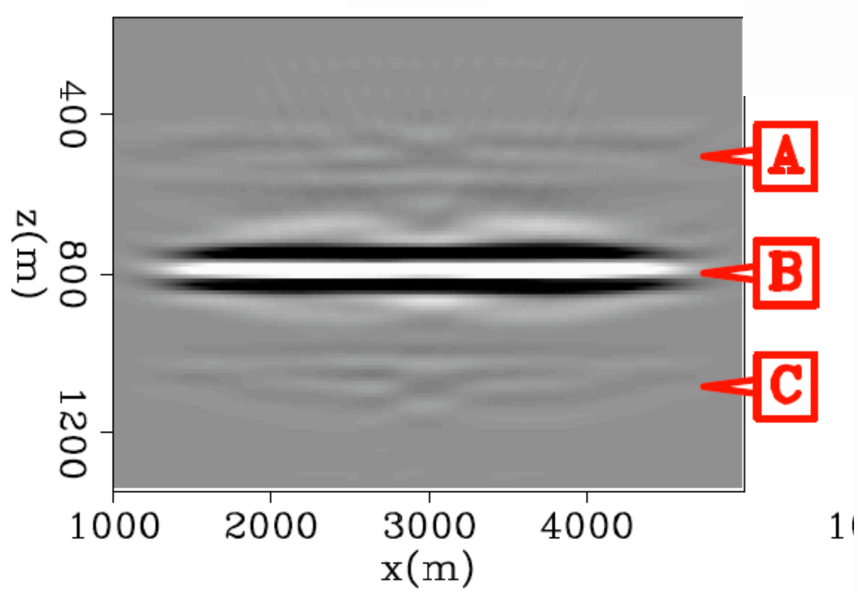
$$\mathbf{F}^T\mathbf{d} =$$

$$= (\mathbf{L}_1^T + \mathbf{L}_2^T + \mathbf{L}_3^T + \ldots)(\mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 + \ldots)$$

$$\hat{\mathbf{m}}_{\text{xtalk}}$$

One reflector model – adjoint image

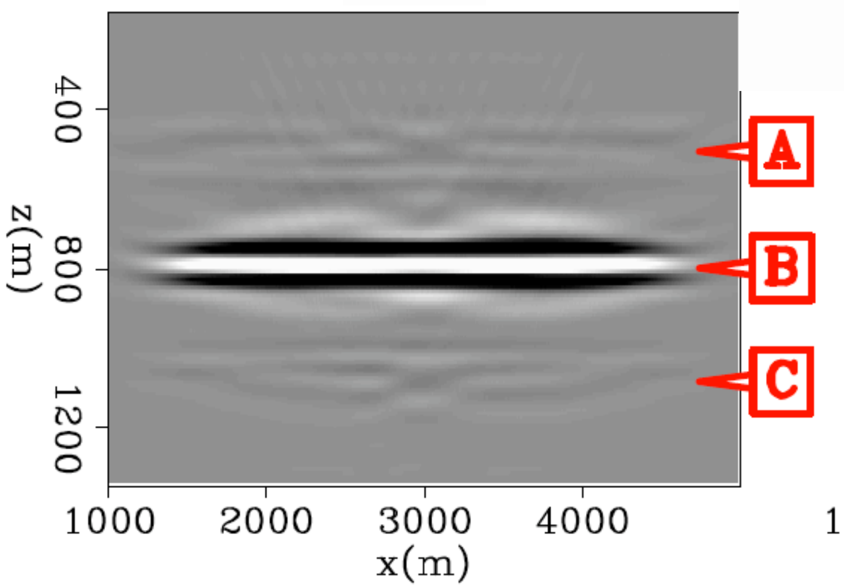
$$\mathbf{F}^T\mathbf{d} = \hat{\mathbf{m}}_{\text{signal}} + \hat{\mathbf{m}}_{\text{xtalk}}$$



One reflector model – adjoint image

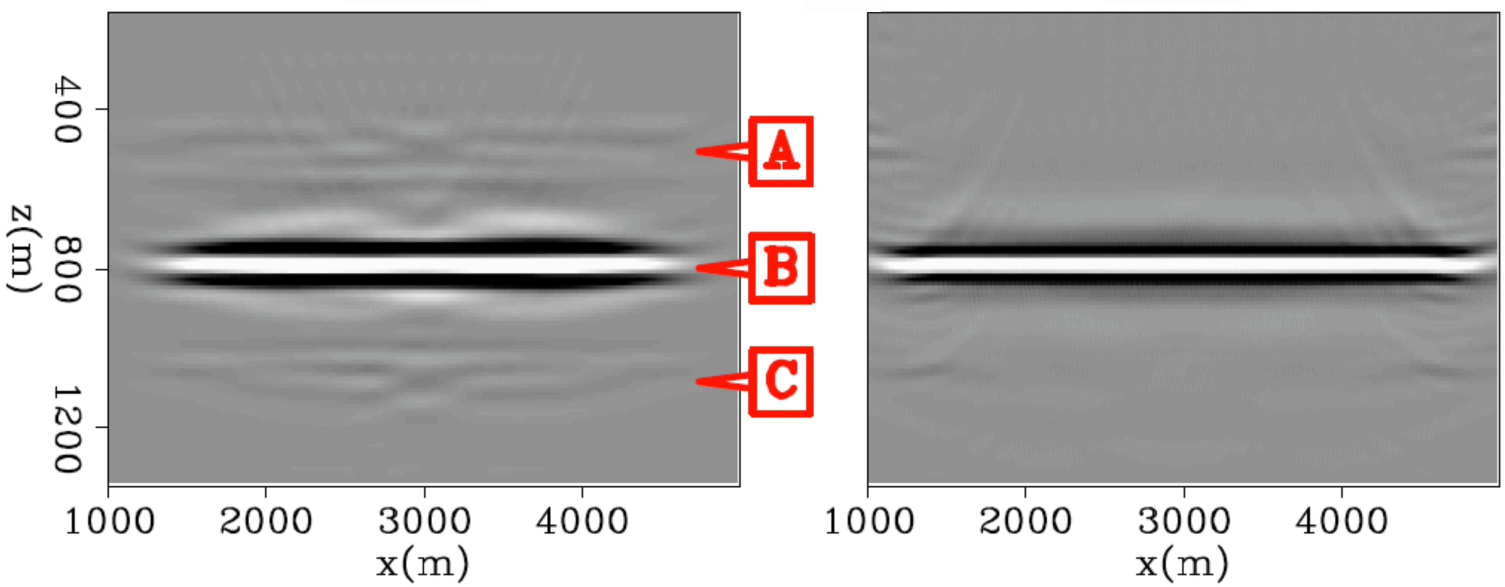
$$\mathbf{F}^T\mathbf{d} = \hat{\mathbf{m}}_{\text{signal}} + \hat{\mathbf{m}}_{\text{xtalk}}$$

$$\begin{aligned} m_A &= \mathrm{L}'_2\mathrm{d}_1 + \mathrm{L}'_3\mathrm{d}_1 + \mathrm{L}'_4\mathrm{d}_1 + \ldots \\ m_B &= \mathrm{L}'_1\mathrm{d}_1 + \mathrm{L}'_2\mathrm{d}_2 + \mathrm{L}'_3\mathrm{d}_3 + \ldots \\ m_C &= \mathrm{L}'_1\mathrm{d}_2 + \mathrm{L}'_1\mathrm{d}_3 + \mathrm{L}'_2\mathrm{d}_3 + \ldots \end{aligned}$$



One reflector model – inversion image

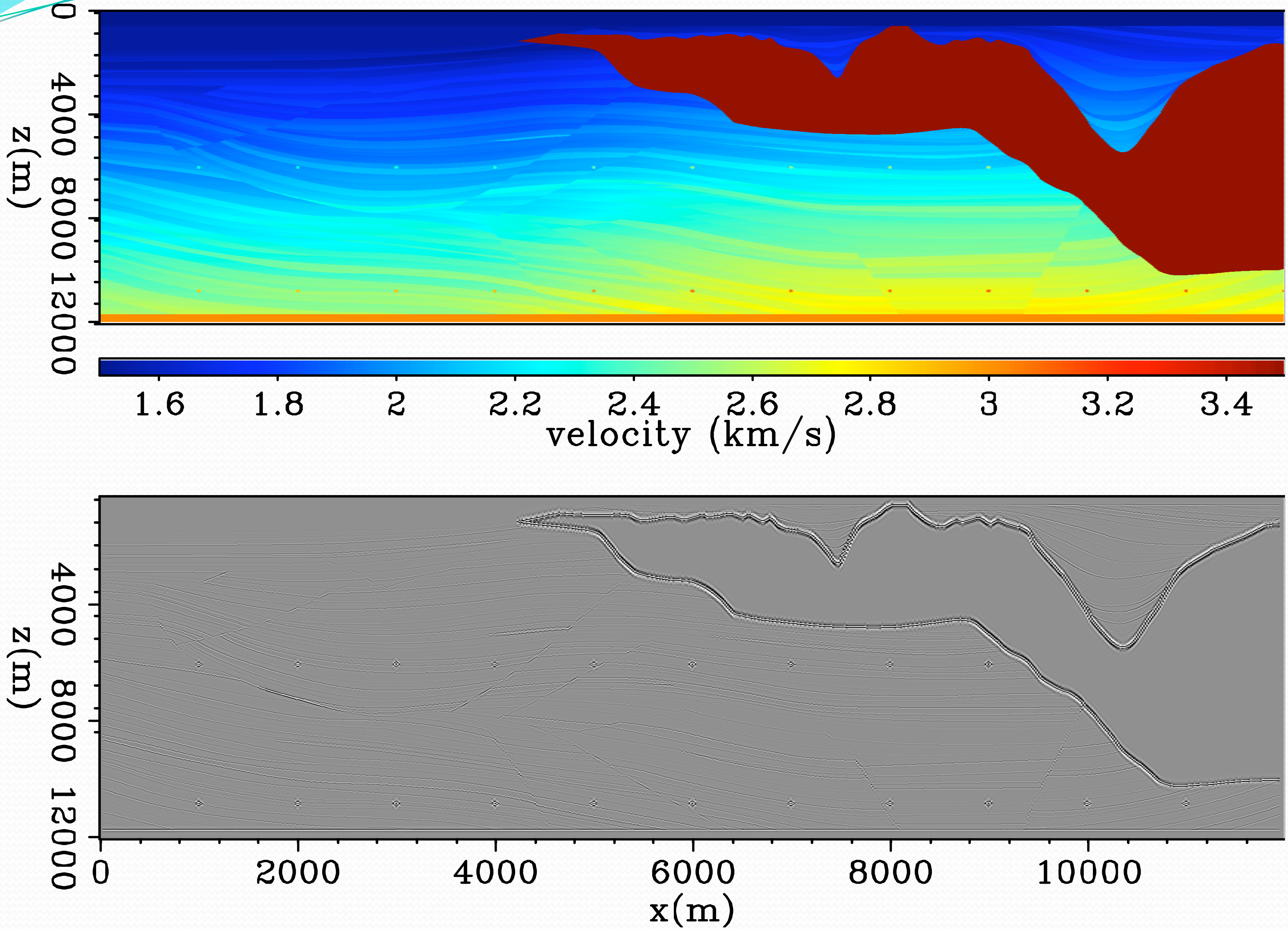
$$\hat{\mathbf{m}}_{\text{inv}} = (\mathbf{F}^T\mathbf{F})^{-1}\mathbf{F}^T\mathbf{d}$$



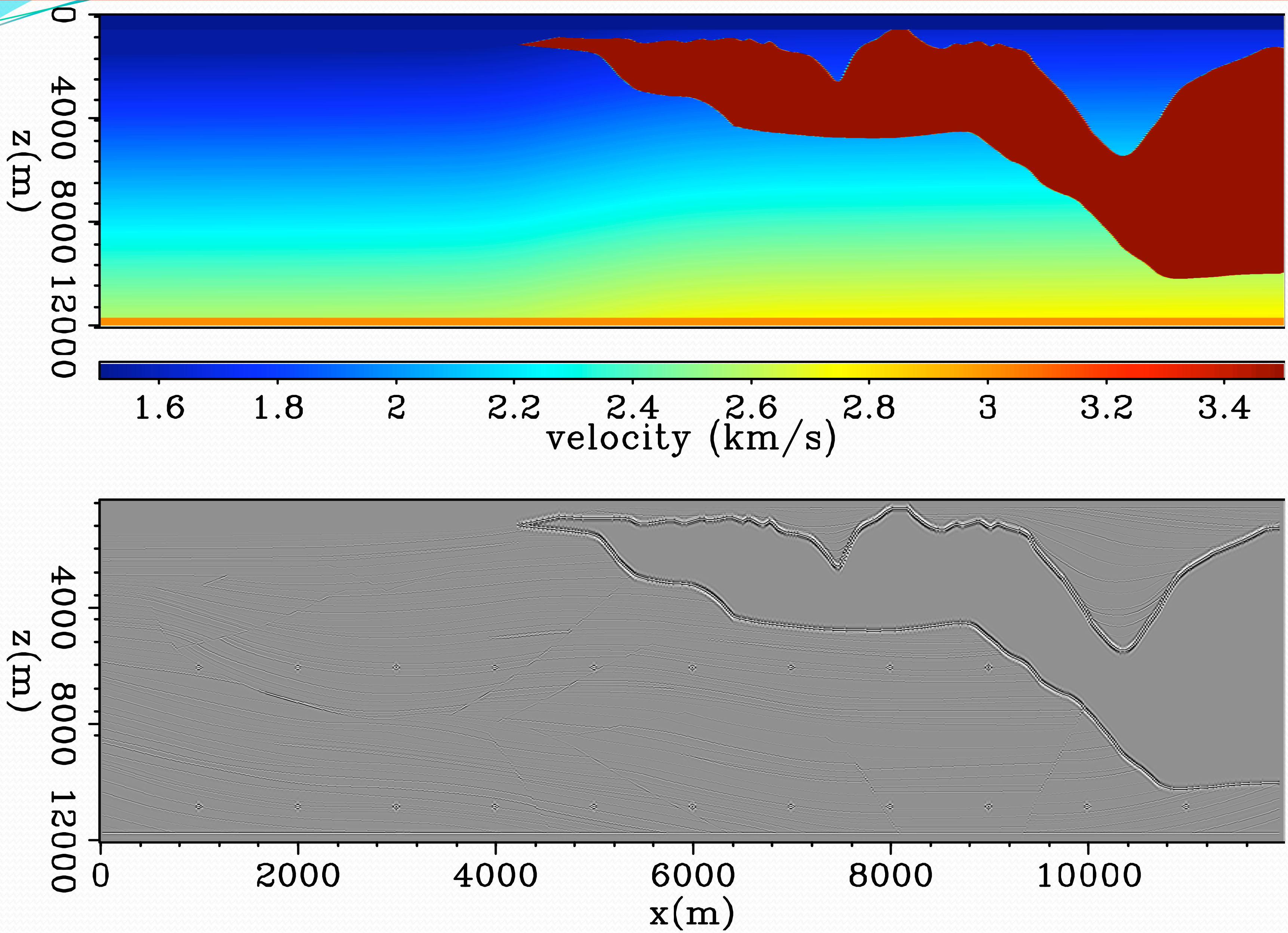
Overview

- Introduction
- Theory
  - Linearized full-wave inversion
- Synthetic examples
  - One reflector model
  - Sigbee2B model
- Discussion and Conclusion

SigBee2B model



SigBee2B model



Primary-only and primary+multiples synthetic data

Primaries + multiples synthetic

$$
\begin{aligned}
\mathbf{d}_{\mathrm{P+M}} &= \mathbf{F}\mathbf{m} \\
&= (\mathbf{L}_1 + \mathbf{L}_2 + \mathbf{L}_3 + \ldots)\mathbf{m} \\
&= \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 + \ldots
\end{aligned}
$$

Primary-only synthetic

$$
\begin{aligned}
\mathbf{d}_{\mathrm{P}} &= \mathbf{L}_p\mathbf{m} \\
&= \mathbf{L}_1\mathbf{m} = \mathbf{d}_1
\end{aligned}
$$

Primary-only and primary+multiples synthetic data

Primaries + multiples synthetic

$$\begin{aligned} \mathbf{d}_{\mathrm{P+M}} &= \mathbf{F}\mathbf{m} \\ &= (\mathbf{L}_1 + \mathbf{L}_2 + \mathbf{L}_3 + \ldots)\mathbf{m} \\ &= \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 + \ldots \end{aligned}$$

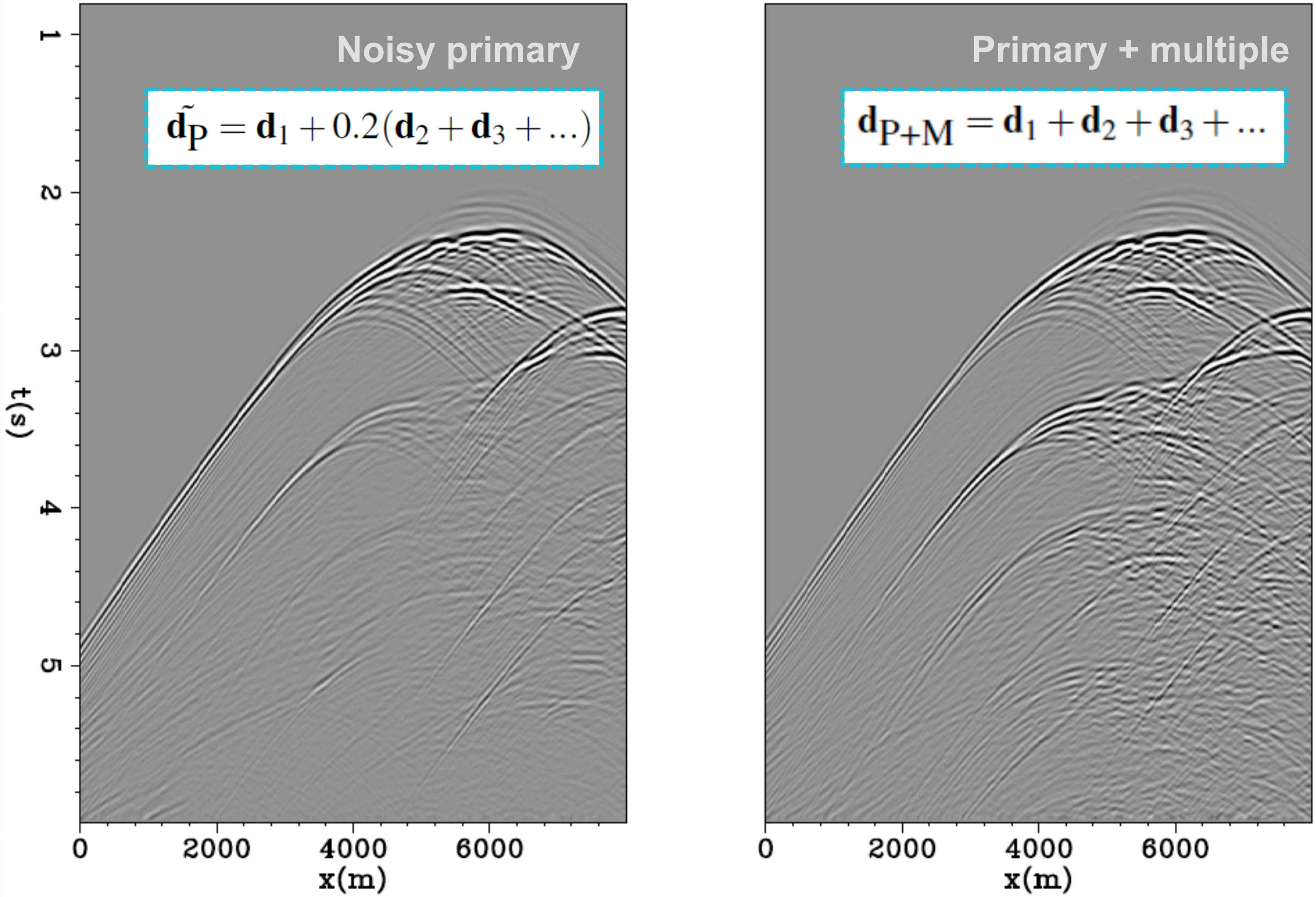
Primary-only synthetic

$$\begin{aligned} \mathbf{d}_{\mathrm{P}} &= \mathbf{L}_p\mathbf{m} \\ &= \mathbf{L}_1\mathbf{m} = \mathbf{d}_1 \end{aligned}$$

Noisy primary

$$\tilde{\mathbf{d}_{\mathrm{P}}} = \mathbf{d}_1 + 0.2(\mathbf{d}_2 + \mathbf{d}_3 + \ldots)$$

SigBee2B model



RTM on noisy primary data

$$\mathbf{L}_{\mathrm{P}}^{T}\tilde{d}_{\mathrm{P}}$$

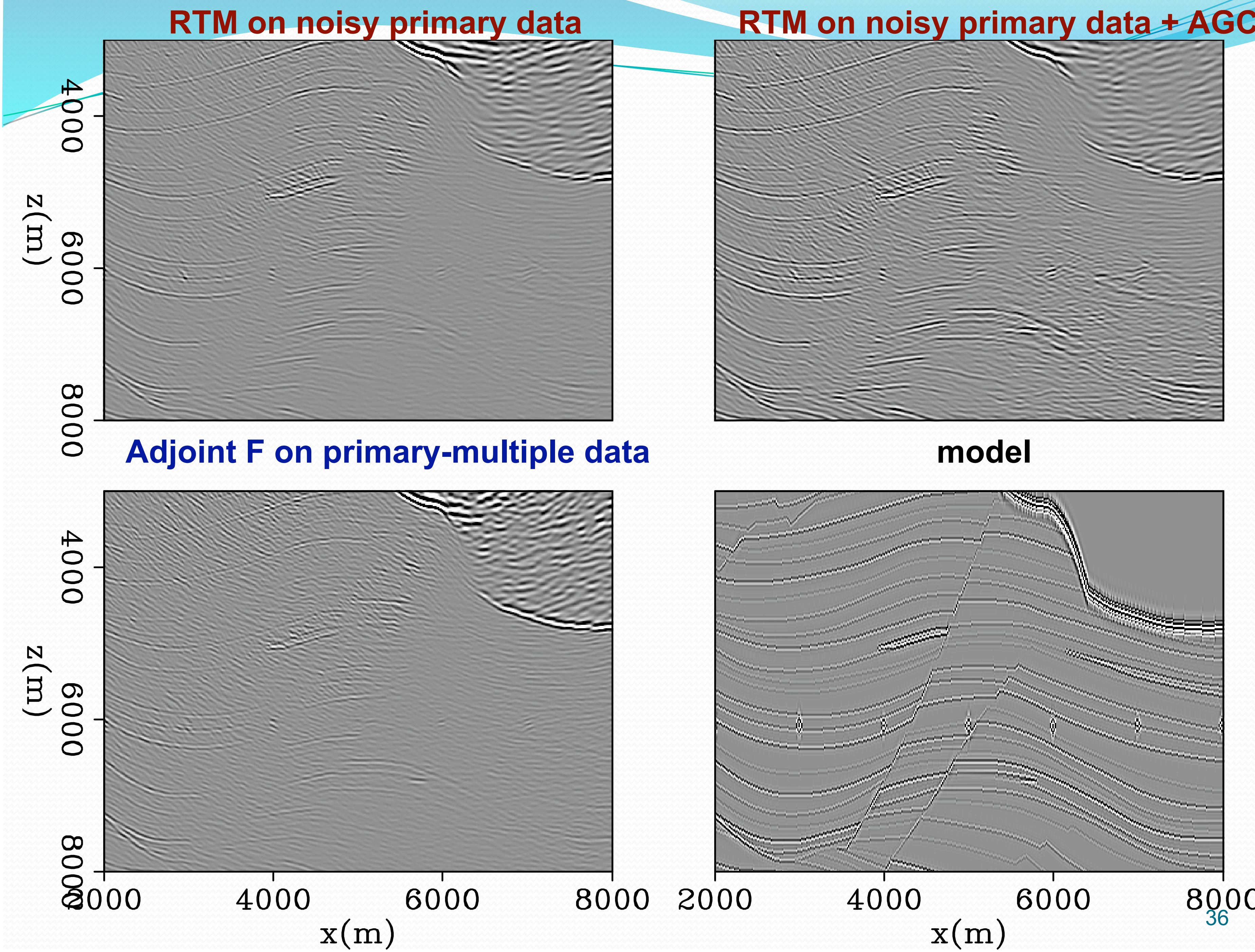
RTM on noisy primary data + AGC

$$\mathbf{L}_{\mathrm{P}}^{T}\tilde{d}_{\mathrm{P}} + \mathrm{AGC}$$

Adjoint F on primary-multiple data

$$\mathbf{F}^{T}d_{\mathrm{P+M}}$$

model


RTM on noisy primary data
RTM on noisy primary data + AGC
Adjoint F on primary-multiple data
model
z(m)
4000
6000
8000
2000
4000
6000
8000
x(m)

Inversion on noisy primary data

$$(\mathbf{L}_P^T \mathbf{L}_P)^{-1} \mathbf{L}_P^T \tilde{\mathbf{d}}_P$$

RTM on noisy primary data + AGC

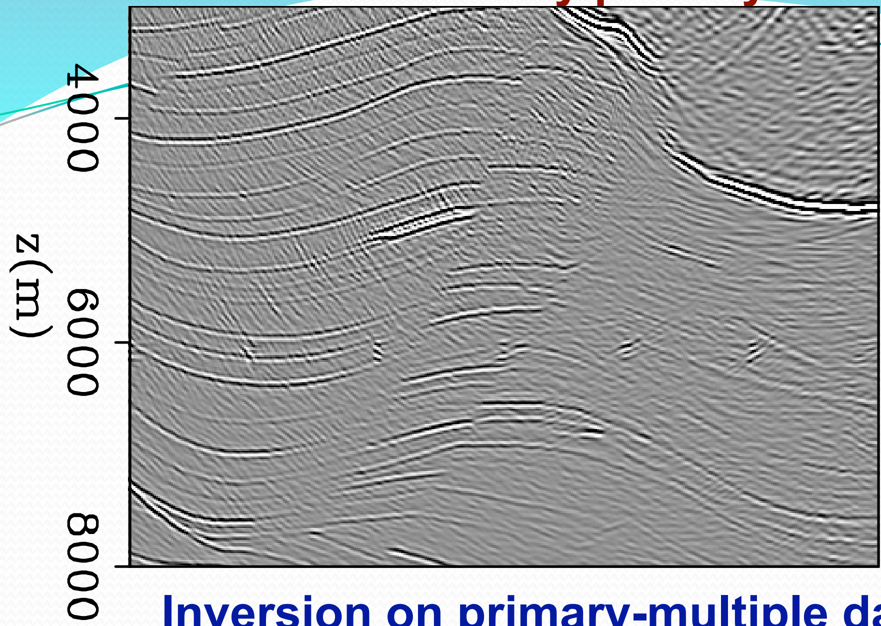
$$\mathbf{L}_P^T \tilde{\mathbf{d}}_P + \mathrm{AGC}$$

Inversion on primary-multiple data

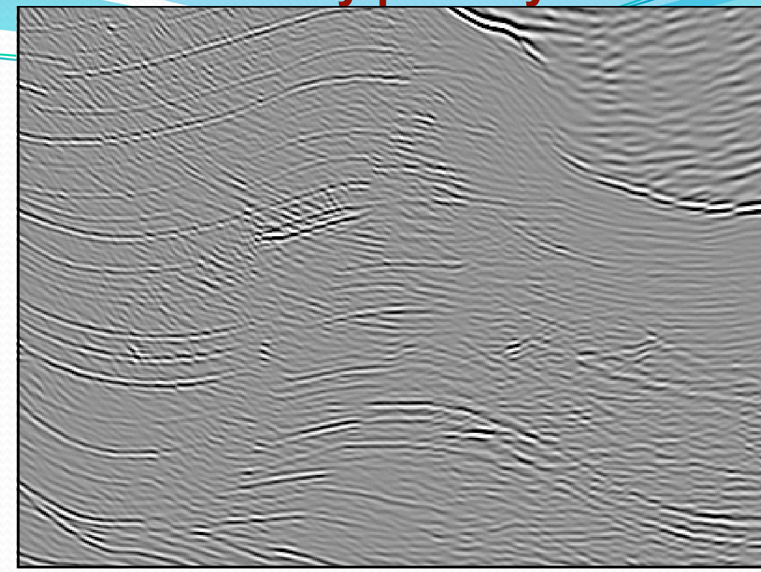
$$(\mathbf{F}^T \mathbf{F})^{-1} \mathbf{F}^T \mathbf{d}_{P+M}$$

model

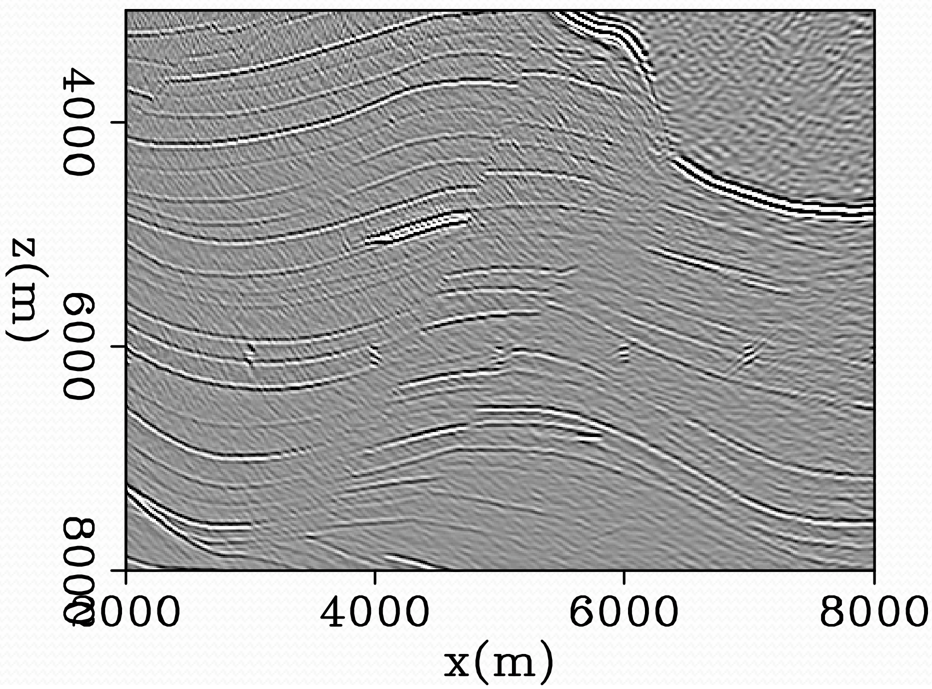
Inversion on noisy primary data



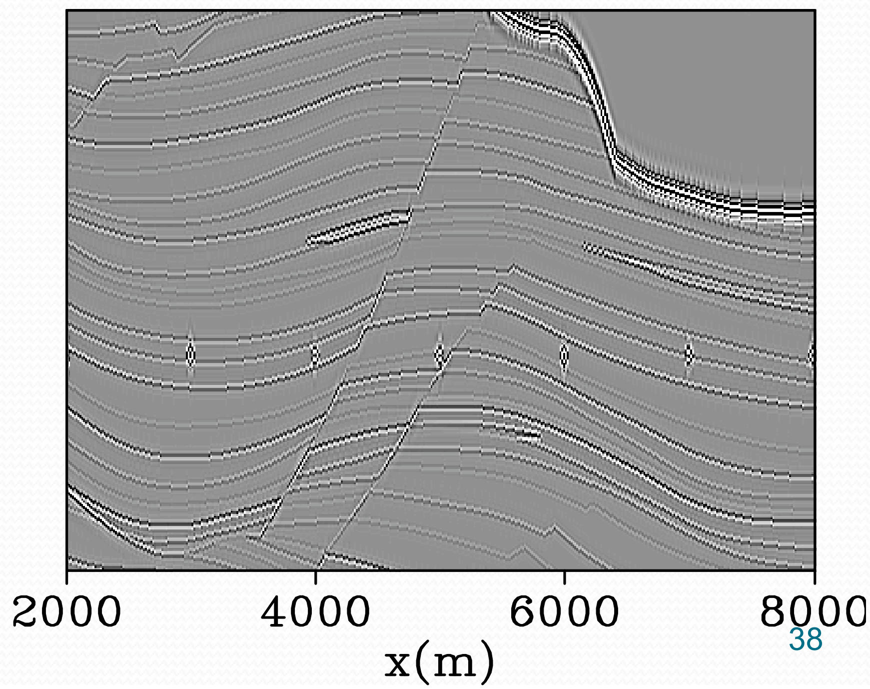
RTM on noisy primary data + AGC

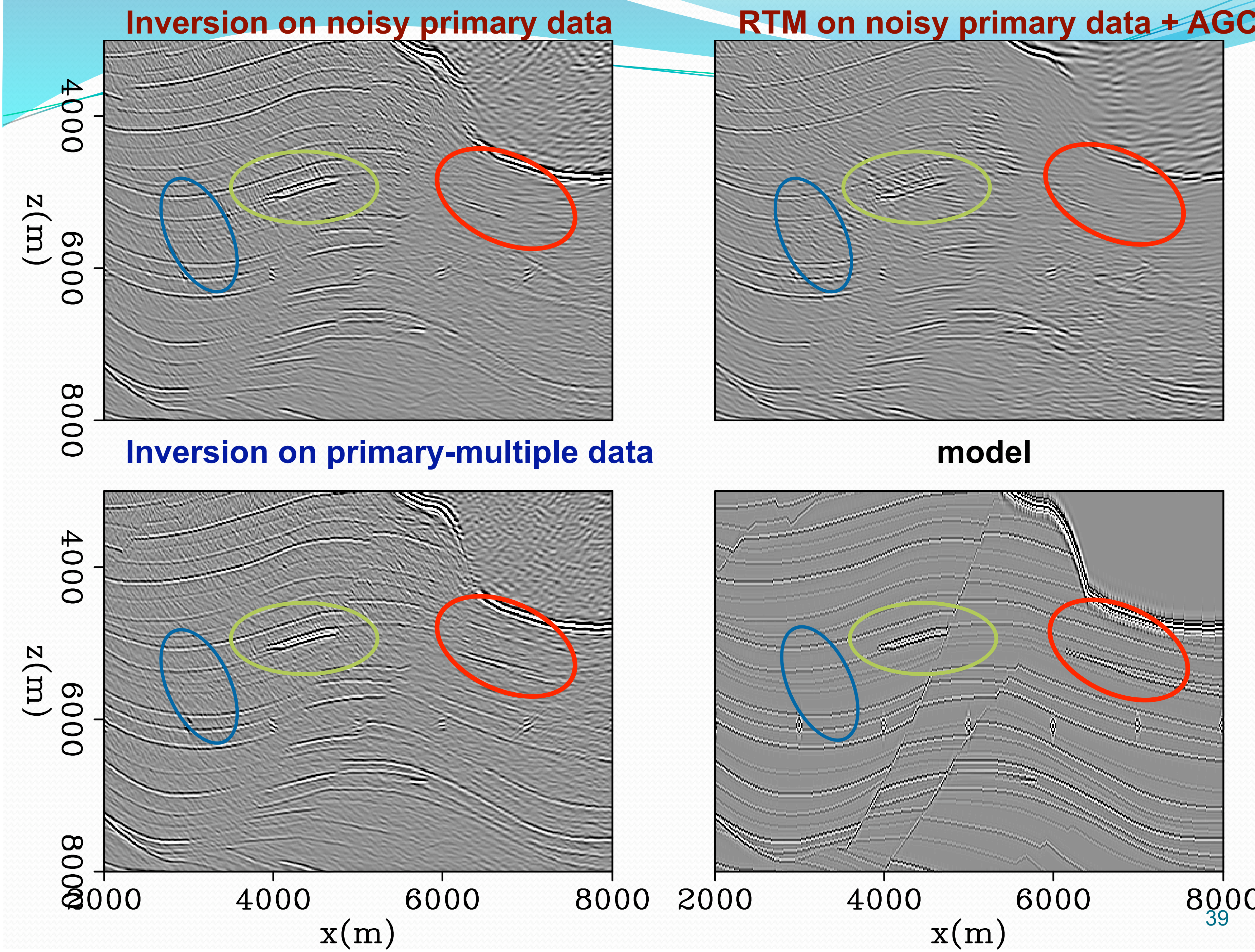


Inversion on primary-multiple data



model




Inversion on noisy primary data
RTM on noisy primary data + AGC
Inversion on primary-multiple data
model
z(m)
4000
6000
8000
x(m)
2000
4000
6000
8000

Overview

- Introduction
- Theory
  - Linearized full-wave inversion
- Synthetic examples
  - One reflector model
  - Sigbee2B model
- Discussion and Conclusion

Discussion

- LFWI does not migrate all orders of multiples
  - Migrates multiples with single scattering off the model
- LFWI depends on the migration velocity
- Can be extended to include variable density, anisotropy and more.
- This technique is appropriate for surveys where multiple removal is an issue

Conclusion

- Linearized full-wave inversion (LFWI) can image both primaries and multiples
- Results from the 2D Sigbee2B model shows that
  - LFWI increases subsurface illumination by using multiple as energy
  - It reduces crosstalks in the final image