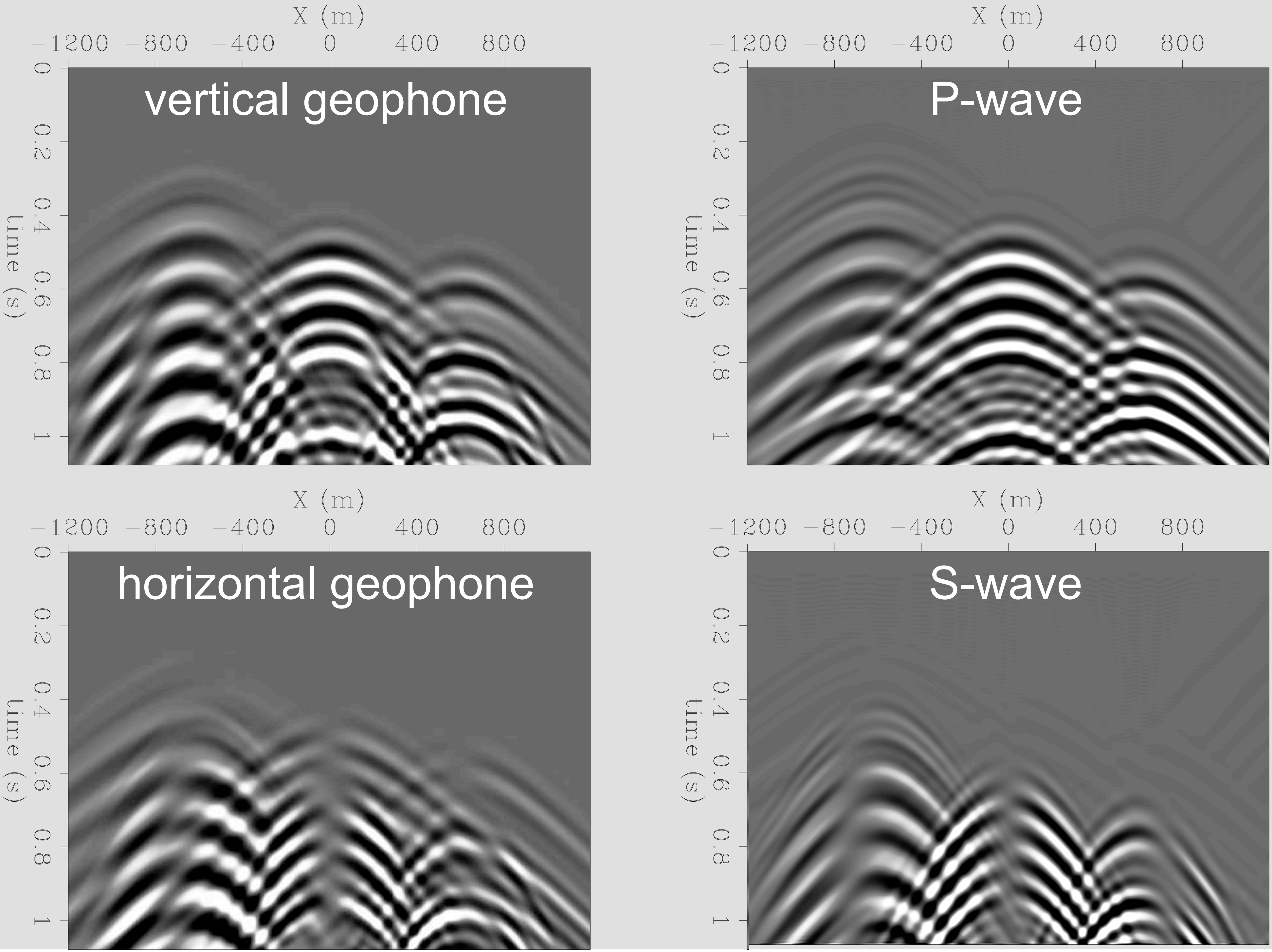
P/S separation of ocean-bottom seismic data by inversion in a homogeneous medium

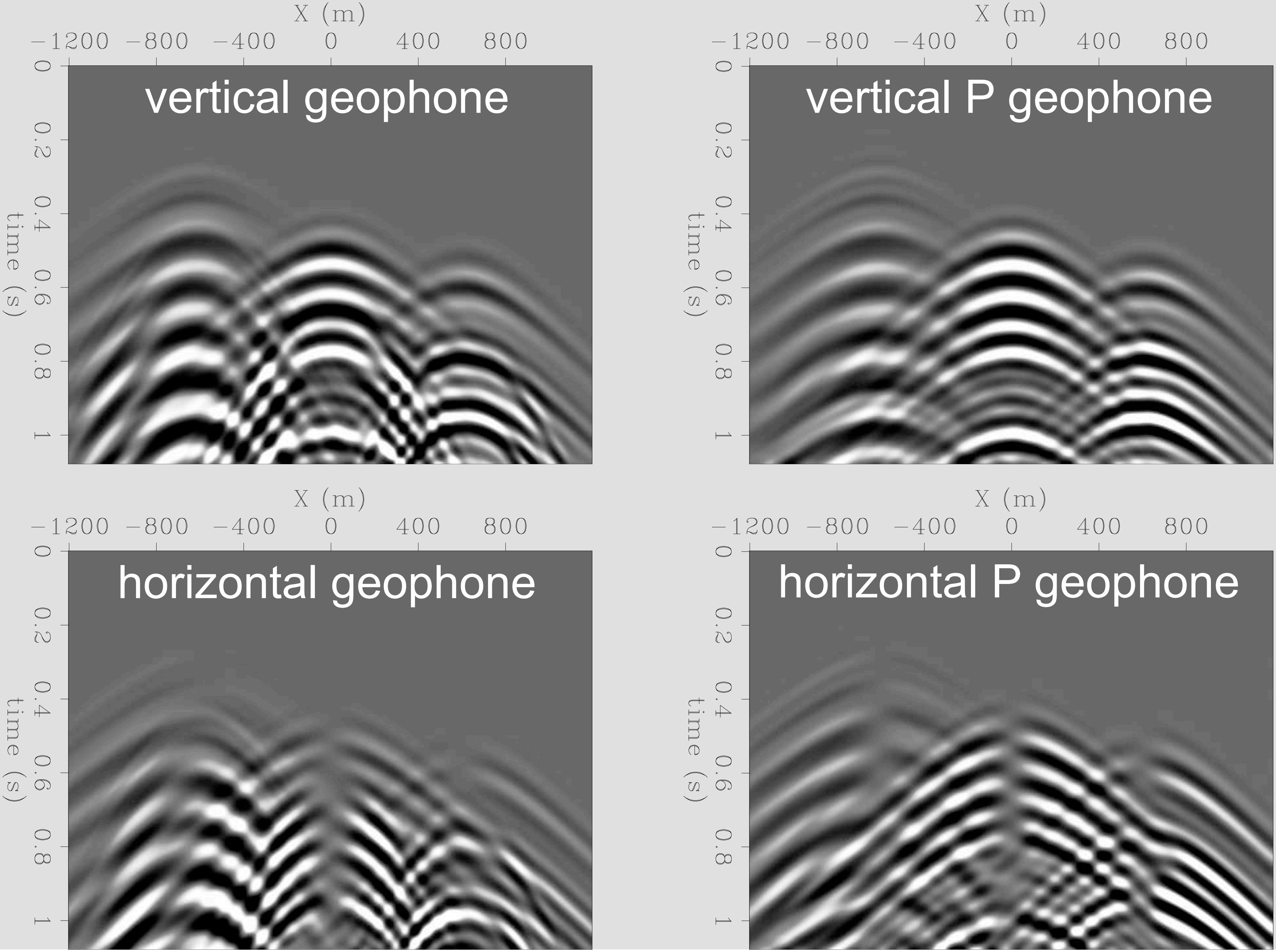
SEP sponsor's meeting
May 23, 2012

Ohad Barak
SEP 147, p. 261

P/S separation



P/S separation



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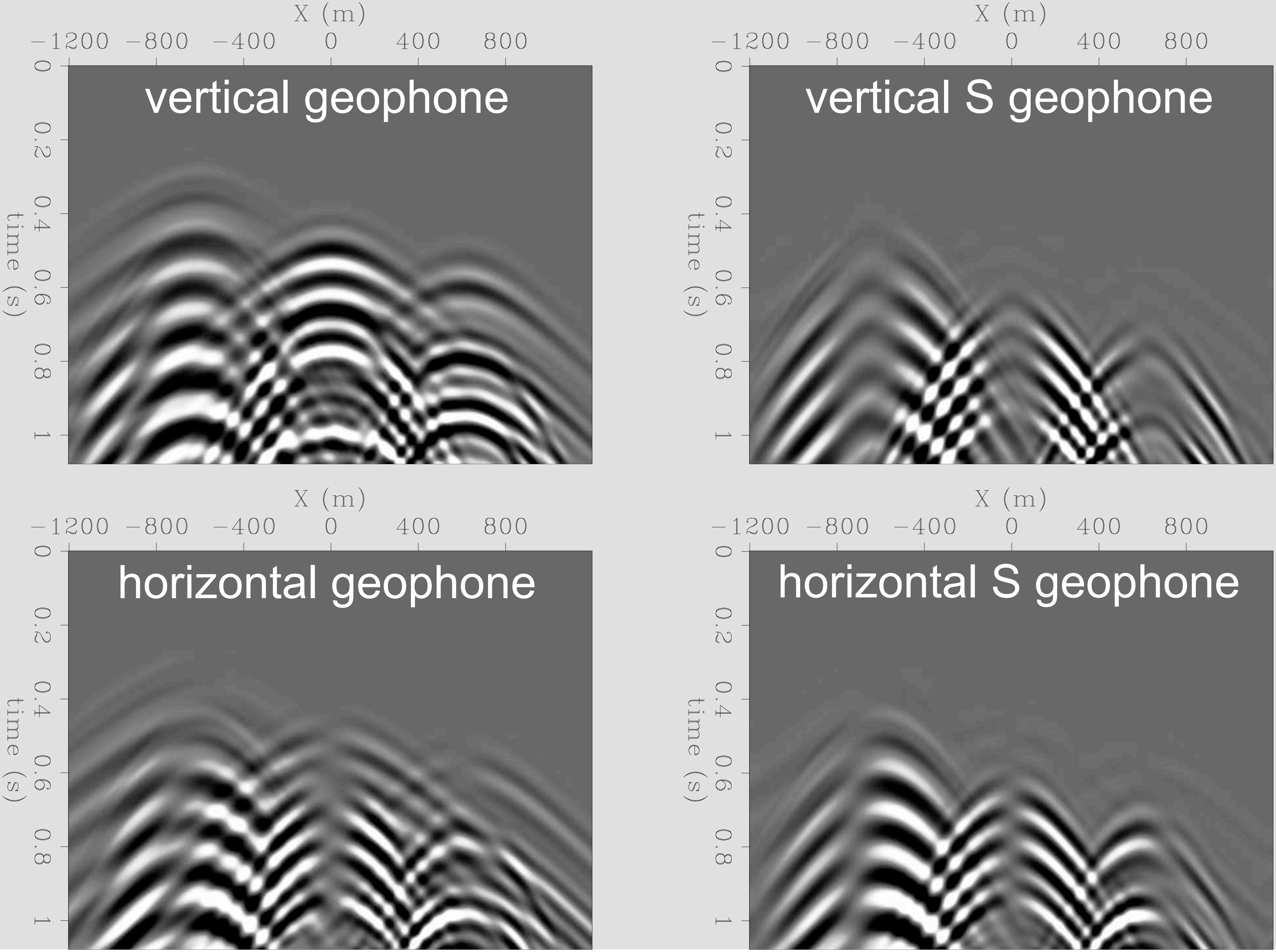

Content

1. Motivation
2. Theory
   - General concept
   - Inversion setup
3. Synthetic examples
   - Land
   - Ocean-bottom seismic
4. Conclusions
5. Road ahead

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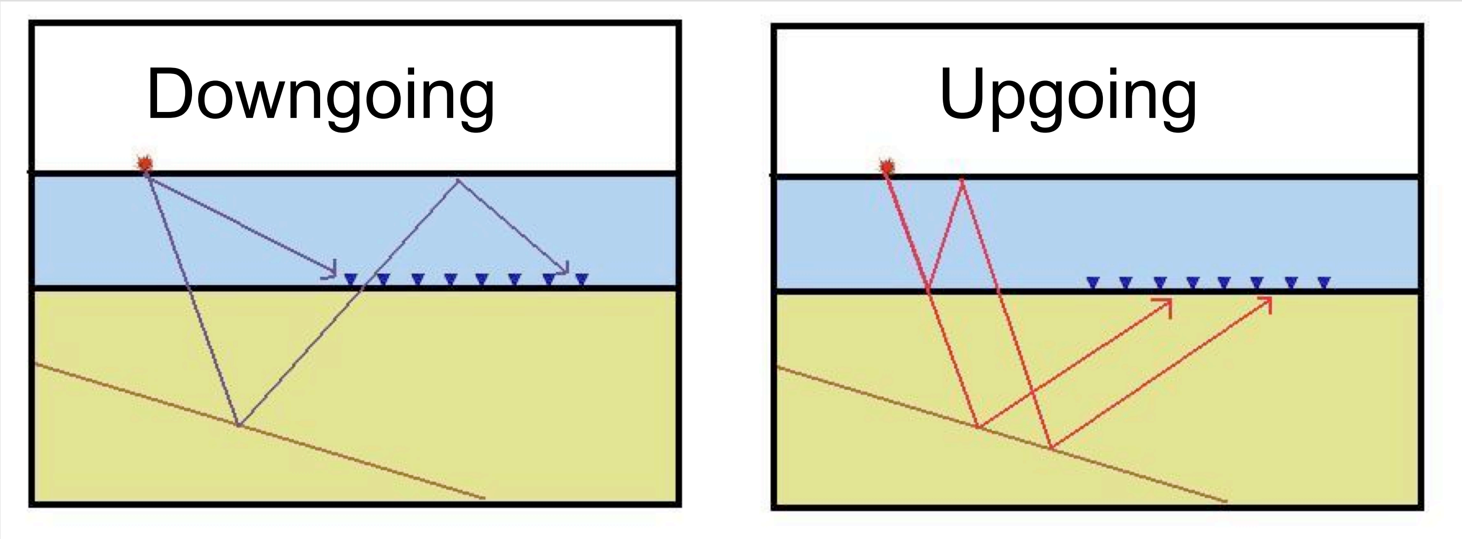
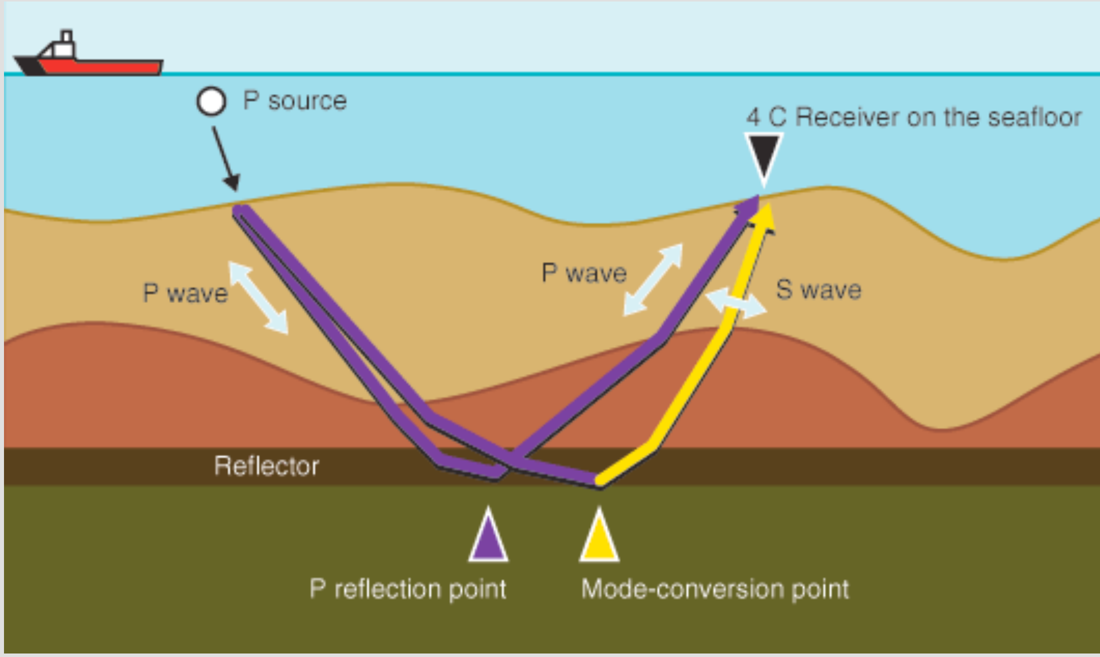
P/S separation of ocean-bottom seismic data



Wong and Ronen, SEP-138



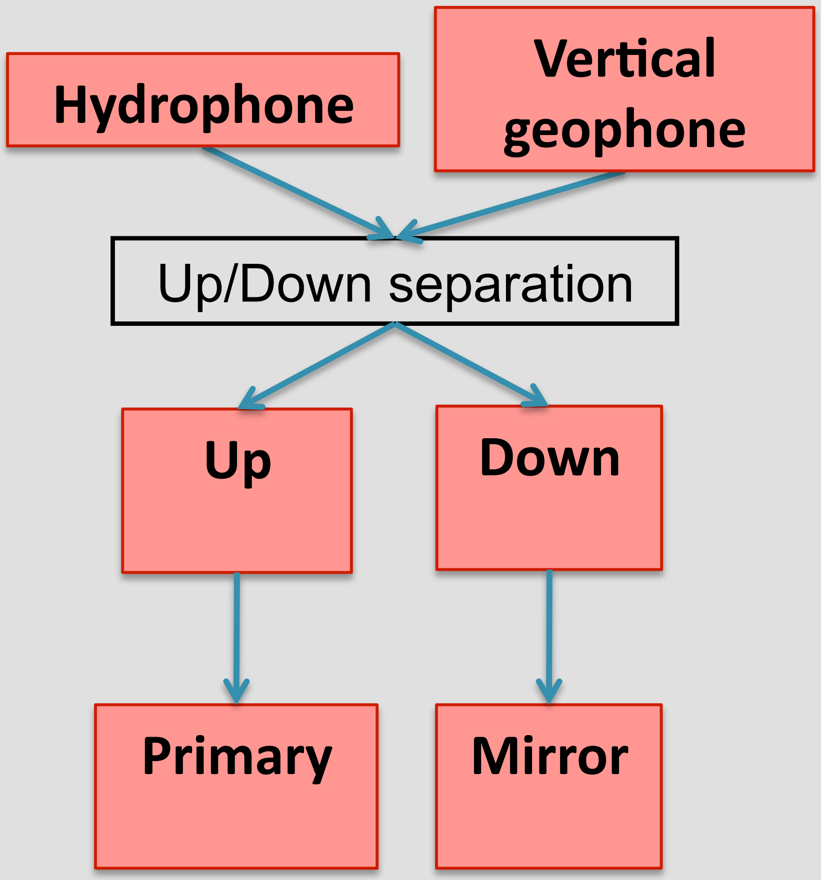
<u>Four-component acquisition</u>

1 Hydrophone: pressure

3 Geophones: vertical and horizontal particle motion

Conventional P/S separation approach

1. Assume vertical geophone is P.



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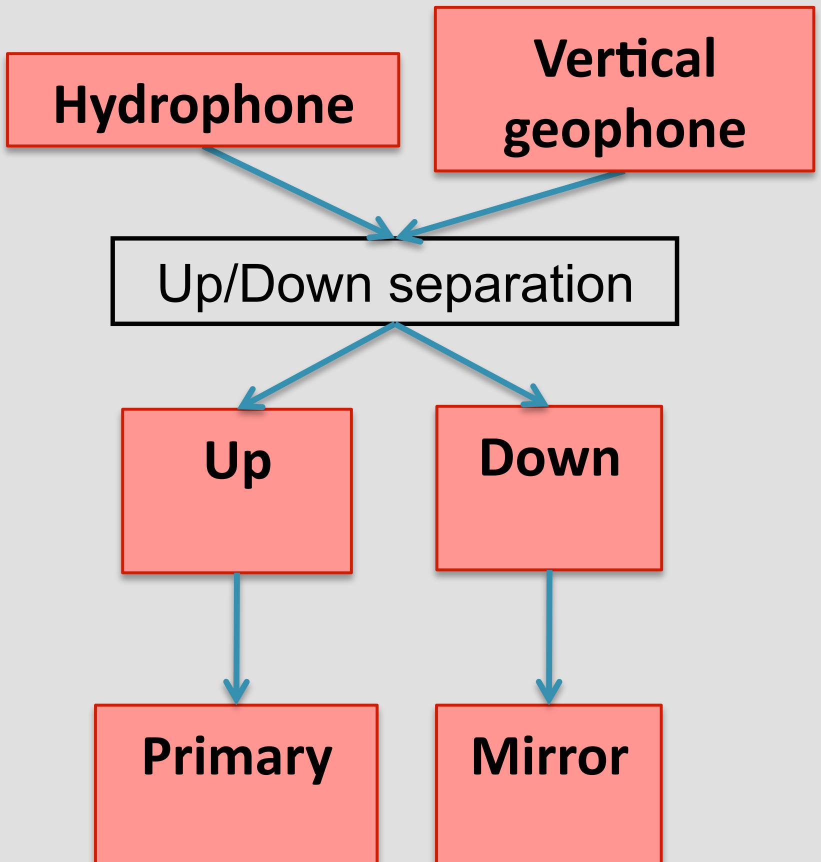

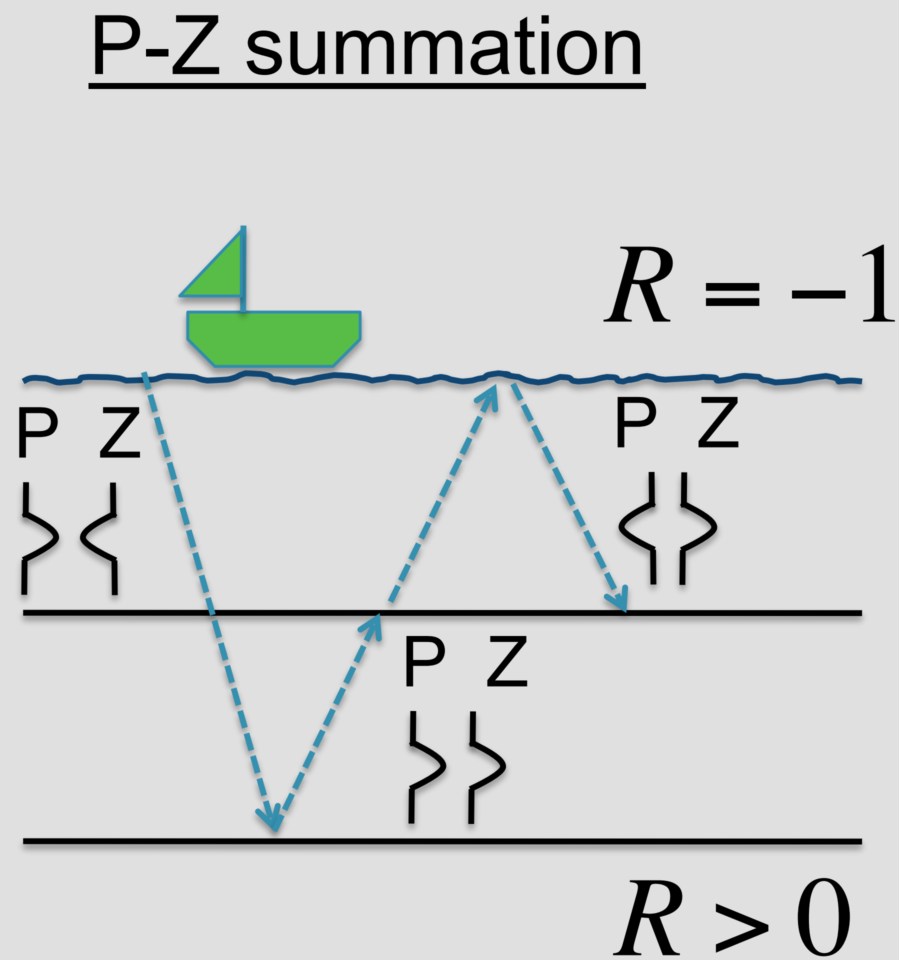

Conventional P/S separation approach

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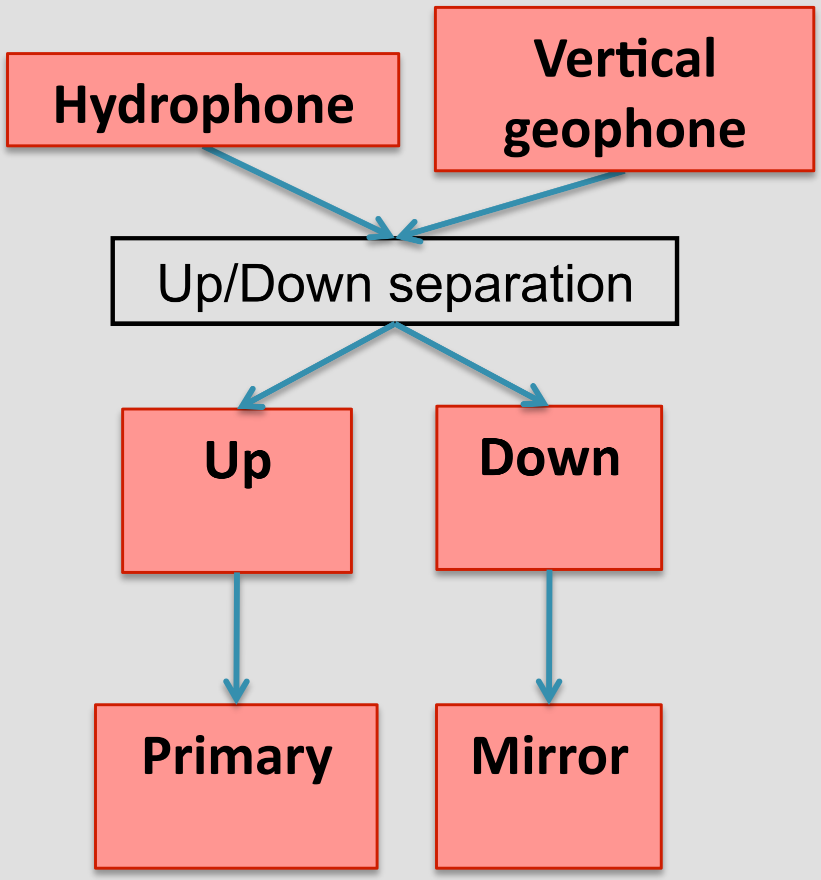
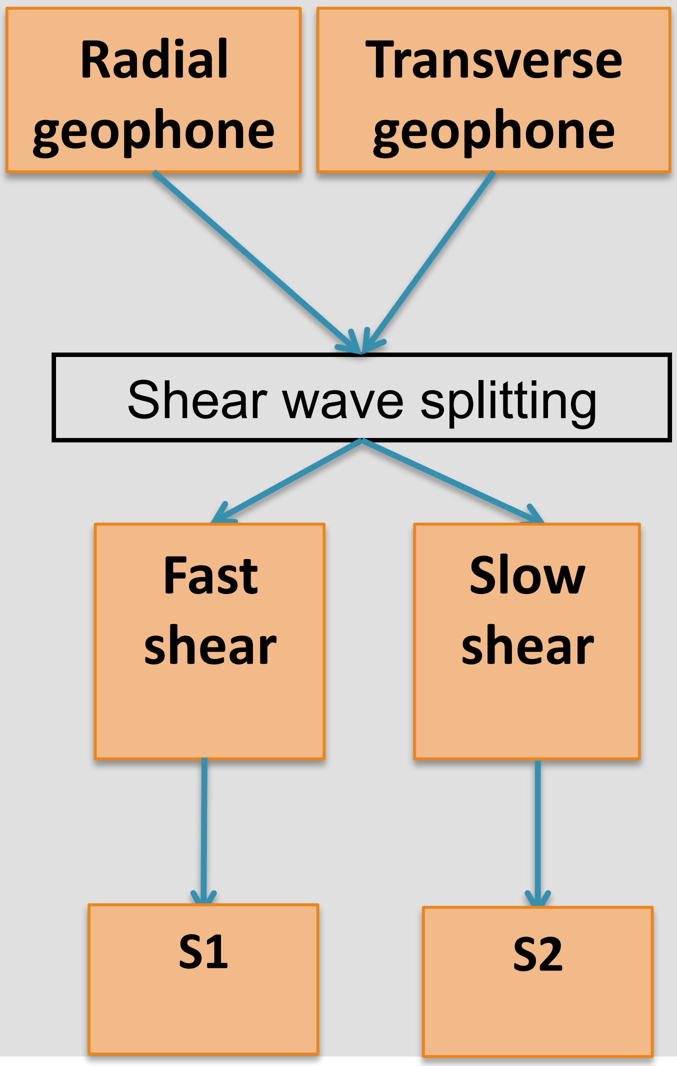


2. Assume horizontal geophones are S.



Conventional P/S separation approach

1. Assume vertical geophone is P.

2. Assume horizontal geophones are S.

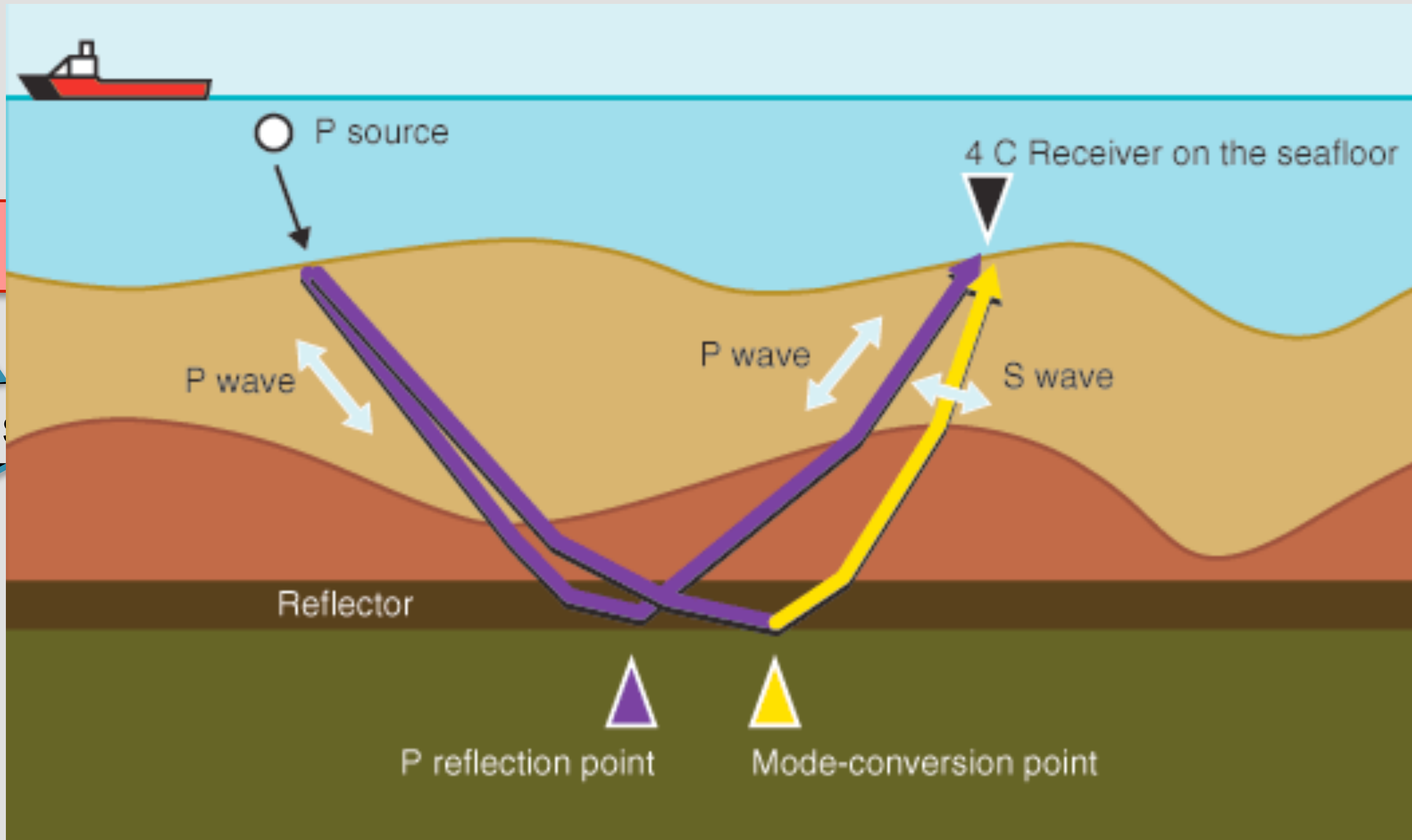
Hydrophone

Up/Down s

Up

Primary

Mirror



Transverse geophone

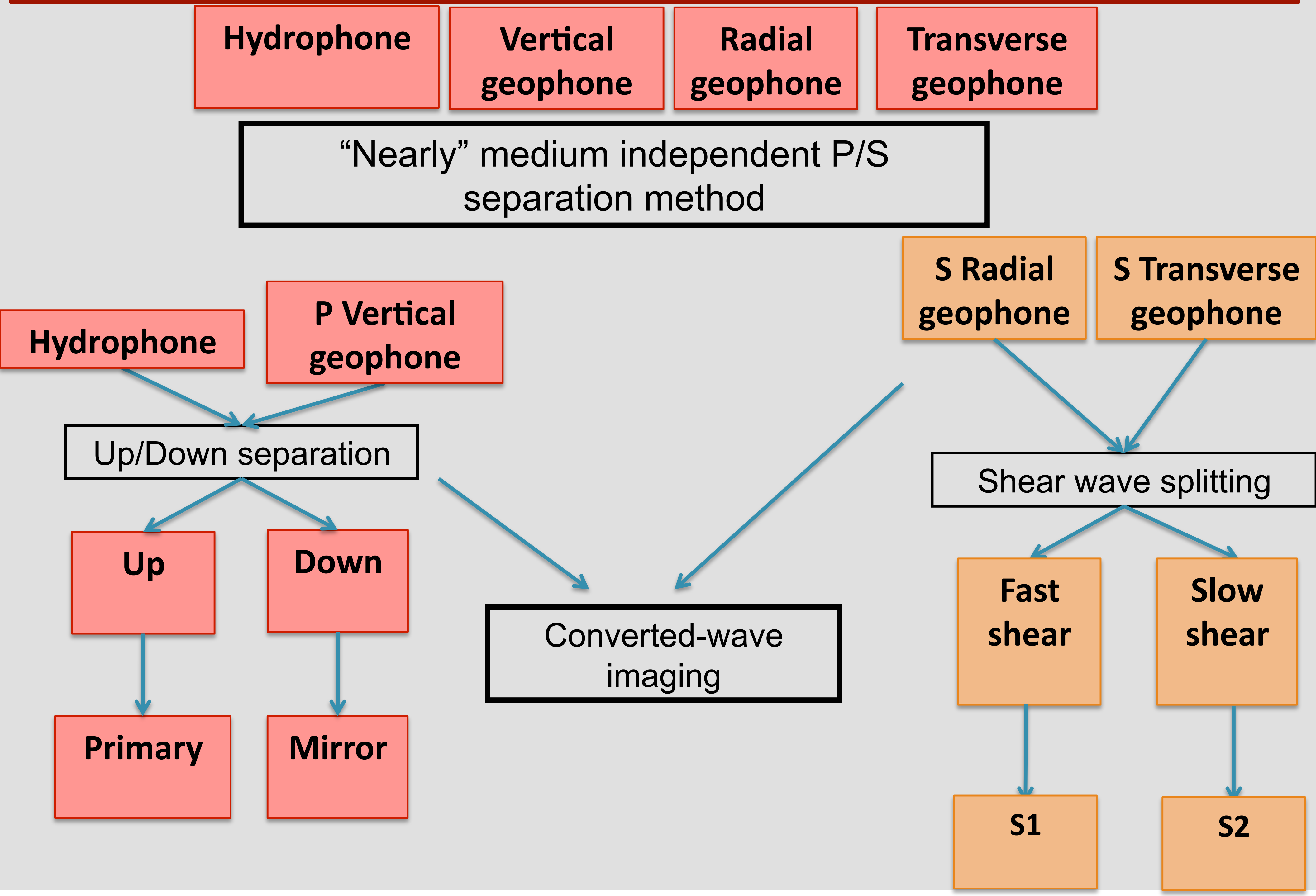
e splitting

Slow shear

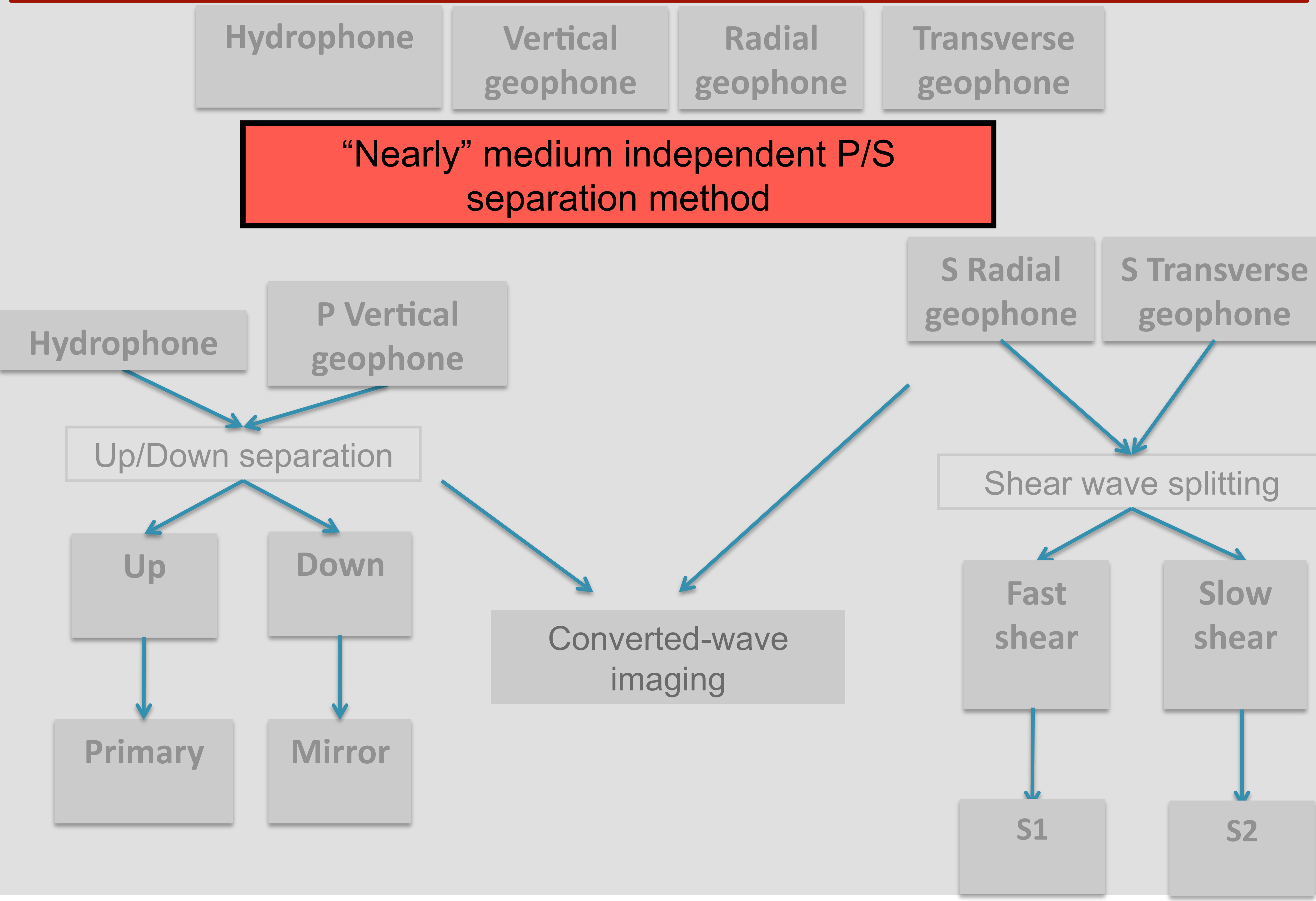
S1

S2

P/S separation of OBS data by inversion



P/S separation of OBS data by inversion



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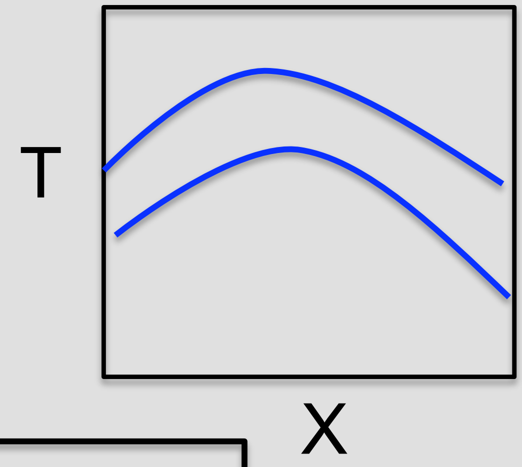
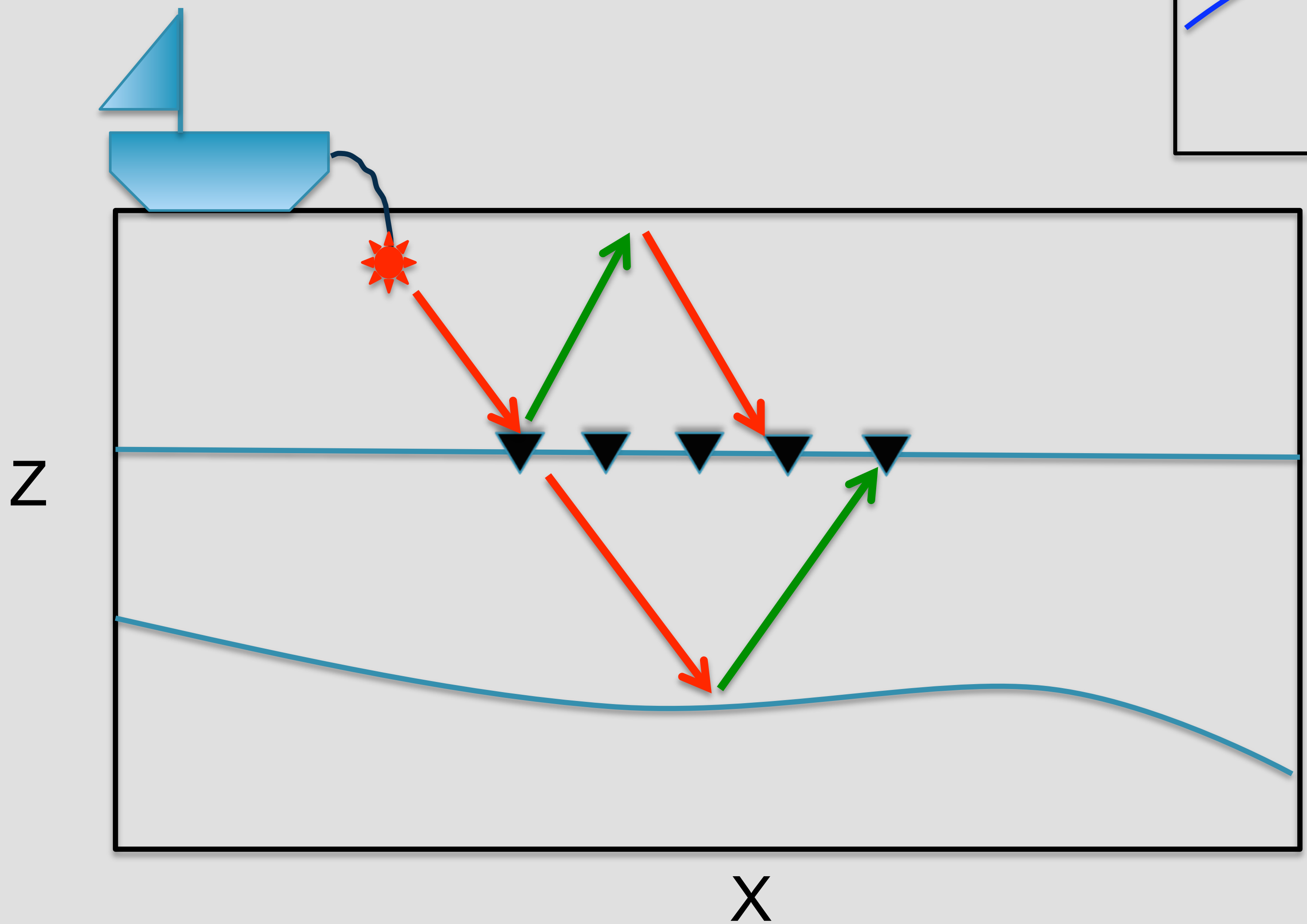
General concept of separation by inversion

Field data was acquired by
a particular experiment:

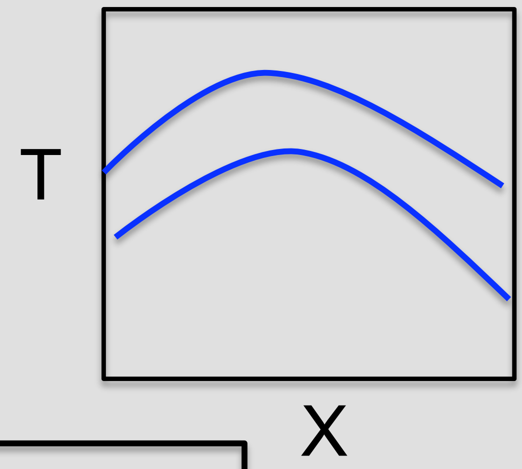


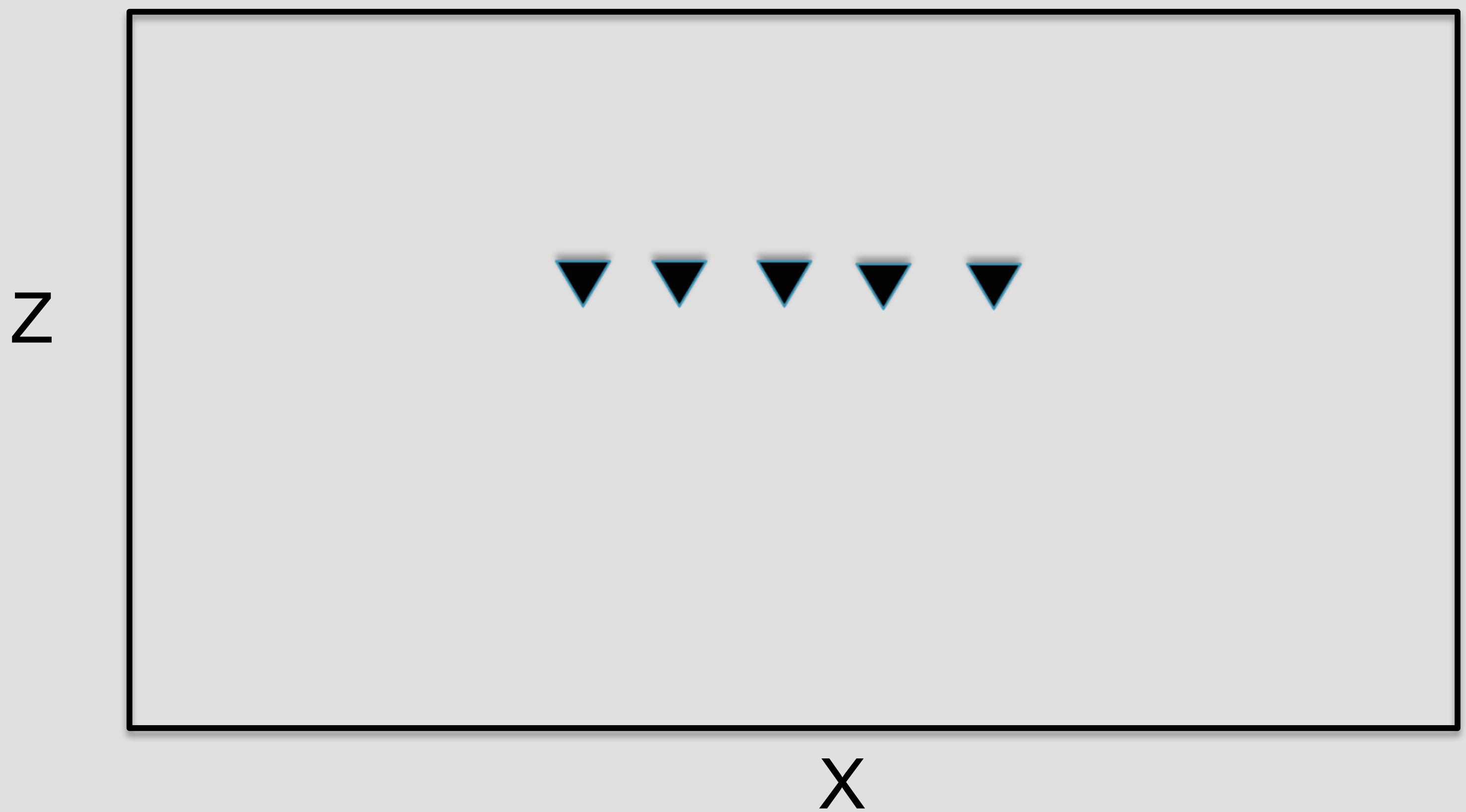


General concept of separation by inversion

The same field data could have been the result of a different experiment.

1. Different medium
2. Different acquisition





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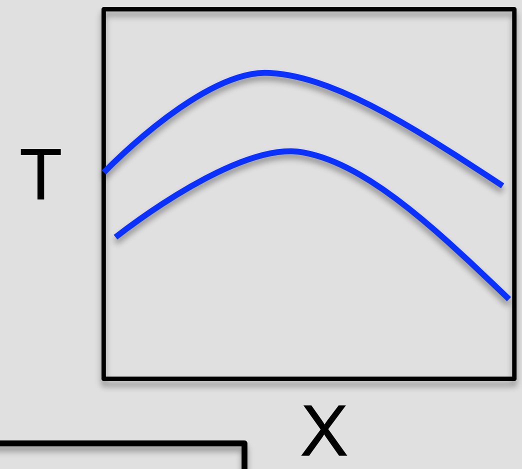

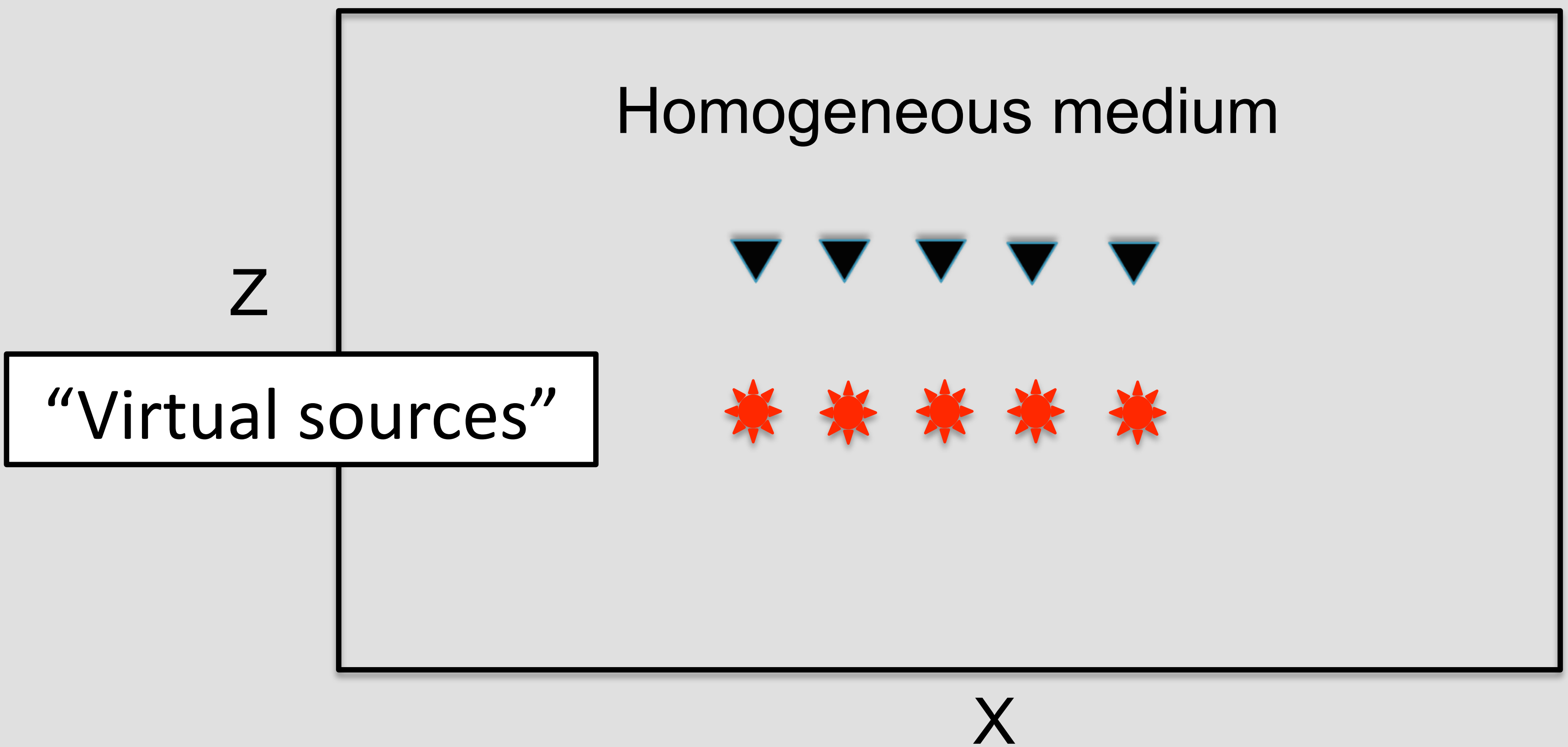

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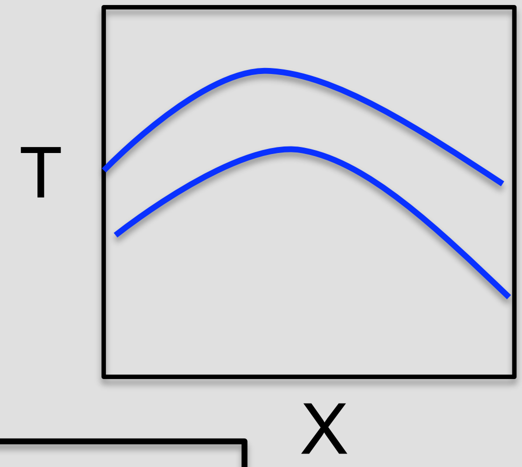
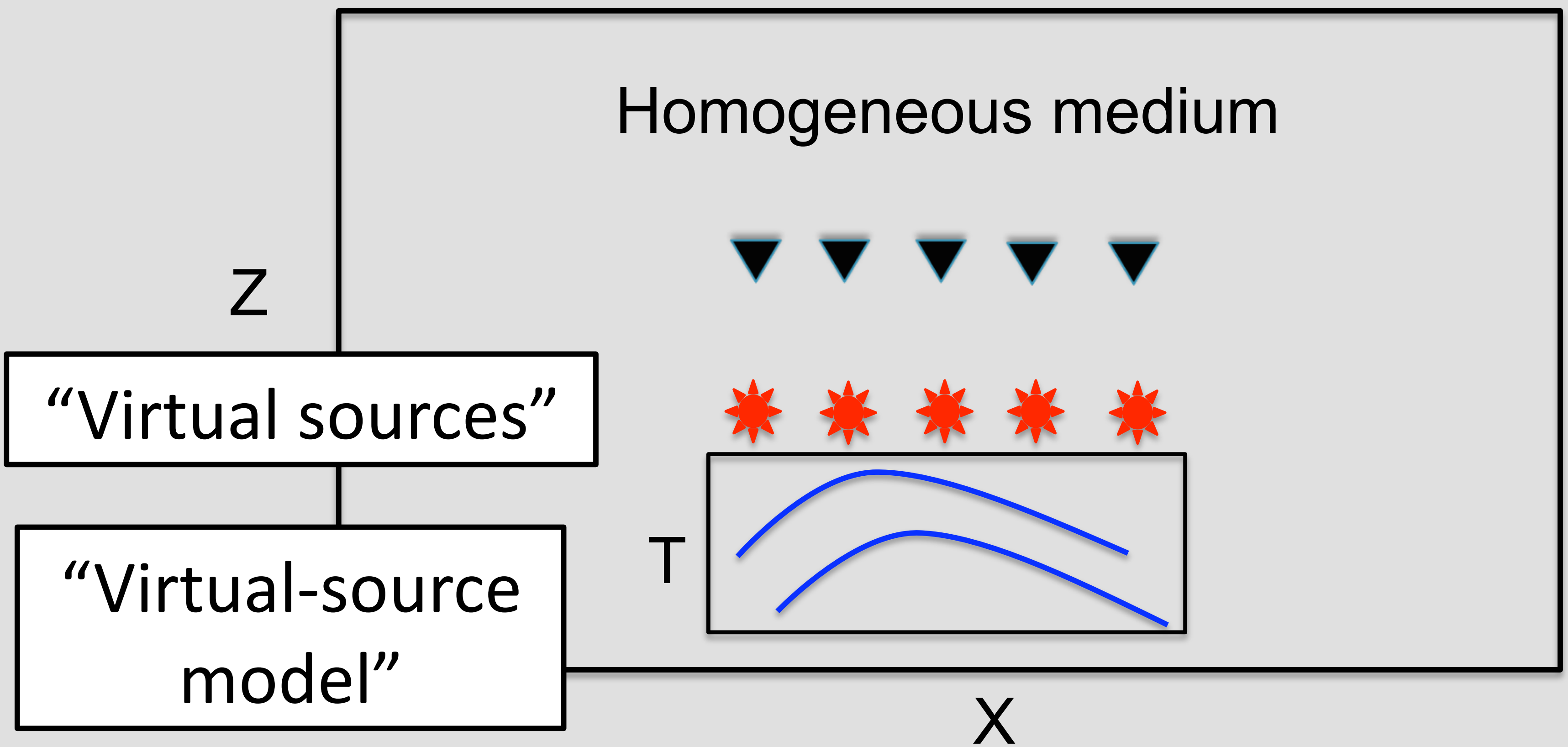




General concept of separation by inversion

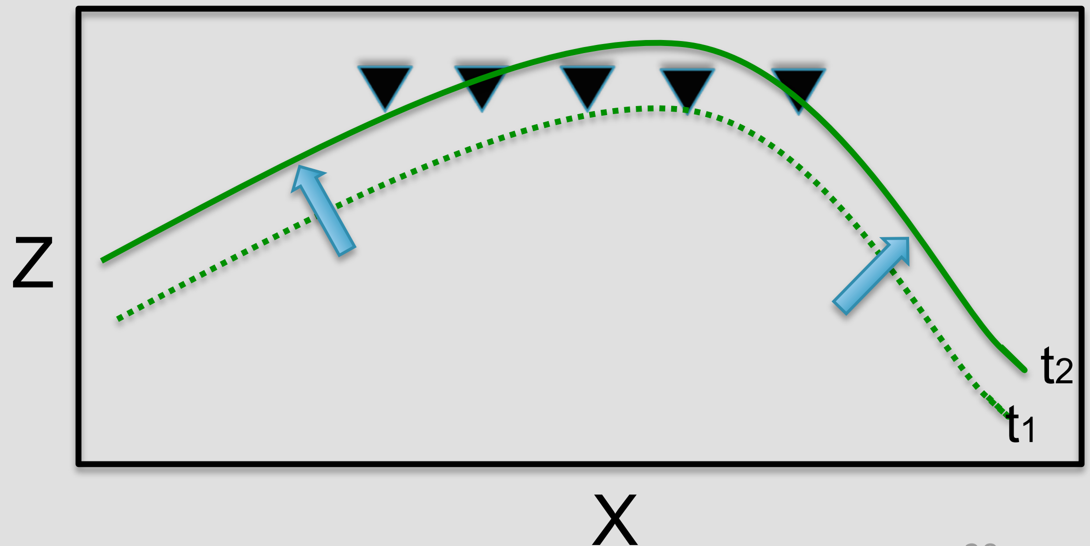
$$d_{\text{obs}}\left(x_r\right) = w(\omega)G\left(\omega, x_s, x_r\right)$$

$$\Updownarrow$$

$$d_{\text{equiv}}\left(x_r\right) = \sum_{x_s^{'}} f\left(\omega, x_s^{'}\right)\tilde{G}\left(\omega, x_s^{'}, x_r\right)$$

General concept of separation by inversion

- Receiver data = a sampling of a field over time.



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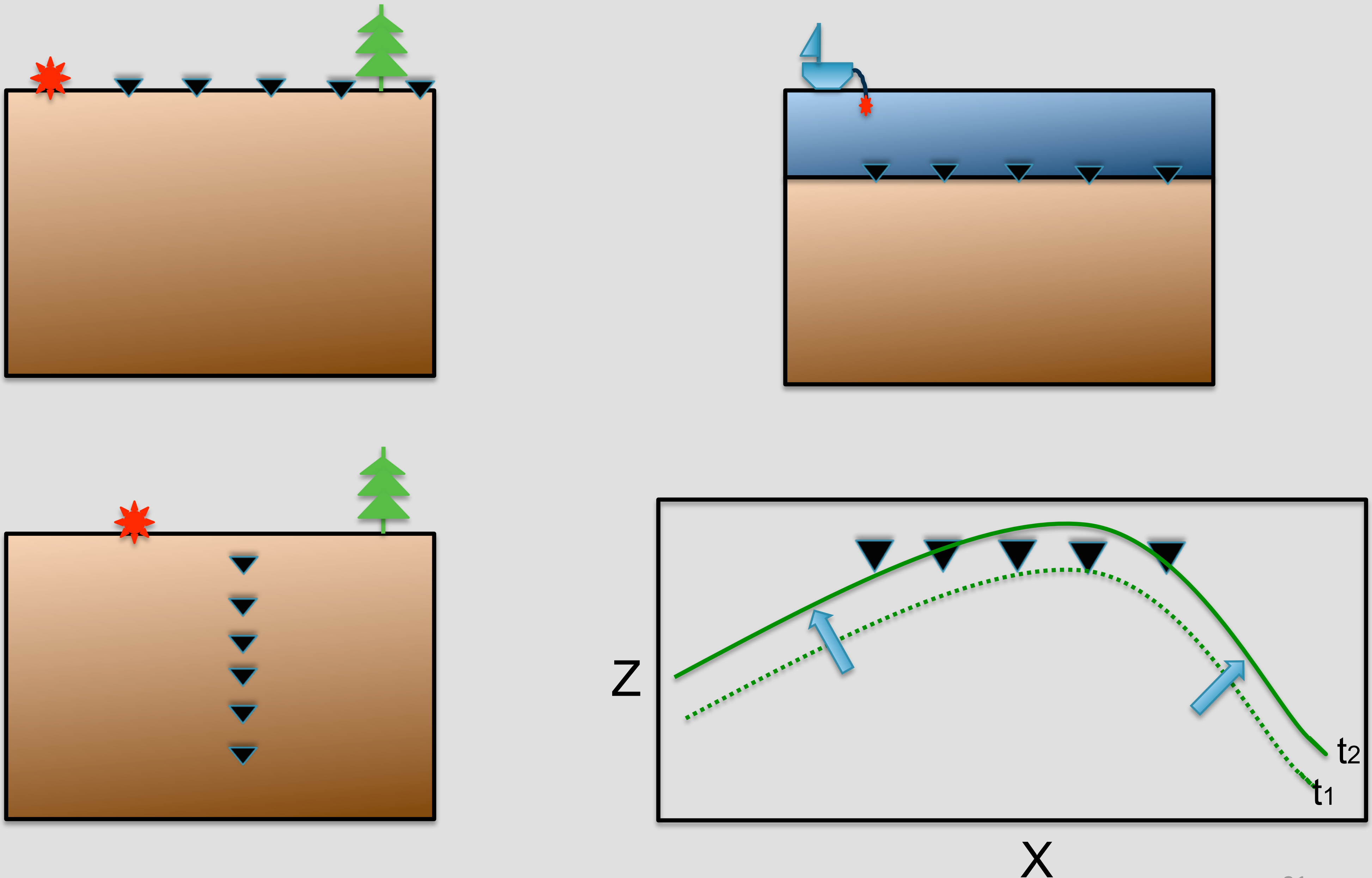
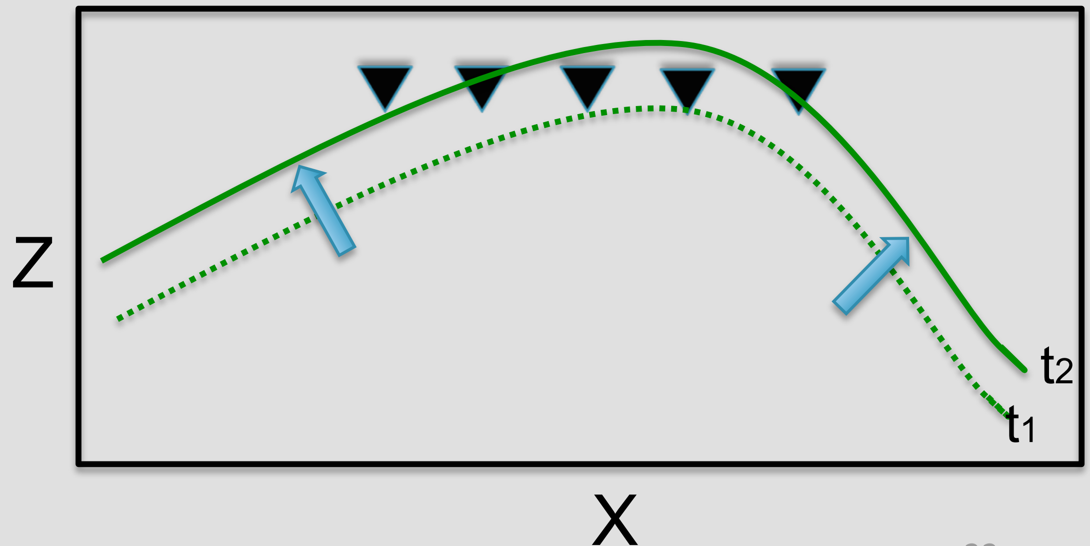


General concept of separation by inversion

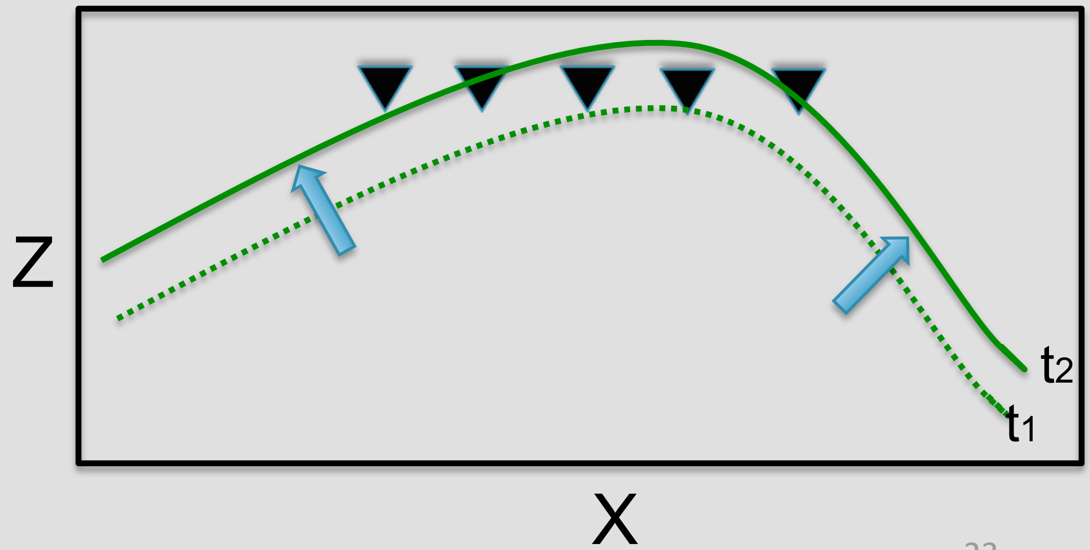
- Receiver data = a sampling of a field over time.
- Matching the recorded data = matching the wavefield at the receiver locations.



General concept of separation by inversion

- Receiver data = a sampling of a field over time.
- Matching the recorded data = matching the wavefield at the receiver locations.
- Assuming wrong medium parameters:

  How accurate is the reconstructed wavefield value one depth level BELOW the receivers?



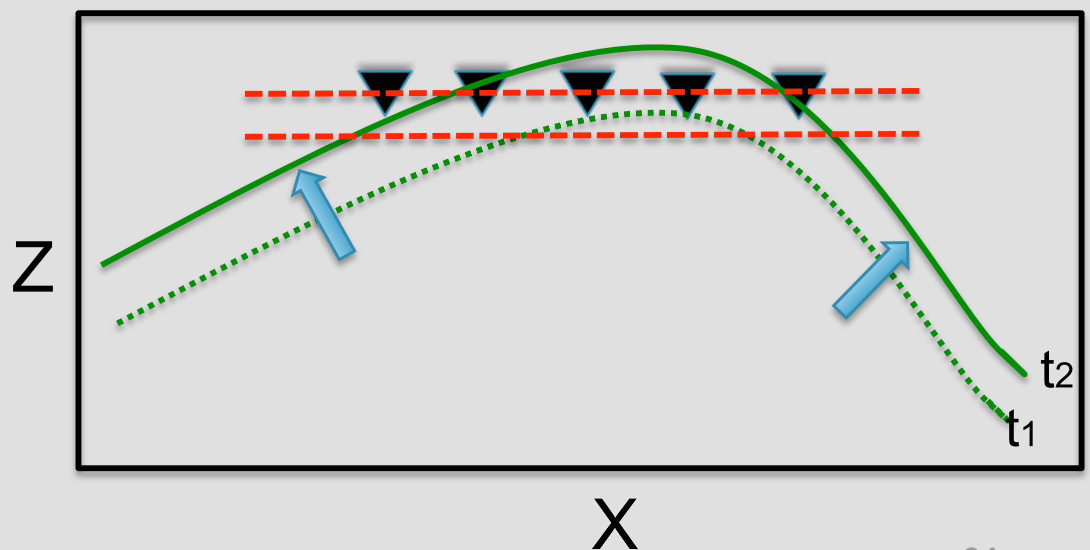
General concept of separation by inversion

- Receiver data = a sampling of a field over time.
- Matching the recorded data = matching the wavefield at the receiver locations.
- Assuming wrong medium parameters:

  How accurate is the reconstructed wavefield value one depth level BELOW the receivers?

1. Typical medium parameters
2. Seismic wavelength
3. Oversampling in space

$$\Delta z = 3m$$



General concept of separation by inversion

How accurate is the reconstructed wavefield value one depth level BELOW the receivers?

$$P = \nabla \cdot u$$

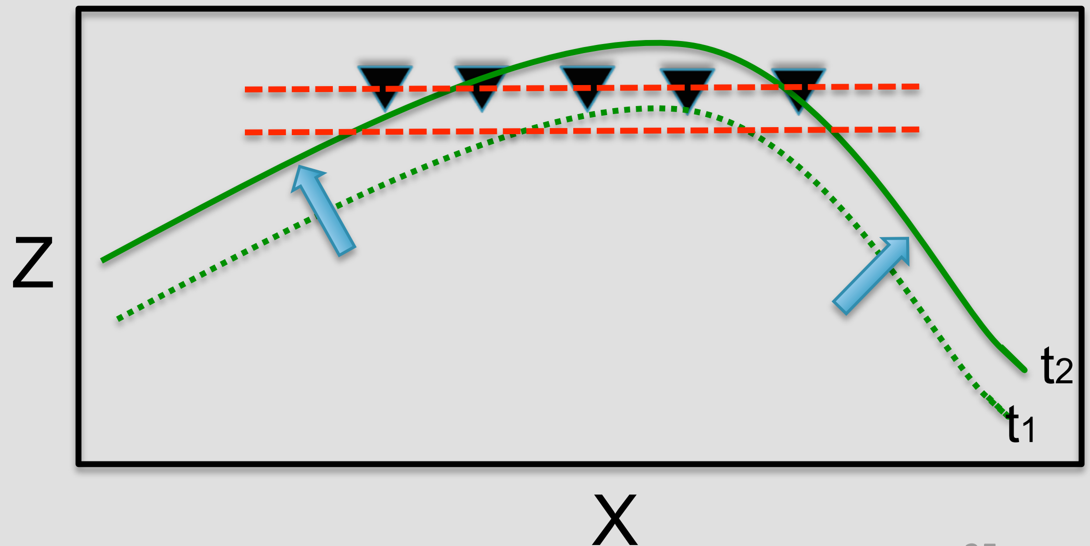
$$S = \nabla \times u$$

$$\tilde{u}_x^p = \frac{1}{|k|}\left(k_x^2 \tilde{u}_x + k_x k_z \tilde{u}_z\right)$$

$$\tilde{u}_z^p = \frac{1}{|k|}\left(k_z^2 \tilde{u}_z + k_x k_z \tilde{u}_x\right)$$

$$\tilde{u}_x^S = \frac{1}{|k|}\left(k_z^2 \tilde{u}_x - k_x k_z \tilde{u}_z\right)$$

$$\tilde{u}_z^S = \frac{1}{|k|}\left(k_x^2 \tilde{u}_z - k_x k_z \tilde{u}_x\right)$$



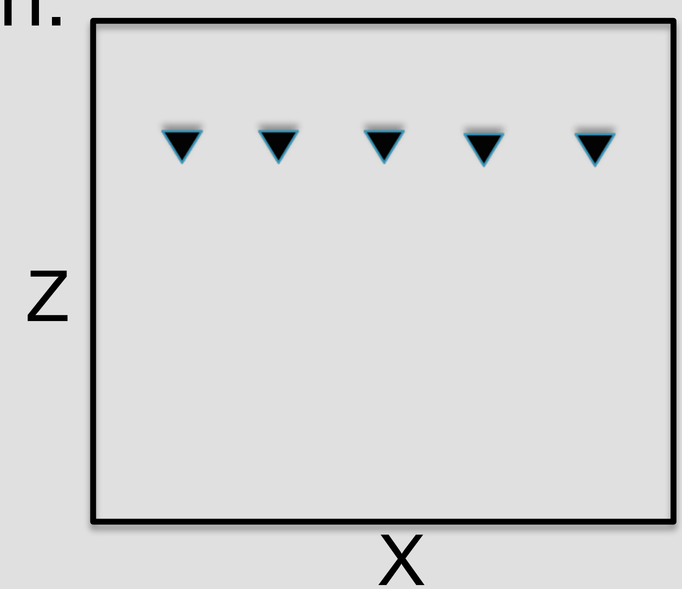
Content

Inversion elements and setup

Observed Data: a shot gather consisting of:

1. Vertical geophone = P_{up} + P_{down} + S + ~~Surface~~ + ~~N_e~~
2. Horizontal geophone = P_{up} + P_{down} + S + ~~Surface~~ + ~~N_e~~

- Rough estimate of medium parameters at receiver's location.
- Constant throughout the inversion.

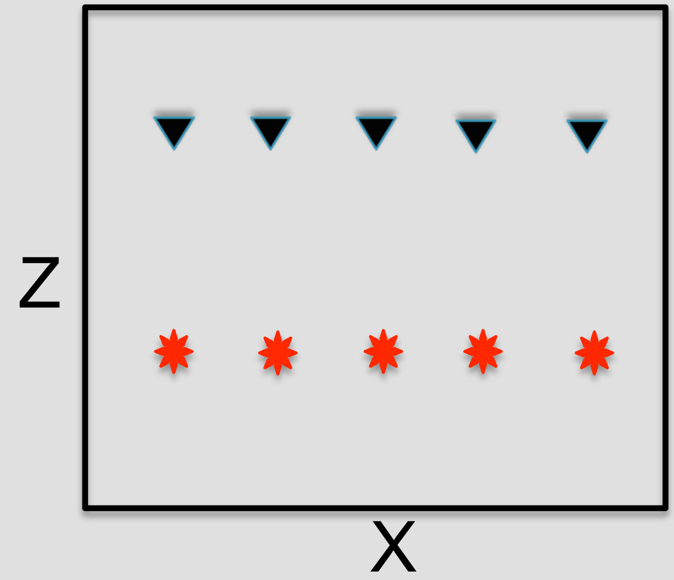


Inversion elements and setup

Model: a virtual source array consisting of:

1. Vertical displacement sources
2. Horizontal displacement sources

- Virtual sources located “somewhere” in medium.

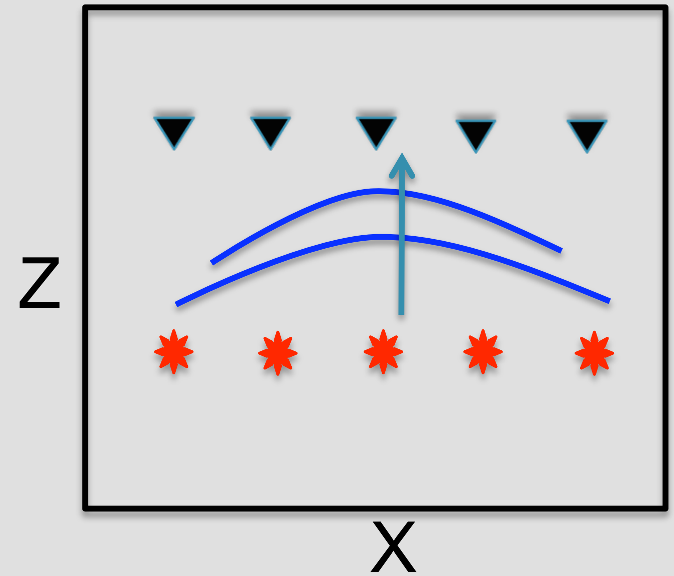


Inversion elements and setup

Model: a virtual source array consisting of:

1. Vertical displacement sources
2. Horizontal displacement sources

- Virtual sources located "somewhere" in medium.
- Virtual sources' functions are propagated; wavefield is recorded at receivers.

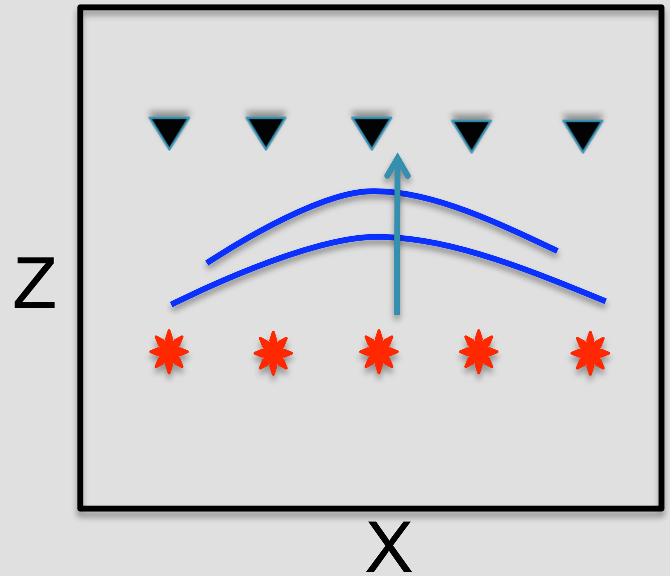


Inversion elements and setup

Model: a virtual source array consisting of:

1. Vertical displacement sources
2. Horizontal displacement sources

- Virtual sources located "somewhere" in medium.
- Virtual sources' functions are propagated; wavefield is recorded at receivers.
- Recorded vertical and horizontal displacements are "reconstructed data"



Inversion elements and setup

Inversion is complete when:
reconstructed displacements ≈ observed displacements.

$$J = \frac{1}{2}\left\| F\vec{m} - \vec{g}_{\text{obs}} \right\|^2$$

$\vec{m}$ - Virtual source functions

F - Propagation operator

$\vec{g}$ - Geophone data

Inversion elements and setup

F - Elastic isotropic propagation operator

$$\nabla\left((\lambda+\mu)\nabla\cdot\vec{u}\right)+\nabla\cdot\left(\mu\nabla\vec{u}\right)+\vec{f}=\rho\ddot{\vec{u}}$$

$$\begin{bmatrix}(\lambda+2\mu)\partial_x^2 u_x + (\lambda+\mu)\partial_x\partial_z u_z + \mu\partial_z^2 u_x + f_x \\ (\lambda+2\mu)\partial_z^2 u_z + (\lambda+\mu)\partial_x\partial_z u_x + \mu\partial_x^2 u_z + f_z\end{bmatrix} = \rho\begin{bmatrix}\partial_t^2 u_x \\ \partial_t^2 u_z\end{bmatrix}$$

$\vec{u}$ - Particle displacement

ρ - Density

$\mu=\rho V_S^2$

$\lambda=\rho V_P^2-2\mu$

- Lame parameters

Inversion elements and setup

F - Elastic isotropic propagation operator

ρ, V_P, V_S - Constant medium parameters.

F is a linear operator.

Inversion elements and setup

After convergence:

1. Forward model using virtual source functions, to generate displacement fields:

$$u_x(x,z)$$

$$u_z(x,z)$$

$$\begin{bmatrix} u_x \\ u_z \end{bmatrix} = F \begin{bmatrix} m_x \\ m_z \end{bmatrix}$$

Inversion elements and setup

<u>After convergence:</u>

1. Forward model using virtual source functions, to generate displacement fields:

$$u_x(x,z)$$

$$u_z(x,z)$$

$$\begin{bmatrix} u_x \\ u_z \end{bmatrix} = F \begin{bmatrix} m_x \\ m_z \end{bmatrix}$$

2. Apply divergence and curl operators on displacement fields at receiver depth:

$$P(x,z_r) = \frac{1}{2\Delta x}\left(u_x\left(x+\Delta x,z_r\right)-u_x\left(x-\Delta x,z_r\right)\right)+\frac{1}{2\Delta z}\left(u_z\left(x,z_r+\Delta z\right)-u_z\left(x,z_r-\Delta z\right)\right)$$

$$S(x,z_r) = \frac{1}{2\Delta x}\left(u_z\left(x+\Delta x,z_r\right)-u_z\left(x-\Delta x,z_r\right)\right)-\frac{1}{2\Delta z}\left(u_x\left(x,z_r+\Delta z\right)-u_x\left(x,z_r-\Delta z\right)\right)$$

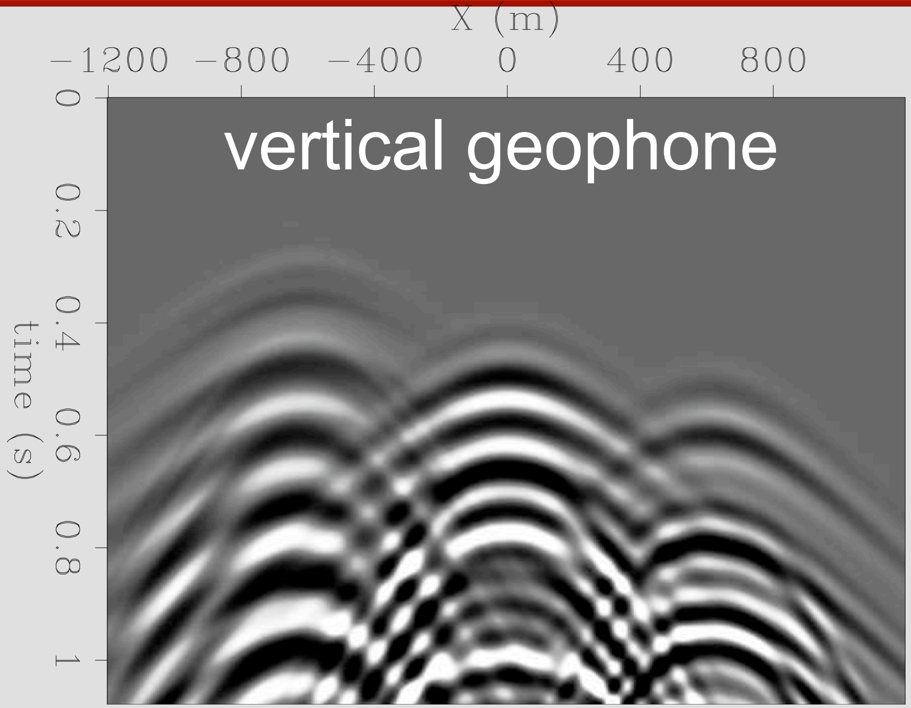
$$J = \frac{1}{2}\left\| F\vec{m} - \vec{g}_{\text{obs}} \right\|^2$$

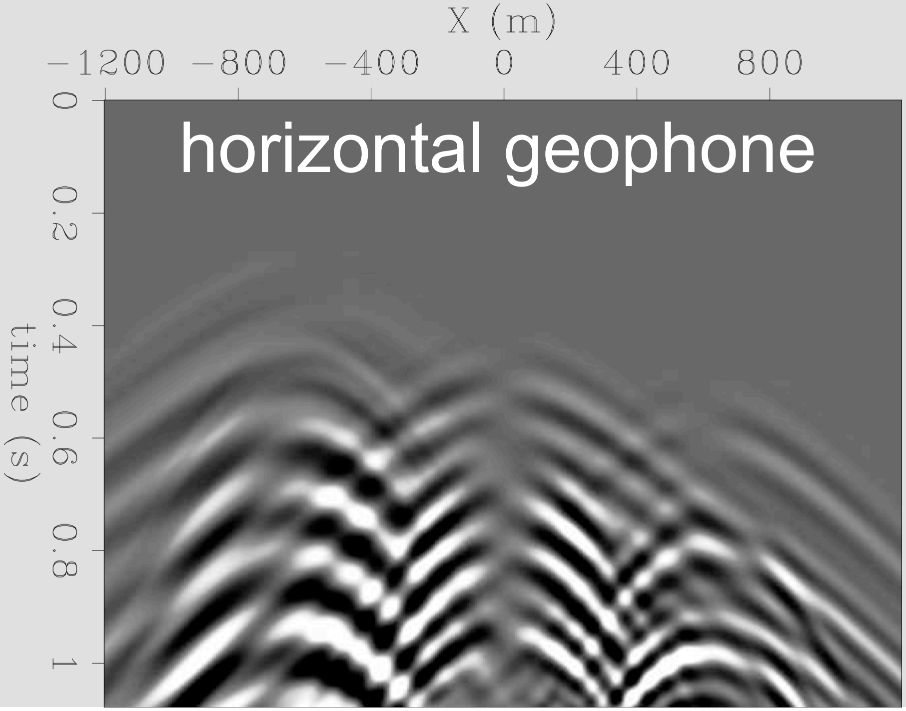
$$\begin{bmatrix} u_x \\ u_z \end{bmatrix} = F \begin{bmatrix} m_x \\ m_z \end{bmatrix}$$

- Objective function matches geophone data only.
- Sufficient for P/S separation of land data.
- Necessary but insufficient for P/S separation of OBS data.

Content

Synthetic land data

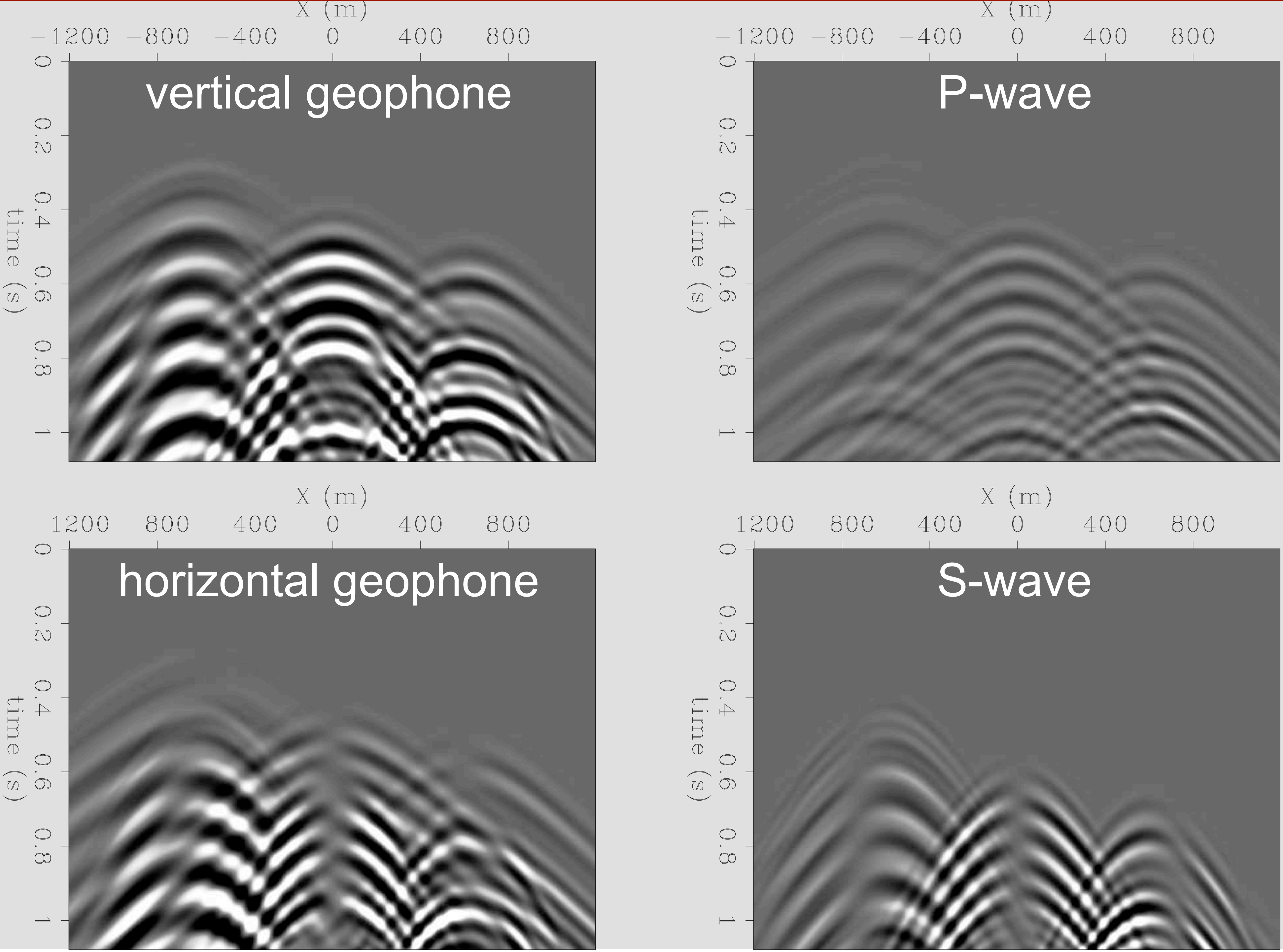




- P and S waves.
- Random reflectivity series.
- Upgoing data only.
- Sum of three shot gathers. with different V_p and V_s:

1. $v_p = 2.0\frac{\text{km}}{\text{s}}, \quad v_s = 1.0\frac{\text{km}}{\text{s}}, \quad \rho = 2.0\frac{\text{gr}}{\text{cm}^3}$
2. $v_p = 1.8\frac{\text{km}}{\text{s}}, \quad v_s = 0.9\frac{\text{km}}{\text{s}}, \quad \rho = 1.8\frac{\text{gr}}{\text{cm}^3}$
3. $v_p = 1.6\frac{\text{km}}{\text{s}}, \quad v_s = 0.8\frac{\text{km}}{\text{s}}, \quad \rho = 1.6\frac{\text{gr}}{\text{cm}^3}$

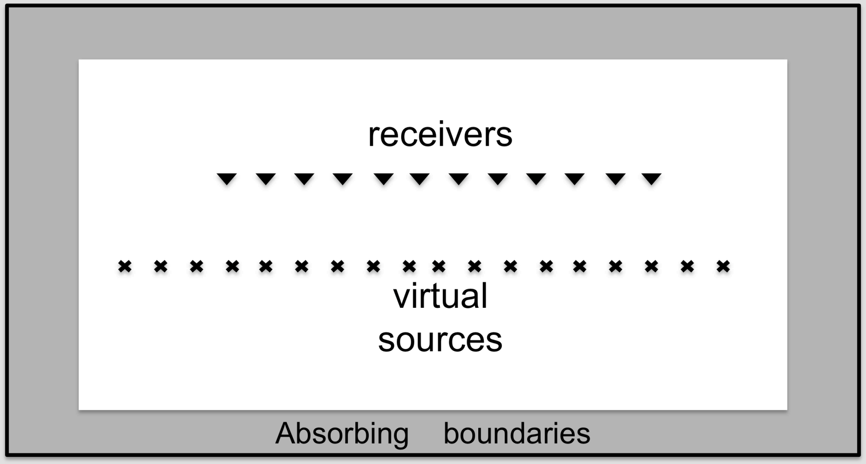
Synthetic land data



Synthetic land data

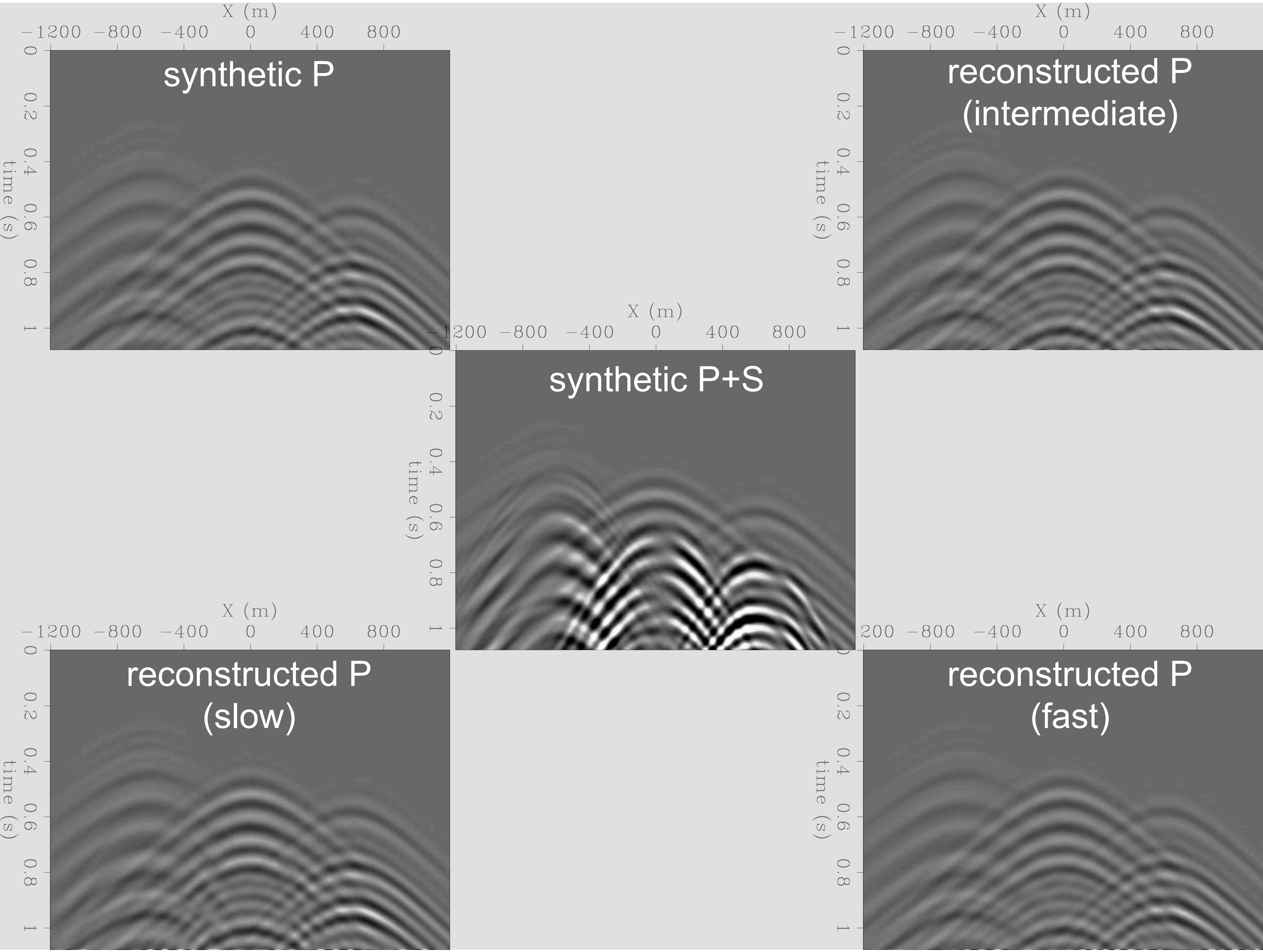
Synthetics generated with:

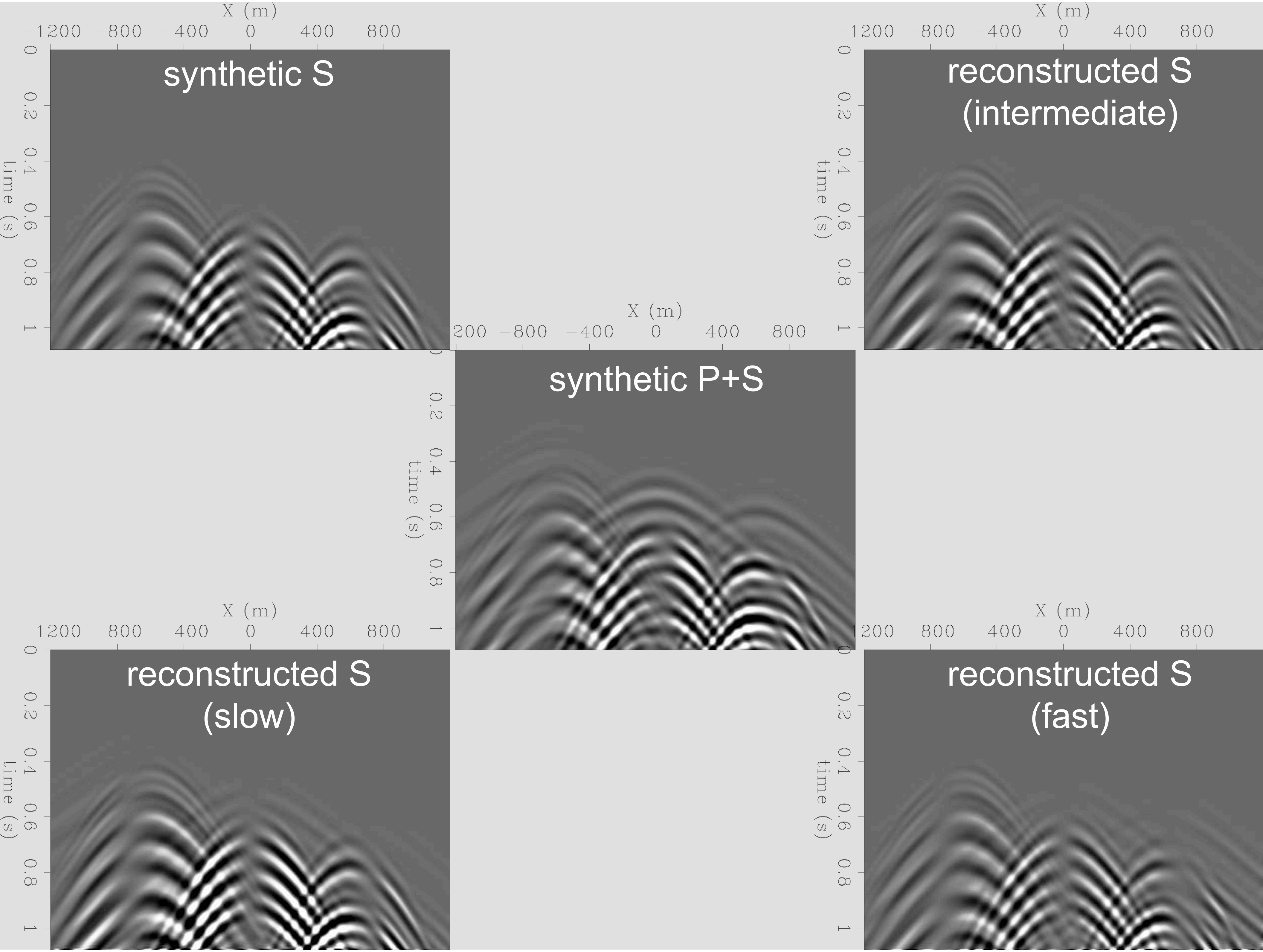
1. $v_p = 2.0 \frac{km}{s}, \quad v_s = 1.0 \frac{km}{s}, \quad \rho = 2.0 \frac{gr}{cm^3}$
2. $v_p = 1.8 \frac{km}{s}, \quad v_s = 0.9 \frac{km}{s}, \quad \rho = 1.8 \frac{gr}{cm^3}$
3. $v_p = 1.6 \frac{km}{s}, \quad v_s = 0.8 \frac{km}{s}, \quad \rho = 1.6 \frac{gr}{cm^3}$



Three separate inversions with:

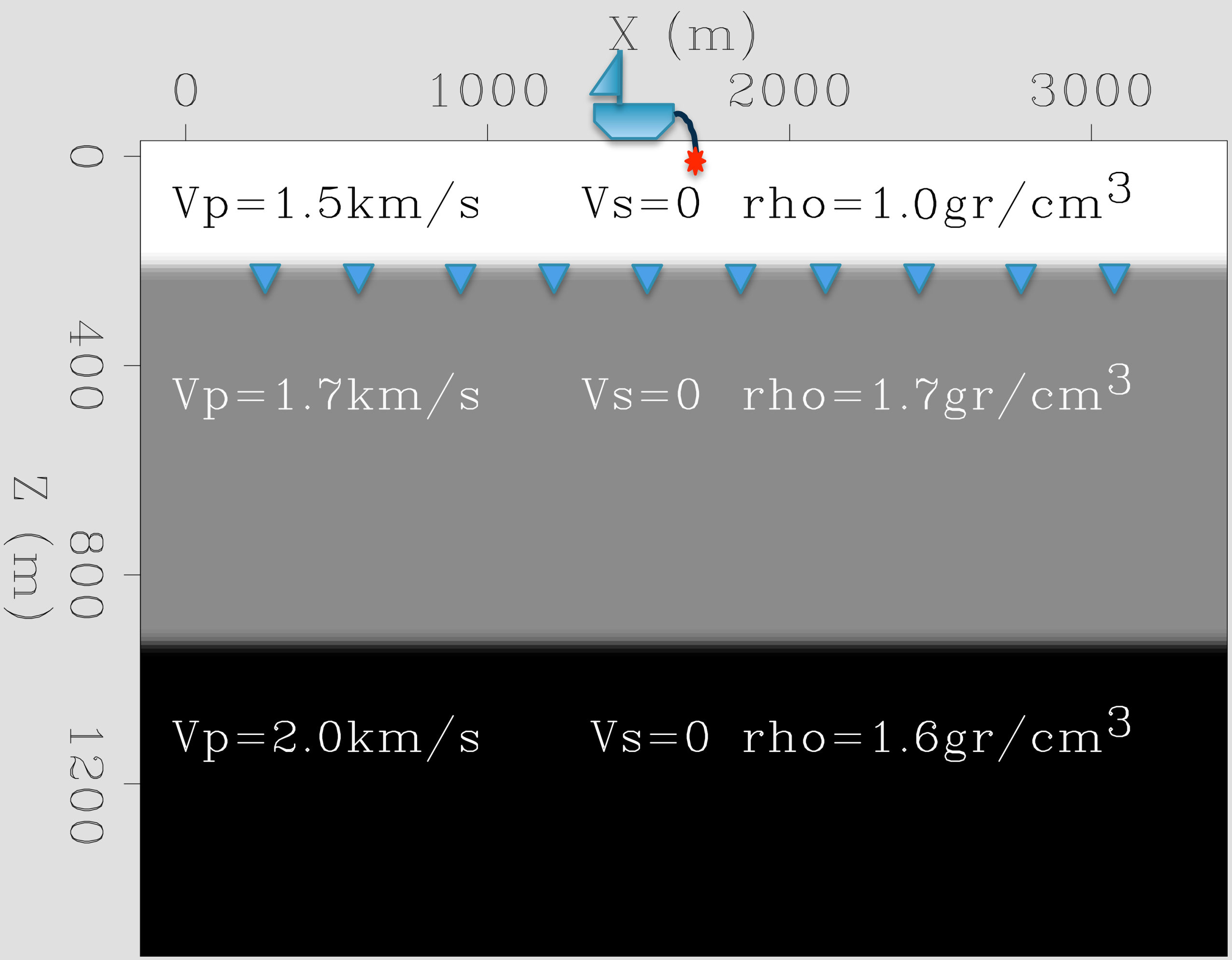
1. intermediate $v_p = 1.7 \frac{km}{s}, \quad v_s = 0.85 \frac{km}{s}, \quad \rho = 1.7 \frac{gr}{cm^3}$
2. slow $v_p = 1.5 \frac{km}{s}, \quad v_s = 0.7 \frac{km}{s}, \quad \rho = 1.5 \frac{gr}{cm^3}$
3. fast $v_p = 2.0 \frac{km}{s}, \quad v_s = 1.0 \frac{km}{s}, \quad \rho = 2.0 \frac{gr}{cm^3}$


synthetic P
reconstructed P
(intermediate)
synthetic P+S
reconstructed P
(slow)
reconstructed P
(fast)
X (m)
−1200 −800 −400 0 400 800
time (s)
0 0.2 0.4 0.6 0.8 1



synthetic S
reconstructed S (intermediate)
synthetic P+S
reconstructed S (slow)
reconstructed S (fast)
X (m)
time (s)

Synthetic OBS data



Inversion medium parameters:

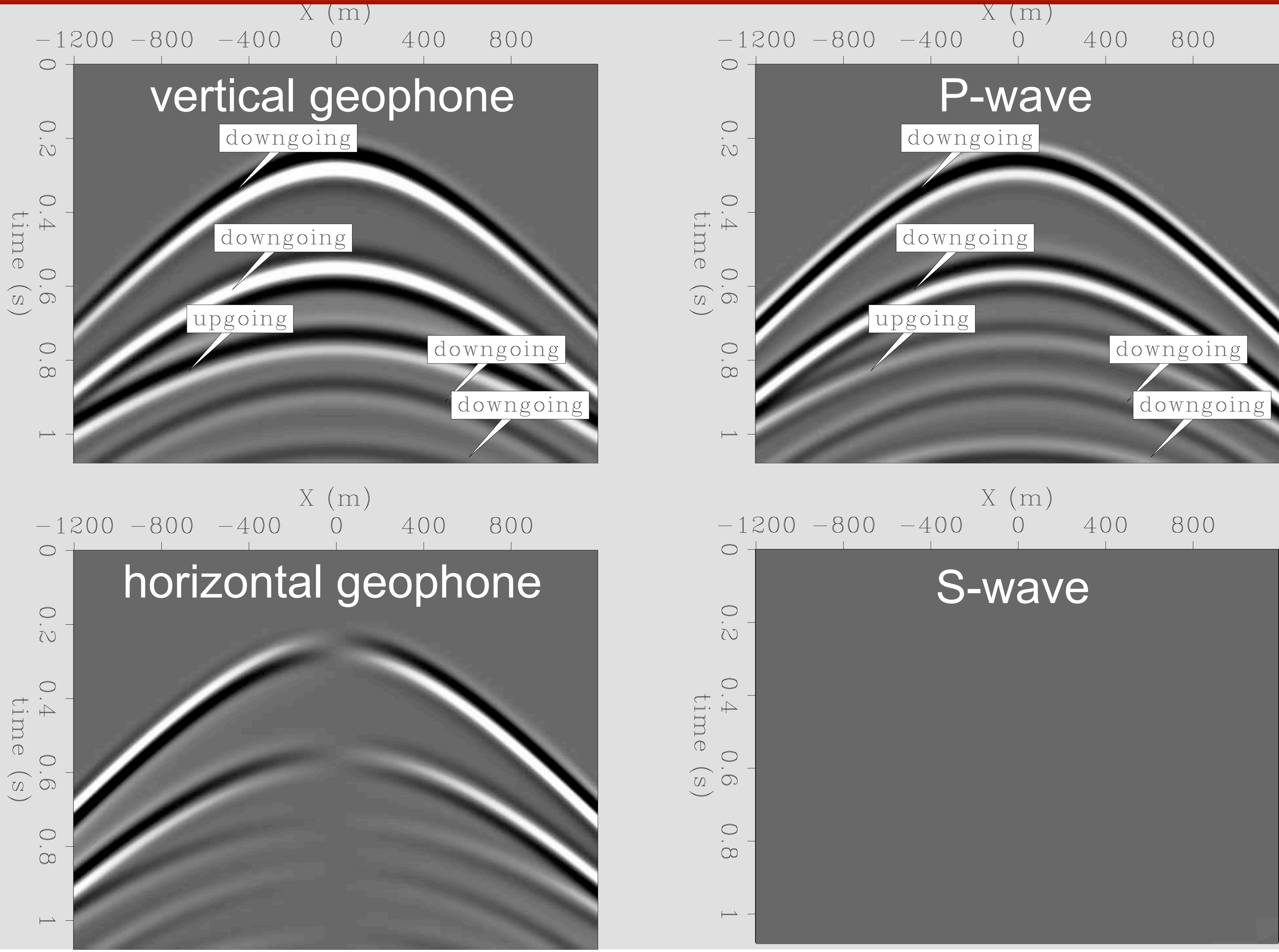
$$v_p = 1.7 \frac{\text{km}}{\text{s}}$$

$$v_s = 0.85 \frac{\text{km}}{\text{s}}$$

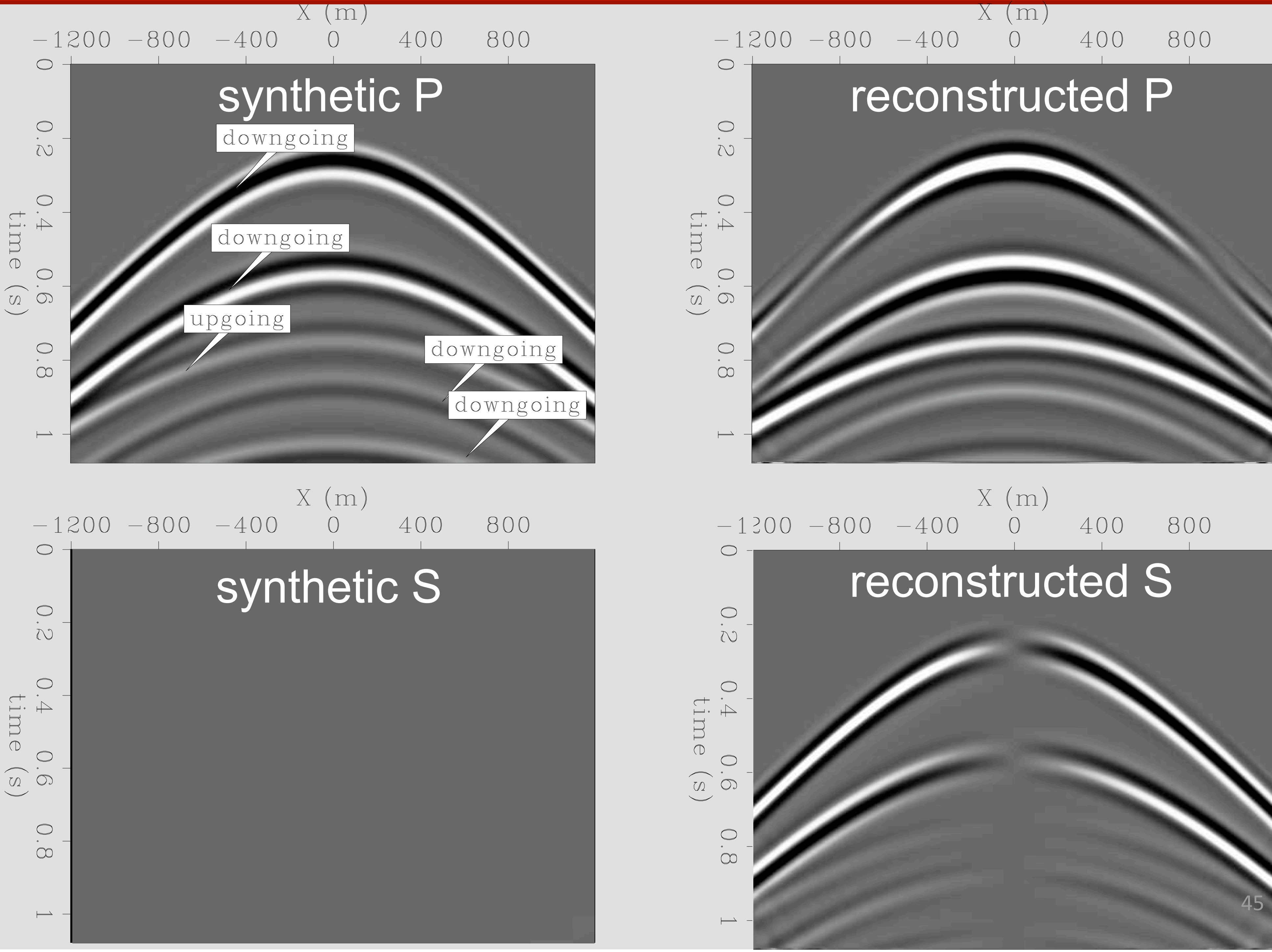
$$\rho = 1.7 \frac{\text{gr}}{\text{cm}^3}$$

Direct arrival muted.

Synthetic OBS data



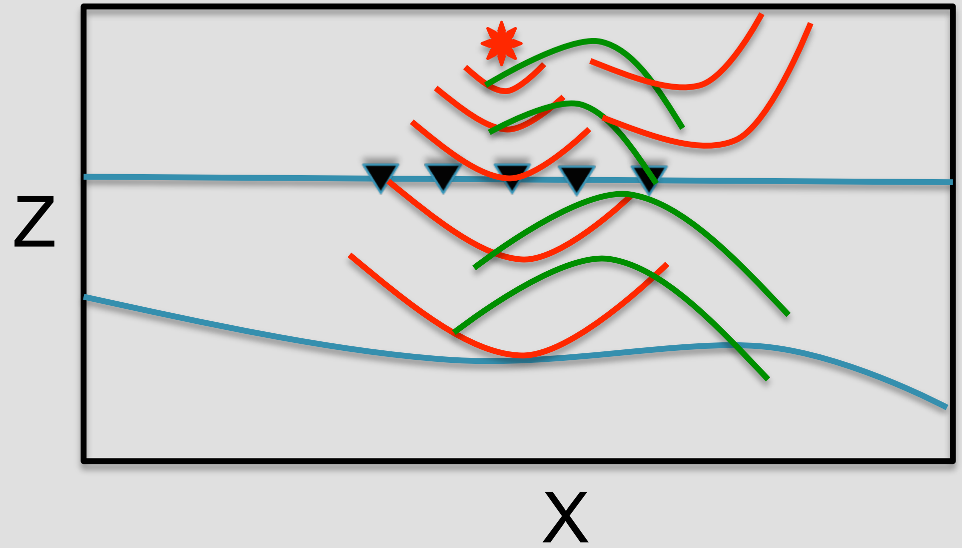
Synthetic OBS data



Null-space issue

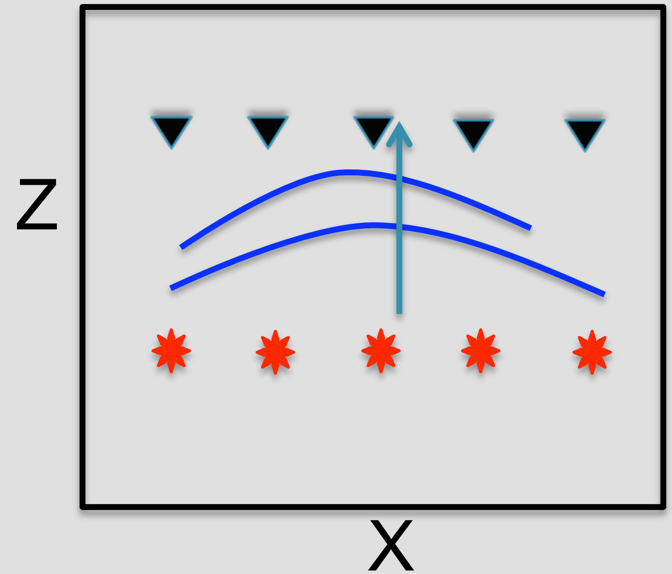
Field data:

Upgoing and downgoing waves.



Reconstructed data:

Upgoing waves only.



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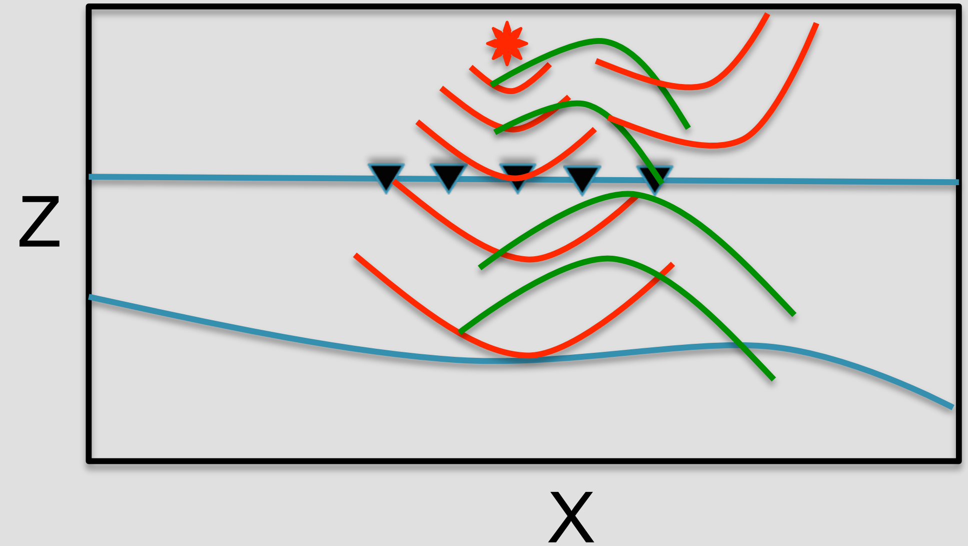

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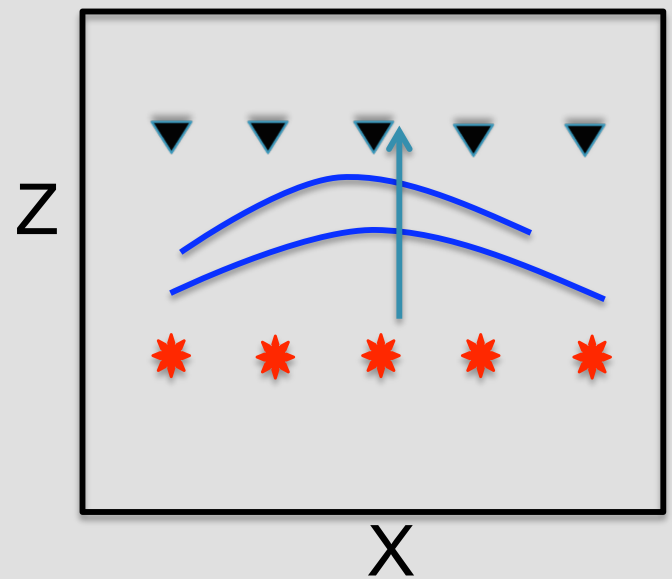
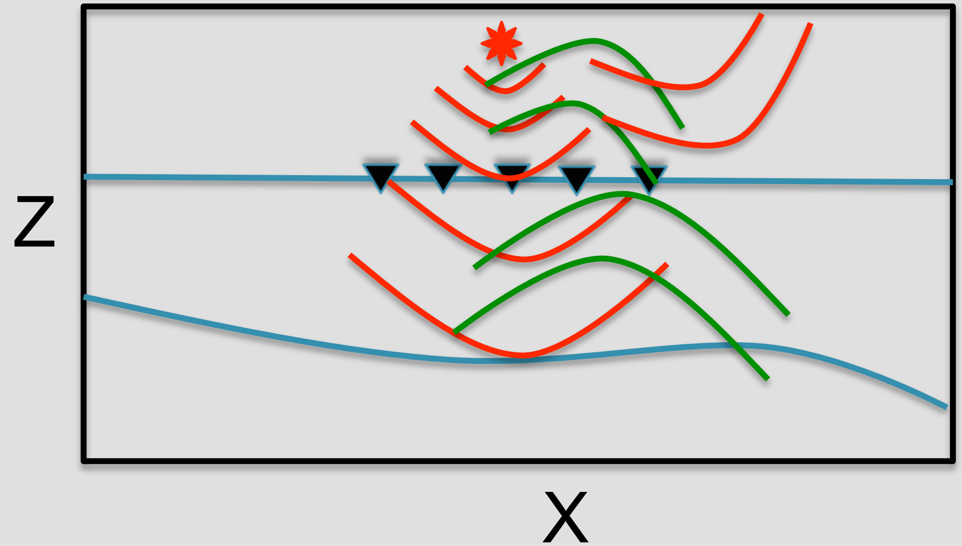


Vertical derivative is unconstrained.

P and S are unconstrained.

Null-space issue



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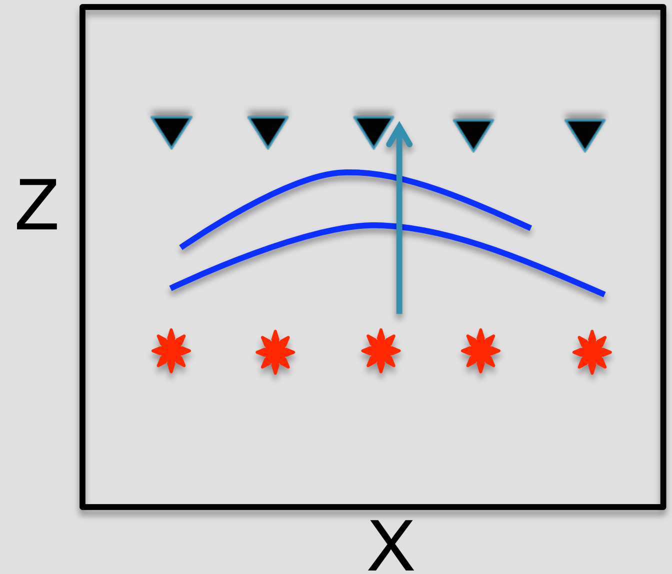
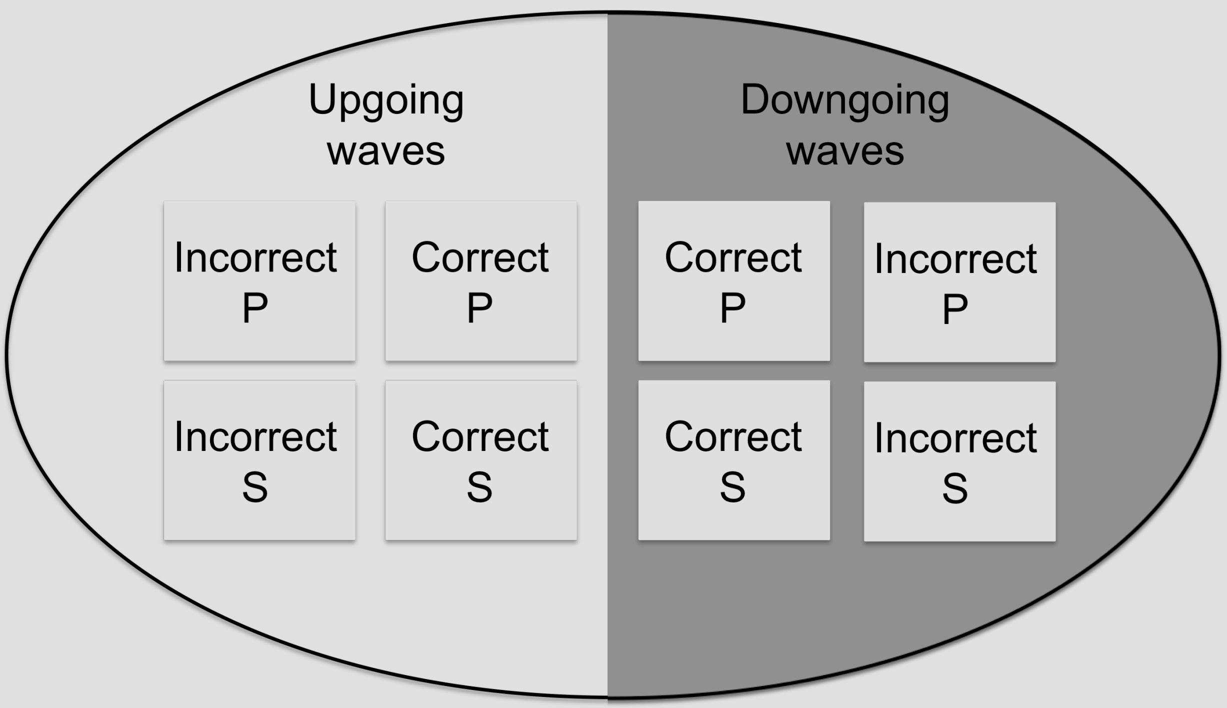
Upgoing waves only.





Managing the solution-space and the null-space

Controls:

1. Virtual-source locations.
2. Additional constraints.

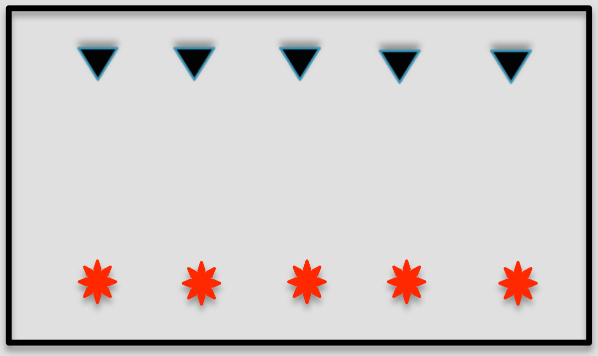
Question:

Can the solution-space and the null-space be controlled in a useful way?

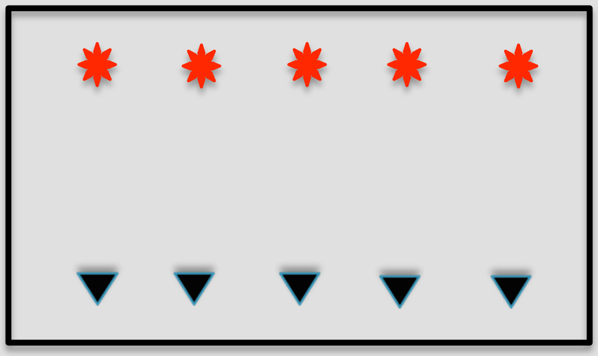
Example: prevent leakage of P-wave energy to S-wave energy.

Virtual-source locations

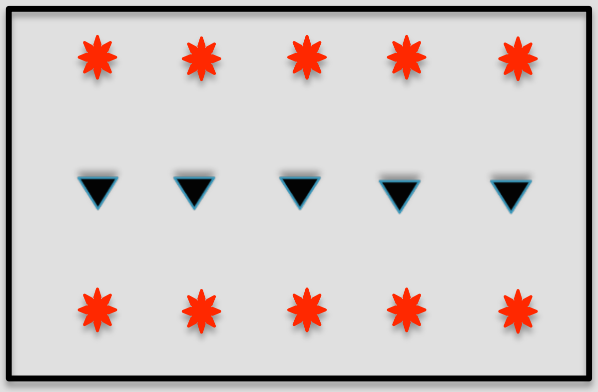
Control which wavenumbers can be reconstructed.



Upgoing only



Downgoing only

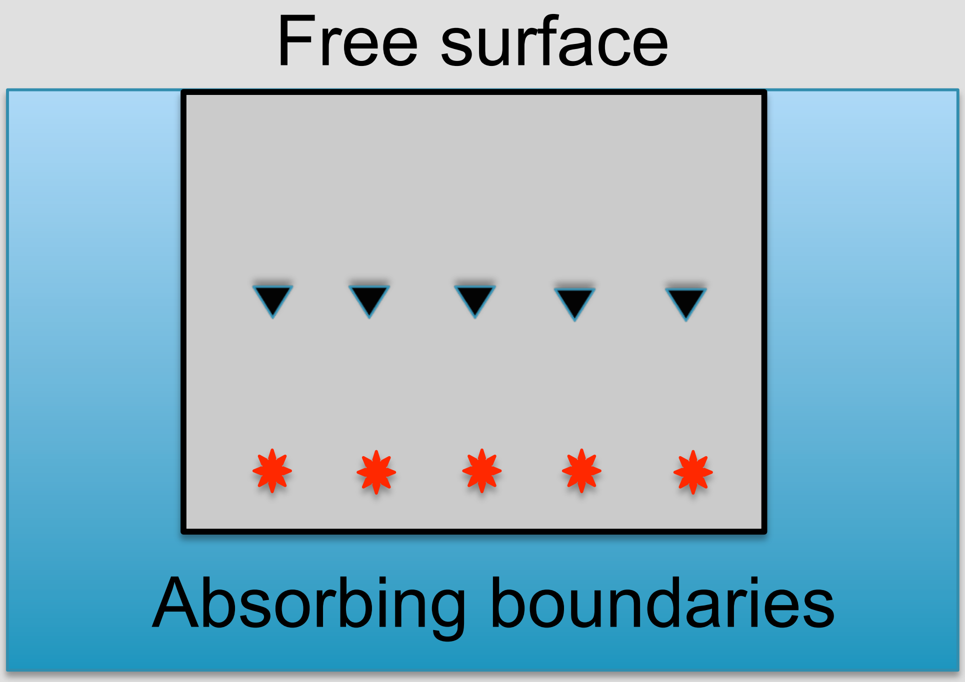


Upgoing and downgoing

Increases null-space.

Virtual-source locations

Use a free upper boundary to generate the downgoing waves.



Additional constraint

Observed hydrophone ***H***.

$$H \Leftrightarrow P = \frac{1}{2}(\lambda + \mu)\nabla \cdot \vec{u}$$

$$\vec{u} \Leftarrow \frac{1}{2}(\lambda + \mu)\nabla P$$

Additional constraint

Observed hydrophone ***H***.

$$H \Leftrightarrow P = \frac{1}{2}(\lambda + \mu)\nabla \cdot \vec{u} \qquad P = \frac{1}{2}(\lambda + \mu)\nabla \cdot \vec{u}^{P}$$

$$\vec{u} \Leftarrow \frac{1}{2}(\lambda + \mu)\nabla P \qquad \vec{u}^{P} \Leftarrow \frac{1}{2}(\lambda + \mu)\nabla P$$

Additional constraint

Observed hydrophone ***H***.

$$H \Leftrightarrow P = \frac{1}{2}(\lambda + \mu)\nabla \cdot \vec{u} \qquad P = \frac{1}{2}(\lambda + \mu)\nabla \cdot \vec{u}^{P}$$

$$\vec{u} \Leftarrow \frac{1}{2}(\lambda + \mu)\nabla P \qquad \vec{u}^{P} \Leftarrow \frac{1}{2}(\lambda + \mu)\nabla P$$

Modified objective function:

$$J = \varepsilon_1 \left\| F\vec{m} - \vec{g}_{\text{obs}} \right\|^2 + \varepsilon_2 \left\| A\vec{m} - H_{\text{obs}} \right\|^2$$

$$A = \nabla \cdot F$$

$$(\lambda + \mu) \Leftrightarrow \varepsilon_2$$

Additional constraint

Observed hydrophone ***H***

$$H \Leftrightarrow P = \frac{1}{2}(\lambda + \mu)\nabla \cdot \vec{u} \qquad P = \frac{1}{2}(\lambda + \mu)\nabla \cdot \vec{u}^{P}$$

$$\vec{u} \Leftarrow \frac{1}{2}(\lambda + \mu)\nabla P \qquad \vec{u}^{P} \Leftarrow \frac{1}{2}(\lambda + \mu)\nabla P$$

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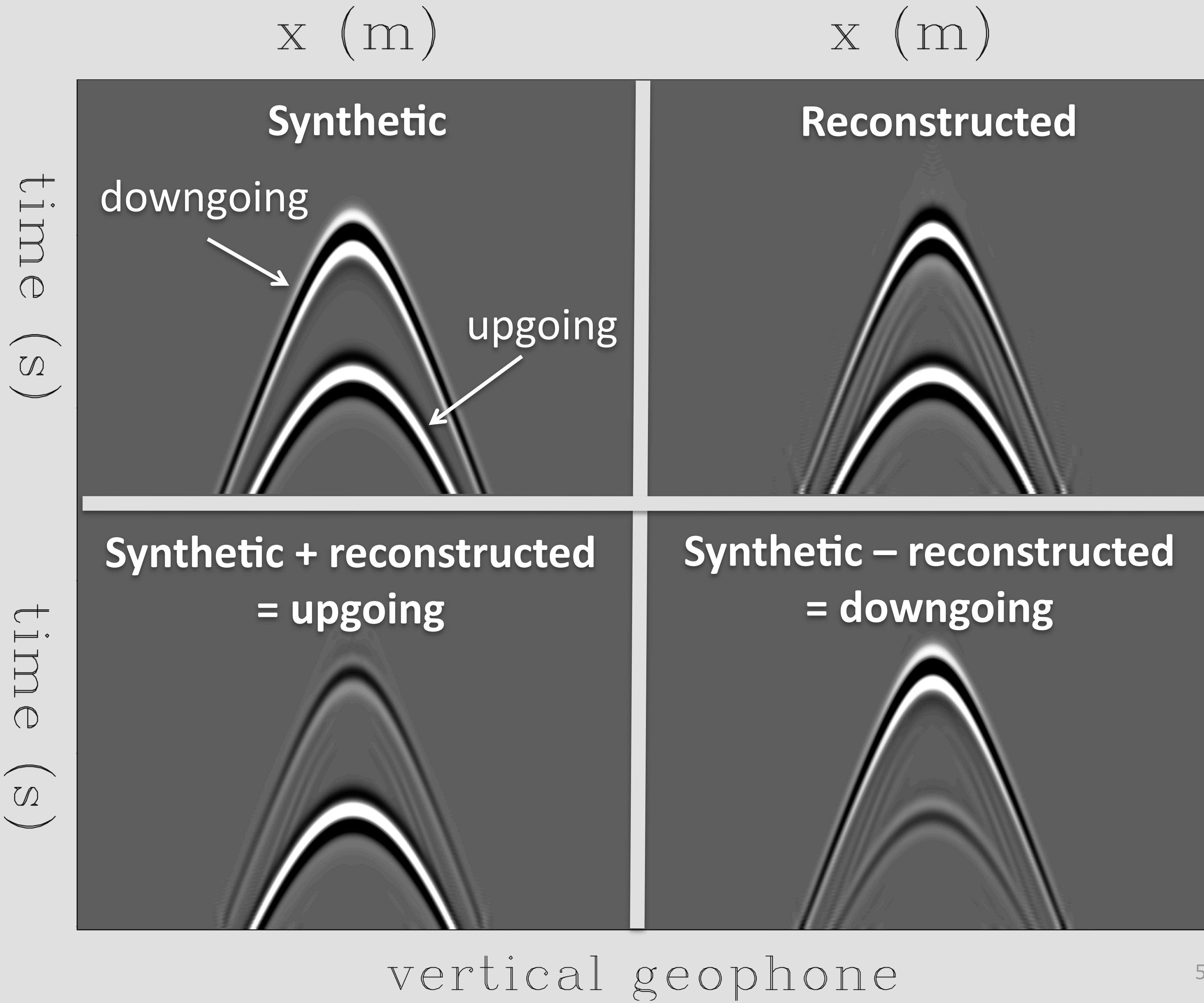
$$A = \nabla \cdot F$$

$$(\lambda + \mu) \Leftrightarrow \varepsilon_2$$

Constrains spatial derivative of P particle motion.


x (m)
x (m)
time (s)
time (s)
Synthetic
downgoing
upgoing
Reconstructed
Synthetic + reconstructed
= upgoing
Synthetic – reconstructed
= downgoing
vertical geophone

Content

P/S separation of ocean-bottom seismic data by inversion

- It is possible to generate equivalent seismic data using virtual-sources and incorrect medium parameters.
- P/S separation is reasonably good even for large medium parameter error.
- Reconstructable wavenumbers depend on virtual sources' locations.
- More constraints are required so that the spatial derivatives of the reconstructed field near the receivers are correctly estimated.

Objective: Solve the OBS problem

1. Utilise different virtual-source / receiver arrangements to enable reconstruction of all wavenumbers (increase null-space).
2. Combine geophone and hydrophone constraints (reduce null-space).

P/S separation of ocean-bottom seismic data by inversion

Thanks for listening