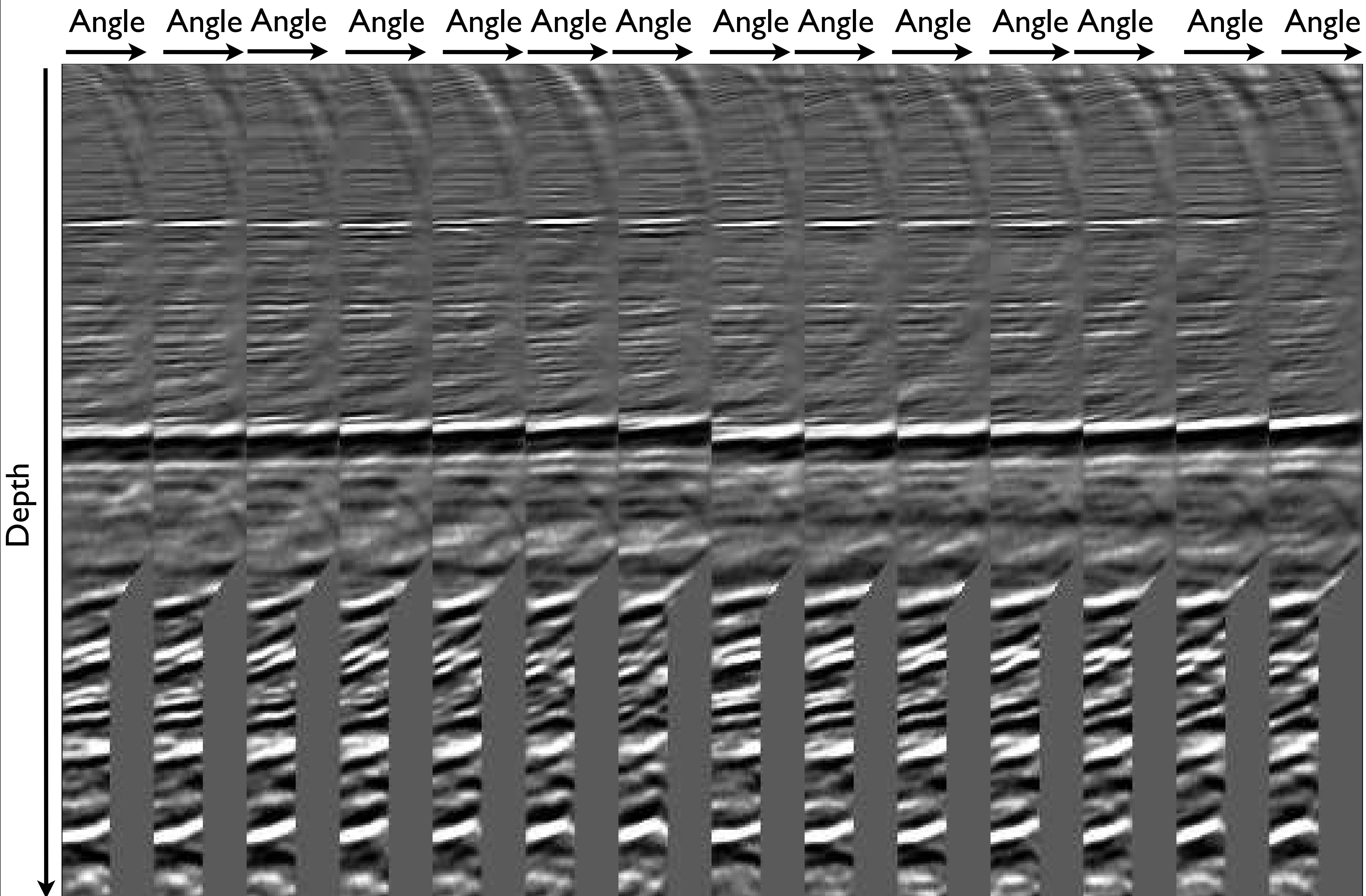
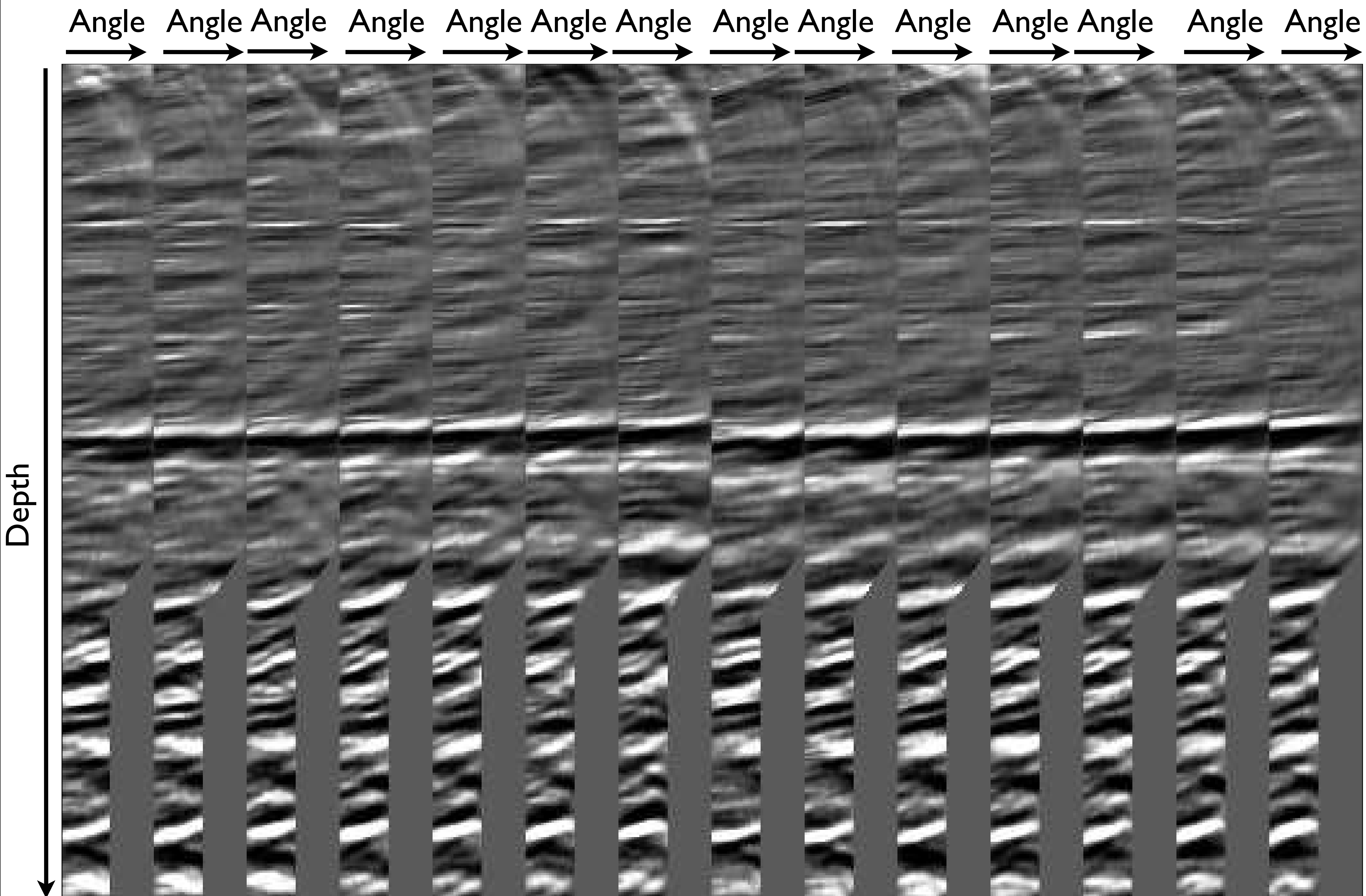
Image gather reconstruction using StOMP

Bob Clapp

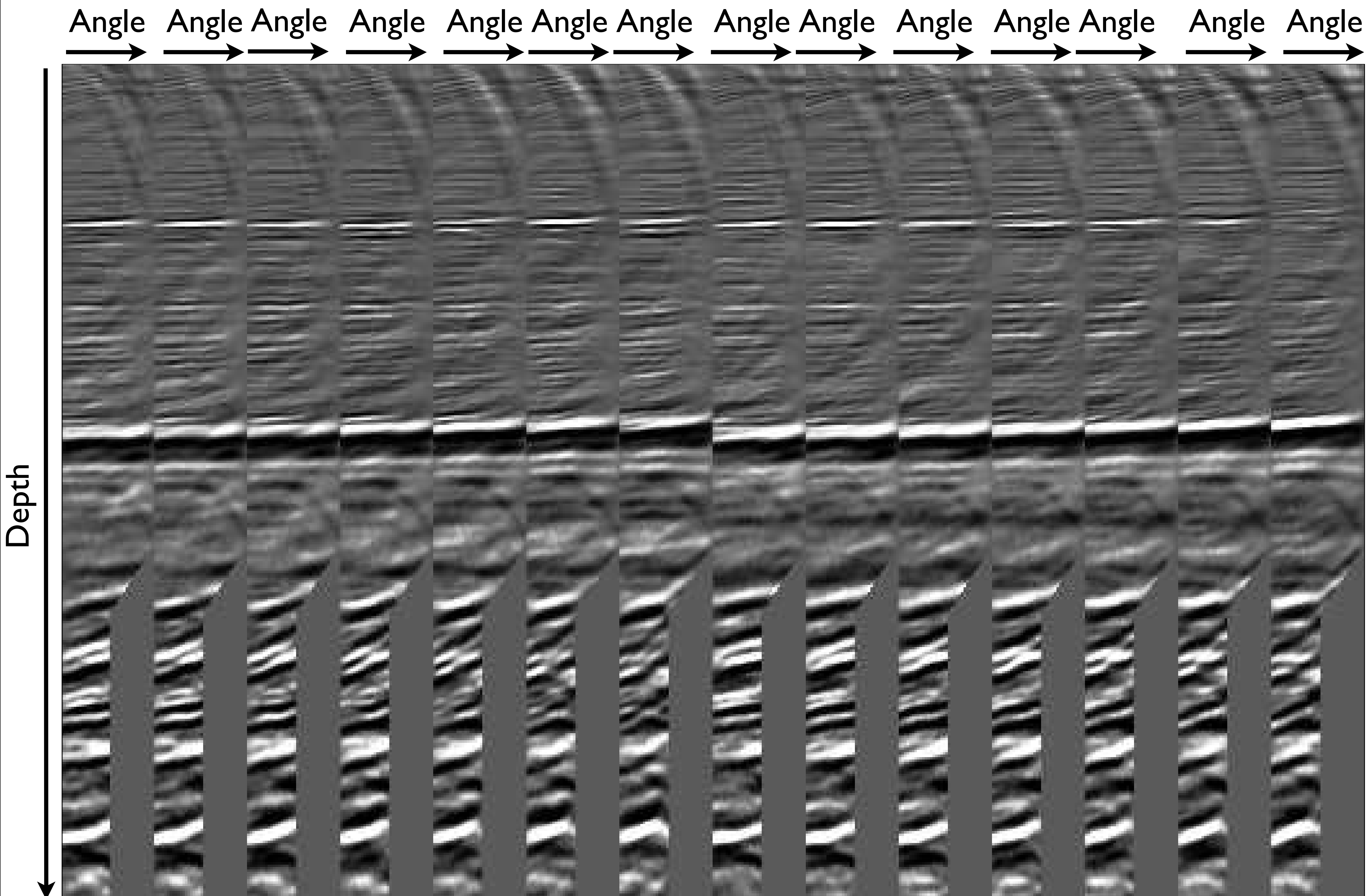
SEP 147-Pg. 127


Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Depth

Method 1

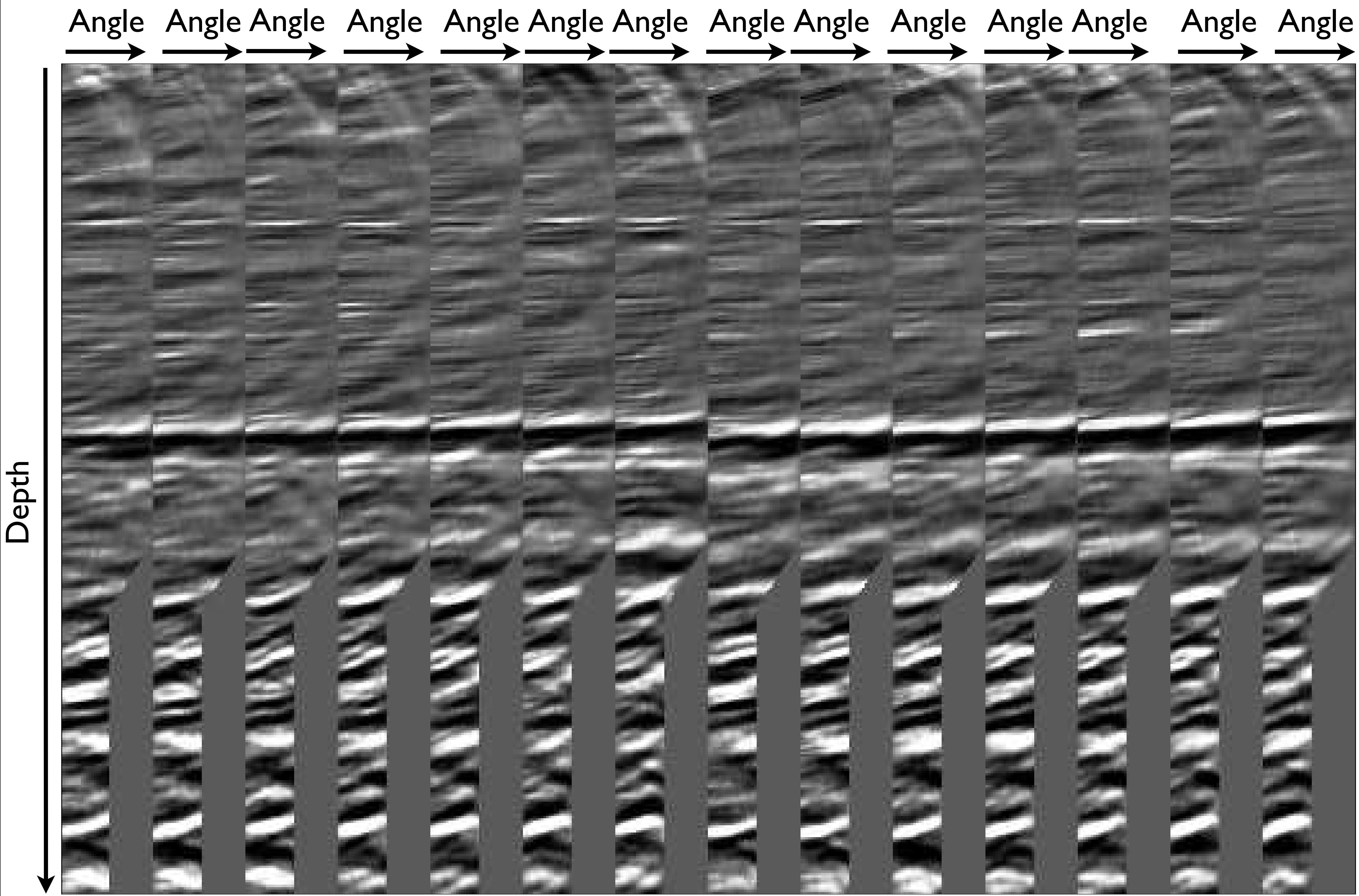

Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Depth

Method 2



Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Angle
Depth

Full volume



1/20 volume size

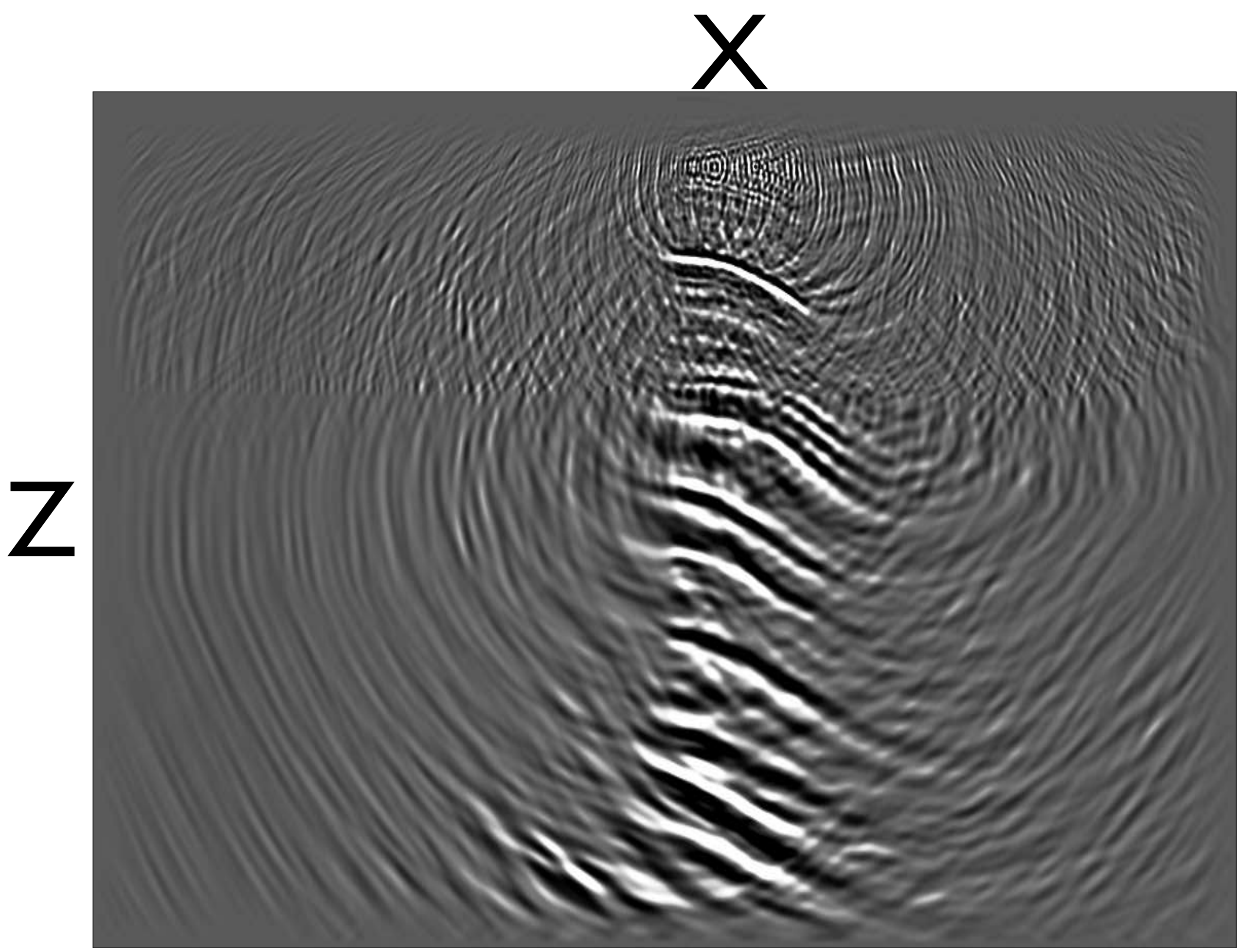
Outline

- Cost of angle gather construction
- Compressive sensing
- Compressibility of seismic data
- StOMP
- Making it work

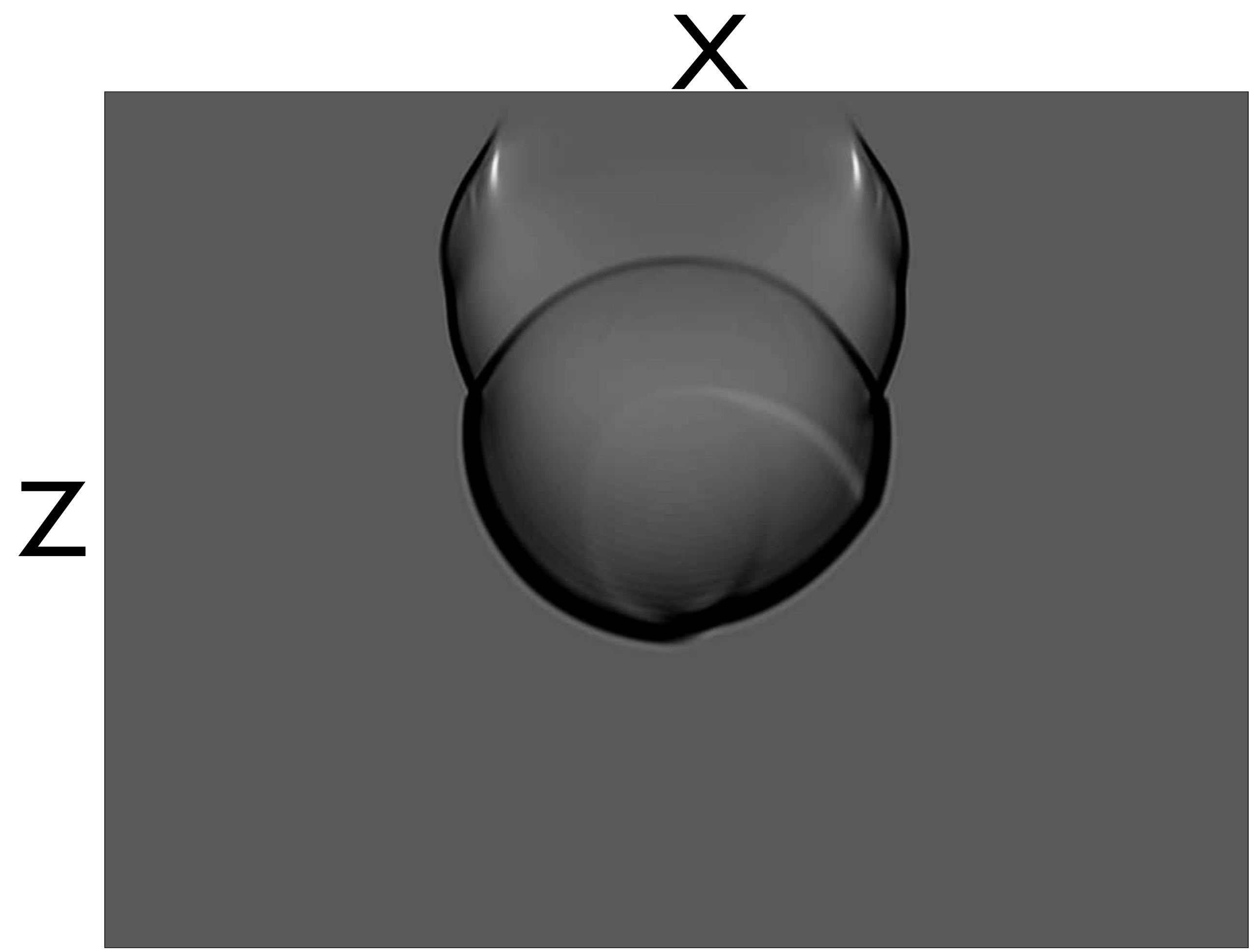
Angle gather construction

- Shift-based gathers
  - Simple to code
  - Tradeoff of massive increase in data volume (cost) or limited azimuthal information
- Gathers dependent on wavefield dips
  - More sophisticated coding
  - Potentially doesn't account for all arrivals

Visual slide on gather construction



Receiver wavefield

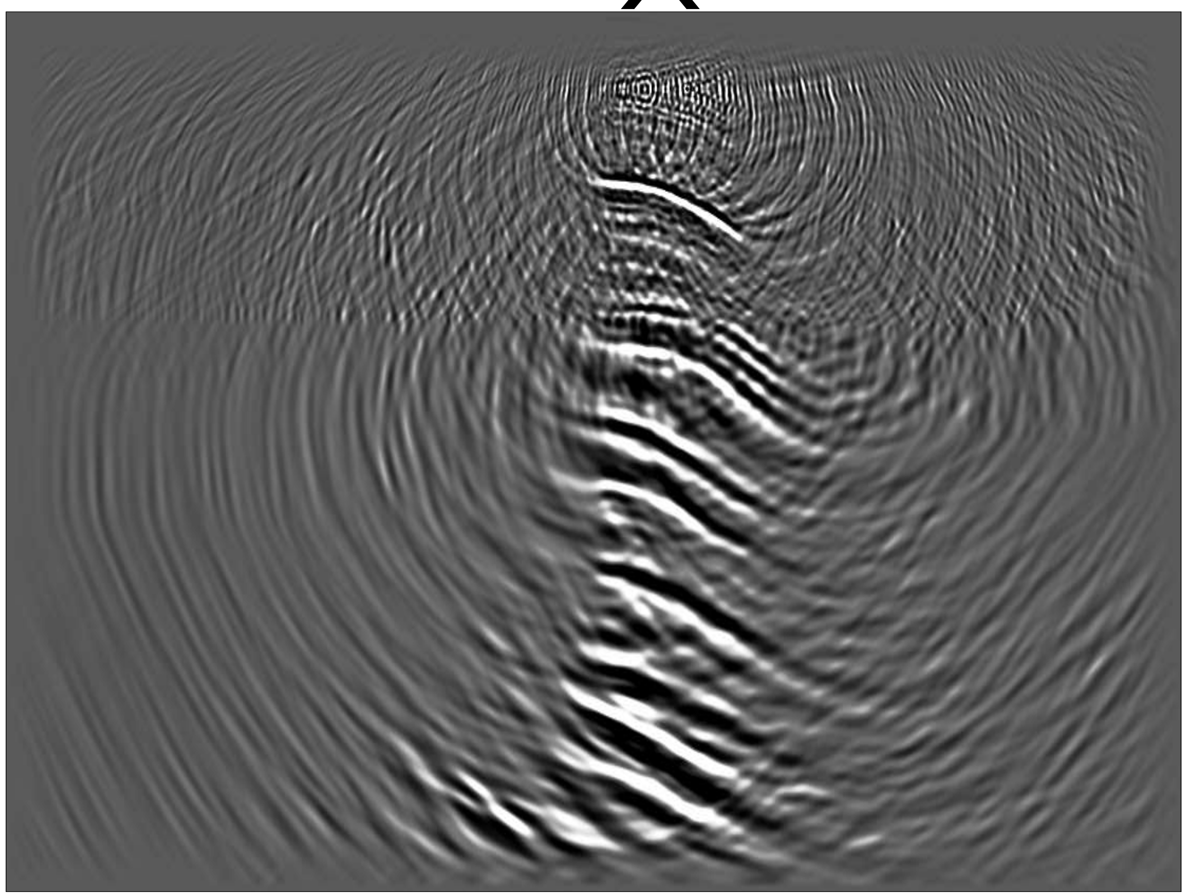


Source wavefield

Visual slide on gather construction

x

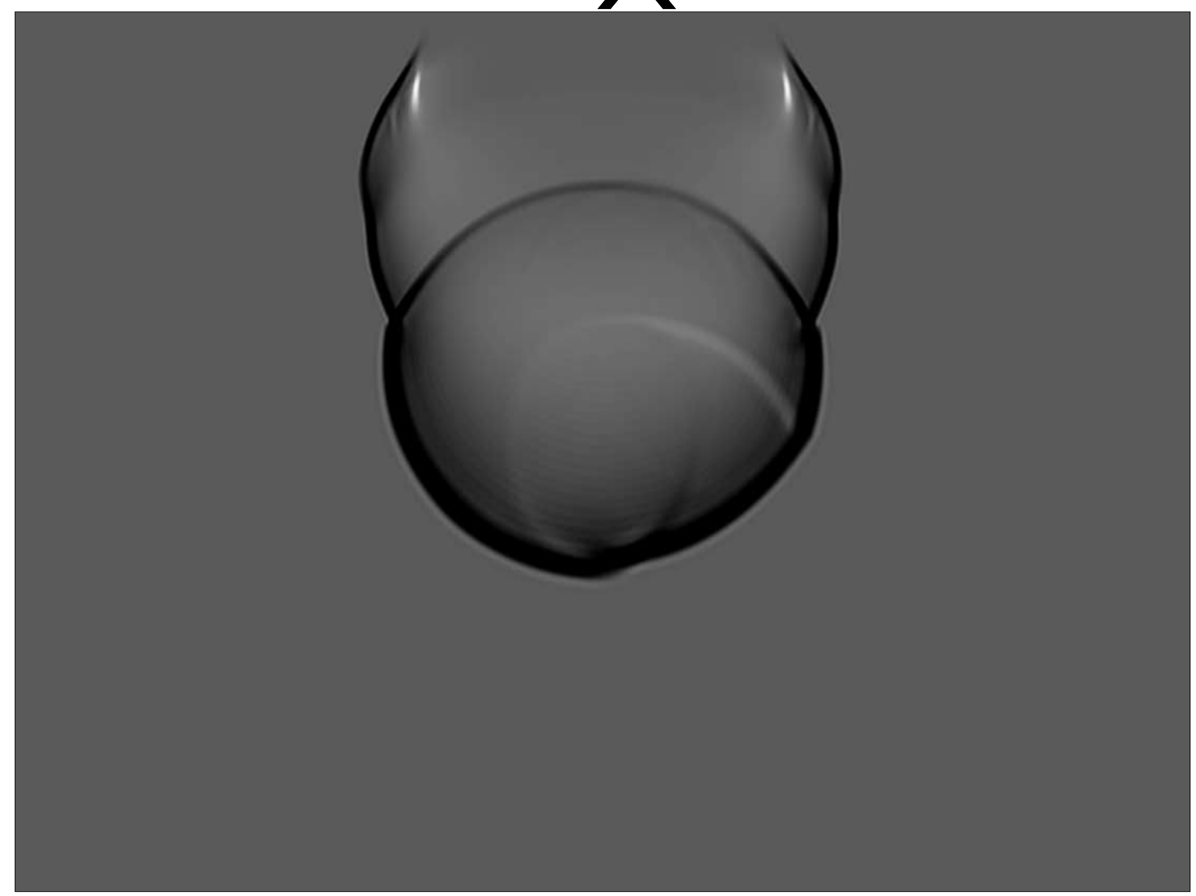
z



Receiver wavefield

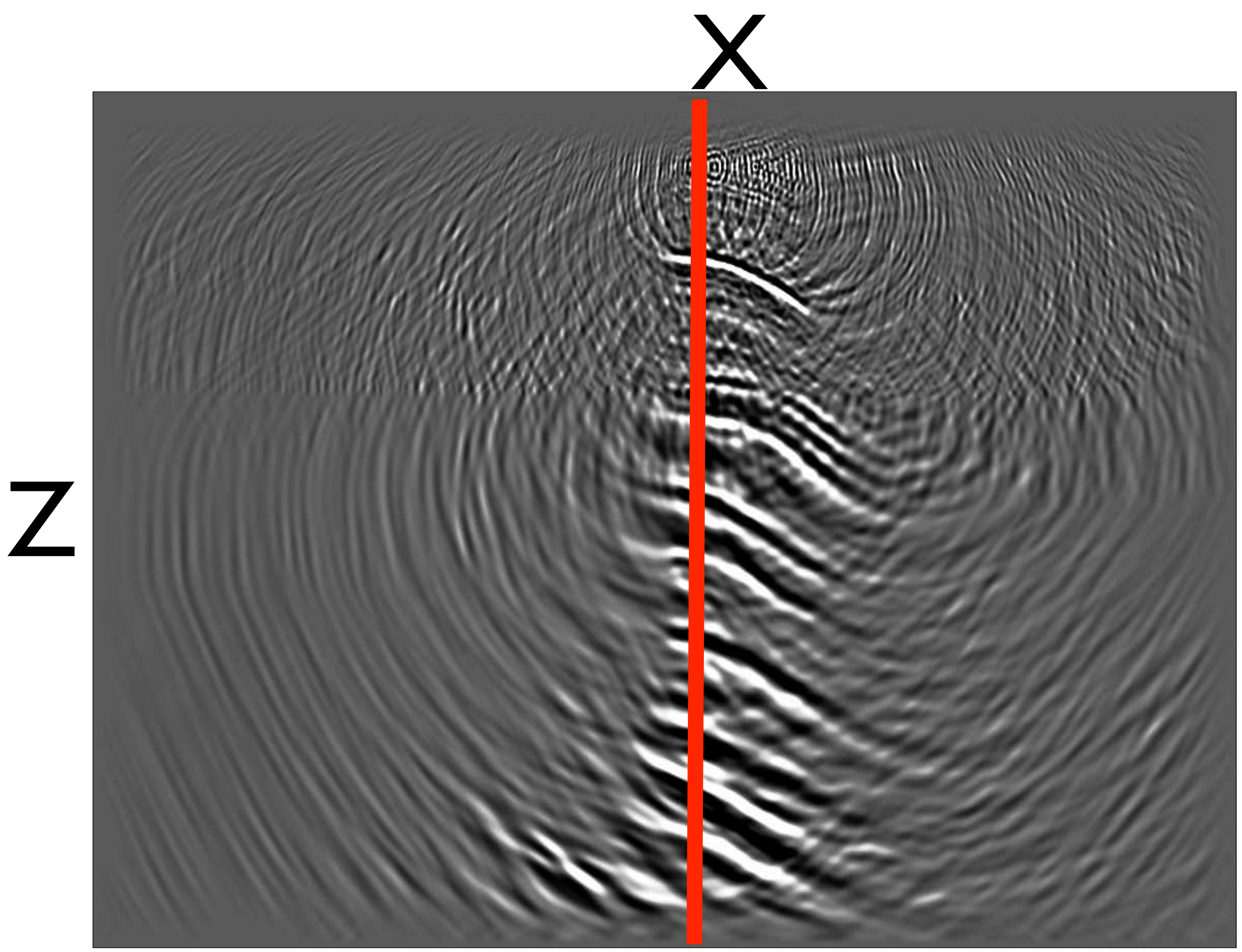
x

z

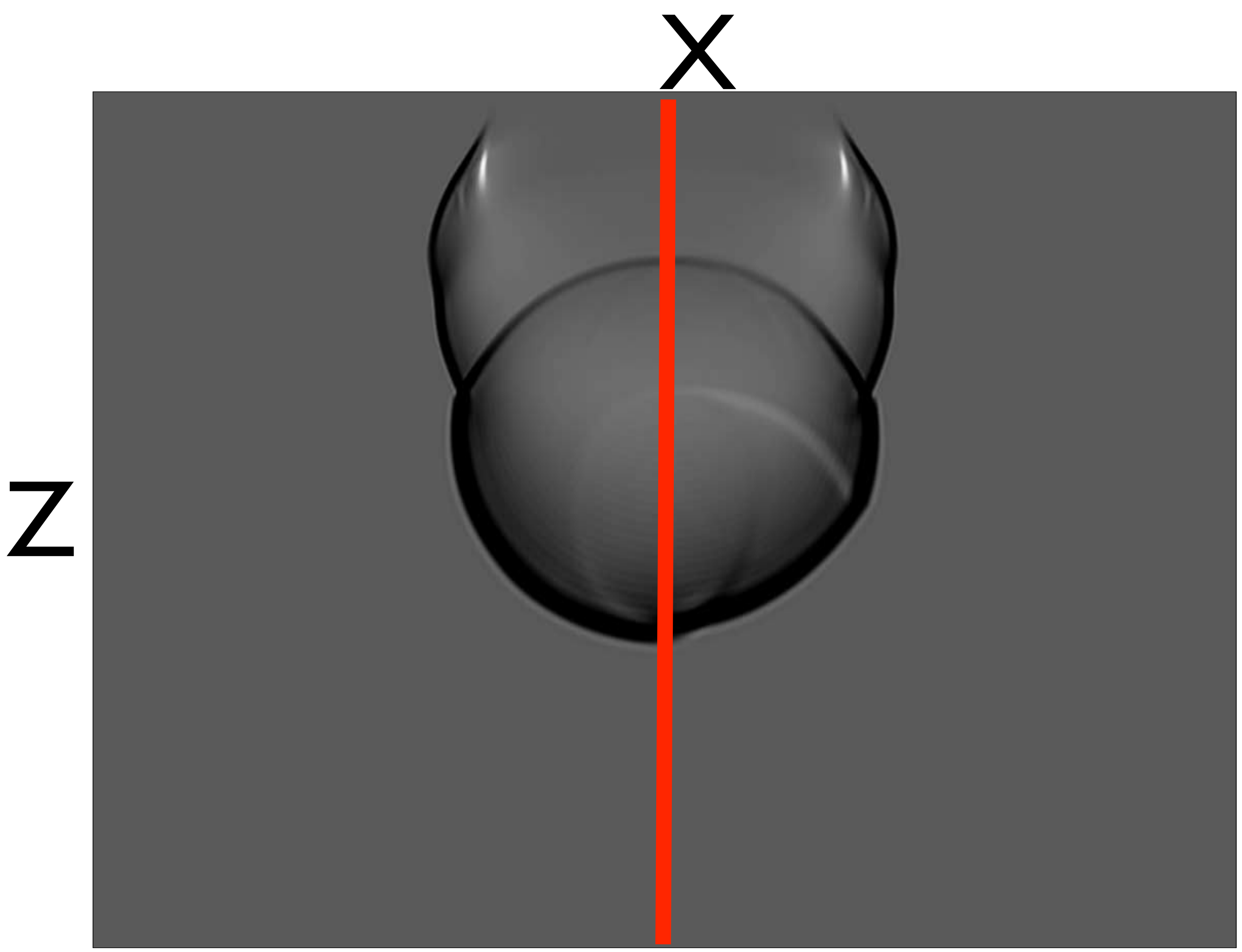


Source wavefield

Visual slide on gather construction

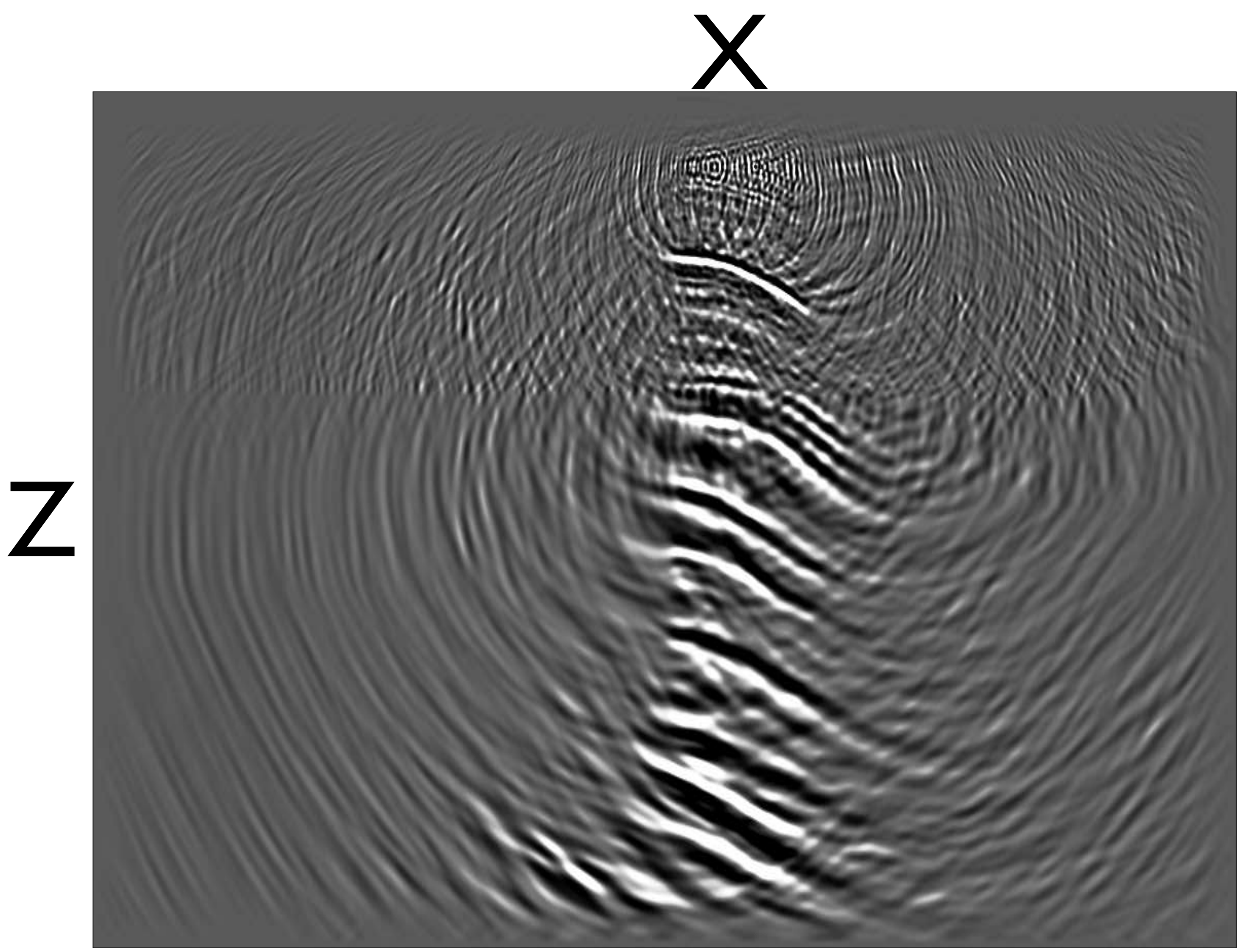


Receiver wavefield

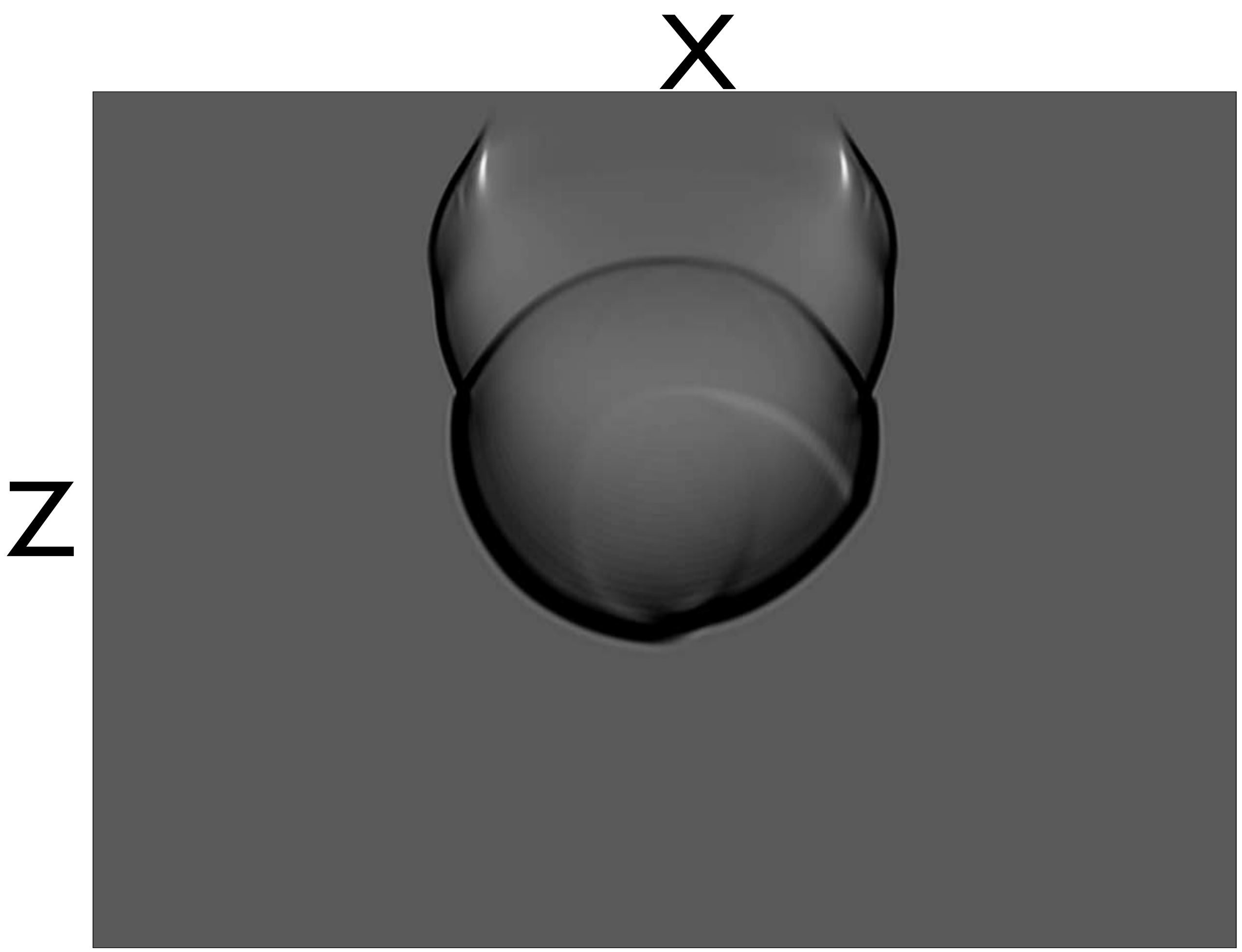


Source wavefield

Visual slide on gather construction



Receiver wavefield

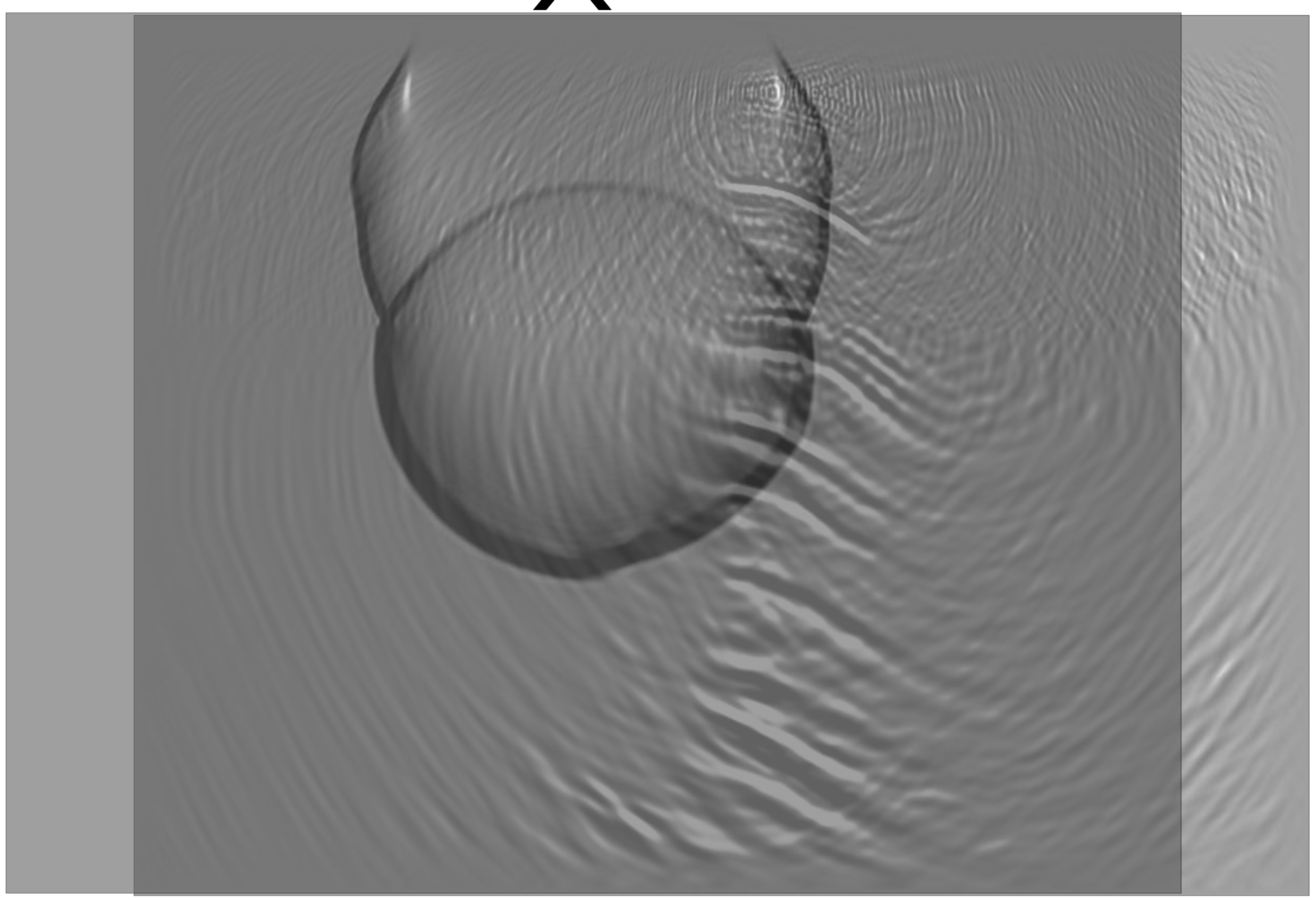


Source wavefield

Visual slide on gather construction

X

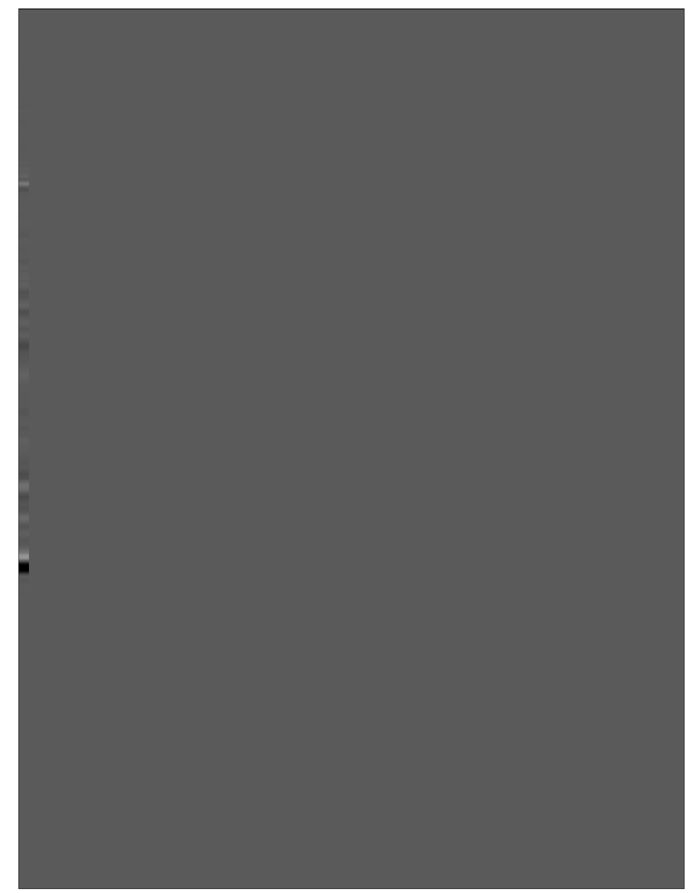
Z



Receiver wavefield

Offset

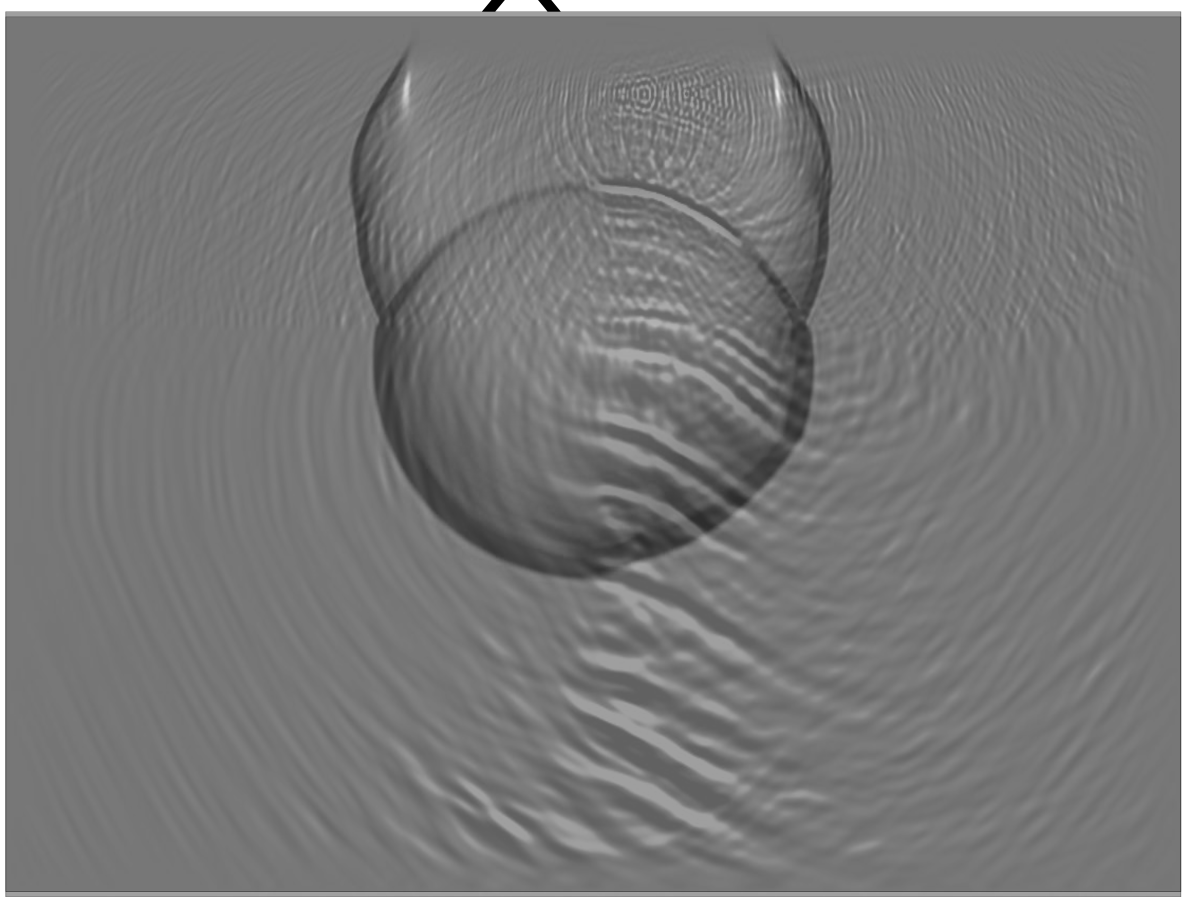
Z



Visual slide on gather construction

X

Z

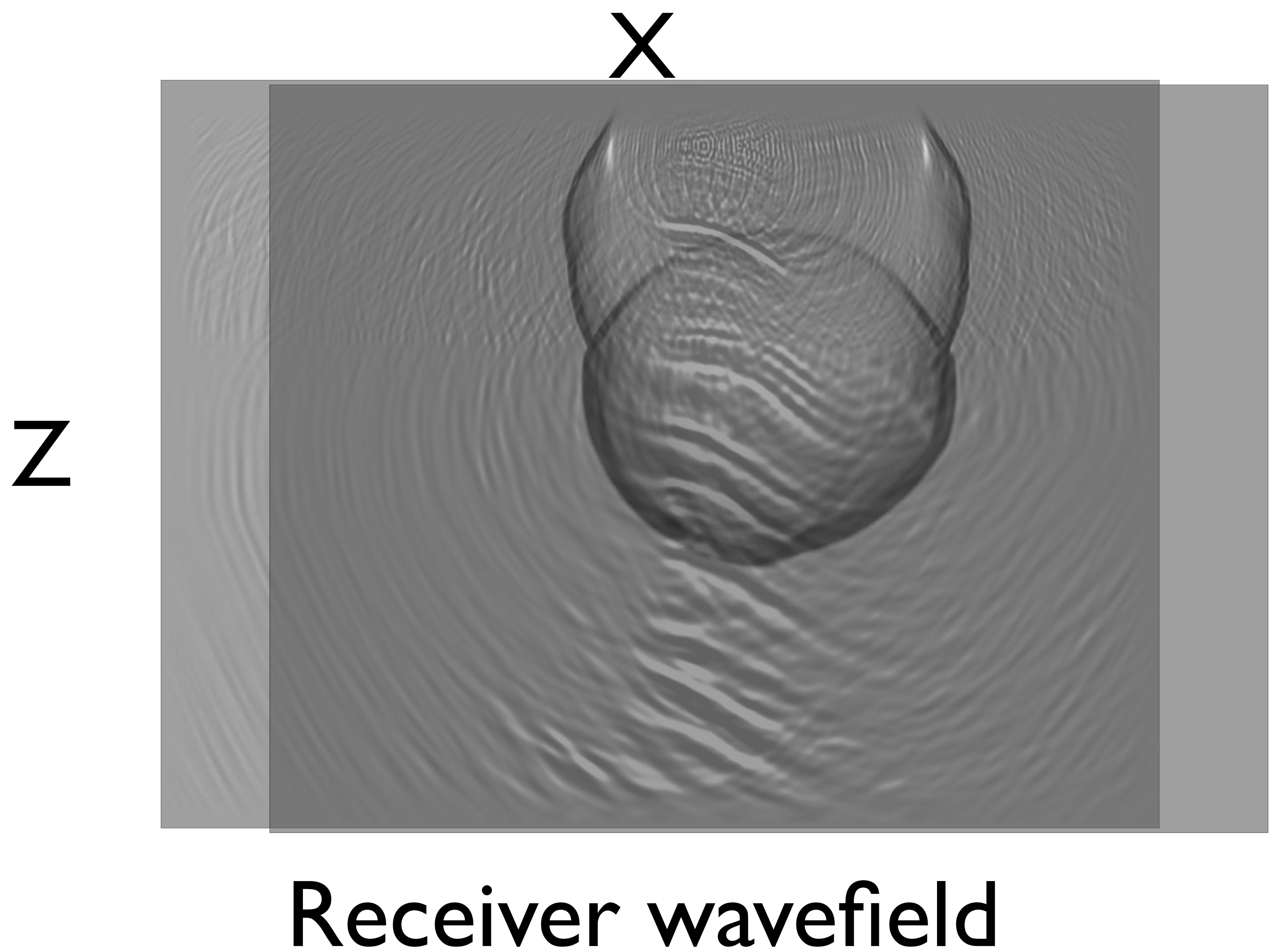


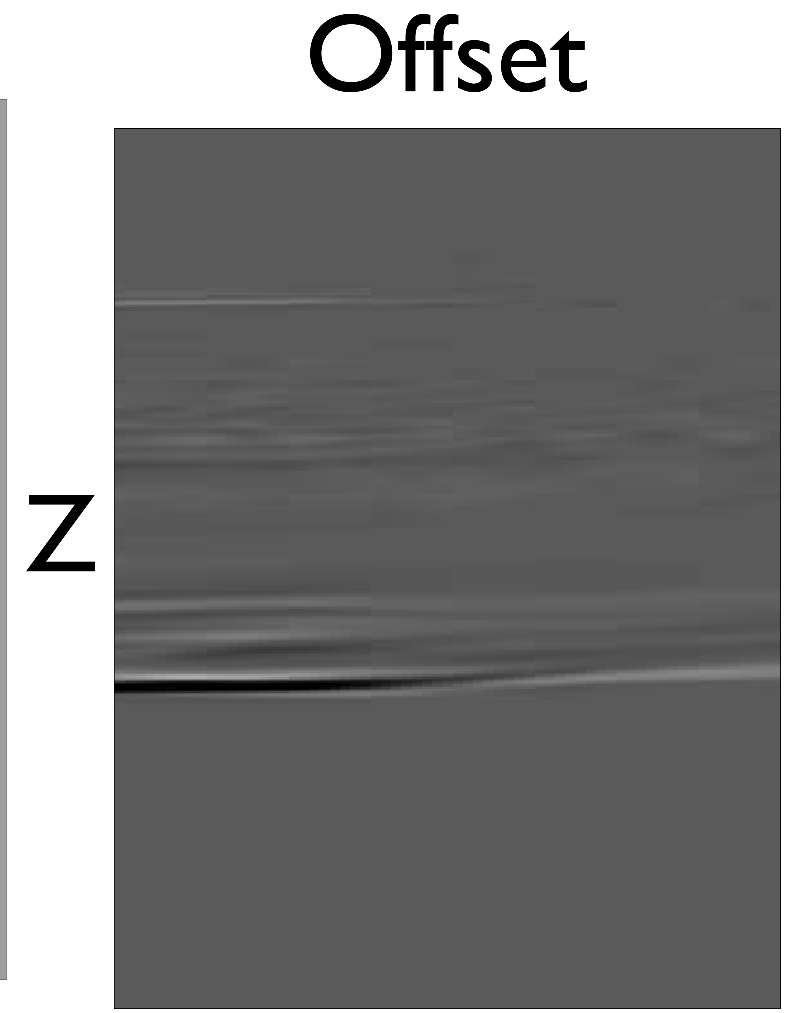
Receiver wavefield

Offset

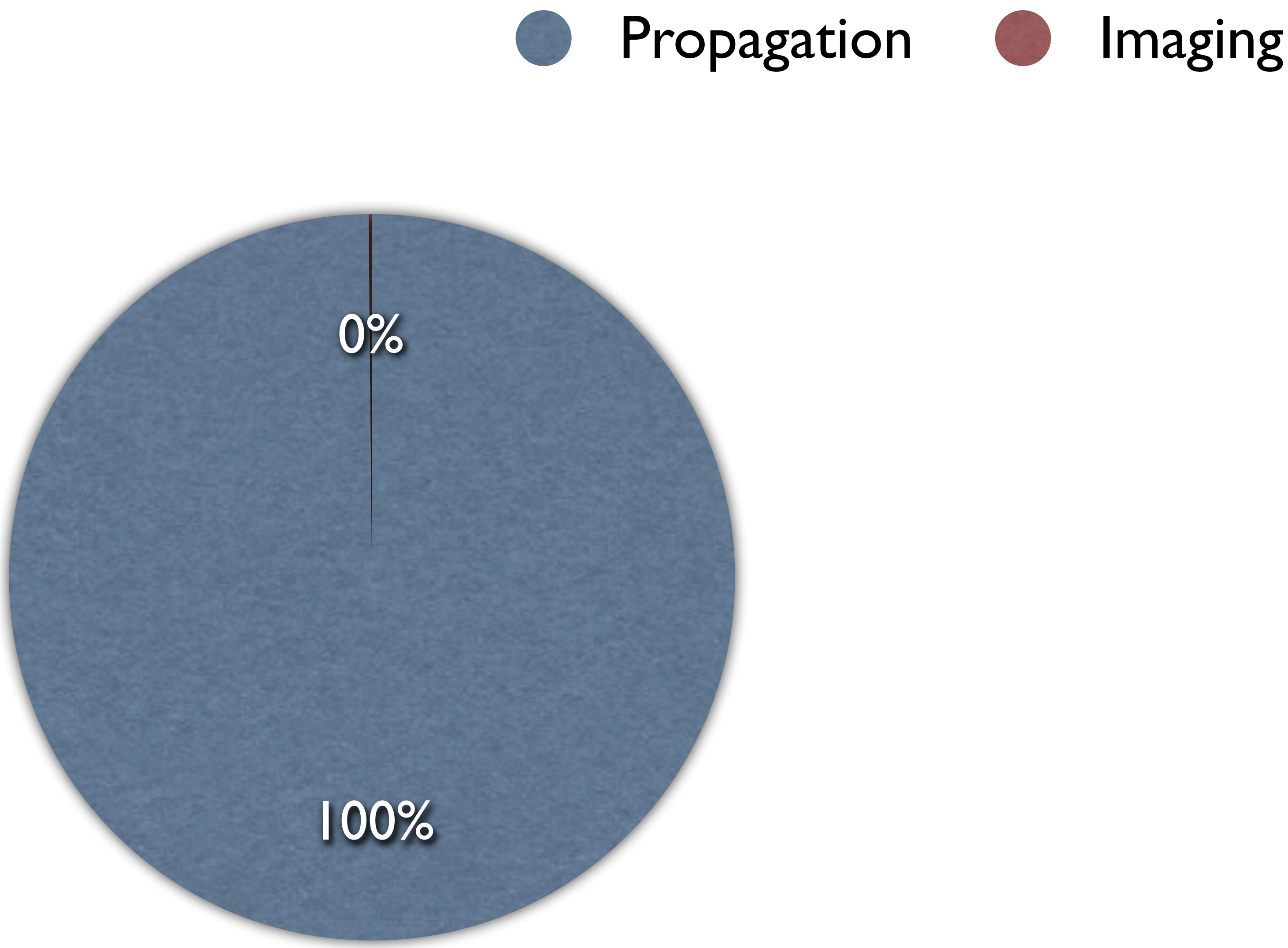
Z

Visual slide on gather construction





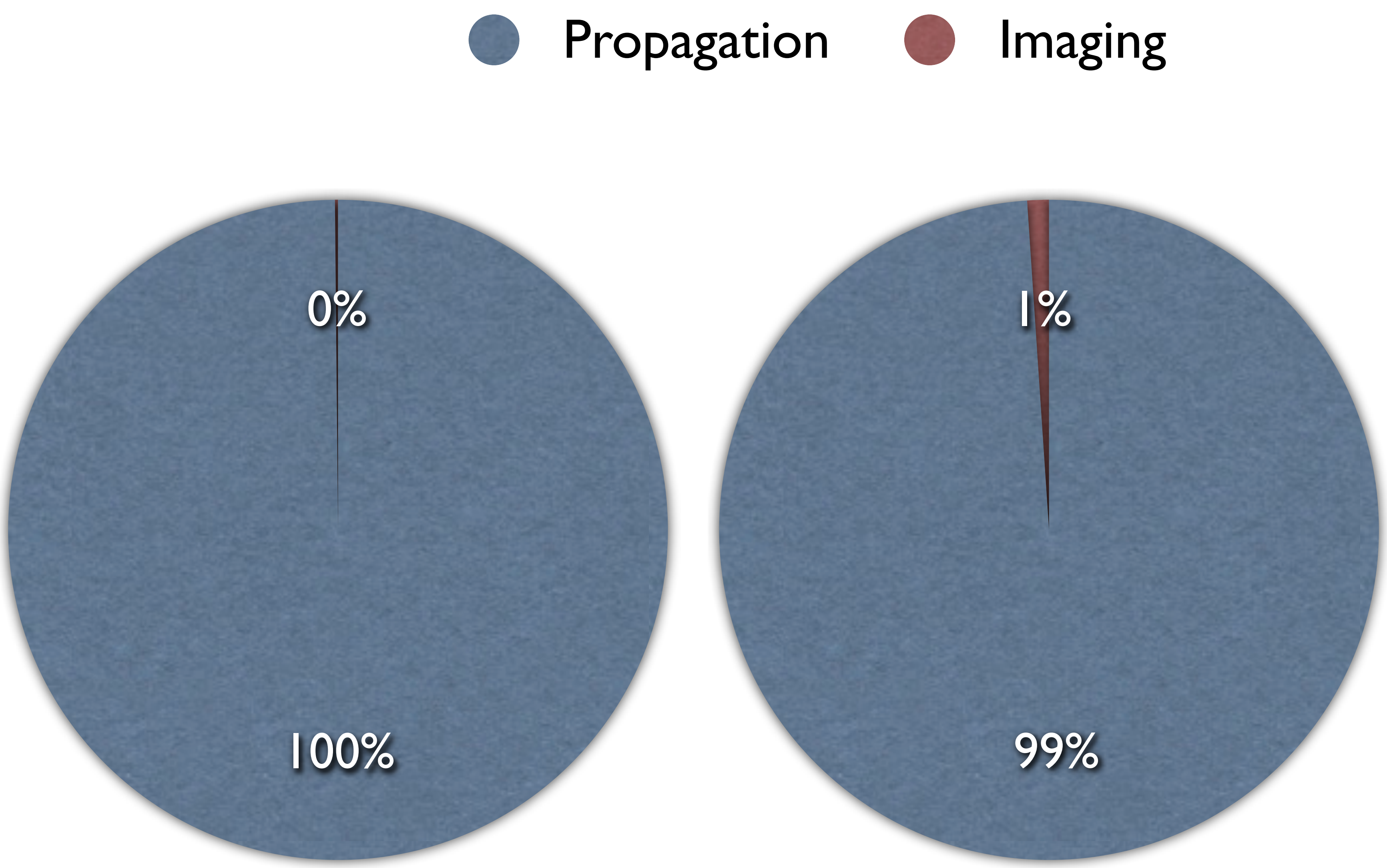
Cost of migration



Read/writes required

Angle gathers | Compressive Sensing | Compression | StOMP | Engineering | Results

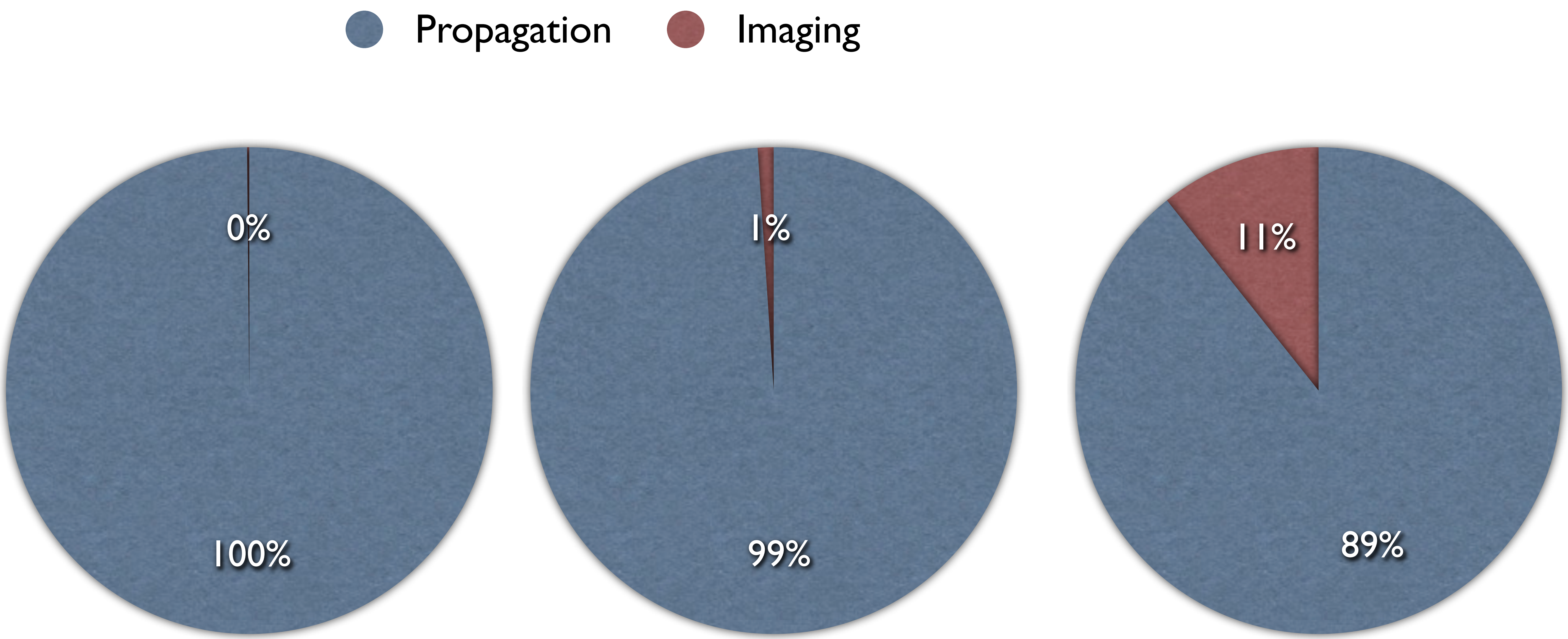
Cost of migration



Read/writes required

Correcting for cache miss ratio

Cost of migration



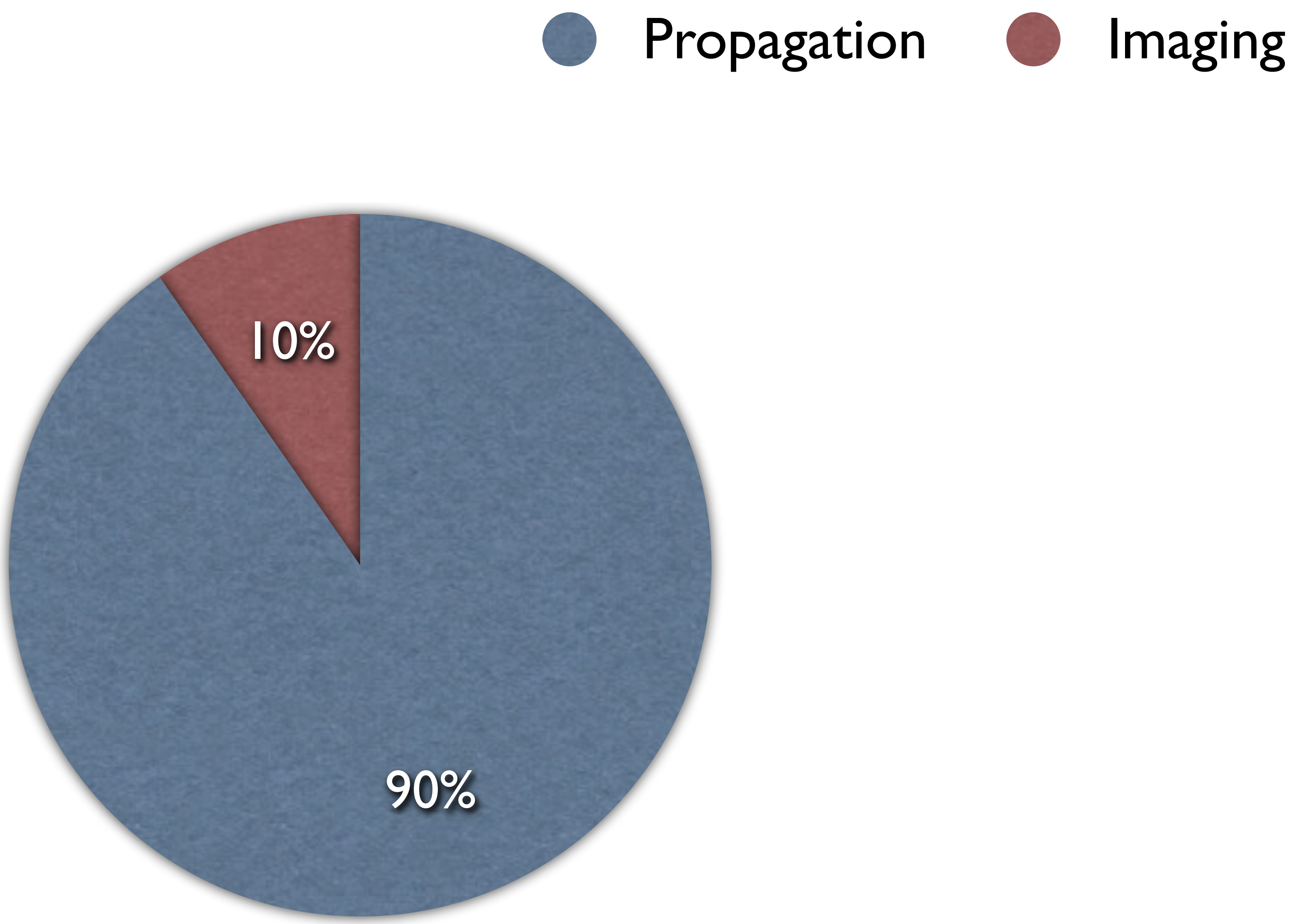
Read/writes required

Correcting for cache miss ratio

Storing checkpoints

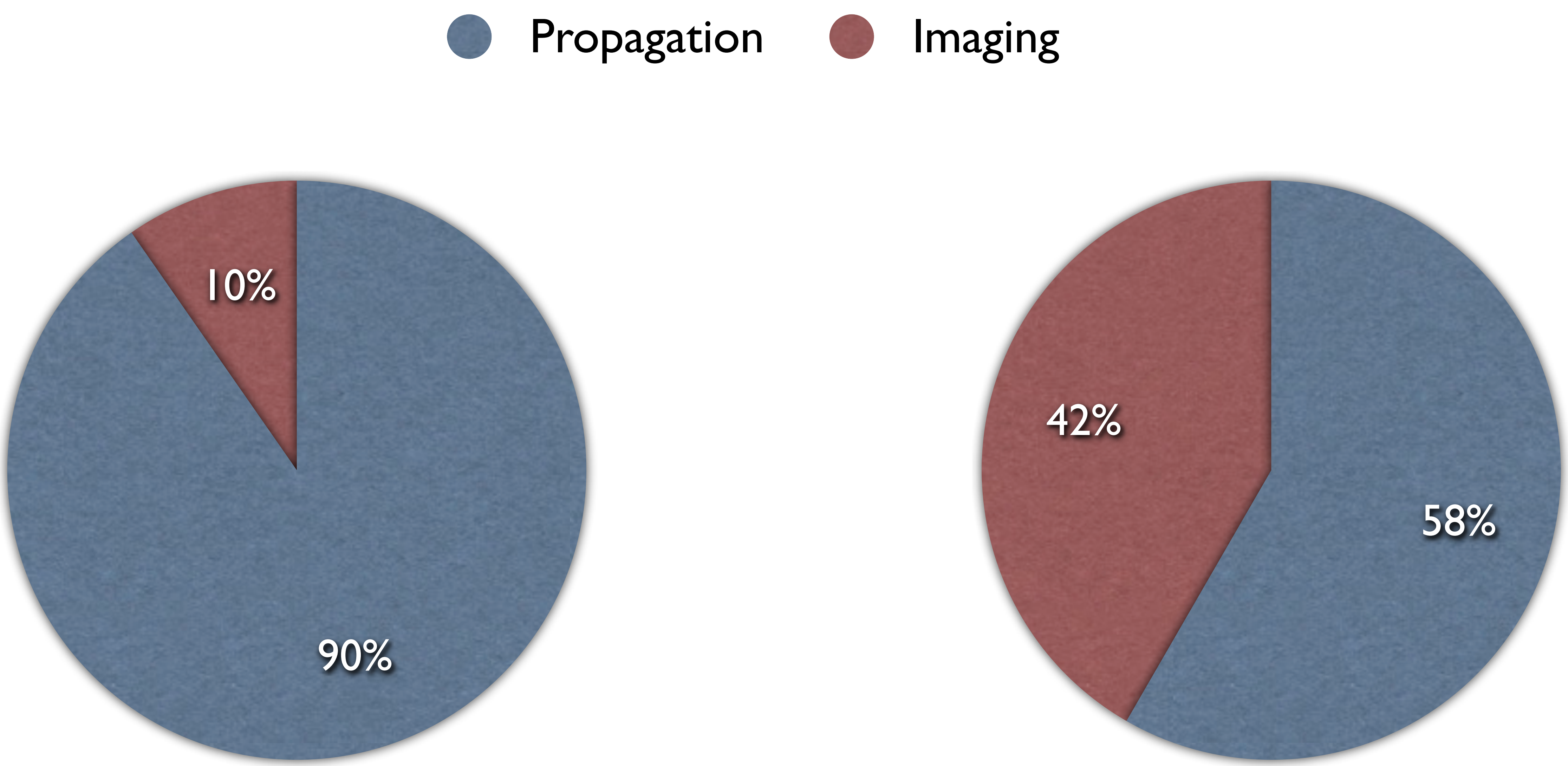
Angle gathers Compressive Sensing Compression StOMP Engineering Results

Cost of single shift gather construction



Hold in same memory

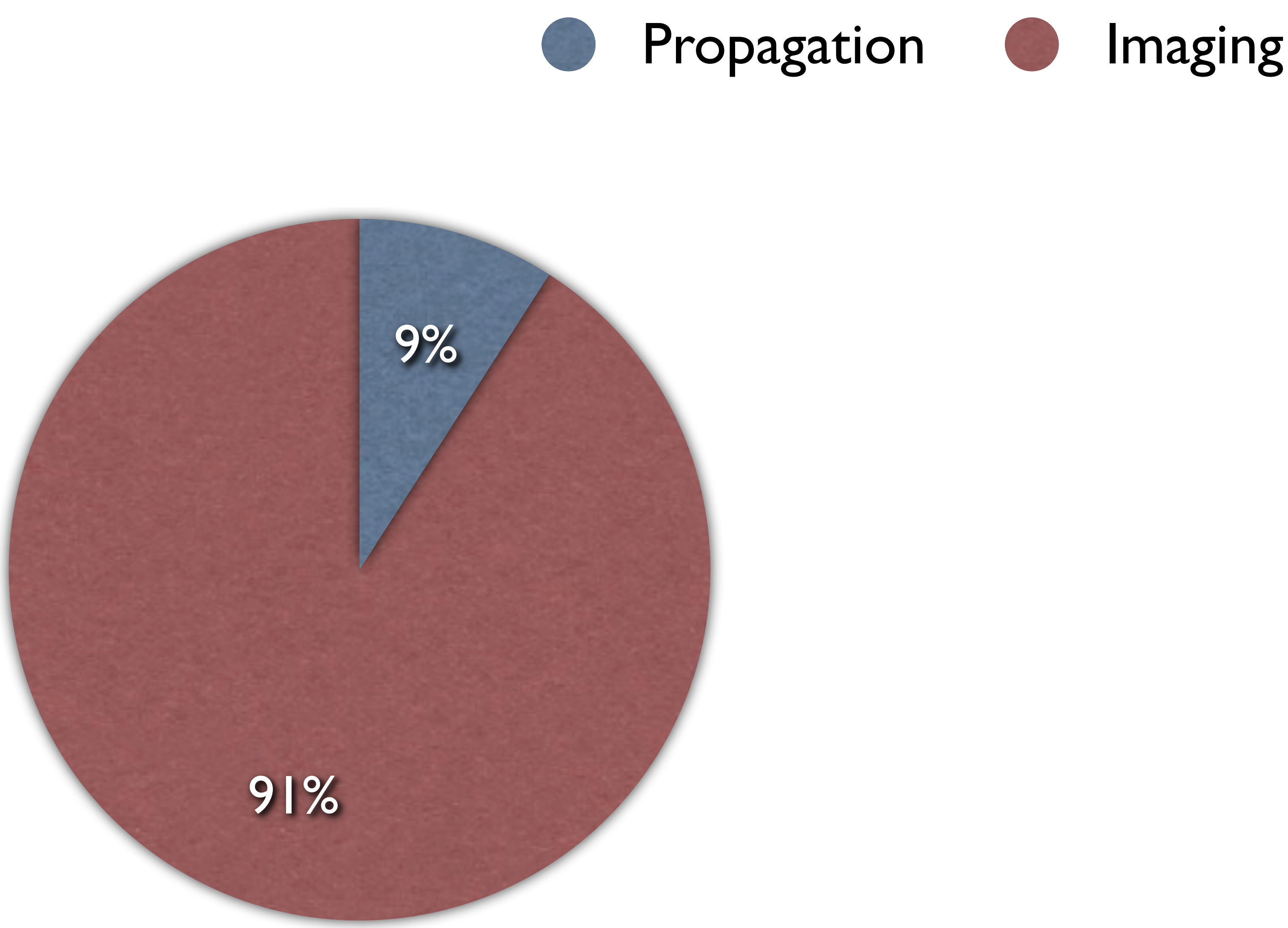
Cost of single shift gather construction



Hold in same memory

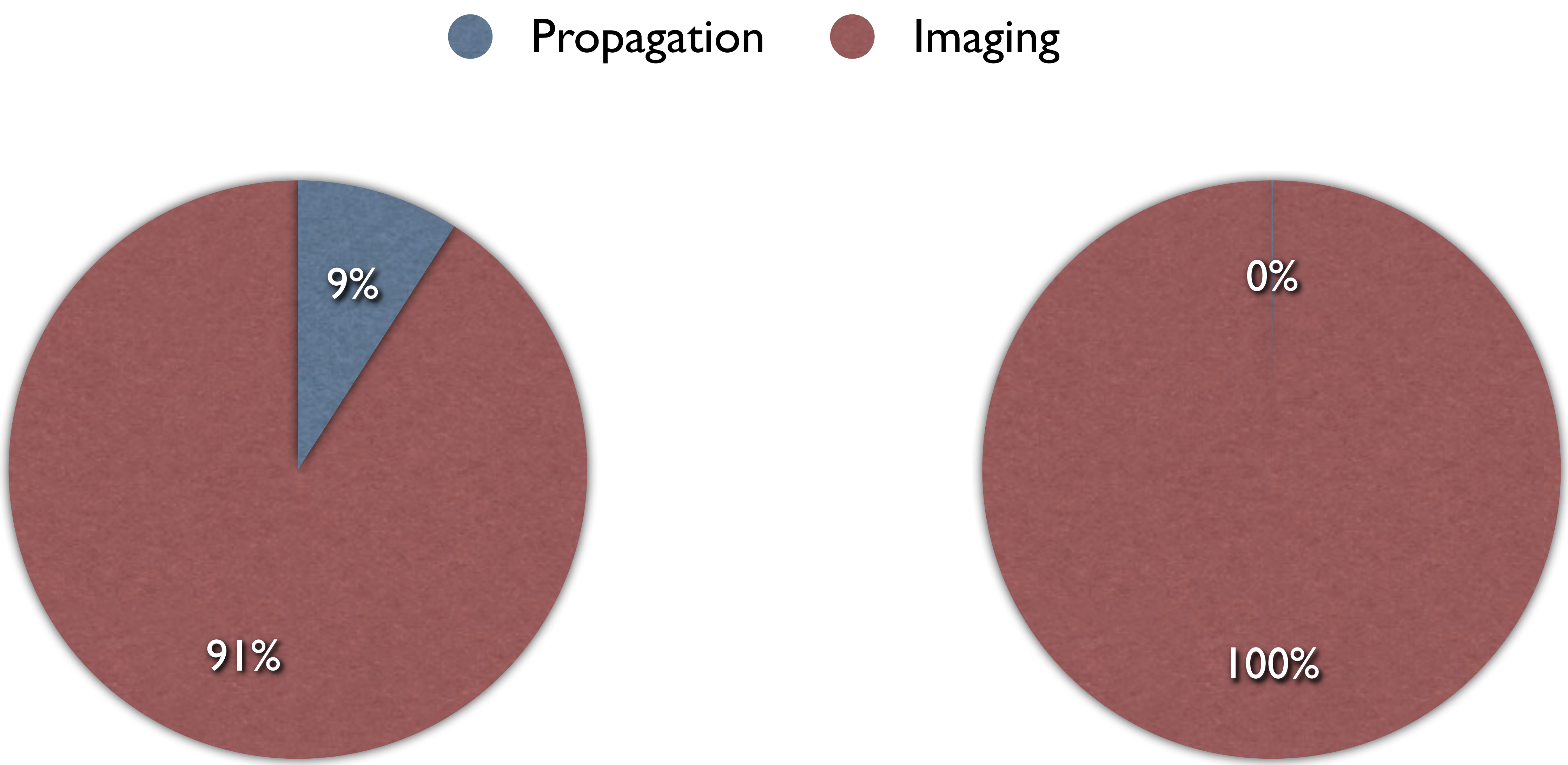
Store to disk/(transpose)/compute

Additional shift dimensions



Hold at same memory level

Additional shift dimensions

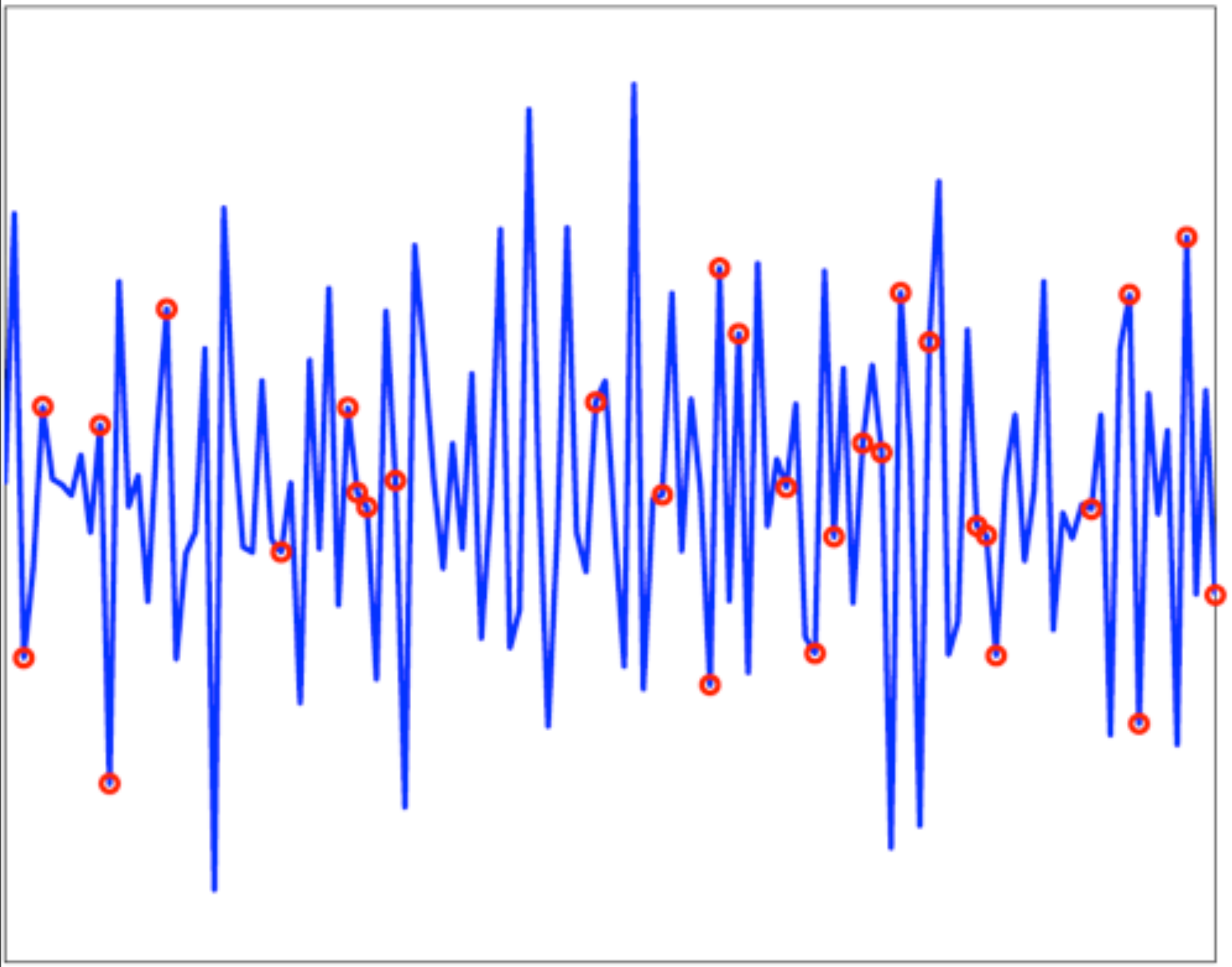


Hold at same memory level

Store to disk/transpose/compute

Sampling Example

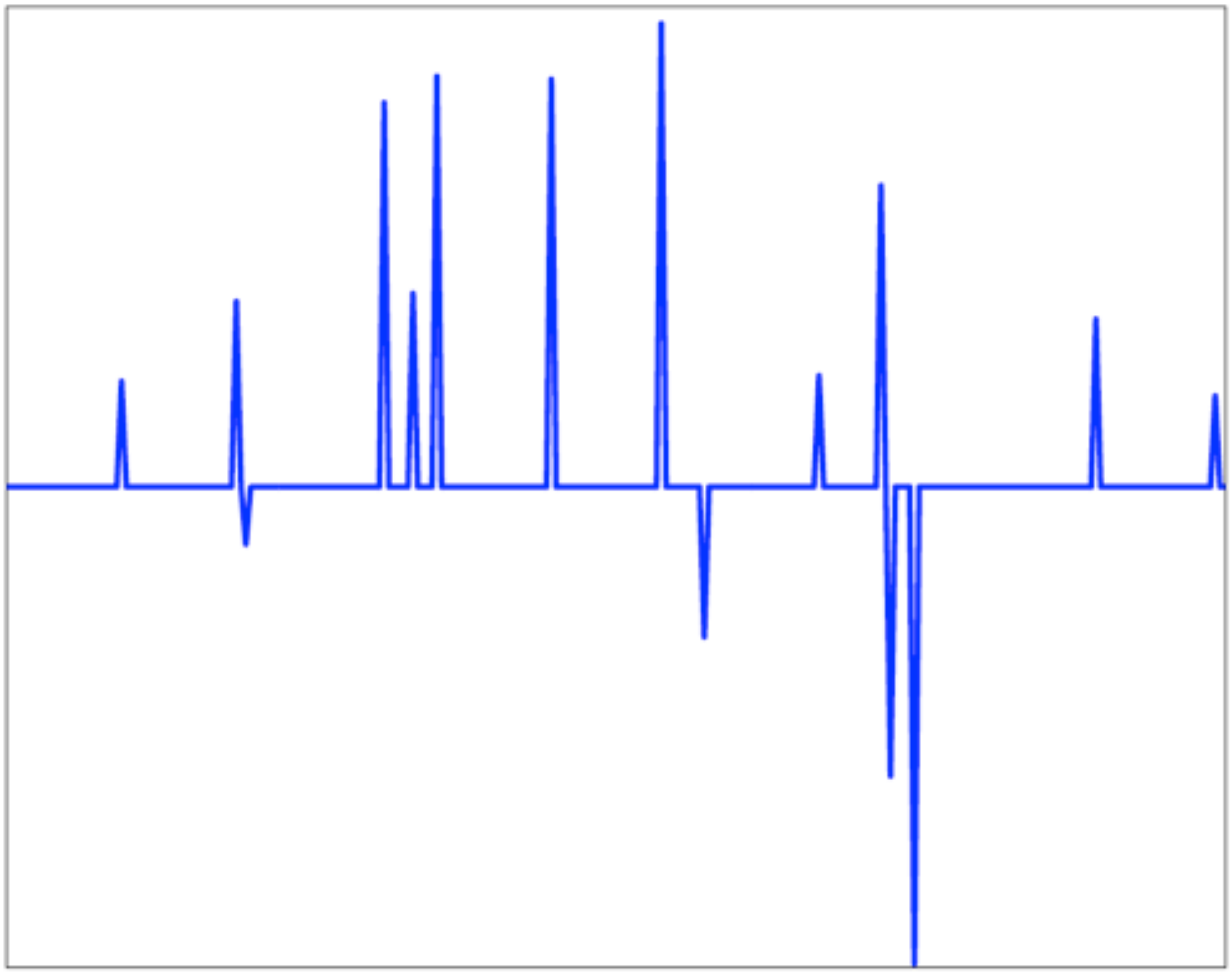
Time domain $f(t)$



Measure M samples

(red circles = samples)

Frequency domain $\hat{f}(\omega)$



K nonzero components

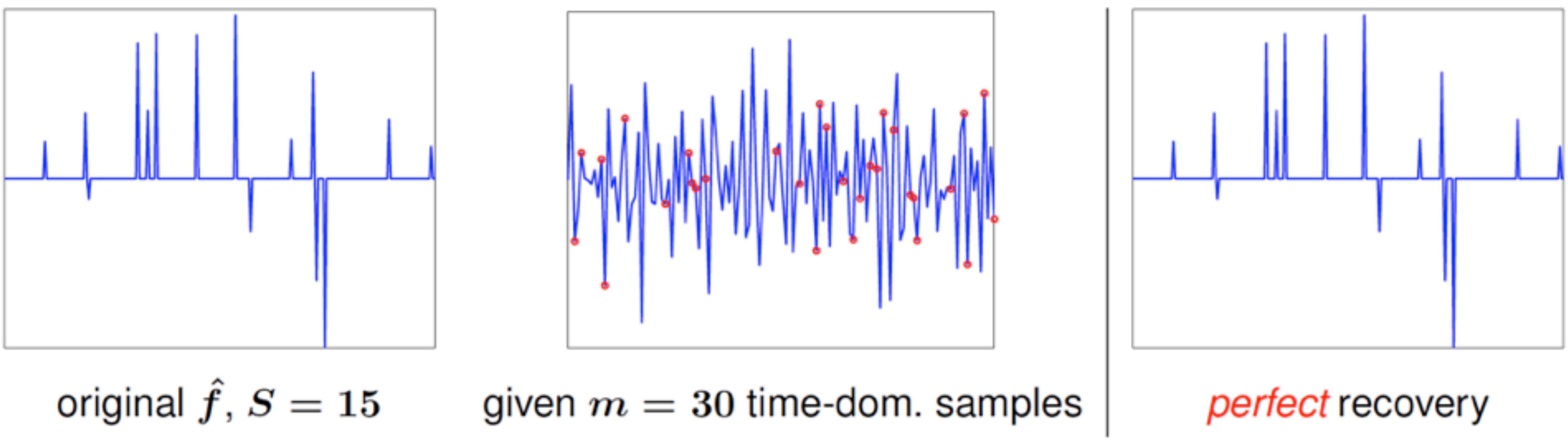
$\#\{\omega : \hat{f}(\omega) \neq 0\} = K$

Romberg & Wakin (2007)

ℓ_1 Reconstruction

Reconstruct by solving

$$\min_{g} \|\hat{g}\|_{\ell_1} := \min \sum_{\omega} |\hat{g}(\omega)| \quad \text{subject to} \quad g(t_m) = f(t_m), \; m = 1, \ldots, M$$



original $\hat{f}$, $S = 15$ | given $m = 30$ time-dom. samples | *perfect* recovery

Romberg & Wakin (2007)

Example: Sparse Image

- Take $M = 100,000$ incoherent measurements $y = \Phi f_a$
- f_a = wavelet approximation (perfectly sparse)
- Solve

$$\min \|\alpha\|_{\ell_1} \quad \text{subject to} \quad \Phi\Psi\alpha = y$$

Ψ = wavelet transform

original (25k wavelets)

perfect recovery

Romberg & Wakin (2007)

Compressive sensing in SEP speak: Basic idea

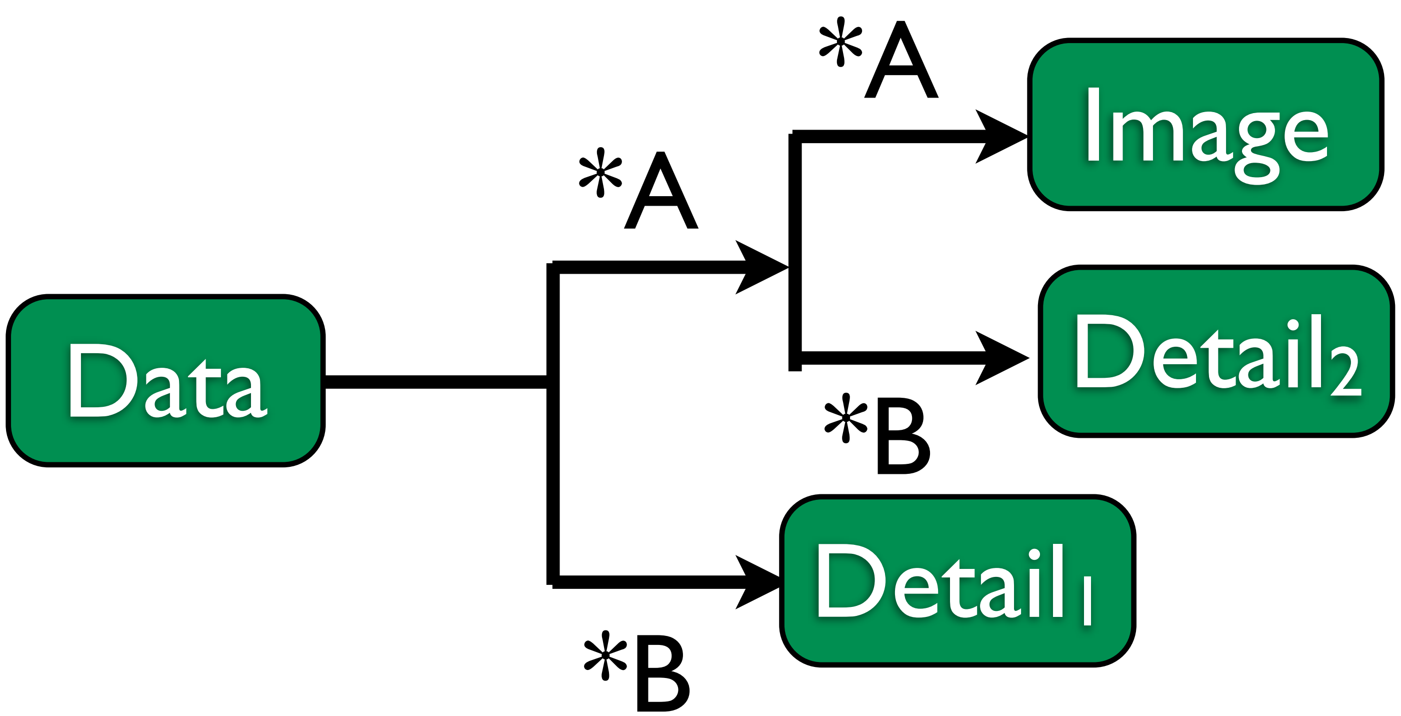
- You want the dataset **d**
- You know that d transforms to something sparse (**m**) by applying the operator **L'**
- You record a random subset of **d**, $\mathbf{d_r}$
- You set up an inverse problem using $\mathbf{d_r}$ to find **m**
- You apply **L** to recover **d**

Compressive sensing in SEP speak: Fitting goals

$$\mathbf{0} \approx \mathbf{r} \underset{1}{=} \mathbf{d_r} - \mathbf{Lm}$$

$\mathbf{r}$	Residual	$\underset{1}{=}$	L1 norm
$\mathbf{d_r}$	Sparse data	$\mathbf{m}$	Sparse model
$\mathbf{L}$	Transform into/from sparse basis		

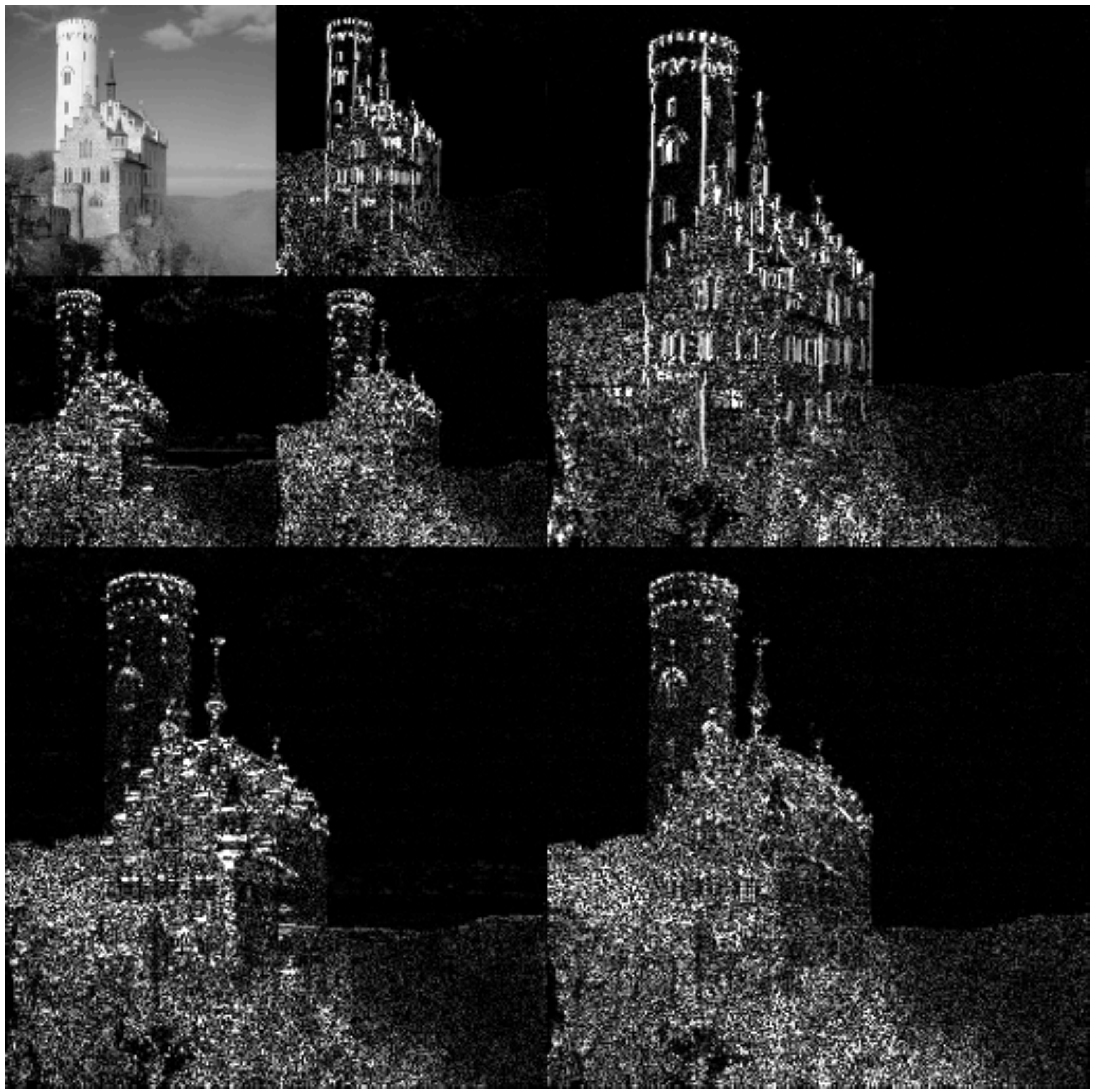
Wavelet transform: 1-D



A - low pass filter (scaling)

B- high pass filter (wavelet)

Wavelet transform

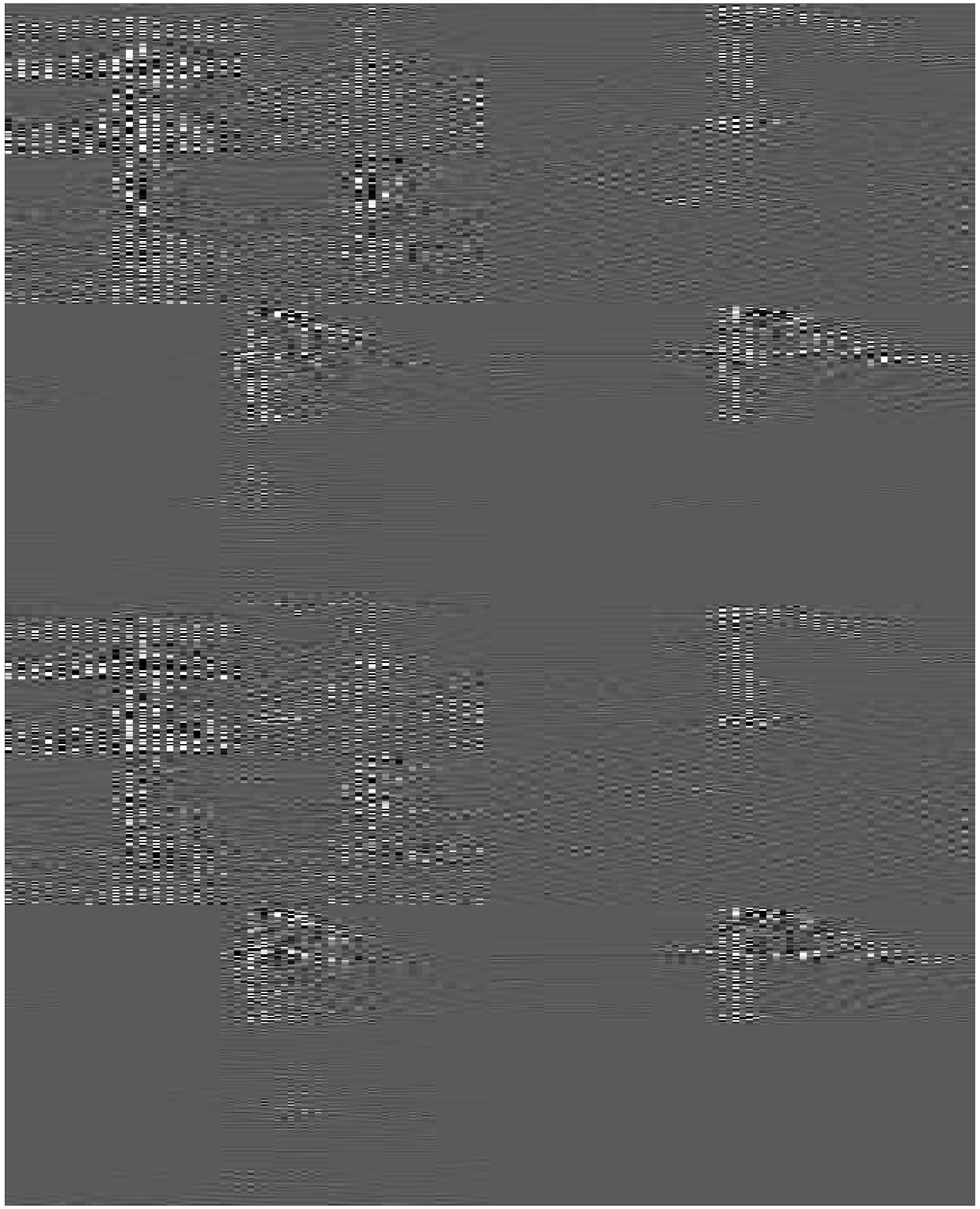


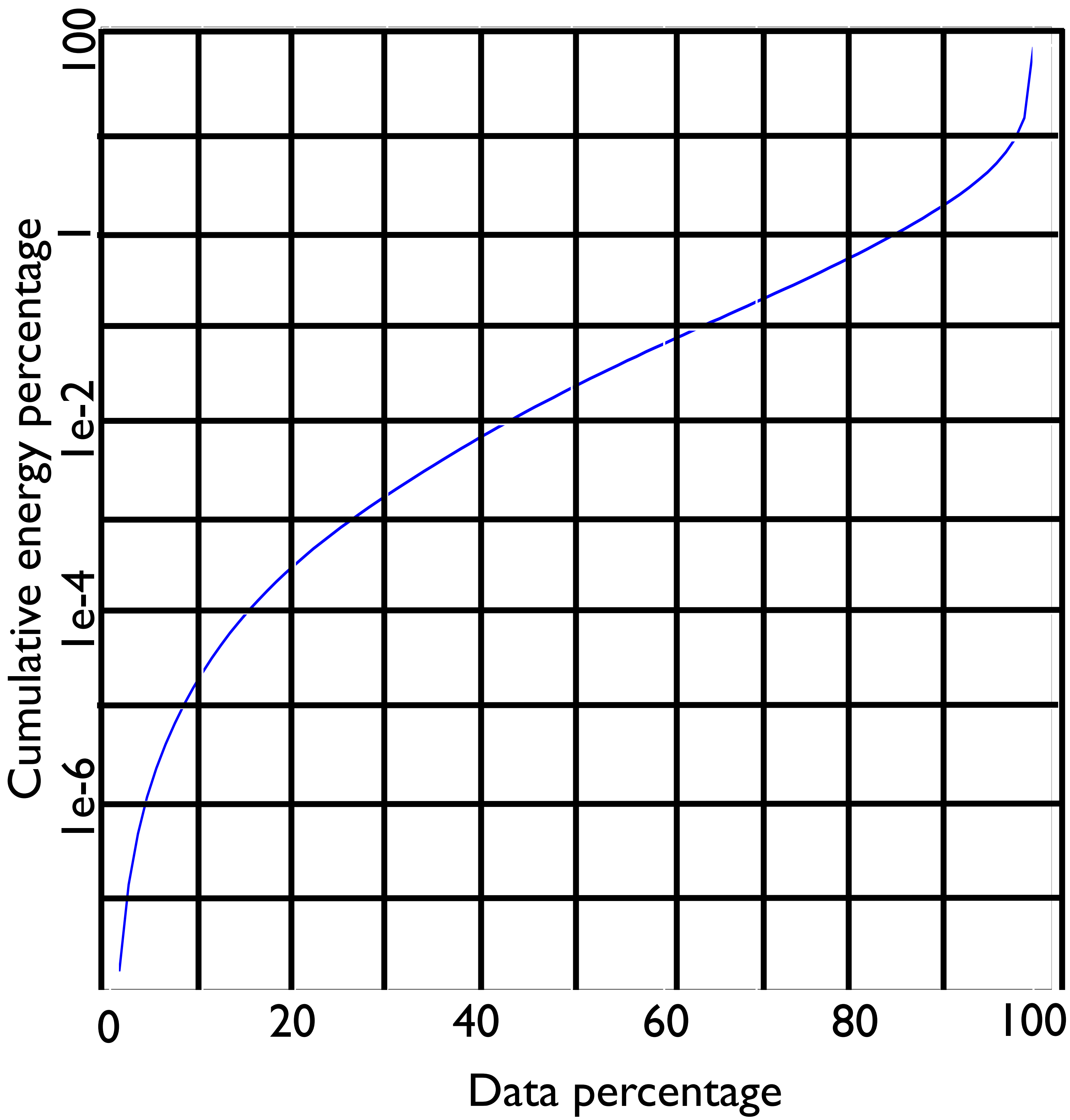
Wikipedia

Test data

- 4-D volume (z,hx,x,y)
- 400,72,32,32
- Wavelet levels 4,2,1,1

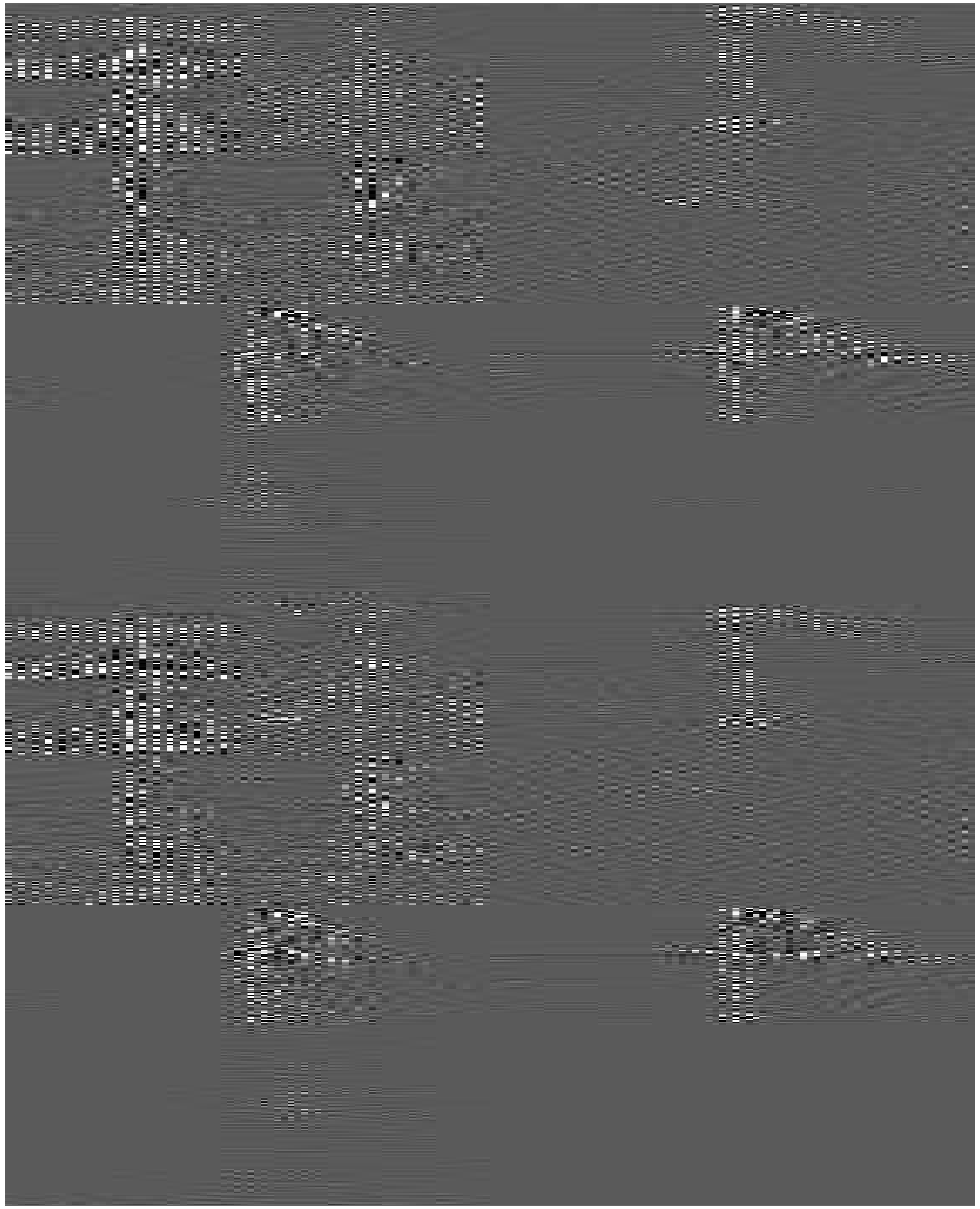
Offset energy sparsity

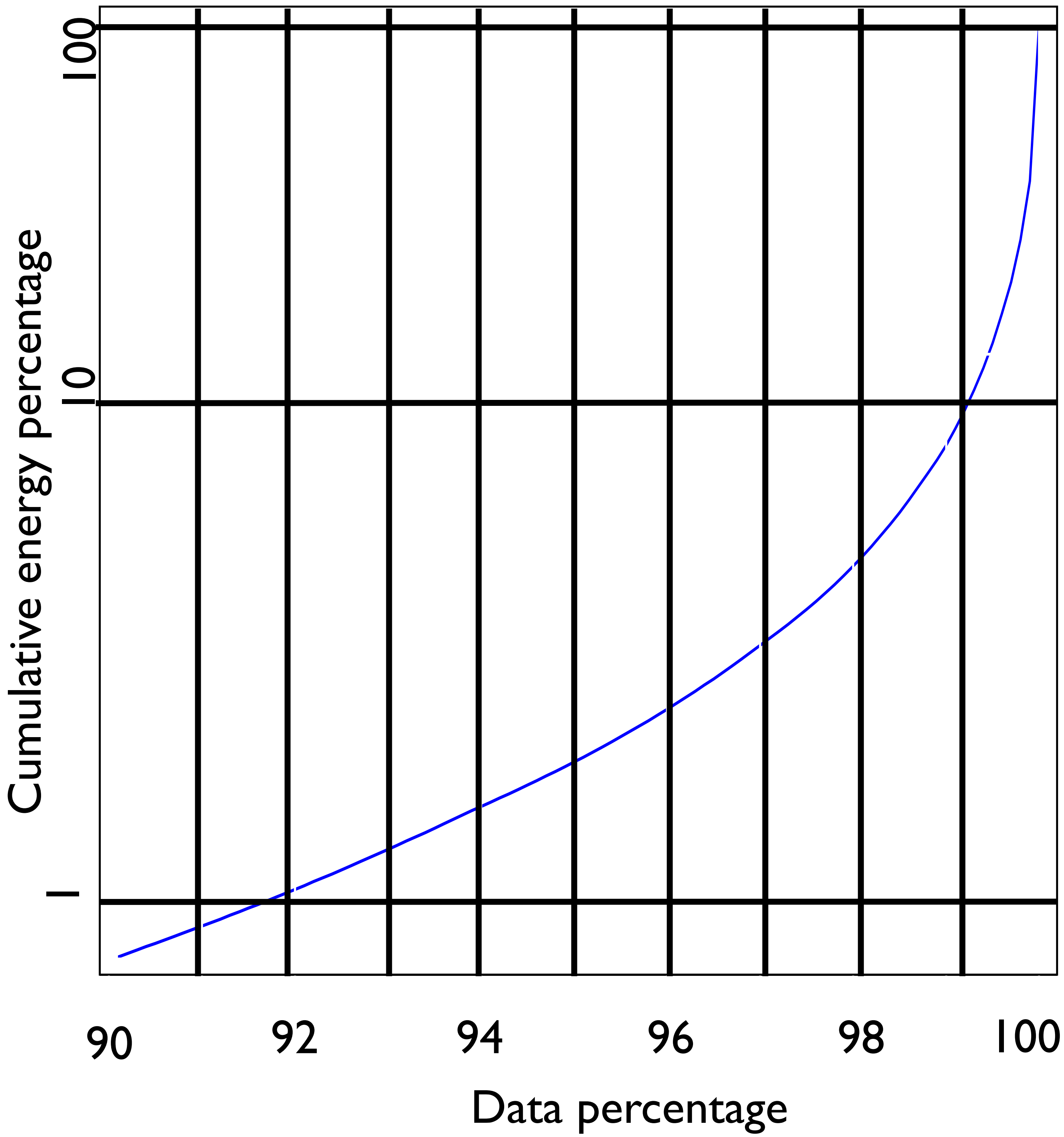




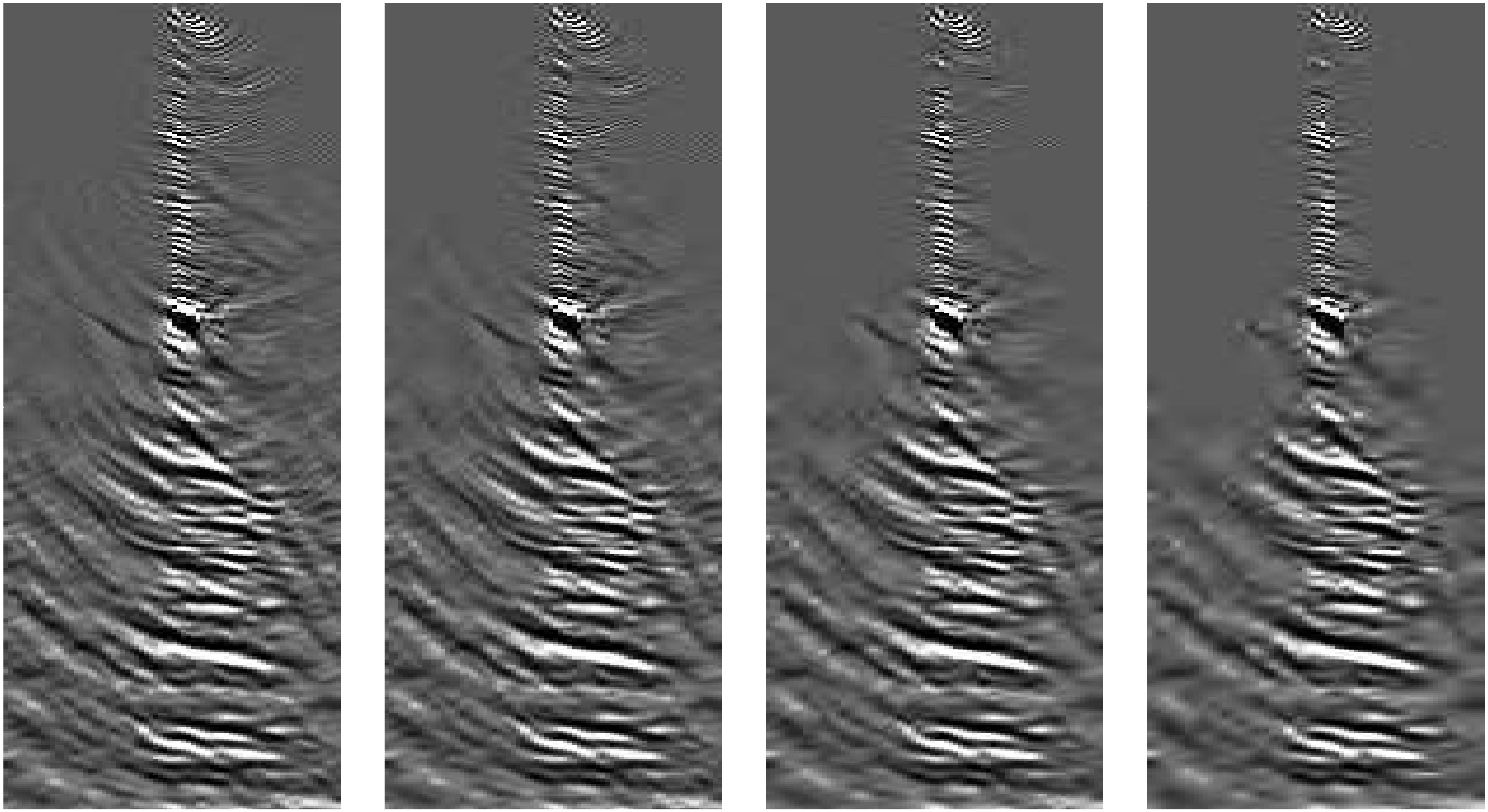
Angle gathers | Compressive Sensing | **Compression** | StOMP | Engineering | Results

Offset energy sparsity



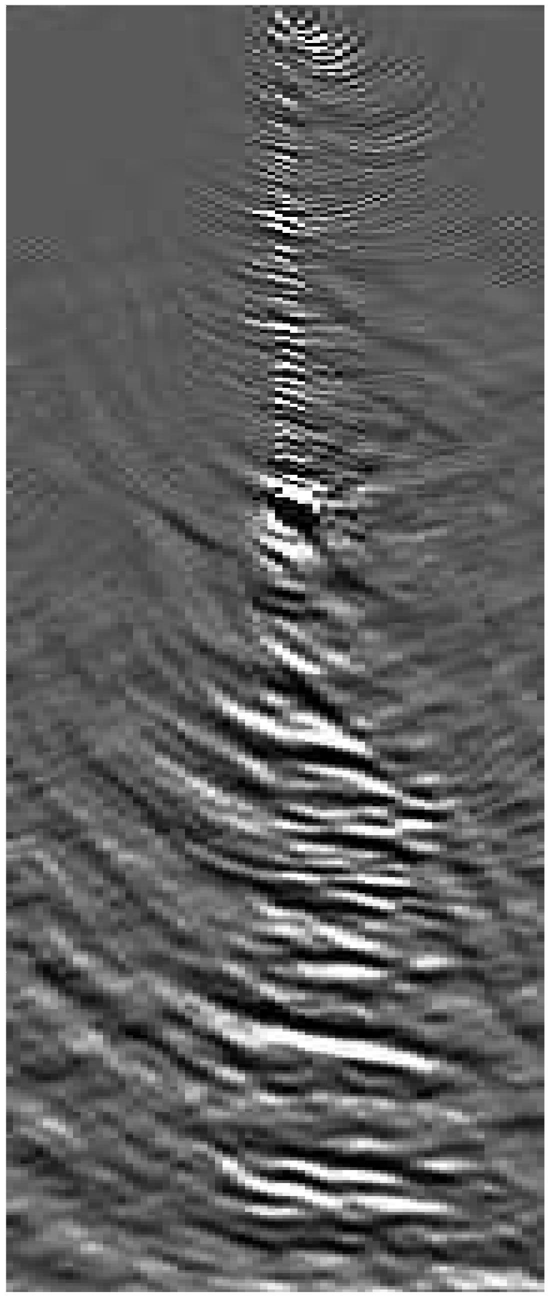


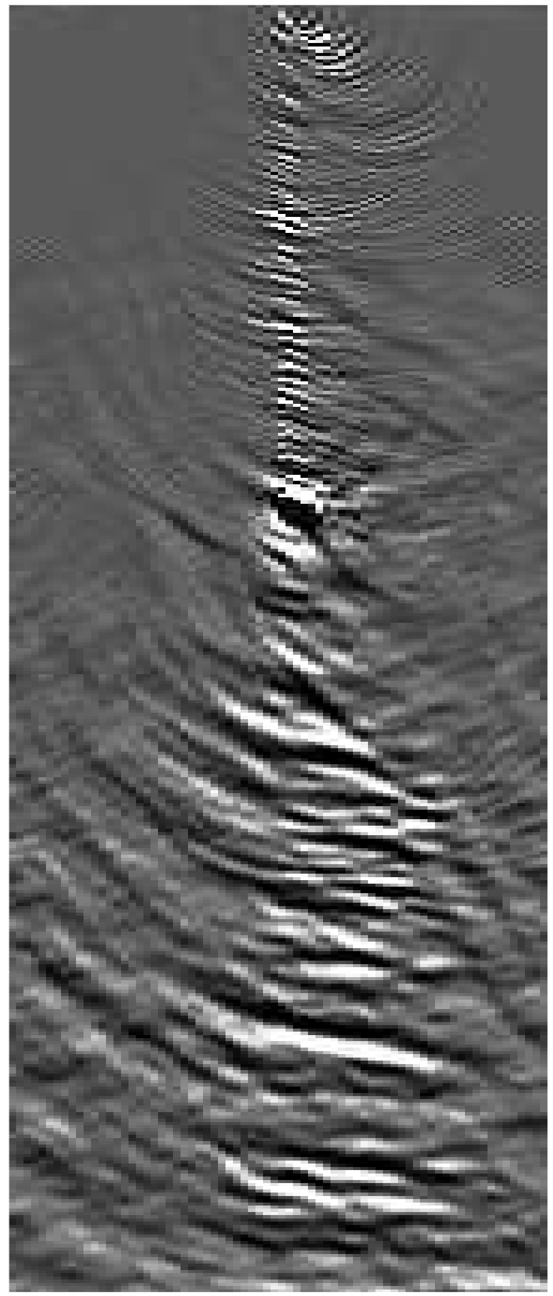
Zeroing wavelet coefficients: Offset

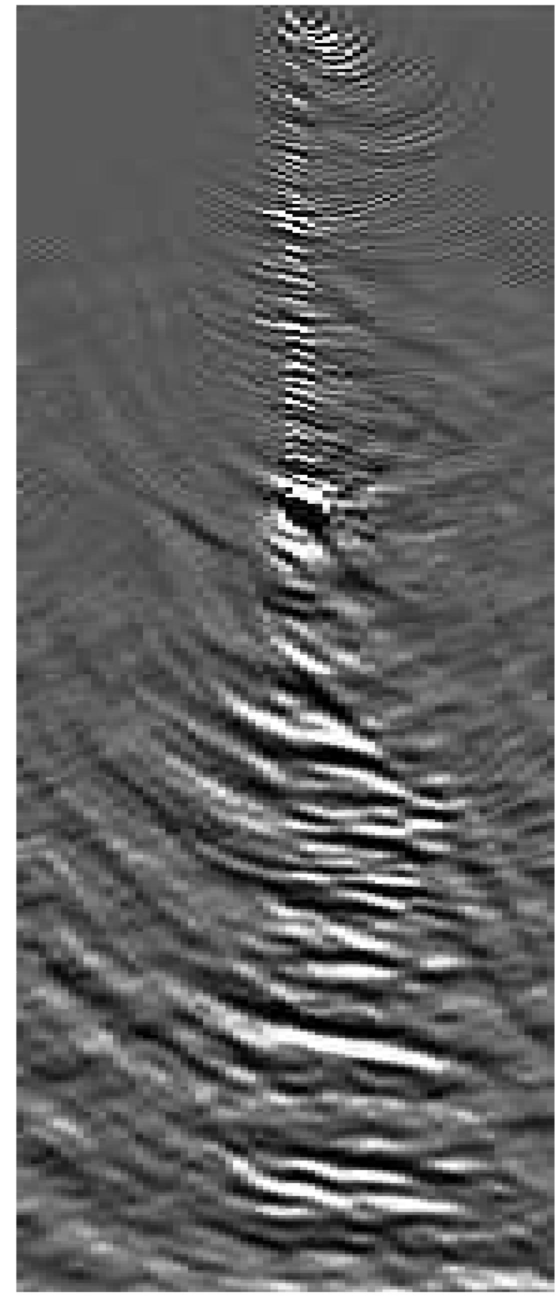


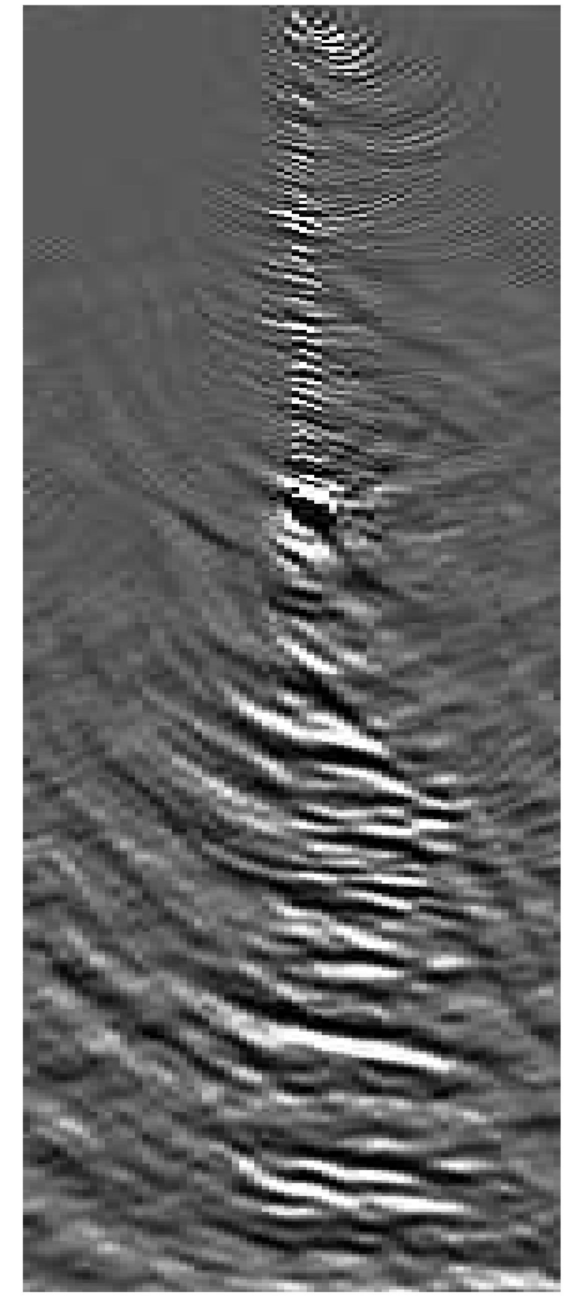
90% 95% 98% 99%

Zeroing wavelet coefficients: Offset


Original


Original


Original


Original

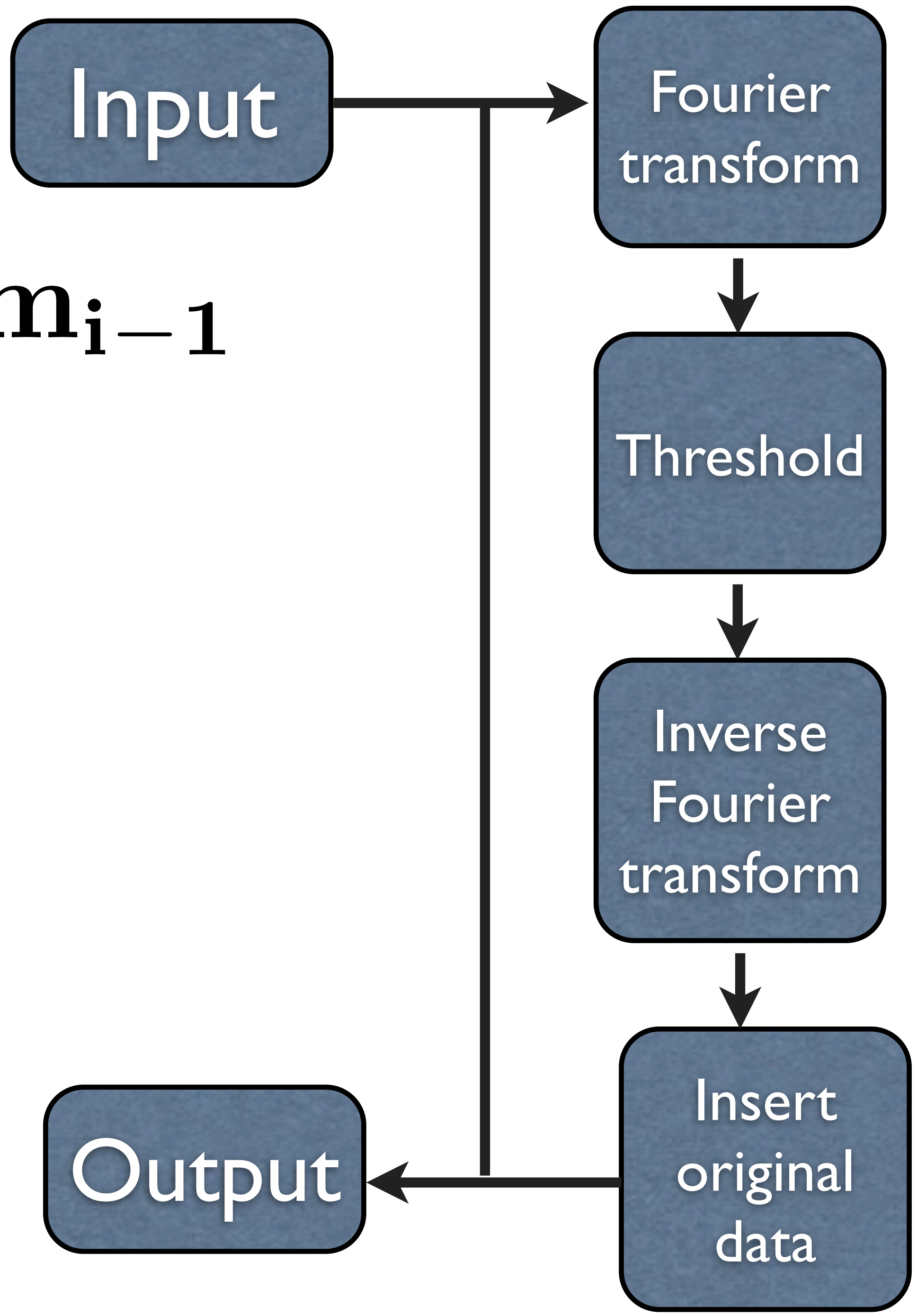
Effective $L_{0/1}$ solvers

- The more L_0 the better
- Has to work, in a reasonable amount of time, on problems with a large model space

Projection on Convex Sets

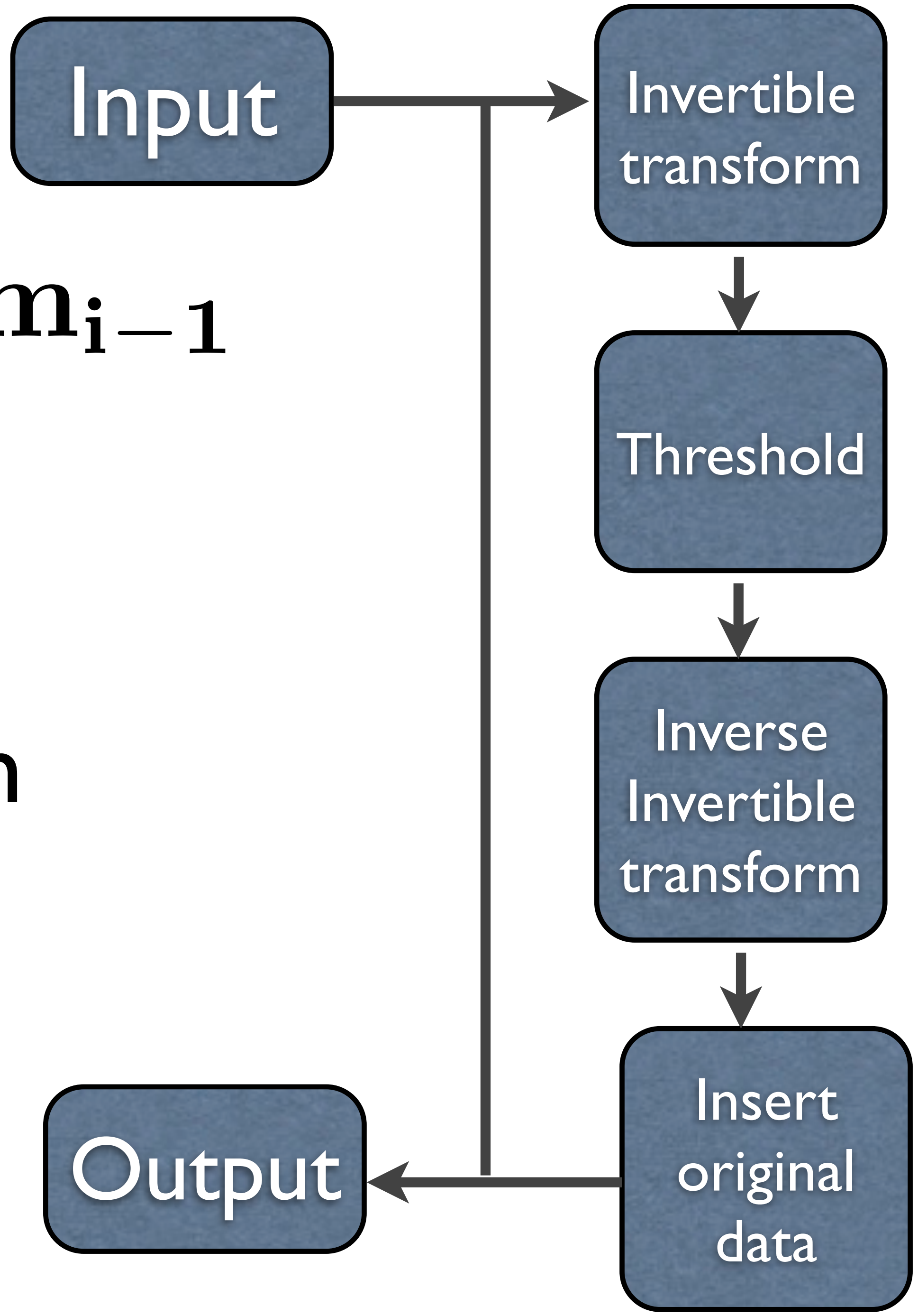
$$\mathbf{m_i} = \mathbf{m_{obs}} + \mathbf{ML^{-1}T_iLm_{i-1}}$$

$\mathbf{m_{obs}}$	Observed data
$\mathbf{m_i}$	model
F	Fourier transform
$\mathbf{T_i}$	Thresholding
M	Masking operator



Projection on Convex Sets

$$\mathbf{m_i} = \mathbf{m_{obs}} + \mathbf{M L^{-1} T_i L m_{i-1}}$$



$\mathbf{m_{obs}}$	Observed data
$\mathbf{m_i}$	model
$\mathbf{L}$	Invertible transform
$\mathbf{T_i}$	Thresholding
$\mathbf{M}$	Masking operator

Problems with POCs

- Has trouble when the sparsity becomes too large

Problems with POCs

- Has trouble when the sparsity becomes too large
- True algorithm says add a single new model component per iteration

Problems with POCs

- Has trouble when the sparsity becomes too large
- True algorithm says add a single new model component per iteration
- Must have a completely invertible transform

Stagewise Orthogonal Matching Pursuit (StOMP)

- Basic pursuit/matching pursuit take too much time

Stagewise Orthogonal Matching Pursuit (StOMP)

- Basic pursuit/matching pursuit take too much time
  - Donoho et al. quote several days for what we would consider a tiny problem

Stagewise Orthogonal Matching Pursuit (StOMP)

- Basic pursuit/matching pursuit take too much time
  - Donoho et al. quote several days for what we would consider a tiny problem
- Proposed StOMP

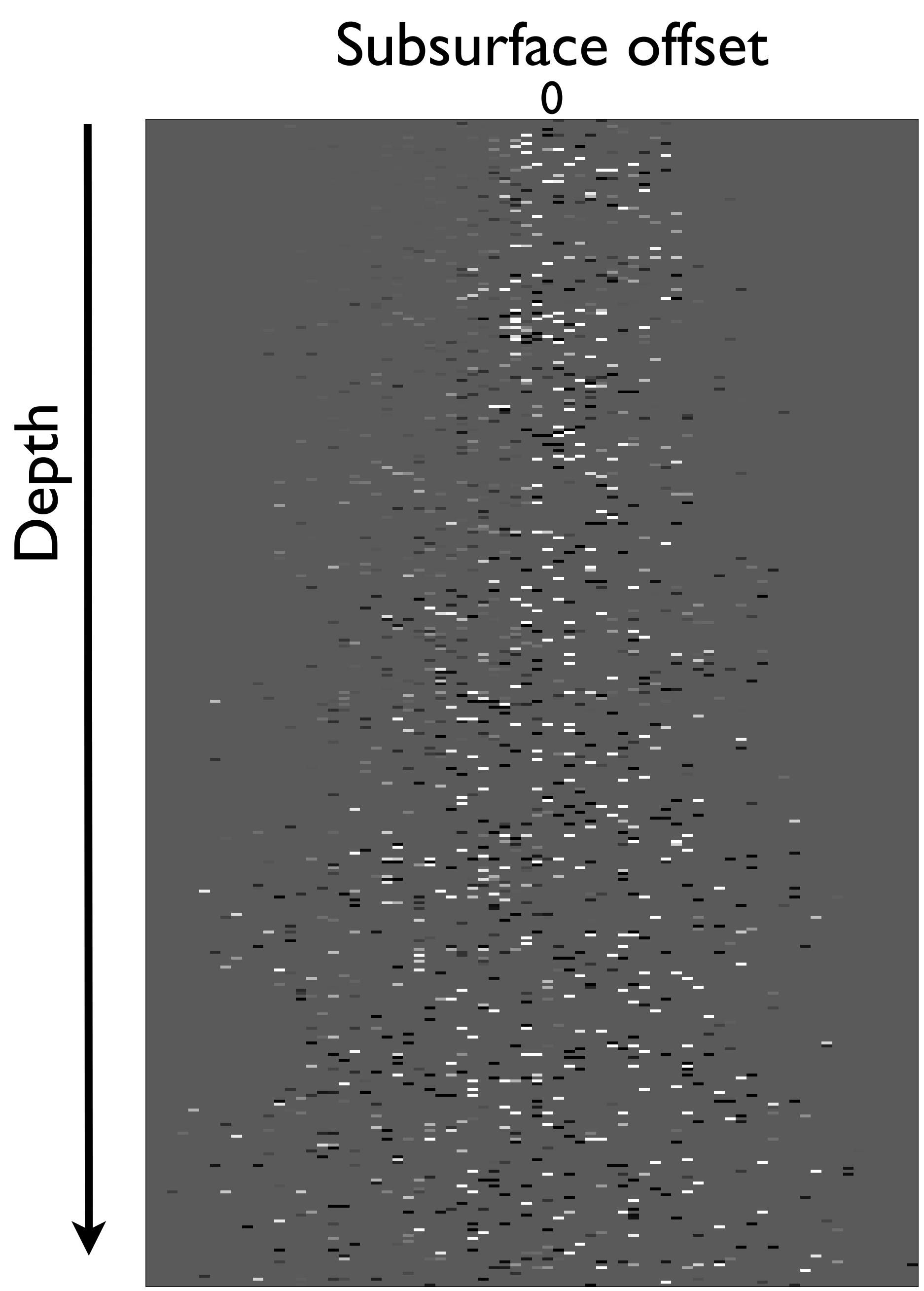
Stagewise Orthogonal Matching Pursuit (StOMP)

- Basic pursuit/matching pursuit take too much time
  - Donoho et al. quote several days for what we would consider a tiny problem
- Proposed StOMP
  - Combination of POCs and matching pursuit

Stagewise Orthogonal Matching Pursuit

$m_0=0$ J=0

$r=d-Lm_{i-1}$

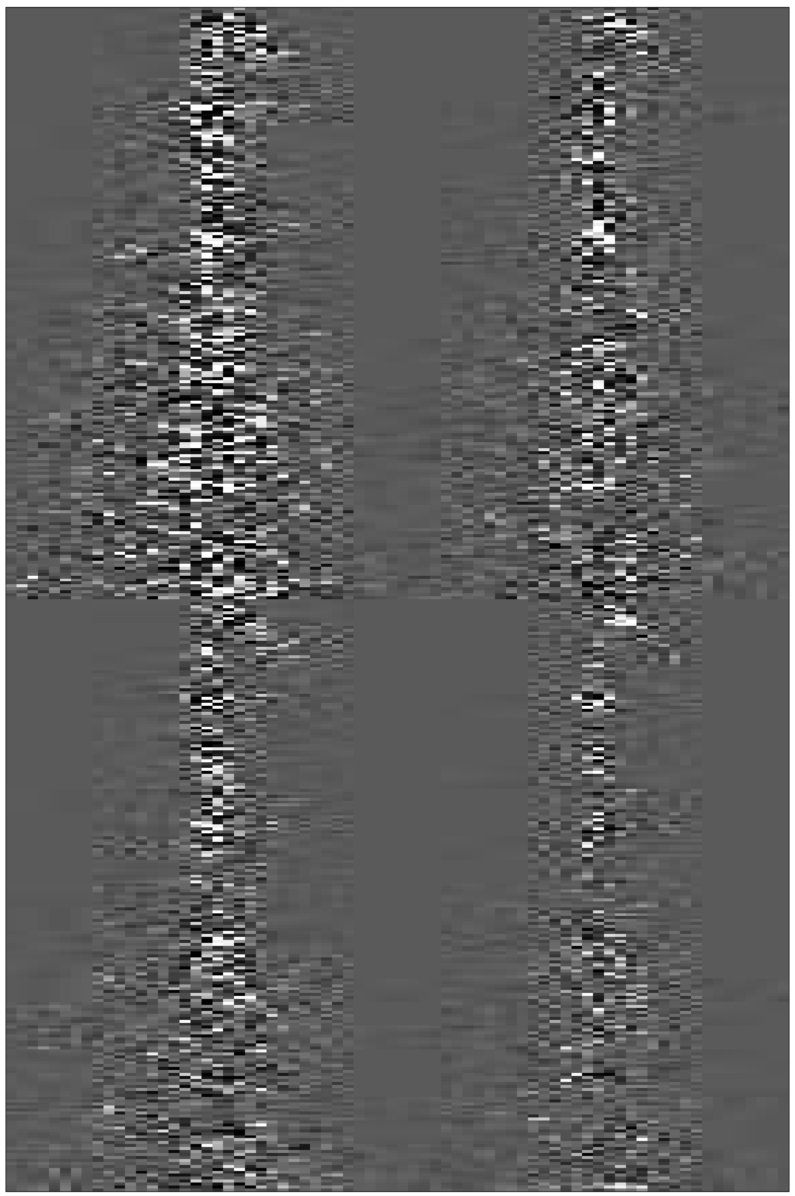


Angle gathers | Compressive Sensing | Compression | **StOMP** | Engineering | Results

Stagewise Orthogonal Matching Pursuit

m_0=0 J=0

Wavelet coefficients

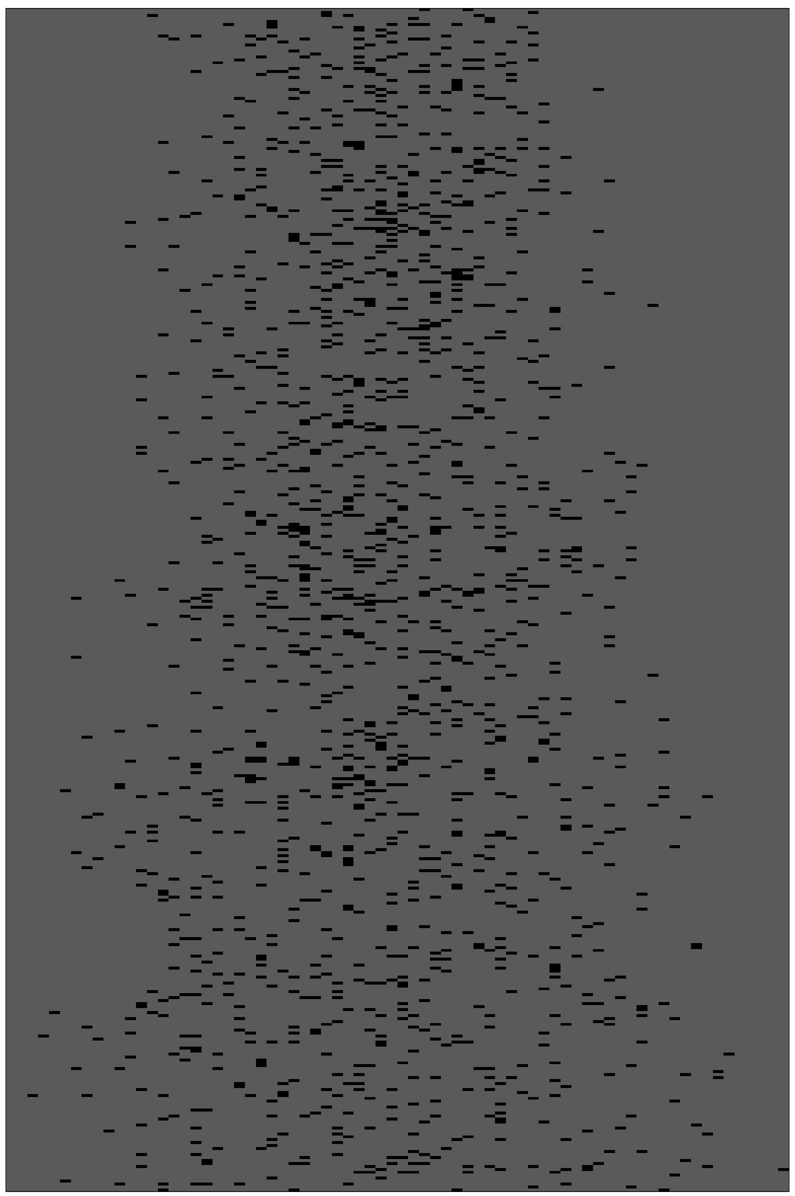


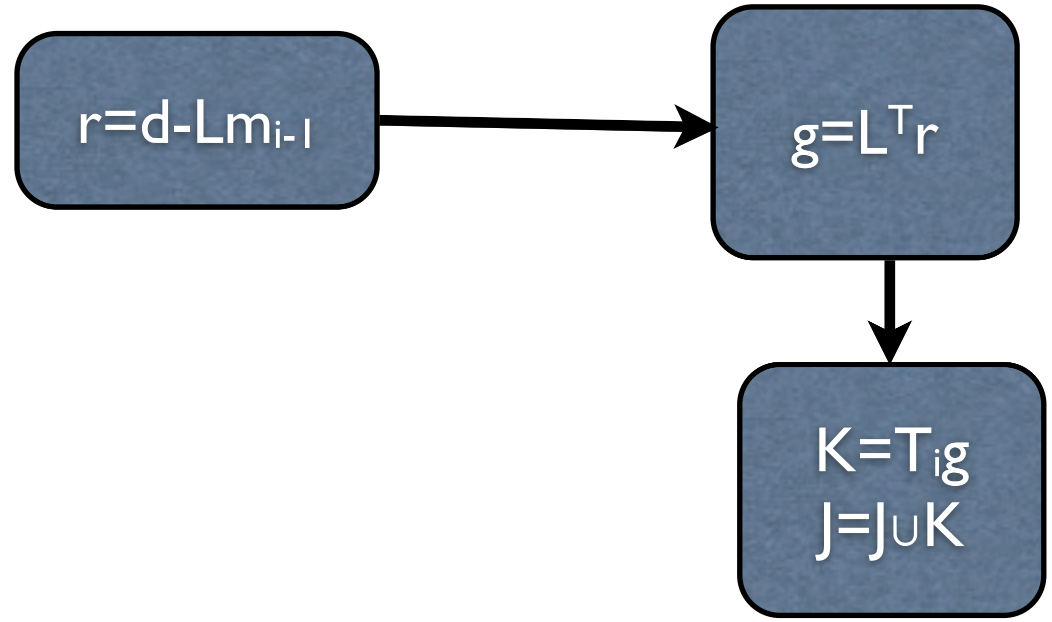
$r=d-Lm_{i-1}$ → $g=L^Tr$

Stagewise Orthogonal Matching Pursuit

m_0=0   J=0

Wavelet coefficients

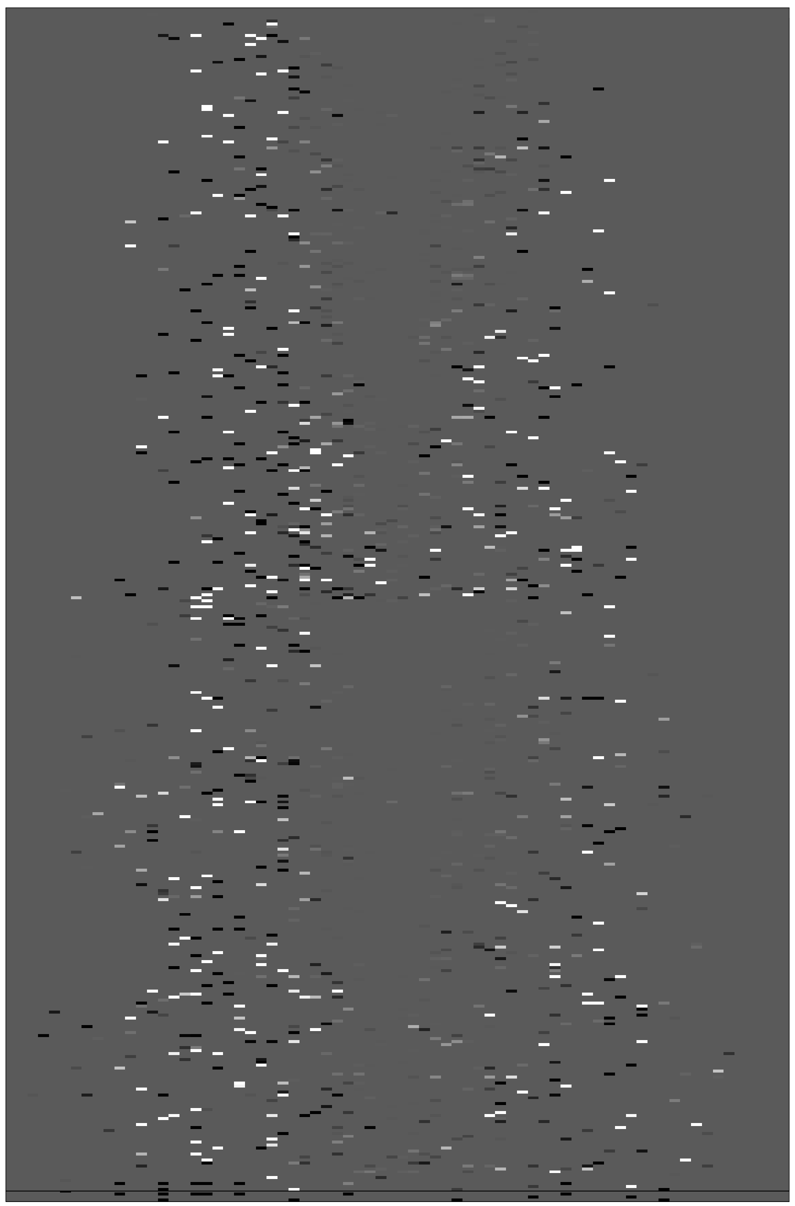


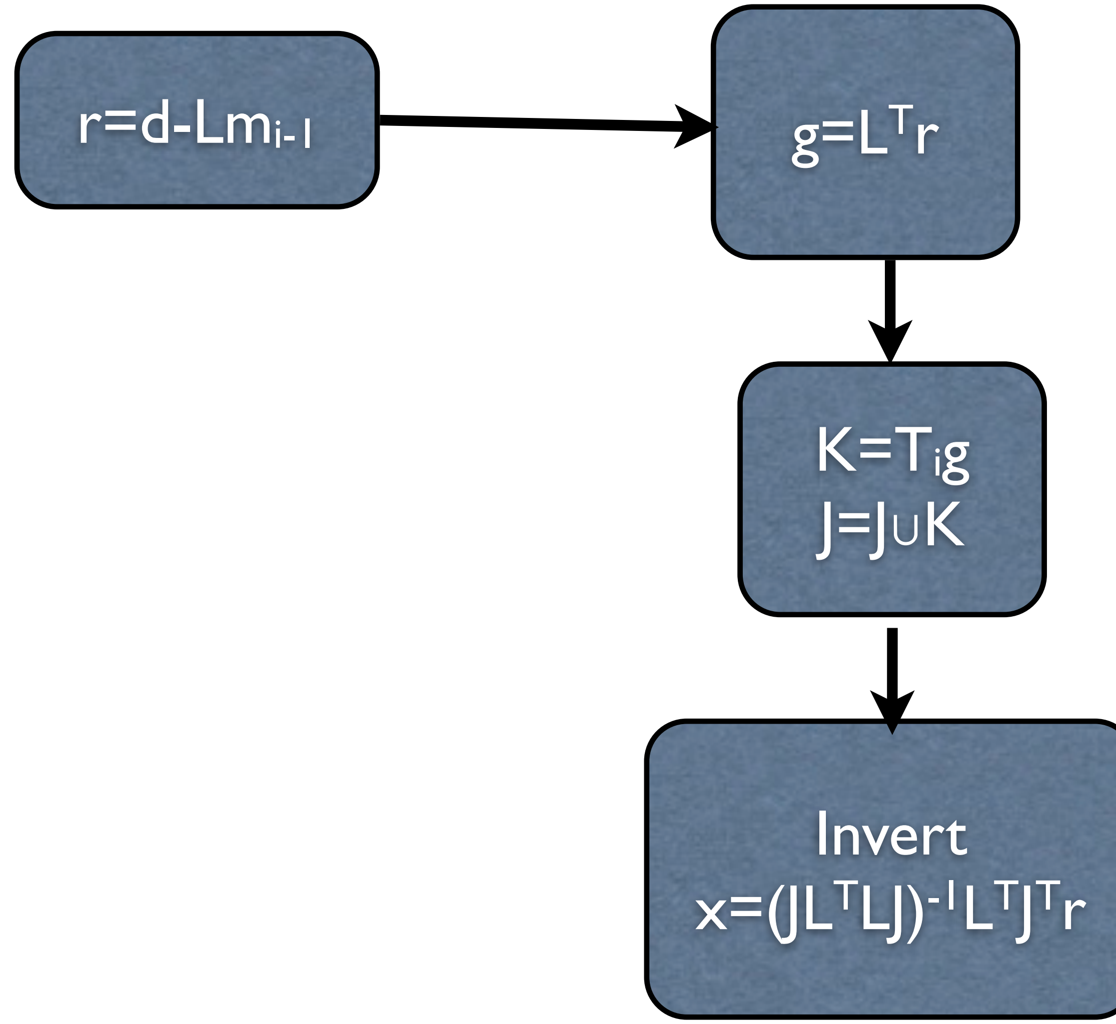


Stagewise Orthogonal Matching Pursuit

$m_0=0 \quad J=0$

Wavelet coefficients

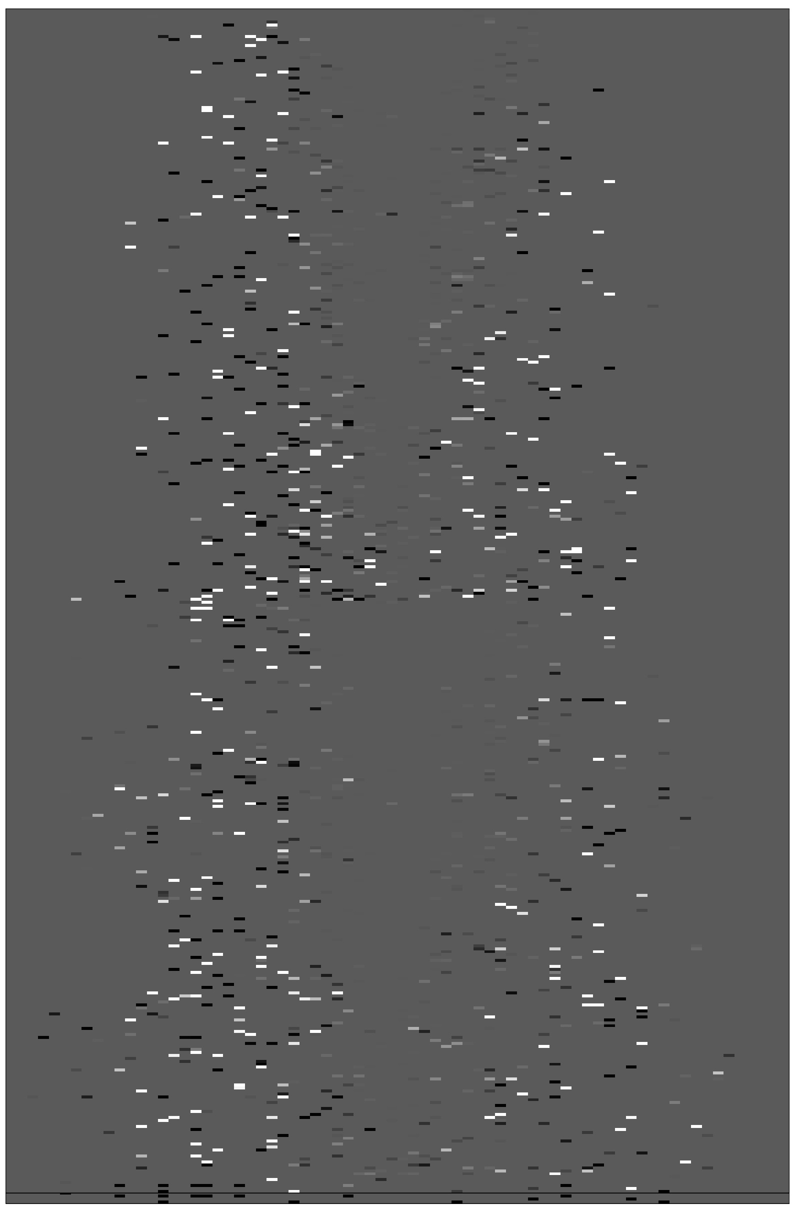


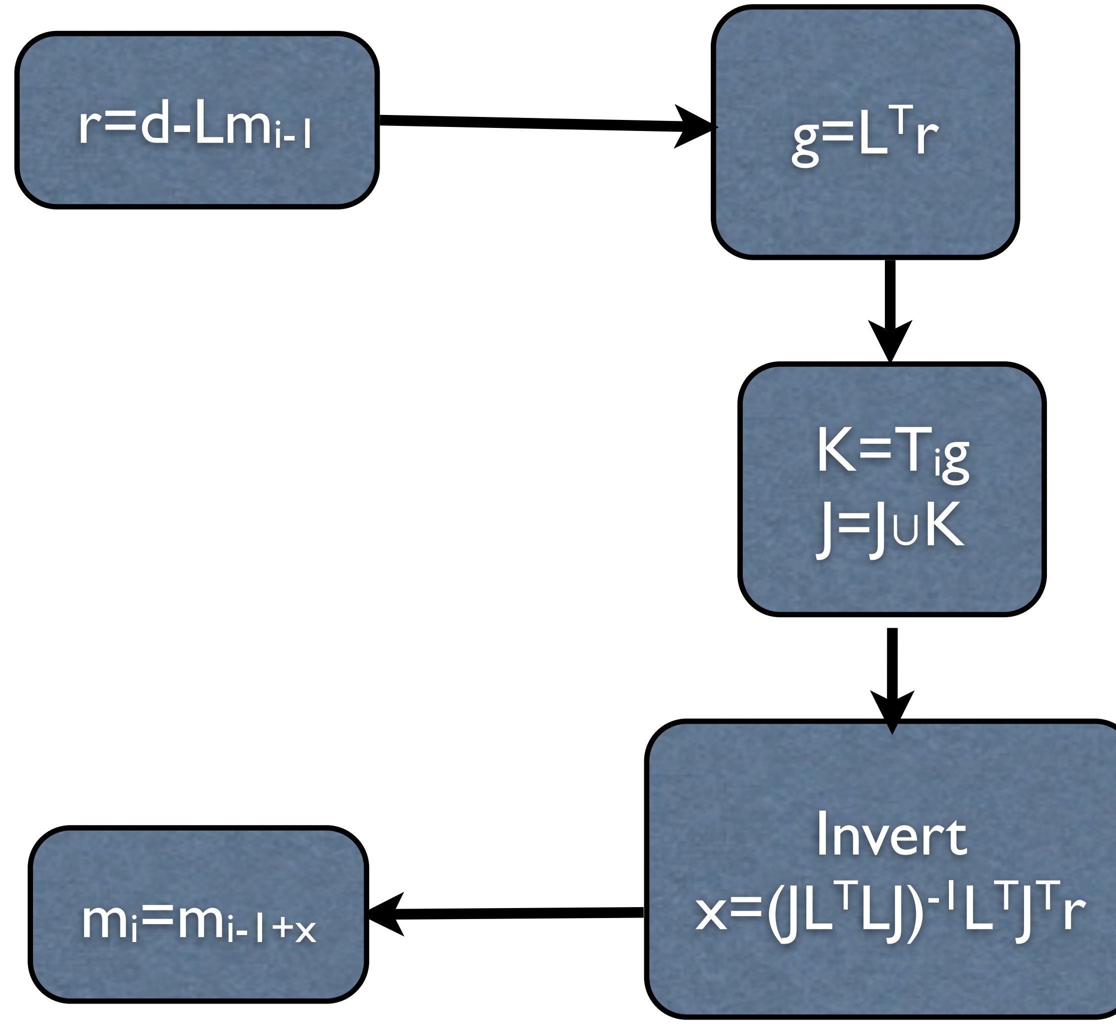


Stagewise Orthogonal Matching Pursuit

$m_0=0$ $J=0$

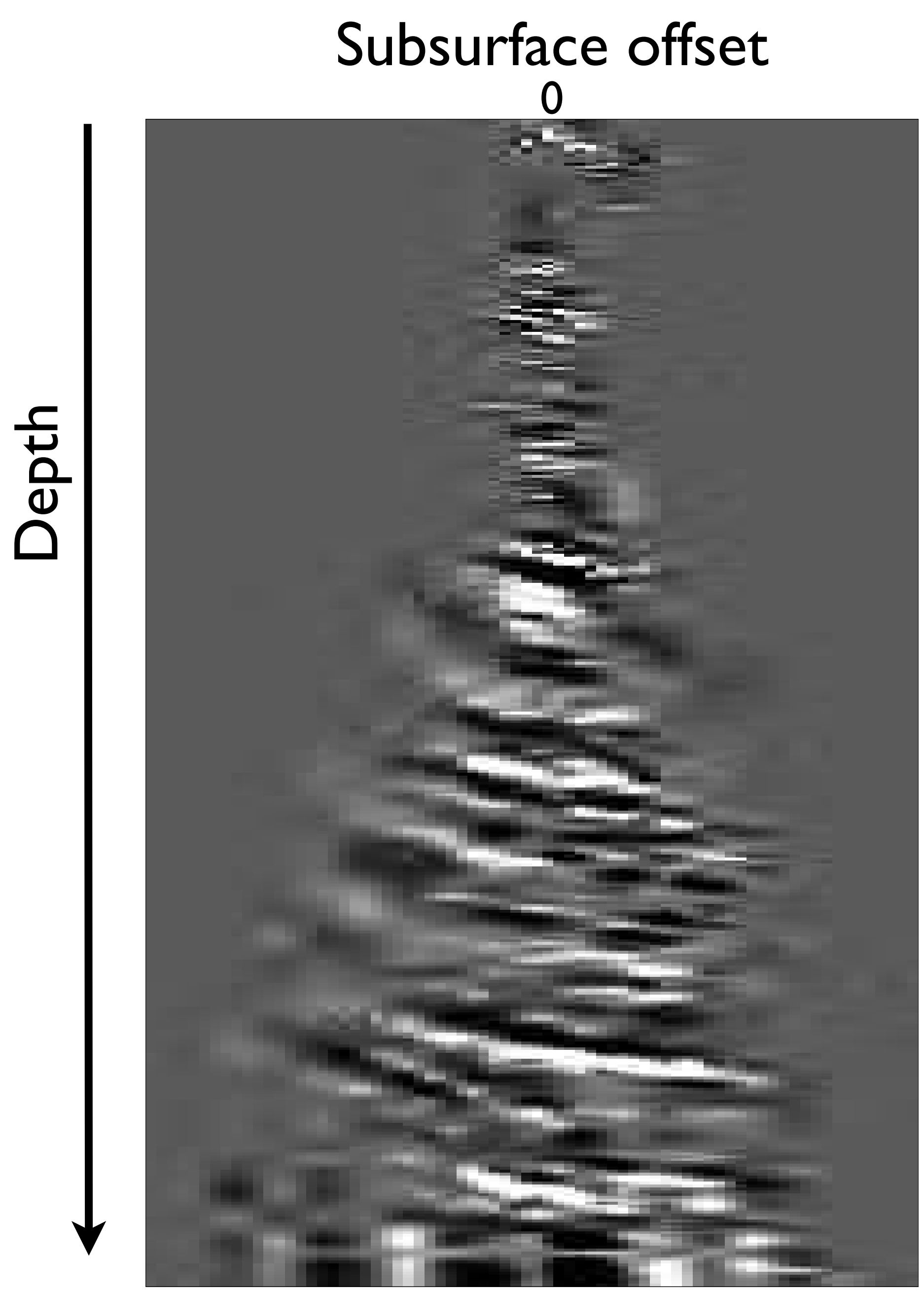
Wavelet coefficients

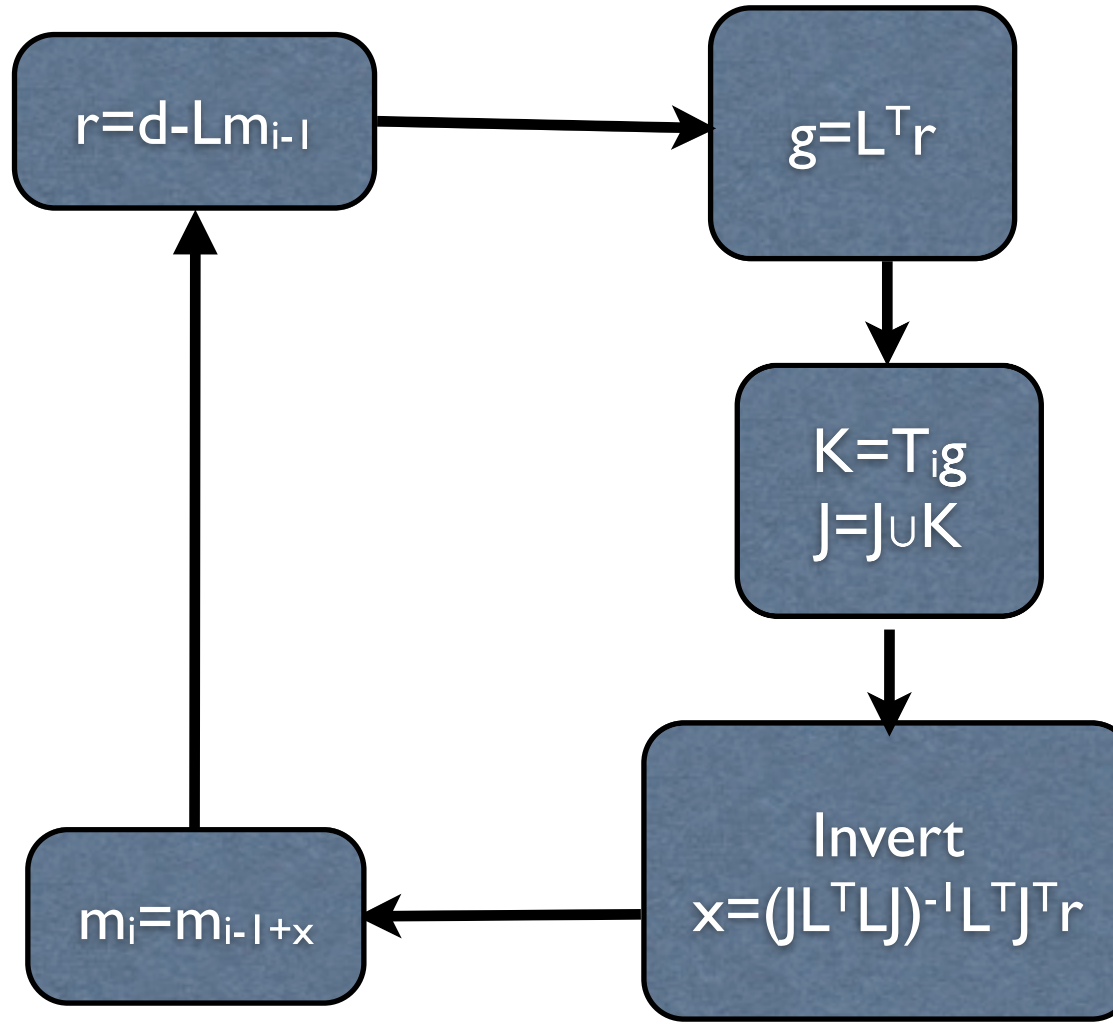




Stagewise Orthogonal Matching Pursuit

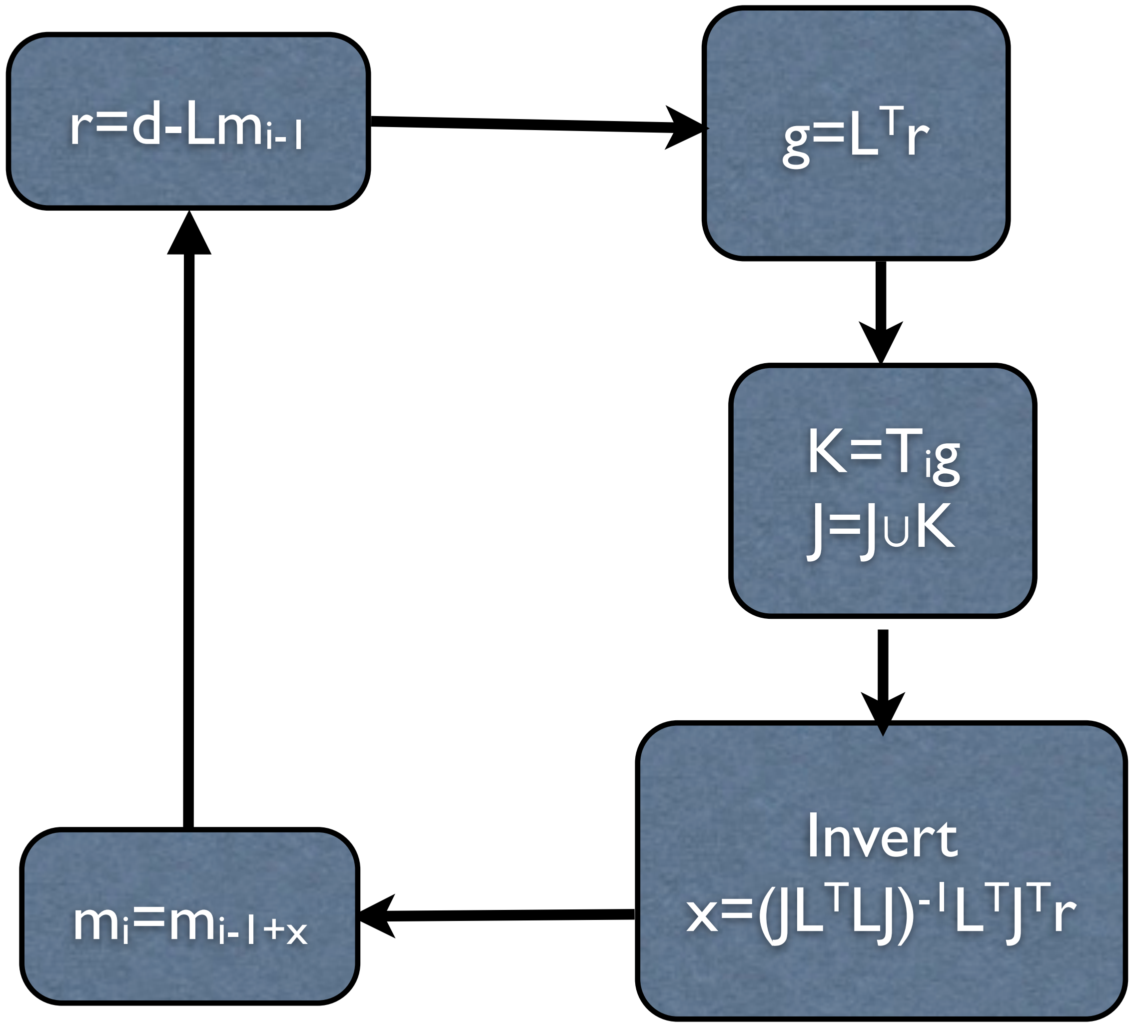
$m_0=0$ $J=0$





Stagewise Orthogonal Matching Pursuit

$\mathbf{m_0=0 \quad J=0}$



d Observed data

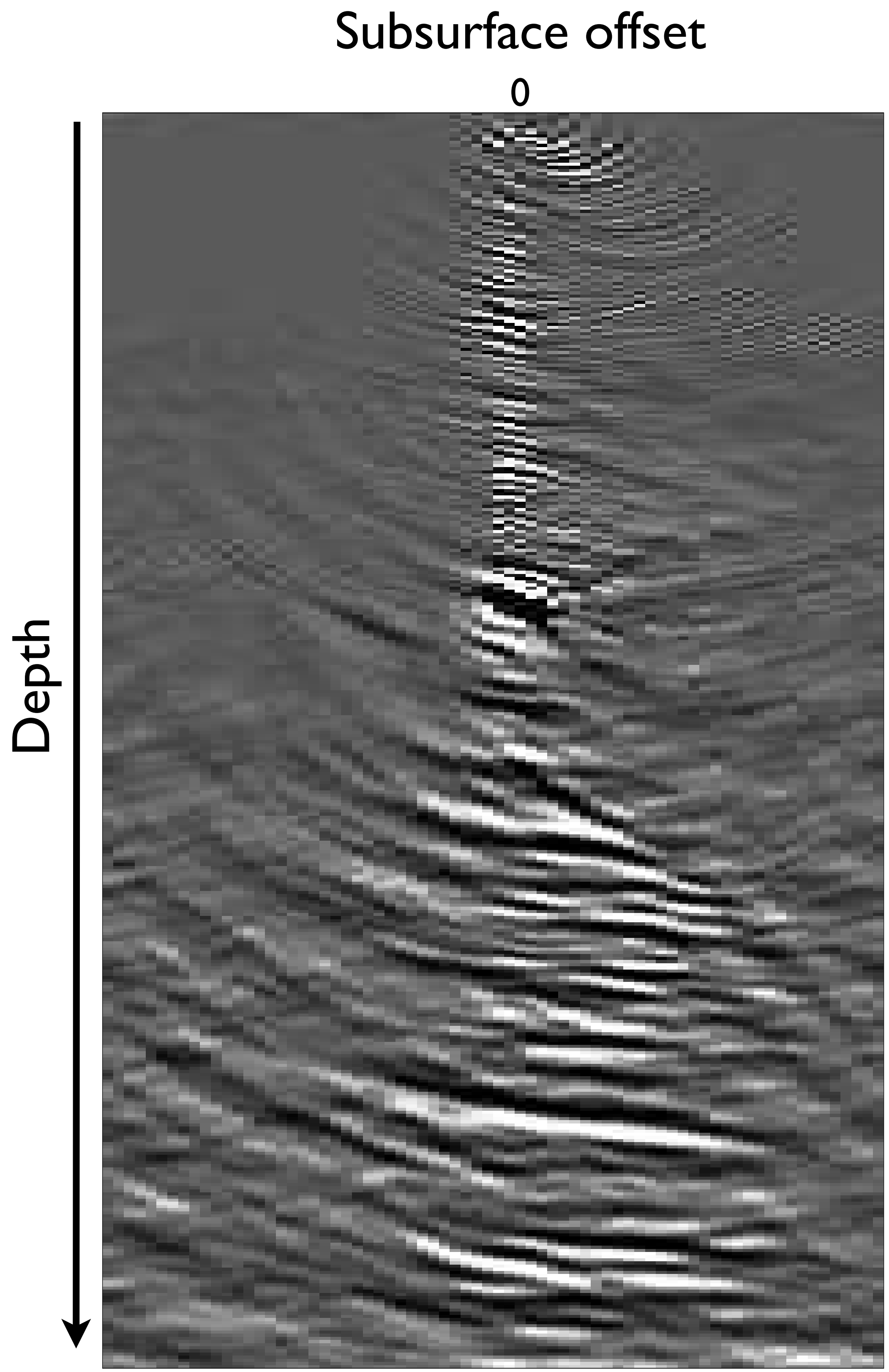
$\mathbf{m_i}$ Model

L Map into a sparse basis

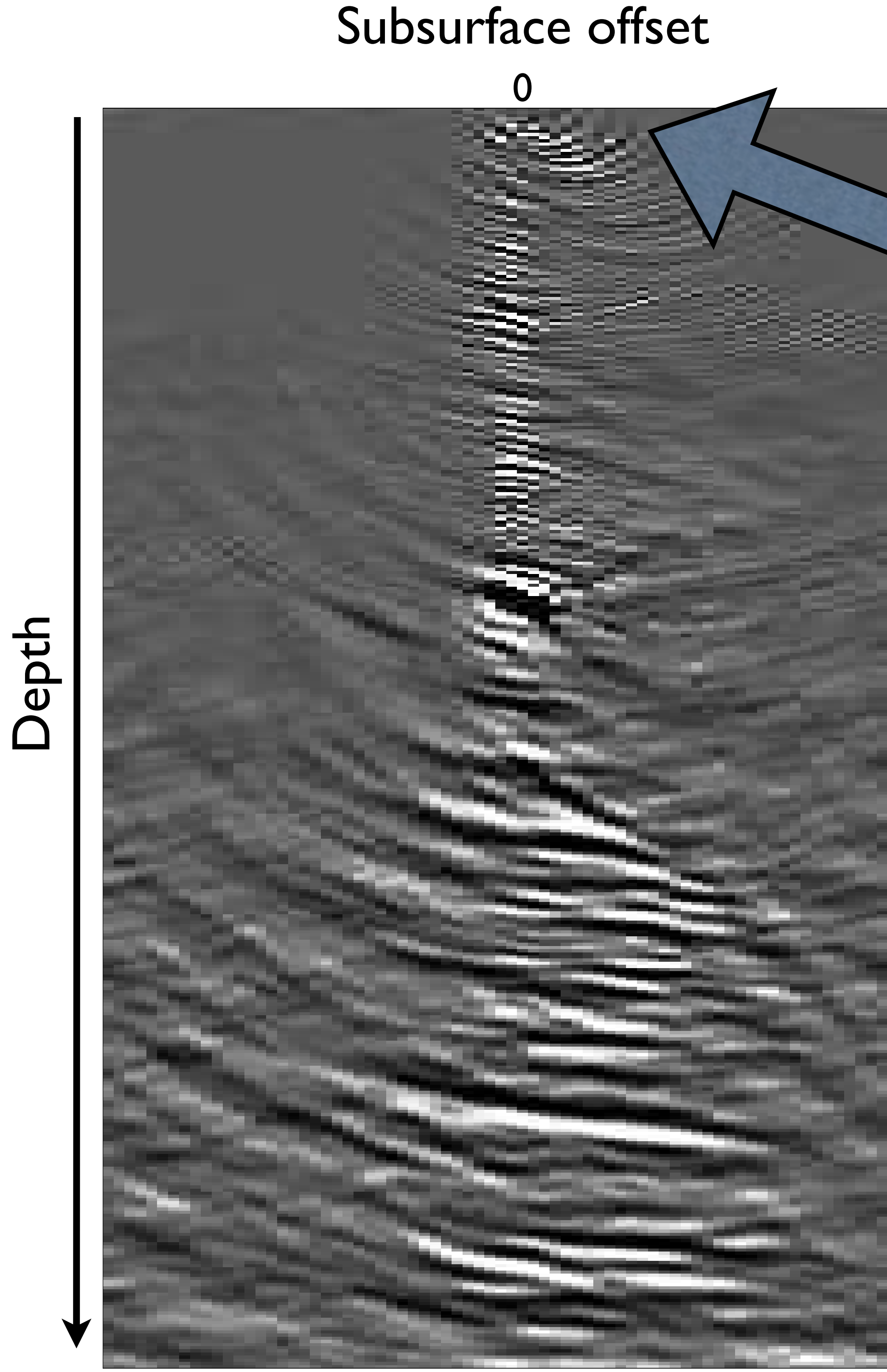
$\mathbf{T_i}$ Thresholding

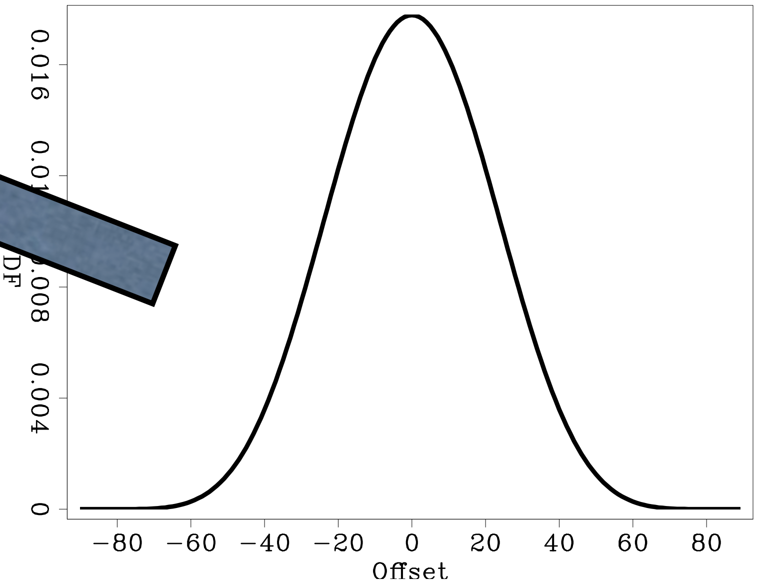
J Non-zero model locations

Random sampling??

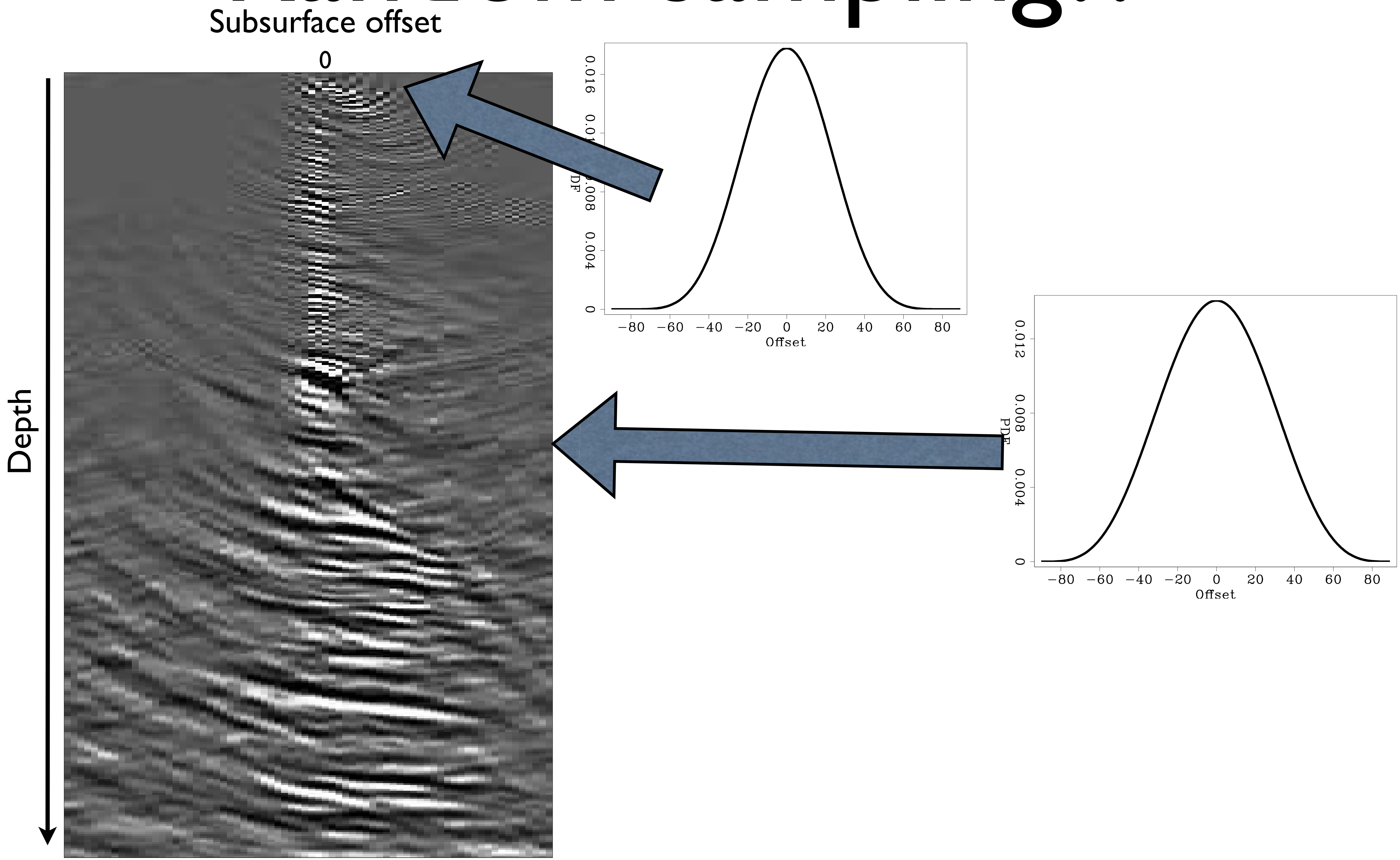


Random sampling??

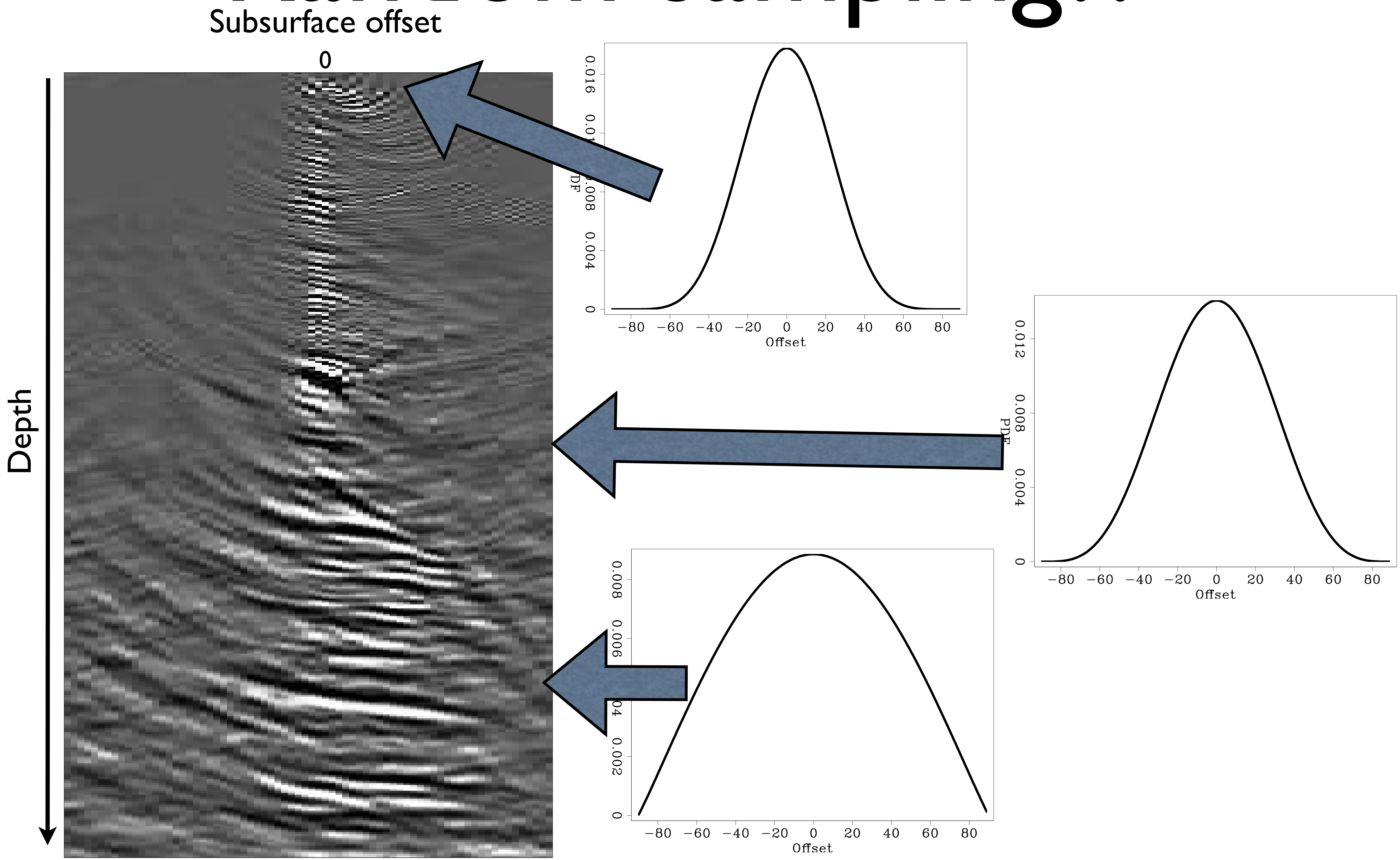




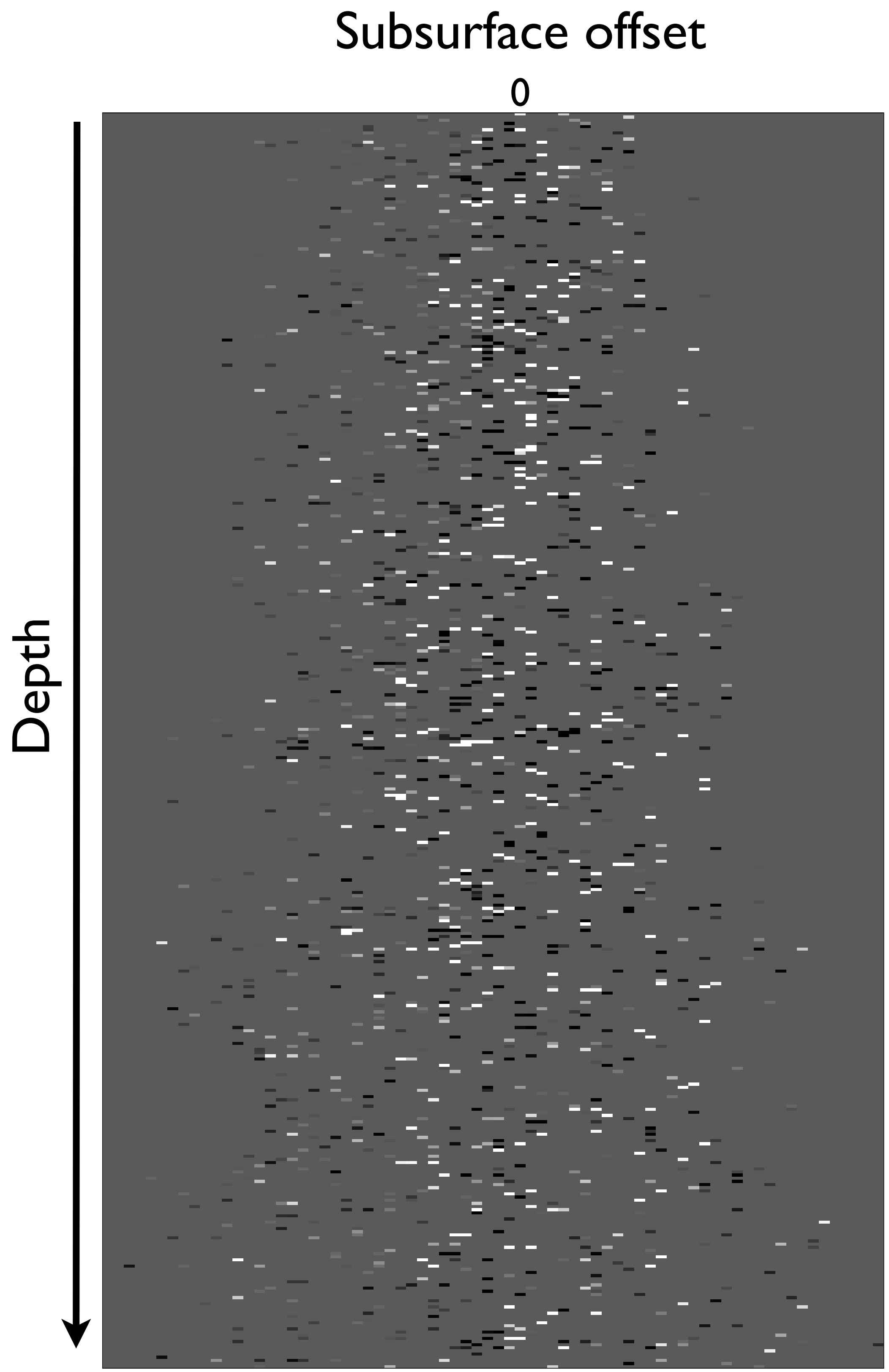
Random sampling??

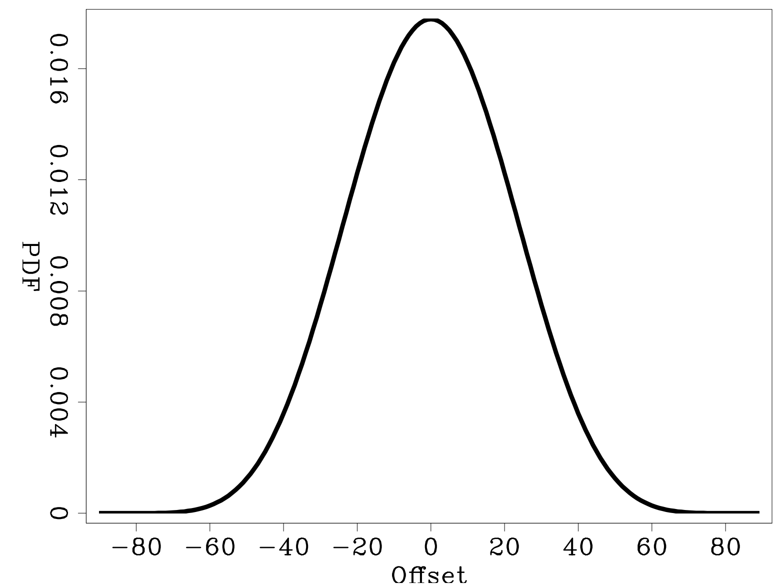


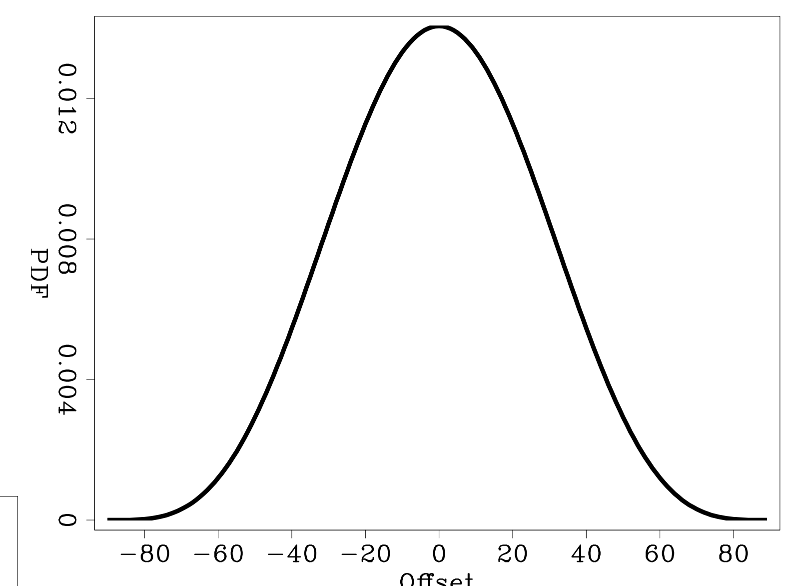
Random sampling??

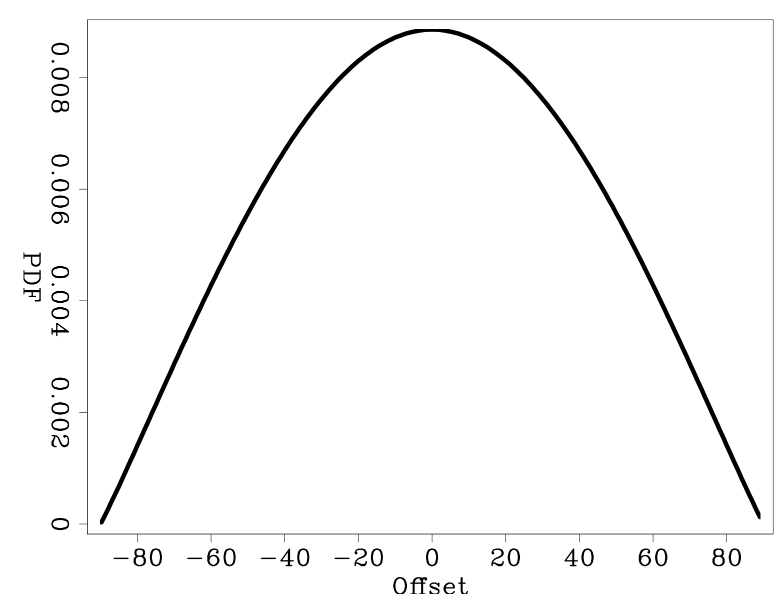


Random sampling??





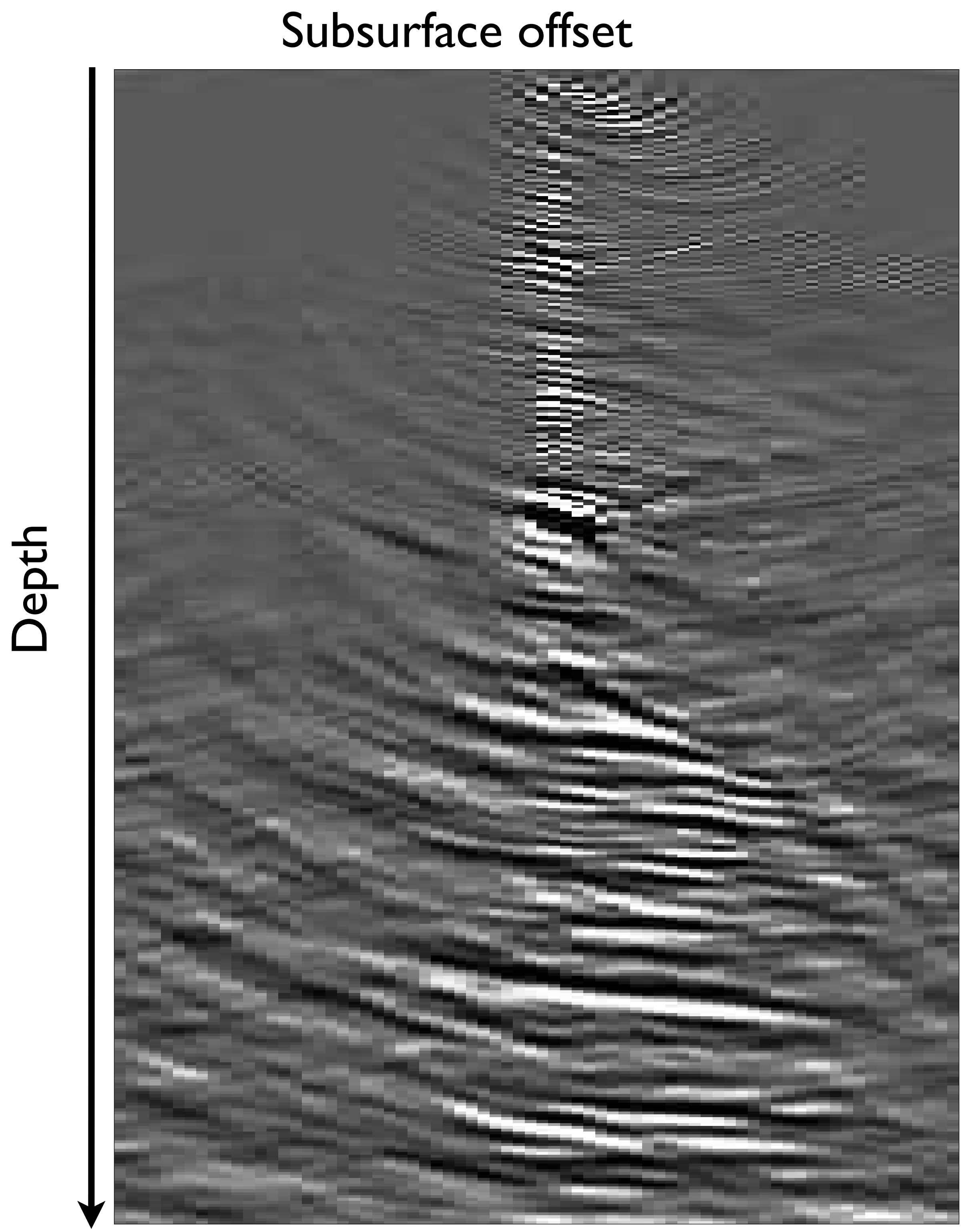




Data acquisition cost

- Purpose of compressive sensing is to reduce the data acquisition cost.
- In angle gather construction the cost is in the storage, rather than the convolutions.
- More convolutions can be added at minimal additional cost.

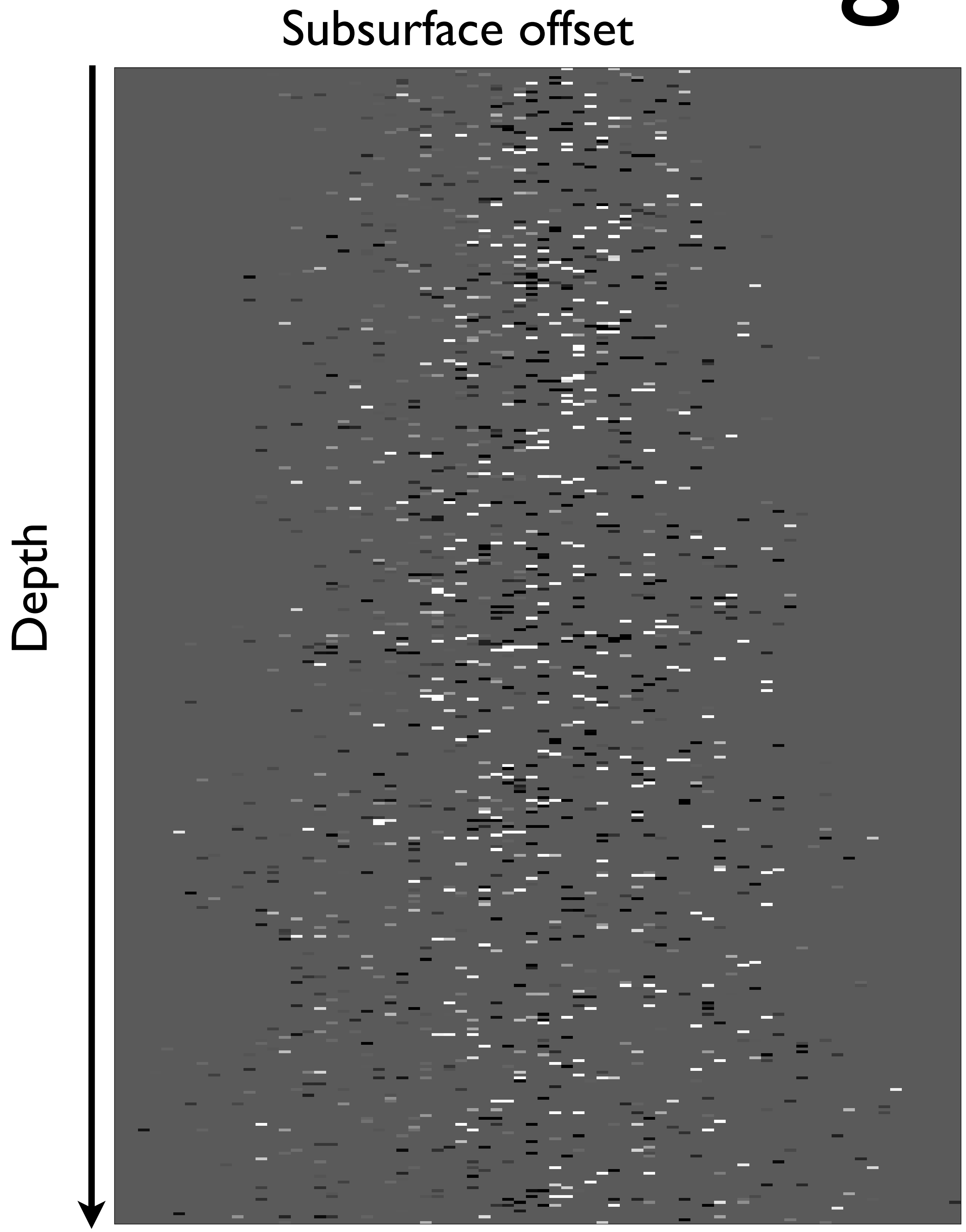
Full image gather



$$\mathbf{d} = \mathbf{m}$$

d - Sampled image gather
m - Full image gather

Pseudo-randomly sampled gather

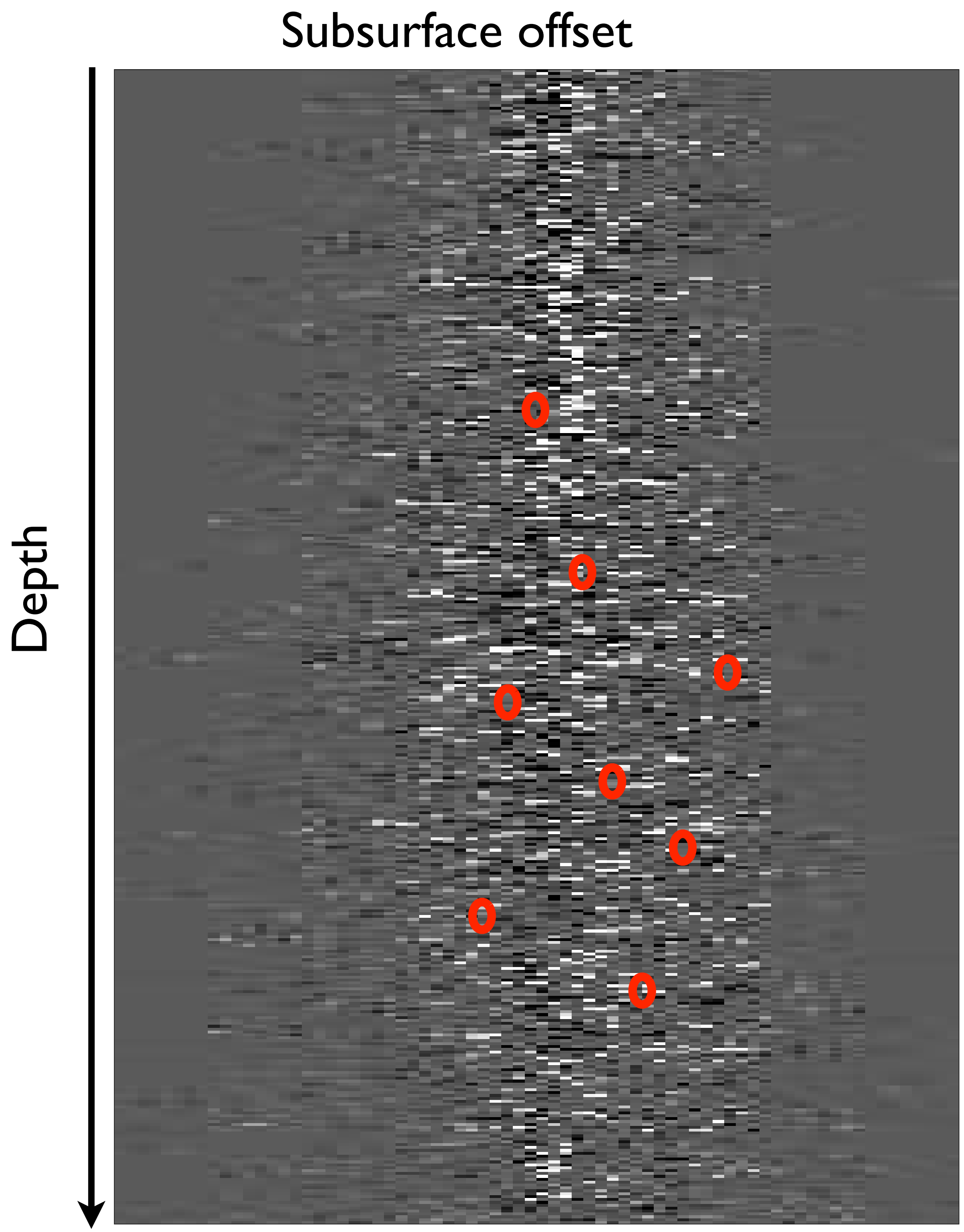


$$\mathbf{d} \approx \mathbf{JWp}$$

$$\mathbf{m} = \mathbf{Wp}$$

J - Selector operator
W - Wavelet transform
p - Basis function
d - Sampled image gather
m - Full image gather

Phase encoding



R - Phase encoding operator

$$r_j = \sum_{i=0}^{n} \alpha_i o\left(\text{Rand}(m)\right)$$

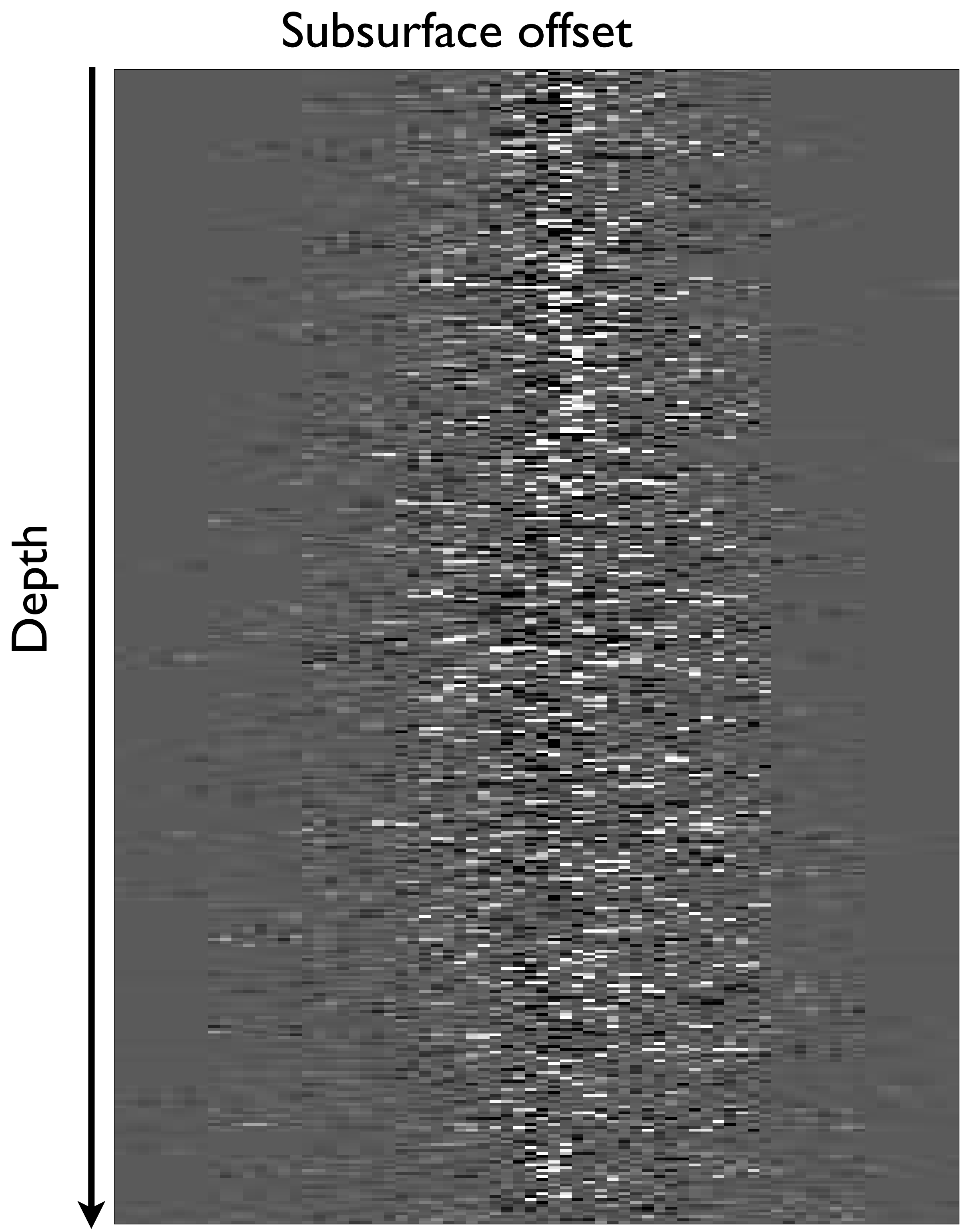
m - number of non-zero correlation locations

o - correlation values

r - output values

α_i - random coefficients

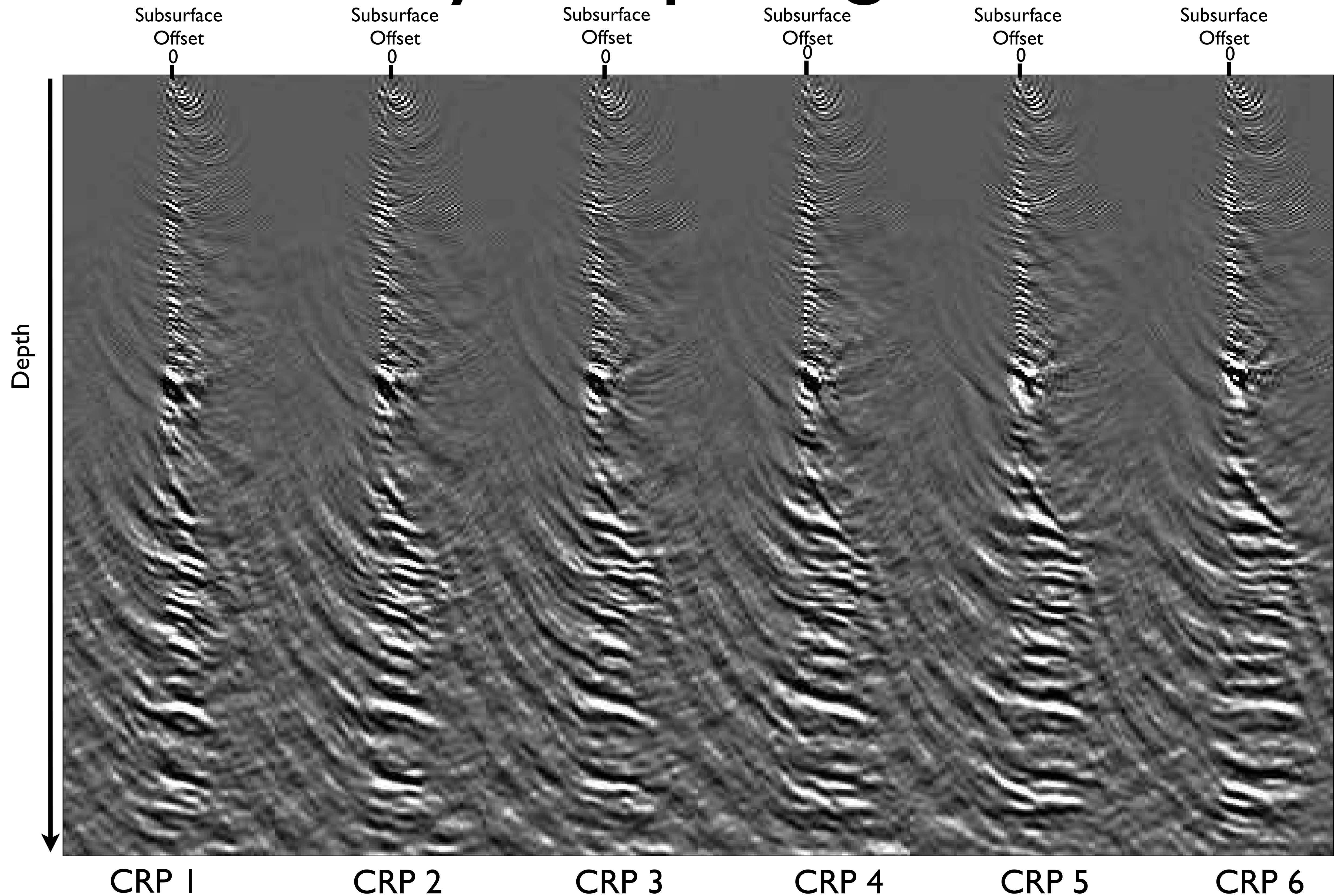
Full image gather



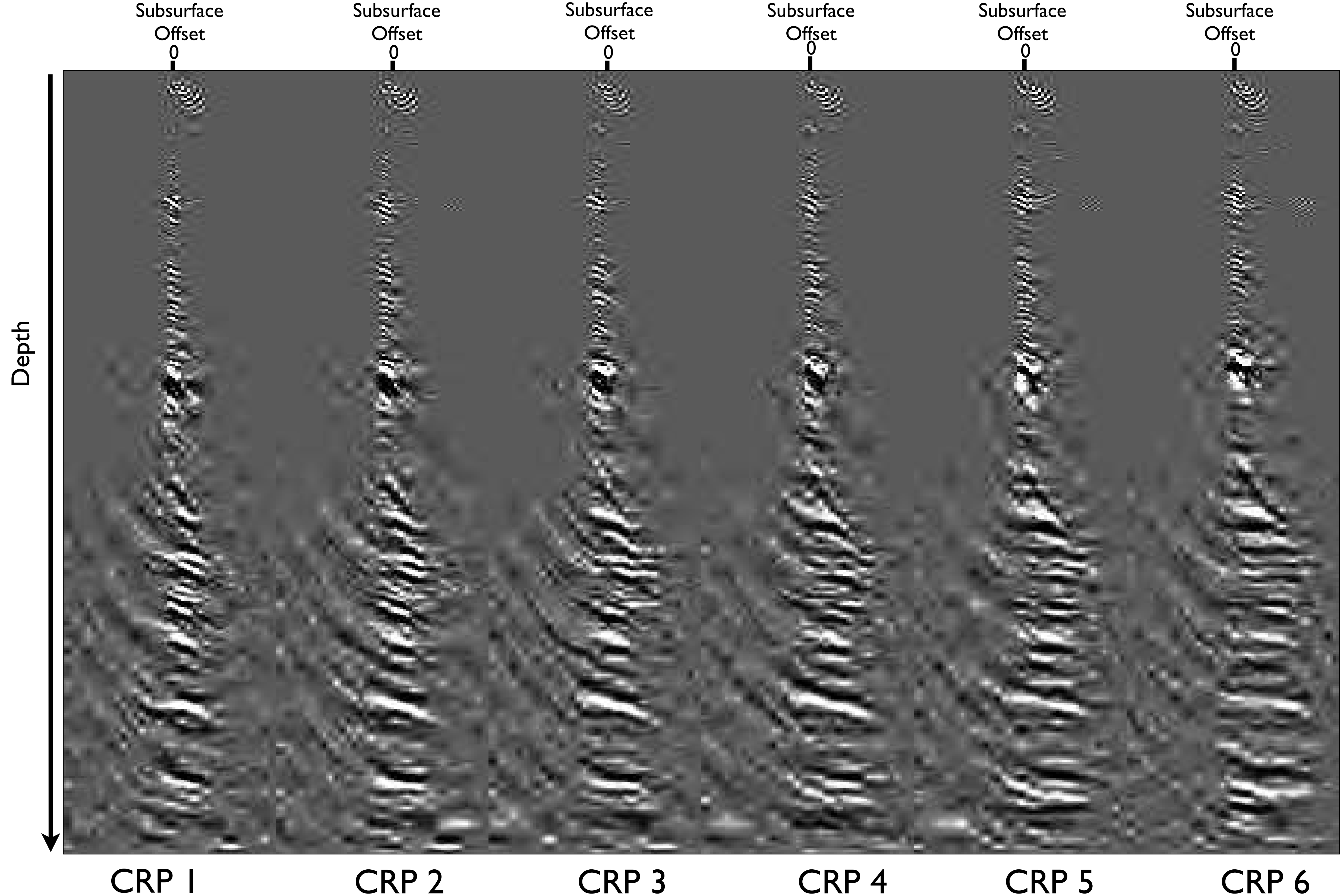
$$\mathbf{d} \approx \mathbf{JRWp}$$
$$\mathbf{m} = \mathbf{Wp}$$

J - Selector operator
W - Wavelet transform
R - Phase encoded sampler
p - Basis function
d - Sampled image gather
m - Full image gather

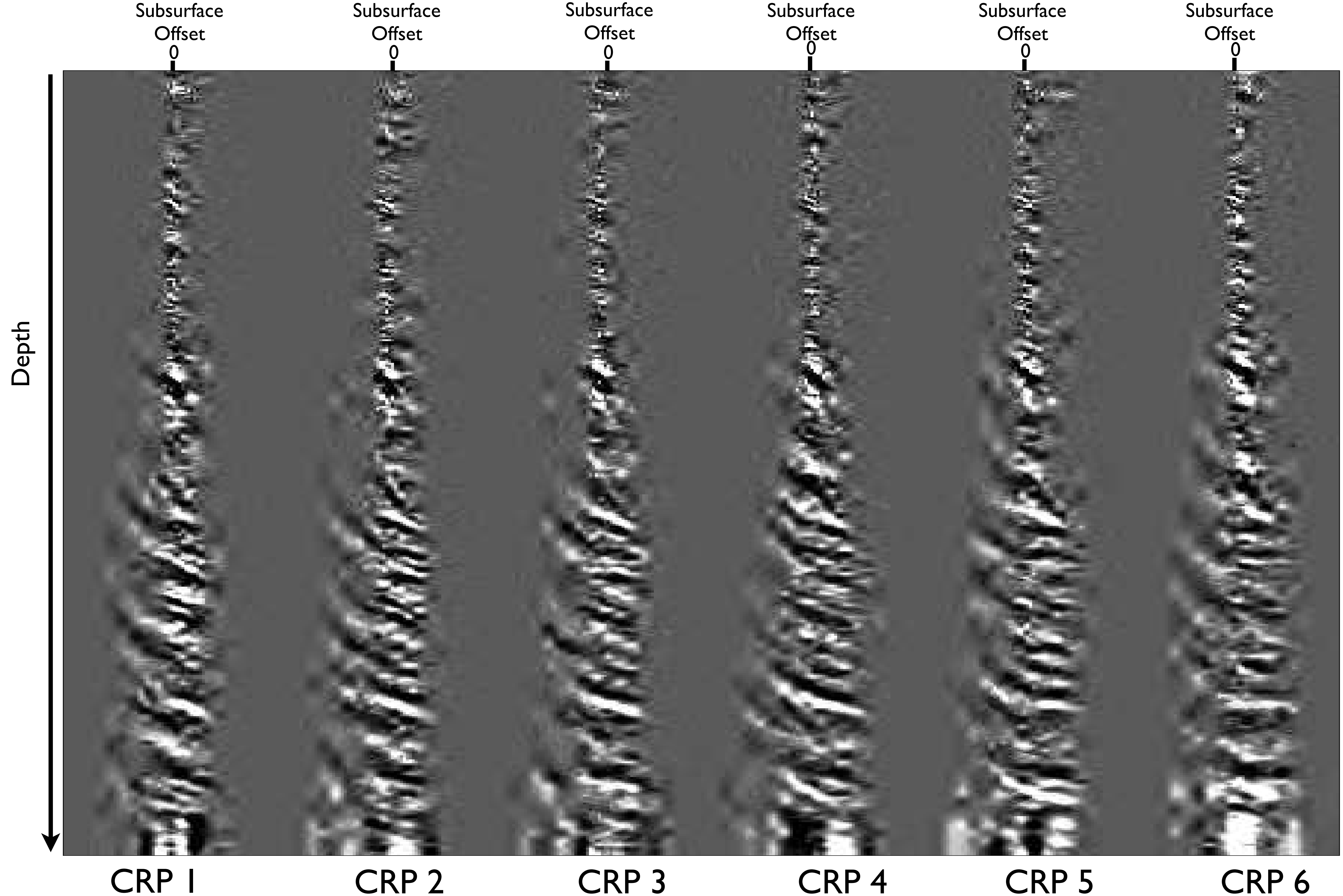
Fully sampled gathers



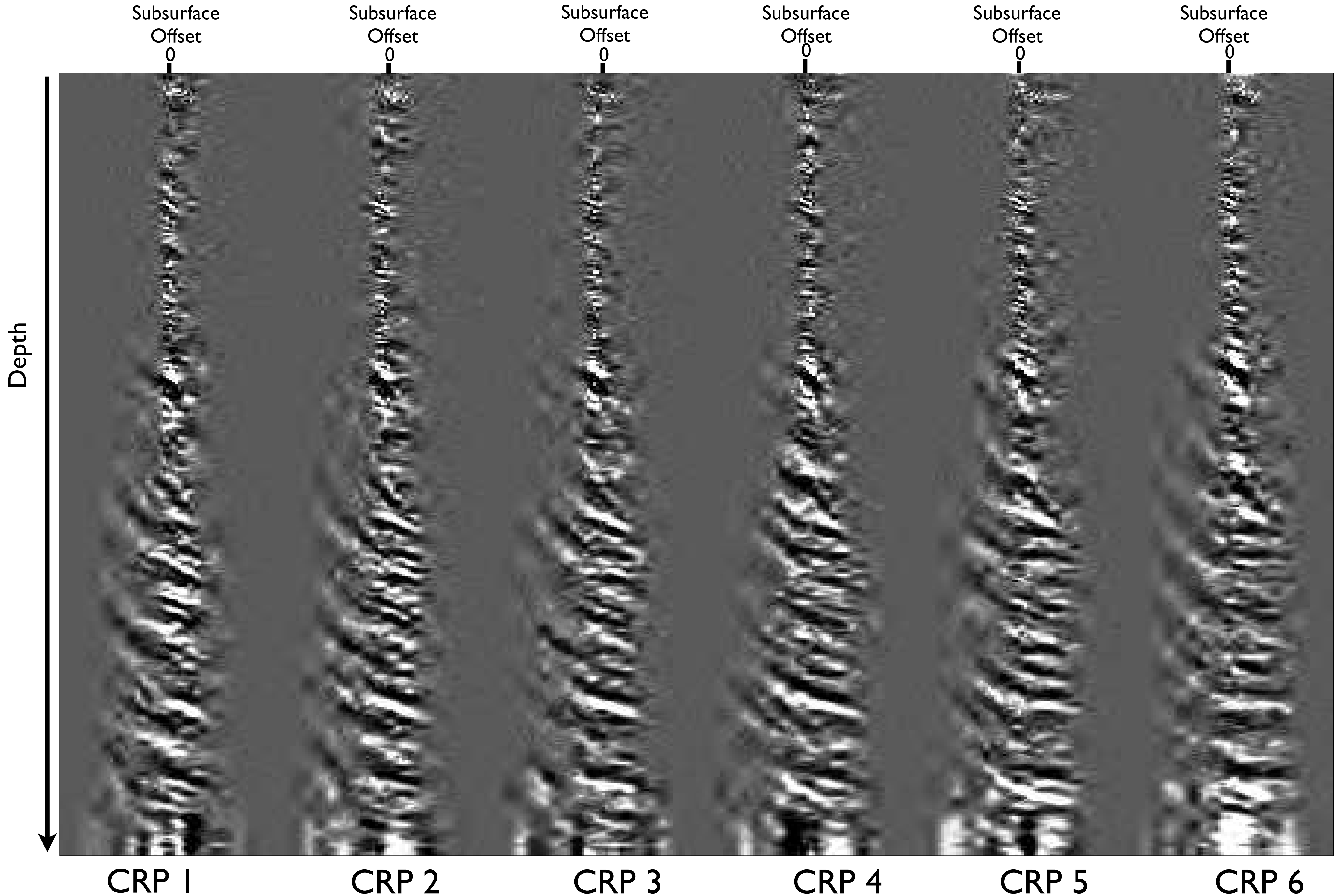
1% wavelet coefficients



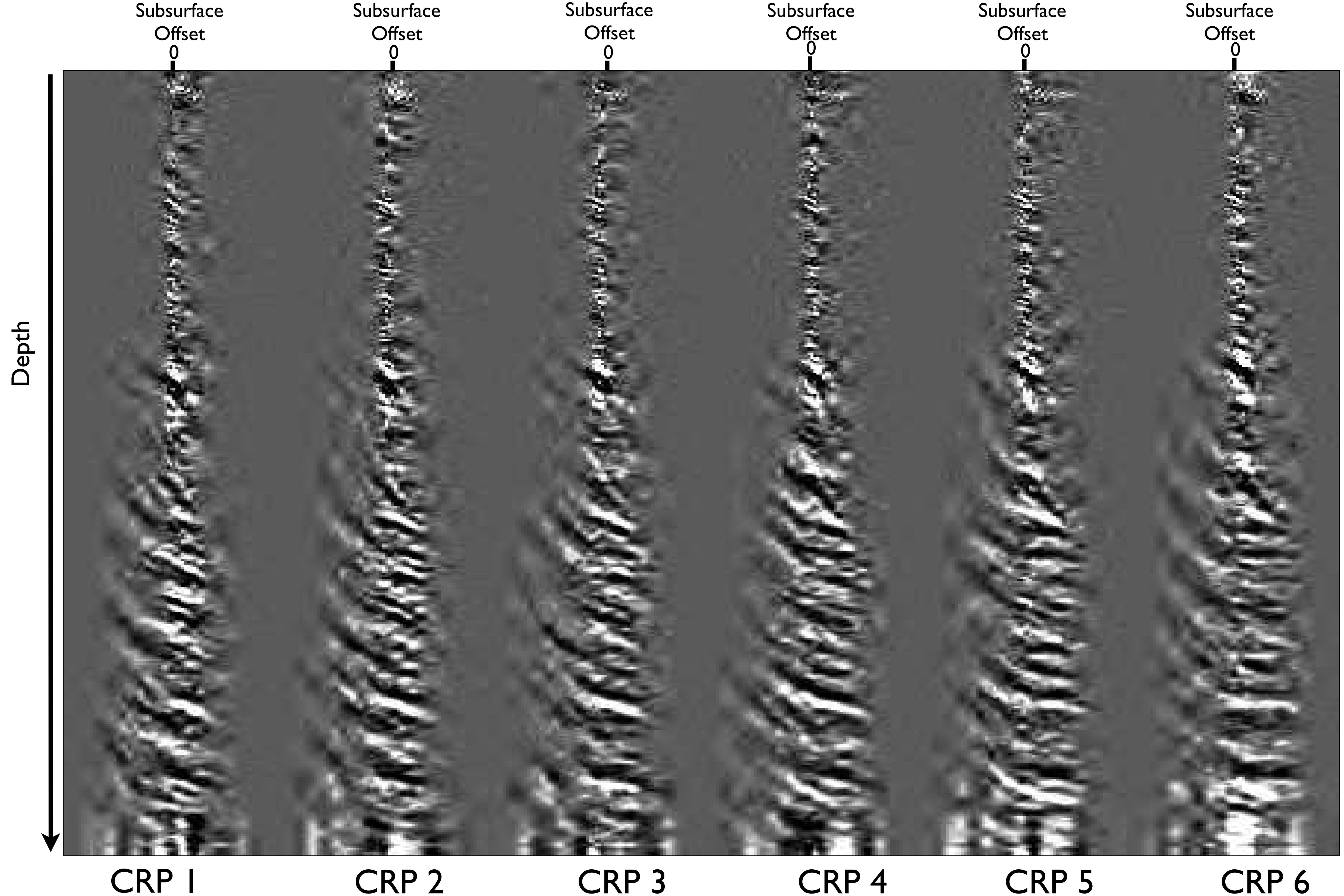
1 iteration of StOMP

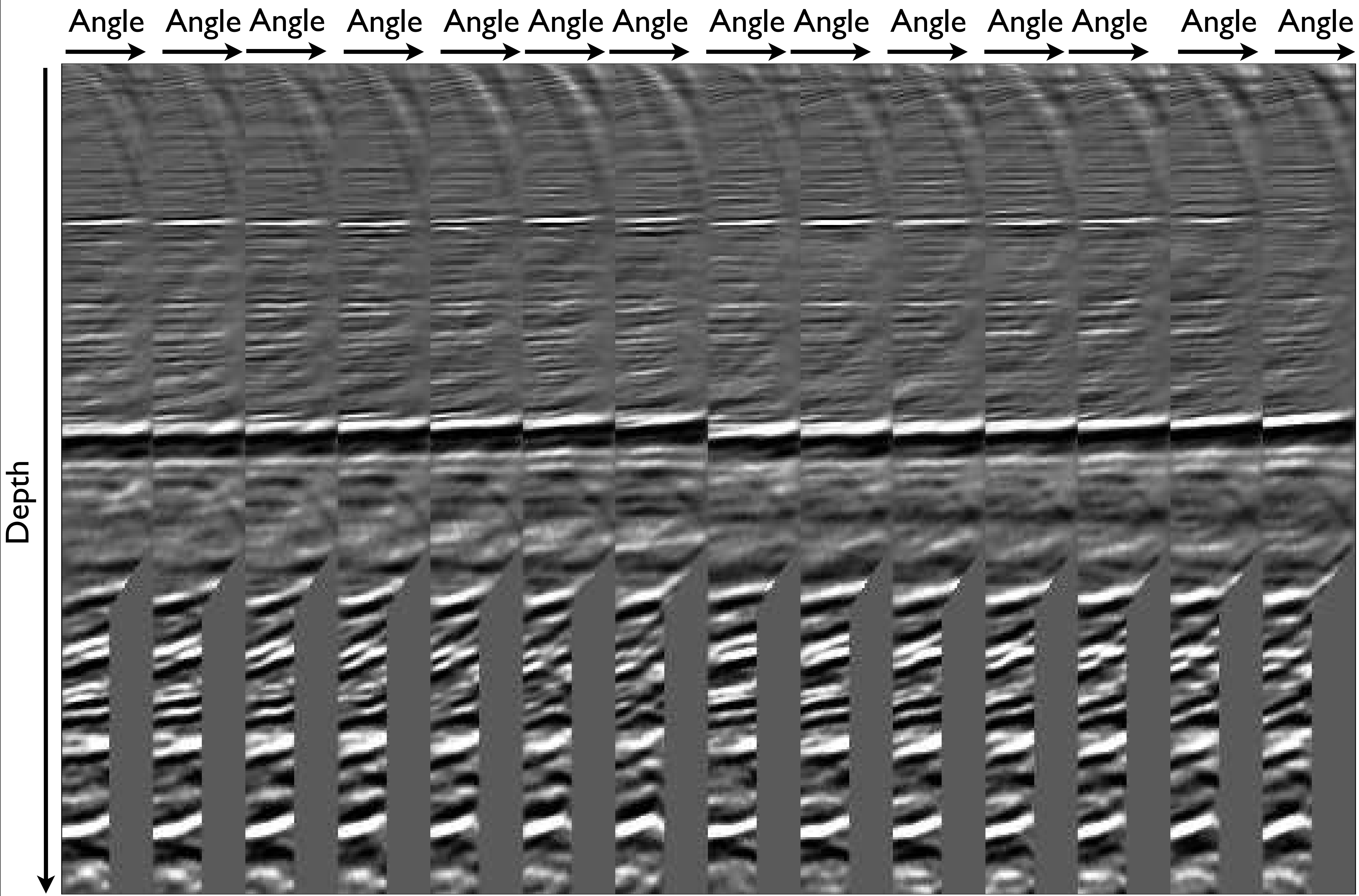


2 iterations of StOMP



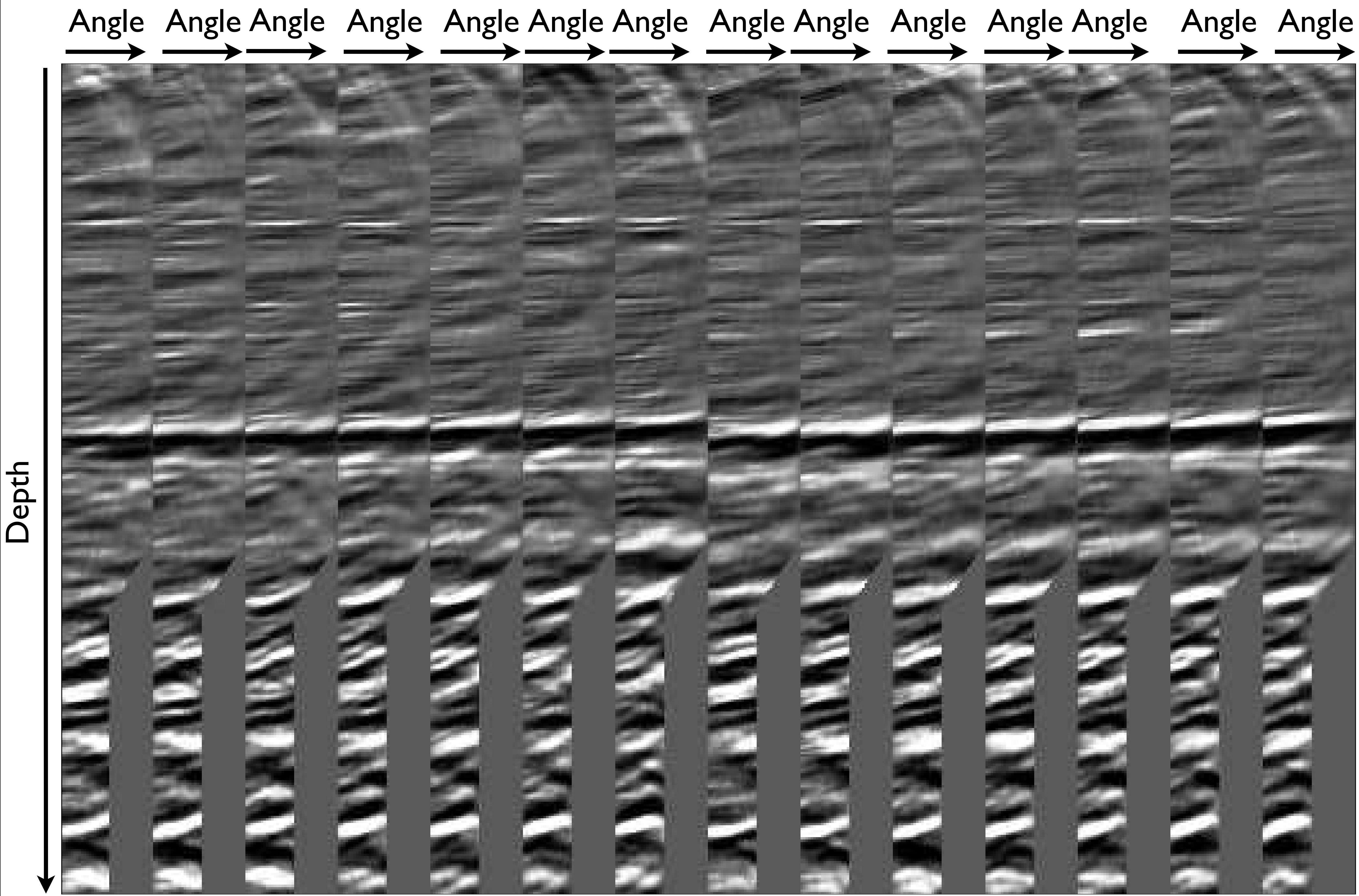
4 iterations of StOMP





Full volume

Angle gathers | Compressive Sensing | Compression | StOMP | Engineering | **Results**



1/20 volume size

Angle gathers | Compressive Sensing | Compression | StOMP | Engineering | **Results**

Conclusions

- The cost and compressibility of offset based image-gather construction fits the compressive sensing guidelines.
- StOMP algorithm appears effective for large-scale inversion problems.
- Including phase encoding further improves results.