Residual moveout based wave-equation migration velocity analysis in 3-D

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SEP-147, p27

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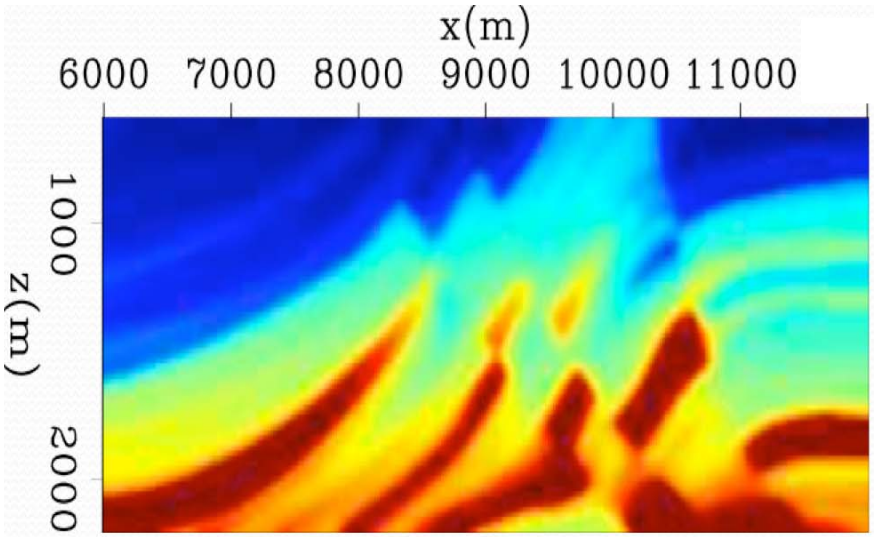
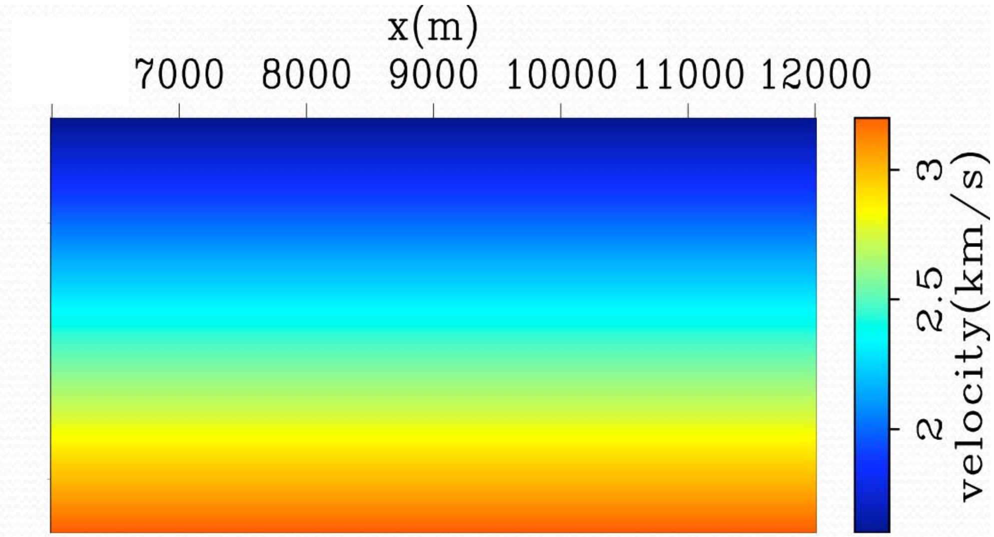
Stanford Exploration Project

Outline

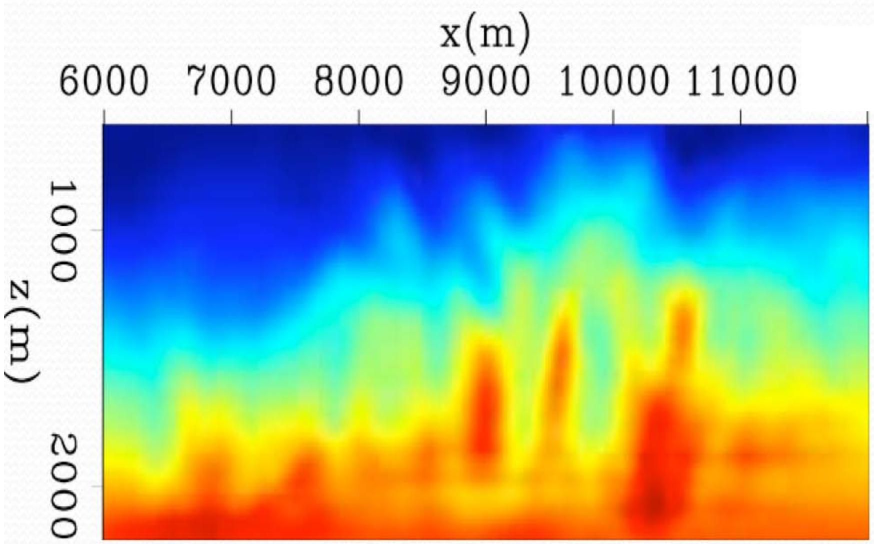
- Motivation of RMO WEMVA
- The 2-D method review
- The extension to 3-D
- Discussion & Conclusion



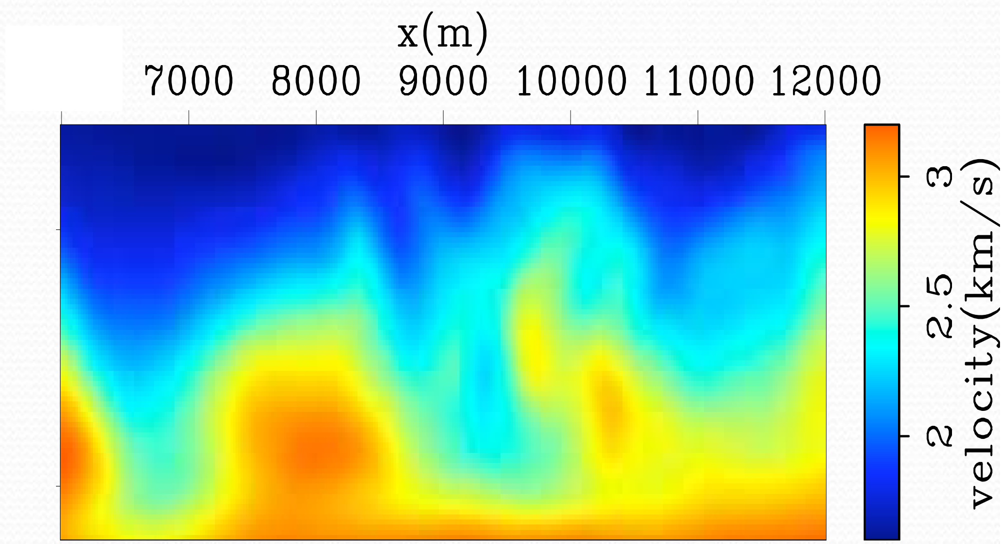
True model



Starting model $v(z) = v_0+\alpha z$



RMO

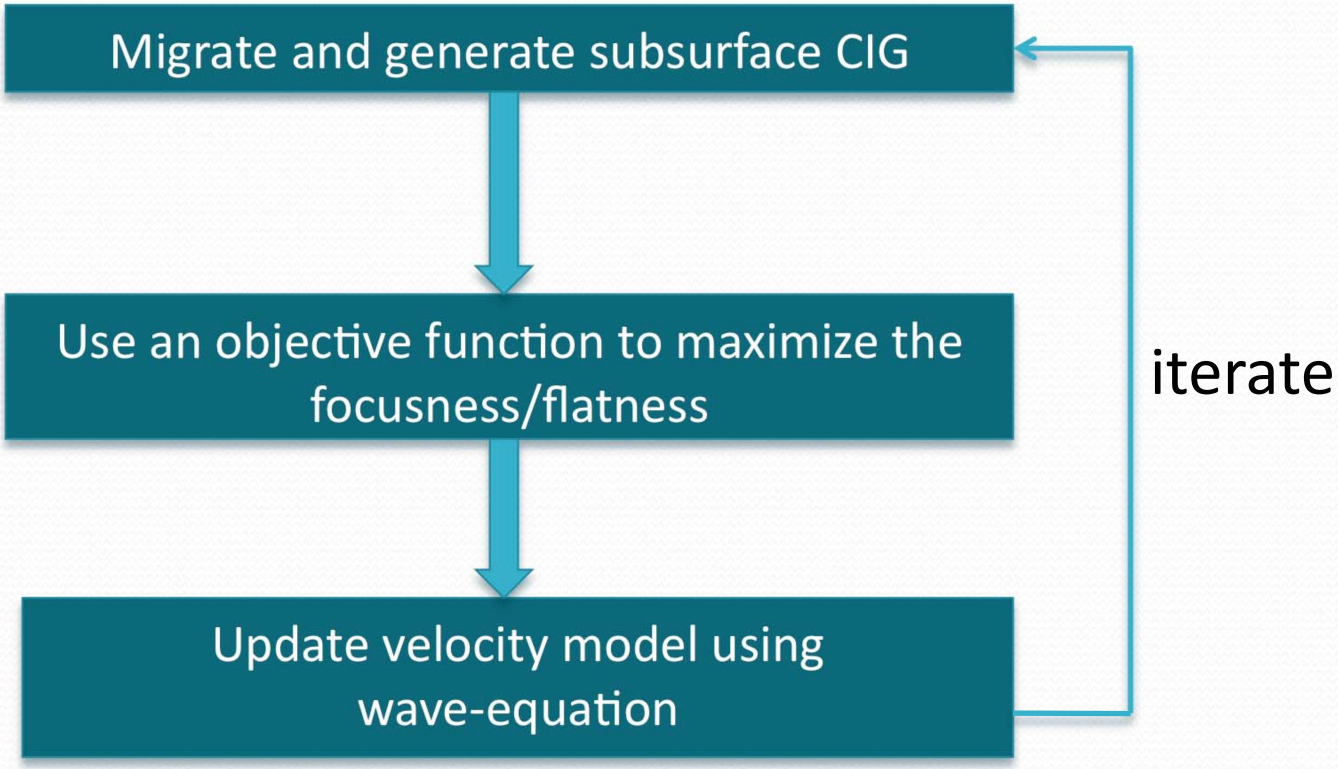


Textbook offset DSO

WEMVA introduction

- Wave-equation based velocity inversion
  - a method of reflection tomography
  - optimize the target in image domain
  - wave-equation based

General WEMVA flow



WEMVA concept

$$I \xrightarrow{J} \Delta I \xrightarrow{\mathbf{T}} \Delta s$$

J : image focusing/flattening objective function

ΔI : image perturbation

$\mathbf{T}$: traditional image space wave equation tomographic operator

Δs : slowness perturbation

Current WEMVA methods

- Maximum stack power
  - cycle-skipping (good local, but no global convergence)
- Differential Semblance Optimization (DSO)
  - create artifacts at angle-gather or reflector discontinuity (slow convergence)
- Sava(2004) constructs image perturbation using Stolt migration
  - ρ parameters need manual picking (non-trivial)

Current WEMVA methods

- Maximum stack power

$$J(s)=\sum_x \sum_z \left[\sum_\gamma I(x,z,\gamma;s) \right]^2$$

I : ADCIGs

γ : reflection angle

s : slowness field

Cycle-skipping

Current WEMVA methods

- Maximum stack power

$$J(s) = \sum_{z,x} \left[S_\gamma I(s) \right]^2$$

I : ADCIGs

γ : reflection angle

s : slowness field

Cycle-skipping

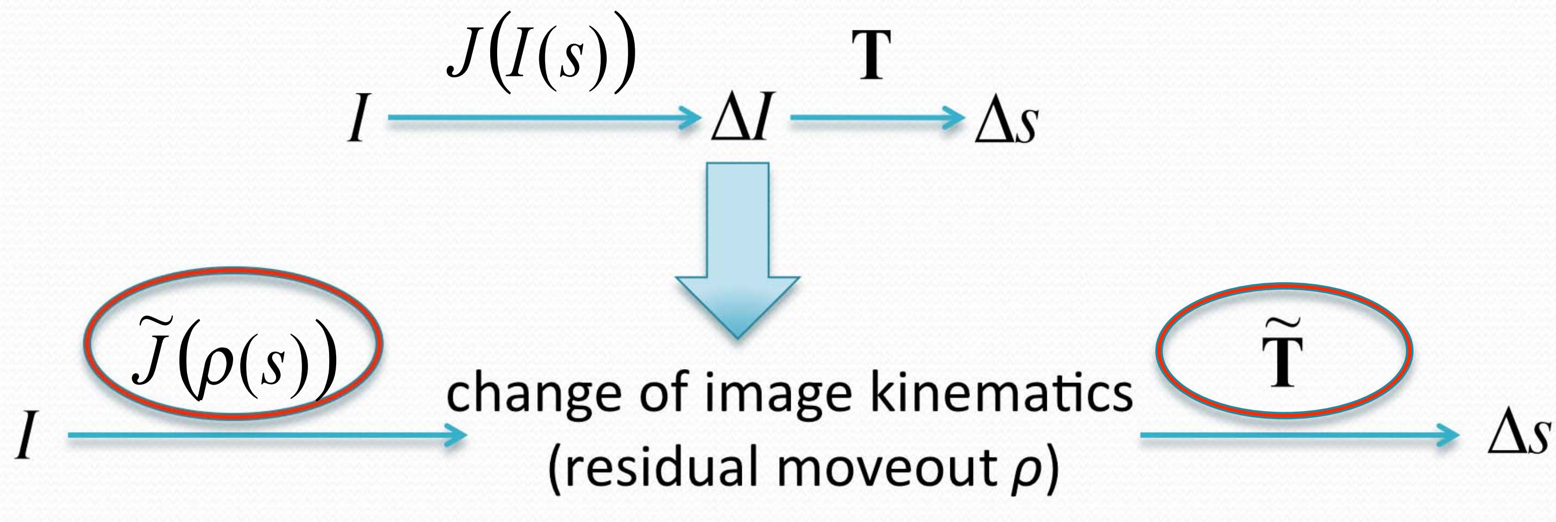
Our goals for WEMVA design

- Be robust against cycle-skipping (global convergence)
- Requires no manual picking
- Convergence faster (fewer DSO type artifacts)
- Be computationally affordable

Key idea

Combining the strength of ray tomography and wave-equation operators

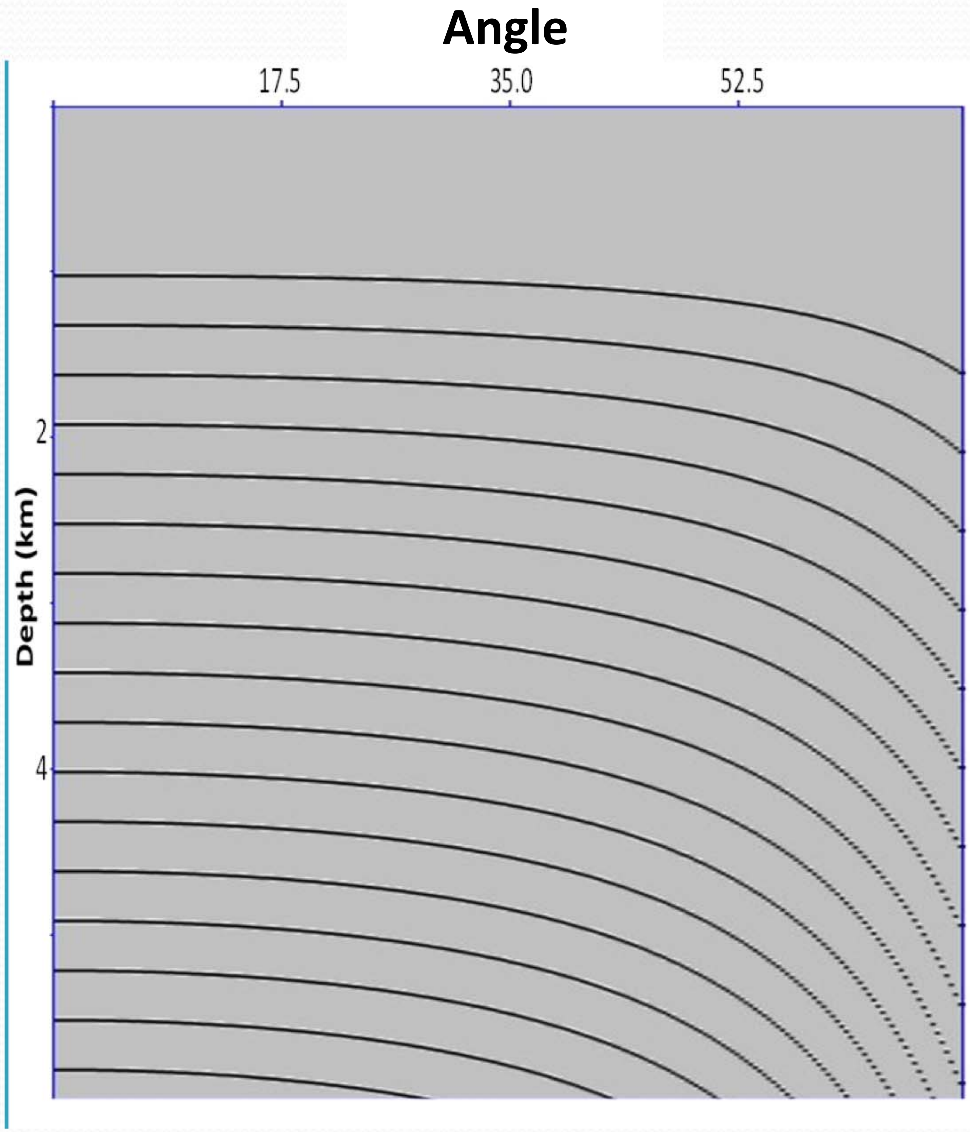
WEMVA flow modification



$\mathbf{T}$: Traditional image space WE tomographic operator

$\tilde{\mathbf{T}}$: New image space WE tomographic operator

Analytic moveout function



- ADCIG moveout with constant velocity error:

$$z = z_0 + \rho \tan^2 \gamma$$

γ : reflection angle

z_0 : depth at $\gamma = 0$

z : depth at angle γ

ρ : the moveout parameter

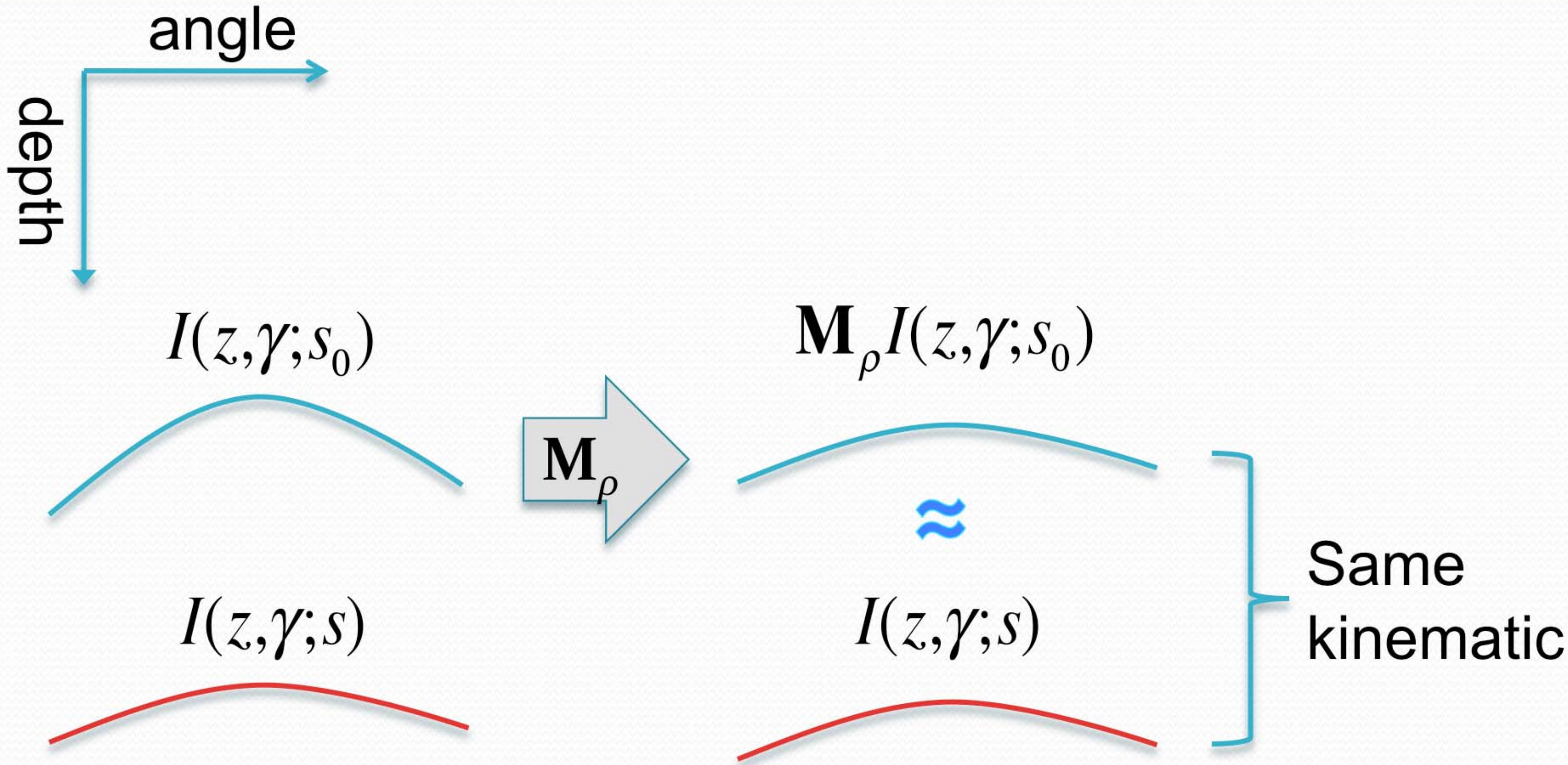
Introducing ρ into objective function

- Define the moveout operator

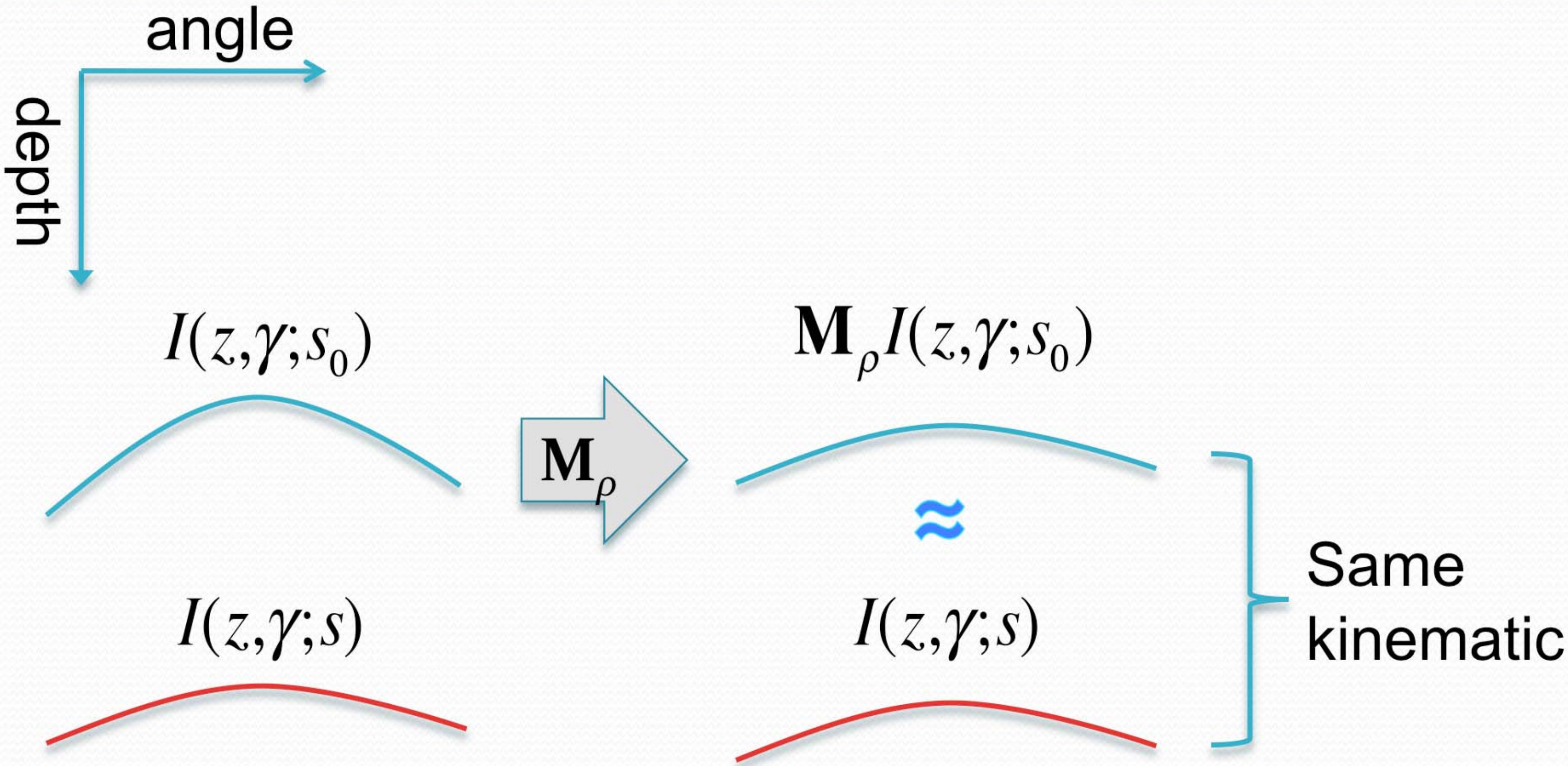
$$\mathbf{M}_{\rho}(I(z,\gamma)) = I(z + \rho \tan^2 \gamma, \gamma)$$

- Figure out ρ(s) relationship

$\rho(s)$


angle
depth
$I(z,\gamma;s_0)$
$\mathbf{M}_\rho$
$\mathbf{M}_\rho I(z,\gamma;s_0)$
≈
$I(z,\gamma;s)$
$I(z,\gamma;s)$
Same
kinematic

$\rho(s)$



$$J(s) = \sum_{z,x} \left[S_\gamma I(s) \right]^2 \quad \Rightarrow \quad J(\rho(s)) = \sum_{z,x} \left[S_\gamma \{ \mathbf{M}_\rho I(s_0) \} \right]^2$$

New objective function

$$J(\rho(s)) = \sum_{z,x} \left[S_\gamma \{ \mathbf{M}_\rho I(s_0) \} \right]^2$$

$$J_{S_m}(\rho(s)) = \frac{1}{2} \sum_{x,z} \frac{\left[S_\gamma \{ \mathbf{M}_\rho I(s_0) \} \right]^2}{S_\gamma \{ \mathbf{M}_\rho I^2(s_0) \}}$$

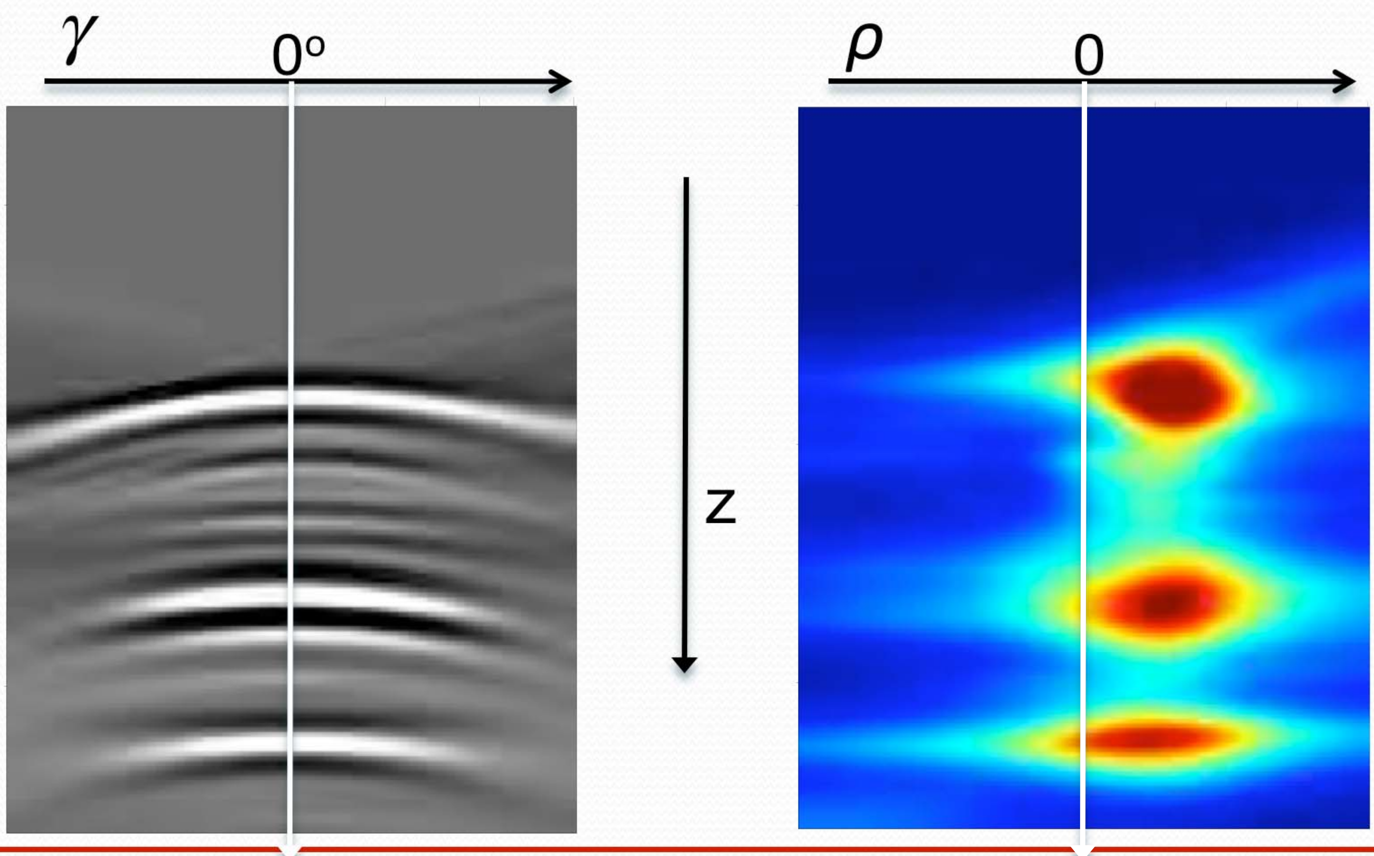
$$\text{semblance}: S_m(\rho(z,x), z, x)$$

Slowness gradient

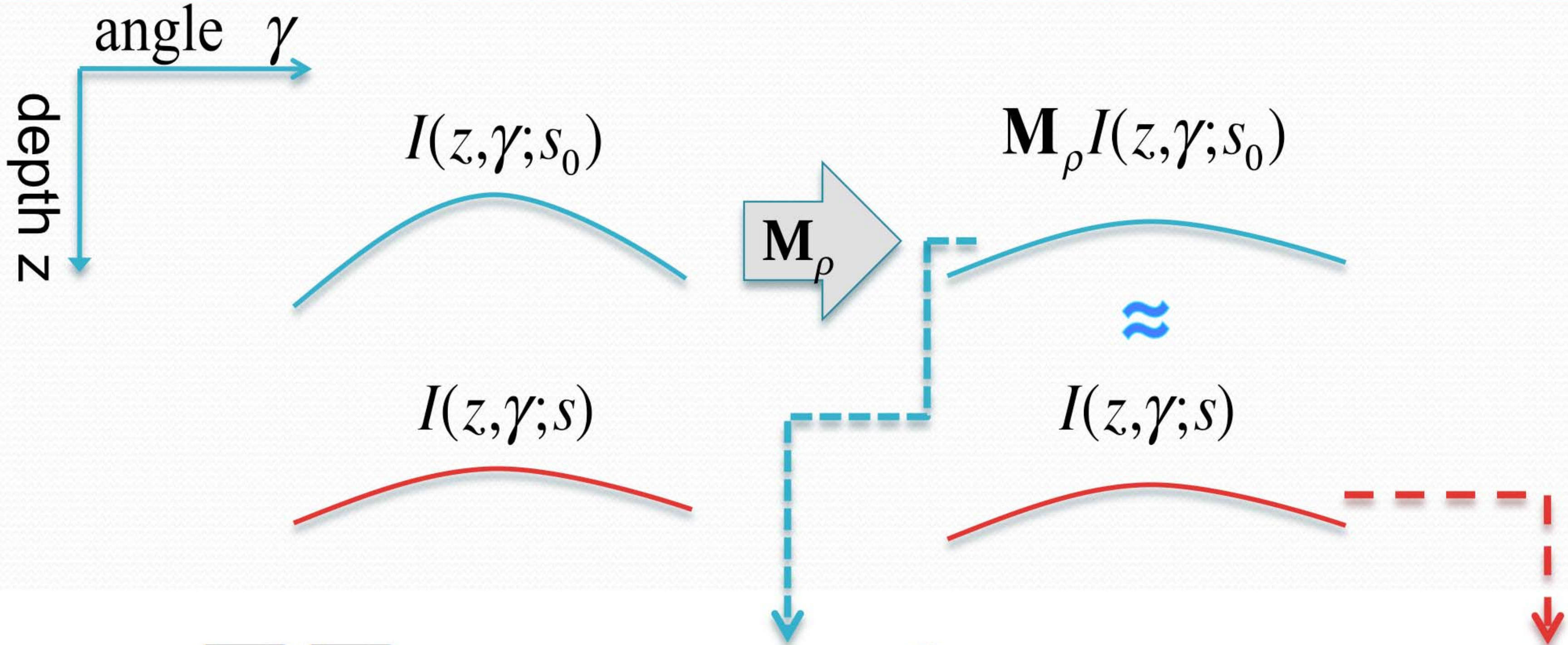
$$\frac{\partial J_{S_m}(\rho(s))}{\partial s} = \sum_{x,z} \frac{\partial S_m(\rho(s))}{\partial \rho} \frac{\partial \rho}{\partial s}$$

Slowness gradient

$$\frac{\partial J_{S_m}(\rho(s))}{\partial s} = \sum_{x,z} \frac{\partial S_m(\rho(s))}{\partial \rho} \frac{\partial \rho}{\partial s}$$



$\rho(s)$, $\beta(s)$



$$J_{\text{aux}} = \sum_{z_w} \sum_{\gamma} I(z + z_w + (\rho \tan^2 \gamma + \beta), \gamma, x; s_0) I(z, \gamma, x; s)$$

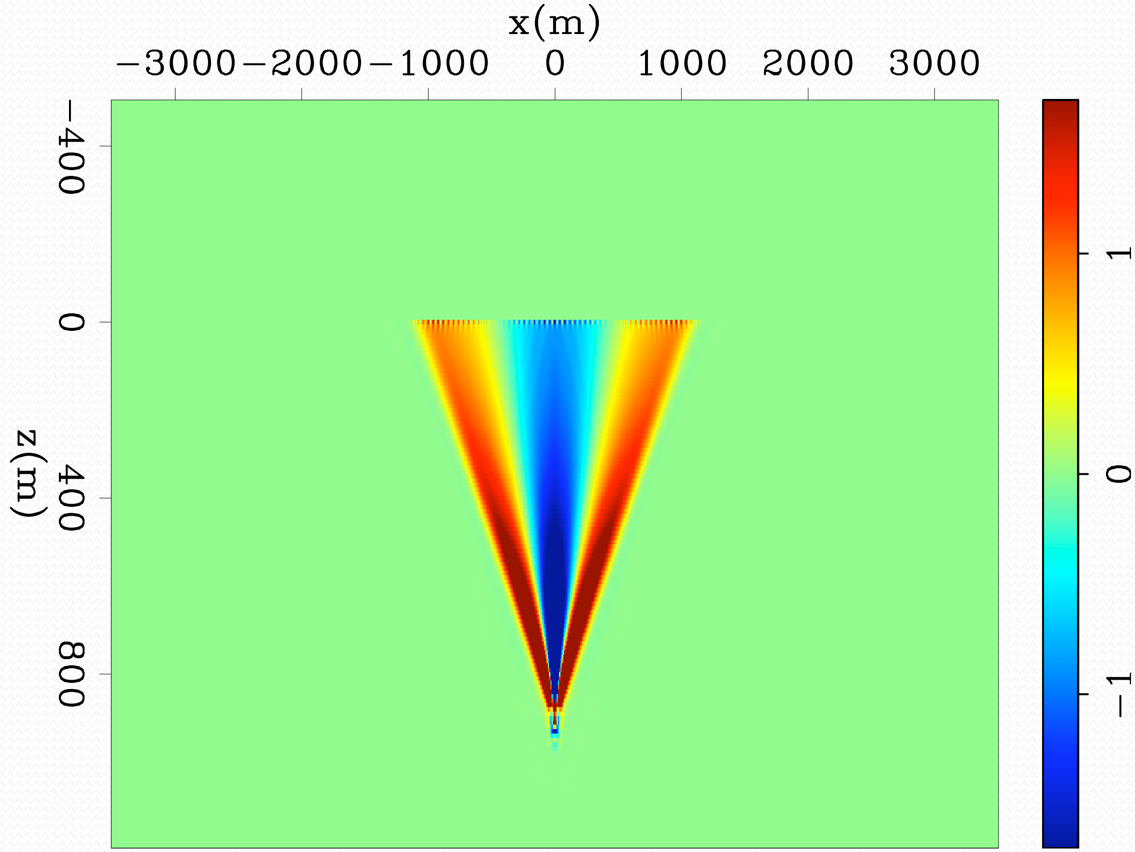
β : a static shift of the gathers caused by the slowness change

Solving $\frac{\partial \rho}{\partial s}$

$$\begin{cases} \frac{\partial J_{\text{aux}}}{\partial \rho} = 0 \\ \frac{\partial J_{\text{aux}}}{\partial \beta} = 0 \end{cases}$$

$$\begin{bmatrix} \frac{\partial^2 J_{\text{aux}}}{\partial \rho^2} & \frac{\partial^2 J_{\text{aux}}}{\partial \rho \partial \beta} \\ \frac{\partial^2 J_{\text{aux}}}{\partial \rho \partial \beta} & \frac{\partial^2 J_{\text{aux}}}{\partial \beta^2} \end{bmatrix} \begin{bmatrix} \frac{\partial \rho}{\partial s} \\ \frac{\partial \beta}{\partial s} \end{bmatrix} = - \begin{bmatrix} \frac{\partial J_{\text{aux}}}{\partial \rho \partial s} \\ \frac{\partial J_{\text{aux}}}{\partial \beta \partial s} \end{bmatrix}.$$

$\partial\rho / \partial s$ sensitivity kernel



Slowness perturbations at near angles and far angles would change ADCIG curvature in opposite directions

Slowness gradient

$$\frac{\partial J_{S_m}(\rho(s))}{\partial s} = \sum_{x,z} \frac{\partial S_m(\rho(s))}{\partial \rho} \frac{\partial \rho}{\partial s}$$

$$=$$

$$-\sum_{z_w} \sum_{\gamma} \sum_{z,x} \frac{\partial I(z+z_w,\gamma,x;s)}{\partial s} (F_{11} \tan^2 \gamma + F_{12}) \frac{\partial J_{Sm}}{\partial \rho}(z,x) \dot{I}(z+z_w,\gamma,x;s_0)$$

Slowness gradient

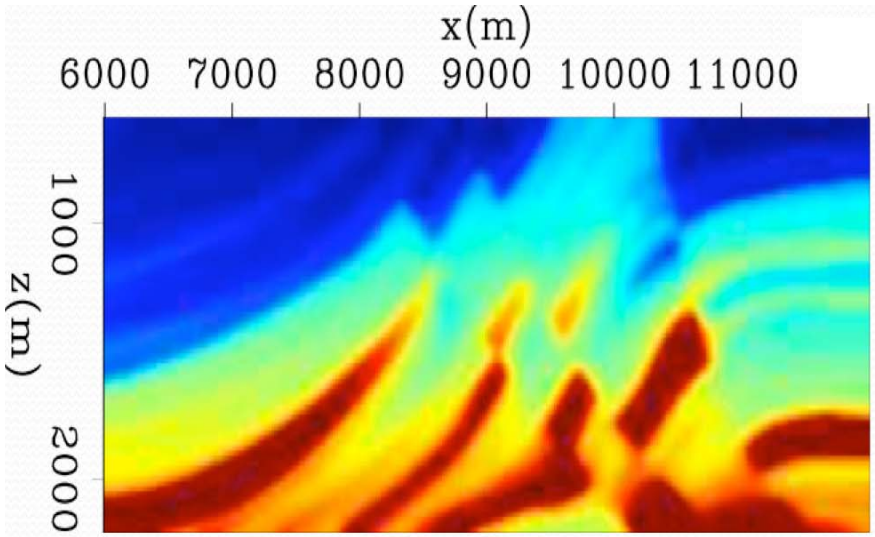
$$-\sum_{z_w}\sum_{\gamma}\sum_{z,x}\frac{\partial I(z+z_w,\gamma,x;s)}{\partial s}(F_{11}\tan^2\gamma+F_{12})\frac{\partial J_{Sm}}{\partial \rho}(z,x)\dot{I}(z+z_w,\gamma,x;s_0)$$

image-space wave equation tomographic operator

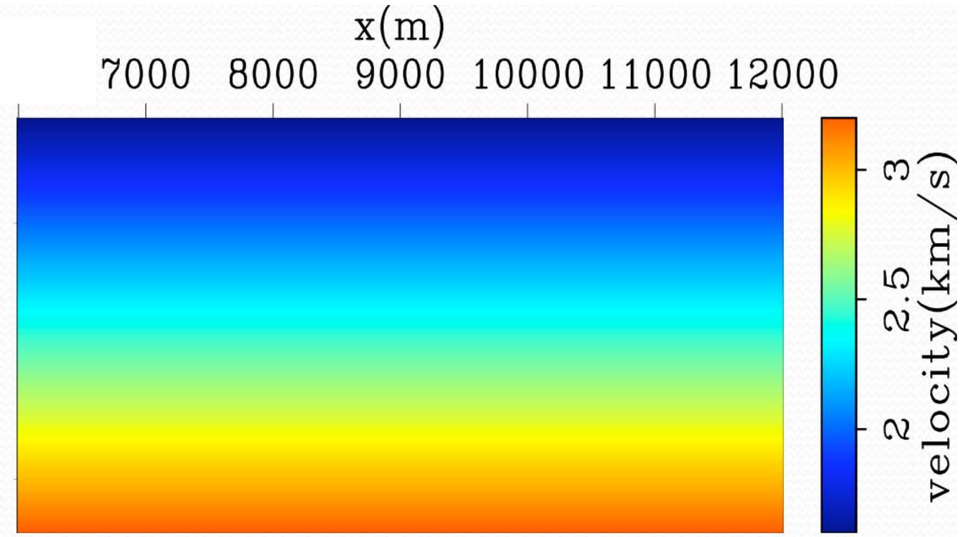
Slowness gradient

$$-\sum_{z_w}\sum_{\gamma}\sum_{z,x}\frac{\partial I(z+z_w,\gamma,x;s)}{\partial s}\boxed{(F_{11}\tan^2\gamma+F_{12})\frac{\partial J_{Sm}}{\partial \rho}(z,x)\dot{I}(z+z_w,\gamma,x;s_0)}$$

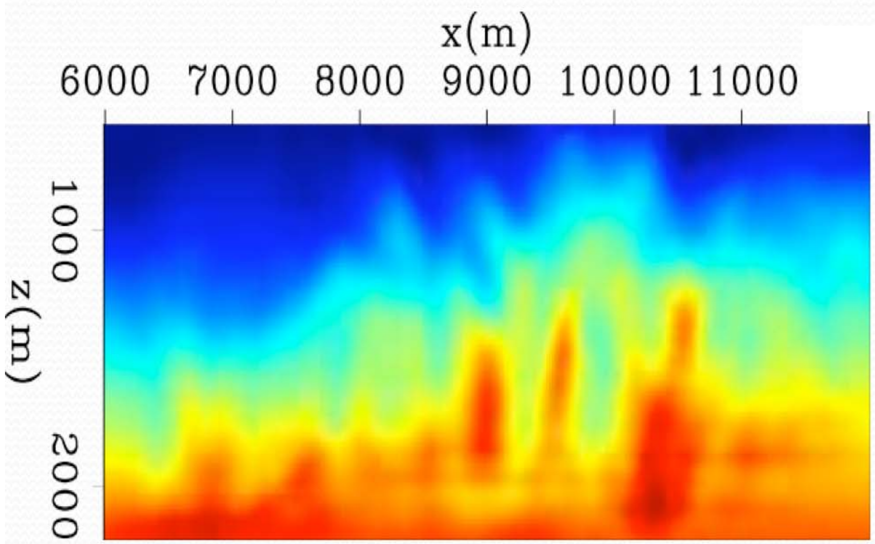
back projected residual image



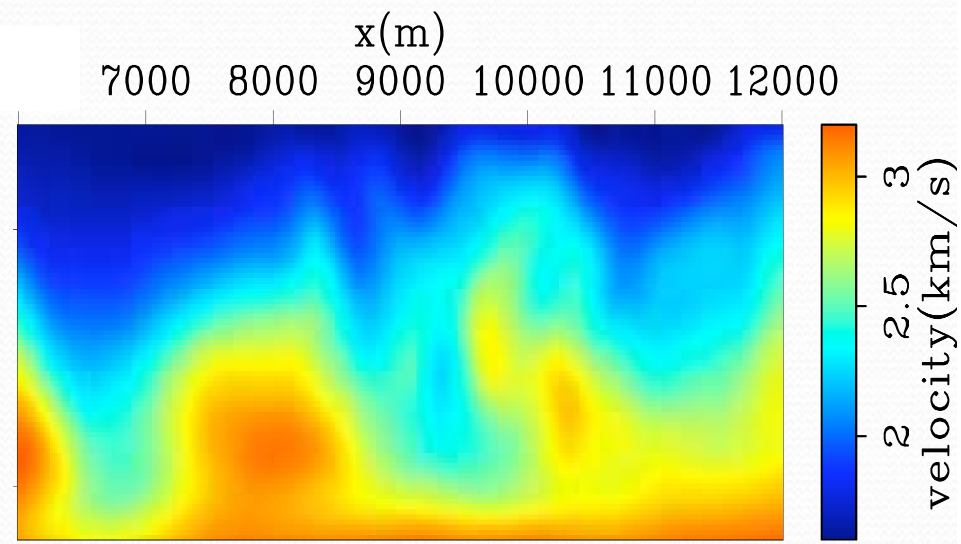
True model



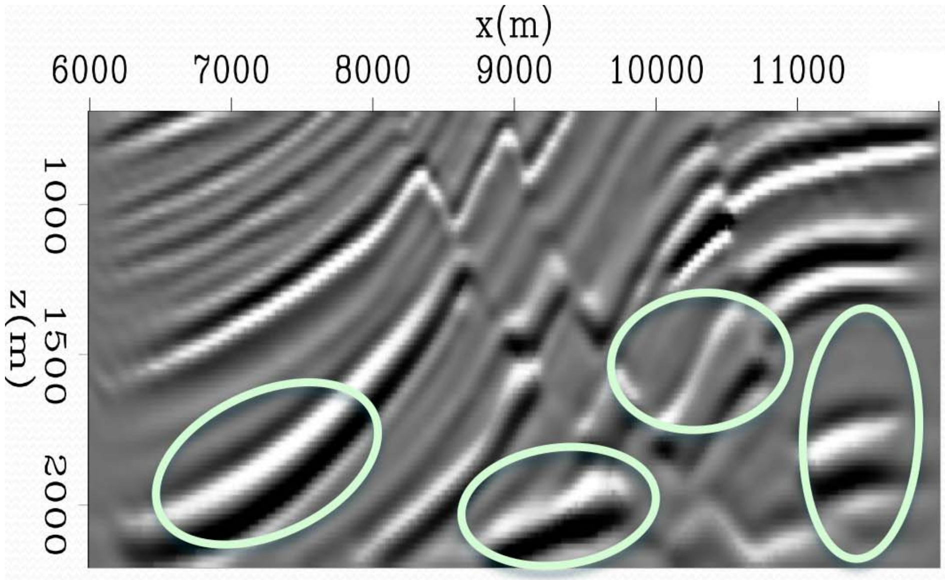
Starting model $v(z) = v_0+\alpha z$



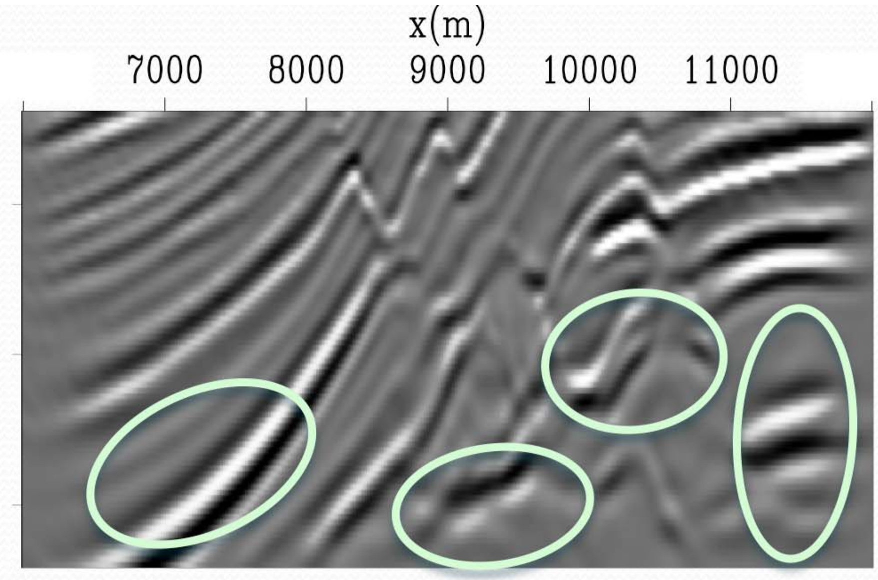
RMO



Textbook offset DSO



True model



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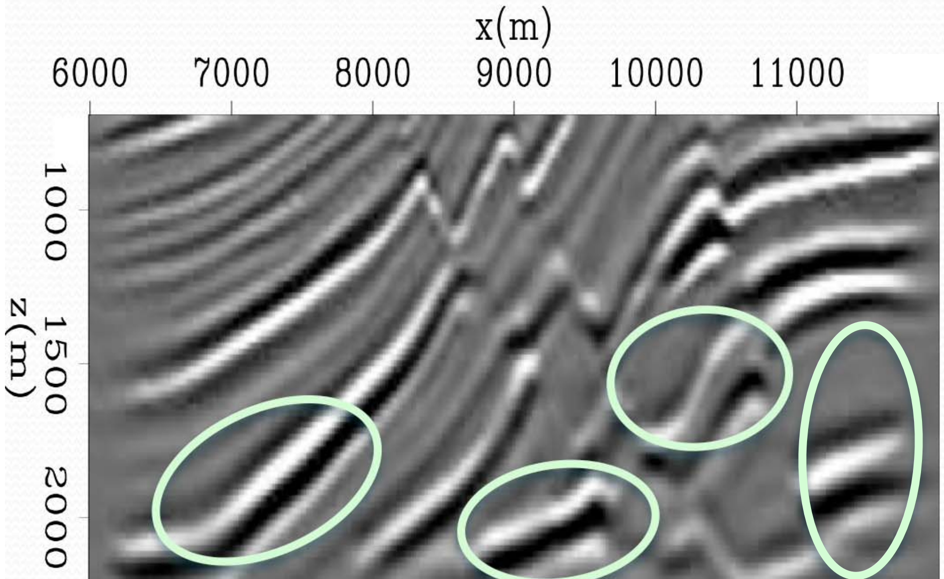
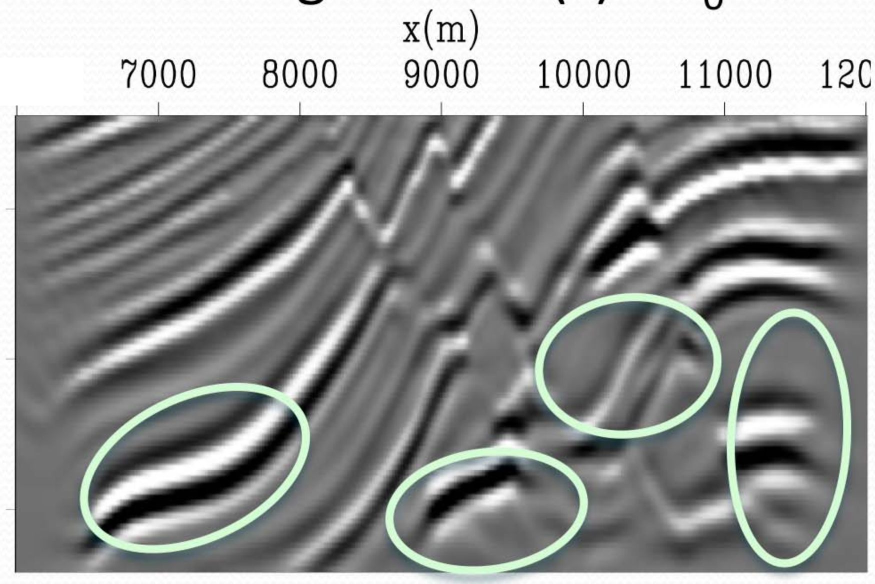
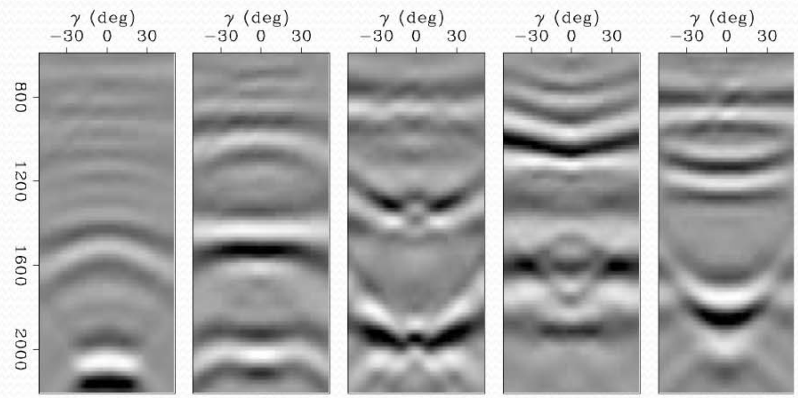


RMO

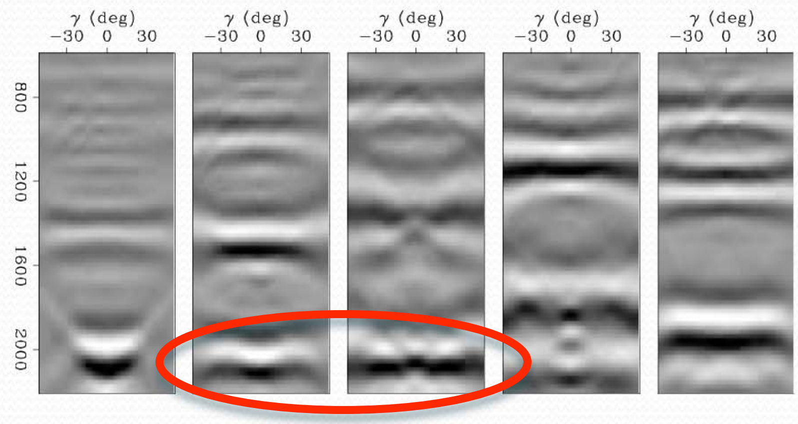


Textbook offset DSO

Angle gathers comparison



Initial model



RMO inverted

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Extend to 3D case (1): Add azimuth

- 3-D ADCIGs $I(z, \gamma, \phi, x, y)$
- Parameterization for 3D moveout surface.
  - Individual curvature for each azimuth: $\rho_i \leftrightarrow \phi_i$

$$\phi \in \{\phi_i : i = 1,2,...,m\}, \;\; \rho = \{\rho_i : i = 1,2,...,m\}$$

$$J_{S_m}(\rho(s)) = \frac{1}{2} \sum_{z,x,y} \sum_{\phi_i} S_m(\rho_i(s), z, \phi_i, x, y)$$

3-D gradient formula

$$\phi \in \{\phi_i : i = 1,2,\ldots,m\}, \;\; \rho = \{\rho_i : i = 1,2,\ldots,m\}$$

$$\frac{\partial J_{S_m}}{\partial s} = \sum_{\rho_i} \frac{\partial J_{S_m}}{\partial \rho_i} \frac{\partial \rho_i}{\partial s}$$

Extend to 3D case (2): $\partial\rho_i / \partial s$

$$J_{\mathrm{aux}} = \sum_{z_w} \sum_{\gamma} I(z + z_w + (\rho \tan^2 \gamma + \beta), \gamma, x; s_0) I(z, \gamma, x; s)$$

$$\Downarrow$$

$$J_{\mathrm{aux}} = \sum_{z_w} \sum_{\gamma} \sum_{\phi_i} I(z + z_w + (\rho_i \tan^2 \gamma + \beta), \gamma, \phi_i, x, y; s_0) I(z, \gamma, \phi_i, x, y; s)$$

We consider all azimuths simultaneously, because the static shift β is shared by all azimuths

$$\begin{bmatrix} \frac{\partial^2 J_{\text{aux}}}{\partial \rho^2} & \frac{\partial^2 J_{\text{aux}}}{\partial \rho \partial \beta} \\ \frac{\partial^2 J_{\text{aux}}}{\partial \rho \partial \beta} & \frac{\partial^2 J_{\text{aux}}}{\partial \beta^2} \end{bmatrix} \begin{bmatrix} \frac{\partial \rho}{\partial s} \\ \frac{\partial \beta}{\partial s} \end{bmatrix} = - \begin{bmatrix} \frac{\partial J_{\text{aux}}}{\partial \rho \partial s} \\ \frac{\partial J_{\text{aux}}}{\partial \beta \partial s} \end{bmatrix}.$$

Jacobian matrix **E**

$$\begin{bmatrix} \frac{\partial^2 J_{\text{aux}}}{\partial {\rho_1}^2} & \frac{\partial^2 J_{\text{aux}}}{\partial \rho_1 \partial \rho_2} & \cdots & \frac{\partial^2 J_{\text{aux}}}{\partial \rho_1 \partial \rho_m} & \frac{\partial^2 J_{\text{aux}}}{\partial \rho_1 \partial \beta} \\ \frac{\partial^2 J_{\text{aux}}}{\partial \rho_2 \partial \rho_1} & \frac{\partial^2 J_{\text{aux}}}{\partial {\rho_2}^2} & \cdots & \frac{\partial^2 J_{\text{aux}}}{\partial \rho_2 \partial \rho_m} & \frac{\partial^2 J_{\text{aux}}}{\partial \rho_2 \partial \beta} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 J_{\text{aux}}}{\partial \rho_m \partial \rho_1} & \frac{\partial^2 J_{\text{aux}}}{\partial \rho_m \partial \rho_2} & \cdots & \frac{\partial^2 J_{\text{aux}}}{\partial {\rho_m}^2} & \frac{\partial^2 J_{\text{aux}}}{\partial \rho_m \partial \beta} \\ \frac{\partial^2 J_{\text{aux}}}{\partial \beta \partial \rho_1} & \frac{\partial^2 J_{\text{aux}}}{\partial \beta \partial \rho_2} & \cdots & \frac{\partial^2 J_{\text{aux}}}{\partial \beta \partial \rho_m} & \frac{\partial^2 J_{\text{aux}}}{\partial \beta^2} \end{bmatrix} \begin{bmatrix} \frac{\partial \rho_1}{\partial s} \\ \frac{\partial \rho_2}{\partial s} \\ \vdots \\ \frac{\partial \rho_m}{\partial s} \\ \frac{\partial \beta}{\partial s} \end{bmatrix} = - \begin{bmatrix} \frac{\partial^2 J_{\text{aux}}}{\partial \rho_1 \partial s} \\ \frac{\partial^2 J_{\text{aux}}}{\partial \rho_2 \partial s} \\ \vdots \\ \frac{\partial^2 J_{\text{aux}}}{\partial \rho_m \partial s} \\ \frac{\partial^2 J_{\text{aux}}}{\partial \beta \partial s} \end{bmatrix}$$

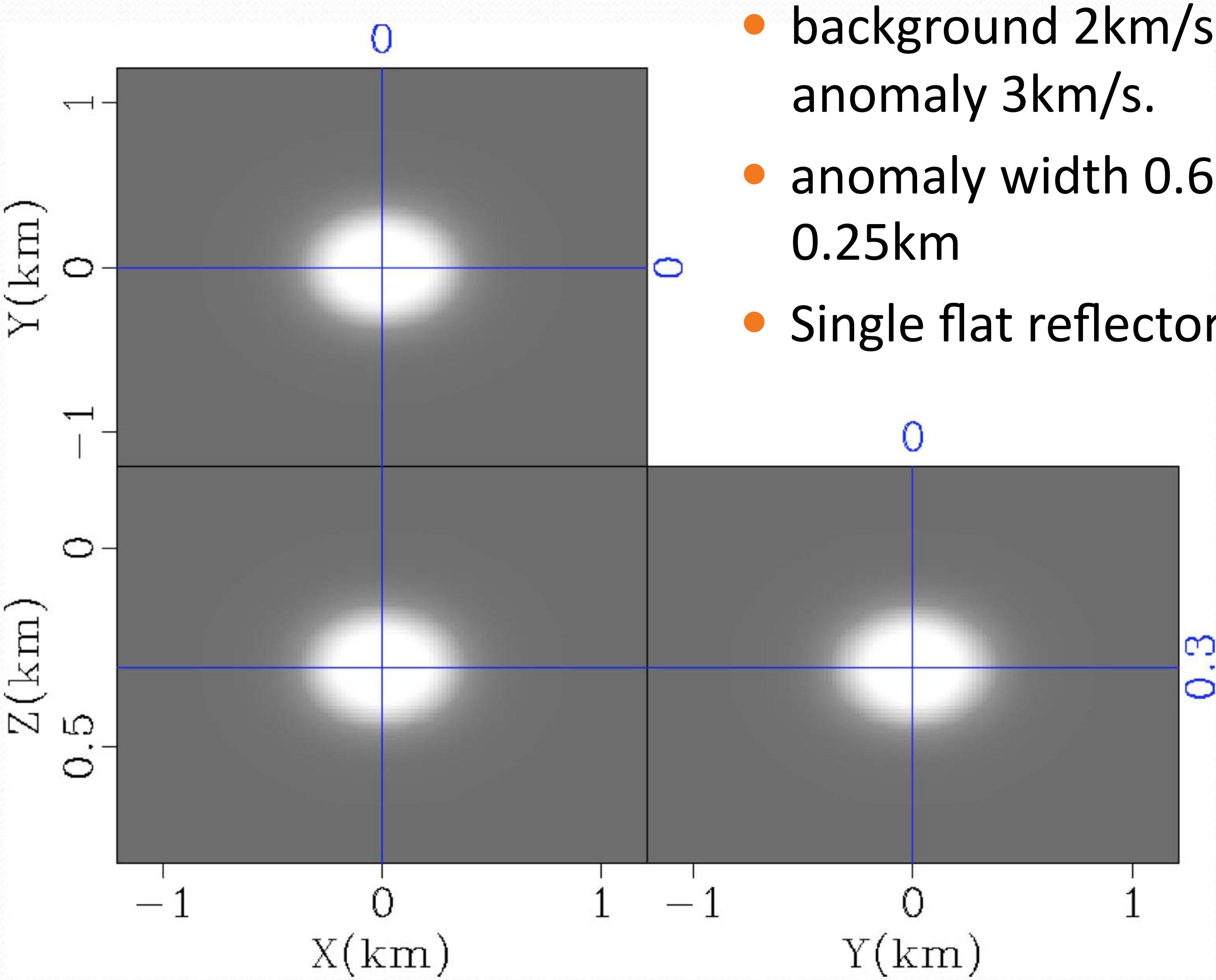
3-D gradient

$$-\sum_{z_w}\sum_{\gamma}\sum_{\phi_i}\frac{\partial I(z+z_w,\gamma,\phi_i,x,y;s)}{\partial s}\boxed{(G_i\tan^2\gamma+G_{m+1})\dot{I}(z+z_w,\gamma,\phi_i,x,y;s_0)}$$

Gaussian anomaly example, basic parameters

- model 2.4km X 2.4km X 1.0km (x,y,z)
- sampling 20m X 20m X 10m (x,y,z)
- fixed receivers, both shots and receivers span (-0.8km,+0.8km) in both x and y direction
  - receiver spacing 20m X 20m,   81 X 81 in total
  - source spacing 100m X 100m, 17 X 17 in total
- one way propagator is used, frequency from 5Hz to 40Hz, 32 frequencies with equal spacing

True velocity model (shown in velocity)

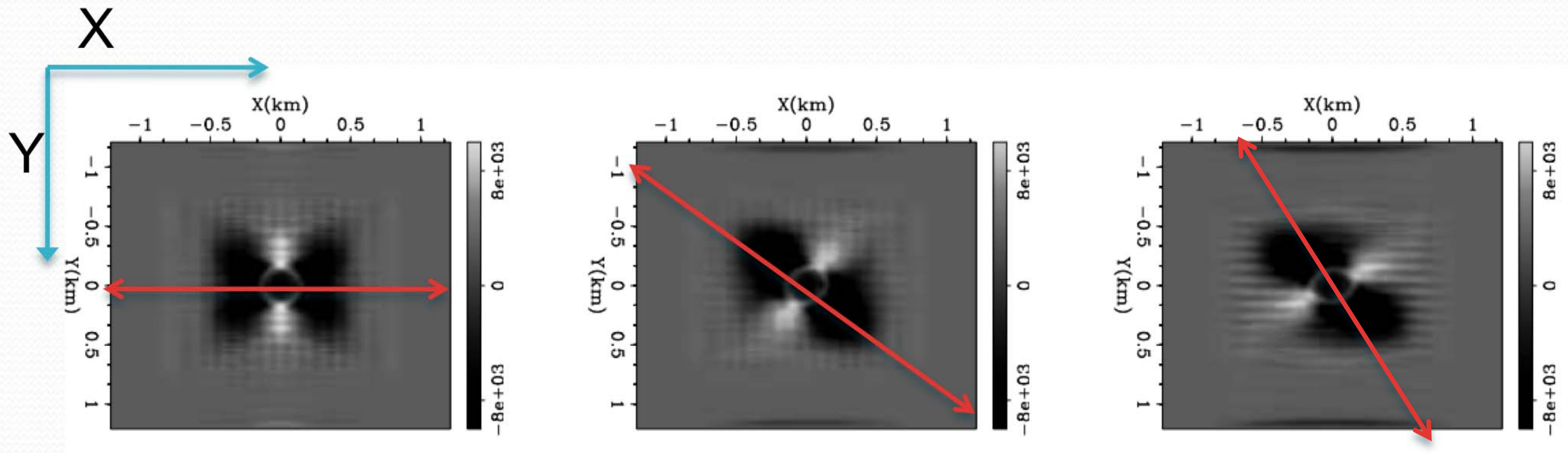


- background 2km/s, center of anomaly 3km/s.
- anomaly width 0.6km, height 0.25km
- Single flat reflector at z=0.6km

Initial migration

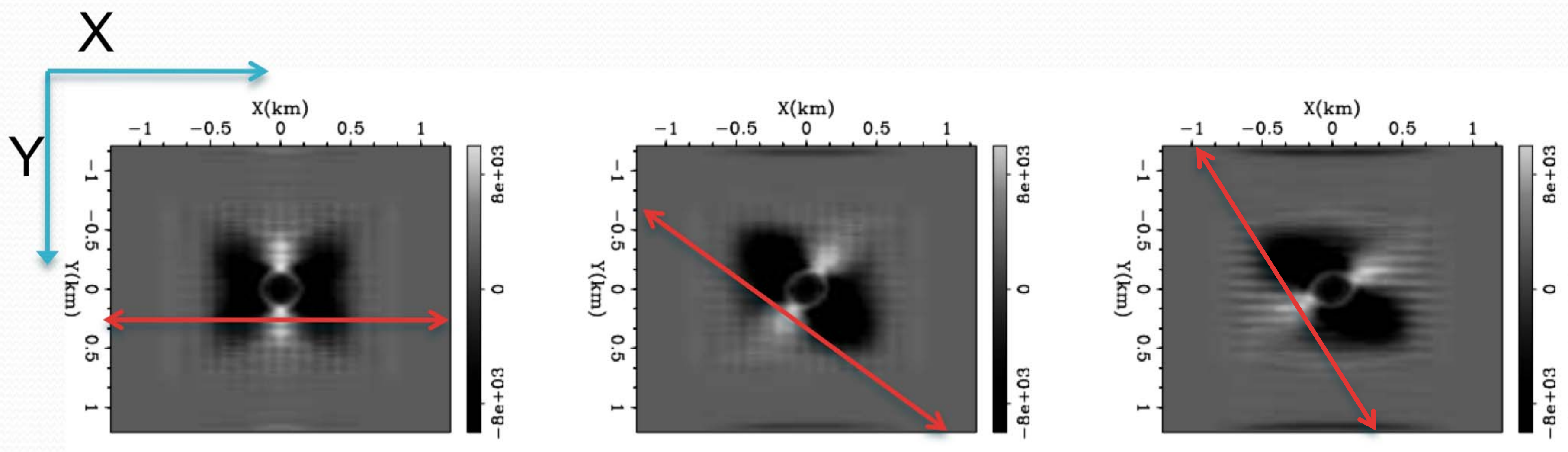
- Starting velocity model, 2km/s constant
- Reflection angle range: [-60º,+60º], spacing 5º .
- Compute only 3 azimuths: 0º, 30º, 60º.

$\partial J_{S_m}(\rho_i)/\partial\rho_i$



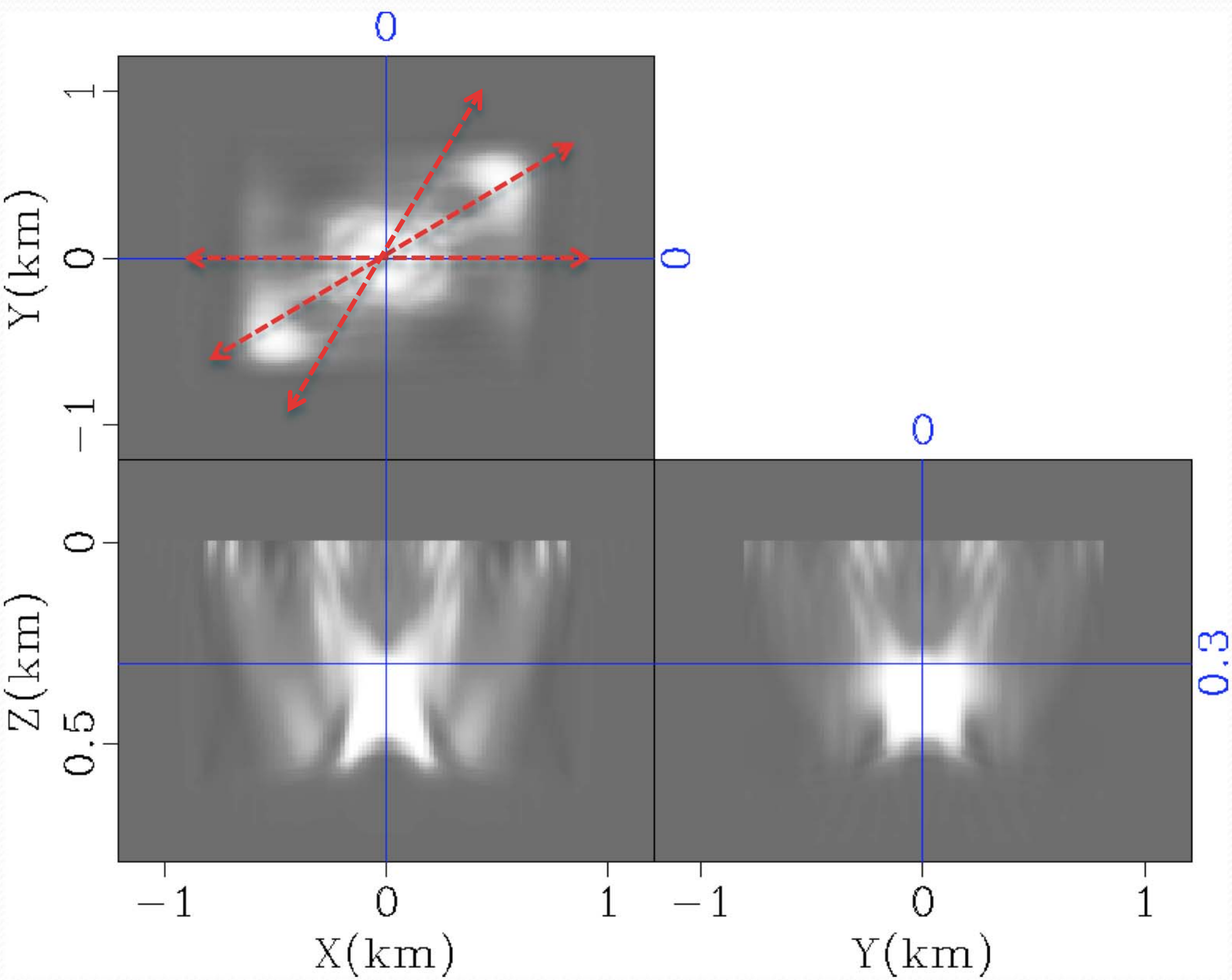
$\phi = 0°$ $\phi = 30°$ $\phi = 60°$

$\partial J_{S_m}(\rho_i)/\partial\rho_i$



$\phi = 0°$ $\phi = 30°$ $\phi = 60°$

First gradient (shown in velocity)



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Subsurface offset vs. angle

$$\frac{\partial J}{\partial s} = -\sum_{z_w}\sum_{\gamma}\sum_{\phi_i}\frac{\partial I(z+z_w,\gamma,\phi_i,x,y;s)}{\partial s}(G_i\tan^2\gamma + G_{m+1})\dot{I}(z+z_w,\gamma,\phi_i,x,y;s_0)$$

- Subsurface offset domain tomography operator is easier for computer implementation

The transform of offset -> angle 3D CIGs

1. Perform Fourier transform $I(hx, hy, x, y, z) \rightarrow I(hx, hy, k_x, k_y, k_z)$.

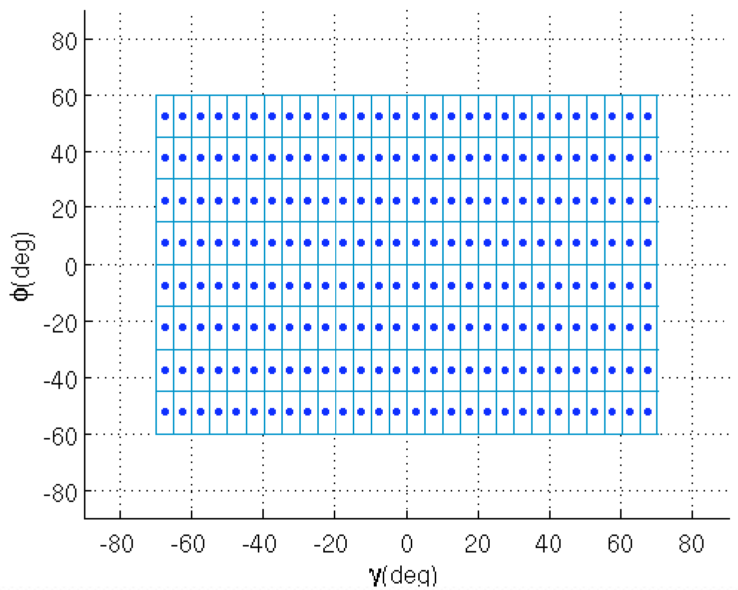
2. For each (k_x, k_y, k_z),

   - apply Fourier transform $I(hx, hy) \rightarrow I(k_{hx}, k_{hy})$
   - map $I(k_{hx}, k_{hy}) \rightarrow I(\gamma, \phi)$ based on the following relations (Tisserant and Biondi, 2003):

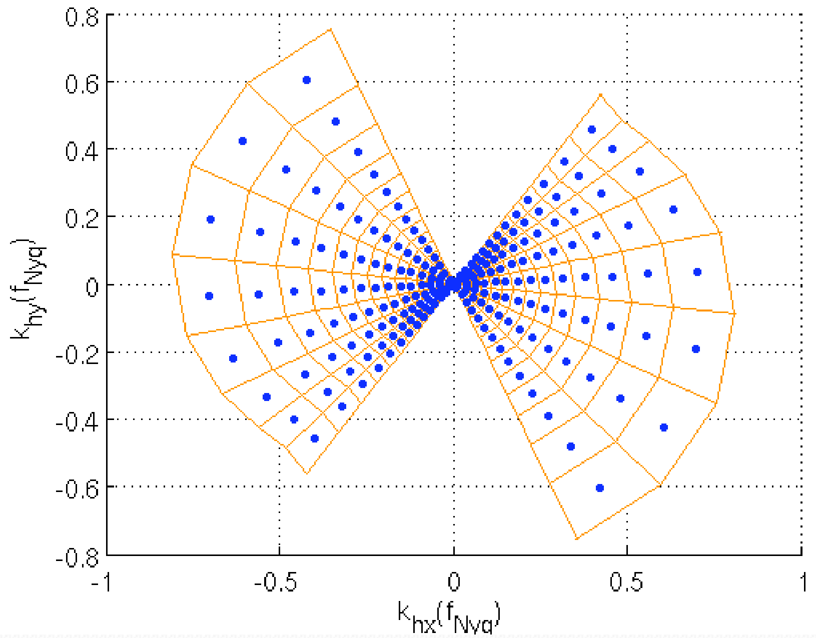
$$
\begin{aligned}
\begin{bmatrix} k'_x \\ k'_y \end{bmatrix} &= \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} k_x \\ k_y \end{bmatrix} \\
k'_{hx} &= k_z \sqrt{1 + (k'_y/k_z)^2} \tan\gamma \\
k'_{hy} &= \frac{k'_y k'_x k'_{hx}}{k'^2_y + k^2_z} \\
\begin{bmatrix} k_{hy} \\ k_{hy} \end{bmatrix} &= \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} k'_{hx} \\ k'_{hy} \end{bmatrix}.
\end{aligned}
$$

3. Apply inverse Fourier transform $I(\gamma, \phi, k_x, k_y, k_z) \rightarrow I(\gamma, \phi, x, y, z)$.

Angle to offset mapping, irregularity



(γ, ϕ)



(k_{hx}, k_{hy})

Conclusion

- The extension of RMO WEMVA to three dimensions is feasible in both theory and practice
- parameterization of the 3-D angle domain CIG moveout
    - single-parameter, more robust but not accurate for complex moveout
    - multi-parameter (Biondi,SEP—145) remains area of investigation
- 3-D offset↔angle transforms need careful handling
- The ranking of the computational cost

1) tomographic operator   2) imaging operator
3) offset ↔ angle transform   4) semblance calculation

Acknowledgment

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