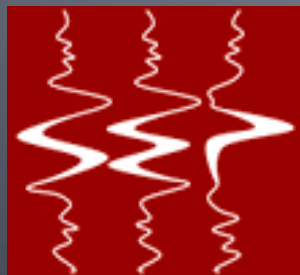


Tomographic full waveform inversion by successive linearization and scale separation



Ali Almomin*, Biondo Biondi
Stanford Exploration Project



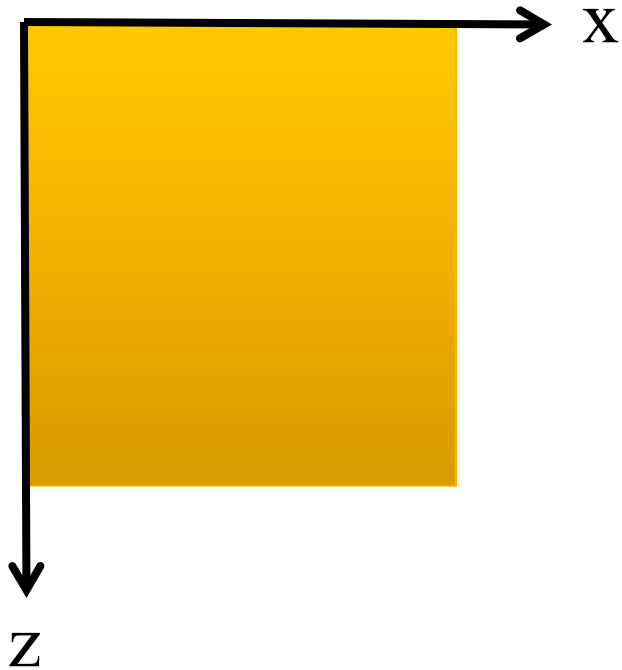
Outline

- Introduction
- Data space extension
- Nested inversion scheme
- Synthetic examples
- Conclusions

Simultaneous scale inversion

- Full waveform inversion (FWI):
 - High resolution results because all scales are inverted simultaneously
 - Local minima
- WE tomography followed by LS migration
 - Lower resolution and accuracy results
 - Global convergence

Extended velocity

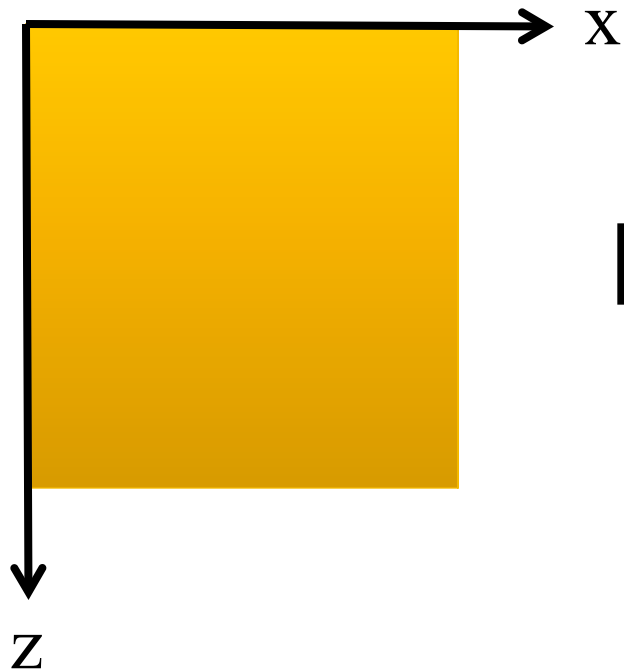


$$v(z,x)$$

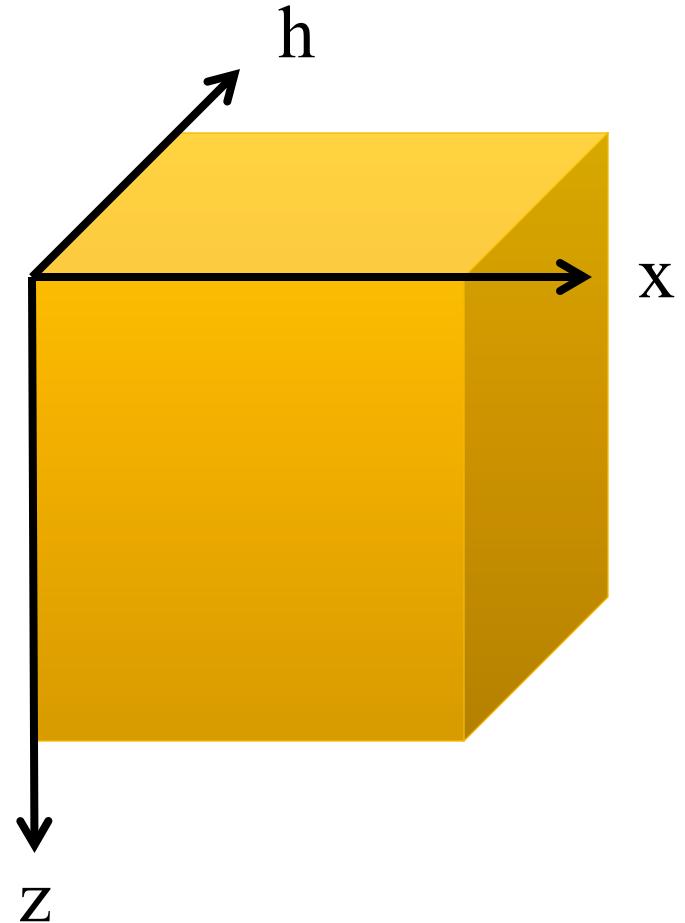
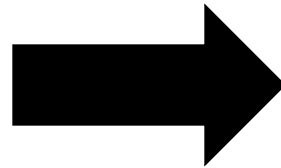
Extended velocity

(Symes, 2008)

(Biondi and Almomin, 2012)



$v(z,x)$



$v(z,x,h)$

Tomographic FWI

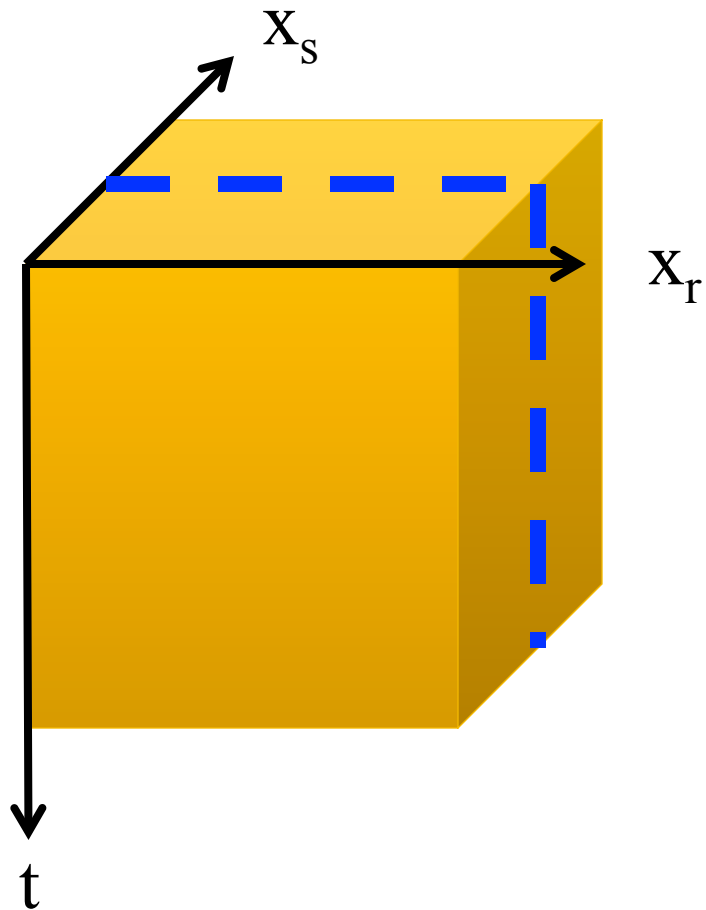
- Tomographic FWI (TFWI) objective function:

$$J(\mathbf{v}(\mathbf{q})) = \left\| \mathcal{L}(\mathbf{v}(\mathbf{q})) - \mathbf{d}_{\text{obs}} \right\| + \left\| \mathbf{E}(\mathbf{v}(\mathbf{q})) \right\|$$

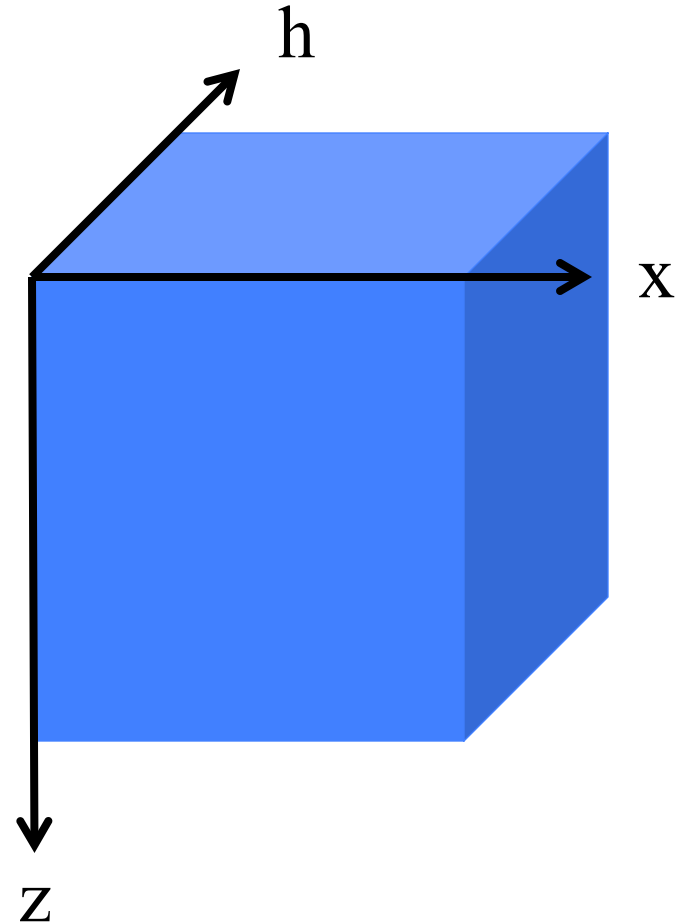
- High resolution
- Energy slowly moves towards zero offset
- Very expensive (and converges slowly)

\mathcal{L} = Extended modeling \mathbf{E} = Enhancing op
 \mathbf{q} = extension axes(\mathbf{h} or $\boldsymbol{\tau}$)

Extended velocity



$$d(t, x_r, x_s)$$



$$v(z, x, h)$$

Cost comparison

Operator	Propagation	Scatter/Image
FWI	10	1
WEMVA	10	1000
TFWI	10000	1000

FWI: Full waveform inversion

WEMVA: Wave-equation migration velocity analysis

TFWI: Tomographic full waveform inversion

Extended model error

- When the model is not correct, a certain behavior with subsurface offset is observed
 - The smooth components (long wavelength) are located mostly around the zero offset
 - The rough components (short wavelength) extend to large offsets

Scale separation

- Separate the velocity into two components:

$$\mathbf{v}(\mathbf{q}) = \mathbf{b}(\mathbf{q} = 0) + \mathbf{p}(\mathbf{q})$$

- \mathbf{b} = background (long wavelength)
 - Affects propagation
- \mathbf{p} = perturbation (short wavelength)
 - Affects scattering/imaging

Linearized TFWI

- LTFWI objective function (SEP147):

$$J(\mathbf{b}, \mathbf{p}) = \|\mathbf{L}(\mathbf{b}) \mathbf{p} - \mathbf{d}_{obs}\| + \|\mathbf{E} \mathbf{p}\|$$

- LTFWI gradients:

$$\frac{\partial J}{\partial \mathbf{p}} = \mathbf{L}^* \Delta \mathbf{d}$$

$$\frac{\partial J}{\partial \mathbf{b}} = \mathbf{T}^* \Delta \mathbf{d}$$

\mathbf{L} = Born modeling op

\mathbf{T} = Tomographic op

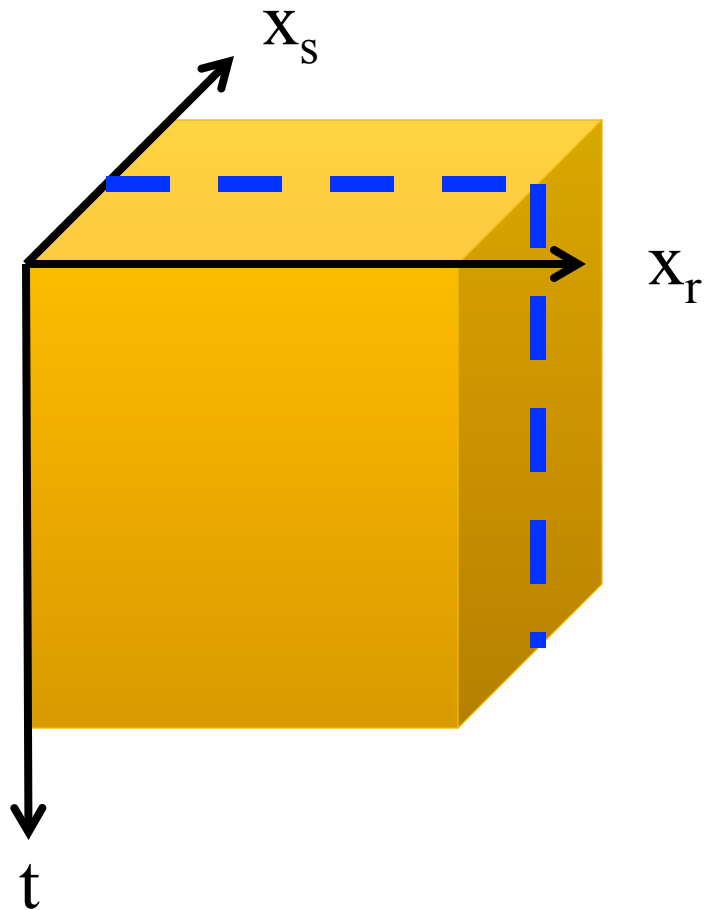
Limitations

- Uses the Born operator for modeling
 - Assumes data to contain primaries only
 - Cannot handle multiples
- Outputs two models
 - Scale mixing becomes sensitive to the separation parameter
 - Combining the outputs might not be trivial

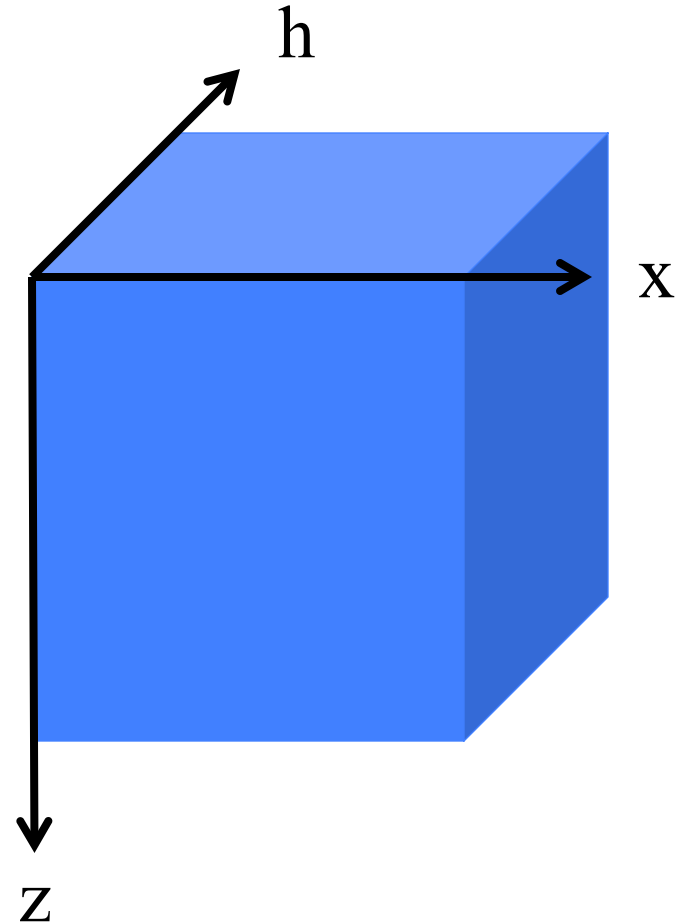
Outline

- Introduction
- Data space extension (SEP148)
- Nested inversion scheme
- Synthetic examples
- Conclusions

Extended velocity

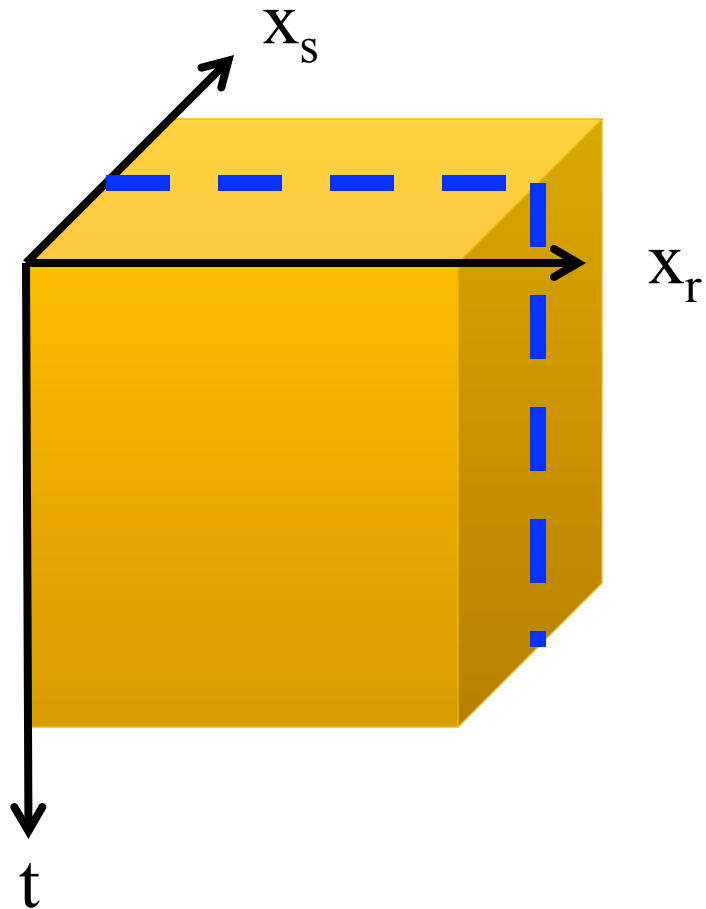


$$d(t, x_r, x_s)$$

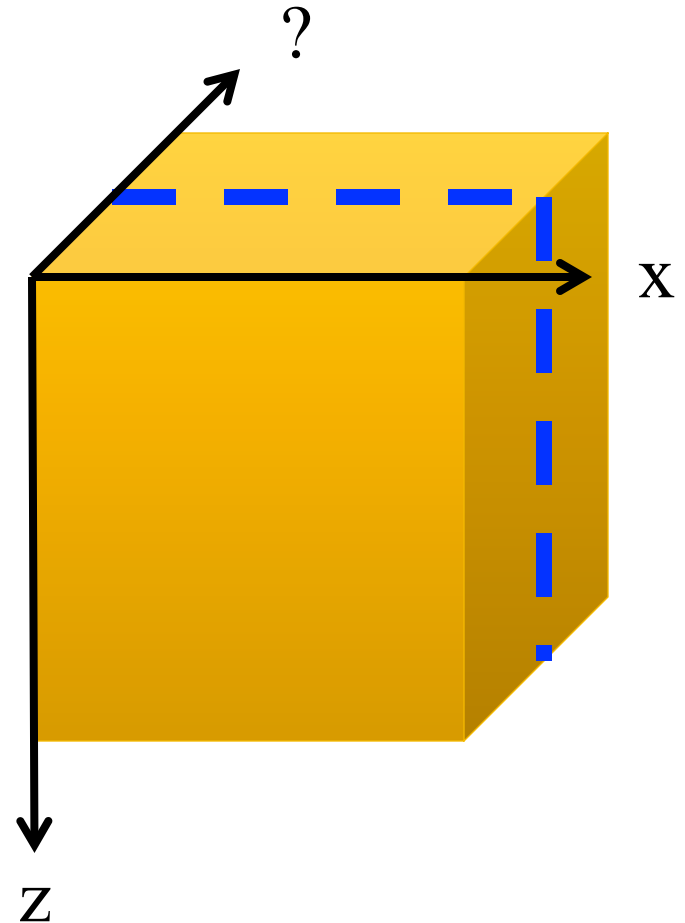


$$v(z, x, h)$$

Extended velocity



$$d(t, x_r, x_s)$$



$$v(z, x, ?)$$

Data space extension

- Extend model over a data space axis:
 - Surface decomposition
 - Model components become independent
 - Cost becomes the same as FWI
- Possible data space extension axes:
 - Source location
 - Source ray parameter

SX

SX

SX

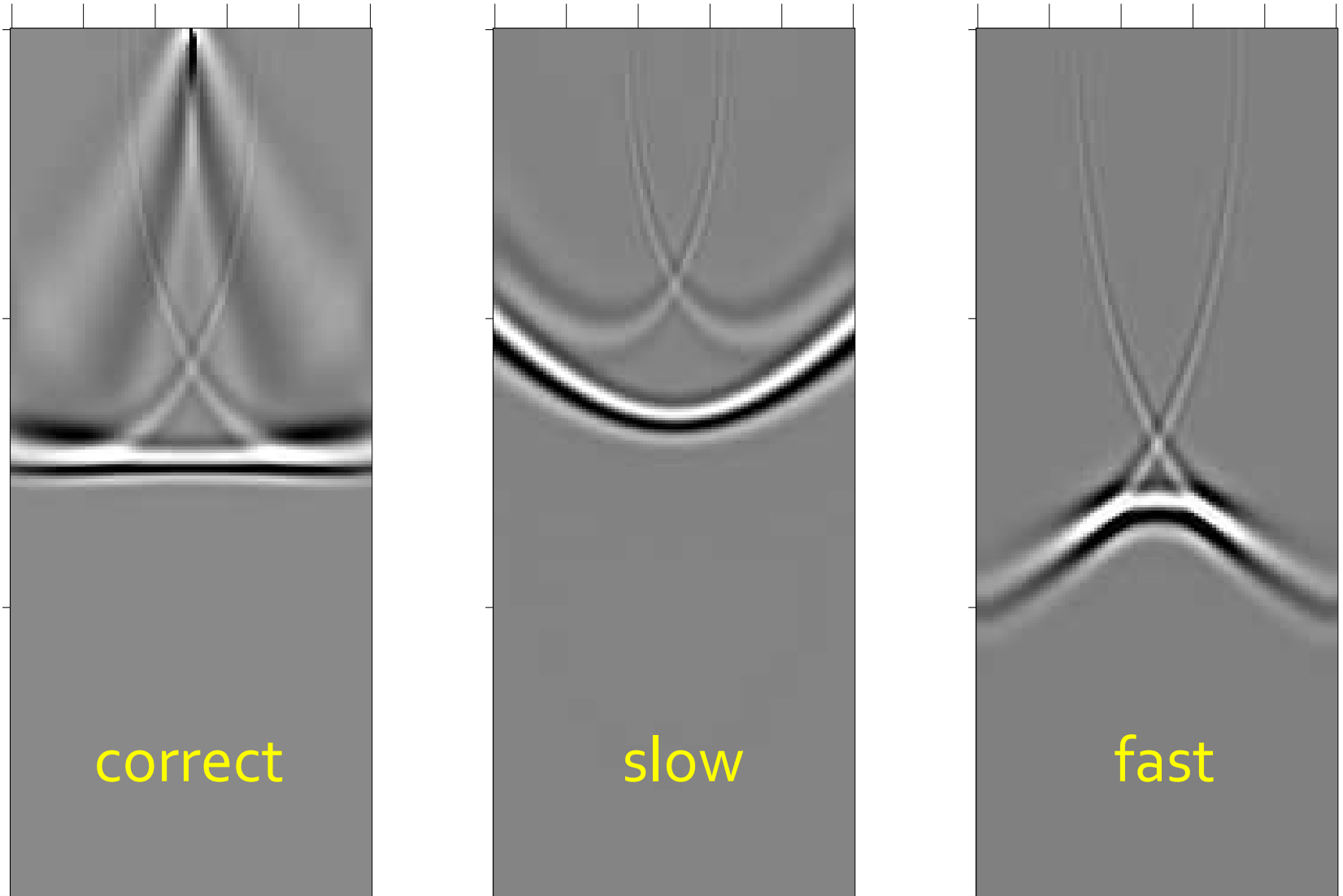
z

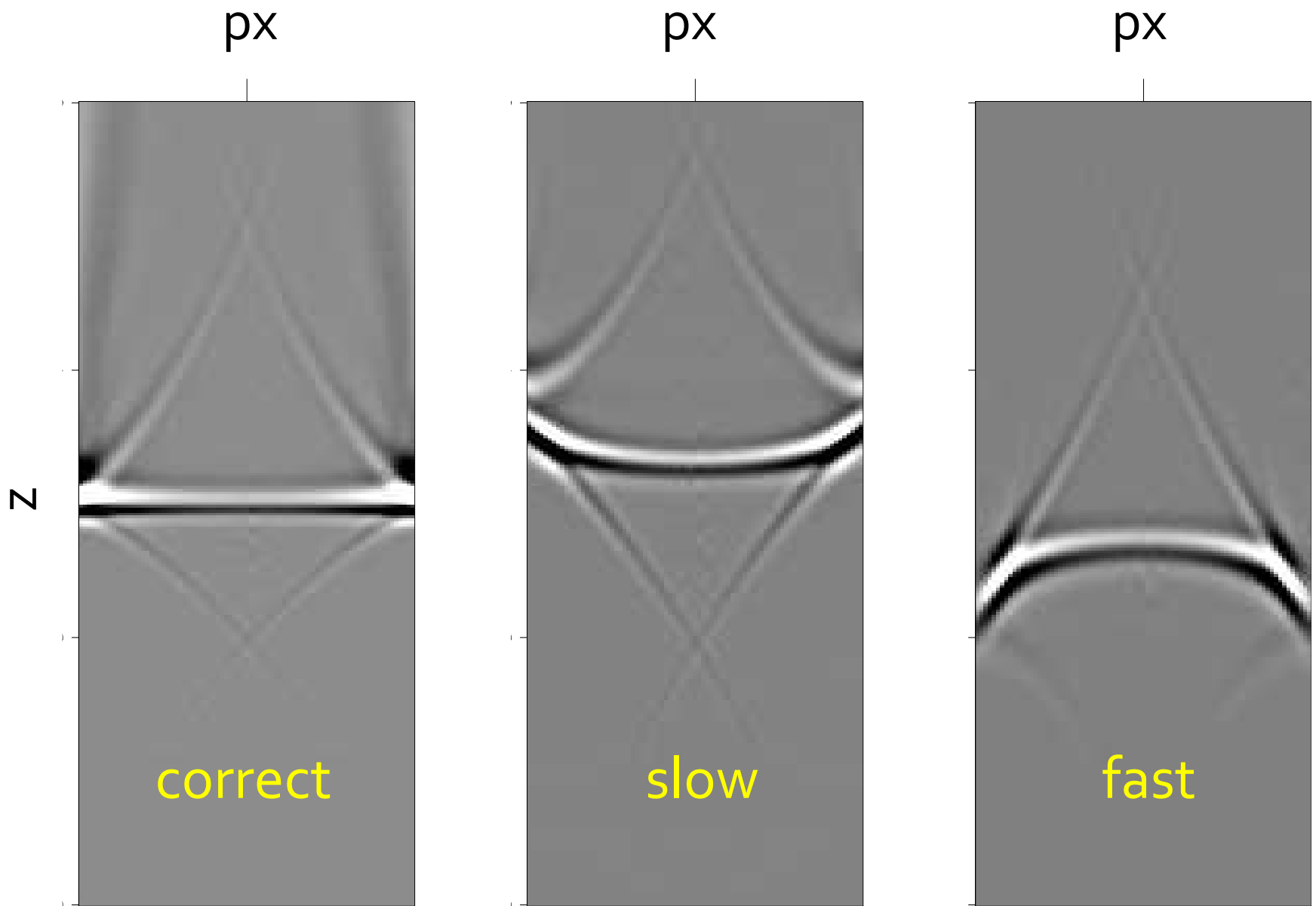
correct

slow

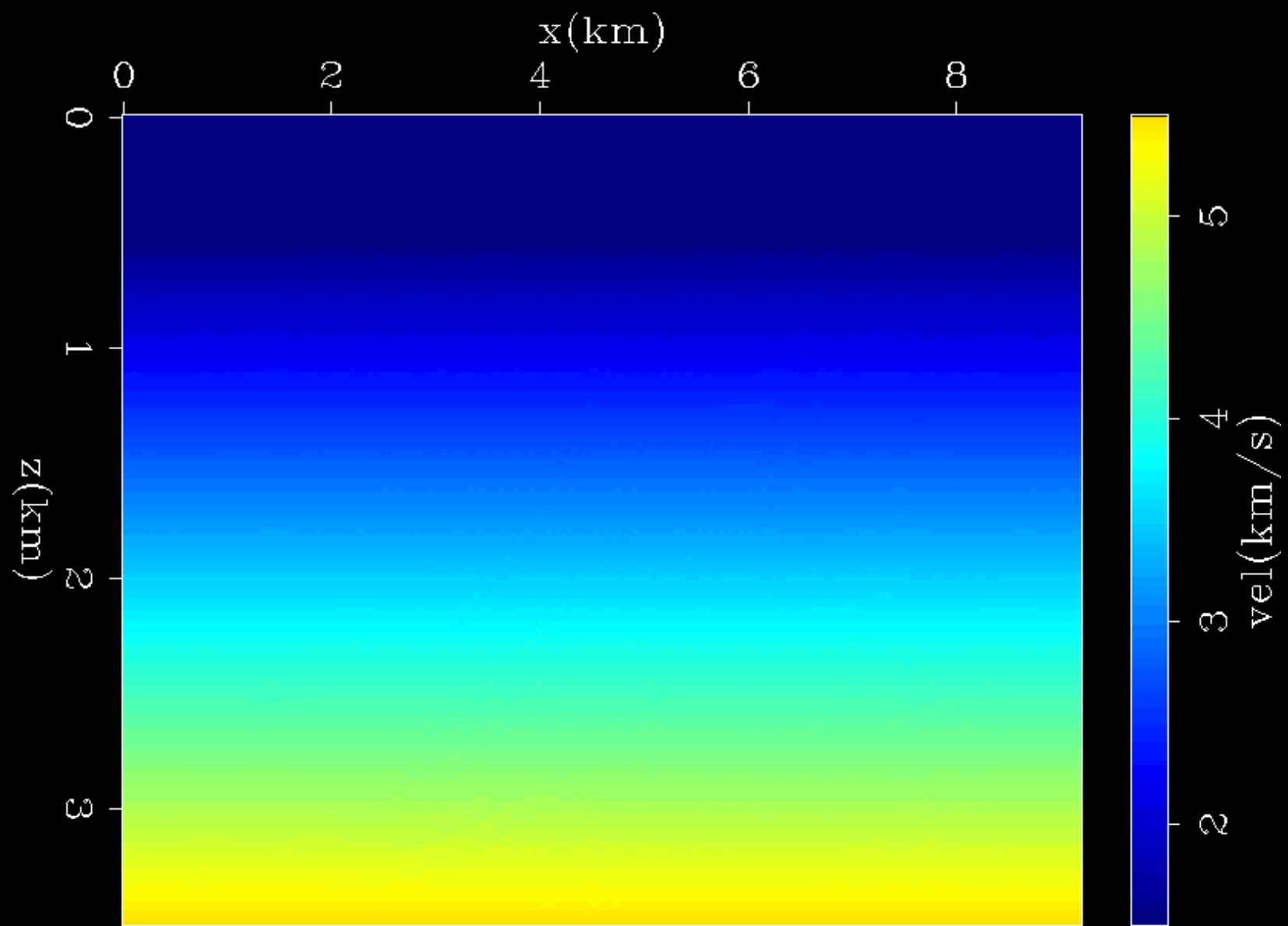
fast

Source location image gathers



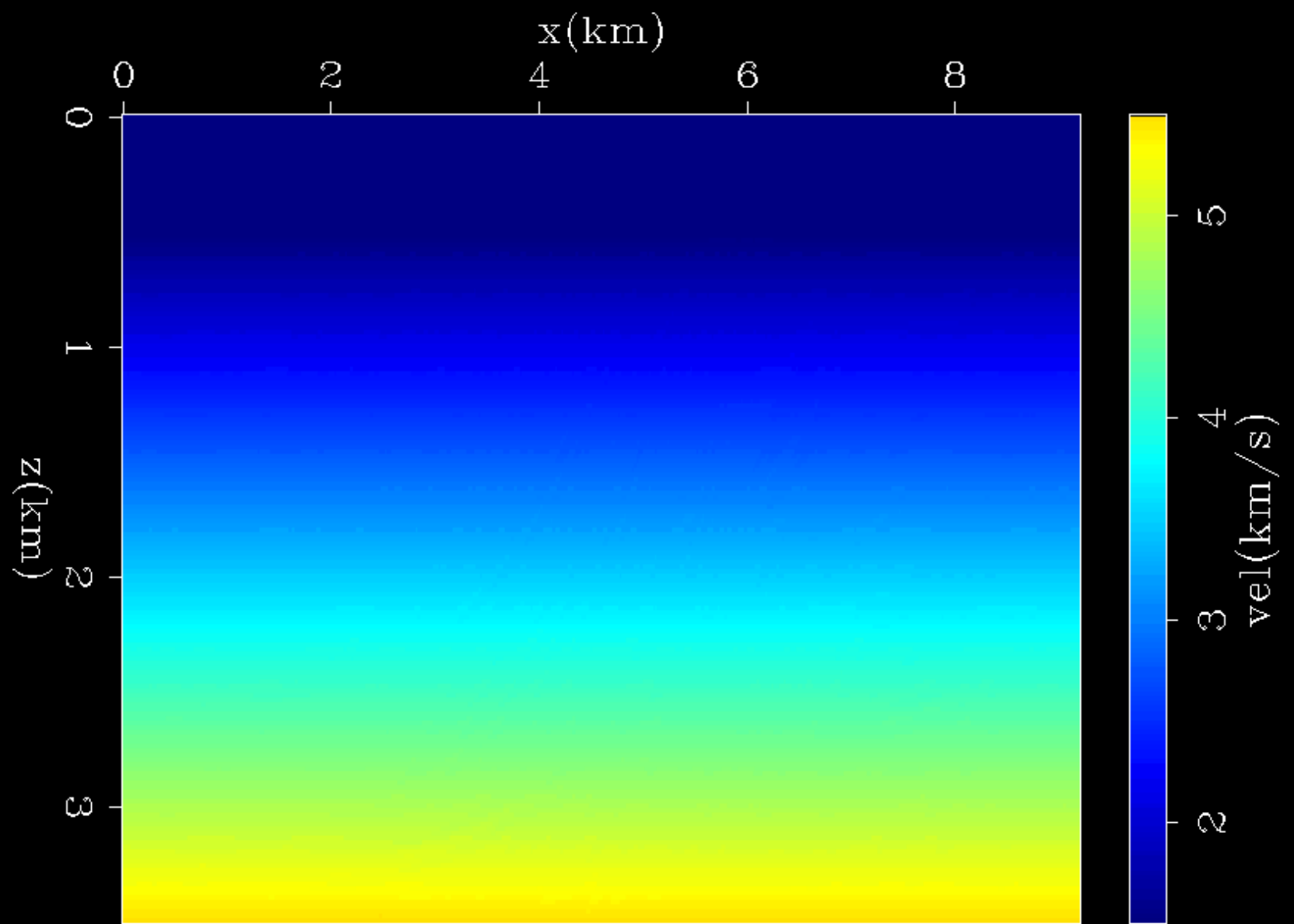


Ray parameter image gathers

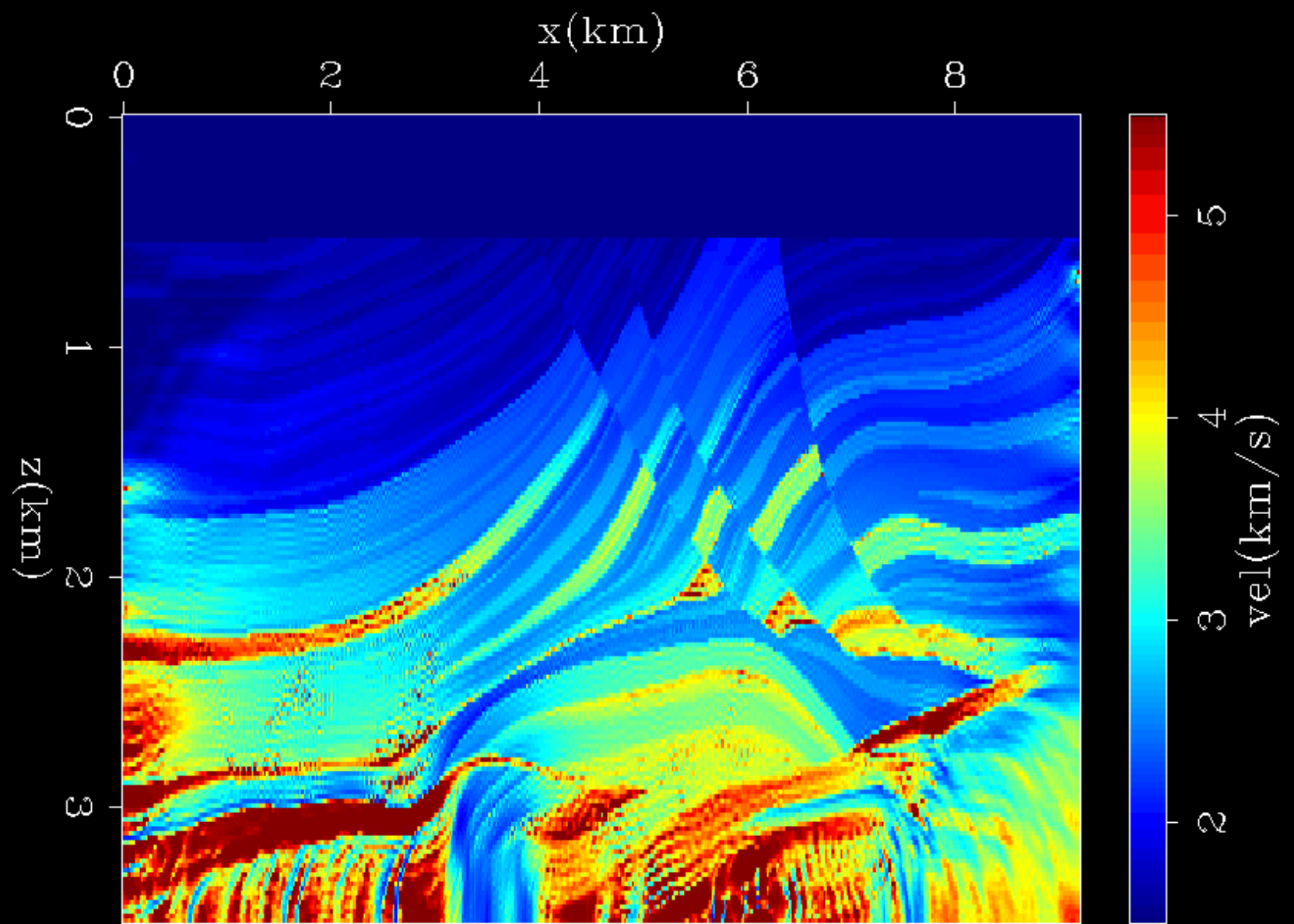


0

Initial model



0 Plane-wave extension



9720

Inverted model

Data space extension

- However, some limitations exist
- Theoretical issues:
 - Surface decomposition can be ambiguous
 - Inaccurate in very complex media
- Practical issues:
 - Shot EFWI have large I/O requirements
 - Ray parameter EFWI have acquisition limitations of plane encoding

Outline

- Introduction
- Data space extension
- Nested inversion scheme (SEP₁₄₉)
- Synthetic examples
- Conclusions

Linearized TFWI

- Primaries LTFWI operator:

$$\mathbf{d}_{\text{primaries}} = \mathbf{L}(\mathbf{b})\mathbf{p}(\mathbf{q})$$

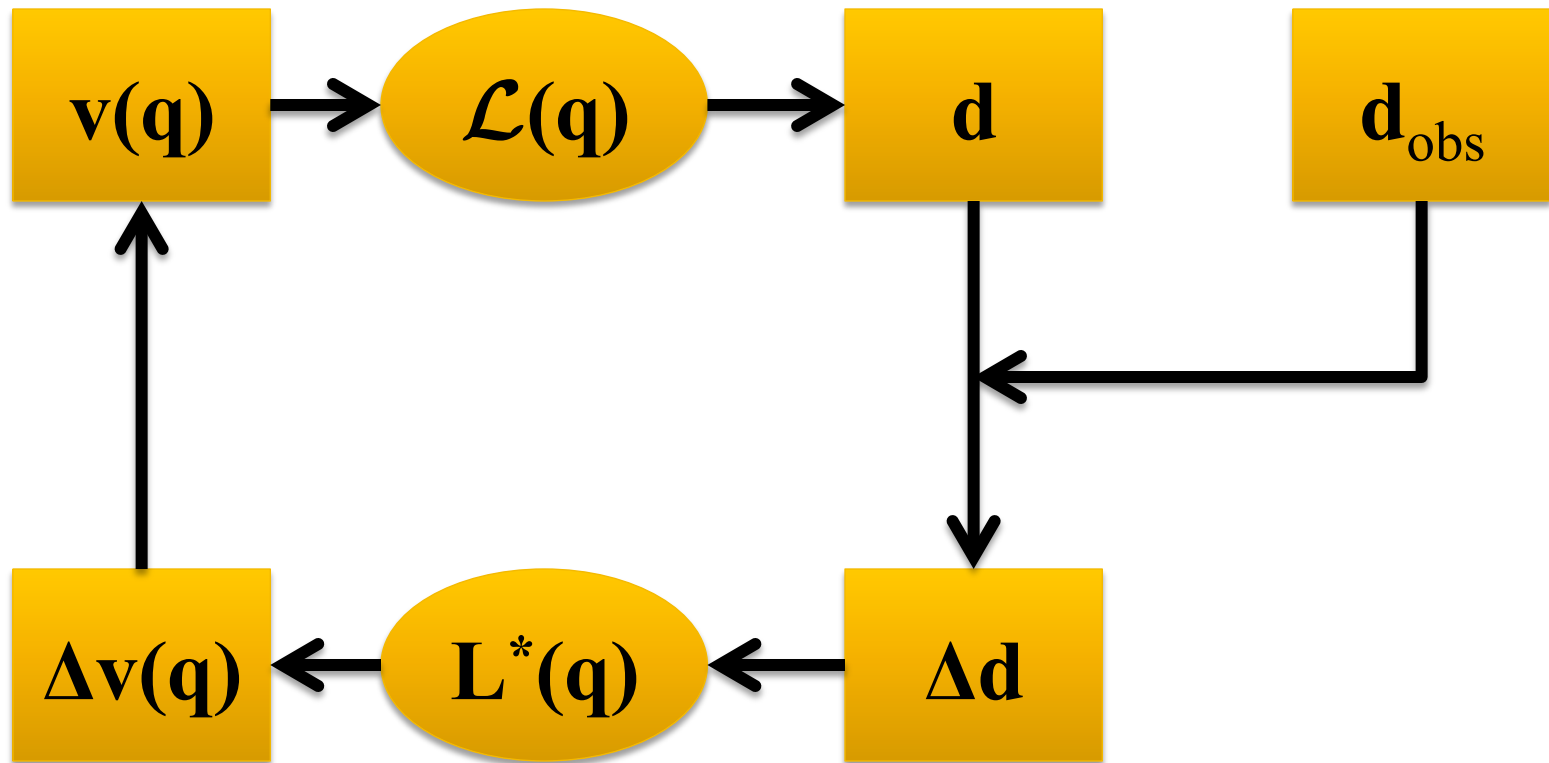
Linearized TFWI

- Primaries LTFWI operator:

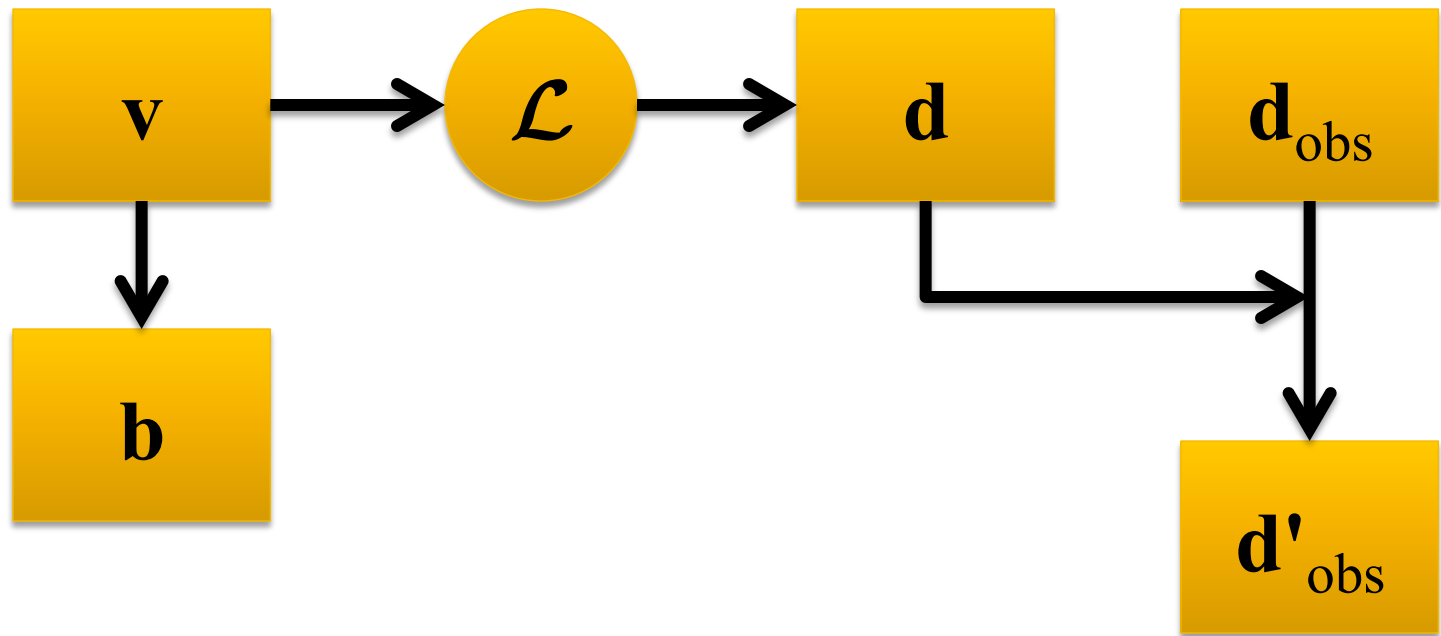
$$\mathbf{d}_{\text{primaries}} = \mathbf{L}(\mathbf{b})\mathbf{p}(\mathbf{q})$$

- First-order scattering LTFWI operator:

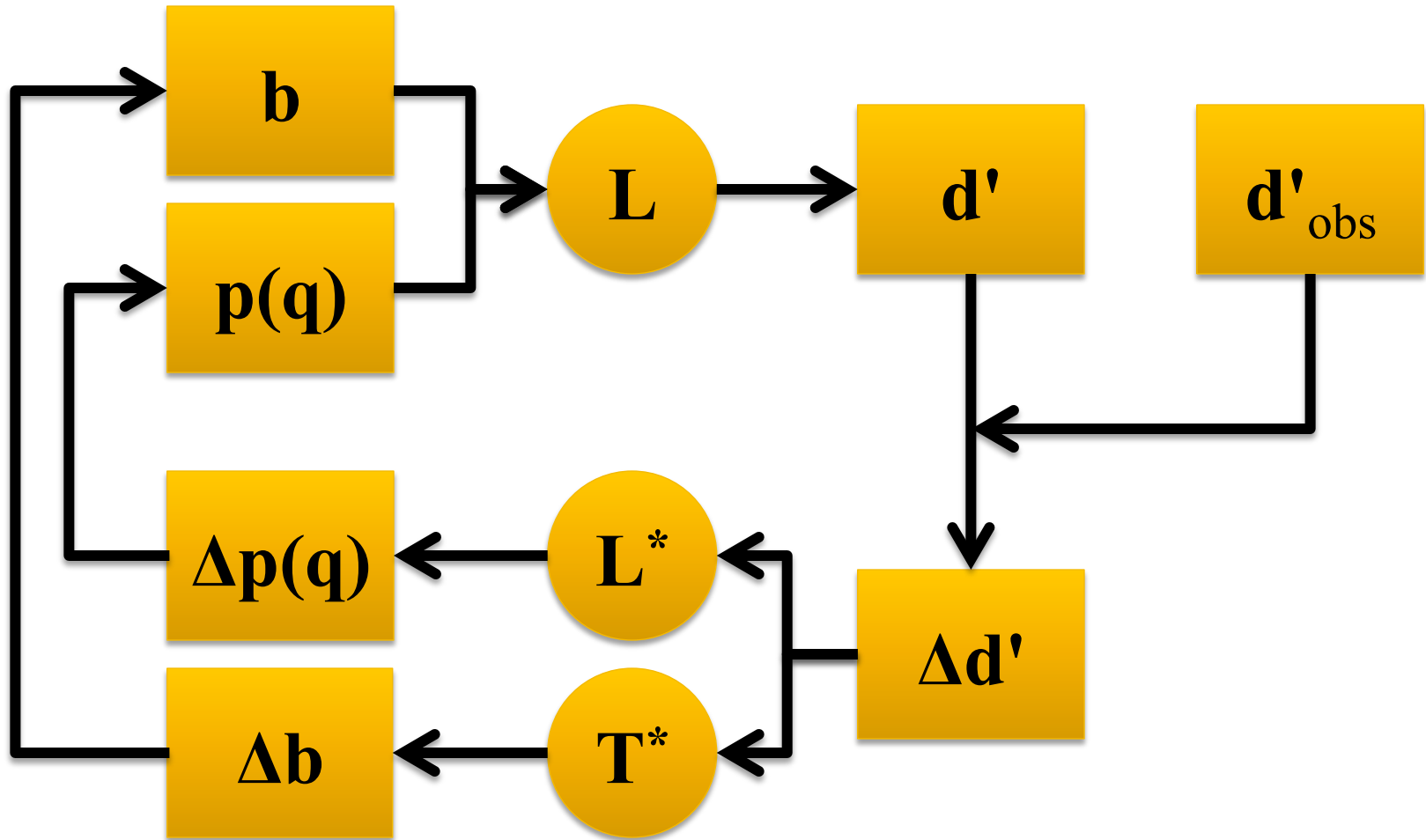
$$\mathbf{d}_{\text{obs}} - \mathcal{L}(\mathbf{b}_0) \approx \mathbf{L}(\mathbf{b})\mathbf{p}(\mathbf{q})$$

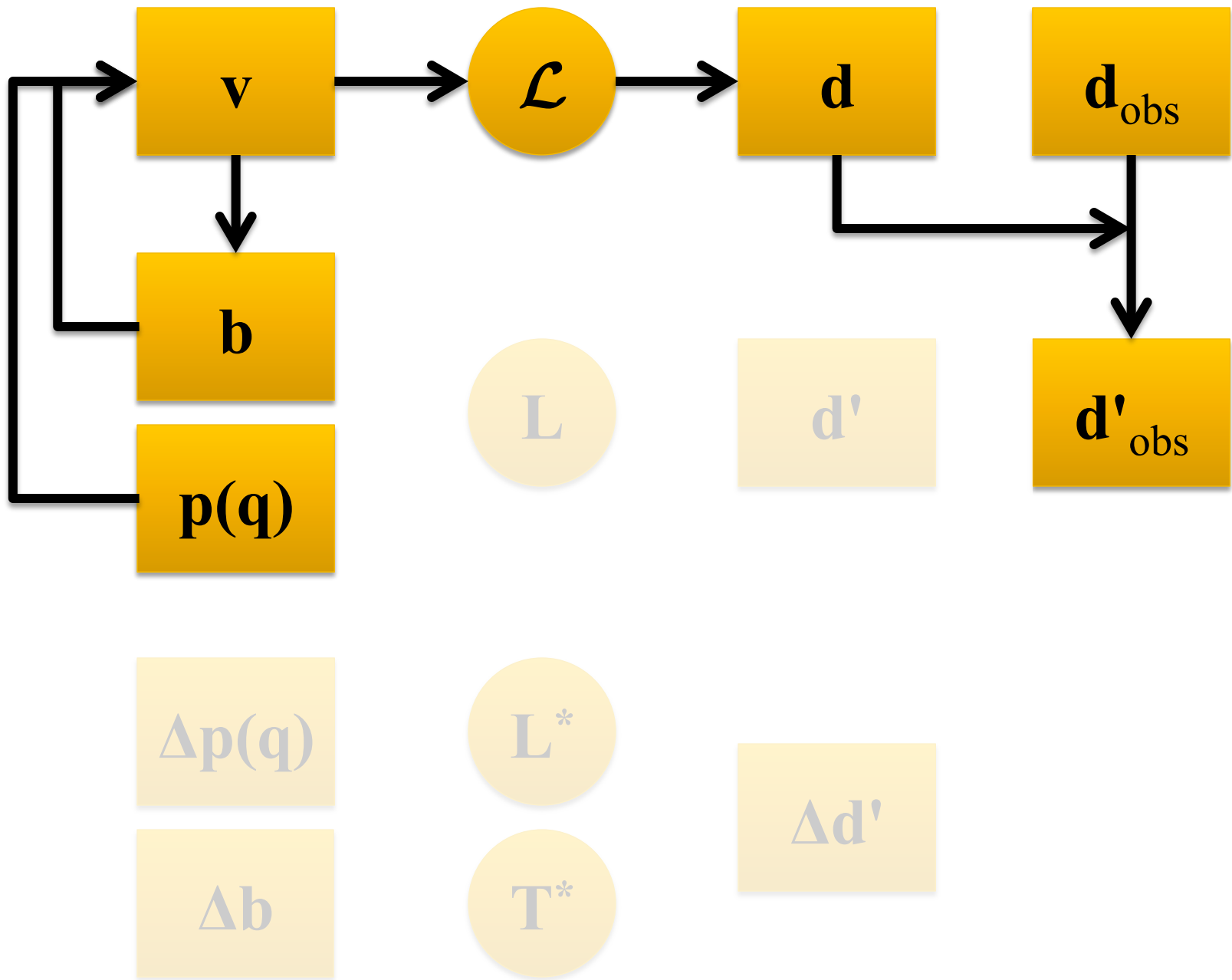


Conventional inversion loop



Nested inversion loop





Linearized TFWI

- LTFWI objective function:

$$J(\mathbf{b}, \mathbf{p}) = \|\mathcal{L}(\mathbf{b}) + \mathbf{L}(\mathbf{b}) \mathbf{p} - \mathbf{d}_{obs}\| + \|\mathbf{E} \mathbf{p}\|$$

- LTFWI gradients

$$\frac{\partial J}{\partial \mathbf{p}} = \mathbf{L}^* \Delta \mathbf{d}'$$

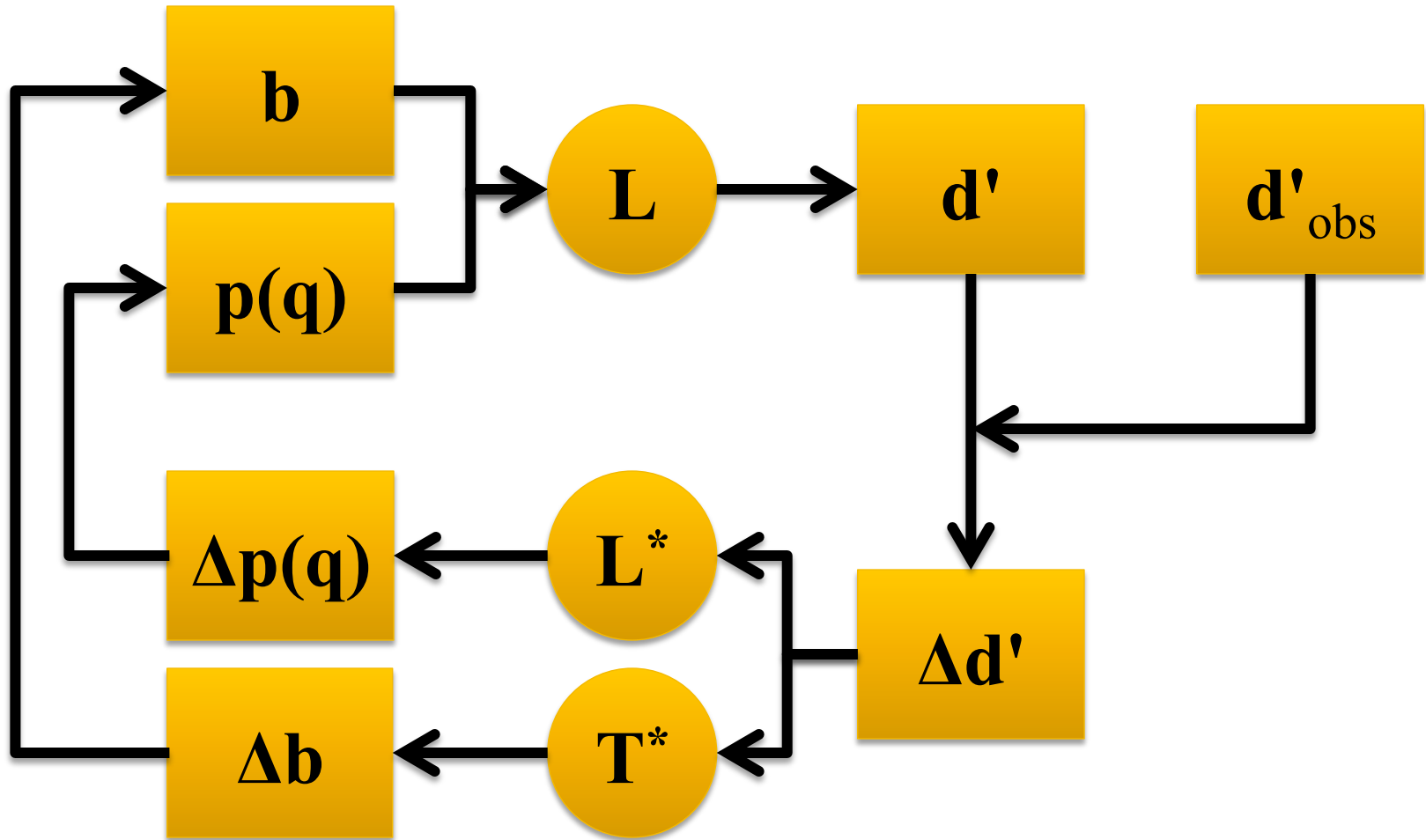
$$\frac{\partial J}{\partial \mathbf{b}} = \mathbf{T}^* \Delta \mathbf{d}'$$

\mathbf{L} = Born modeling op

\mathbf{T} = Tomographic op

Scale mixing

- Separating the two models might hinder the simultaneous inversion accuracy
- The two models are indirectly connected:
 - Data residuals
 - Perturbation
- Not directly connected in model space

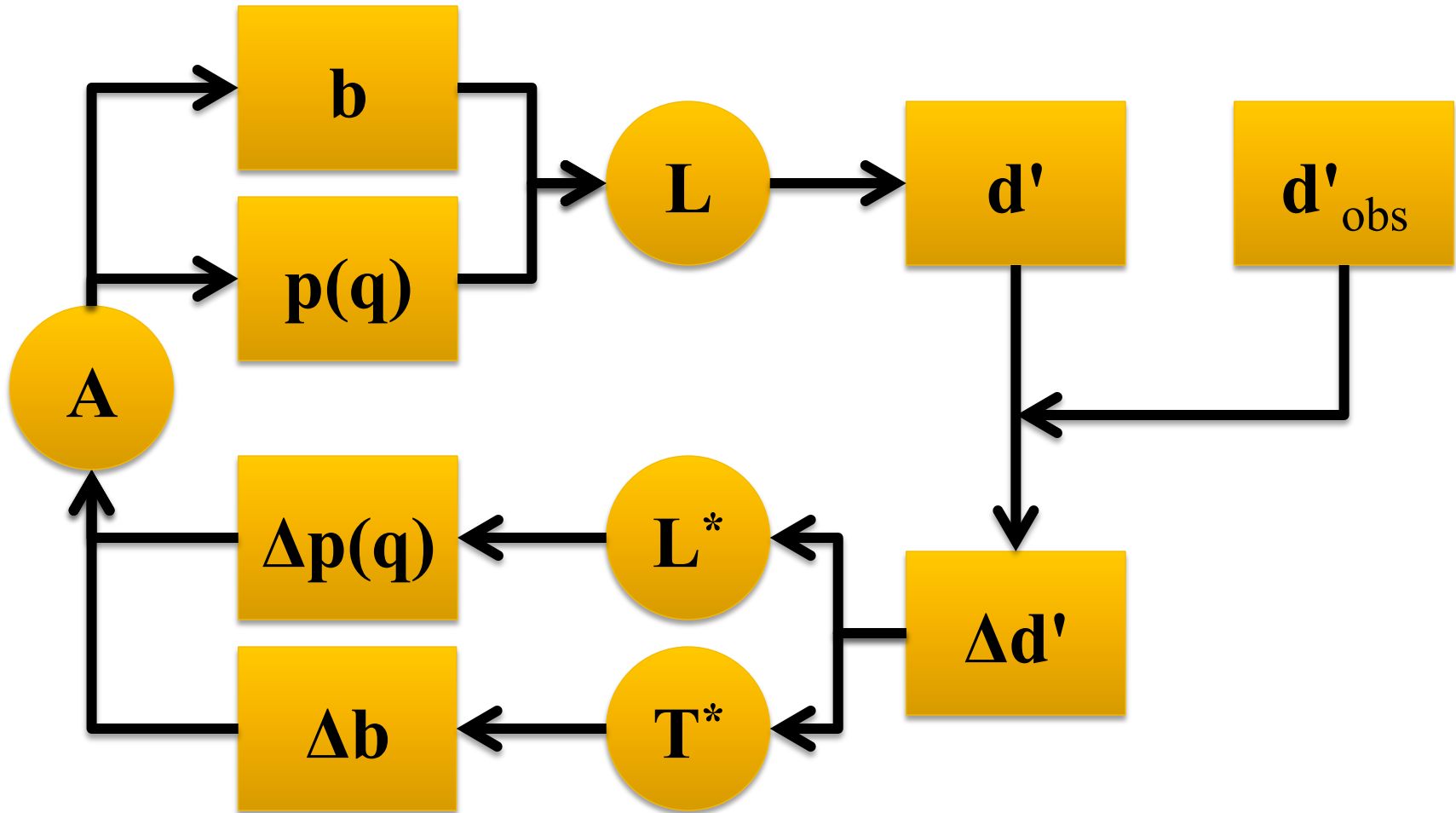


v

\mathcal{L}

d

d_{obs}



Scale mixing

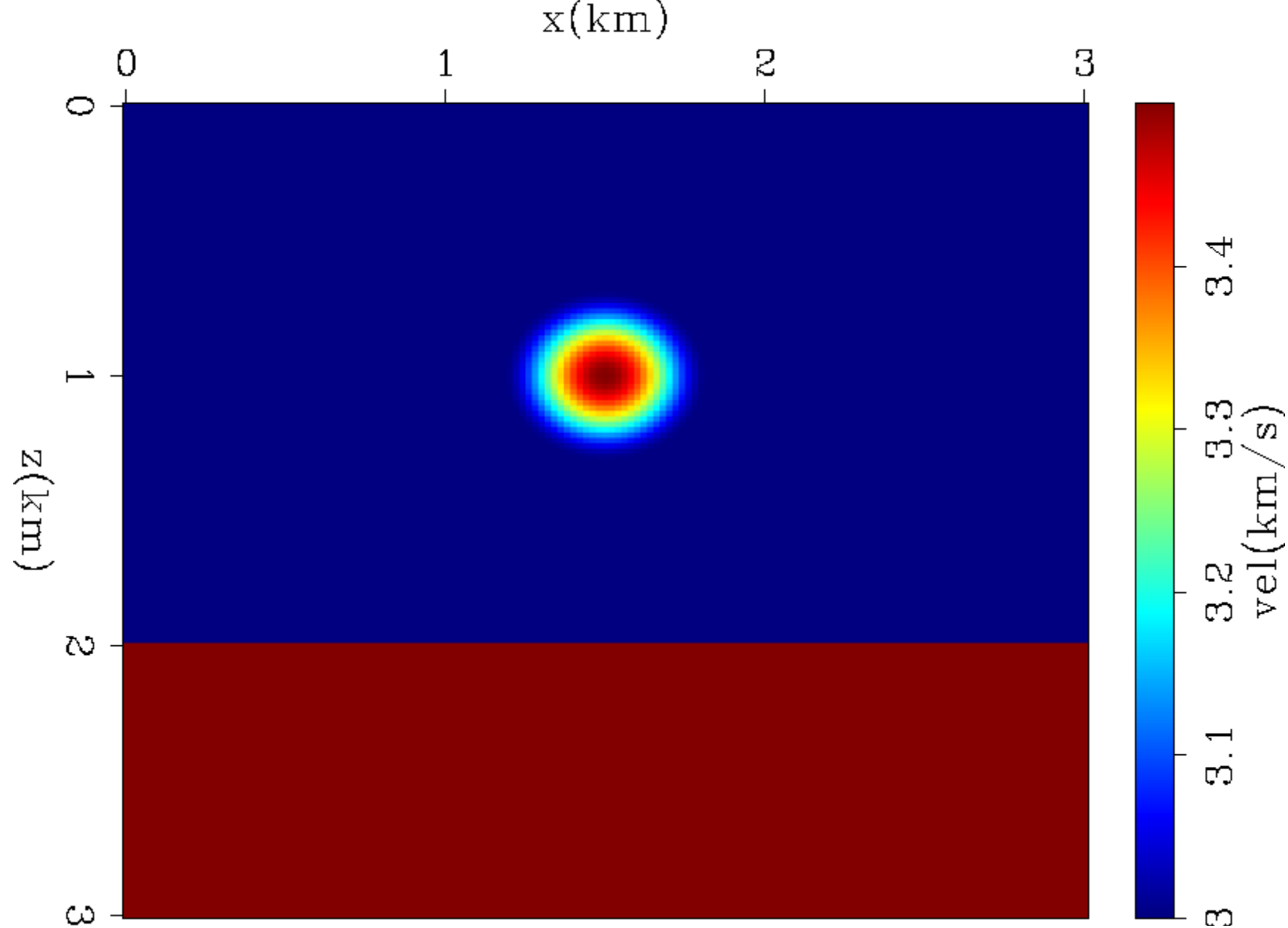
- \mathbf{A} is a scale separation operator which separates the gradient into smooth and rough components
- Gradient separation can be done in Fourier domain
- Both gradients need to have the same units

Nested inversion scheme

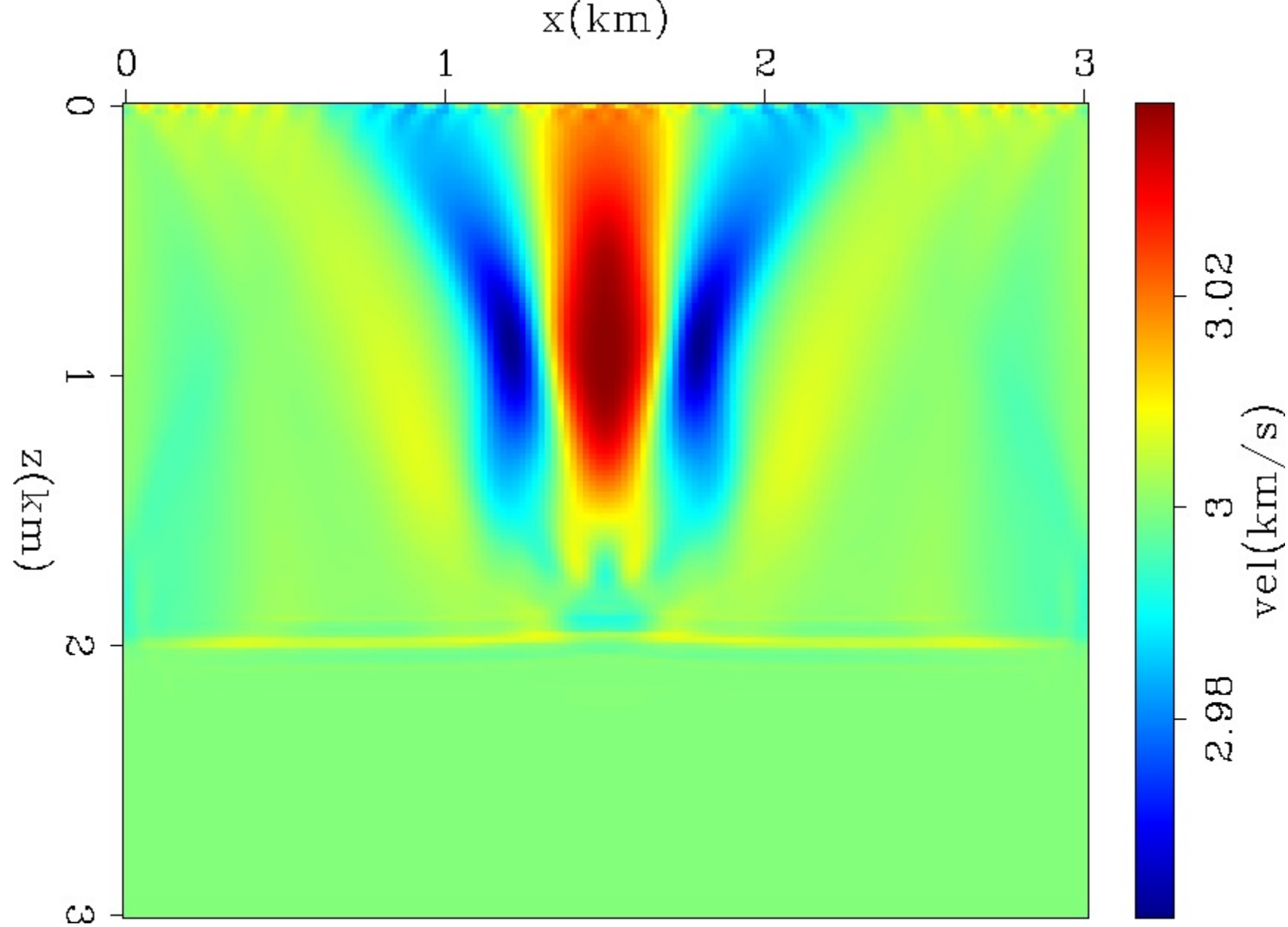
- Not limited by the Born operator
 - Can handle multiples
- Background data get updated
 - Correctly compensate for nonlinear effects
- Both models get eventually pushed into the velocity model
- Inversion is less sensitive to scale separation

Outline

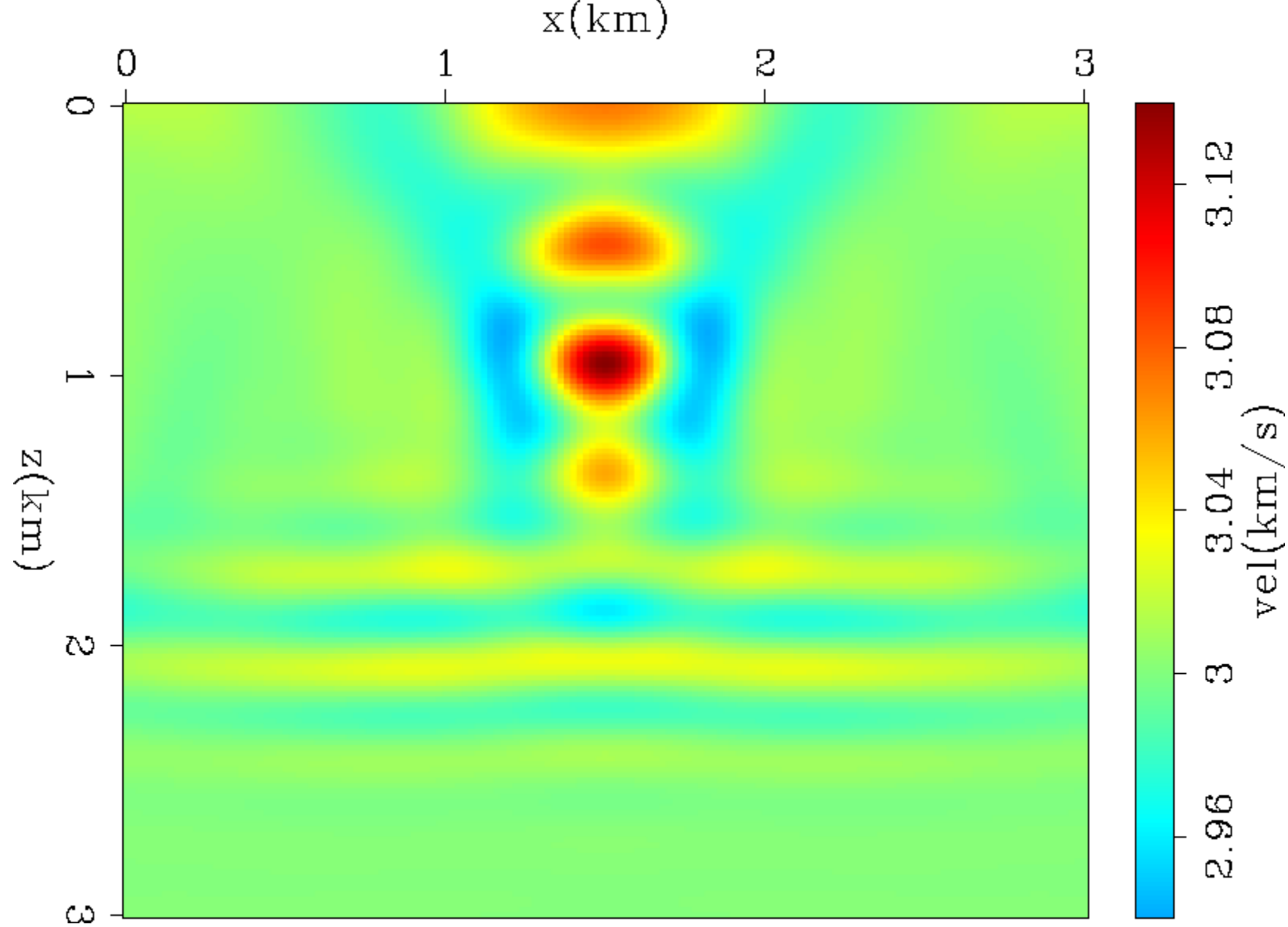
- Introduction
- Data space extension
- Nested inversion scheme
- **Synthetic examples**
- Conclusions



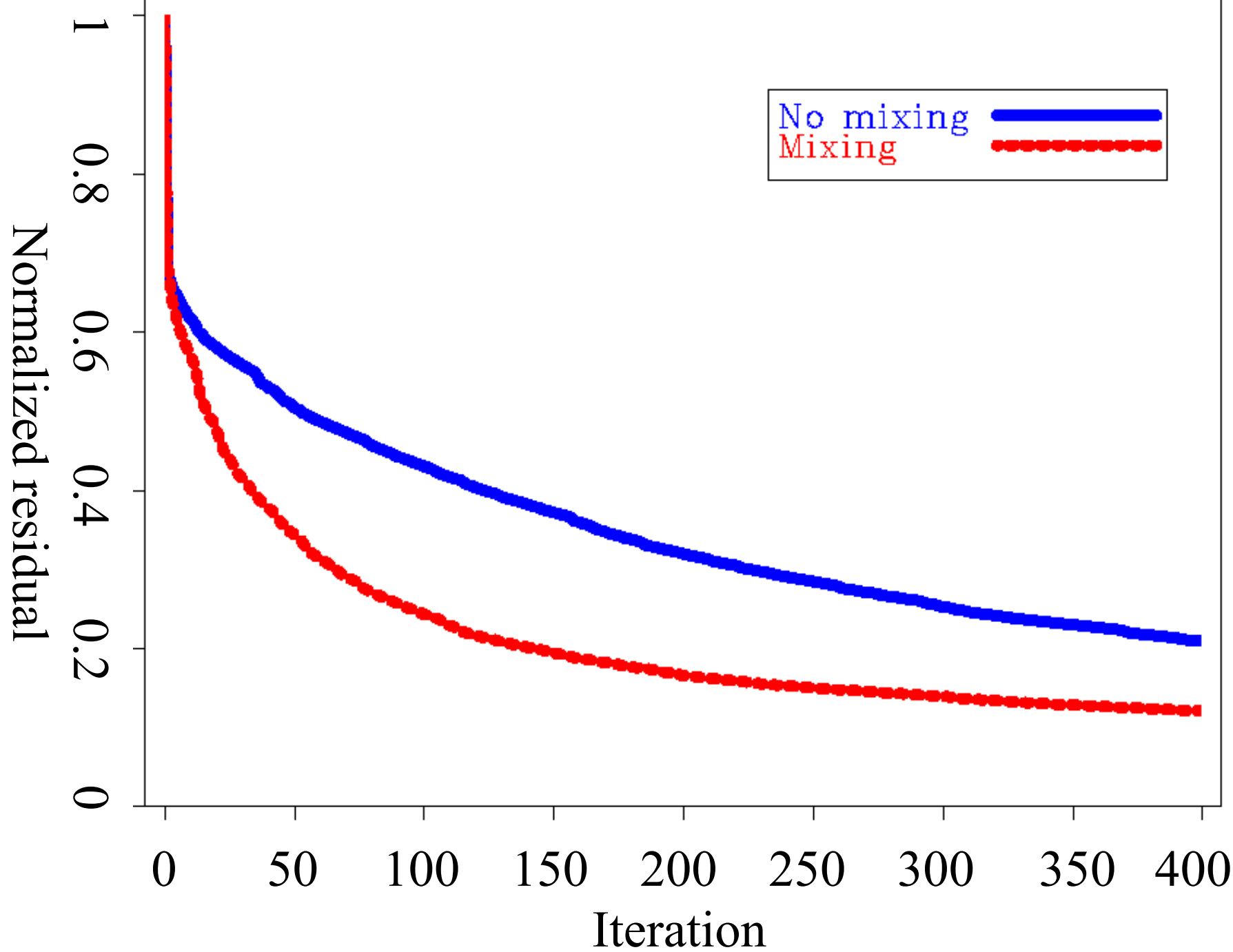
True Gaussian velocity

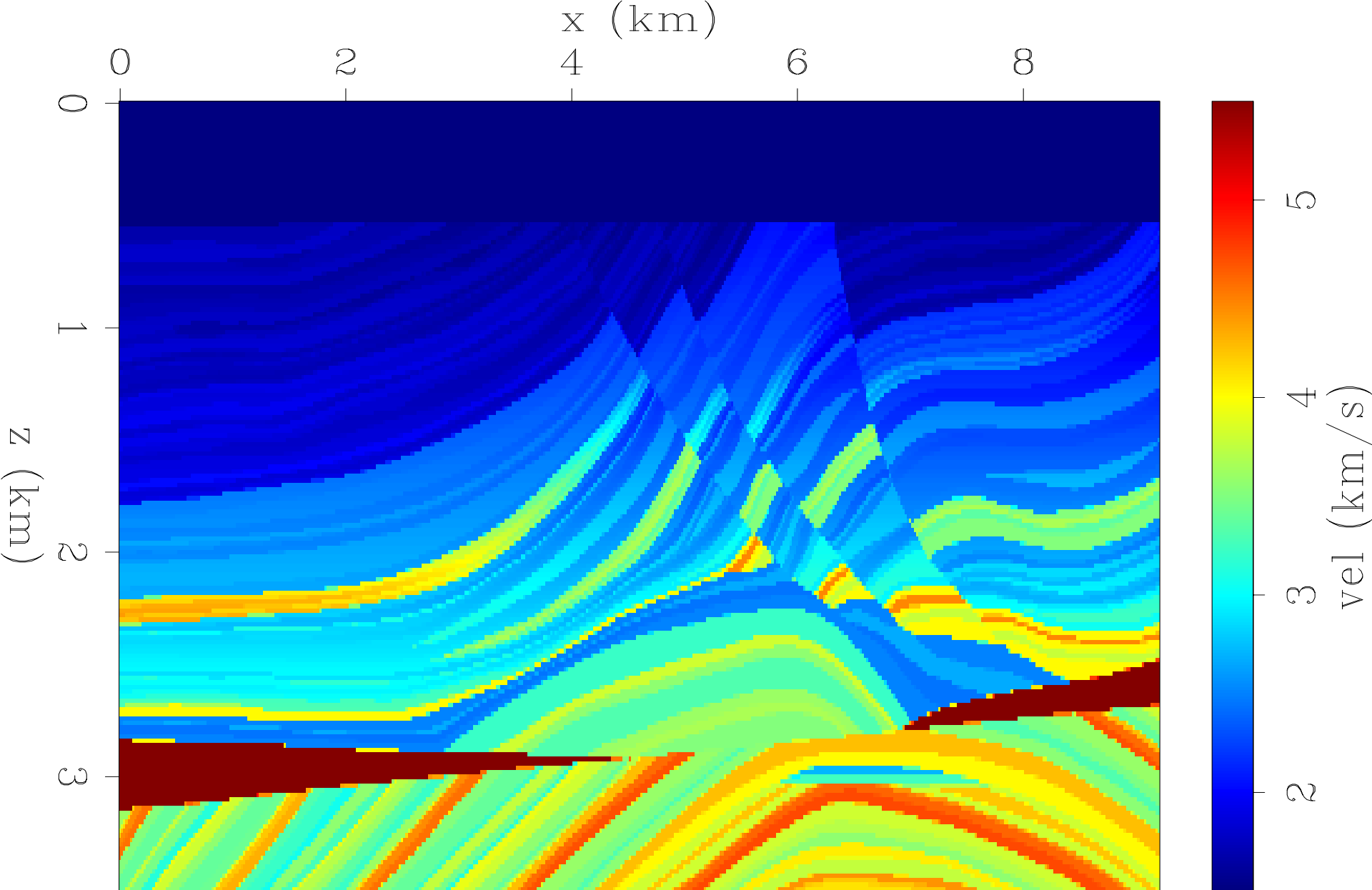


Inverted background without mixing

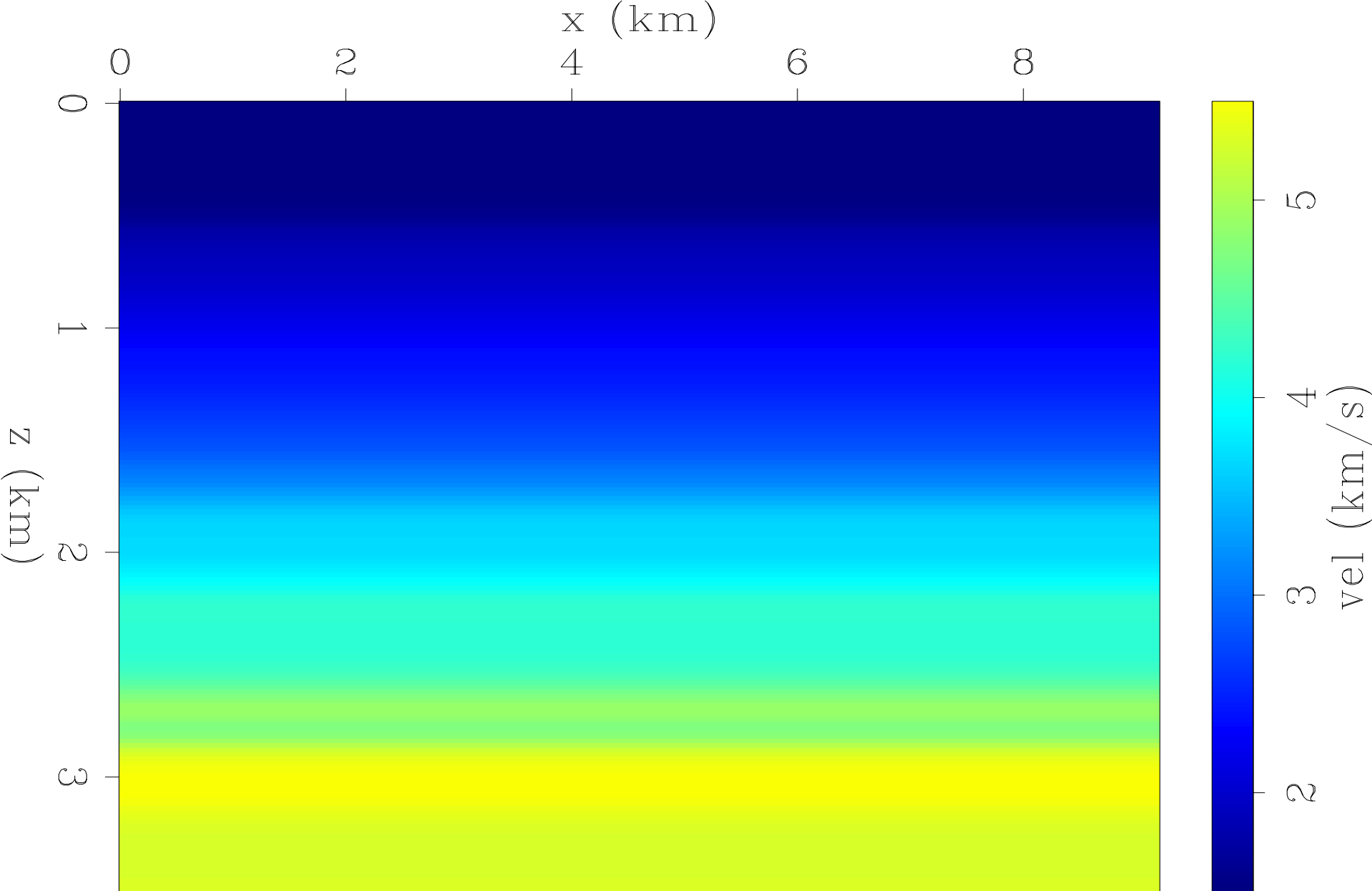


Inverted background with mixing

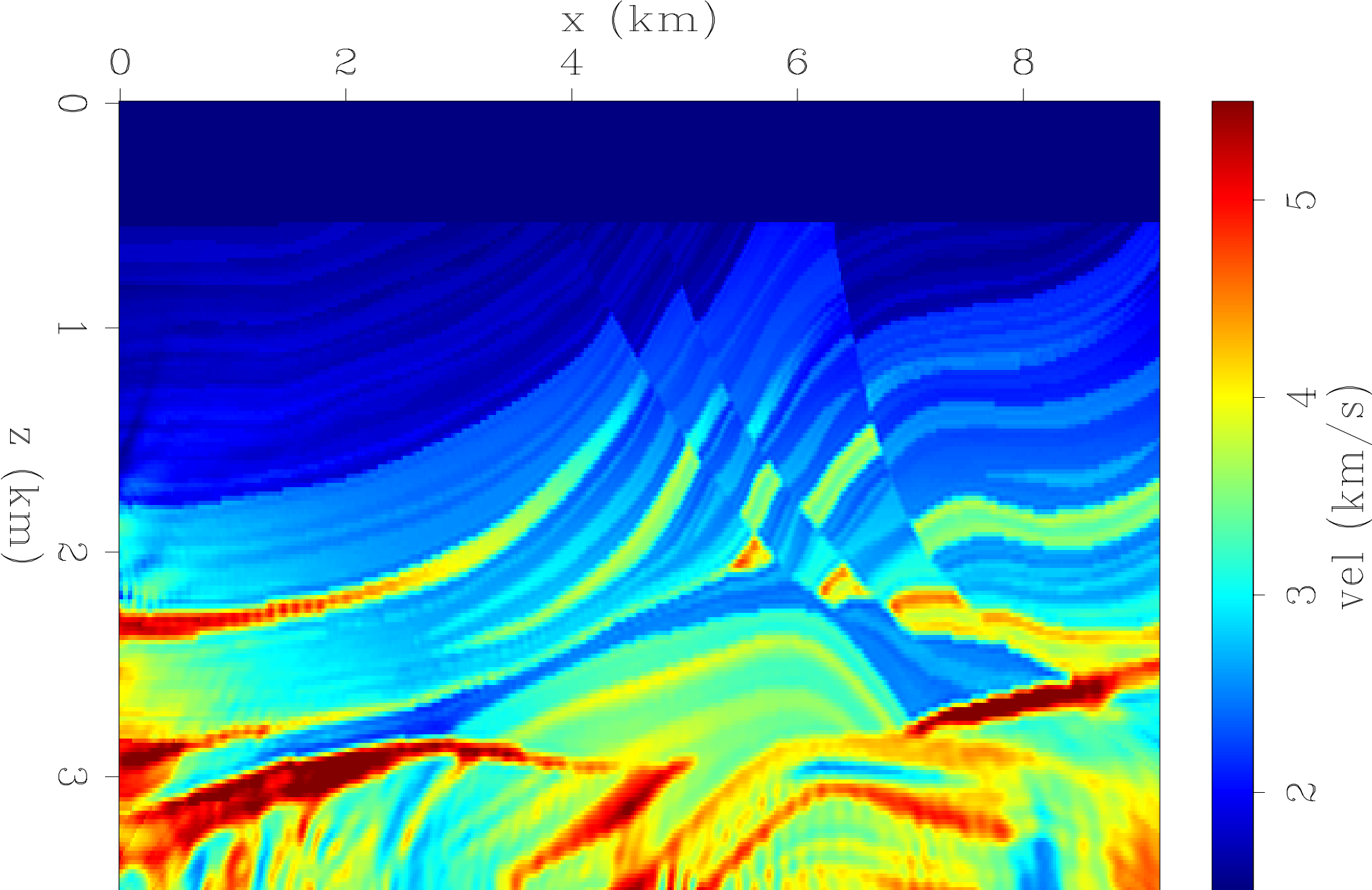




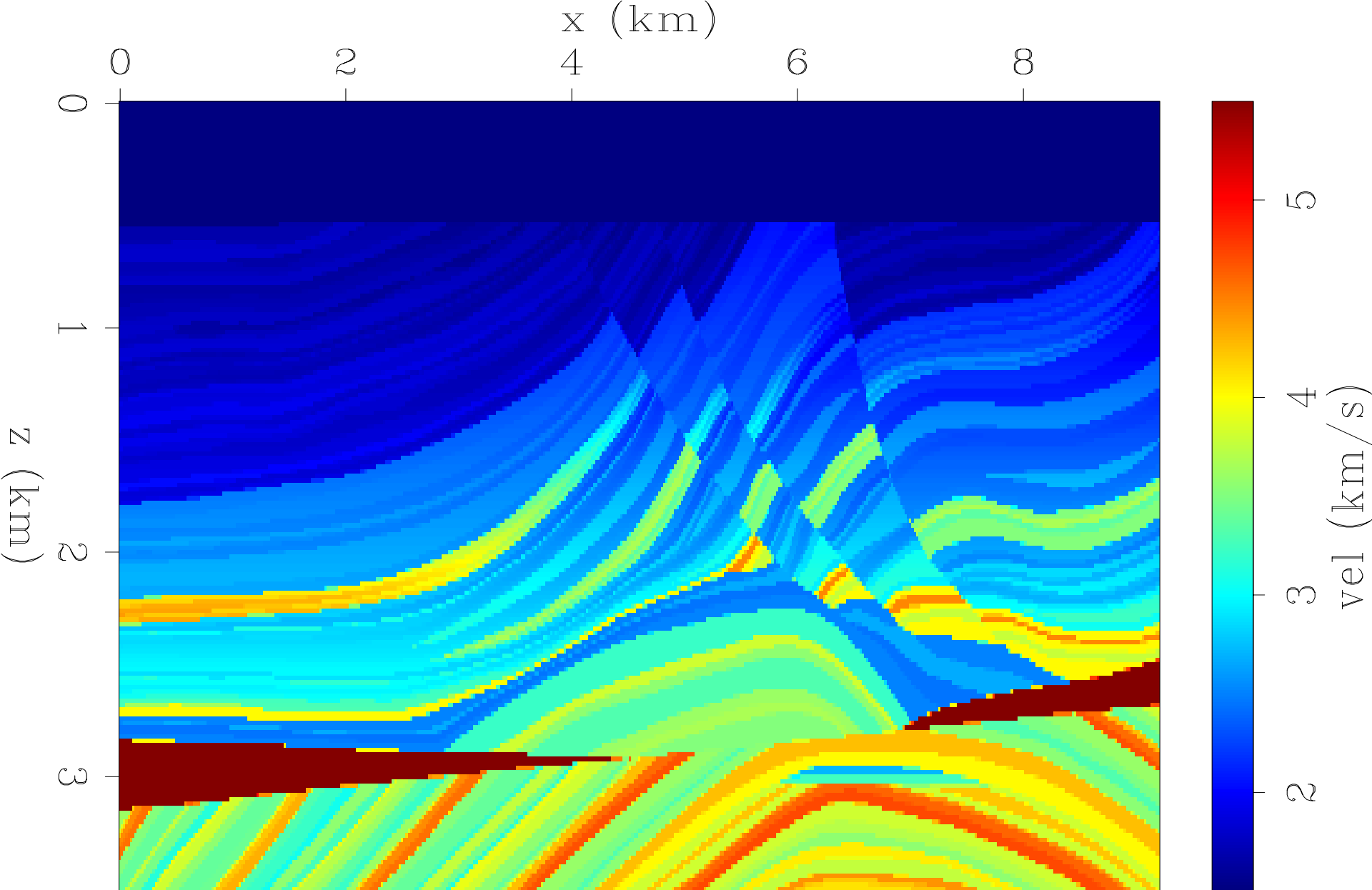
True marmousi model



Initial marmousi model



Inverted marmousi model



True marmousi model

Outline

- Introduction
- Data space extension
- Nested inversion scheme
- Synthetic examples
- **Conclusions**

Conclusions

- Data space extension has the same cost as FWI but can might fail in complex media and has I/O and encoding issues
- By using Born approximation, we cut the cost by separating the model into background and perturbation

Conclusions

- The nested scheme allows proper comparison to the full dataset
- The model separation hinders the simultaneous inversion of scales
- We regain the high resolution results by scale mixing of parameters in Fourier domain

Future work

- Improve enhancing operators
 - DSO operators are poorly conditioned and propagates information slowly
- Improve convergence rate
 - Diagonal Hessian preconditioning
 - Data weighting

Acknowledgment

- Saudi Aramco for supporting my studies

Thanks!