

Tomographic Full Waveform Inversion (TFWI) by τ velocity extension

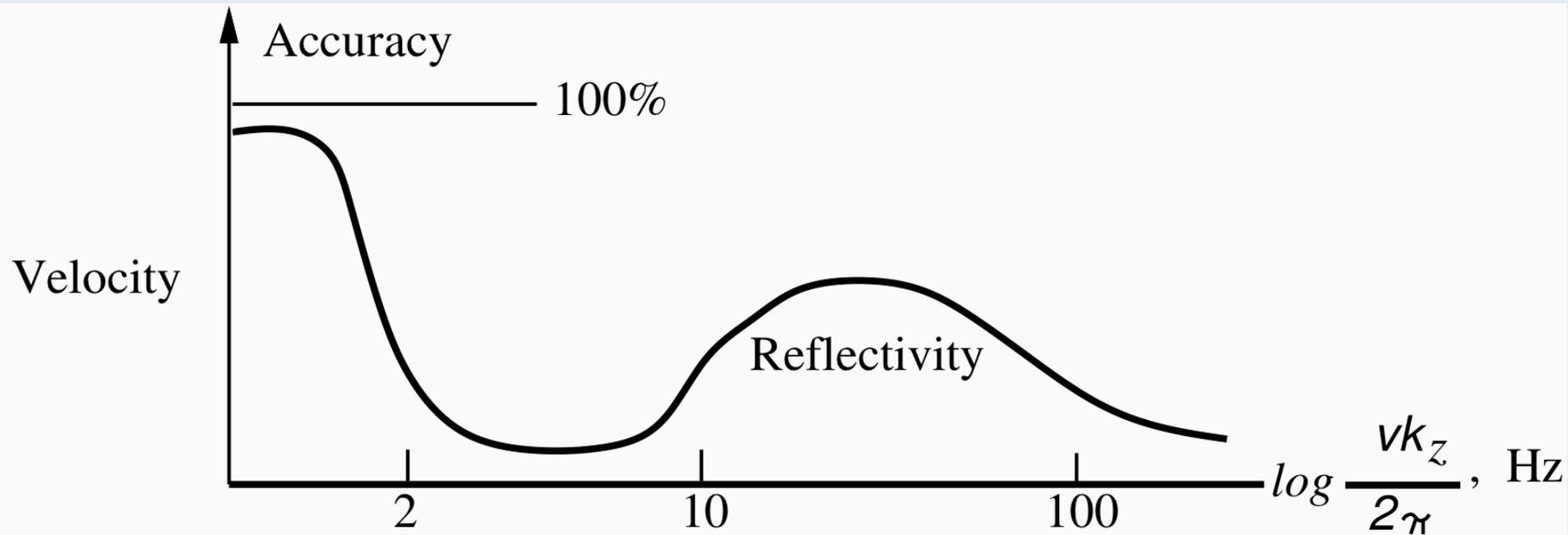
Biondo Biondi & Ali Almomin

SEP Meeting

June 18, 2013

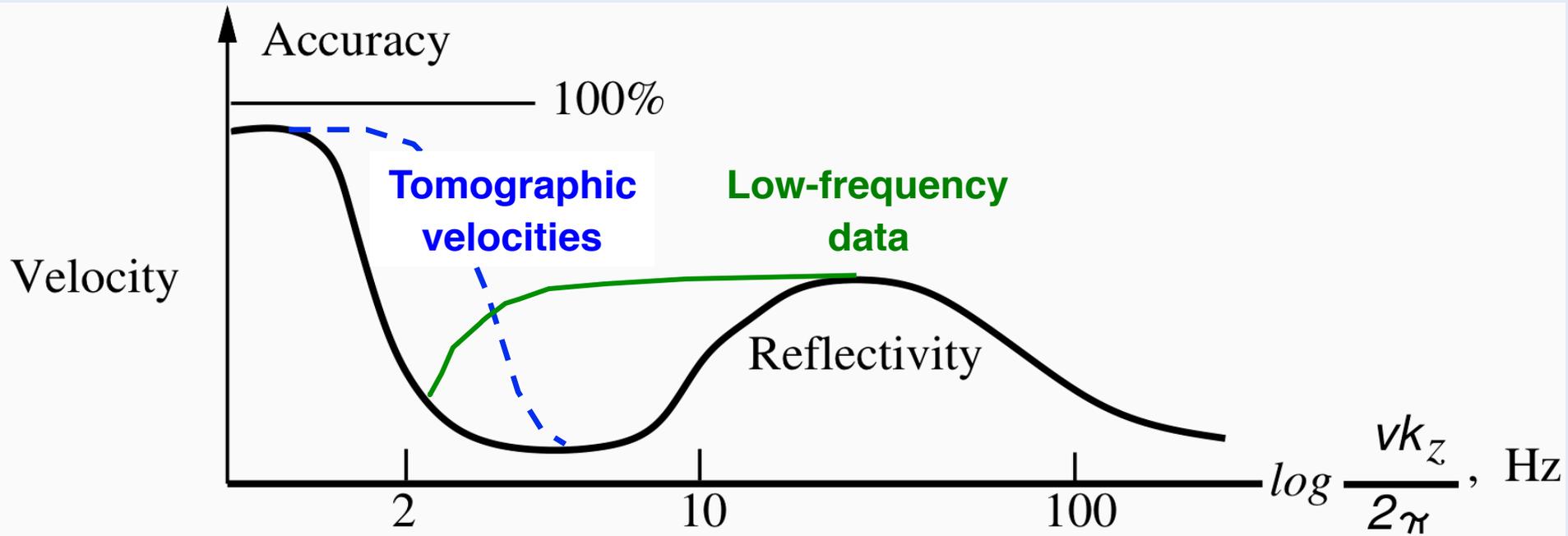


Scale separation in seismic imaging

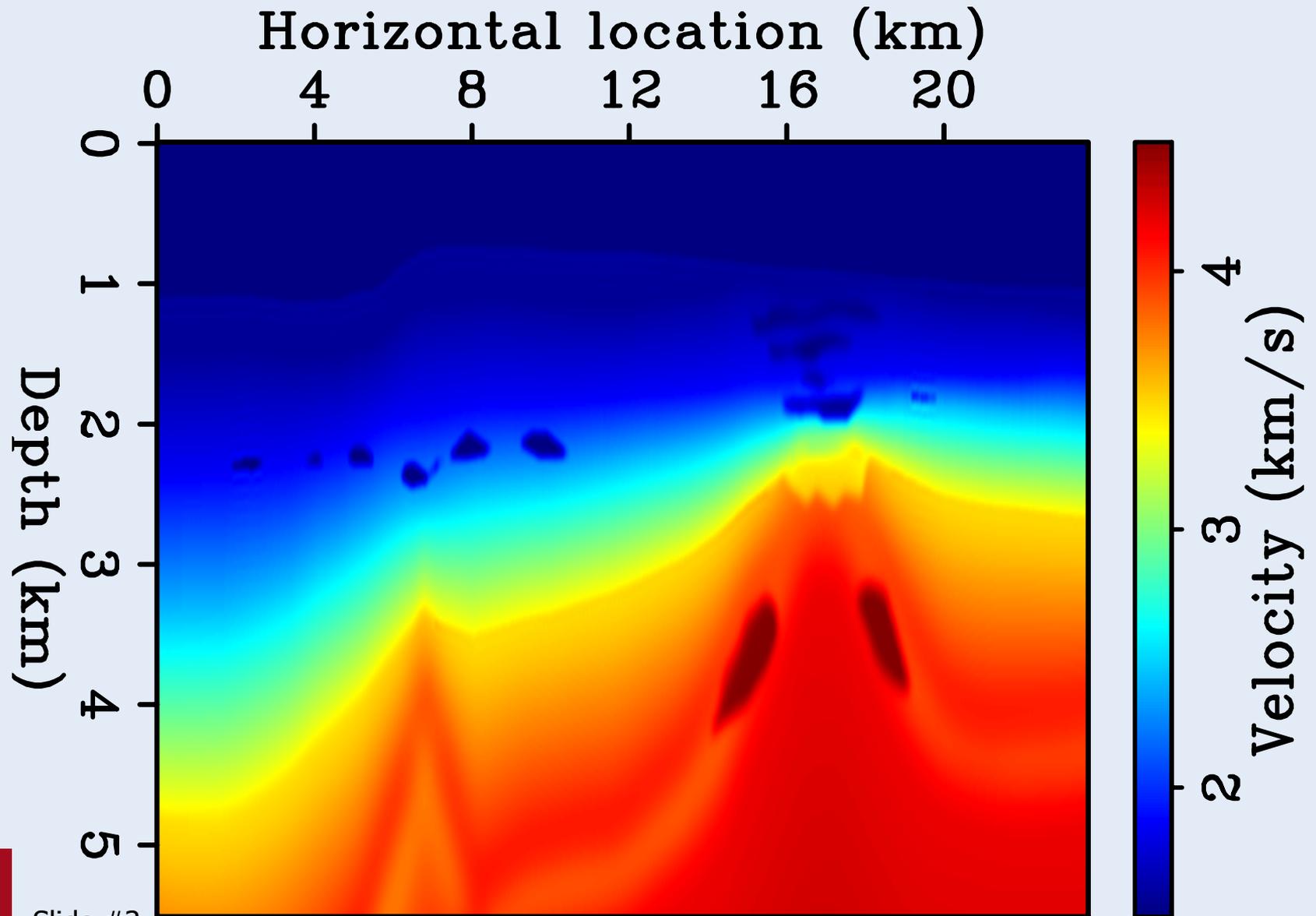


From Jon Claerbout's "Imaging the Earth Interior"

Gap is closing

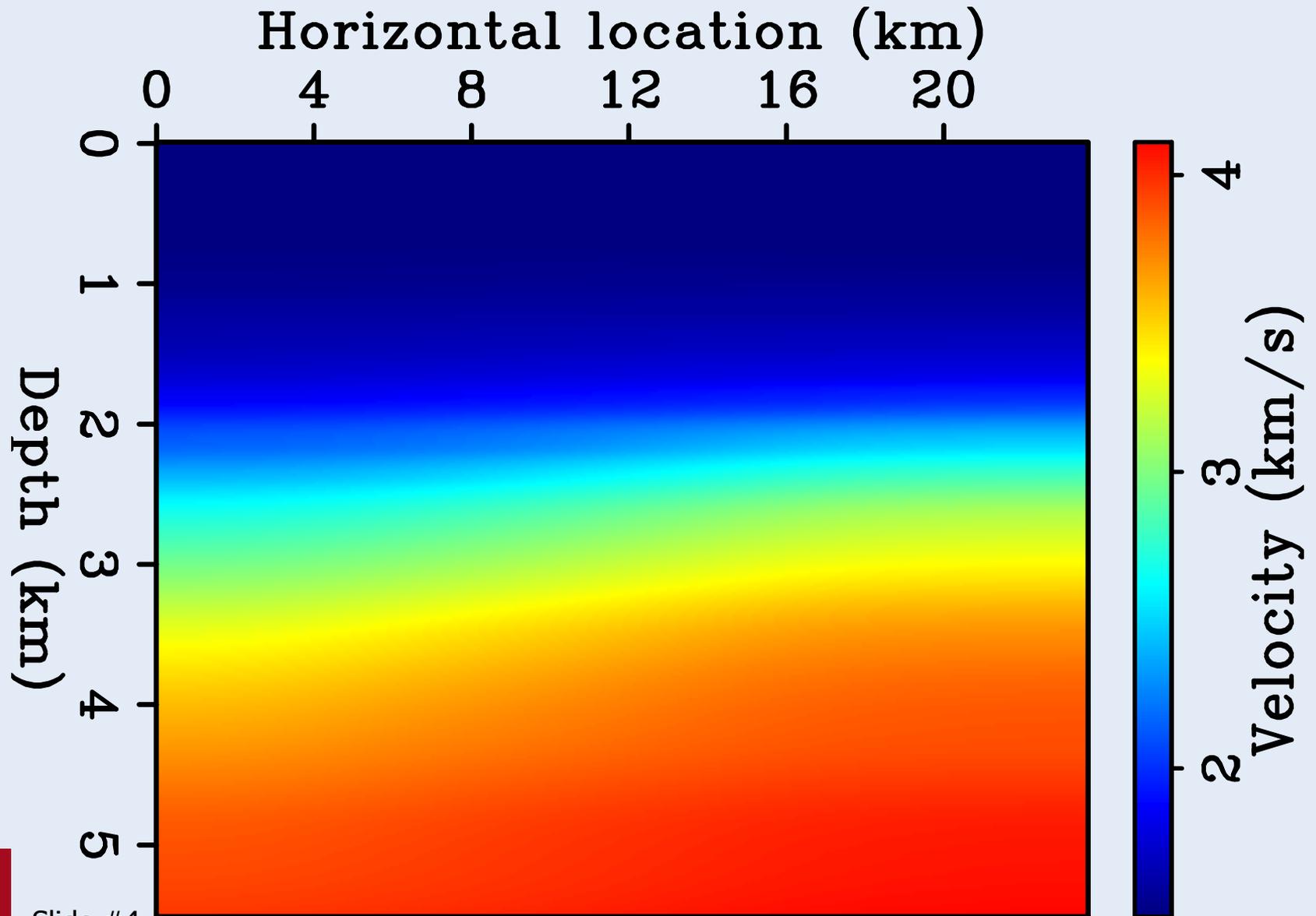


“Mud volcano” in BP 2005 model



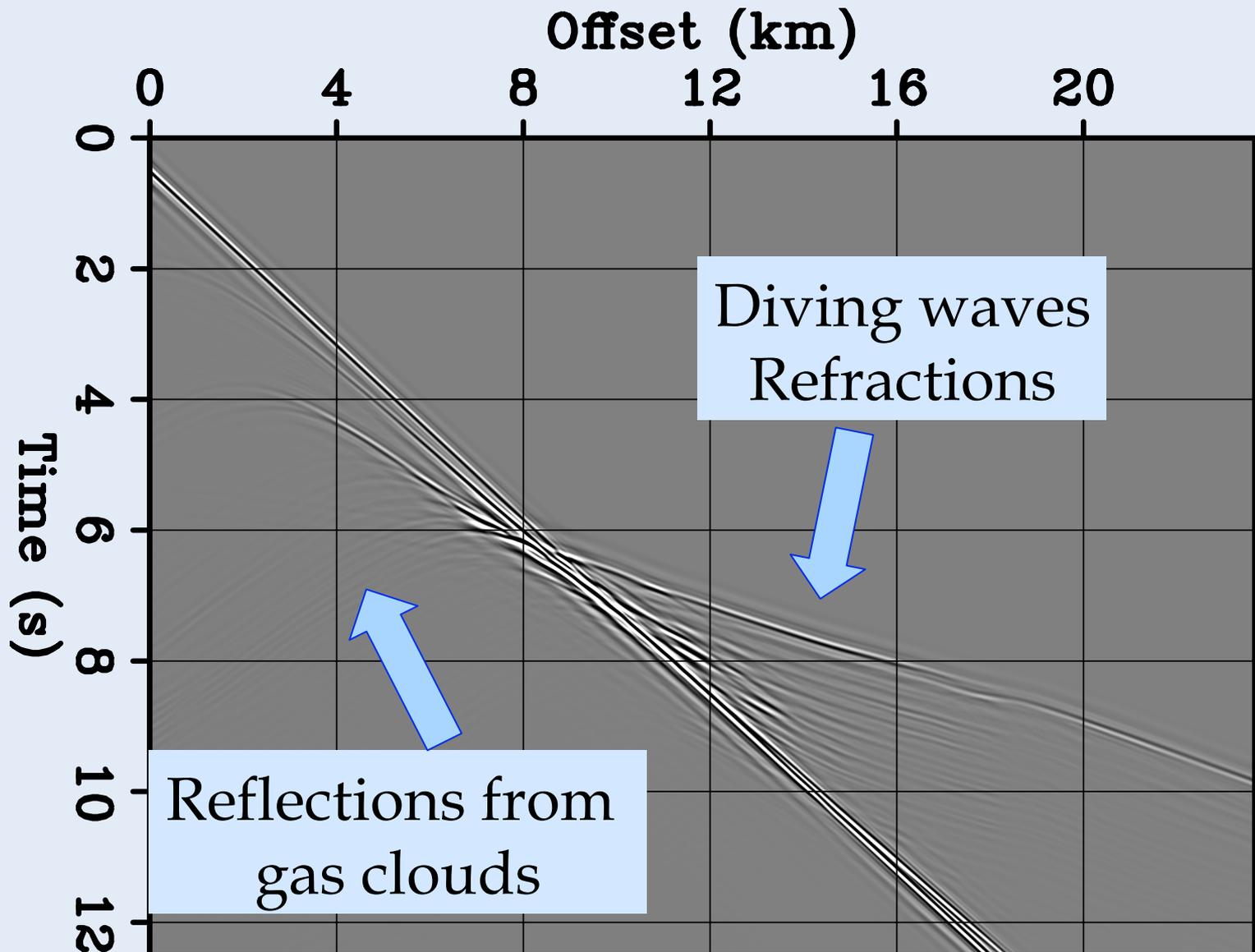
Slide #3

Starting model for TFWI

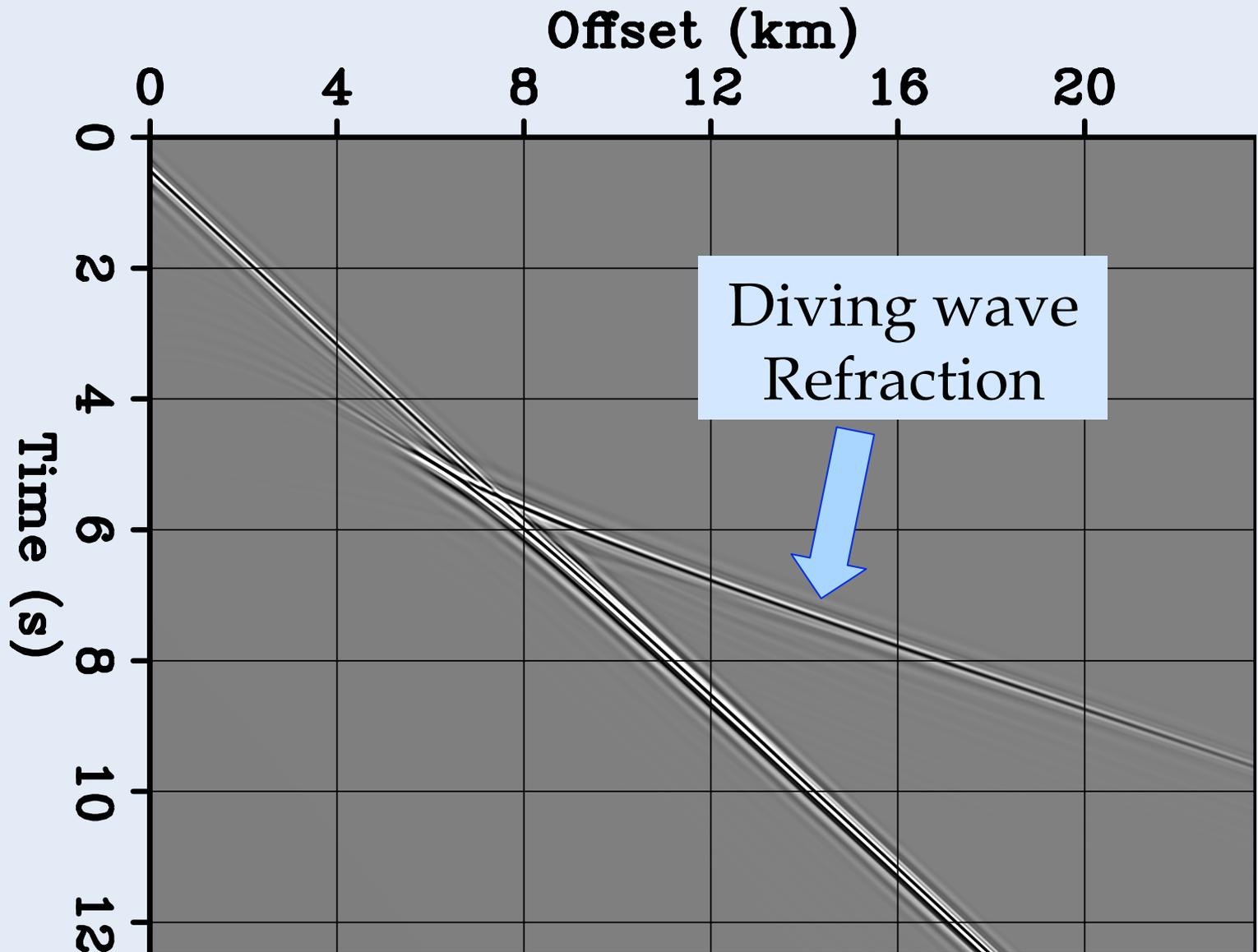


Slide #4

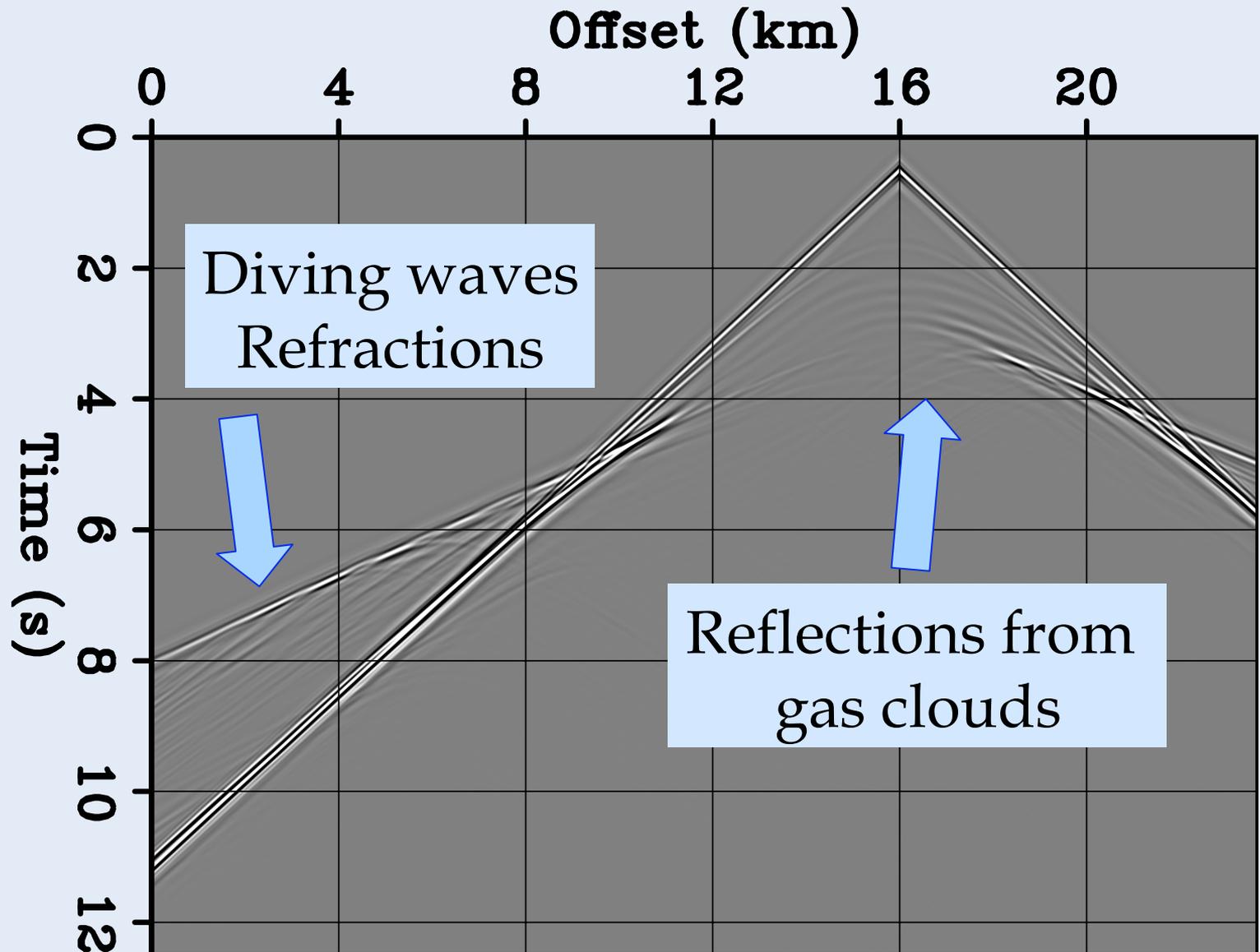
Leftmost shot gather - True model



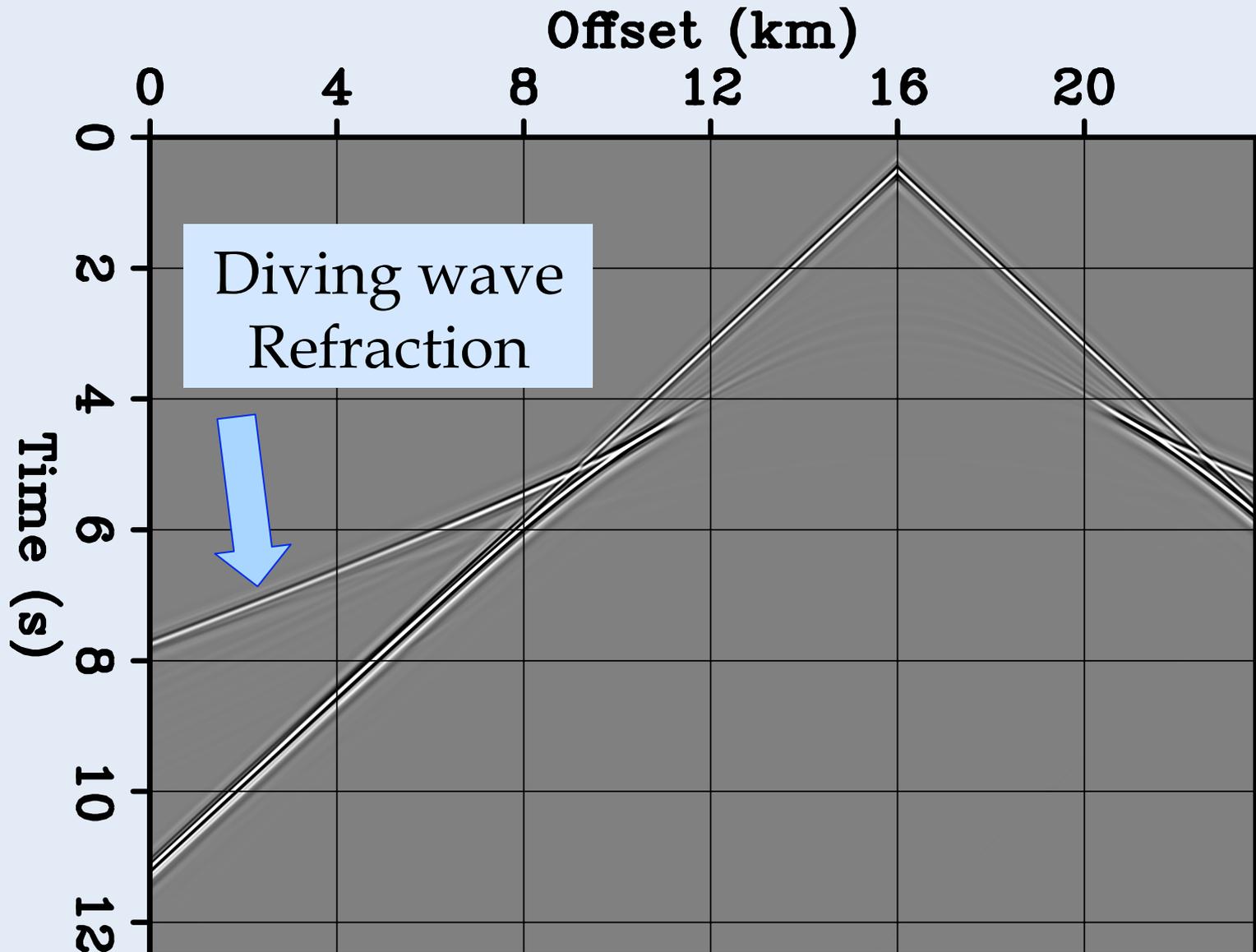
Leftmost shot gather - Initial model



Middle shot gather – True model



Middle shot gather – Initial model

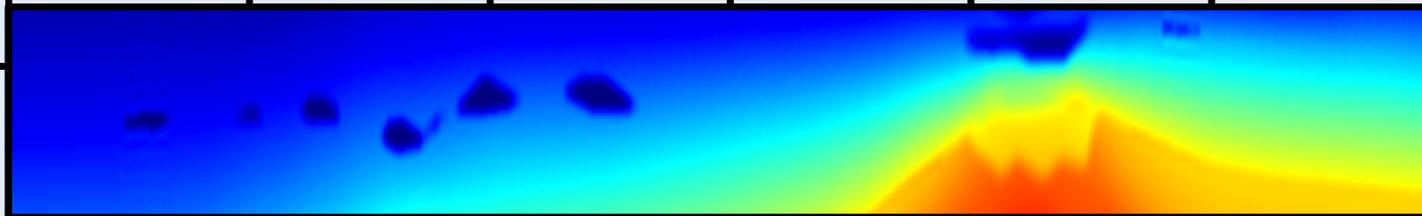


Imaging of gas clouds

Horizontal location (km)

0 4 8 12 16 20

2

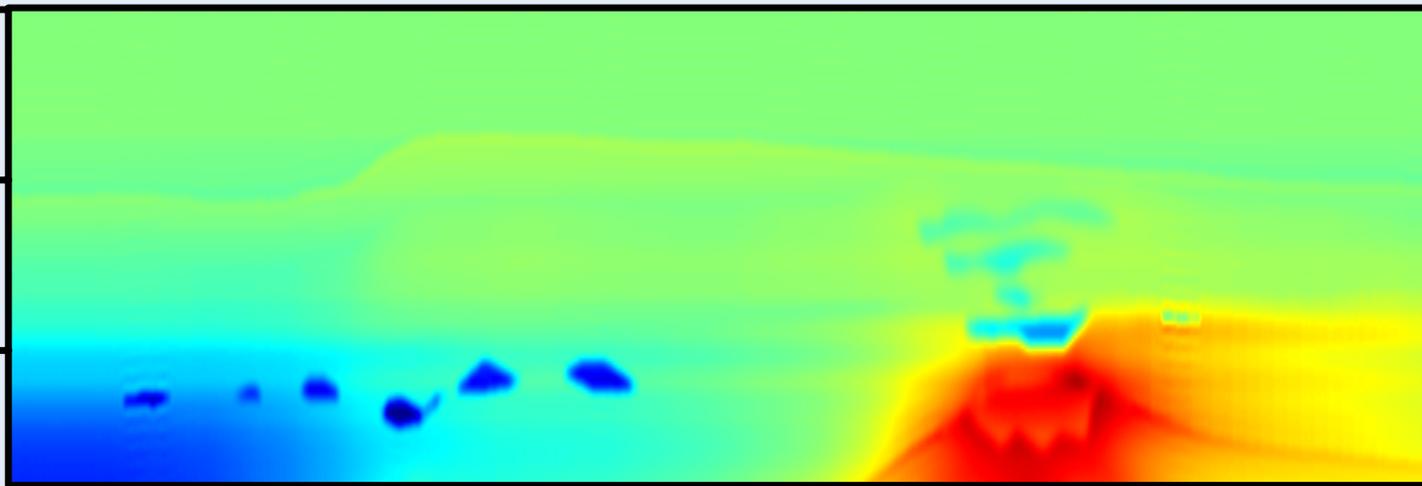


True
model

0

1

2



True
-
Initial

-0.8

0

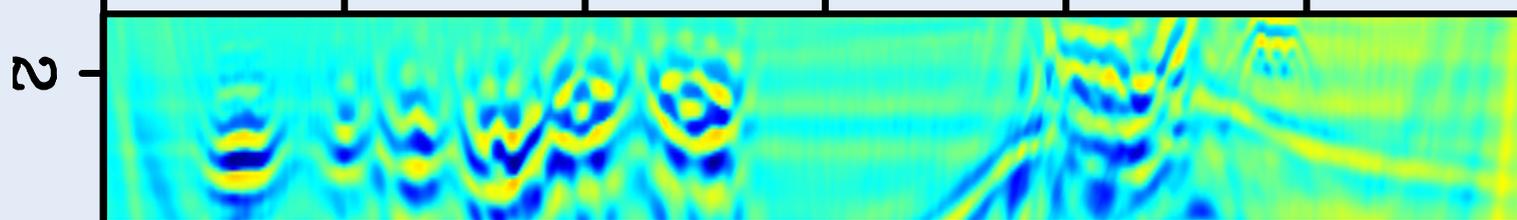
0.8

Vel. (km/s)

Gas clouds after 10 iterations

Horizontal location (km)

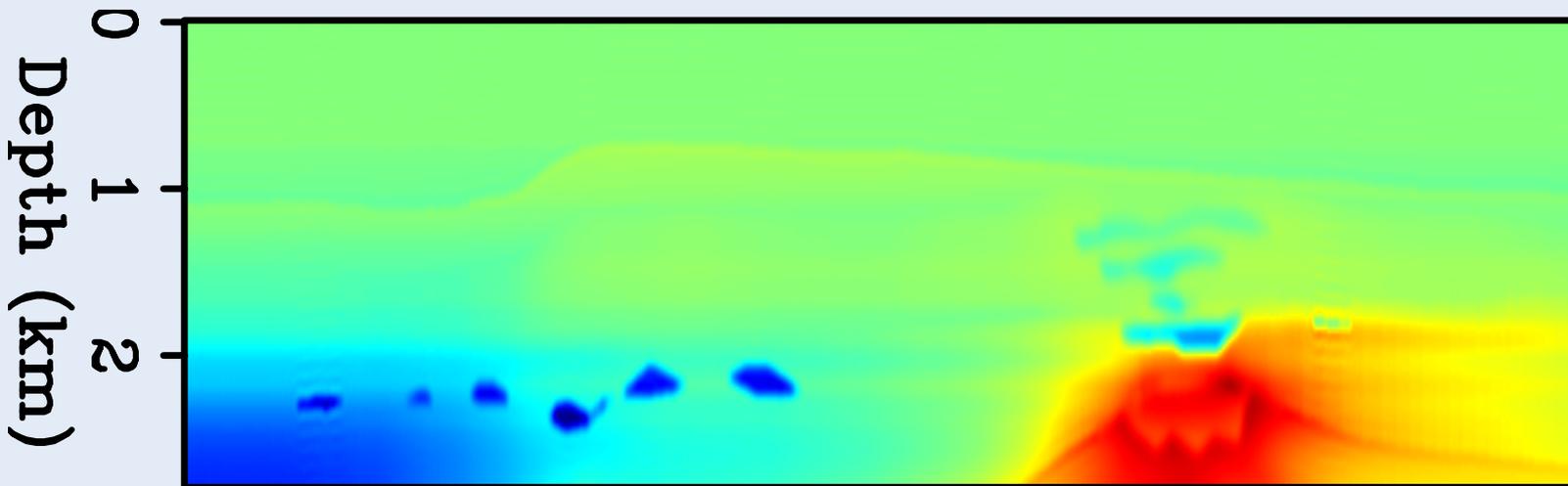
0 4 8 12 16 20



10th iter.

—

Initial



True

—

Initial



-0.8

0

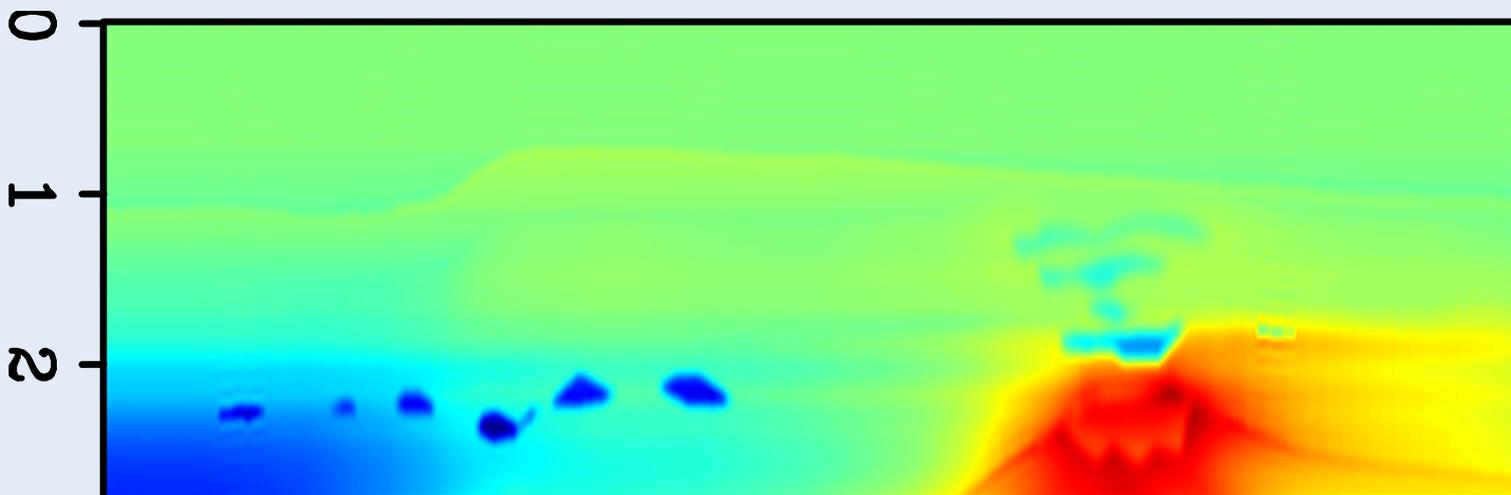
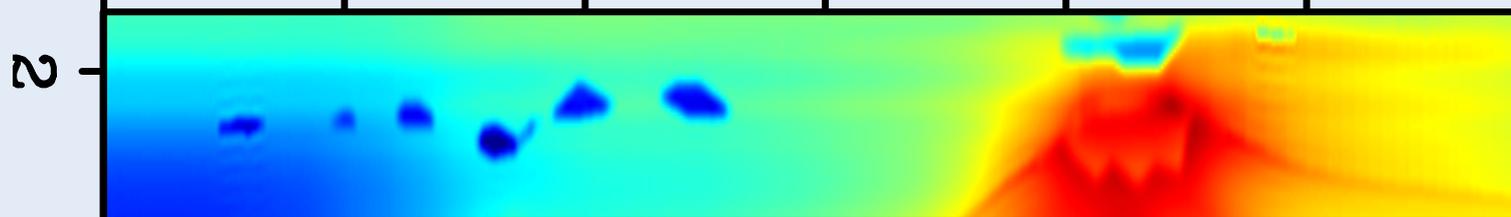
0.8

Vel. (km/s)

Gas clouds in true model

Horizontal location (km)

0 4 8 12 16 20



-0.8

0

0.8

Vel. (km/s)

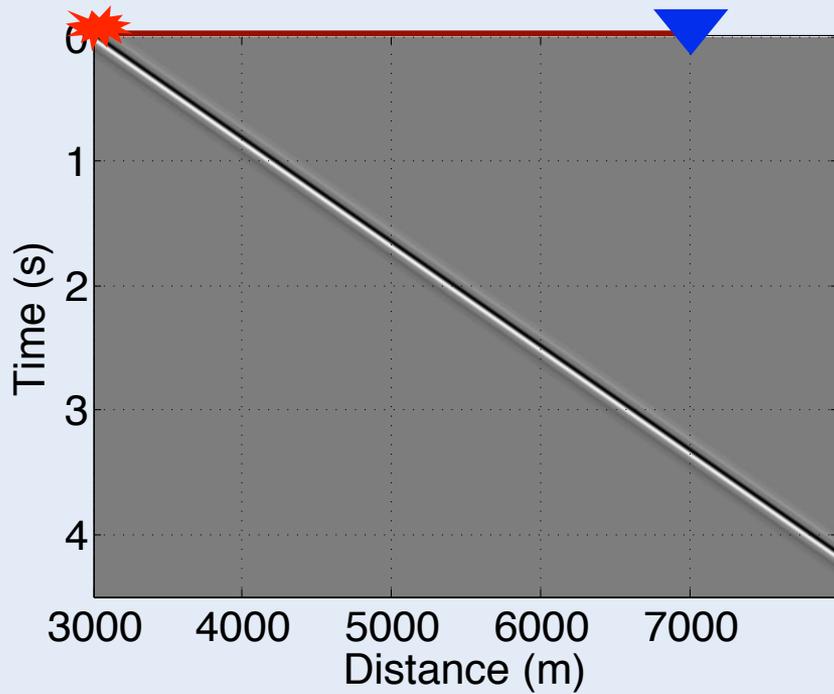
FWI

$$J_{\text{FWI}}(\mathbf{v}) = \frac{1}{2} \|\mathcal{L}(\mathbf{v}) - \mathbf{d}\|_2^2$$

where: J is the objective function to optimize,
 \mathcal{L} is non-linear modeling operator,
 \mathbf{v} is velocity model,
 \mathbf{d} are data.

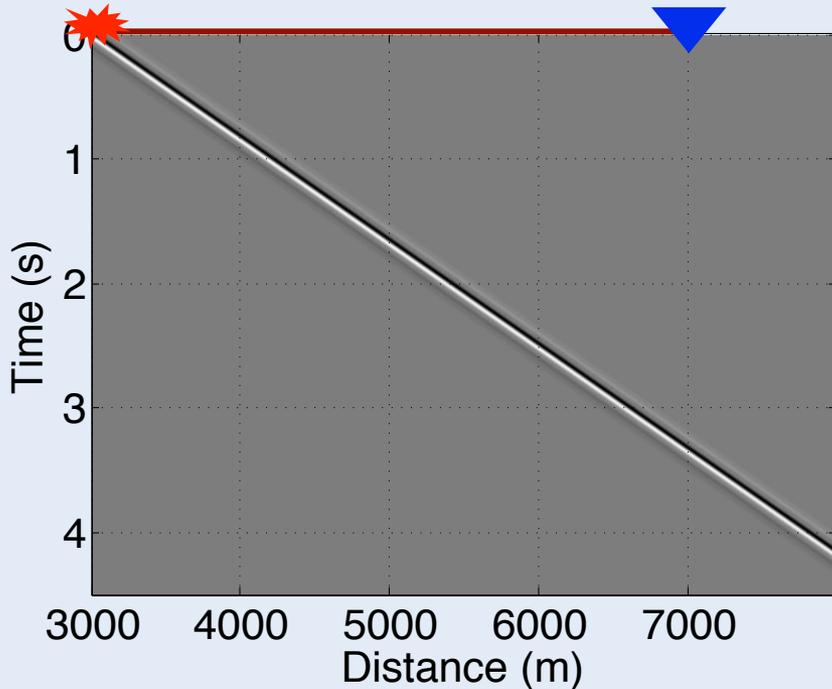
1D transmission: $V_{\text{true}} = C$

Wavefield: $V = V_{\text{true}}$

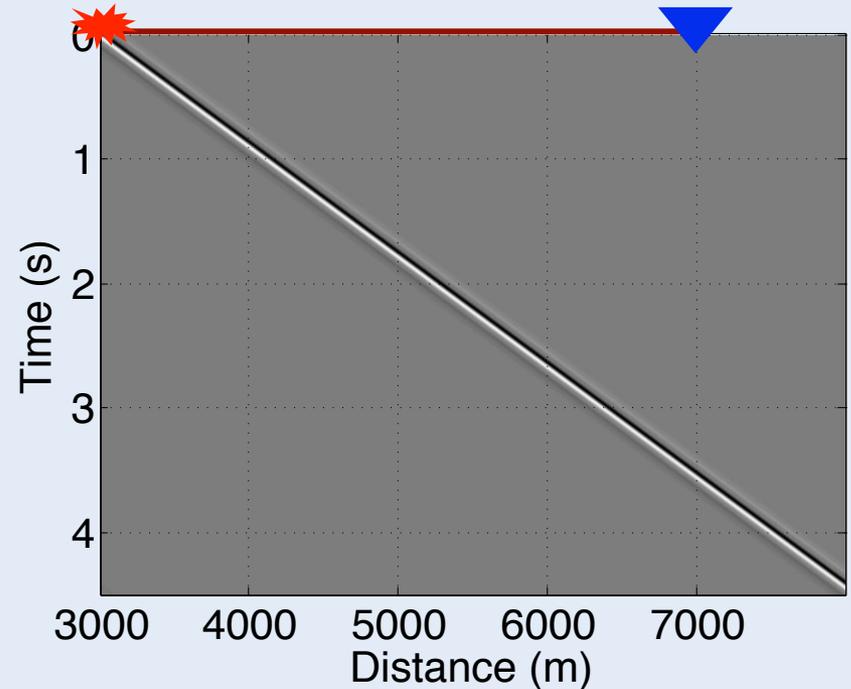


1D transmission: $V_{\text{true}} = C; V_{\text{start}} < V_{\text{true}}$

Wavefield: $V = V_{\text{true}}$

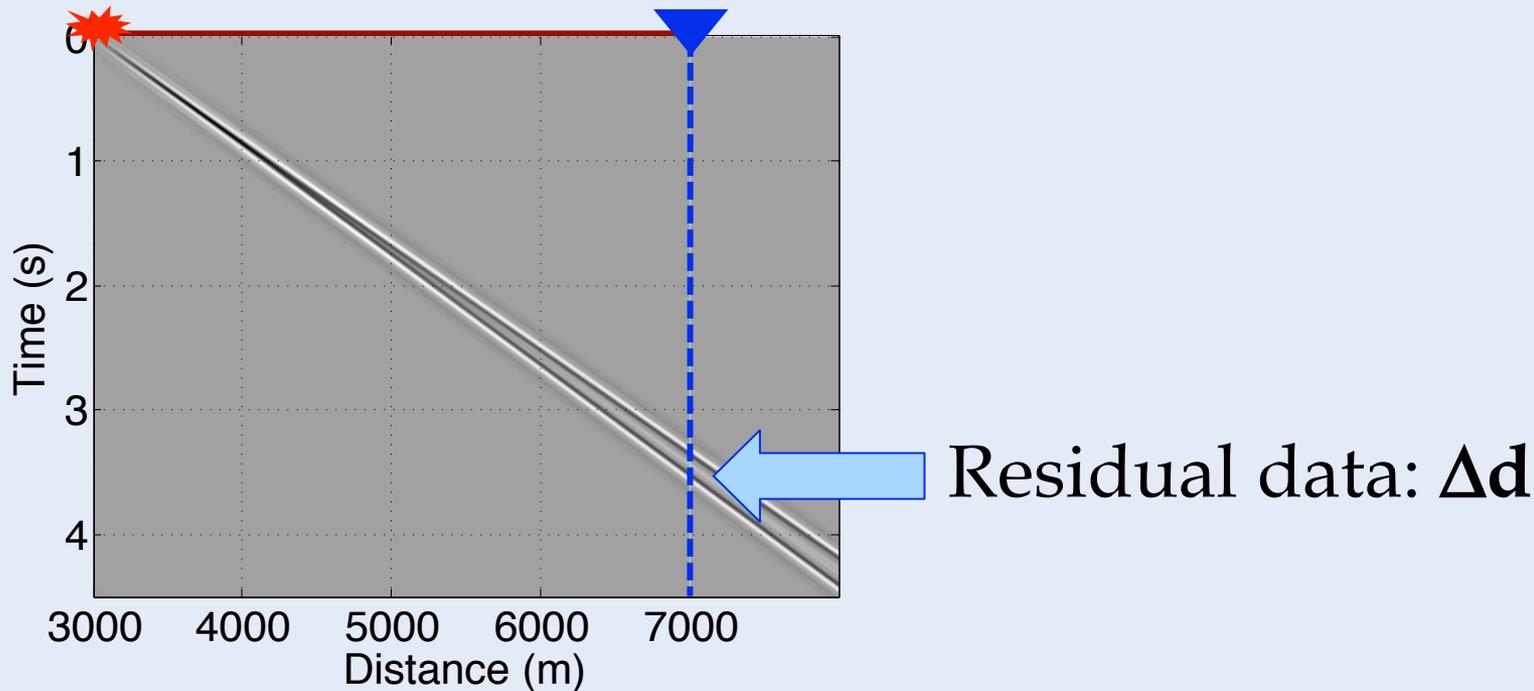


Wavefield: $V = V_{\text{start}}$



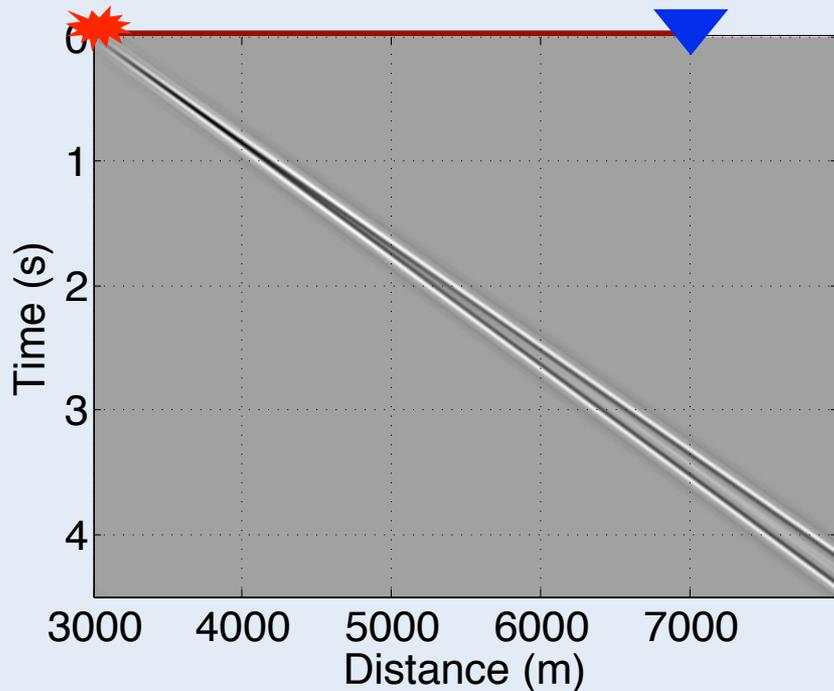
1D transmission: $V_{\text{true}} = C$; $V_{\text{start}} < V_{\text{true}}$

Residual wavefield: ΔP

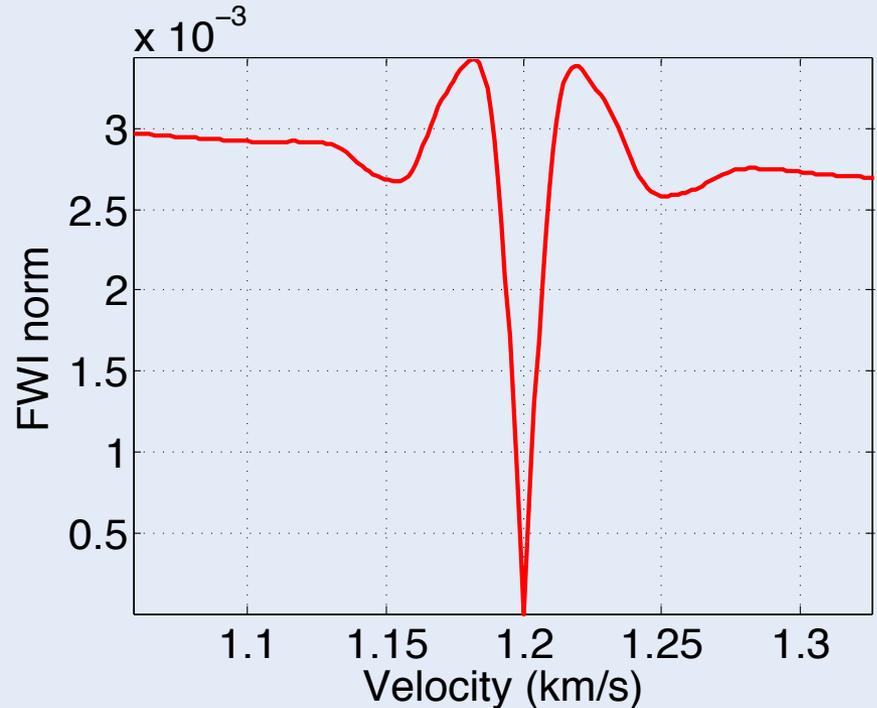


1D transmission: FWI norm

Residual wavefield: ΔP



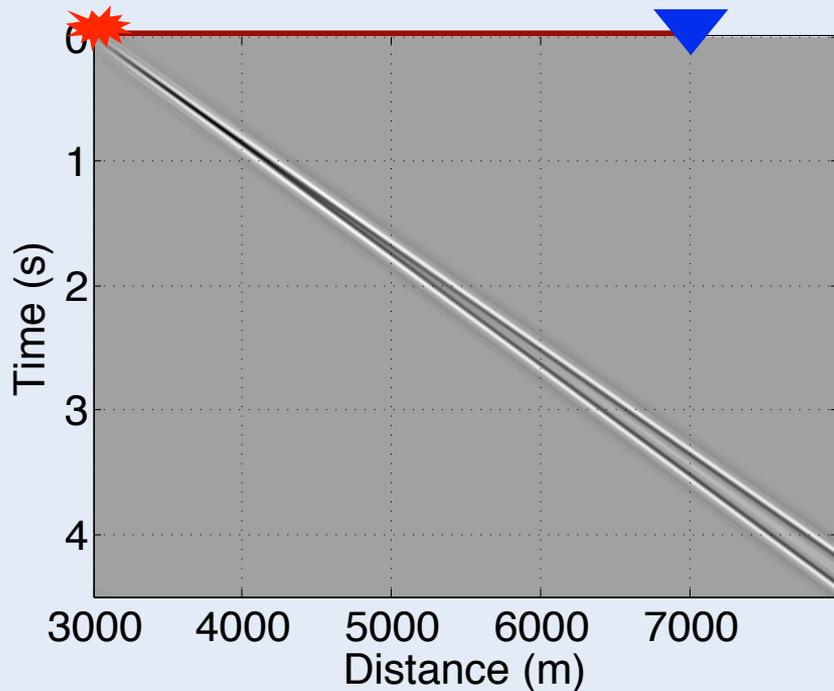
FWI norm



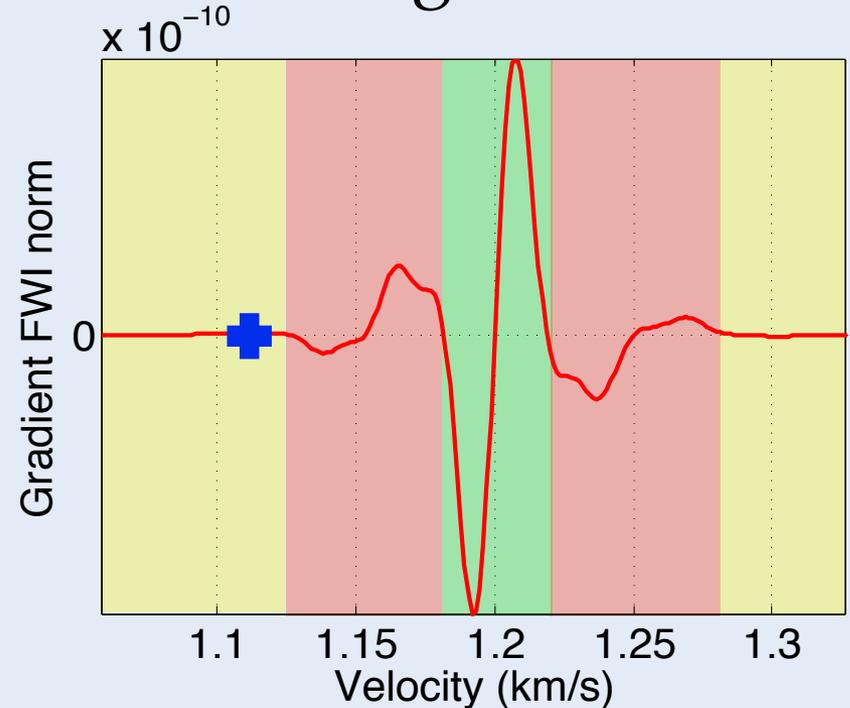
$$\|\mathcal{L}(\mathbf{v}) - \mathbf{d}\|_2^2$$

1D transmission: FWI gradient

Residual wavefield



FWI gradient



$$\nabla \left\| \mathcal{L}(\mathbf{v}) - \mathbf{d} \right\|_2^2$$

Derivation of first-order Born scattering

Full non-linear scattering

$$\left[\partial_{tt} - \mathbf{v}_0^2 \nabla^2 \right] \mathbf{P}_0 = \mathbf{f}$$

$$\left[\partial_{tt} - \mathbf{v}_0^2 \nabla^2 \right] \delta \mathbf{P} = \delta \mathbf{v}^2 \nabla^2 (\mathbf{P}_0 + \delta \mathbf{P})$$

\mathbf{v}_0 : Background velocity

\mathbf{P}_0 : Background wavefield

$\delta \mathbf{v}$: Velocity perturbation

$\delta \mathbf{P}$: Scattered wavefield

\mathbf{f} : Source function

First-order Born scattering

$$\left[\partial_{tt} - \mathbf{v}_0^2 \nabla^2 \right] \mathbf{P}_0 = \mathbf{f}$$

$$\left[\partial_{tt} - \mathbf{v}_0^2 \nabla^2 \right] \delta \hat{\mathbf{P}} = \delta \mathbf{v}^2 \nabla^2 \mathbf{P}_0$$

$\delta \hat{\mathbf{P}}$: Born scattered wavefield



Linearized τ extension $\tilde{\mathcal{L}}(\tilde{\mathbf{v}}) = \mathcal{L}(\mathbf{v}_0) + \tilde{\mathbf{L}}\delta\tilde{\mathbf{v}}^2(\tau)$

Full non-linear scattering

$$\left[\partial_{tt} - \mathbf{v}_0^2 \nabla^2 \right] \mathbf{P}_0 = \mathbf{f}$$

$$\left[\partial_{tt} - \mathbf{v}_0^2 \nabla^2 \right] \delta\mathbf{P} = \delta\mathbf{v}^2 \nabla^2 (\mathbf{P}_0 + \delta\mathbf{P})$$

\mathbf{v}_0 : Background velocity

\mathbf{P}_0 : Background wavefield

$\delta\mathbf{v}$: Velocity perturbation

$\delta\mathbf{P}$: Scattered wavefield

\mathbf{f} : Source function

Linearized τ extension

$$\left[\partial_{tt} - \mathbf{v}_0^2 \nabla^2 \right] \mathbf{P}_0 = \mathbf{f}$$

$$\left[\partial_{tt} - \mathbf{v}_0^2 \nabla^2 \right] \delta\tilde{\mathbf{P}} = \delta\tilde{\mathbf{v}}(\tau)^2 * \nabla^2 \mathbf{P}_0$$

$\delta\tilde{\mathbf{v}}(\tau)$: **Extended-velocity**
perturbation

$\delta\tilde{\mathbf{P}}$: **New** scattered wavefield



Linearized τ extension $\tilde{\mathcal{L}}(\tilde{\mathbf{v}}) = \mathcal{L}(\mathbf{v}_0) + \tilde{\mathbf{L}}\delta\tilde{\mathbf{v}}^2(\tau)$

Full non-linear scattering

$$\left[\partial_{tt} - \mathbf{v}_0^2 \nabla^2 \right] \mathbf{P}_0 = \mathbf{f}$$

$$\left[\partial_{tt} - \mathbf{v}_0^2 \nabla^2 \right] \delta\mathbf{P} = \delta\mathbf{v}^2 \nabla^2 (\mathbf{P}_0 + \delta\mathbf{P})$$

\mathbf{v}_0 : Background velocity

\mathbf{P}_0 : Background wavefield

$\delta\mathbf{v}$: Velocity perturbation

$\delta\mathbf{P}$: Scattered wavefield

\mathbf{f} : Source function

Linearized τ extension

$$\left[\partial_{tt} - \mathbf{v}_0^2 \nabla^2 \right] \mathbf{P}_0 = \mathbf{f}$$

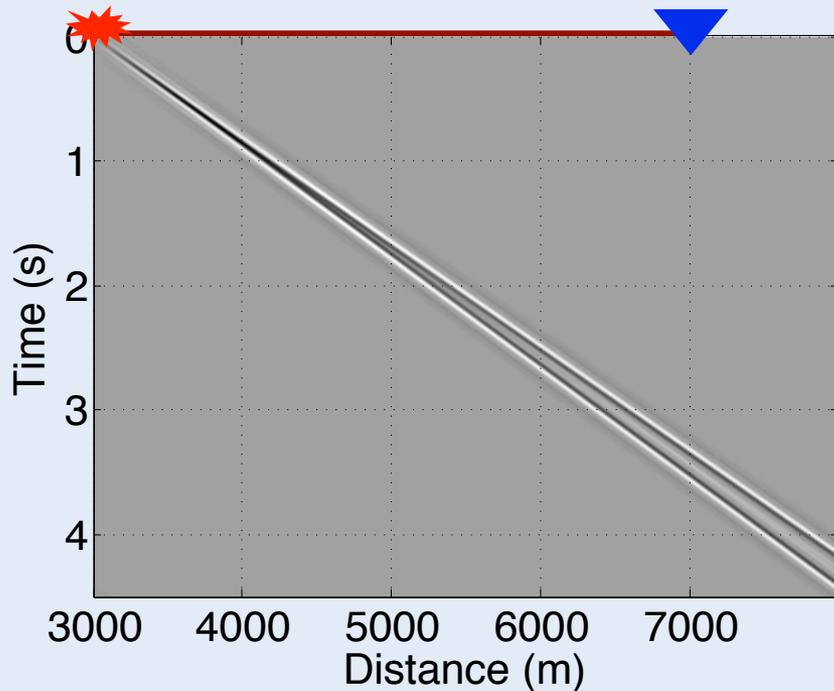
$$\left[\partial_{tt} - \mathbf{v}_0^2 \nabla^2 \right] \delta\tilde{\mathbf{P}} = \delta\tilde{\mathbf{v}}(\tau)^2 * \nabla^2 \mathbf{P}_0$$

$\delta\tilde{\mathbf{v}}(\tau)$: Extended-velocity
perturbation

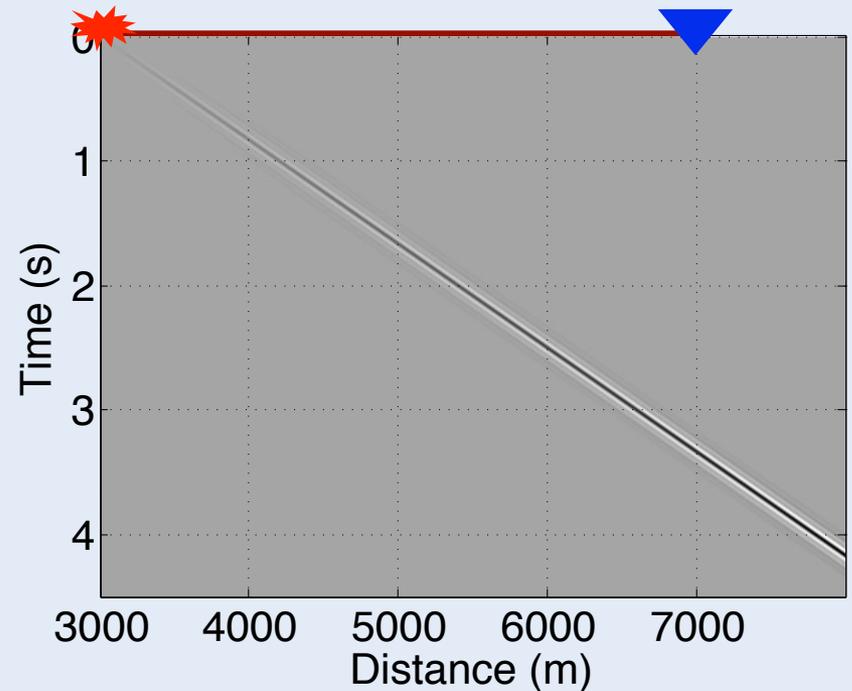
$\delta\tilde{\mathbf{P}}$: New scattered wavefield

1D example: Born failure

Residual wavefield



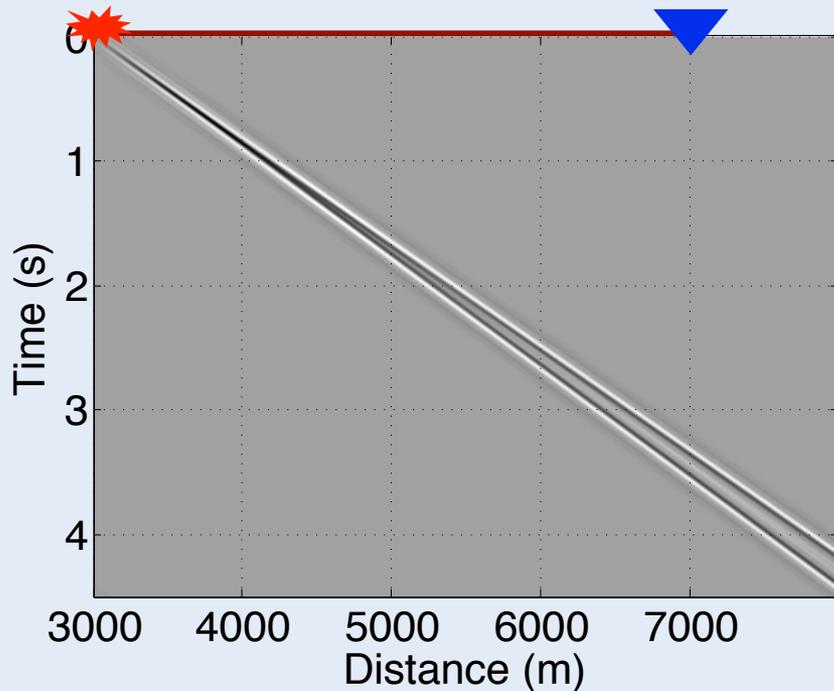
First-order Born scattered



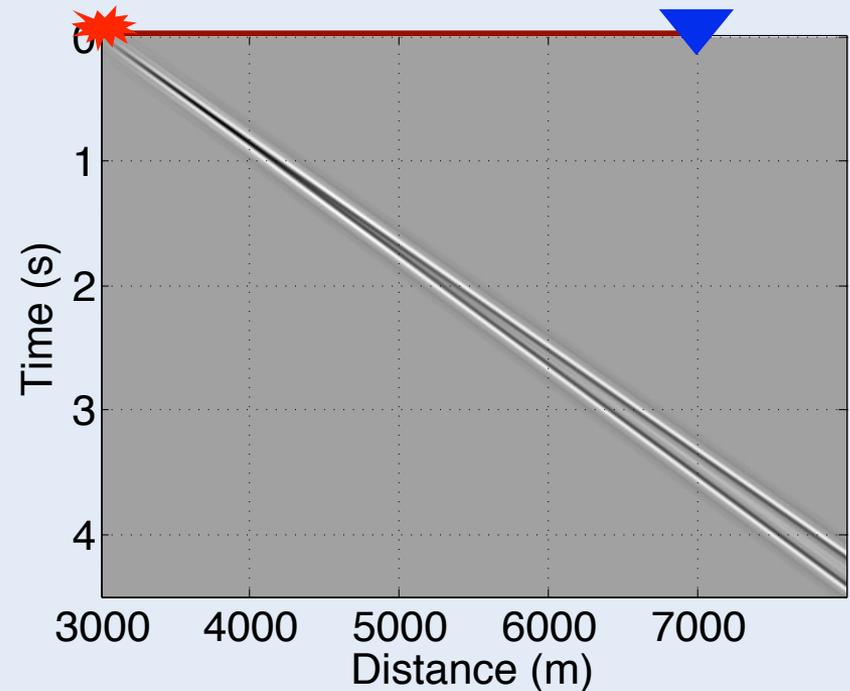
$$\left[\partial_{tt} - v_0^2 \nabla^2 \right] \delta \hat{\mathbf{P}} = \delta v^2 \nabla^2 \mathbf{P}_0$$

1D example: τ -extension success

Residual wavefield



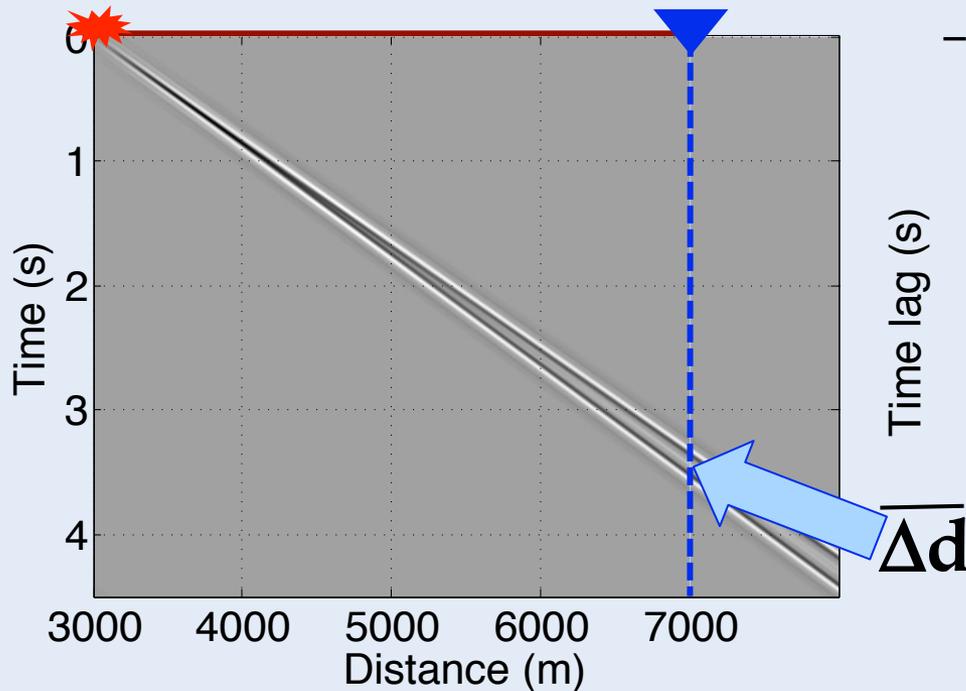
Linearized τ extension



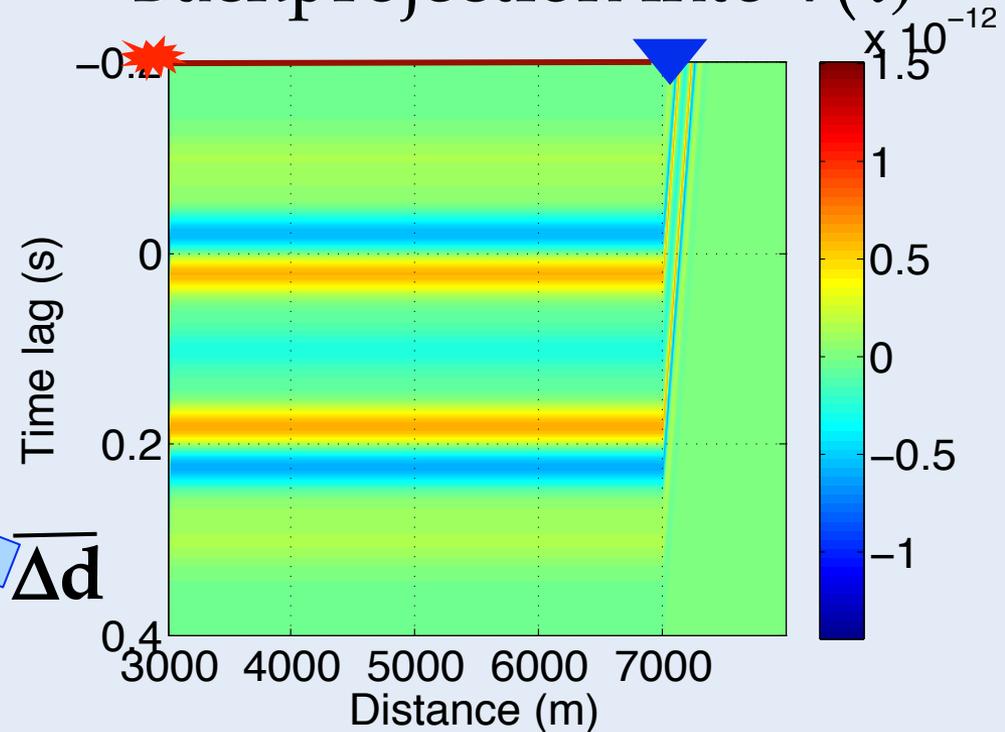
$$\left[\partial_{tt} - v_0^2 \nabla^2 \right] \delta \tilde{\mathbf{P}} = \delta \tilde{\mathbf{v}} (\tau)^2 * \nabla^2 \mathbf{P}_0$$

1D example: τ -extended gradient

Residual wavefield



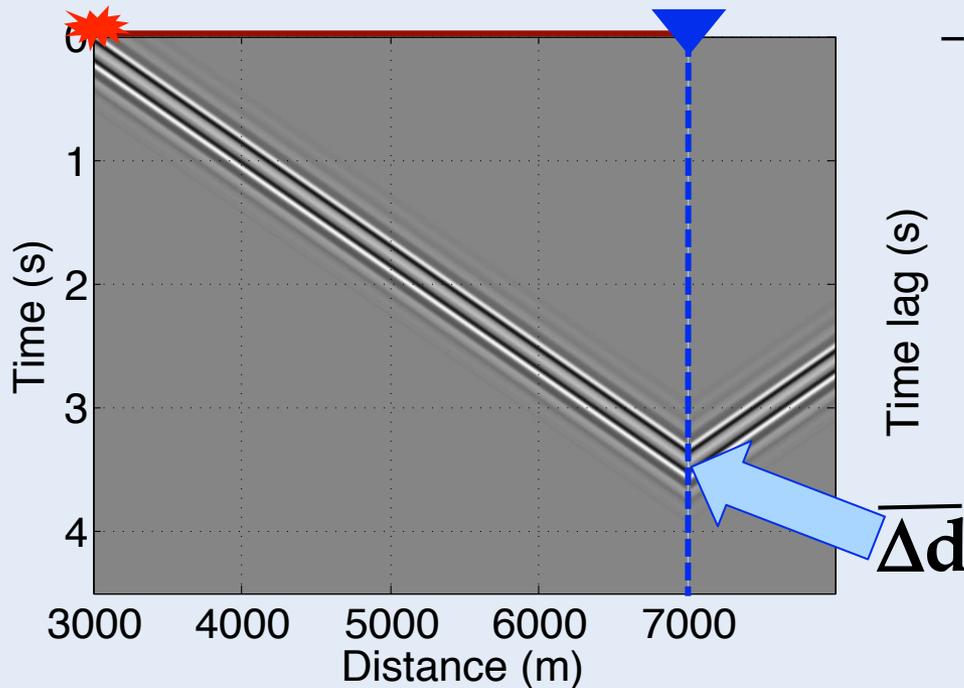
Backprojection into $\mathbf{v}(\tau)$



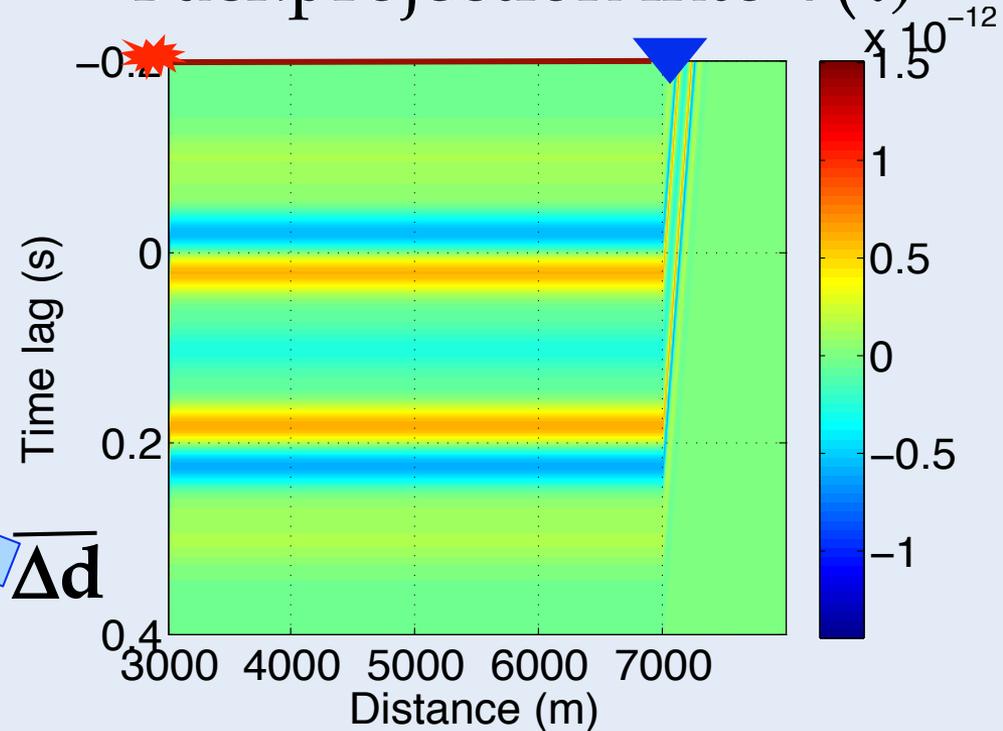
$$\Delta \tilde{\mathbf{v}}(\tau) = \tilde{\mathbf{L}}' \Delta \mathbf{d}$$

1D example: τ -extended modeling

Modeled data from $\Delta \mathbf{v}(\tau)$



Backprojection into $\mathbf{v}(\tau)$



$$\overline{\Delta \mathbf{d}} = \tilde{\mathbf{L}} \tilde{\mathbf{L}}' \Delta \mathbf{d}$$

$$\Delta \tilde{\mathbf{v}}(\tau) = \tilde{\mathbf{L}}' \Delta \mathbf{d}$$

TFWI

$$J_{\text{TFWI}}(\tilde{\mathbf{v}}) = \frac{1}{2} \left\| \tilde{\mathcal{L}}(\tilde{\mathbf{v}}) - \mathbf{d} \right\|_2^2 + \varepsilon \left\| \tilde{\mathcal{F}}(\tilde{\mathbf{v}}) \right\|_2^2$$

$\tilde{\mathcal{F}}(\tilde{\mathbf{v}})$: measures focusing of $\tilde{\mathbf{v}}$
– Stacking after RMO,
+ $\left\| \tau | \tilde{\mathbf{v}} \right\|_2^2$.

Symes, Geophysical Prospective, 2008
V(h) and DSO

TFWI

$$J_{\text{TFWI}}(\tilde{\mathbf{v}}) = \frac{1}{2} \left\| \tilde{\mathcal{L}}(\tilde{\mathbf{v}}) - \mathbf{d} \right\|_2^2 + \varepsilon \left\| \tilde{\mathcal{F}}(\tilde{\mathbf{v}}) \right\|_2^2$$

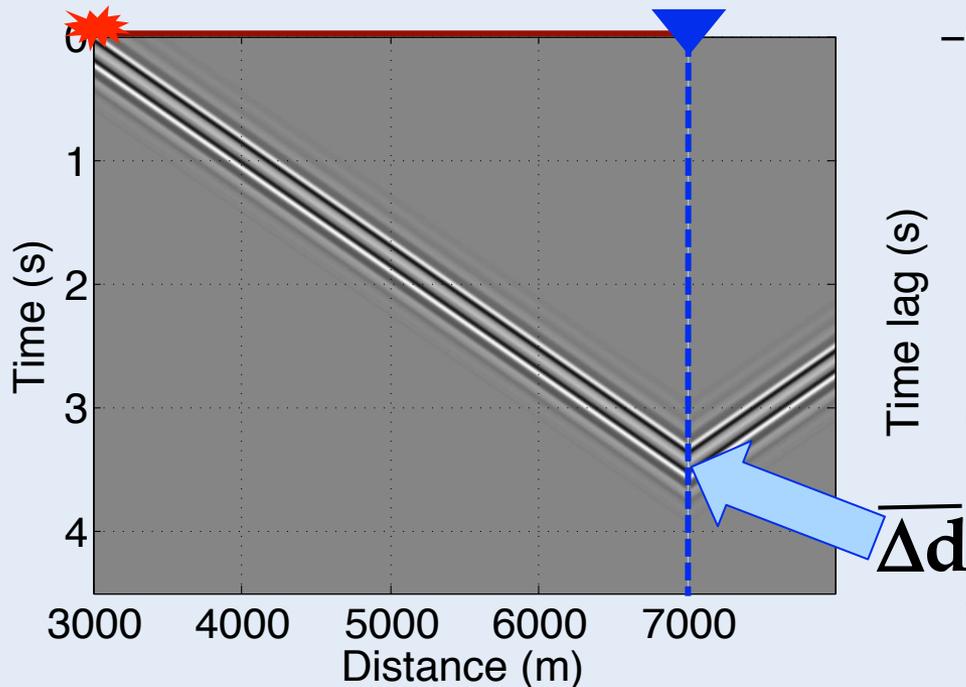
$\tilde{\mathcal{F}}(\tilde{\mathbf{v}})$: measures focusing of $\tilde{\mathbf{v}}$

– Stacking after RMO,

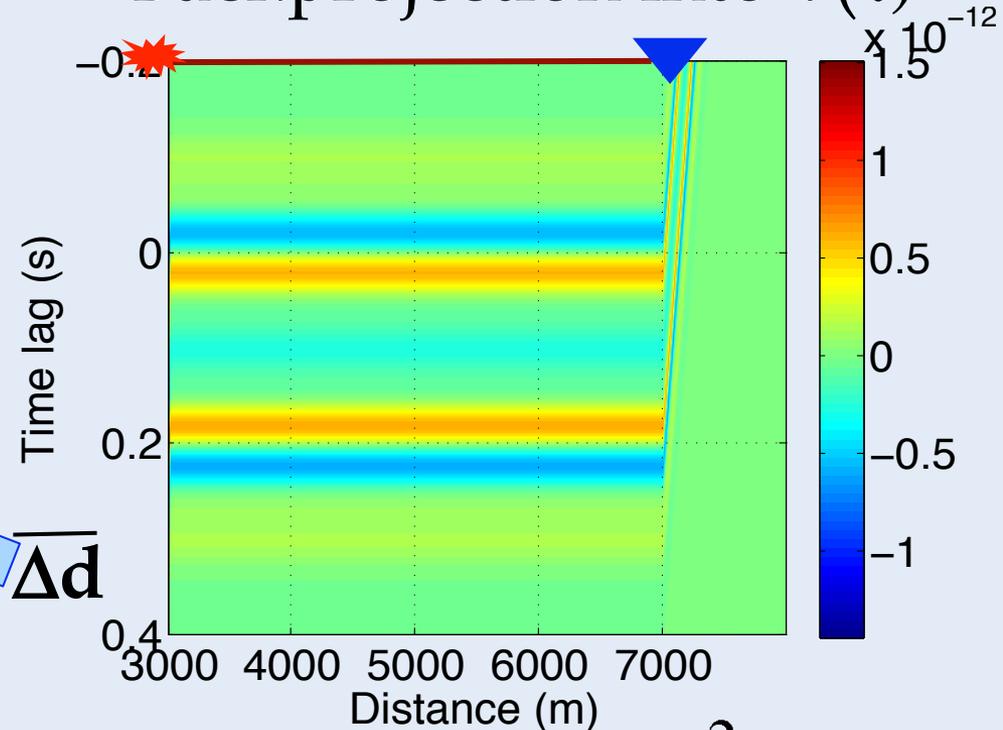
$$+ \left\| \tau \tilde{\mathbf{v}} \right\|_2^2.$$

1D example: gradient of focusing

Modeled data from $\Delta v(\tau)$



Backprojection into $v(\tau)$

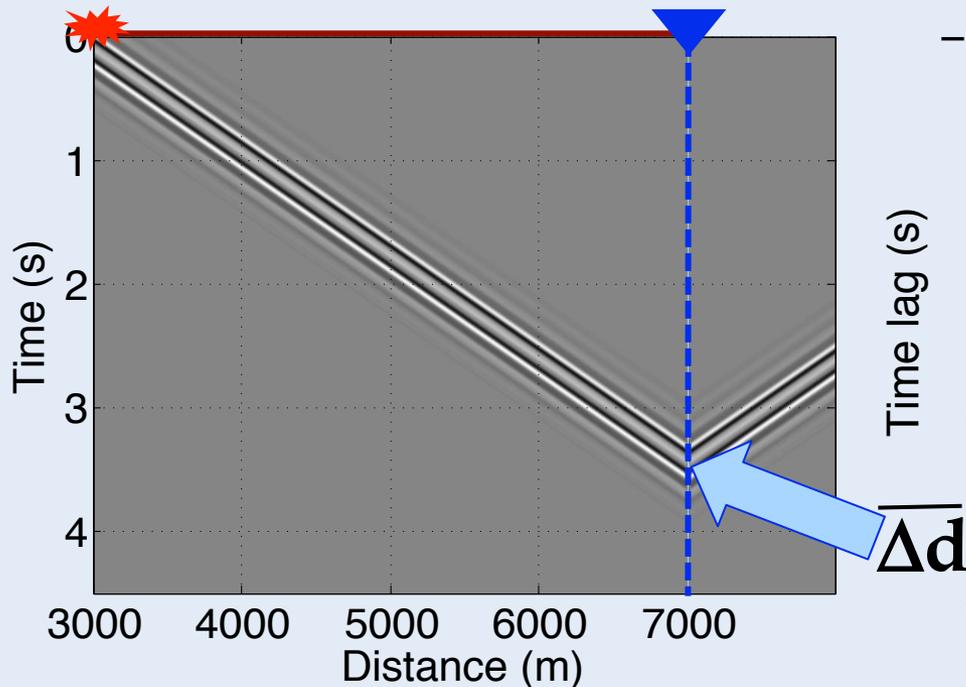


$$J_{\text{TFWI}}(\tilde{\mathbf{v}}_1) = \nabla J_d(\tilde{\mathbf{v}}_1) - \varepsilon \nabla \left\| |\boldsymbol{\tau}| \tilde{\mathbf{v}}_1 \right\|_2^2$$

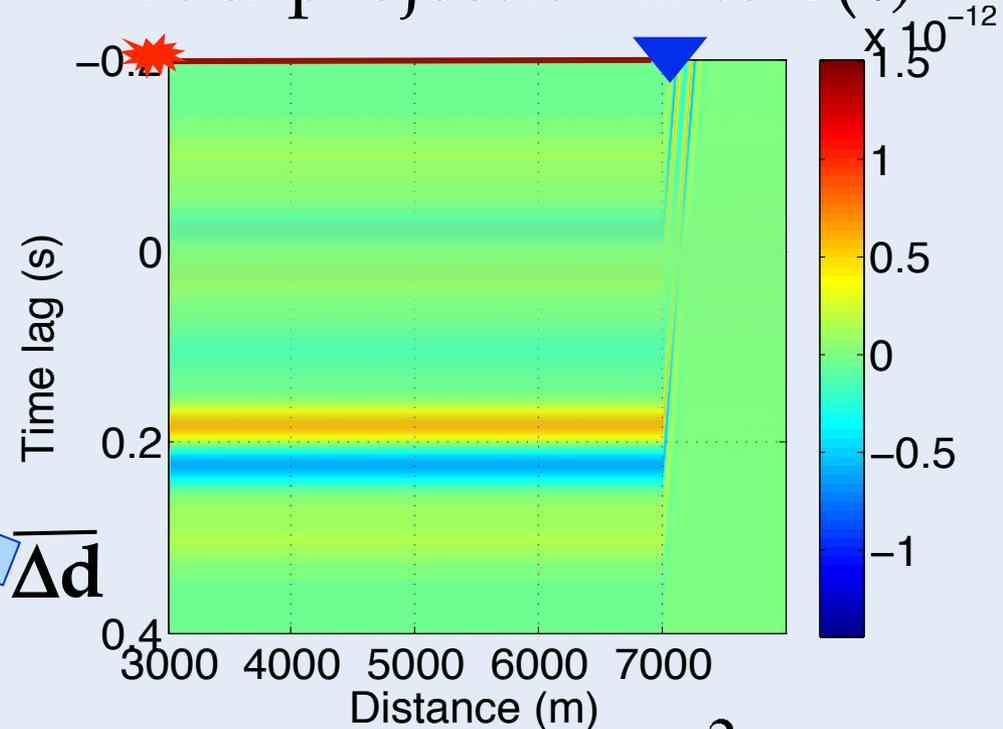
$$\text{with: } \tilde{\mathbf{v}}_1 = \tilde{\mathbf{v}}_0 + \alpha \Delta \tilde{\mathbf{v}}_0$$

1D example: gradient of focusing

Modeled data from $\Delta v(\tau)$



Backprojection into $v(\tau)$

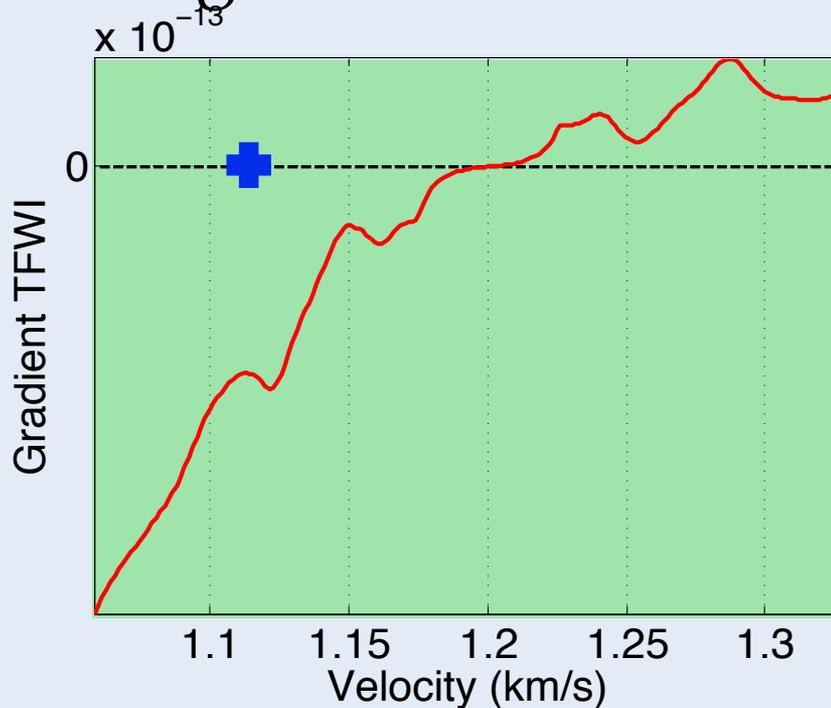


$$J_{\text{TFWI}}(\tilde{\mathbf{v}}_1) = \nabla J_d(\tilde{\mathbf{v}}_1) - \varepsilon \nabla \left\| |\boldsymbol{\tau}| \tilde{\mathbf{v}}_1 \right\|_2^2$$

$$\text{with: } \tilde{\mathbf{v}}_1 = \tilde{\mathbf{v}}_0 + \alpha \Delta \tilde{\mathbf{v}}_0$$

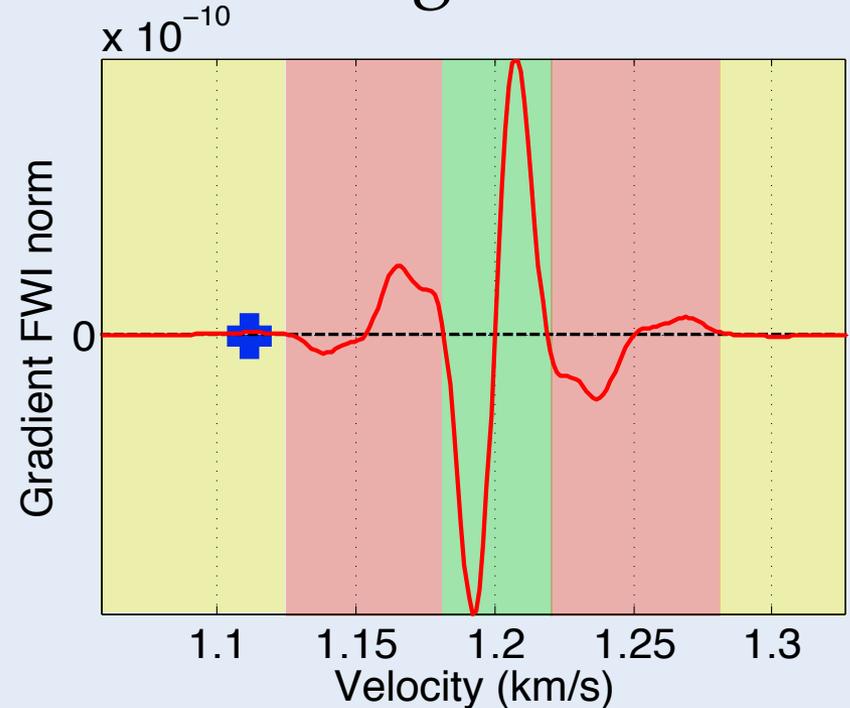
1D example: TFWI vs. FWI gradient

TFWI gradient second iter.



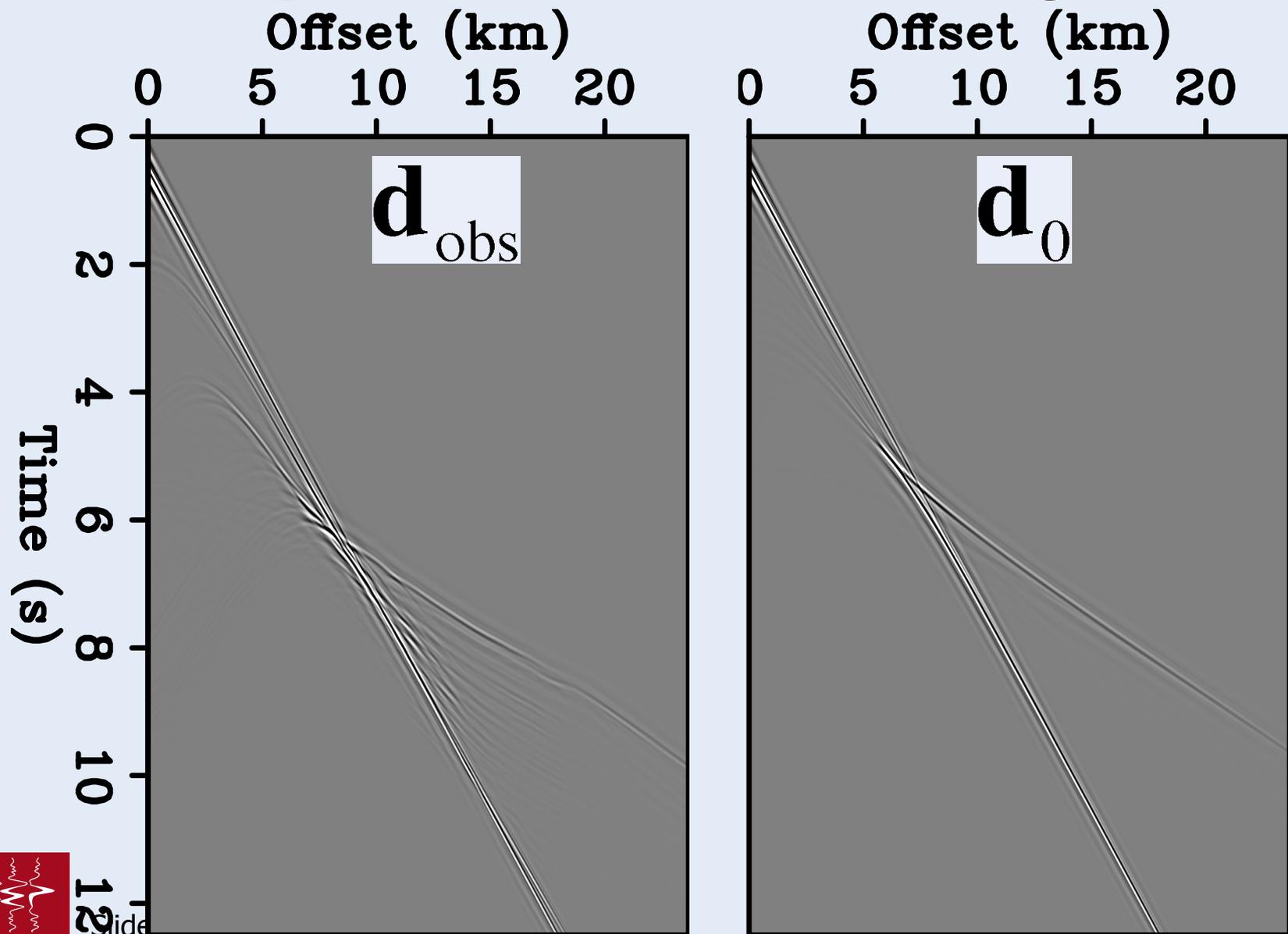
$$\nabla J_{\text{TFWI}}(\tilde{\mathbf{v}}_2)$$

FWI gradient

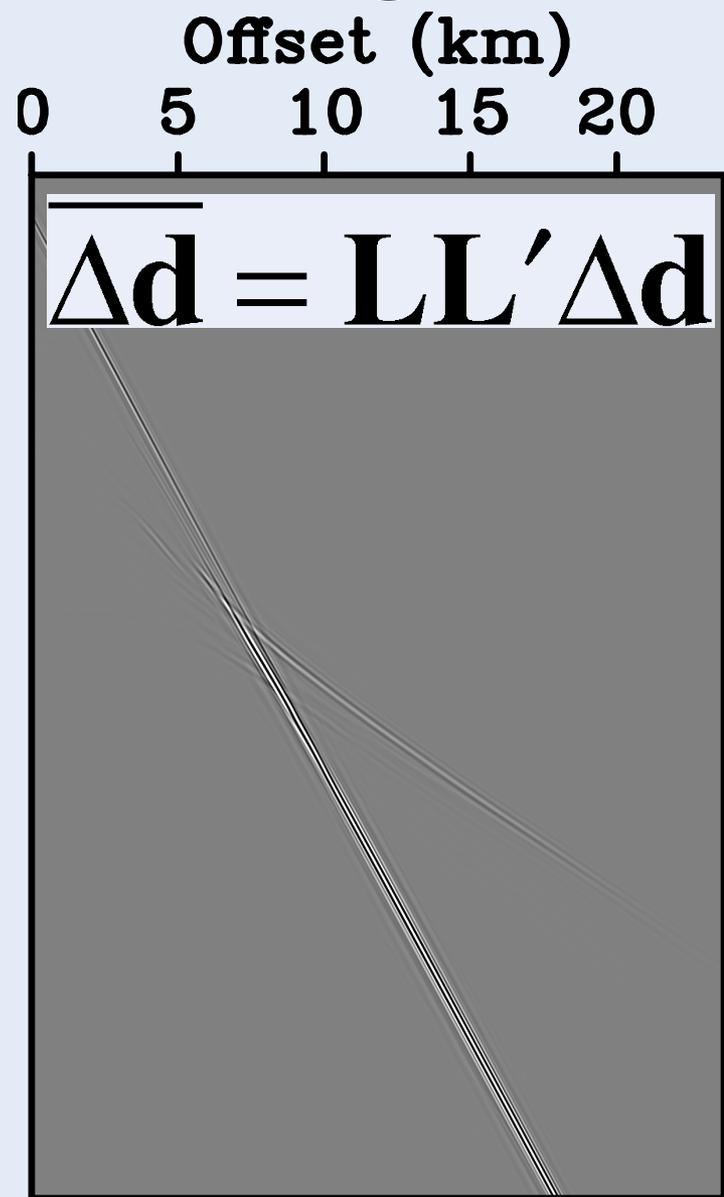
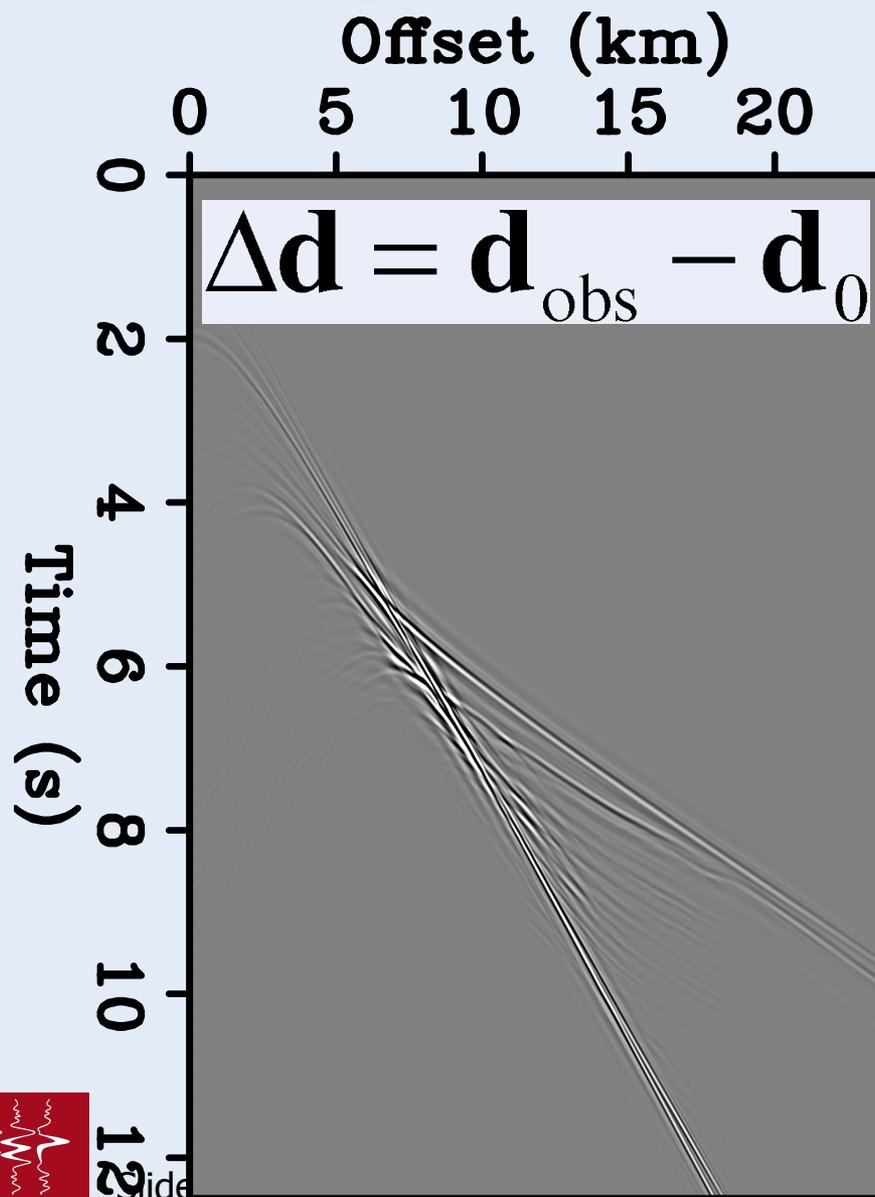


$$\nabla J_{\text{FWI}}(\mathbf{v}_0)$$

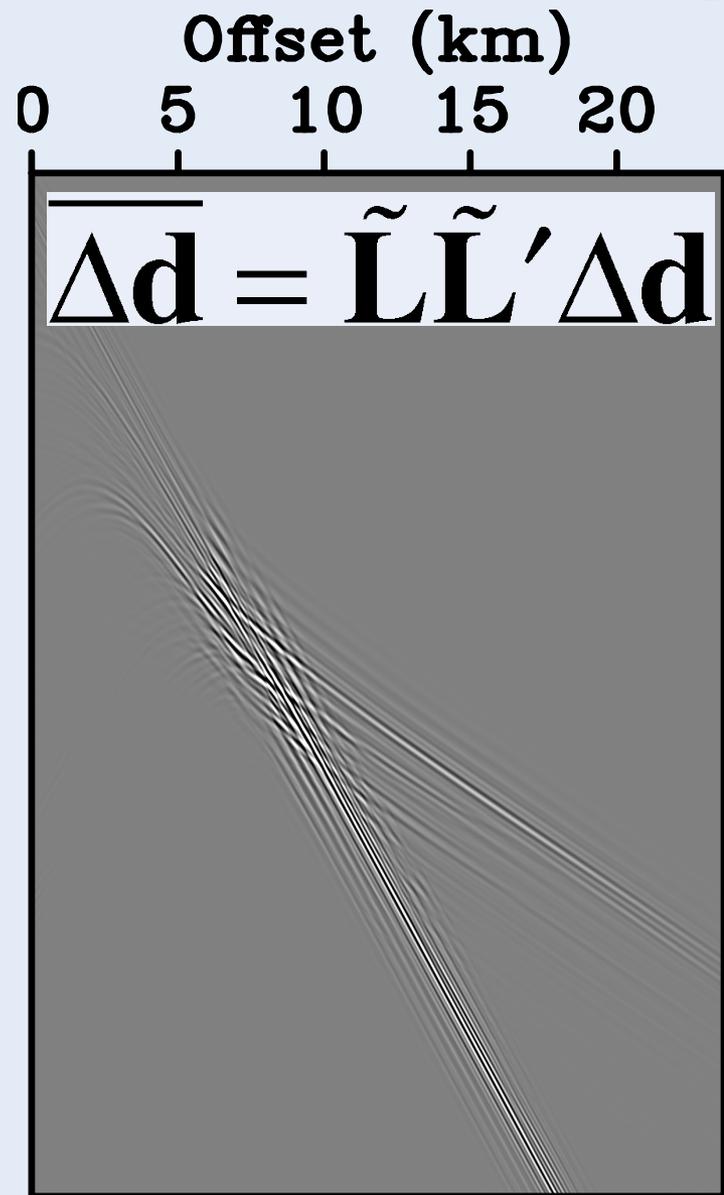
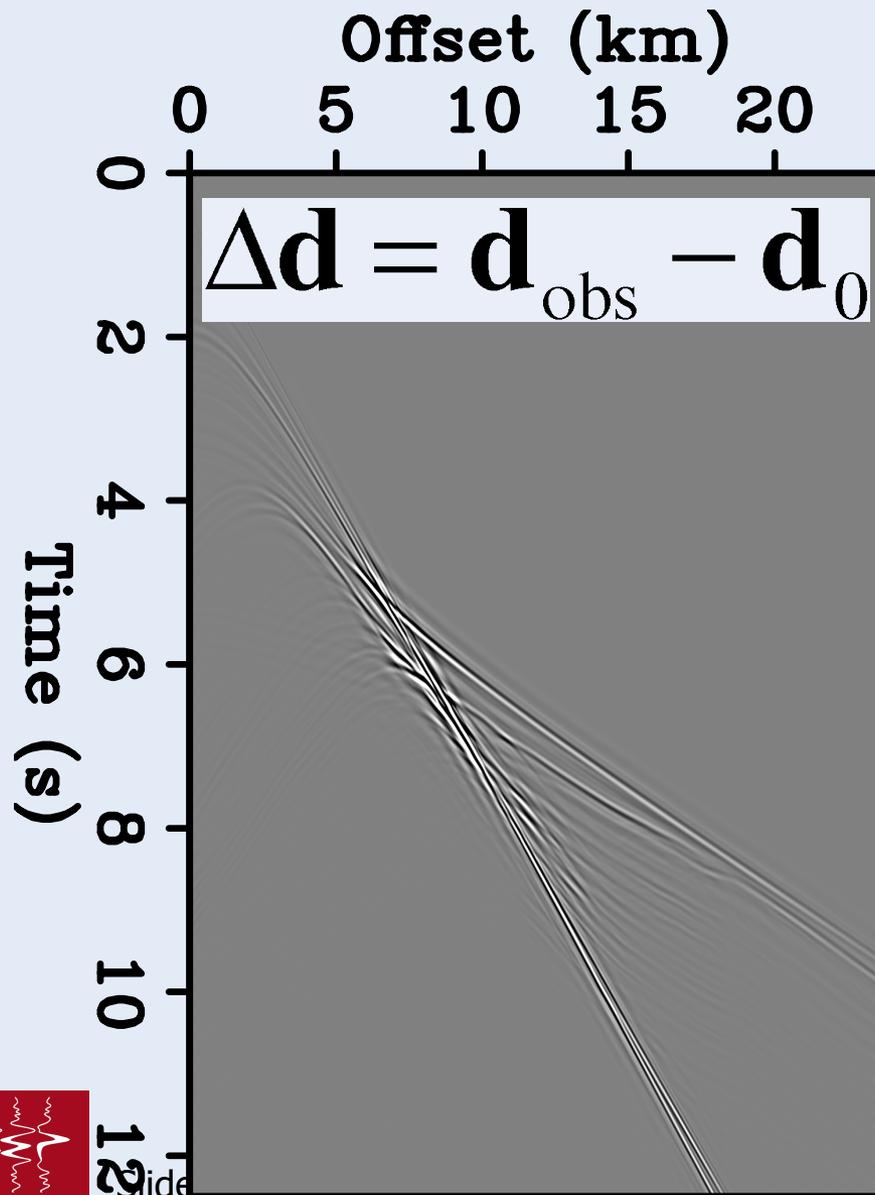
BP example: Leftmost shot gather



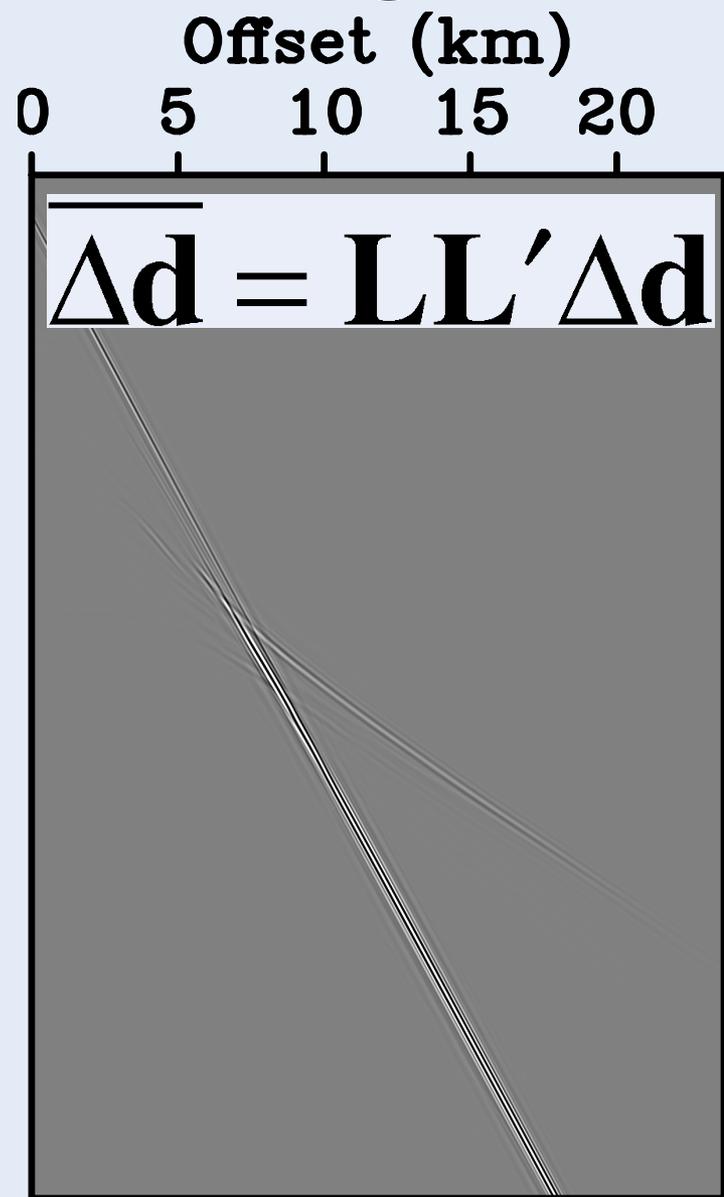
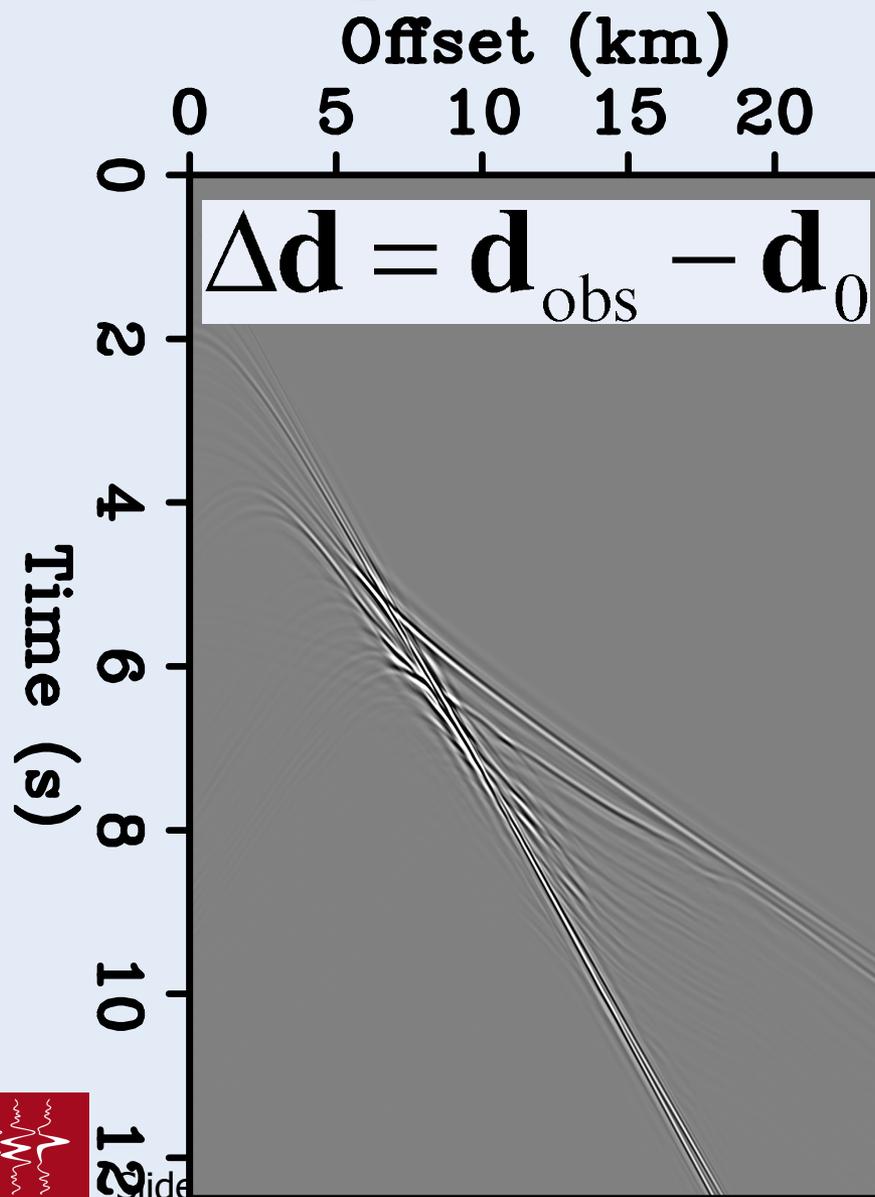
BP example: Born modeling



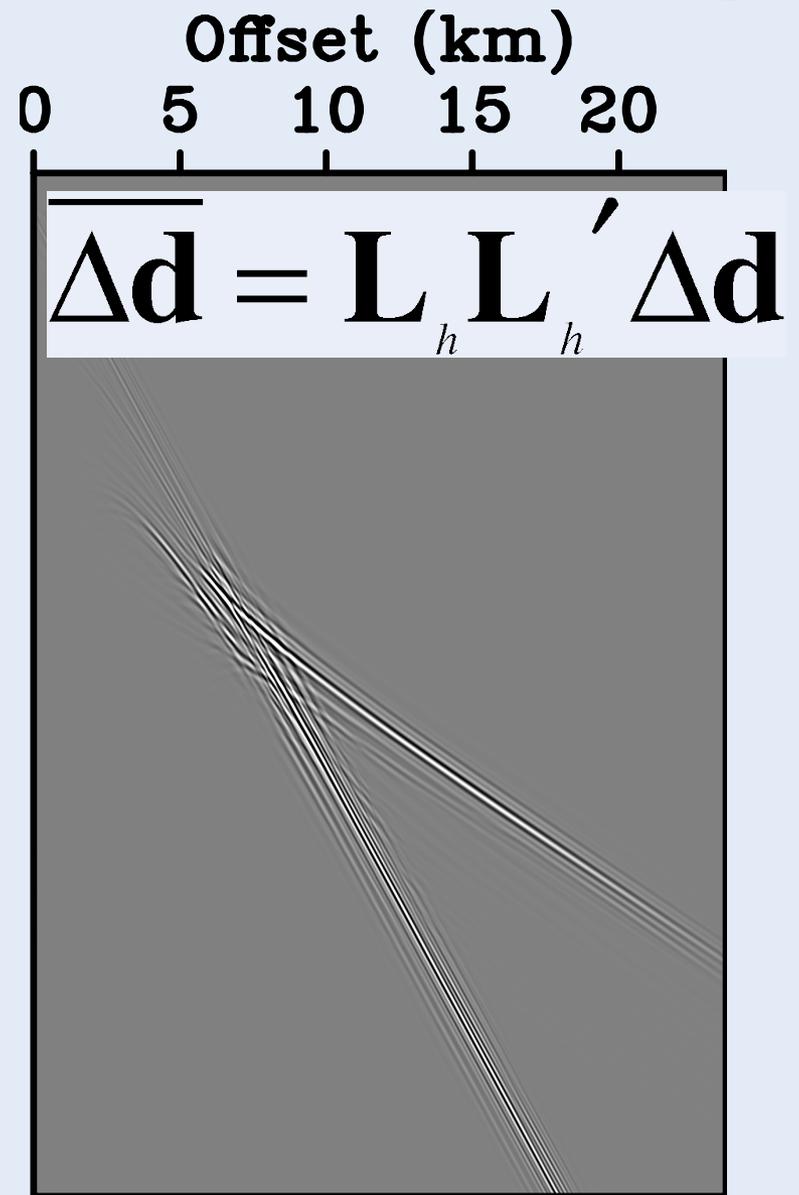
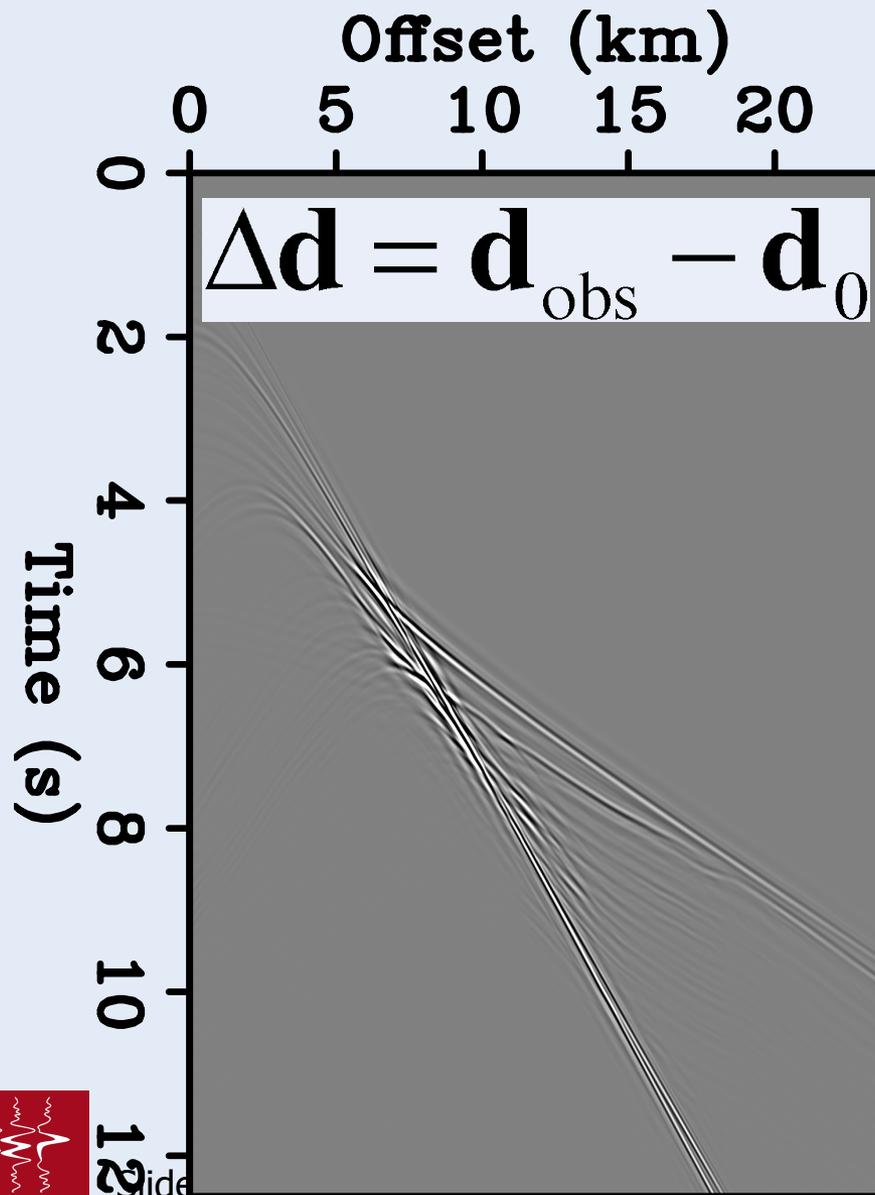
BP example: τ -extended modeling



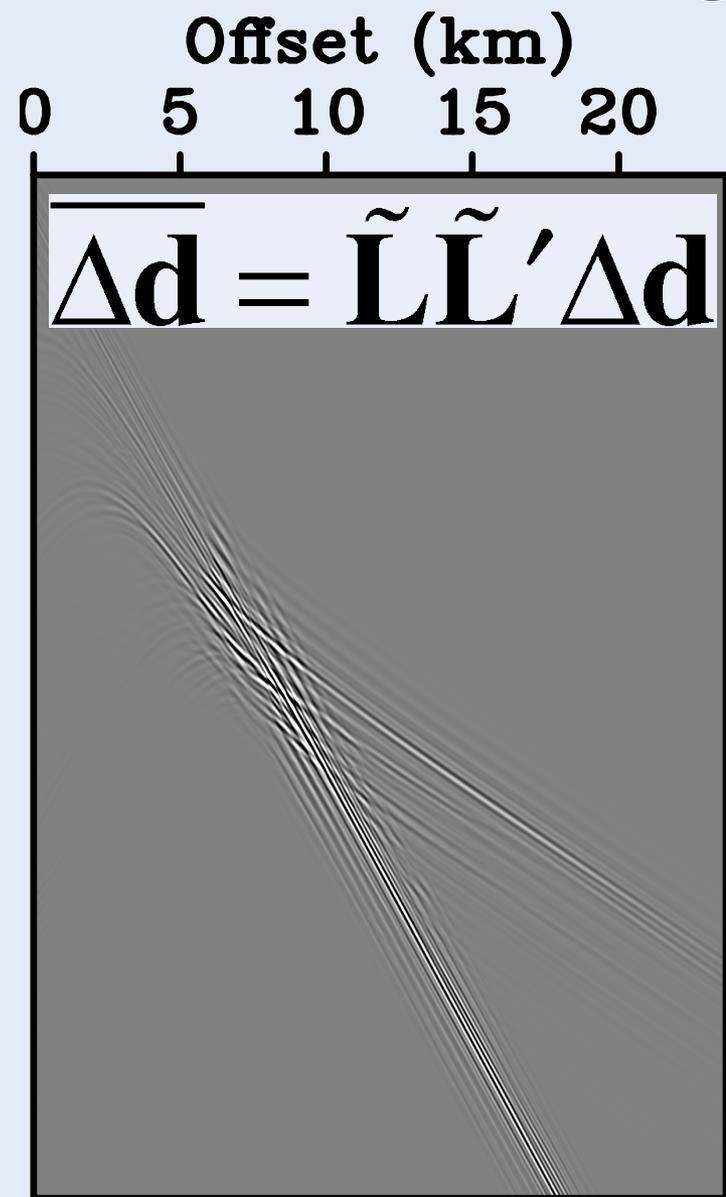
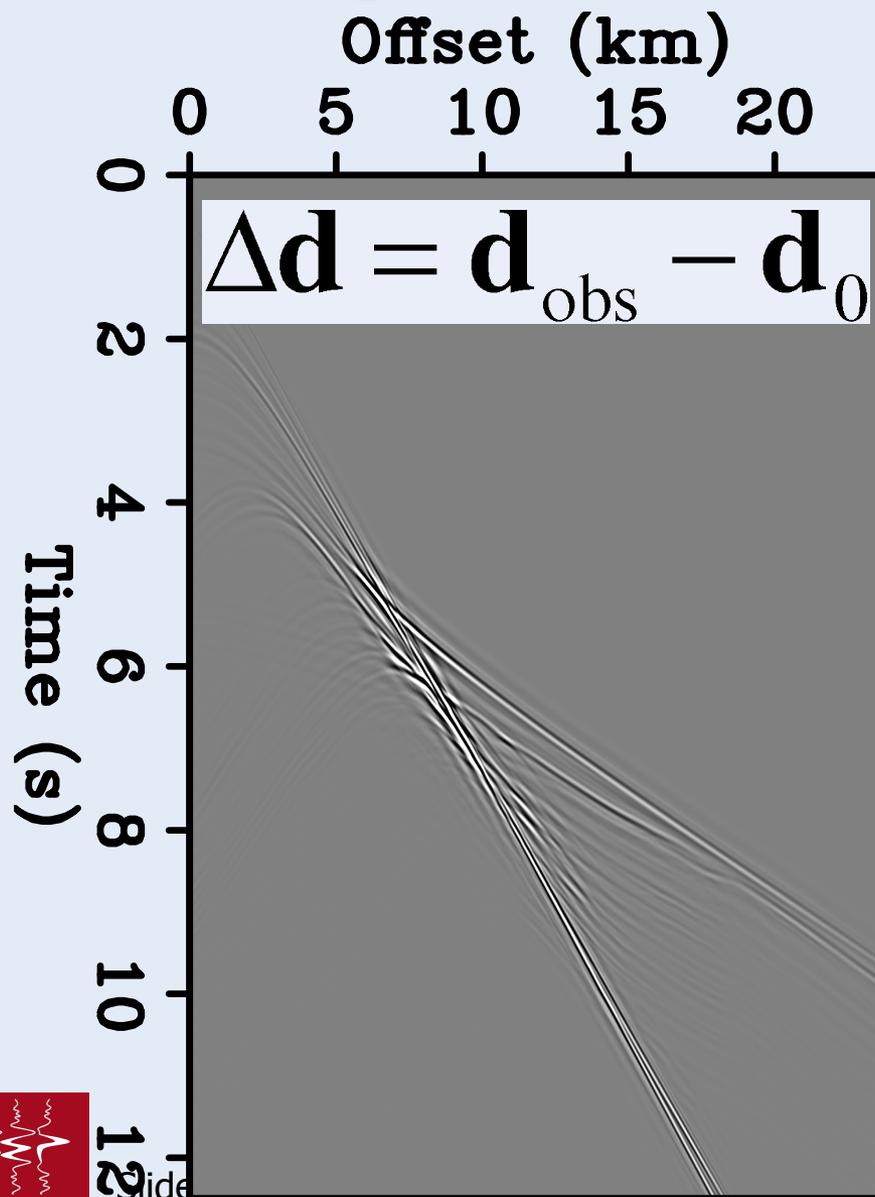
BP example: Born modeling



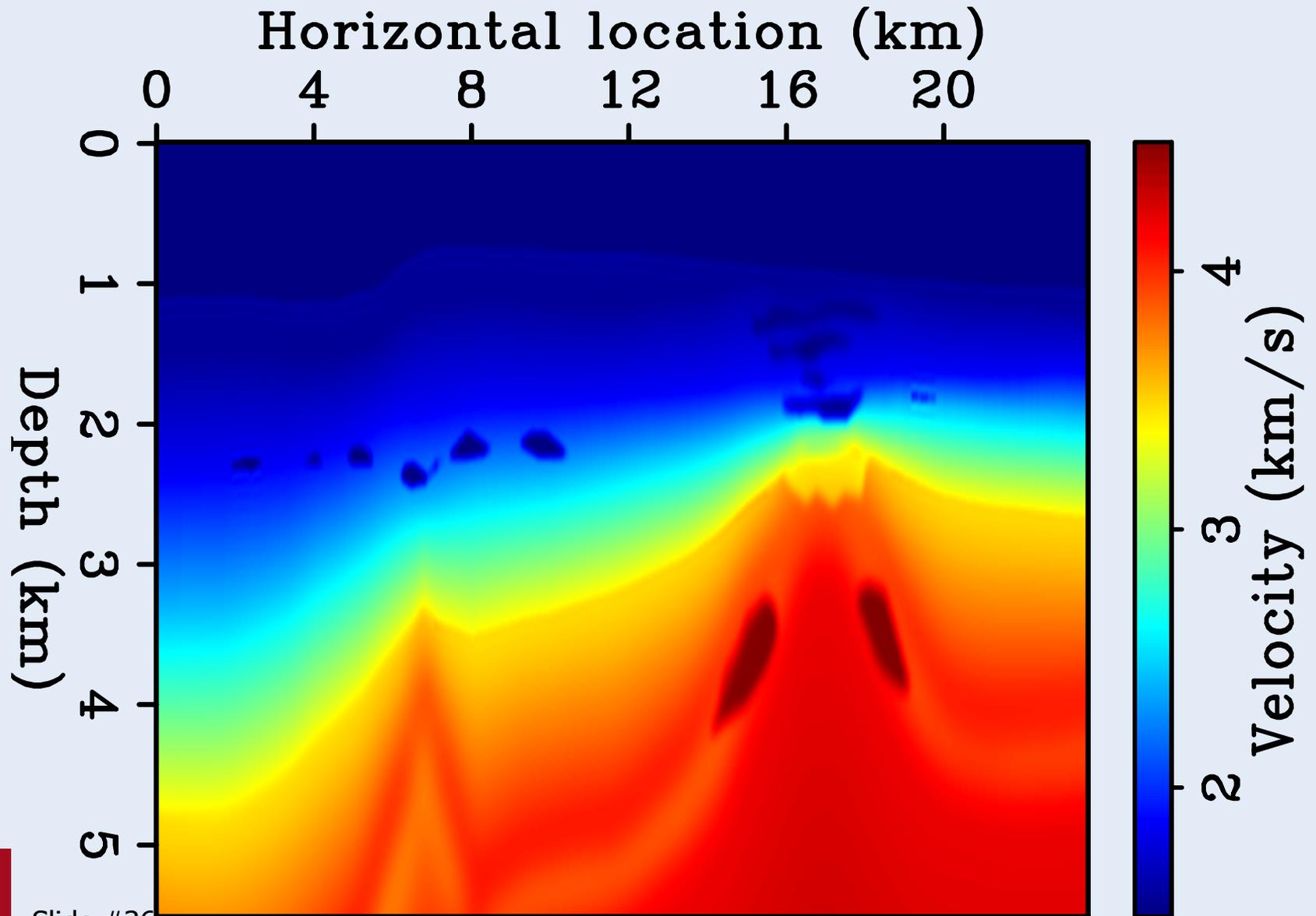
BP example: h-extended modeling



BP example: τ -extended modeling

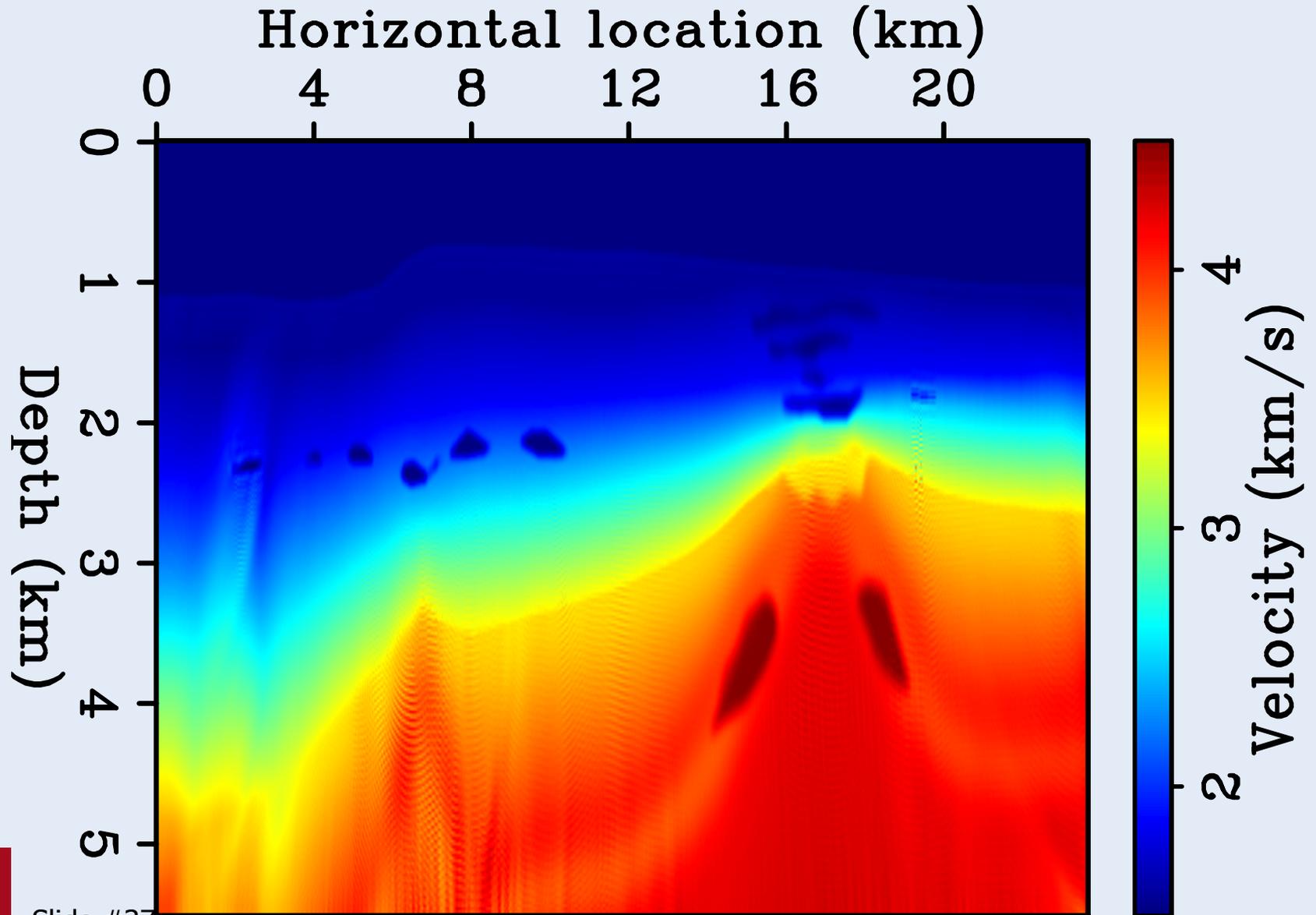


“Mud volcano” in BP 2005 model



Slide #36

Inverted model after last iteration

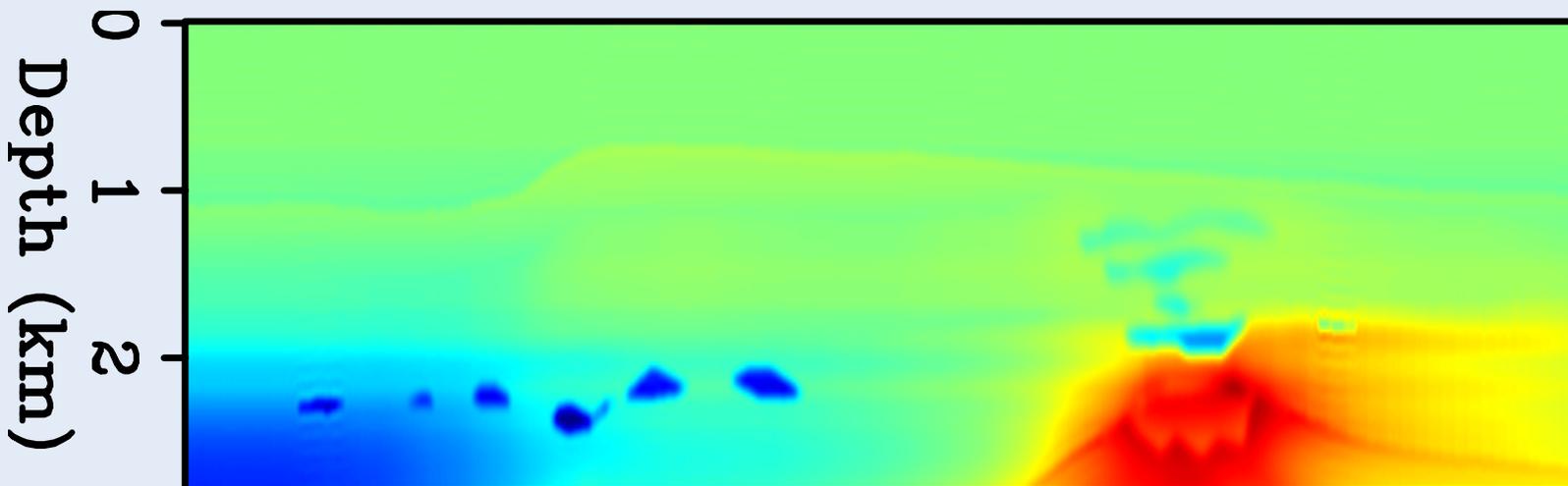
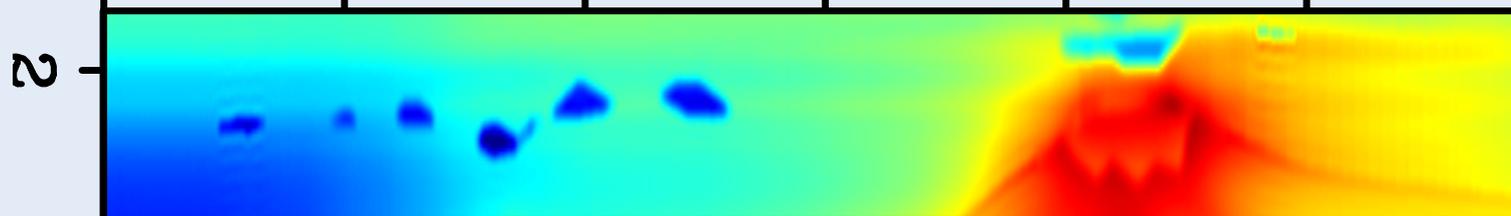


Slide #37

Gas clouds in true model

Horizontal location (km)

0 4 8 12 16 20

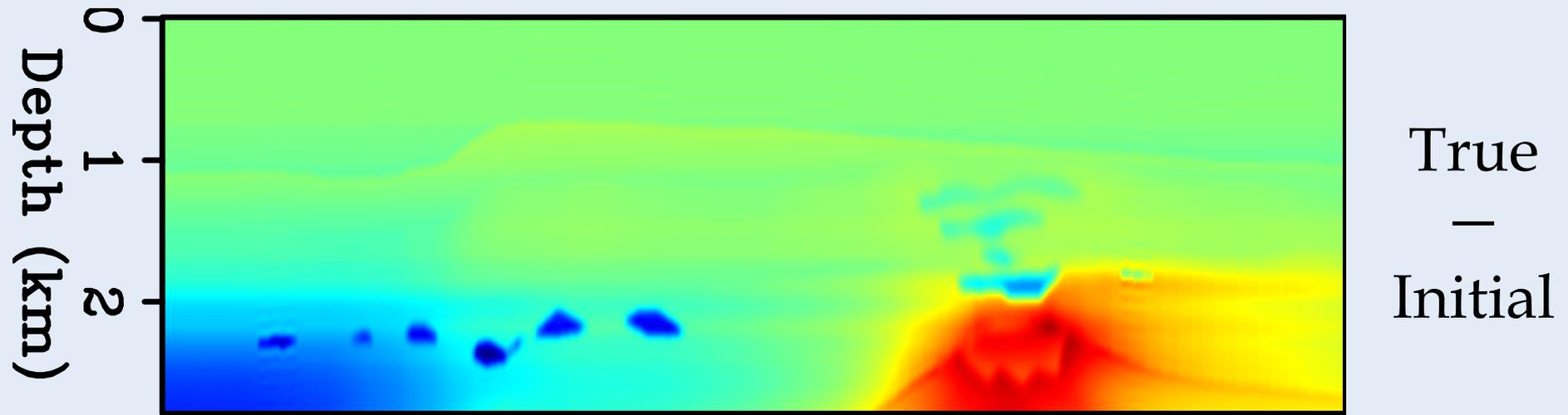
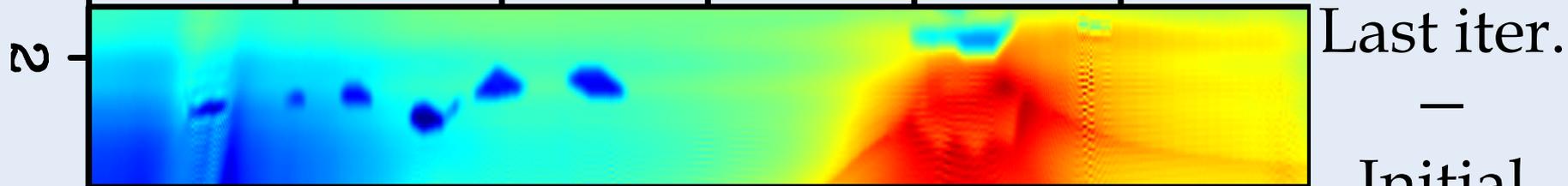


Vel. (km/s)

Gas clouds after last iteration

Horizontal location (km)

0 4 8 12 16 20



-0.8

0

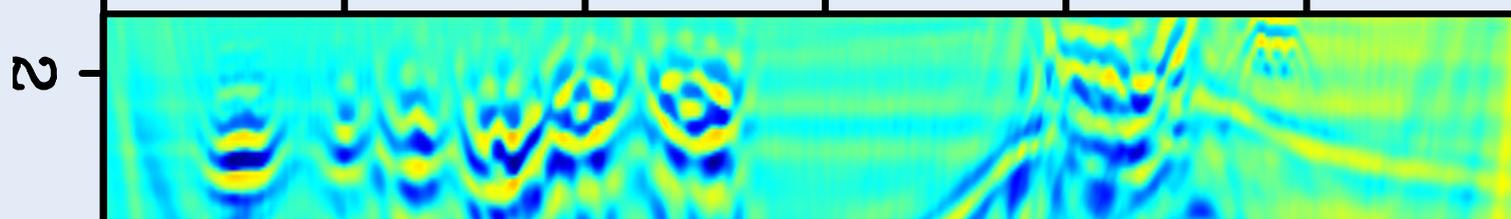
0.8

Vel. (km/s)

Gas clouds after 10 iterations

Horizontal location (km)

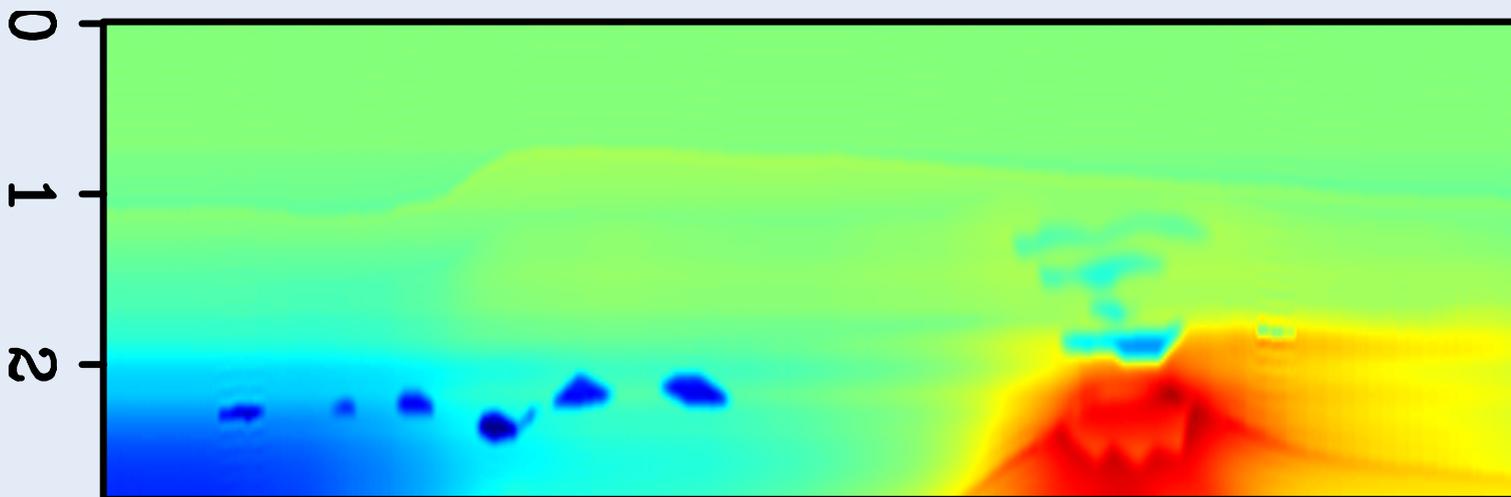
0 4 8 12 16 20



10th iter.

—

Initial



True

—

Initial



-0.8

0

0.8

Vel. (km/s)

Summary and previews

- TFWI is substantially less sensitive to accuracy of starting model than FWI is.
- At the core of TFWI convergence advantage is a new linearization that enables modeling of large time shifts in both reflected and transmission data.
- A lot work needs to be done to make TFWI computationally efficient by (see Ali's talk):
 1. Reducing number of iterations,
 2. Making each iteration cheaper.



Slide #42