Angle gather recovery using iterative thresholding

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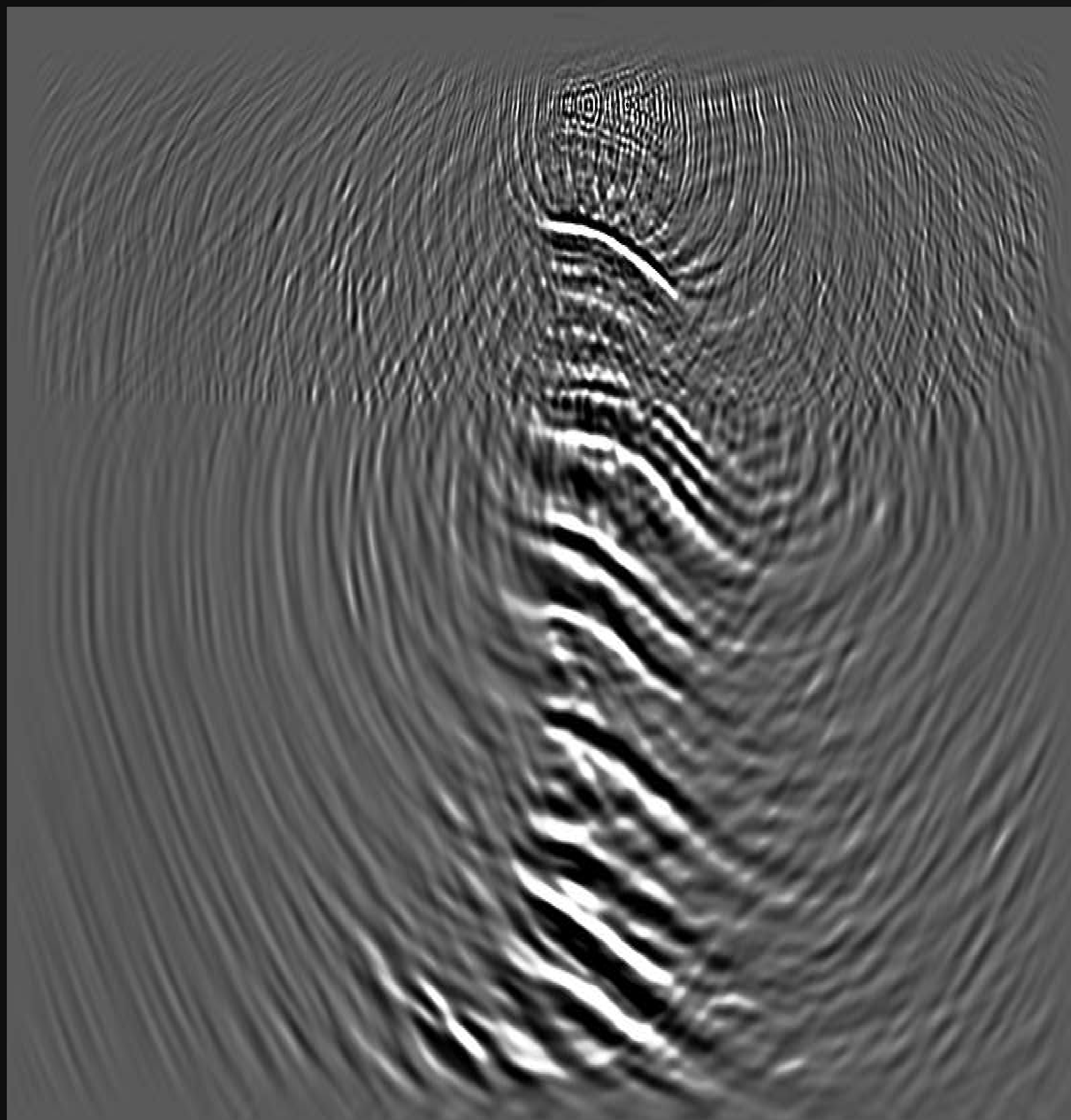
SEP149-79

Outline

- Background
- IST algorithm
- Modifications

Receiver wavefield

Source wavefield



Z

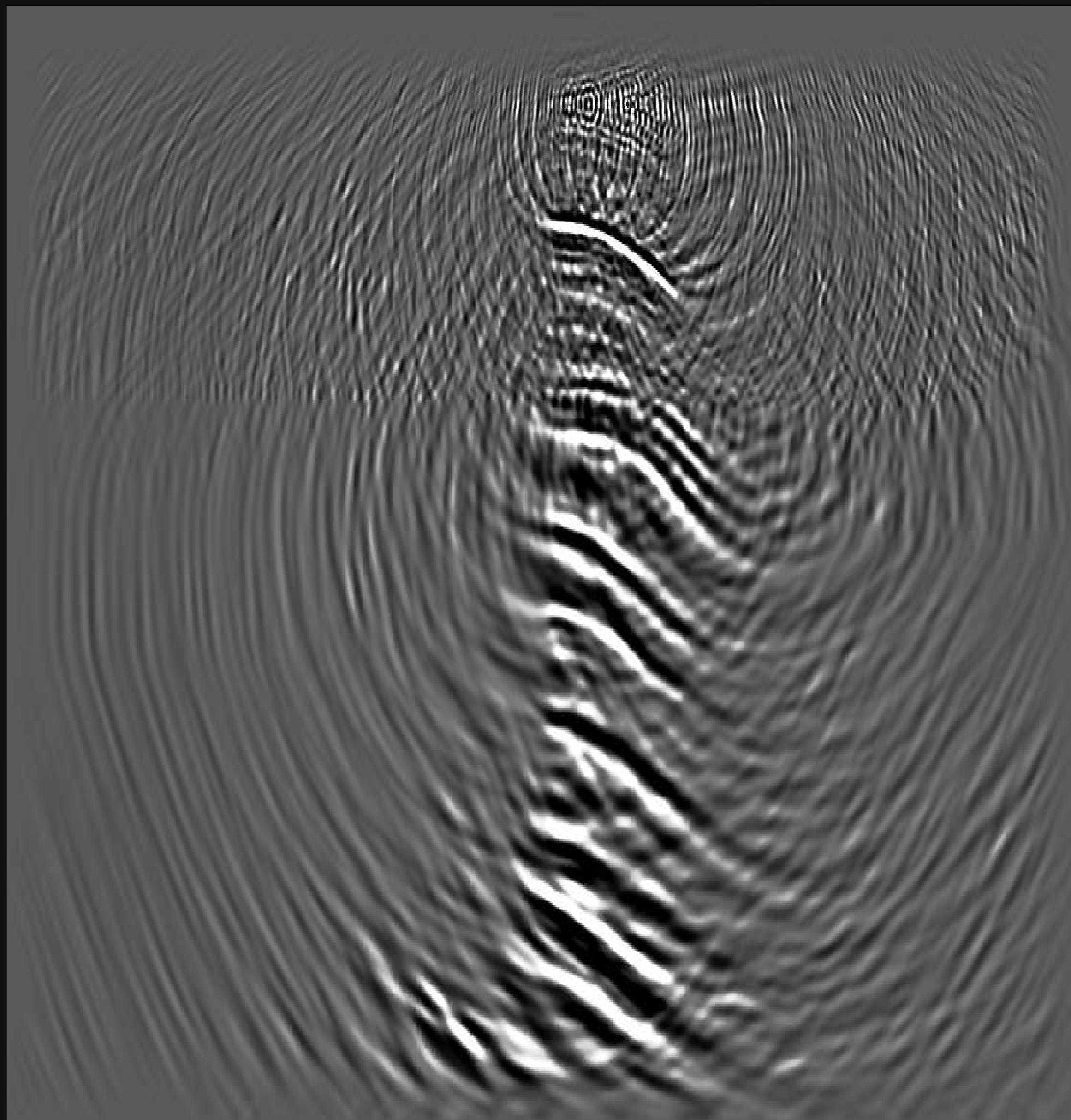
X

X

Visual for angle gather construction

Receiver wavefield

Source wavefield

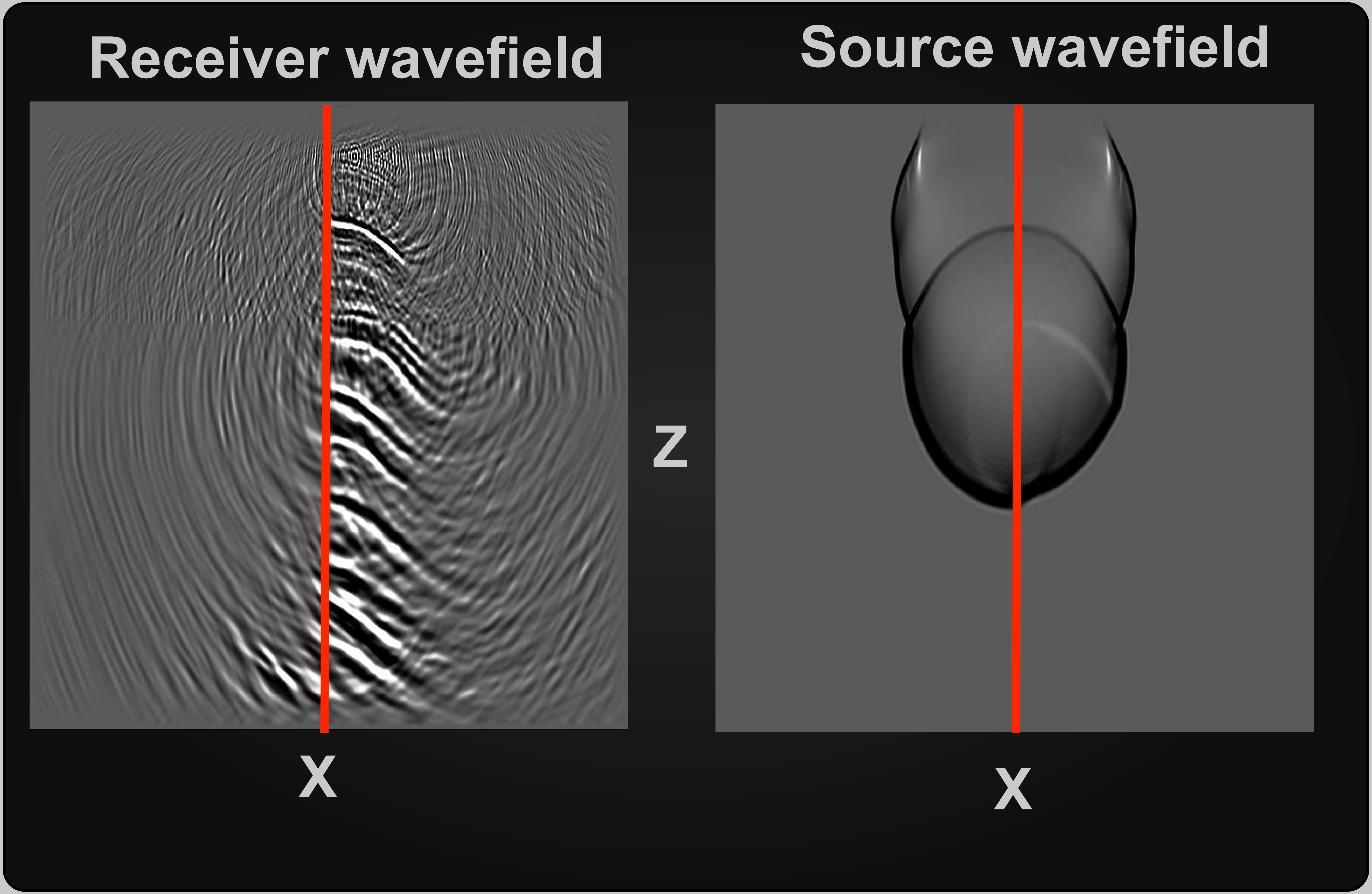


Z

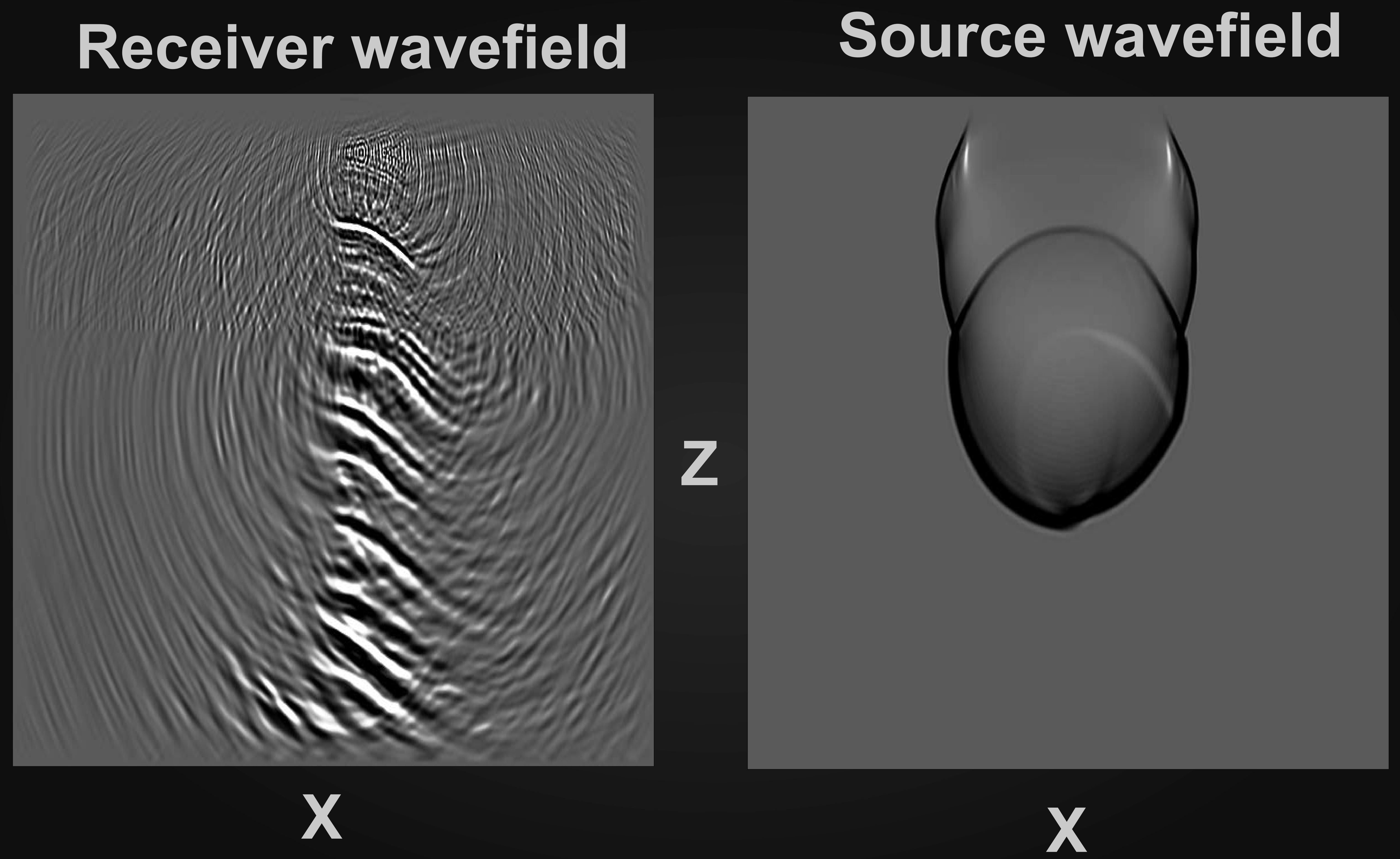
X

X

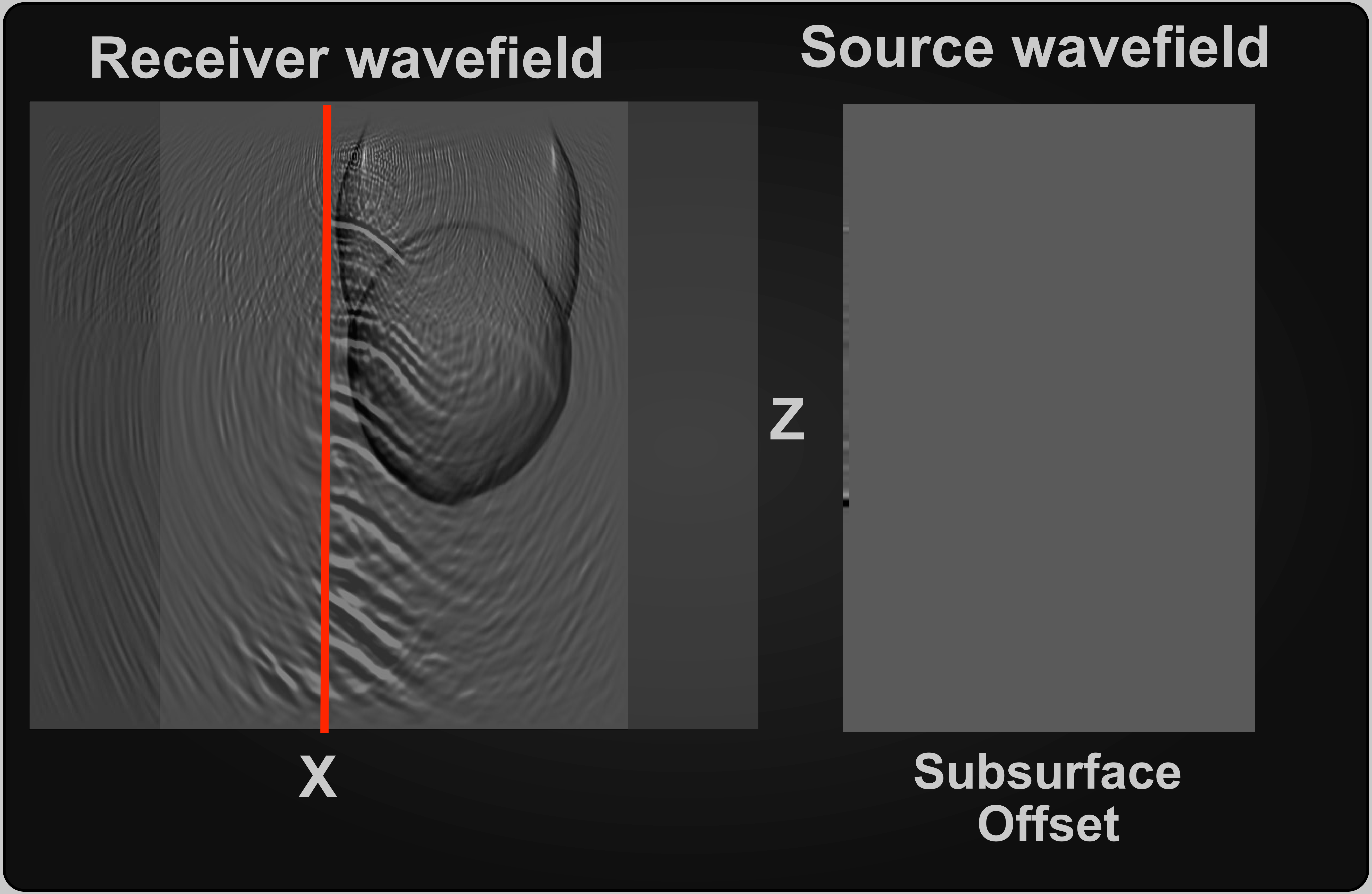
Visual for angle gather construction



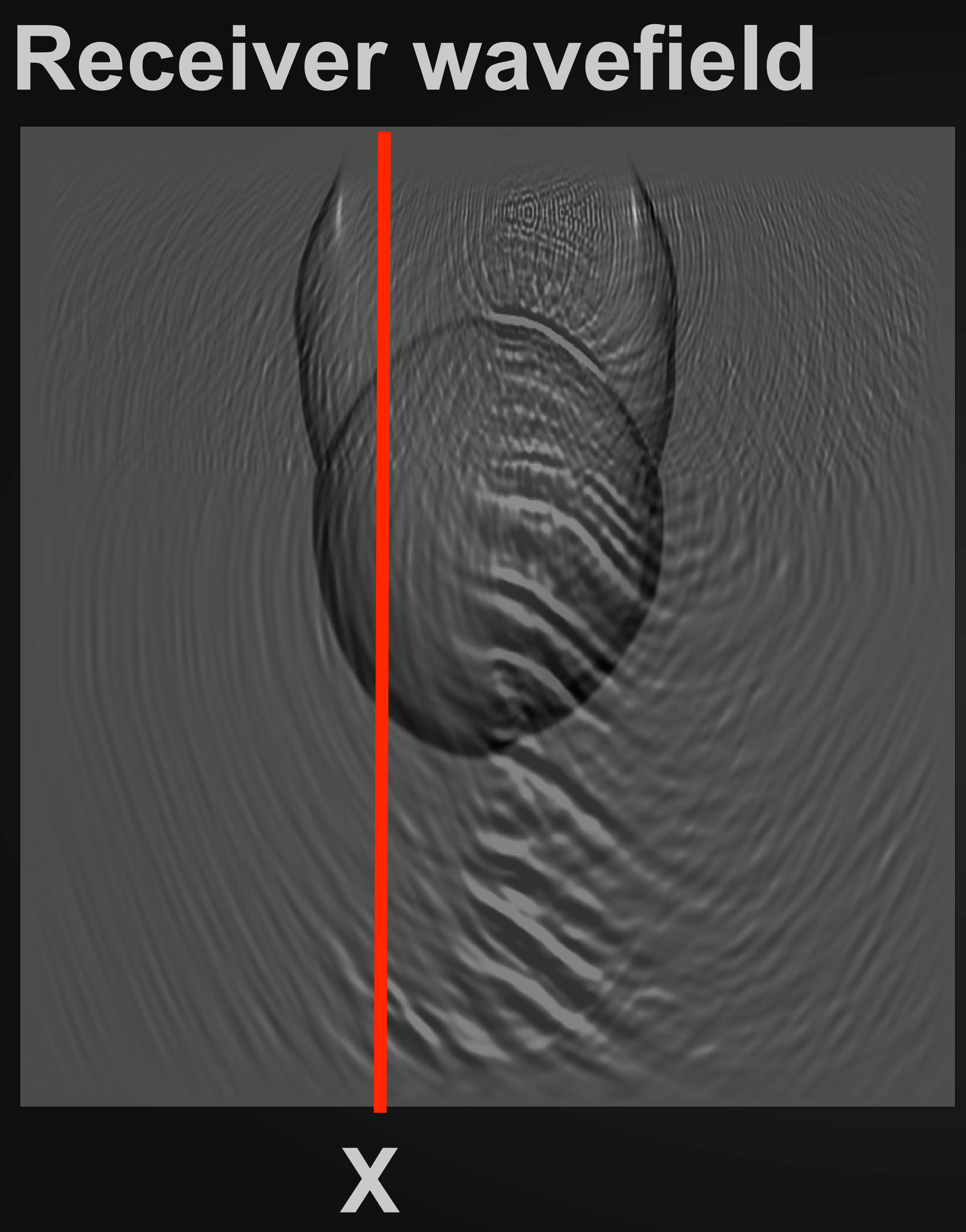
Visual for angle gather construction



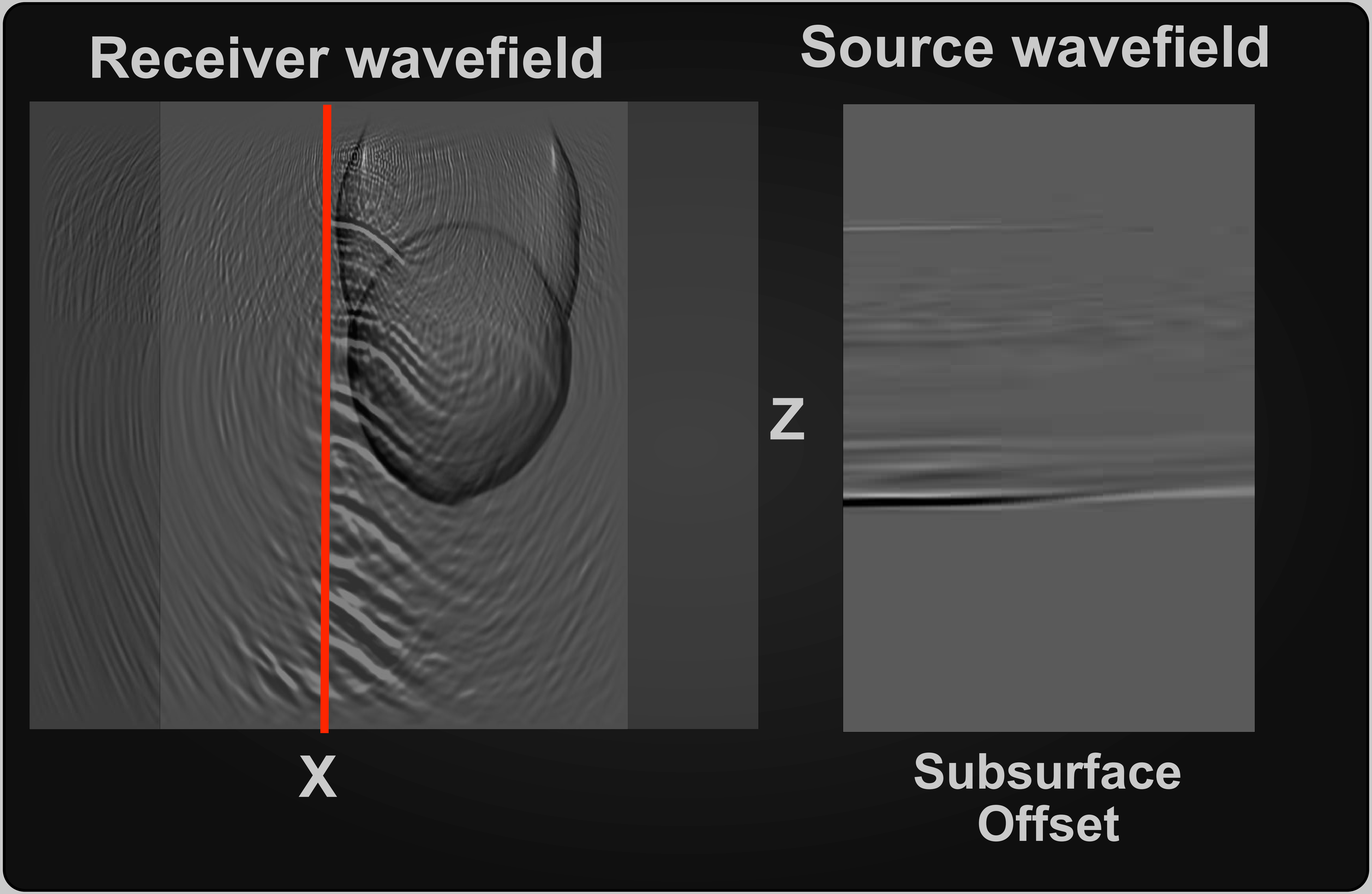
Visual for angle gather construction



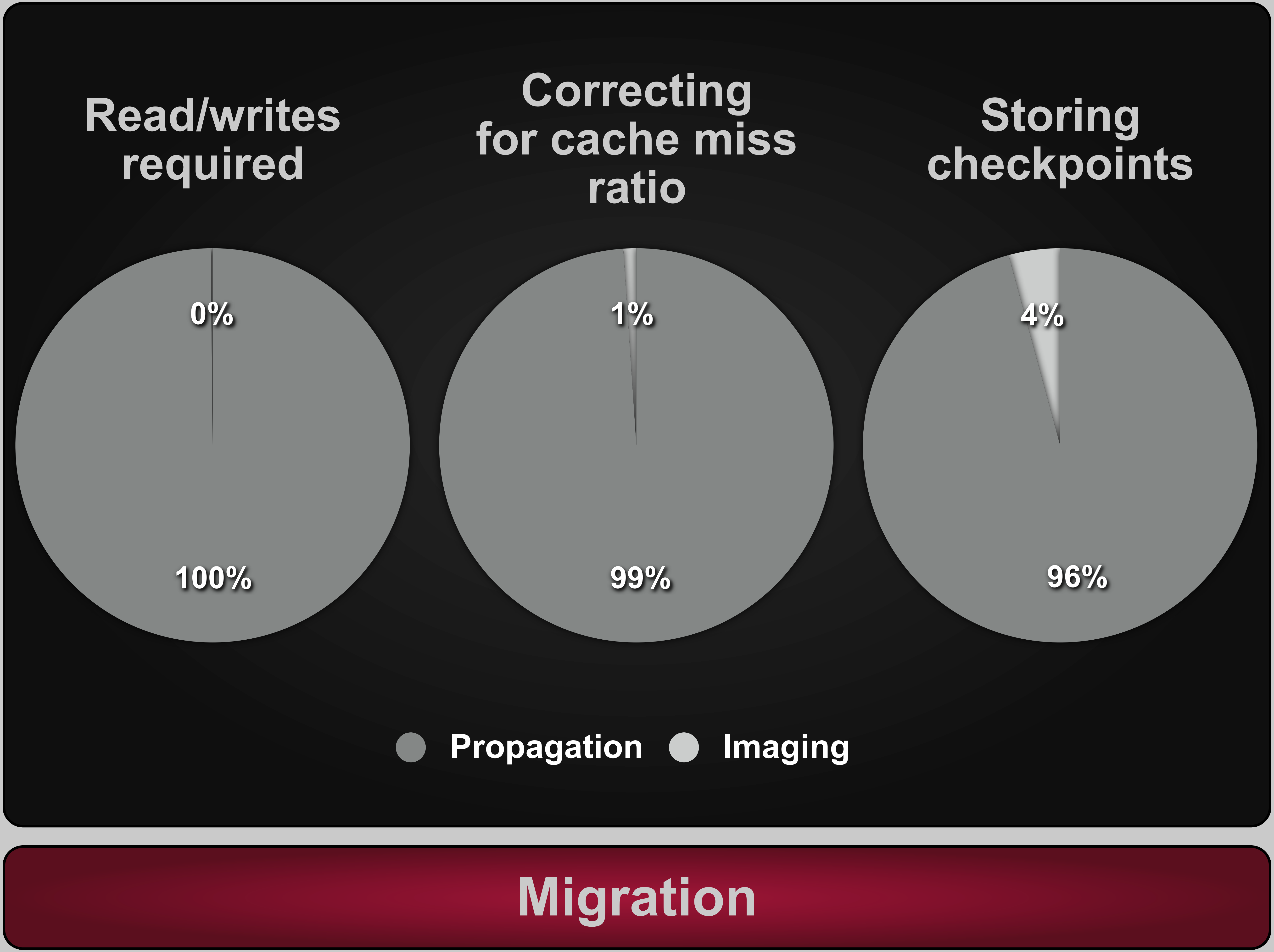
Visual for angle gather construction



Visual for angle gather construction



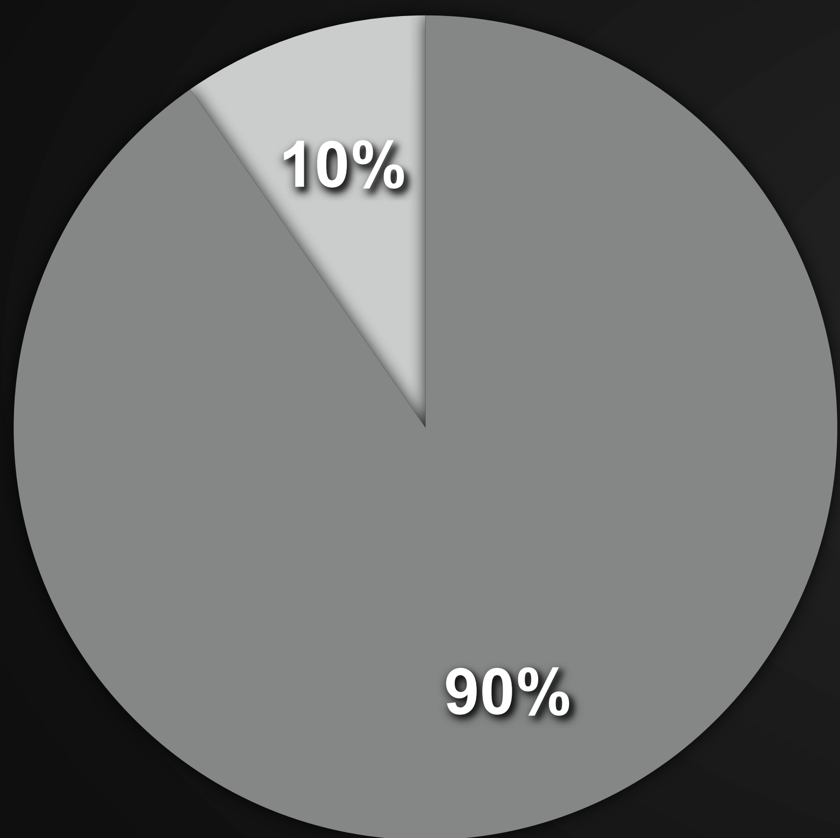
Visual for angle gather construction

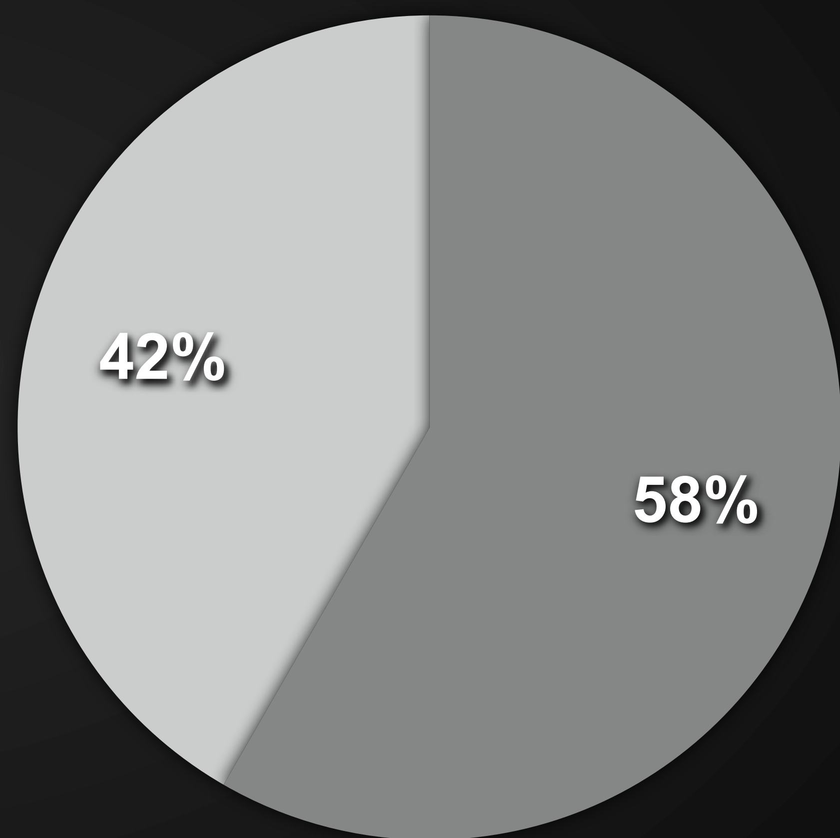


Thursday, June 13, 13

Hold in same memory

Store to disk/(transpose)/compute

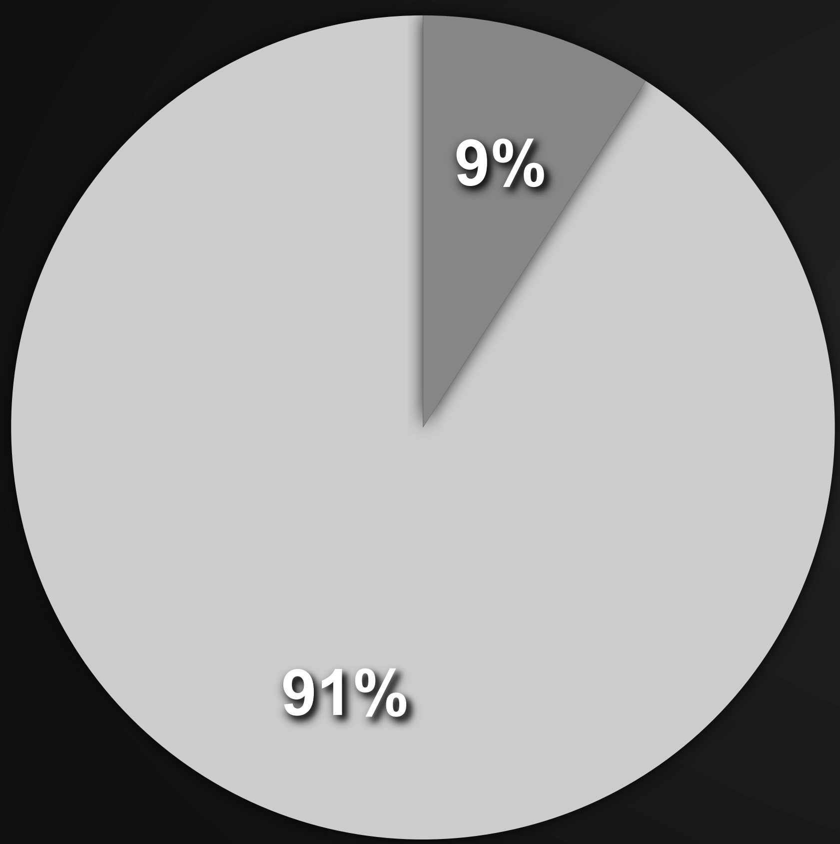




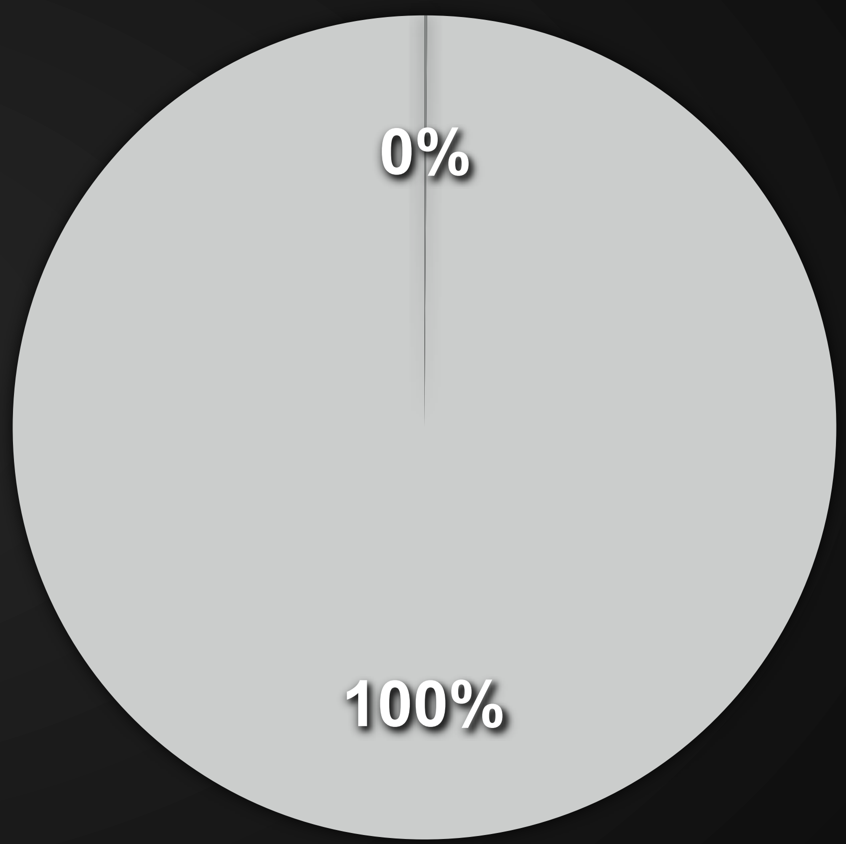
Propagation Imaging

Single shift gathers

Hold at same memory level



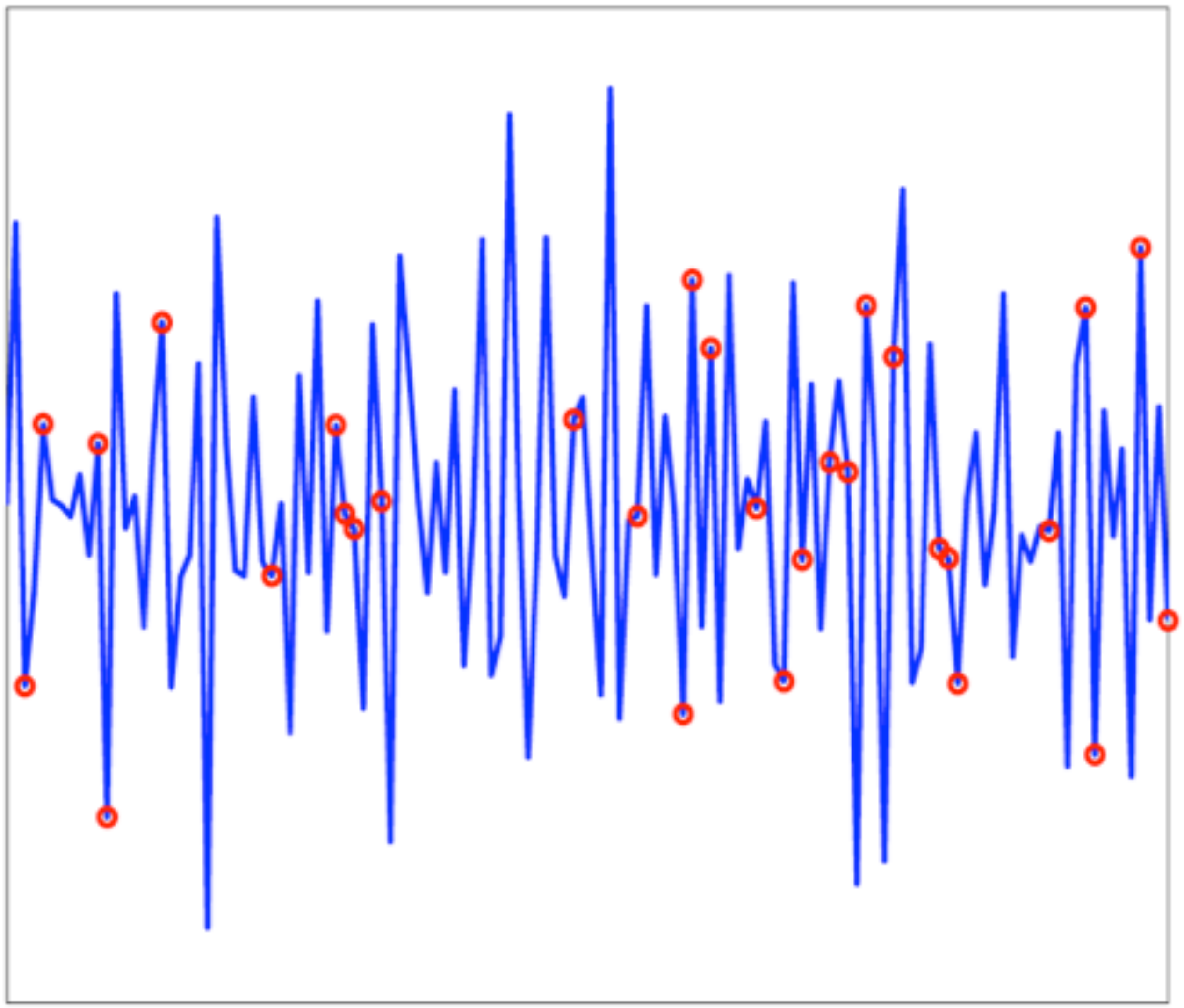
Store to disk/transpose/compute



Multi-shift gathers

Sampling Example

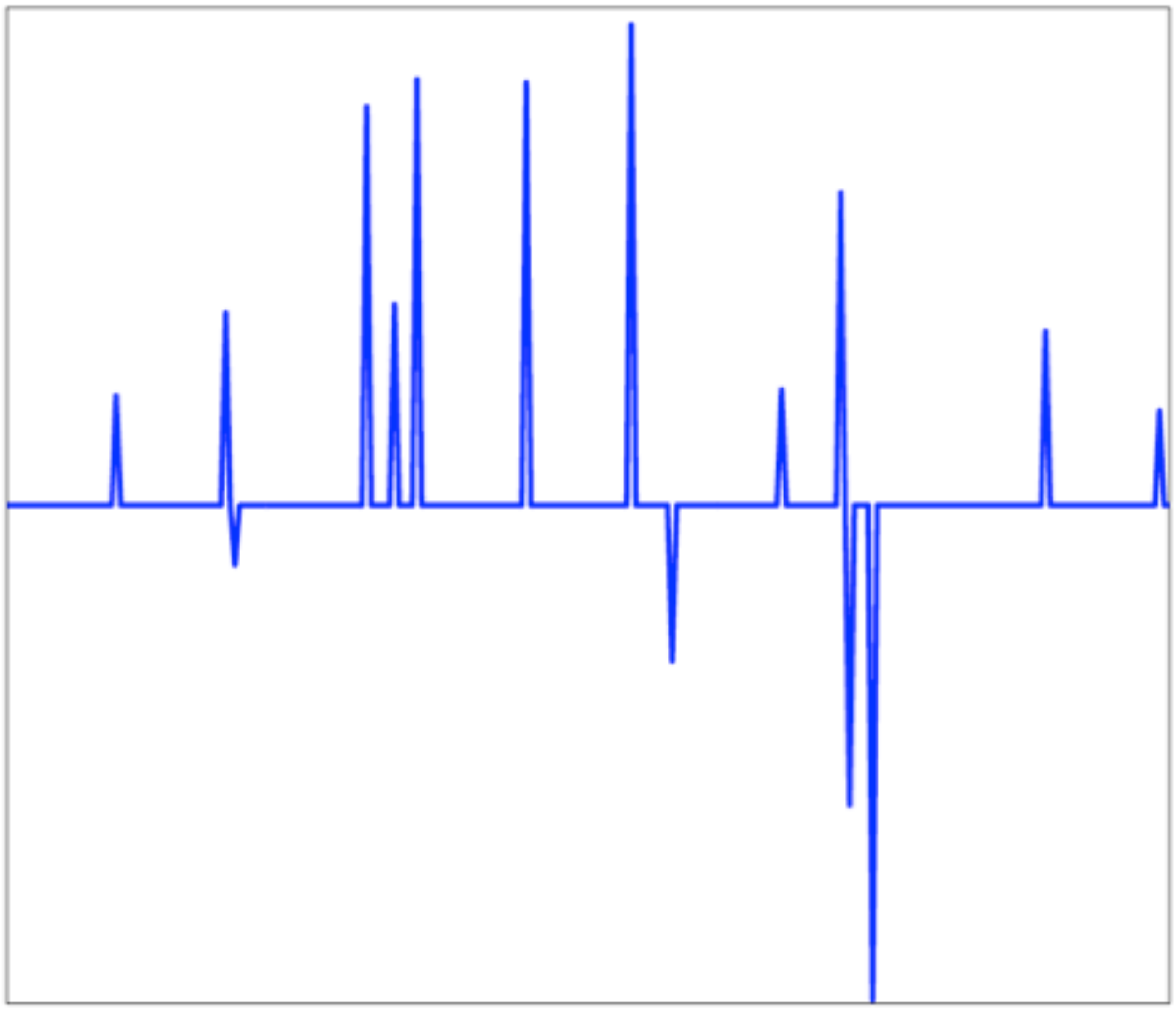
Time domain $f(t)$



Measure M samples

(red circles = samples)

Frequency domain $\hat{f}(\omega)$



K nonzero components

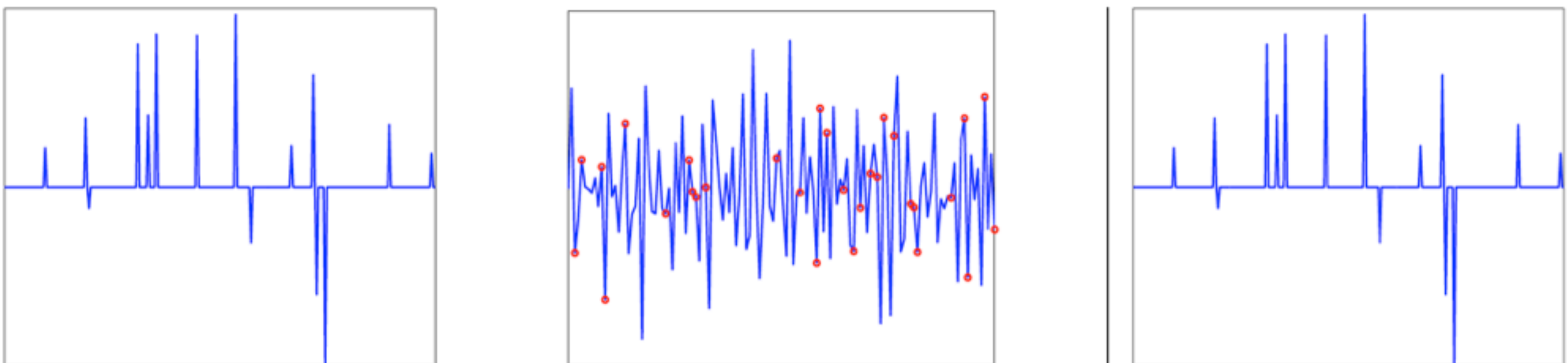
$$\#\{\omega : \hat{f}(\omega) \neq 0\} = K$$

Romberg & Wakin (2007)

ℓ_1 Reconstruction

Reconstruct by solving

$$\min_g \|\hat{g}\|_{\ell_1} := \min \sum_\omega |\hat{g}(\omega)| \quad \text{subject to} \quad g(t_m) = f(t_m), \; m = 1, \ldots, M$$



original $\hat{f}$, $S = 15$ | given $m = 30$ time-dom. samples | *perfect* recovery

Romberg & Wakin (2007)

Example: Sparse Image

- Take $M = 100,000$ incoherent measurements $y = \Phi f_a$
- f_a = wavelet approximation (perfectly sparse)
- Solve

$$\min \ \|\alpha\|_{\ell_1} \quad \text{subject to} \quad \Phi\Psi\alpha = y$$

Ψ = wavelet transform

original (25k wavelets)

perfect recovery

Romberg & Wakin (2007)

- You want the dataset d
- You know that d transforms to something sparse (m) by applying the operator L’
- You record a random subset of d, d_r

Compressive sensing

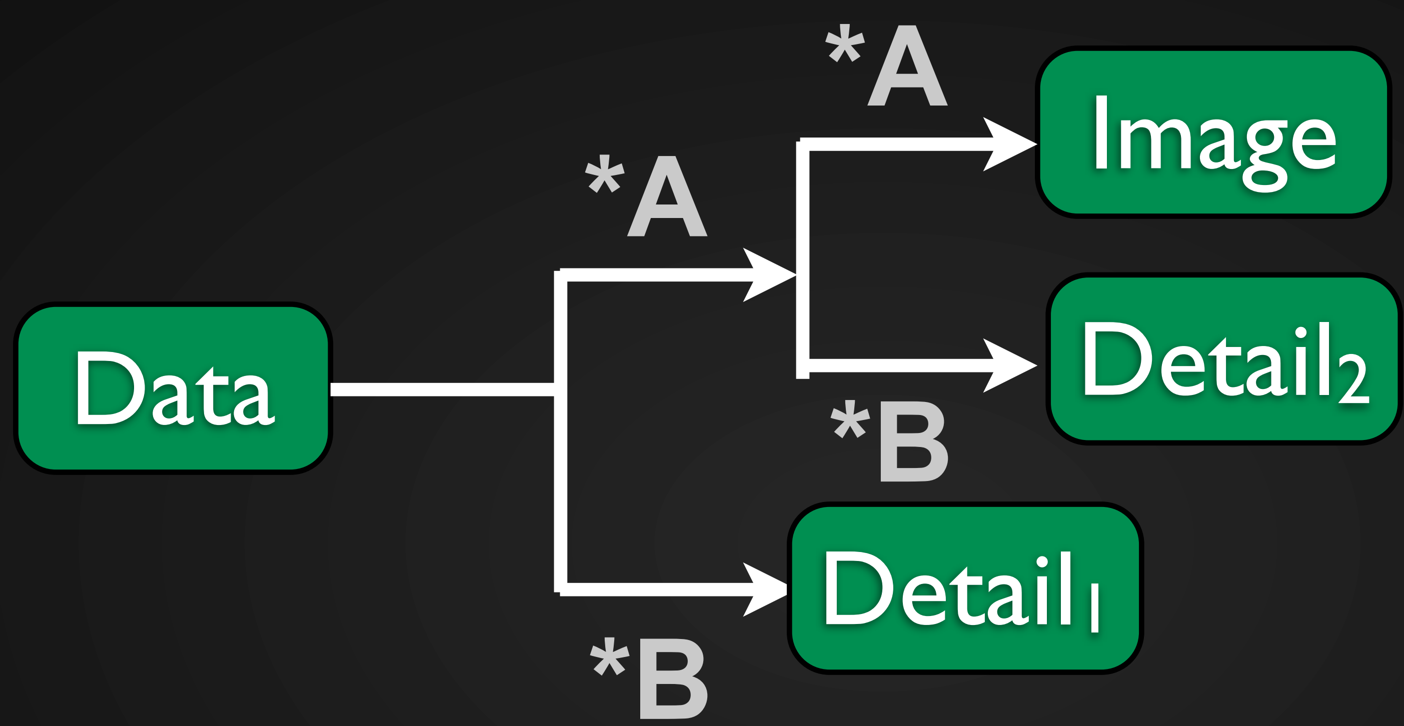
$$\mathbf{0} \approx \mathbf{r} \underset{1}{=} \mathbf{d_r} - \mathbf{Lm}$$

$\mathbf{r}$ Residual $\quad$ $\underset{1}{=}$ L1 norm

$\mathbf{d}$ Sparse data $\quad$ $\mathbf{m}$ Sparse model

$\mathbf{L}$ Transform into/from sparse basis

Compressive sensing in SEP speak



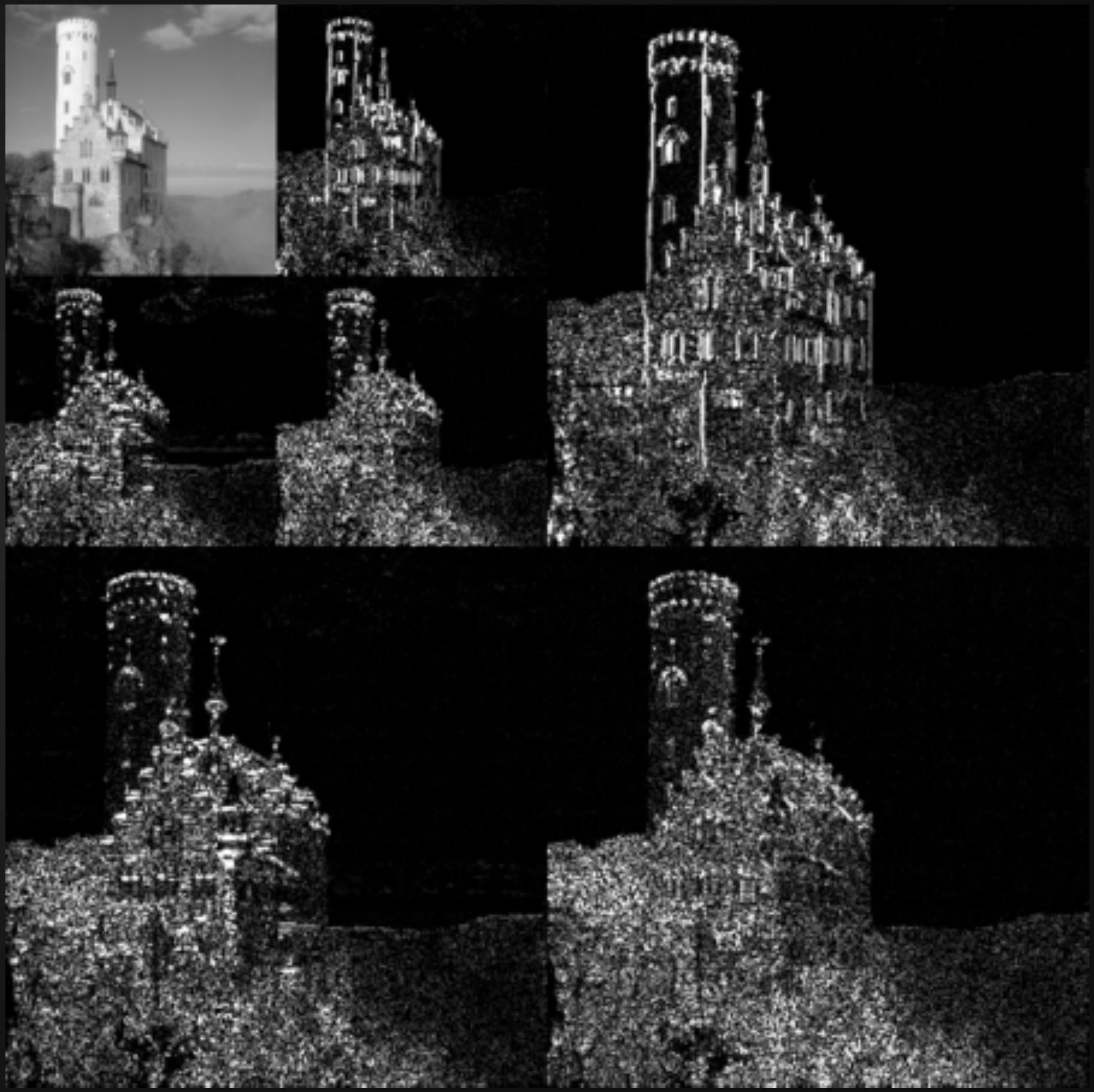
A - low pass filter (scaling)

B- high pass filter (wavelet)

Wavelet transform

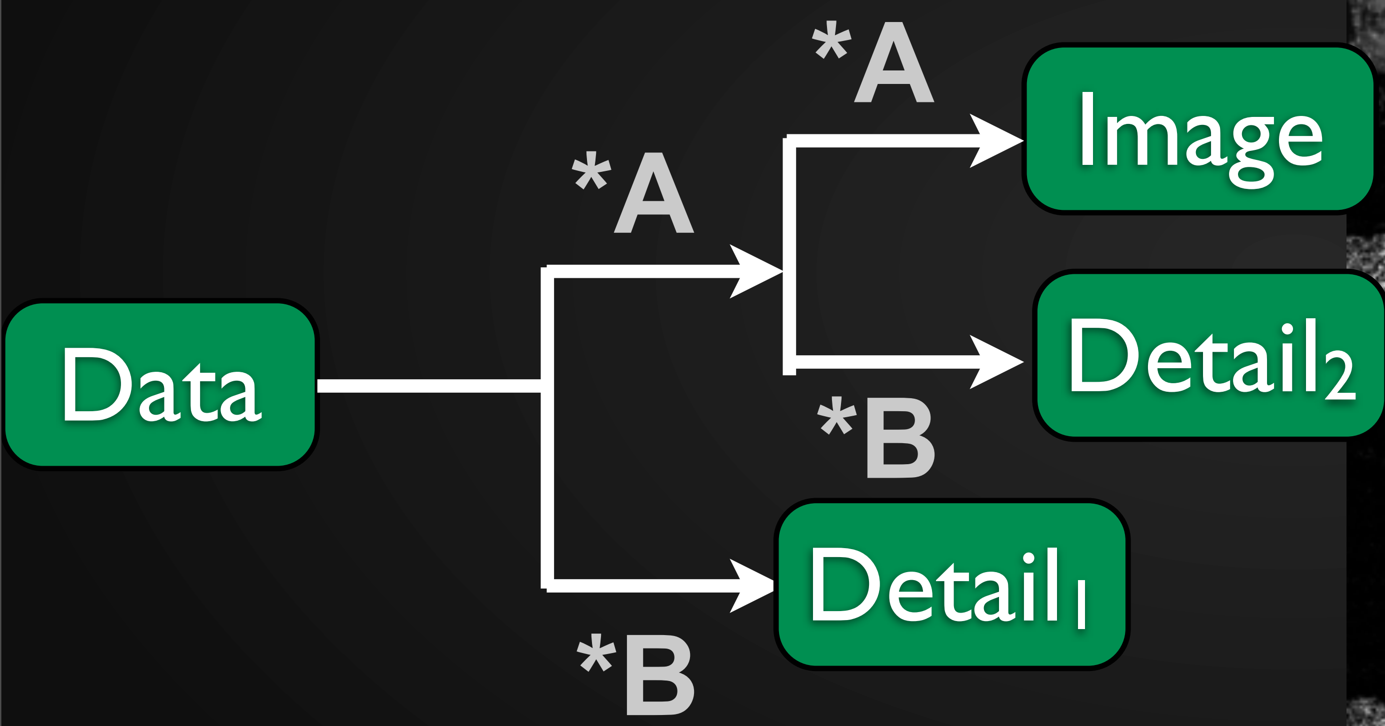
Original

Multi-D wavelet transform

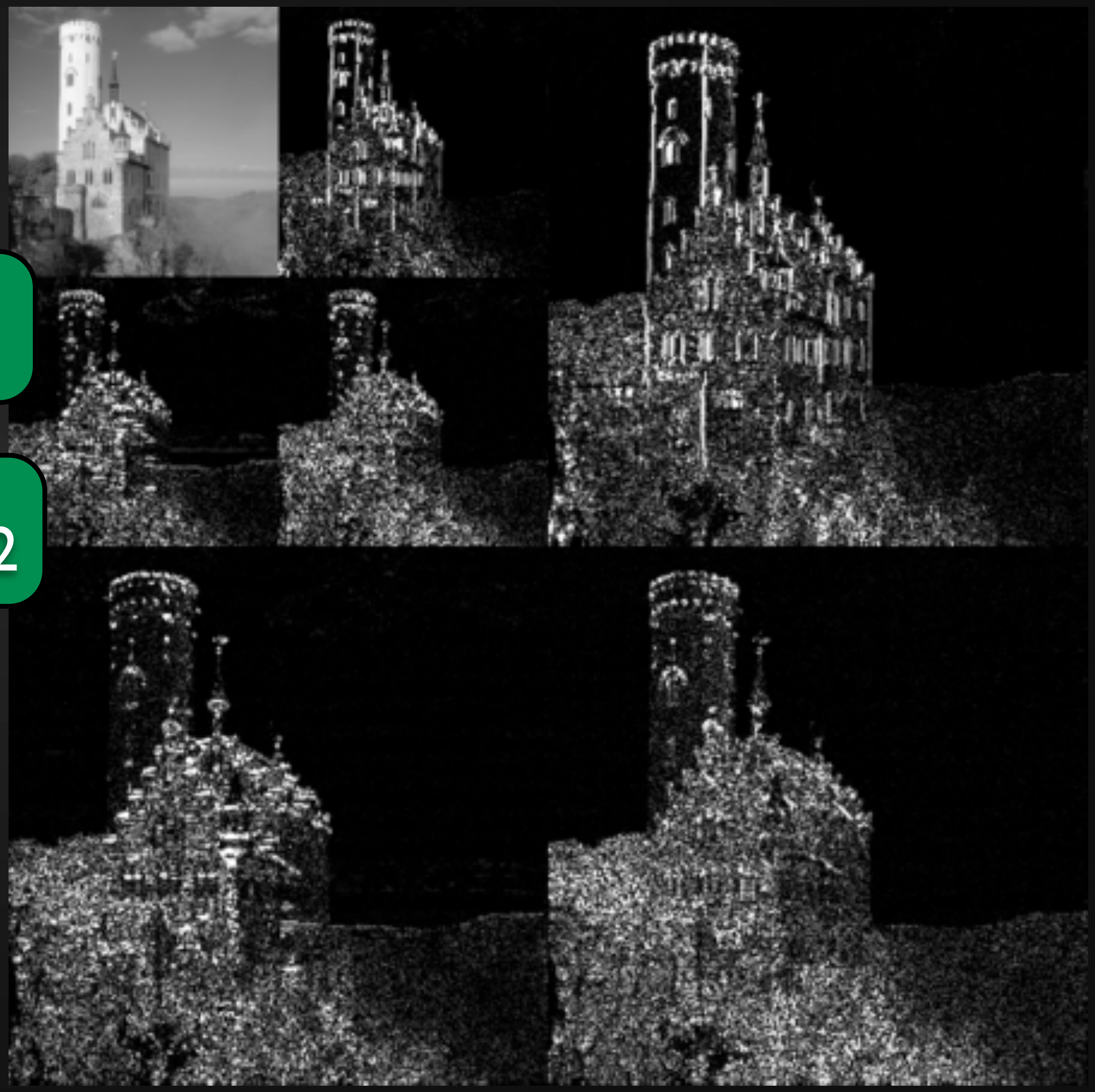


Multi-D wavelet transform

Wavelet transform

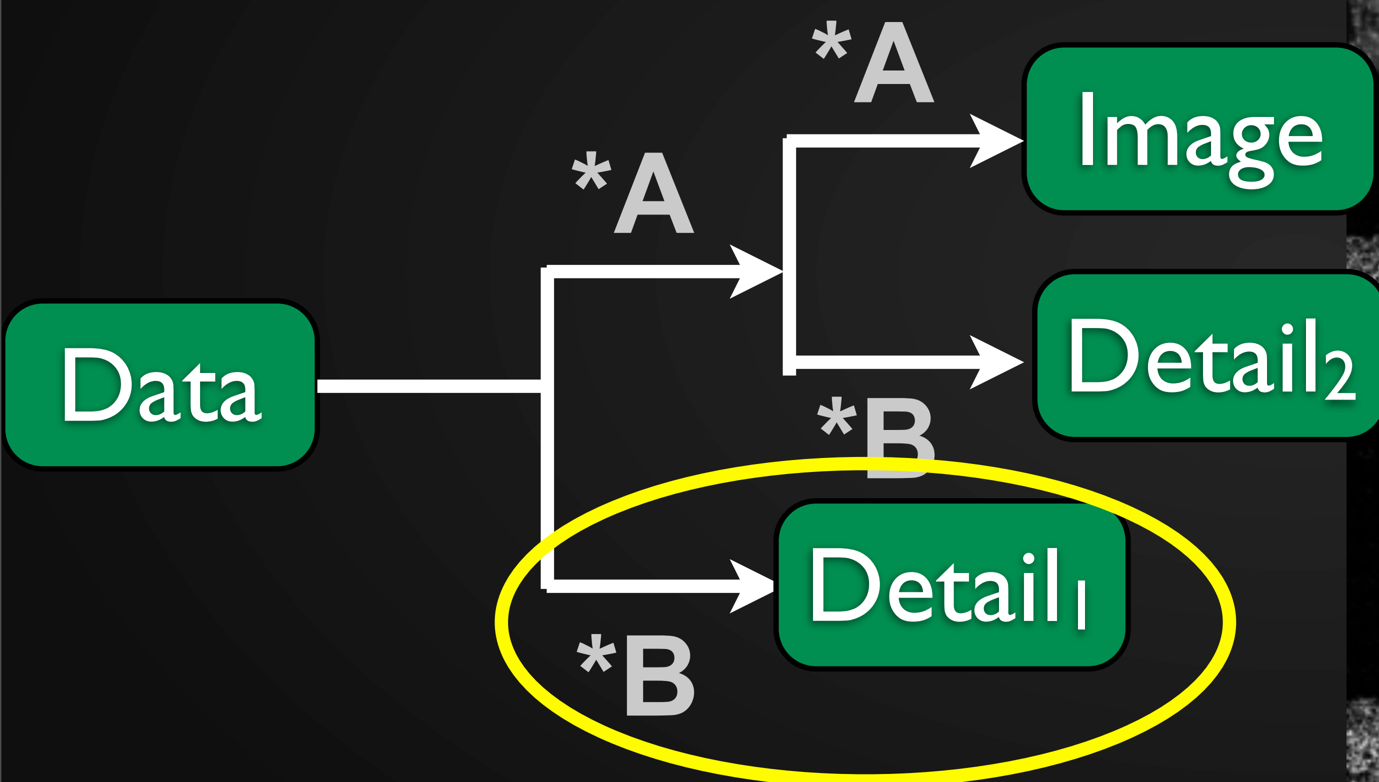


Multi-D wavelet transform

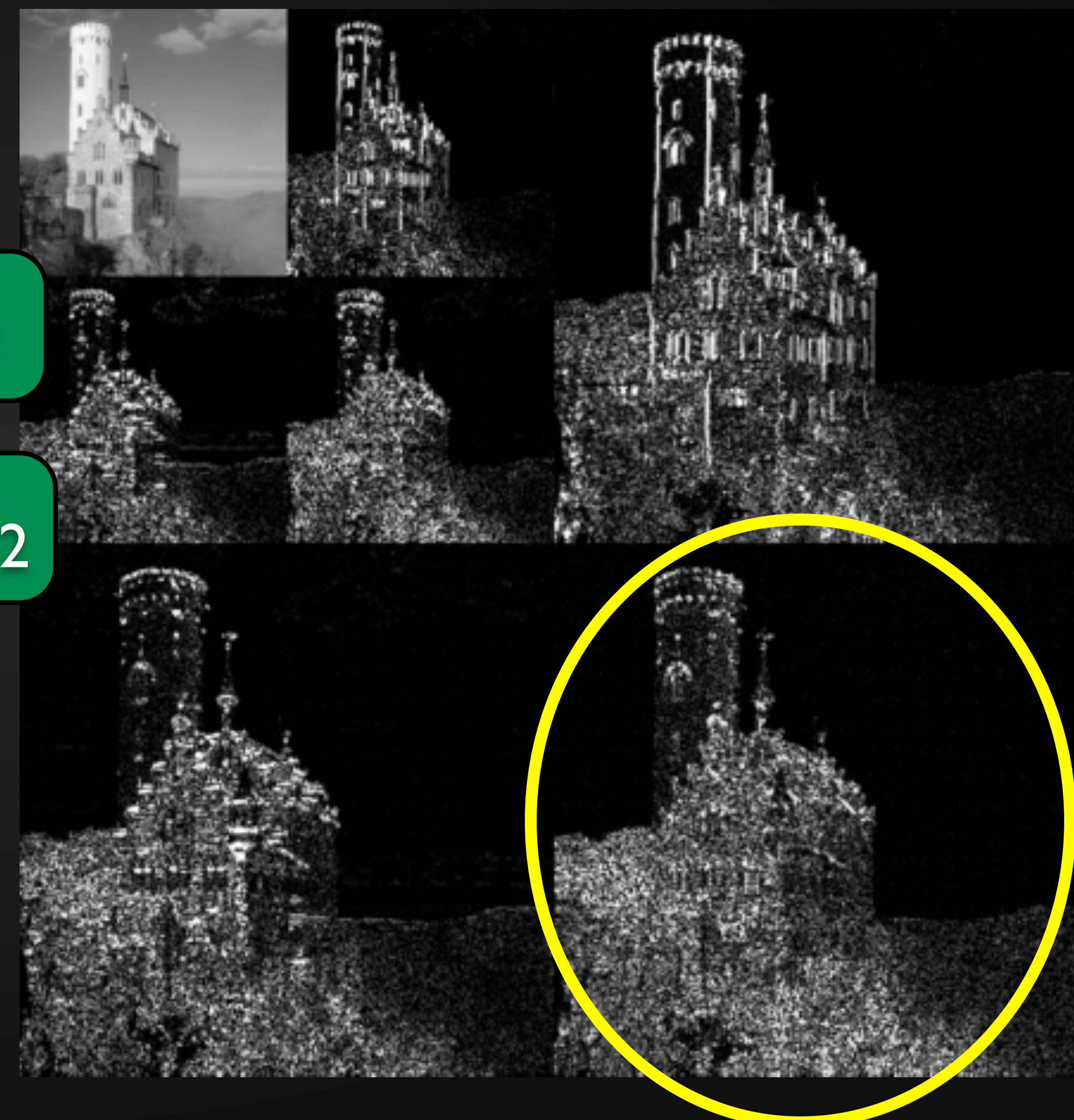


Multi-D wavelet transform

Wavelet transform



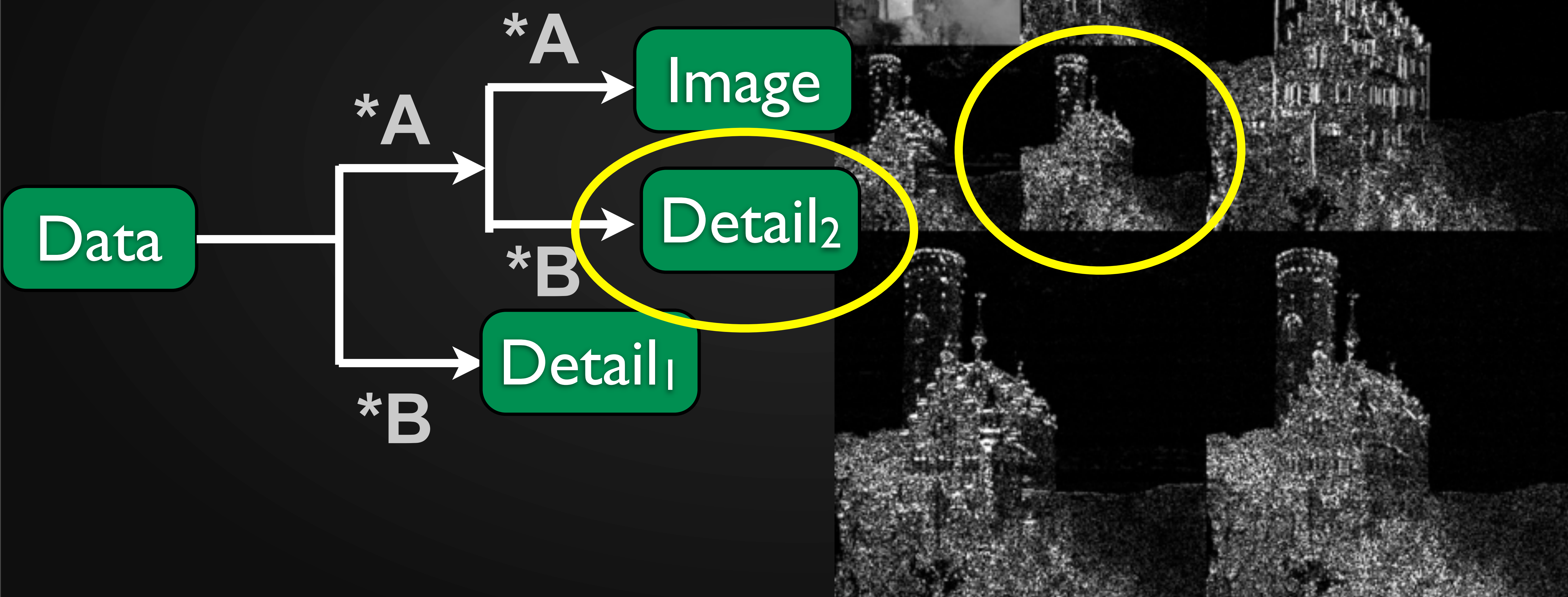
Multi-D wavelet transform



Multi-D wavelet transform

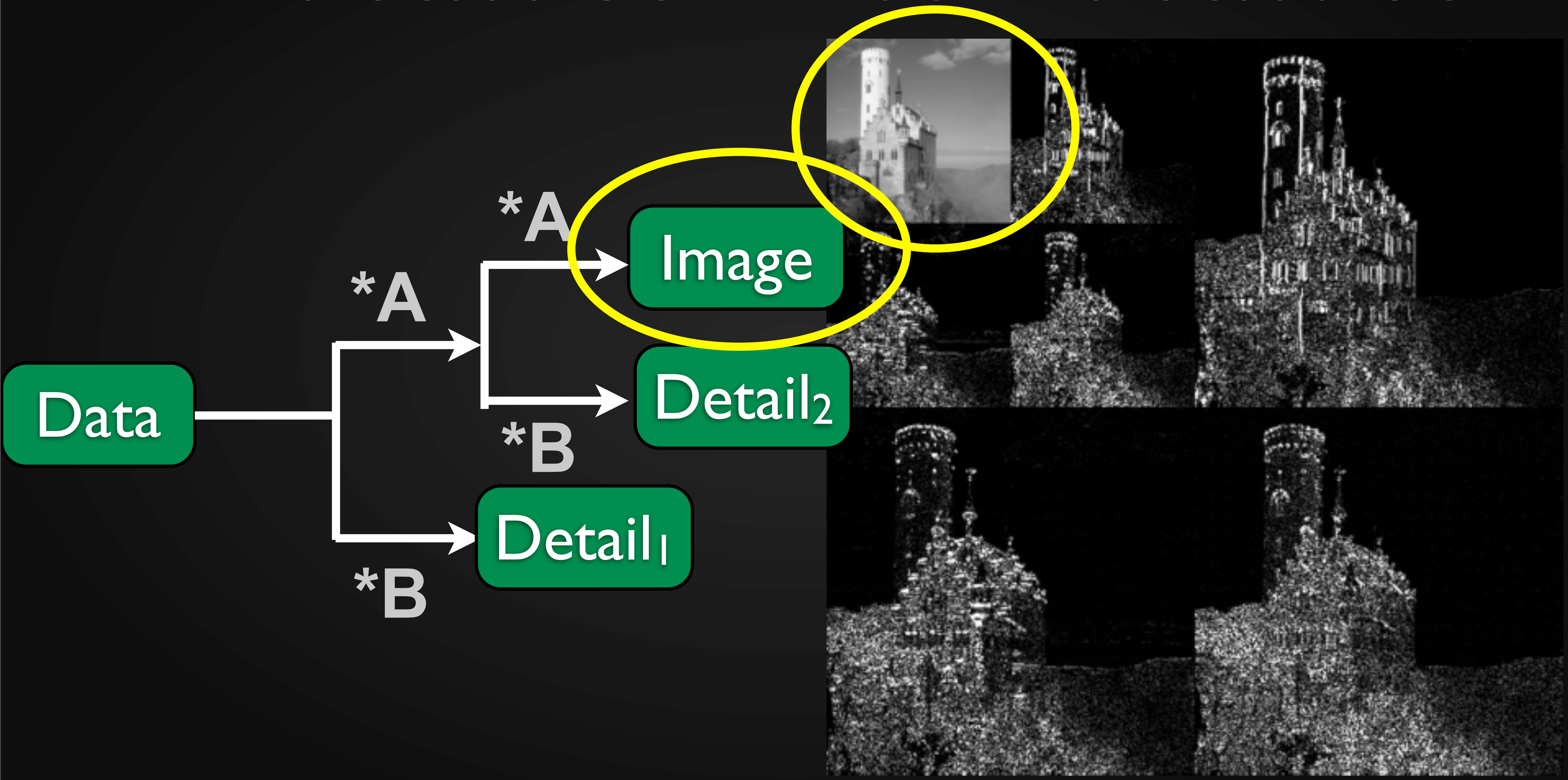
Wavelet transform

Multi-D wavelet transform



Multi-D wavelet transform

Wavelet transform — Multi-D wavelet transform



Multi-D wavelet transform

$$\mathbf{r} = \mathbf{d} - \mathbf{L}\mathbf{x_0}$$

$$\mathbf{g} = \mathbf{L^T}\mathbf{r}$$

$$\mathbf{h} = \mathbf{L}\mathbf{g}$$

$$\alpha = -\frac{\mathbf{r}\mathbf{h^T}}{\mathbf{h}\mathbf{h^T}}$$

$$\mathbf{r} += \alpha\mathbf{h}$$

$$\mathbf{x_i} += \alpha\mathbf{g}$$

Linear iteration

Steepest descent iteration

$$\mathbf{r} = \mathbf{d} - \mathbf{L}\mathbf{x_0}$$

$$\mathbf{g} = \mathbf{L^T}\mathbf{r}$$

$$\mathbf{h} = \mathbf{L}\mathbf{g}$$

$$\alpha = -\frac{\mathbf{r}\mathbf{h^T}}{\mathbf{h}\mathbf{h^T}}$$

$$\mathbf{r} + = \alpha\mathbf{h}$$

$$\mathbf{x_i} + = \alpha\mathbf{g}$$

Linear iteration

$\mathbf{d}$ **randomly sampled data**

Steepest descent iteration

Steepest descent iteration

Linear iteration

$$\mathbf{r} = \mathbf{d} - \mathbf{L}\mathbf{x_0}$$

$$\mathbf{g} = \mathbf{L^T}\mathbf{r}$$

$$\mathbf{h} = \mathbf{L}\mathbf{g}$$

$$\alpha = -\frac{\mathbf{r}\mathbf{h^T}}{\mathbf{h}\mathbf{h^T}}$$

$$\mathbf{r} + = \alpha\mathbf{h}$$

$$\mathbf{x_i} + = \alpha\mathbf{g}$$

$\mathbf{L}$

n-d wavelet transform followed by masking

i

Linear iteration

$$\mathbf{h} = \mathbf{d} - \alpha \mathbf{L} \mathbf{x_i}$$

$$\mathbf{x_{i+1}} = \mathbf{x_i} + \alpha \mathbf{L^T} \mathbf{h}$$

Landweber iteration

Landweber iteration

Linear iteration

i

$$\mathbf{h} = \mathbf{d} - \alpha \mathbf{L} \mathbf{x_i}$$

$$\mathbf{x_{i+1}} = \mathbf{x_i} + \alpha \mathbf{L^T} \mathbf{h}$$

α Scale by inverse of largest eigenvalue

$$\mathbf{x} \longrightarrow \text{random}$$

Power iteration

$$\mathbf{y} = \mathbf{L}\mathbf{x}$$

$$\mathbf{g} = \mathbf{L}^{\mathbf{T}}\mathbf{y}$$

$$\alpha = \frac{1}{\mathbf{g}^{\mathbf{T}}\mathbf{g}}$$

$$\mathbf{x} = \alpha\mathbf{g}$$

α Scale by inverse of largest eigenvalue

Power iteration

i

Inner iteration

$$\mathbf{h} = \mathbf{d} - \alpha \mathbf{L} \mathbf{x_i}$$

$$\mathbf{x_{i+1}} = \mathbf{x_i} + \alpha \mathbf{L^T} \mathbf{h}$$

where $x_{i+1}[j] < \lambda$

$$x_{i+1}[j] = 0$$

Thresholding

Iterative thresholding

k

Thresholding iteration

i

Inner iteration

$$\mathbf{h} = \mathbf{d} - \alpha \mathbf{L} \mathbf{x_i}$$

$$\mathbf{x_{i+1}} = \mathbf{x_i} + \alpha \mathbf{L^T} \mathbf{h}$$

where $x_{i+1}[j] < \lambda_k$

$$x_{i+1}[j] = 0$$

$$\mathbf{x} = \mathbf{L}^{\mathbf{T}}\mathbf{d}$$

$$p = .005$$

k

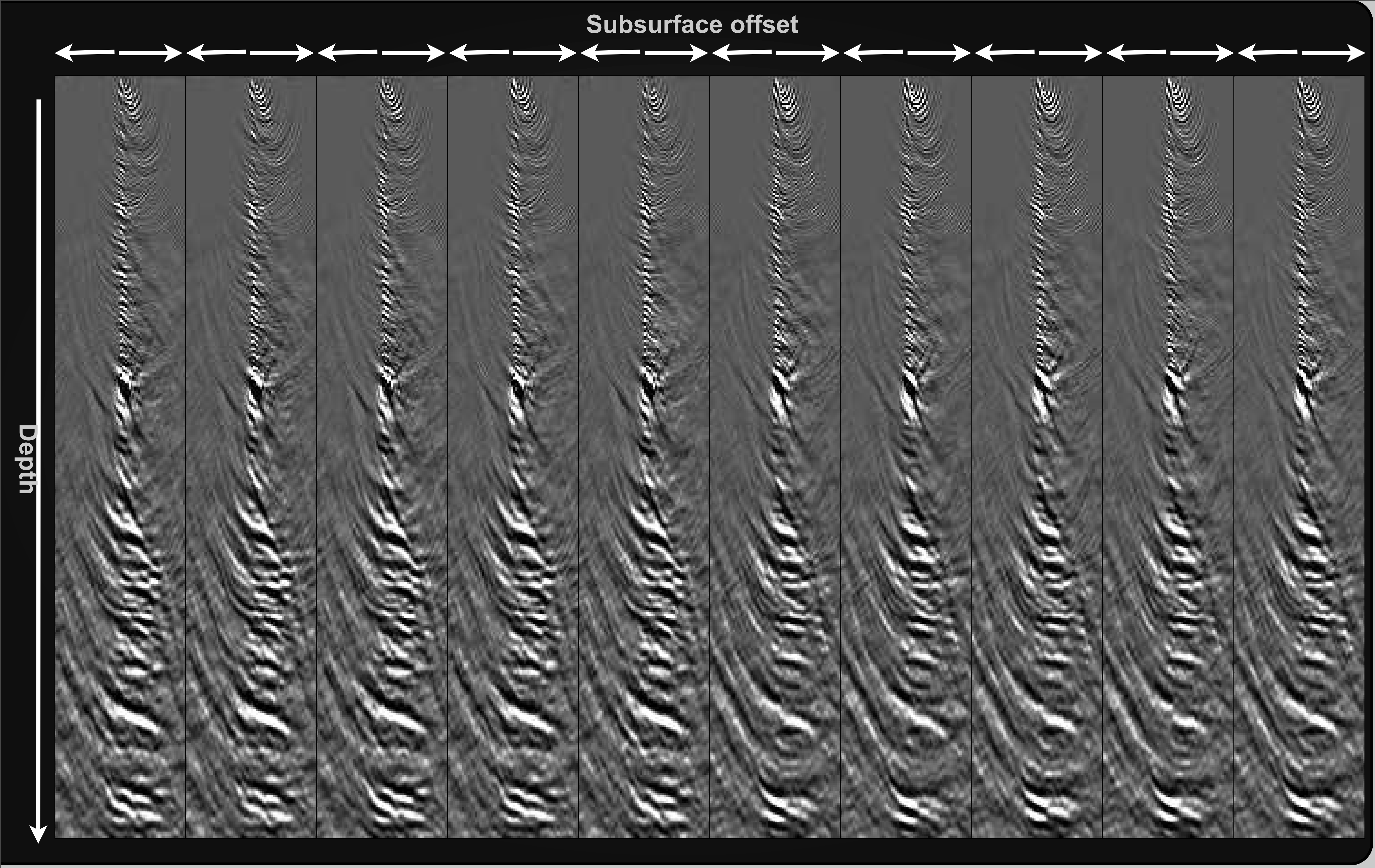
Thresholding iteration

$$\lambda_k = Q(\mathbf{x}, \mathbf{1.} - \mathbf{p})$$

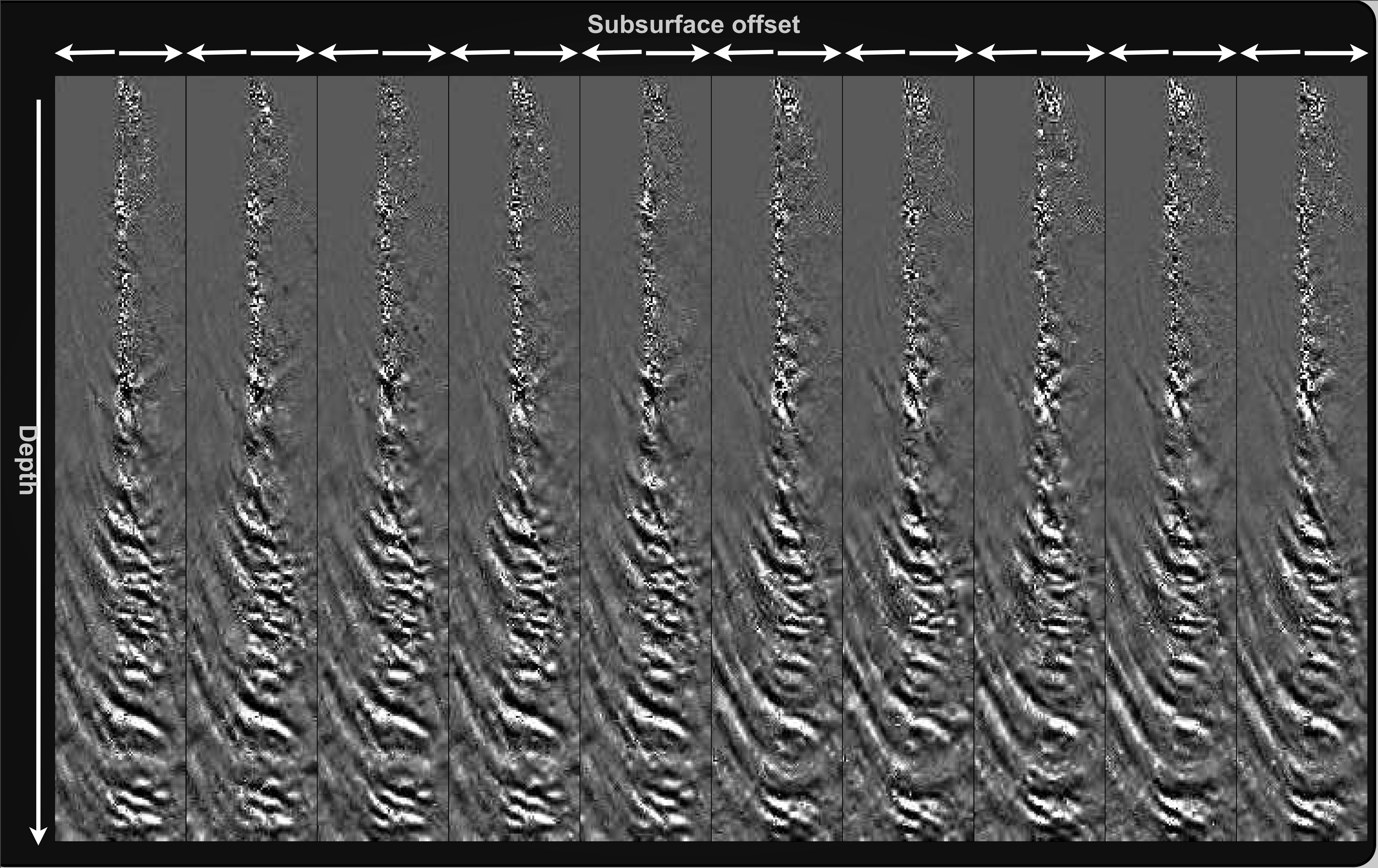
$$p = p * 1.8$$

$Q(\mathbf{x}, \mathbf{m})$
Return the m value percentile value of $\mathbf{x}$

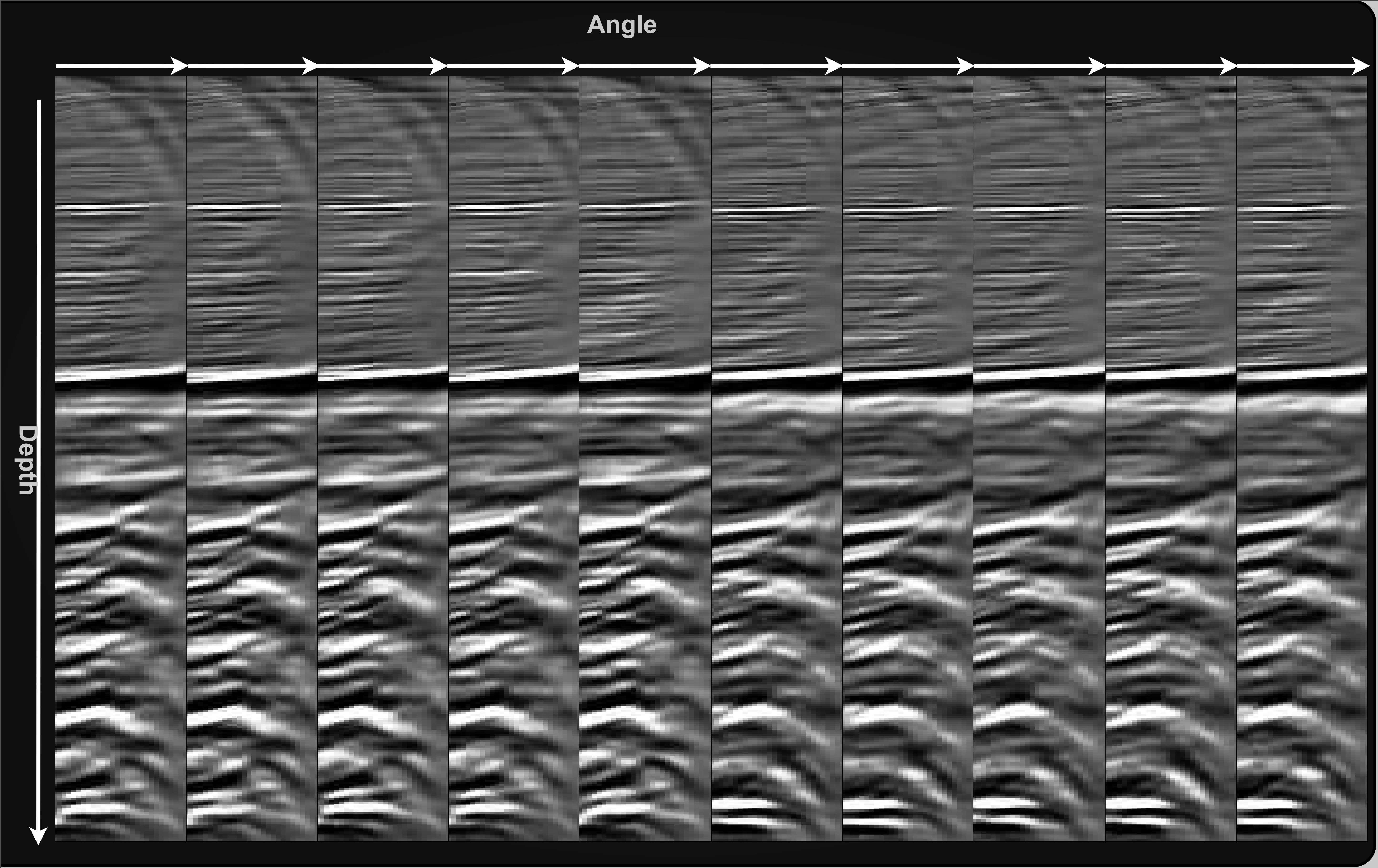
Thresholding scheme



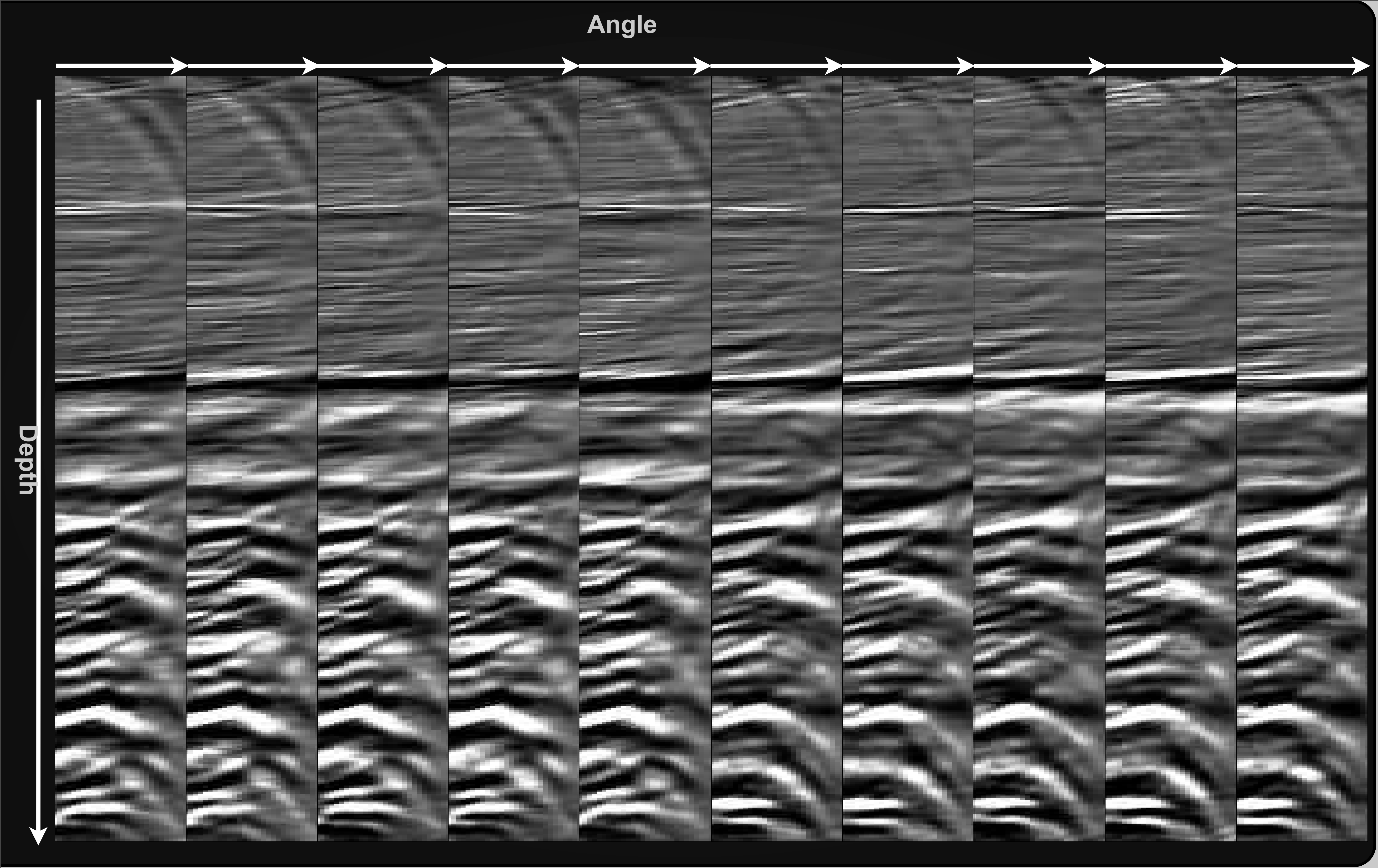
100% offsets



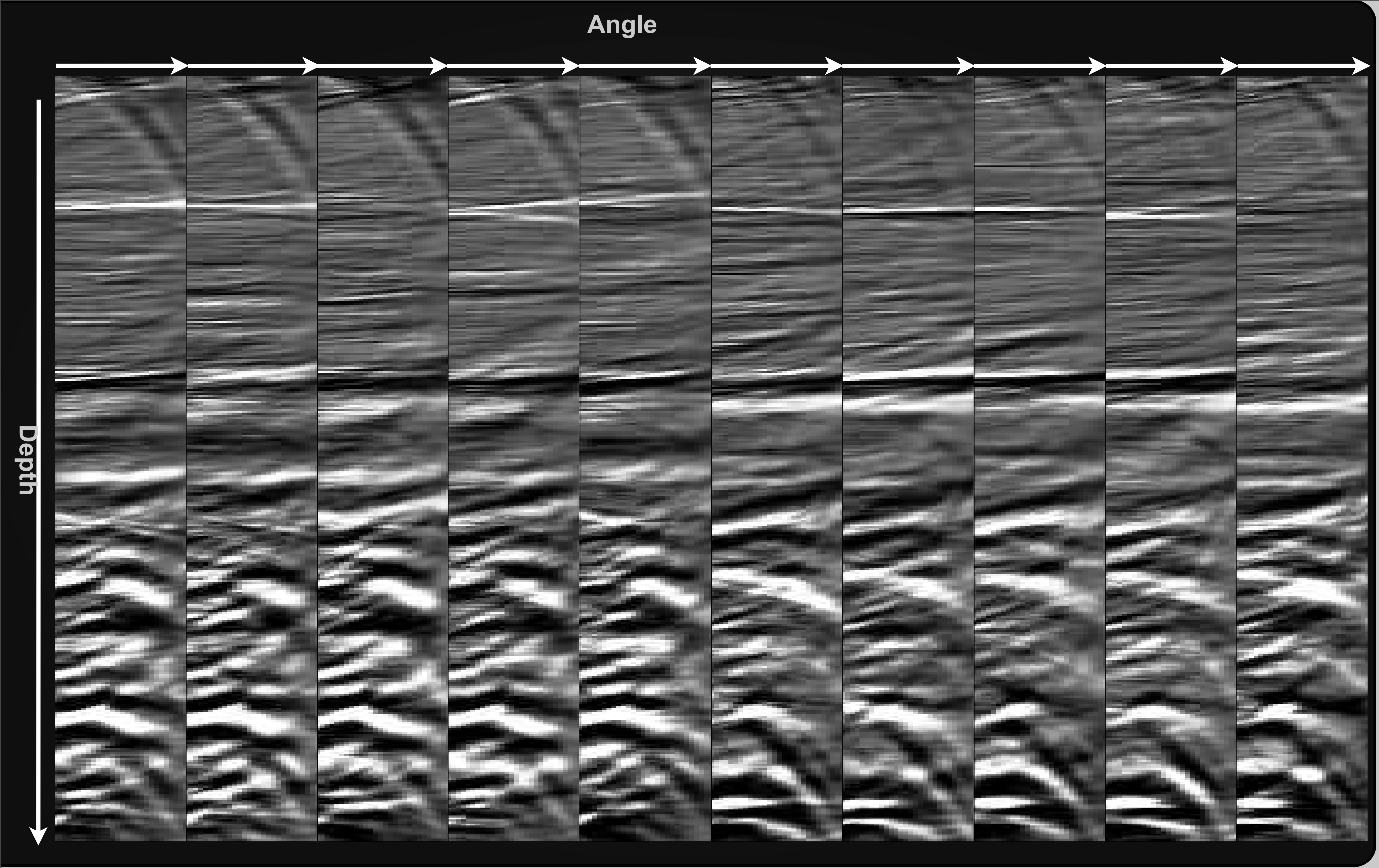
20% recovery result



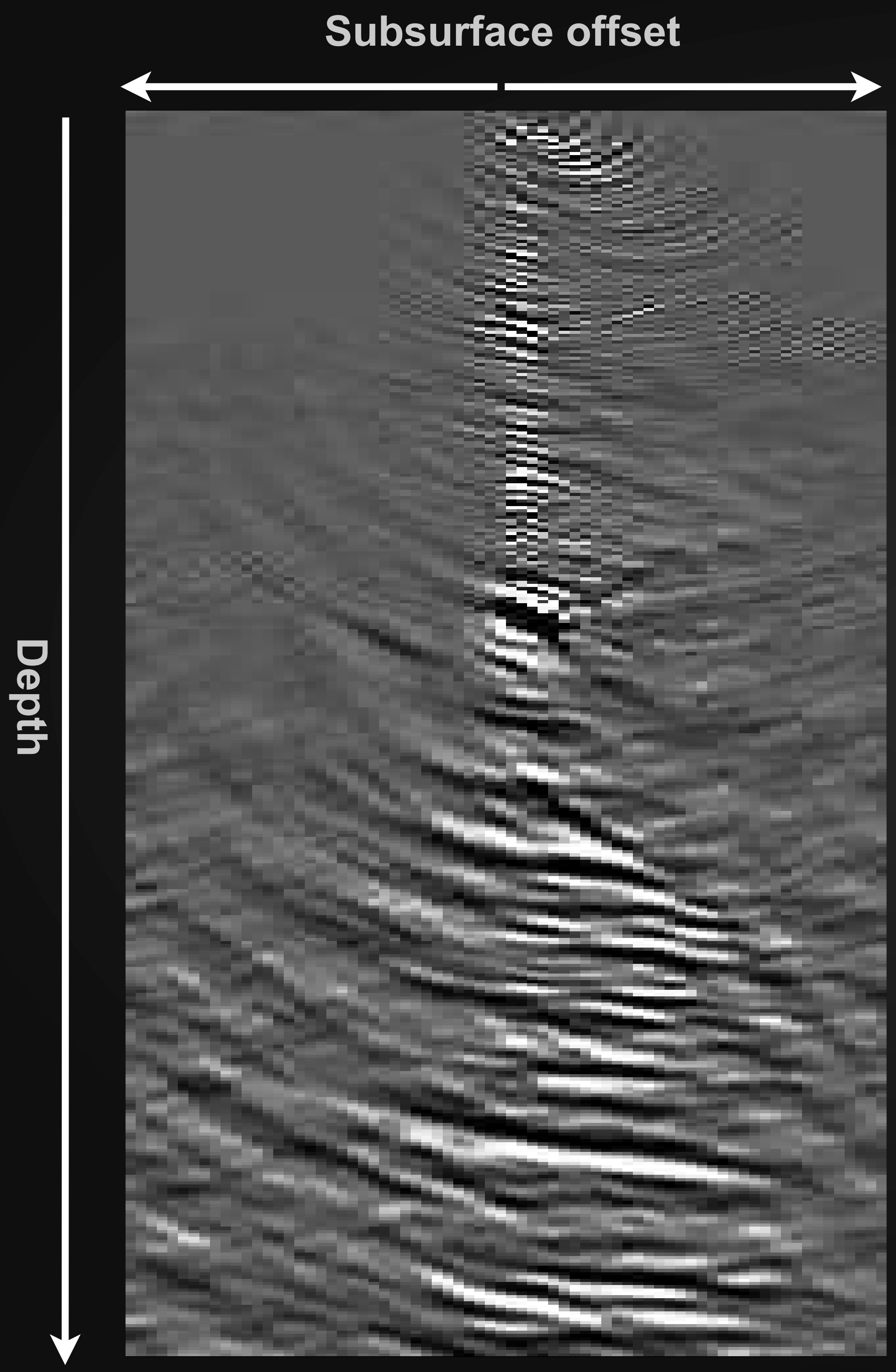
100% offset angle result



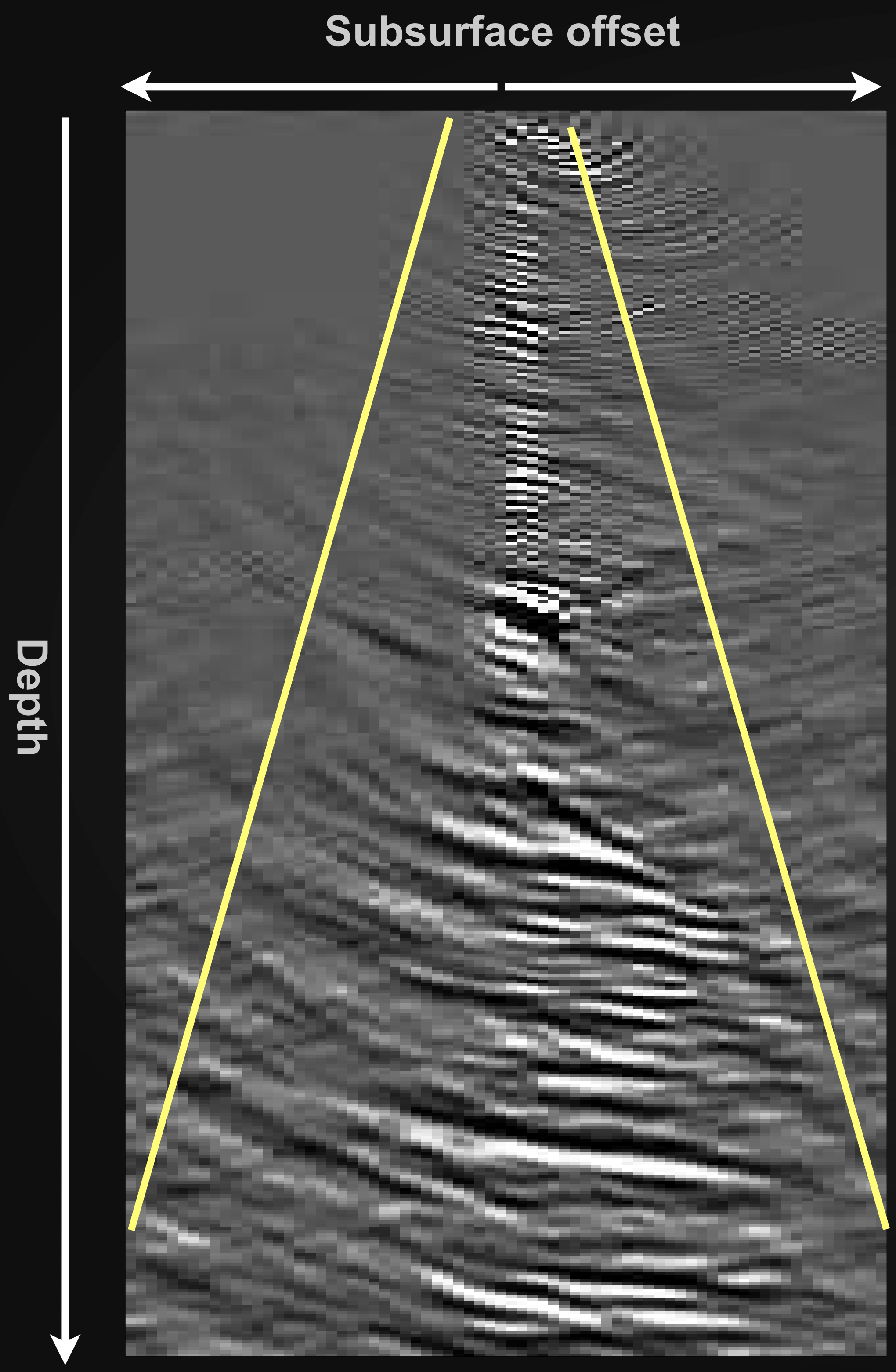
20% recovery (angle domain)



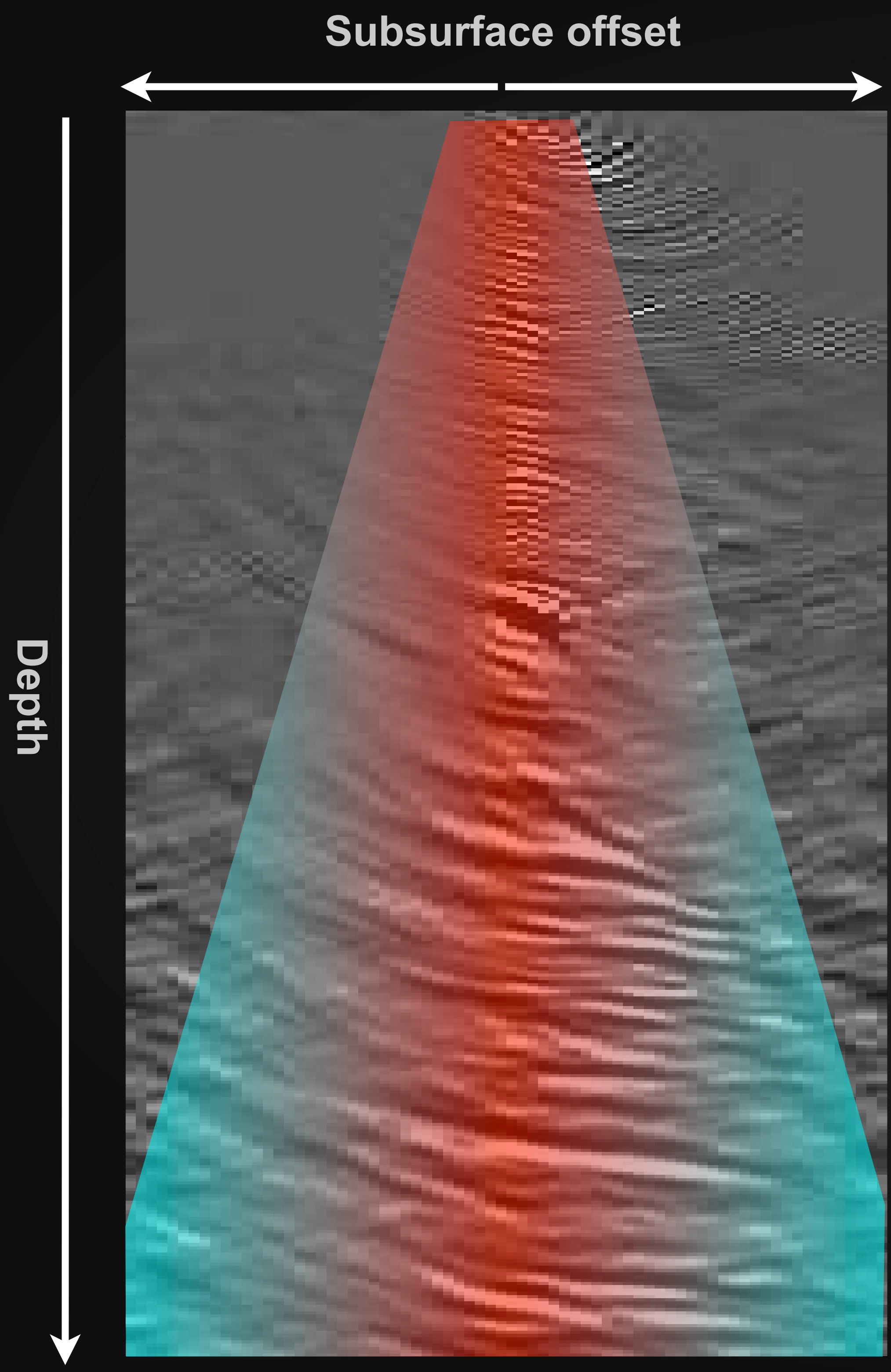
10% recovery (angle domain)



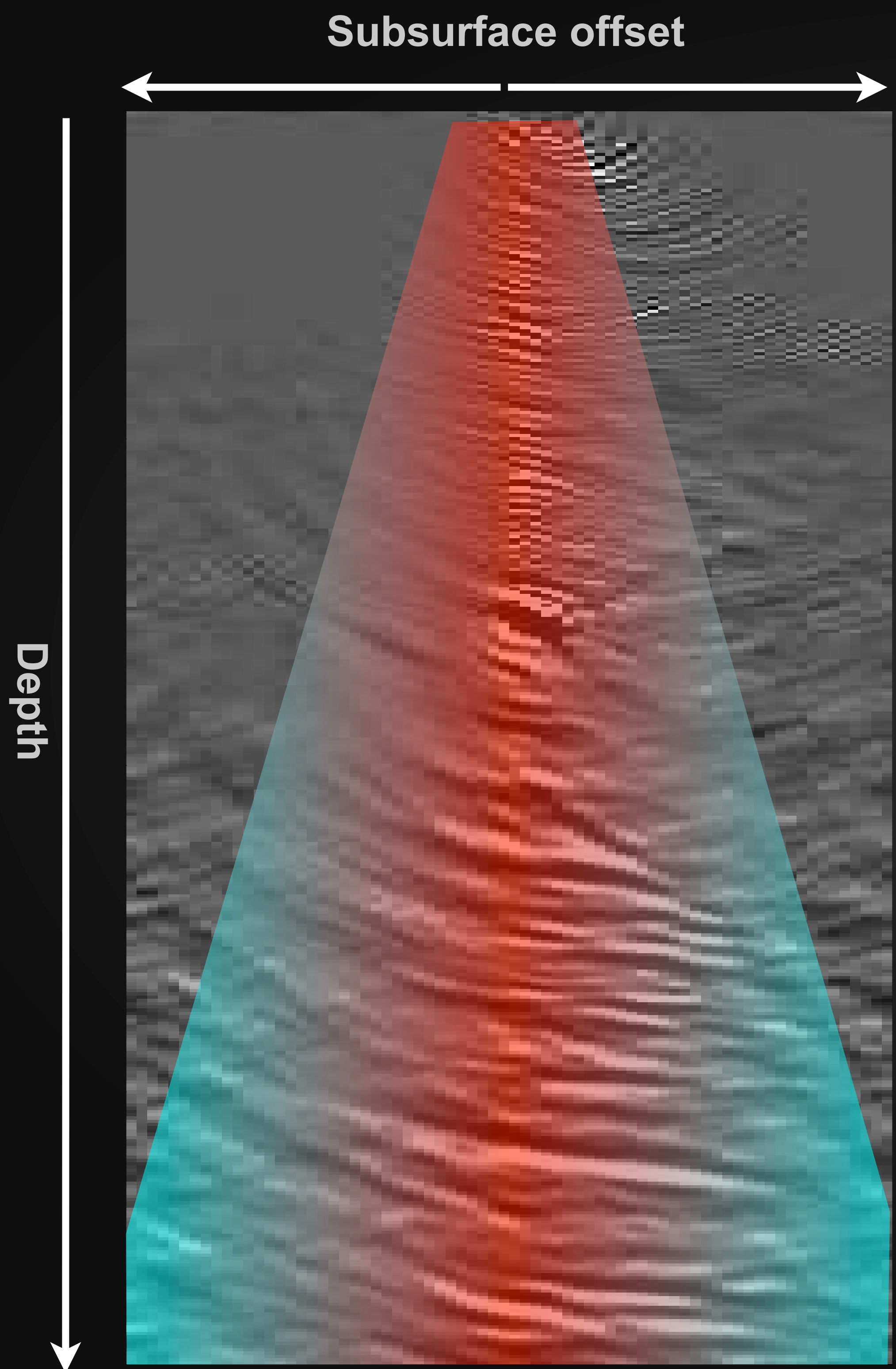
Thresholding scheme

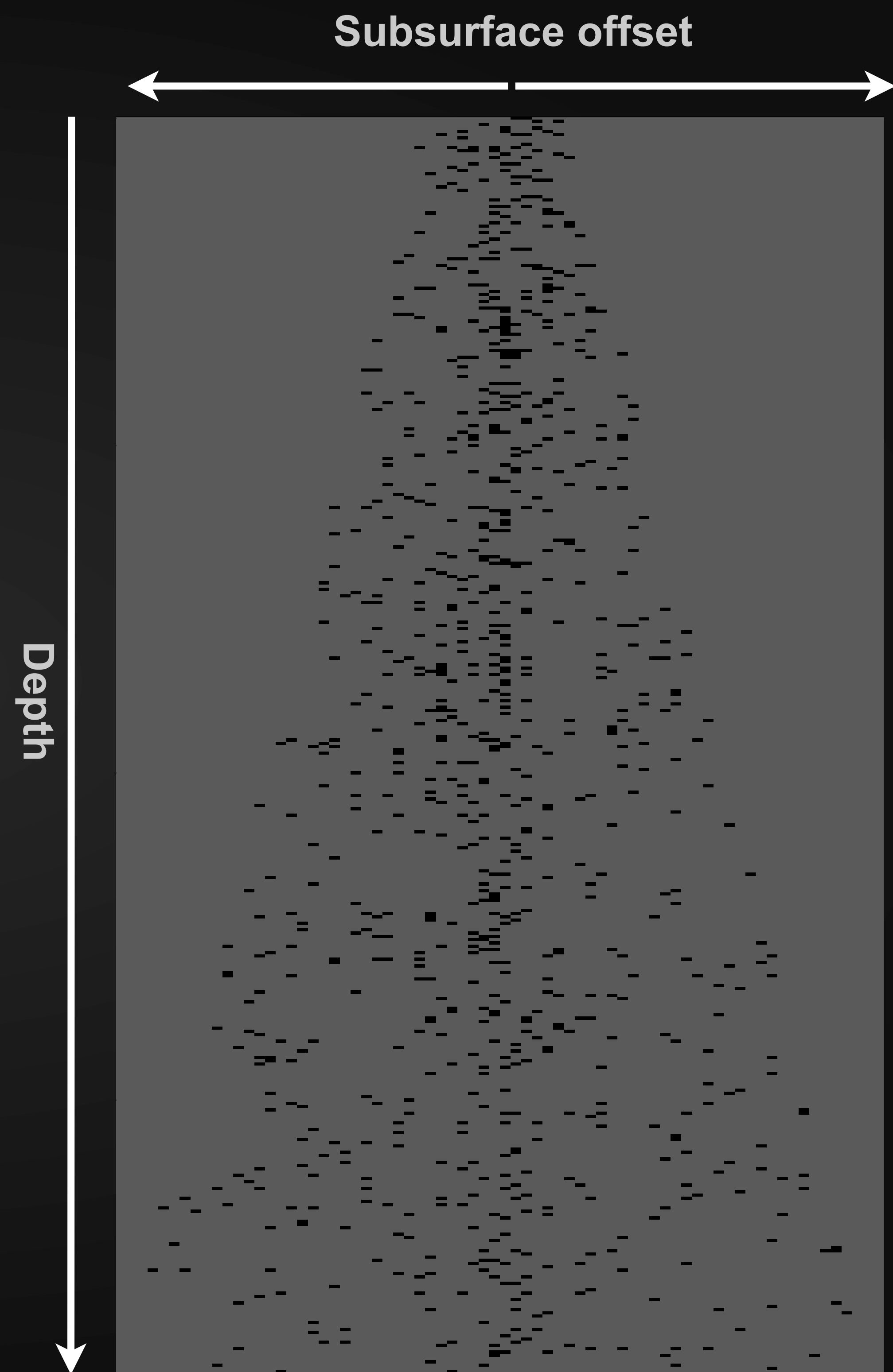


Thresholding scheme

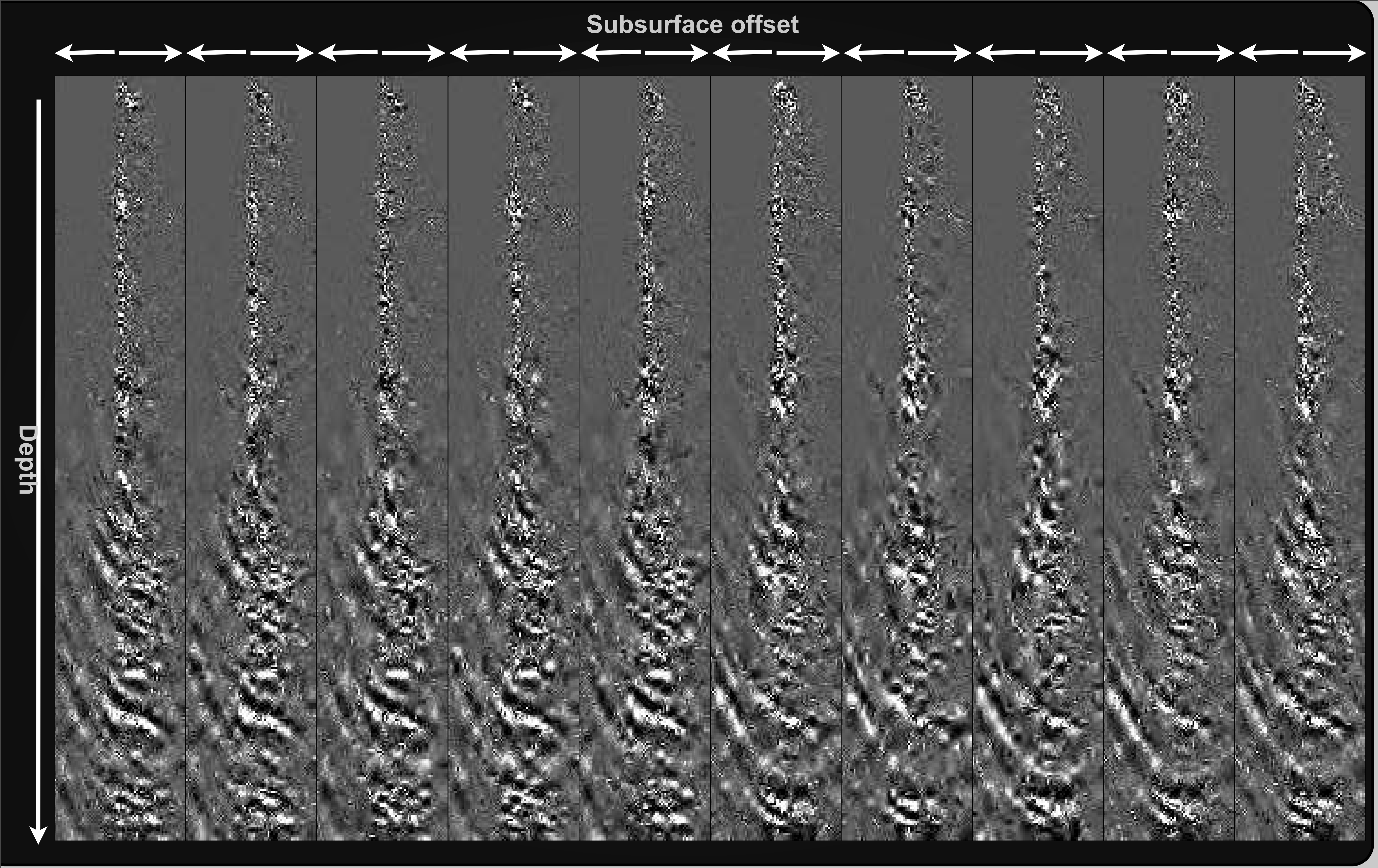


Thresholding scheme

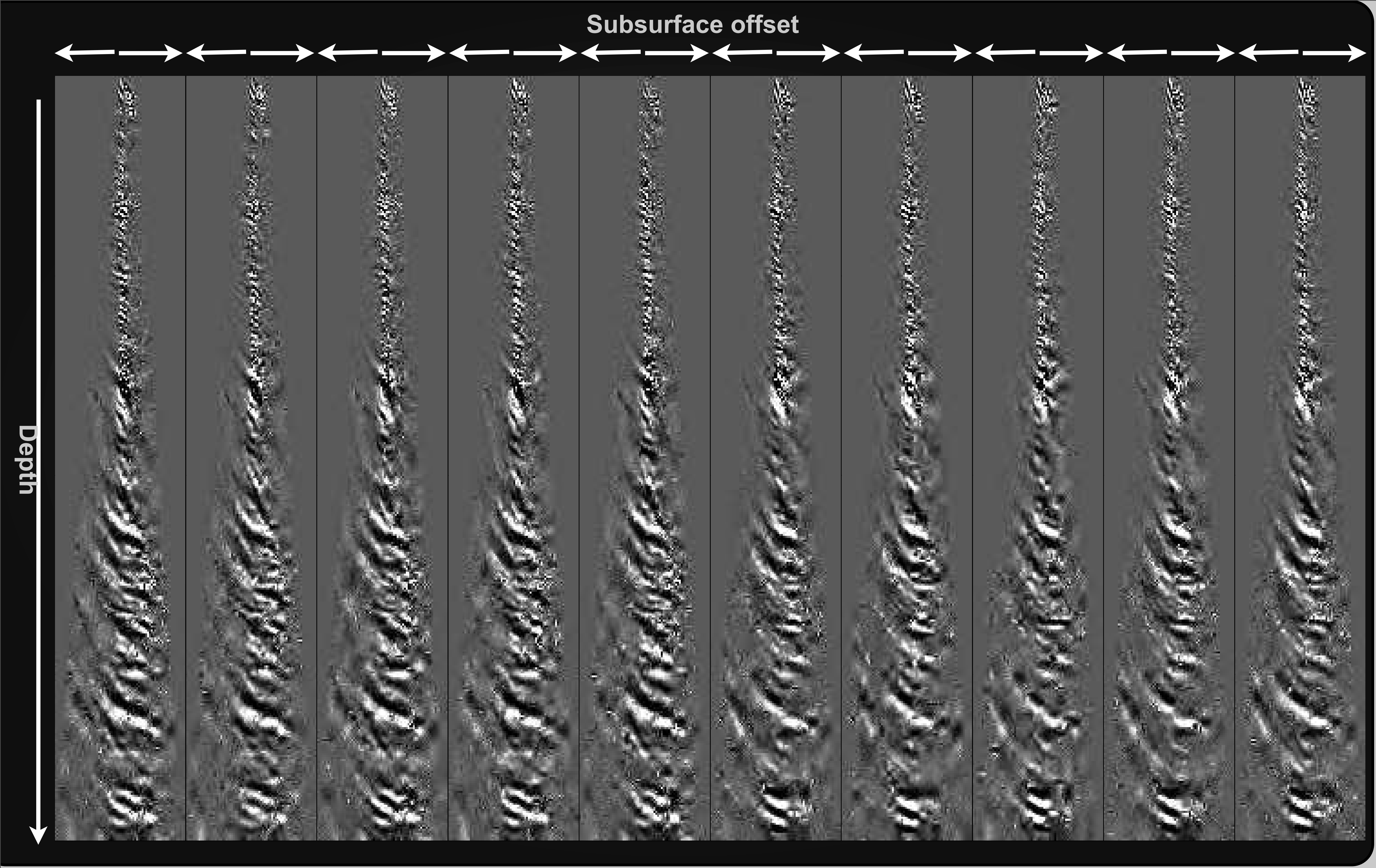




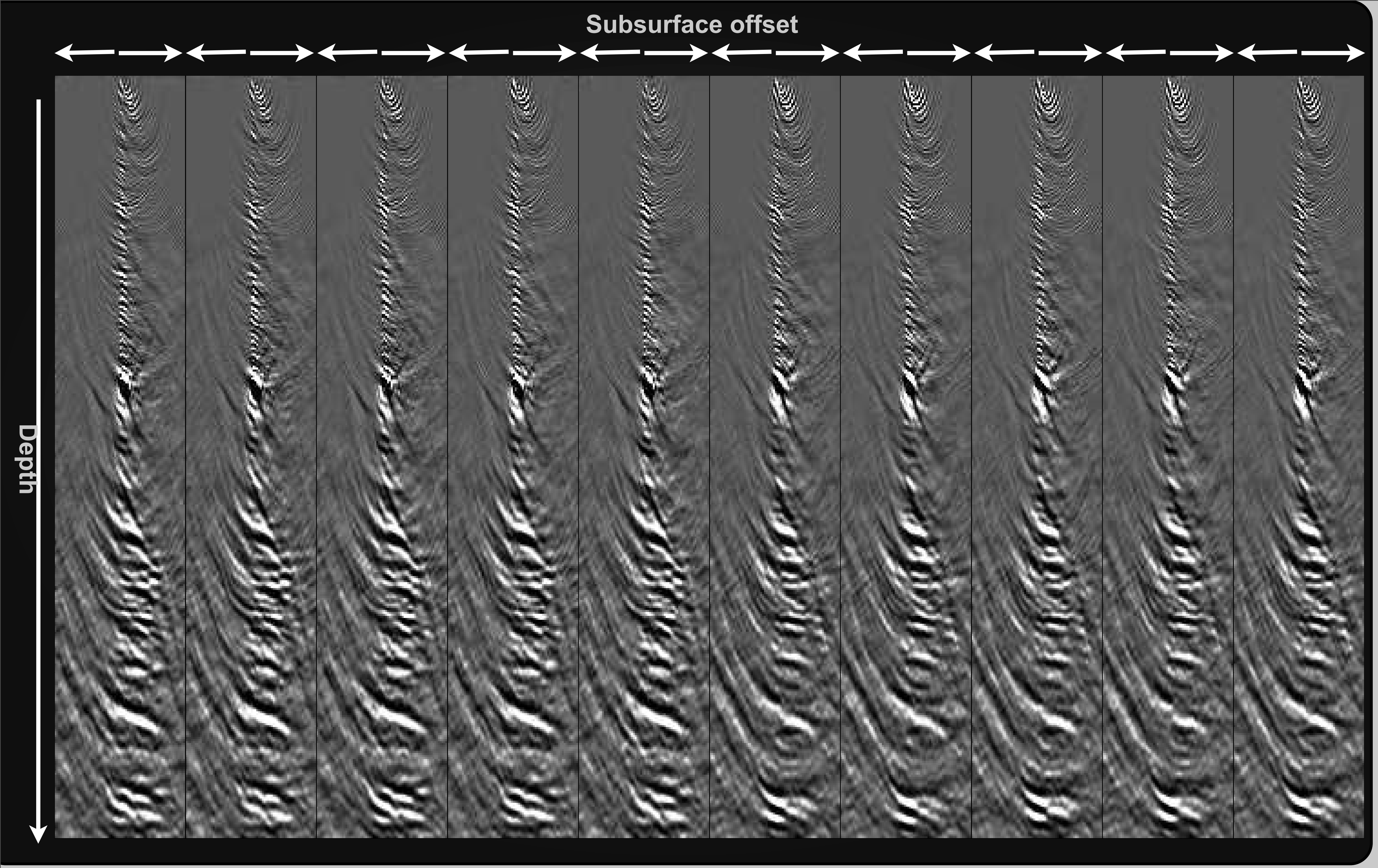
Thresholding scheme



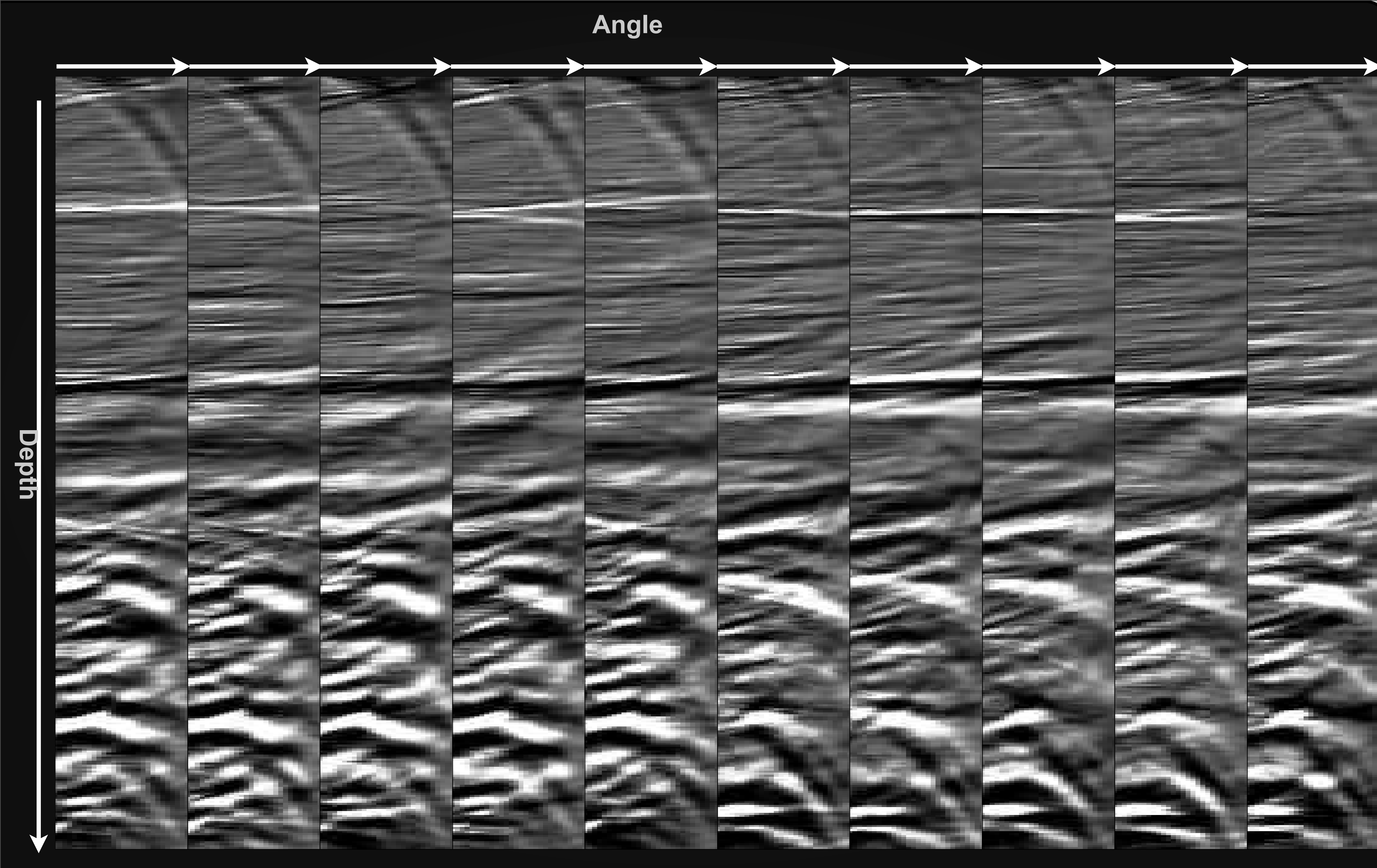
10% standard



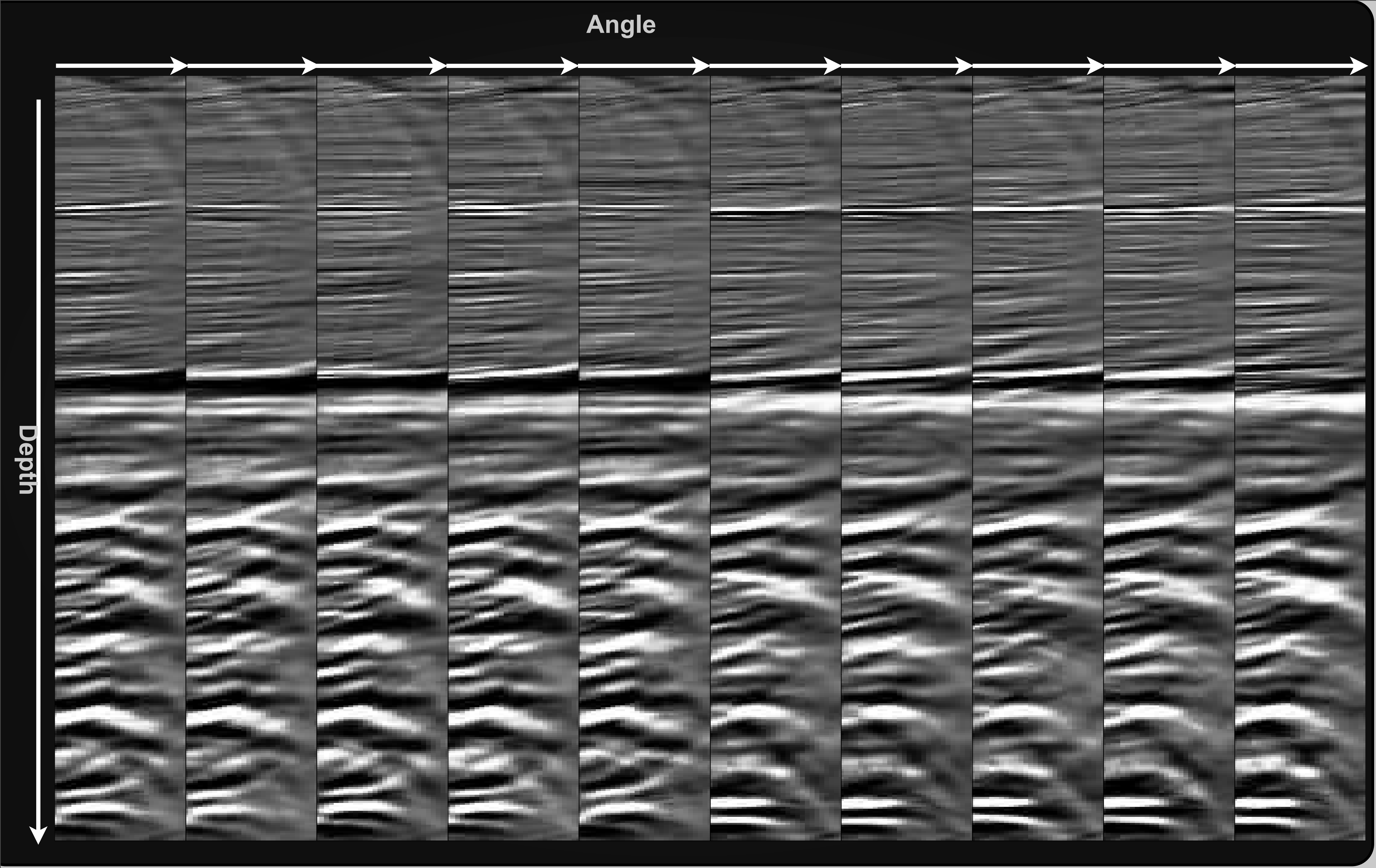
10% cone



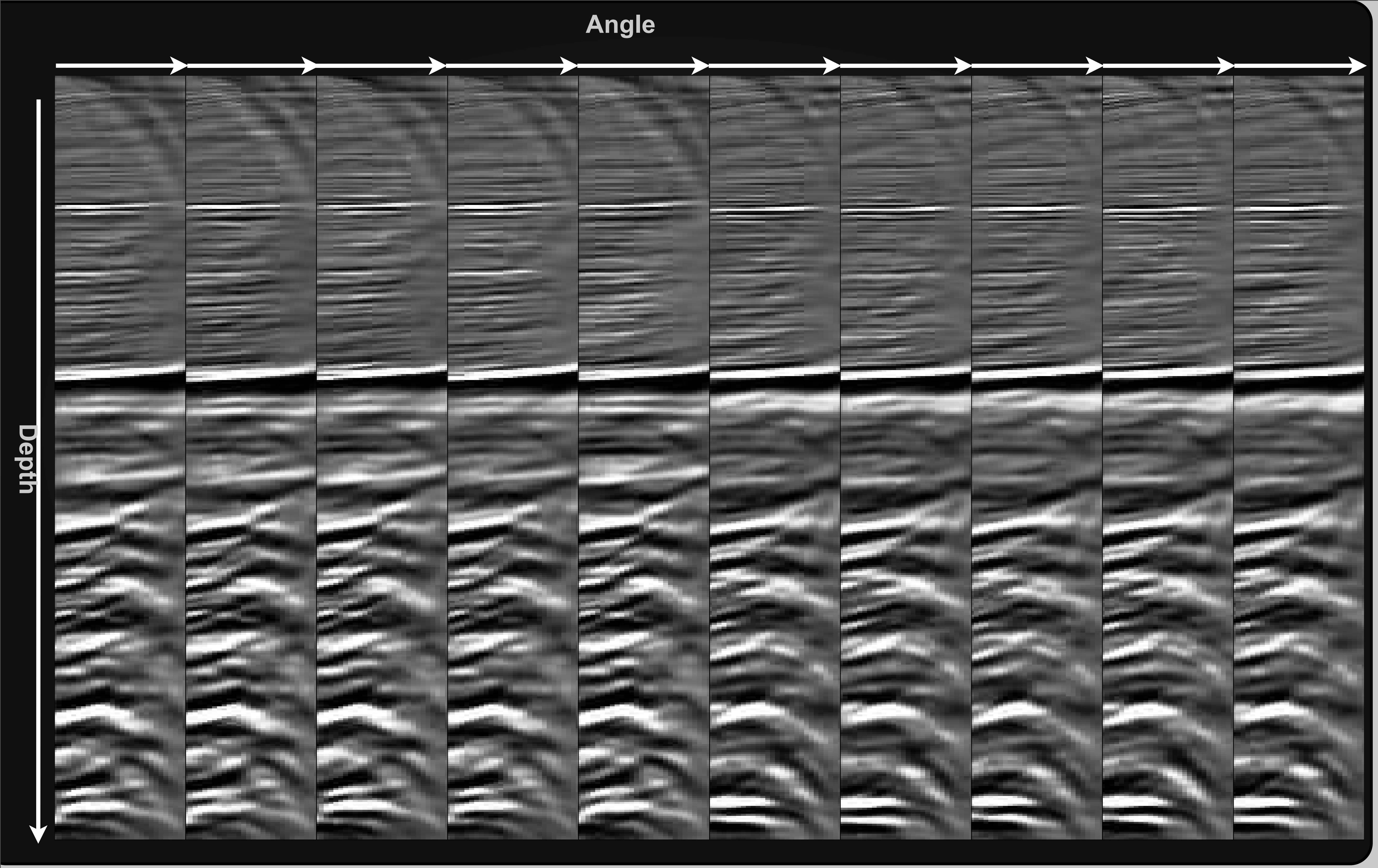
Full offsets



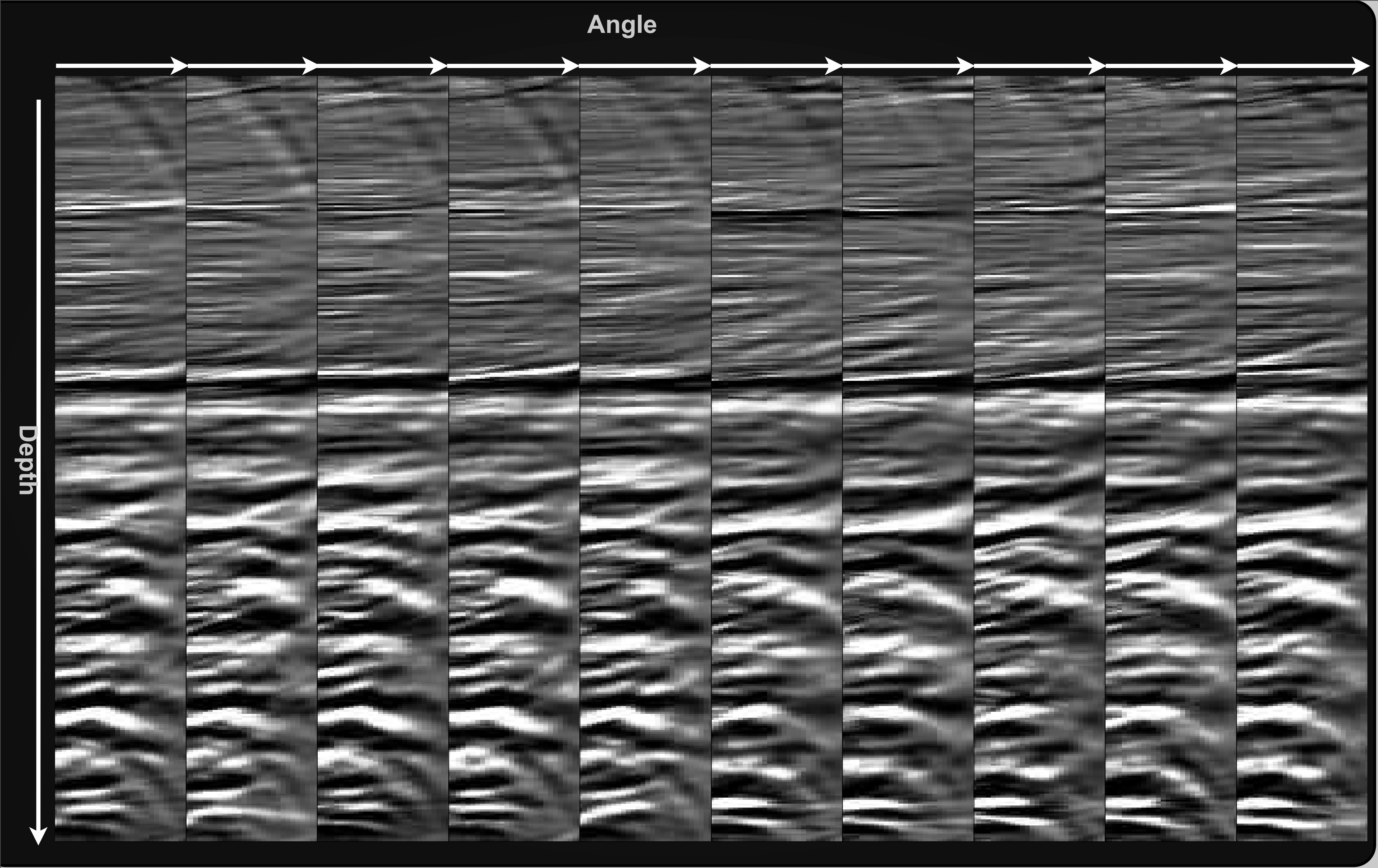
10% standard (angle domain)



10% cone (angle domain)



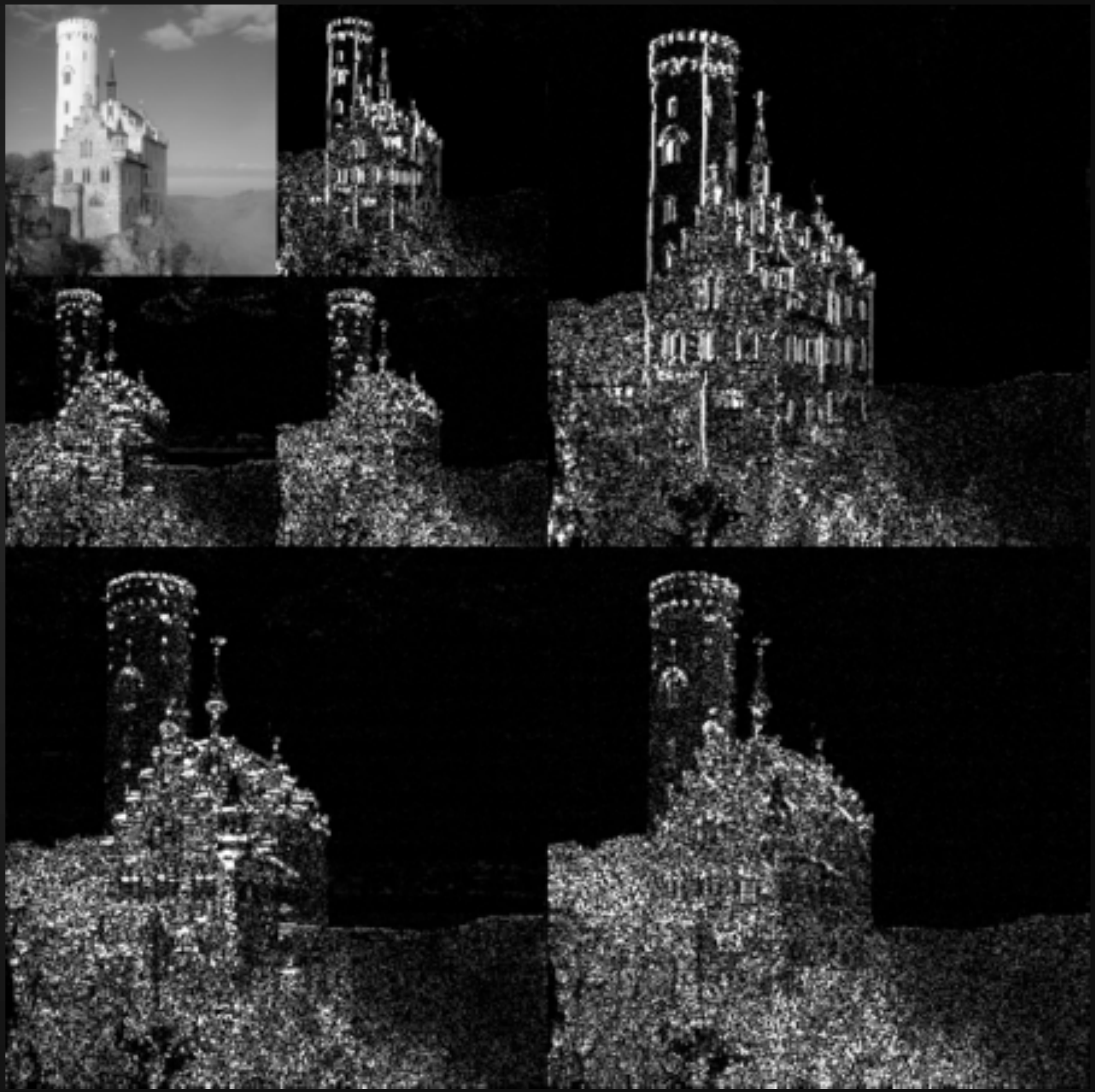
100% angle result



5% cone angle result

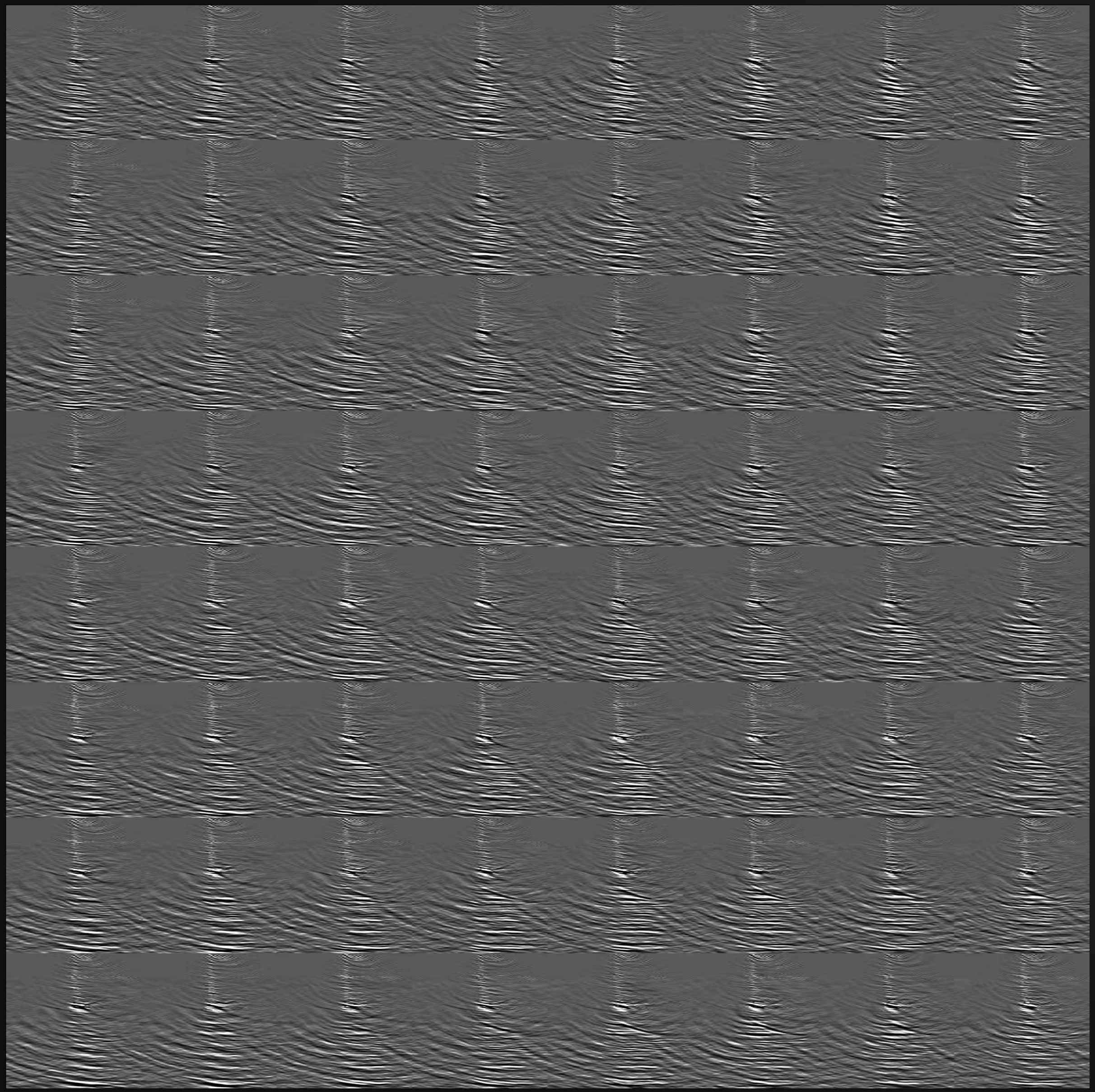
Original

Multi-D wavelet transform

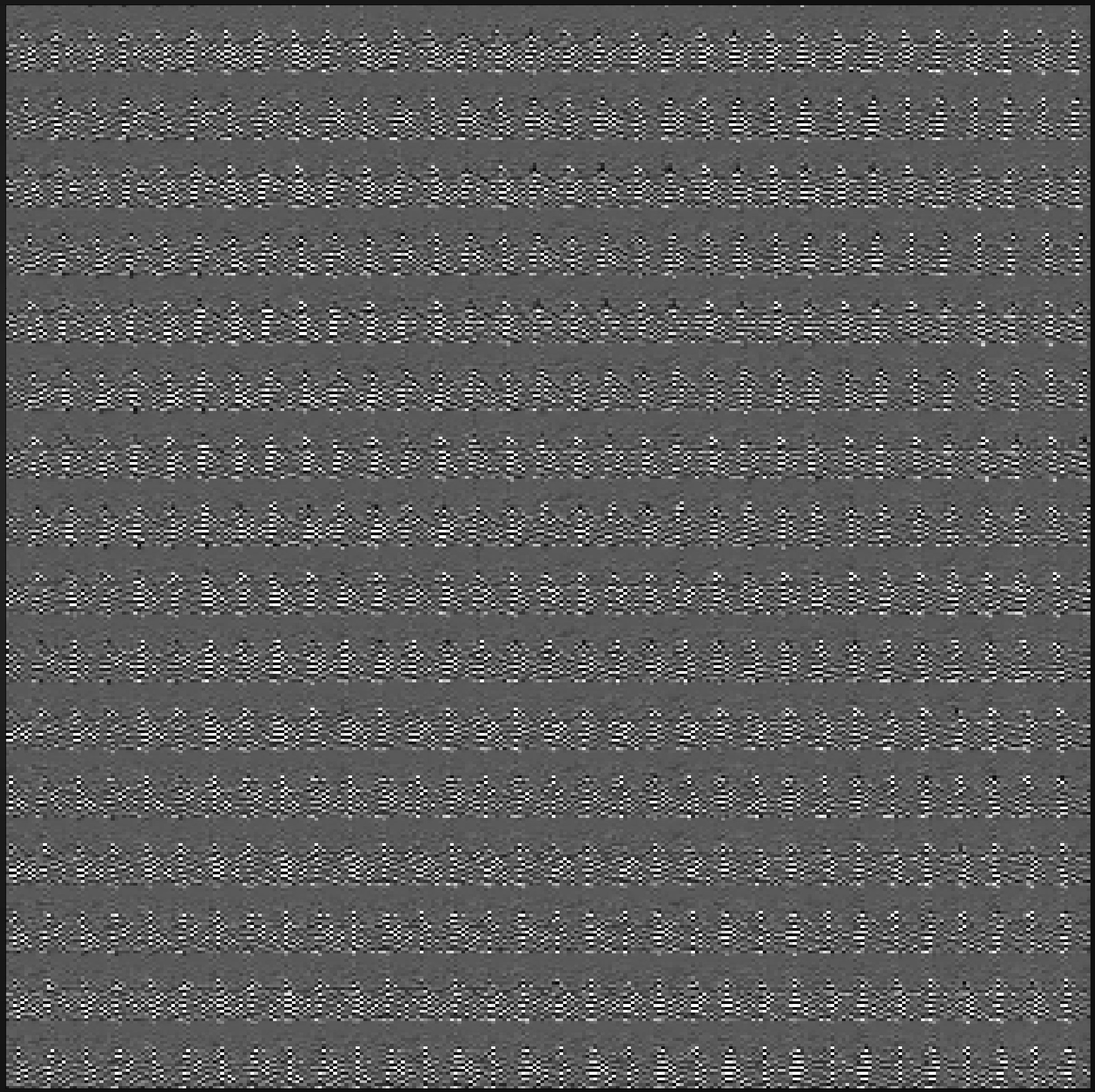


Multi-D wavelet transform

Original



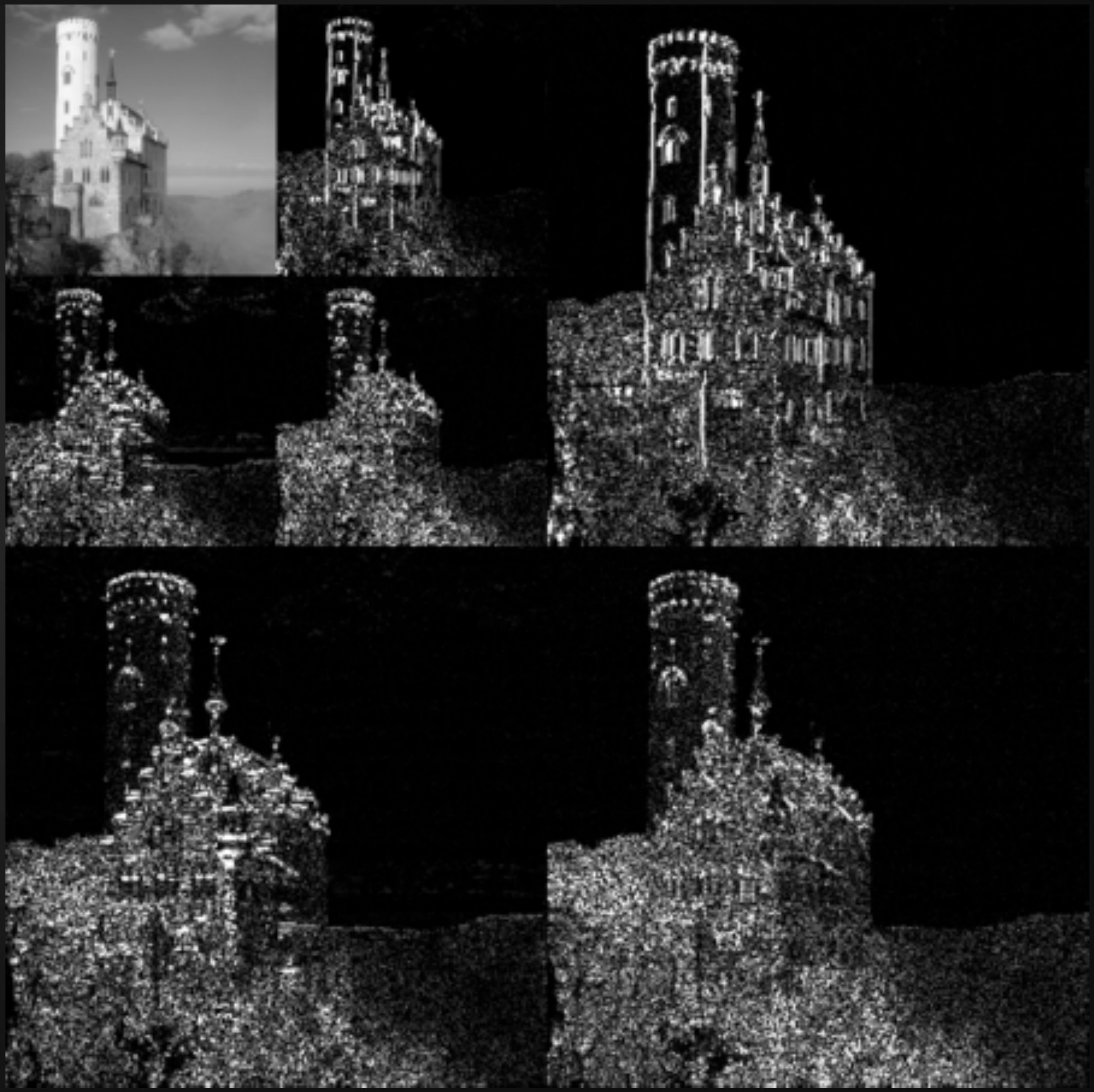
Lowest pass



Multi-D wavelet transform

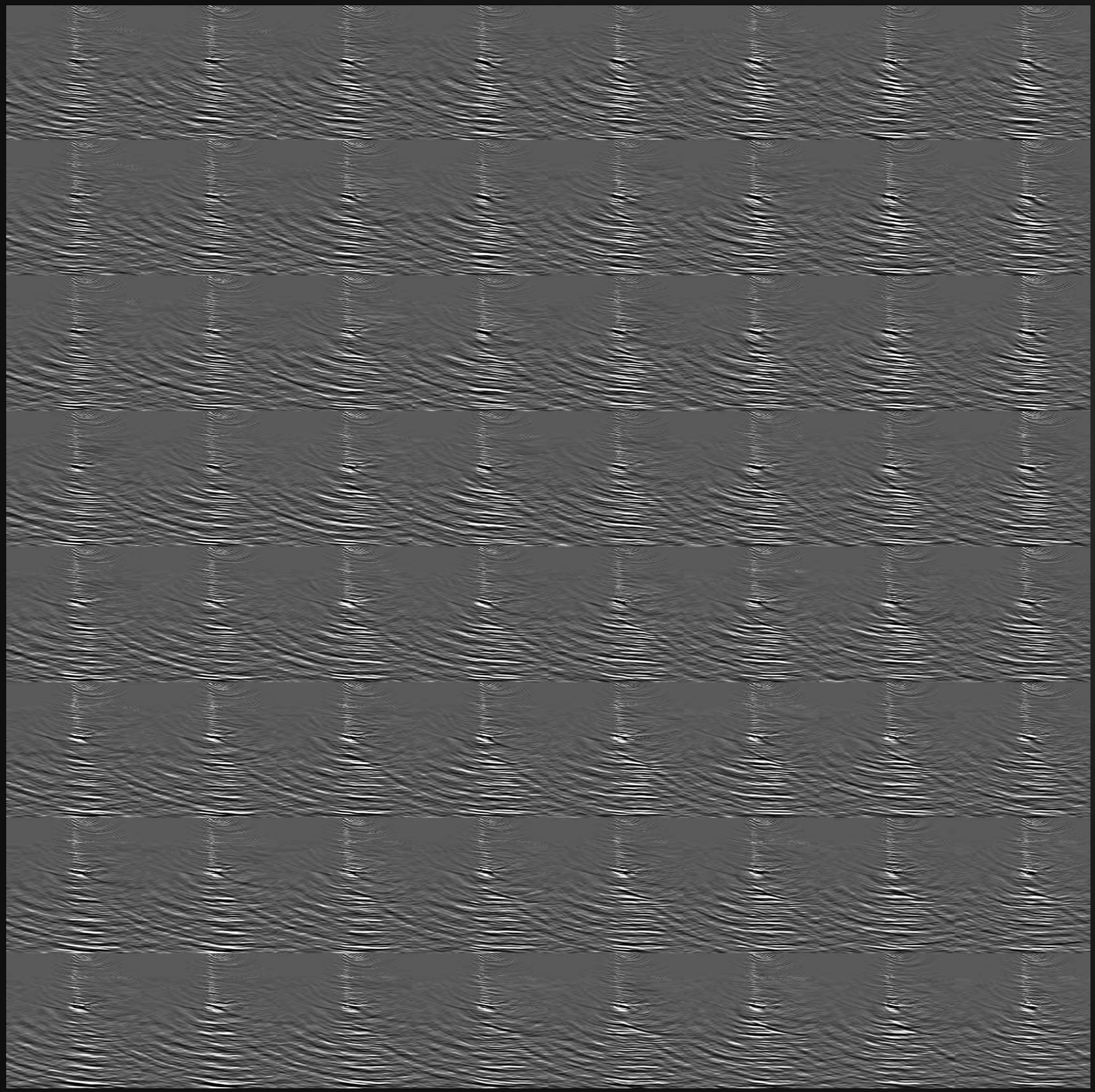
Original

N-D wavelet transform

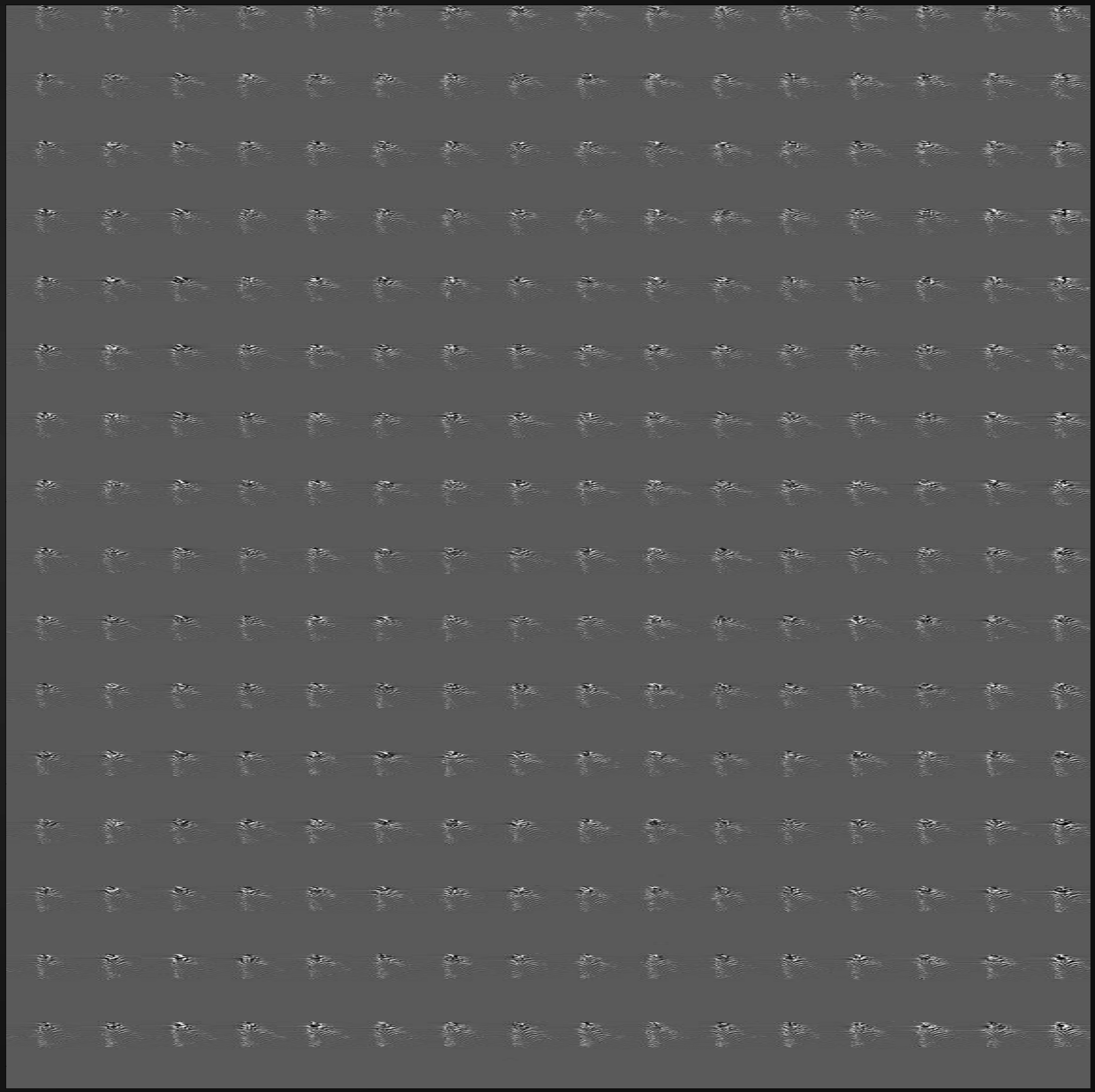


Multi-D wavelet transform

Original

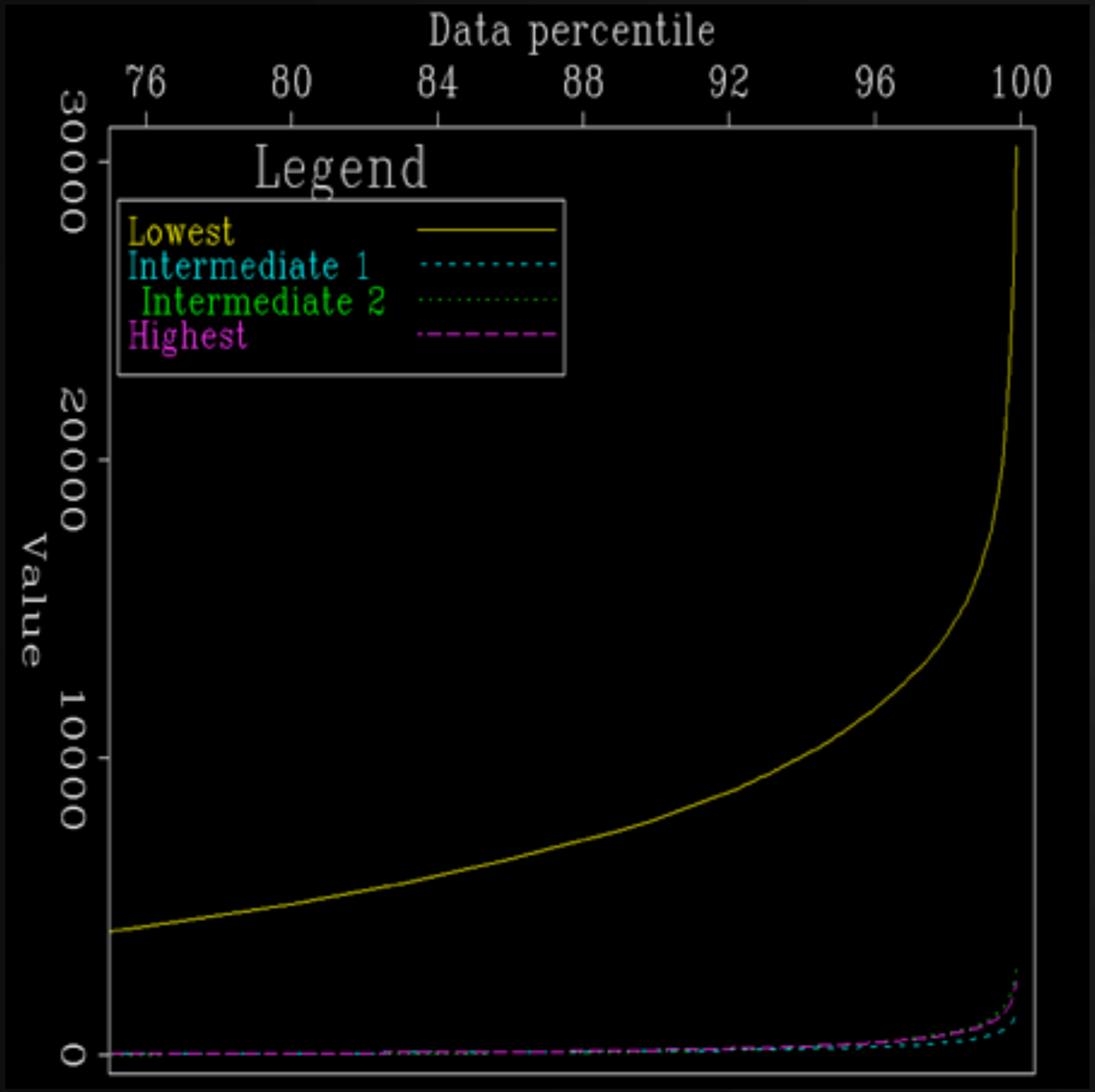


Highest pass

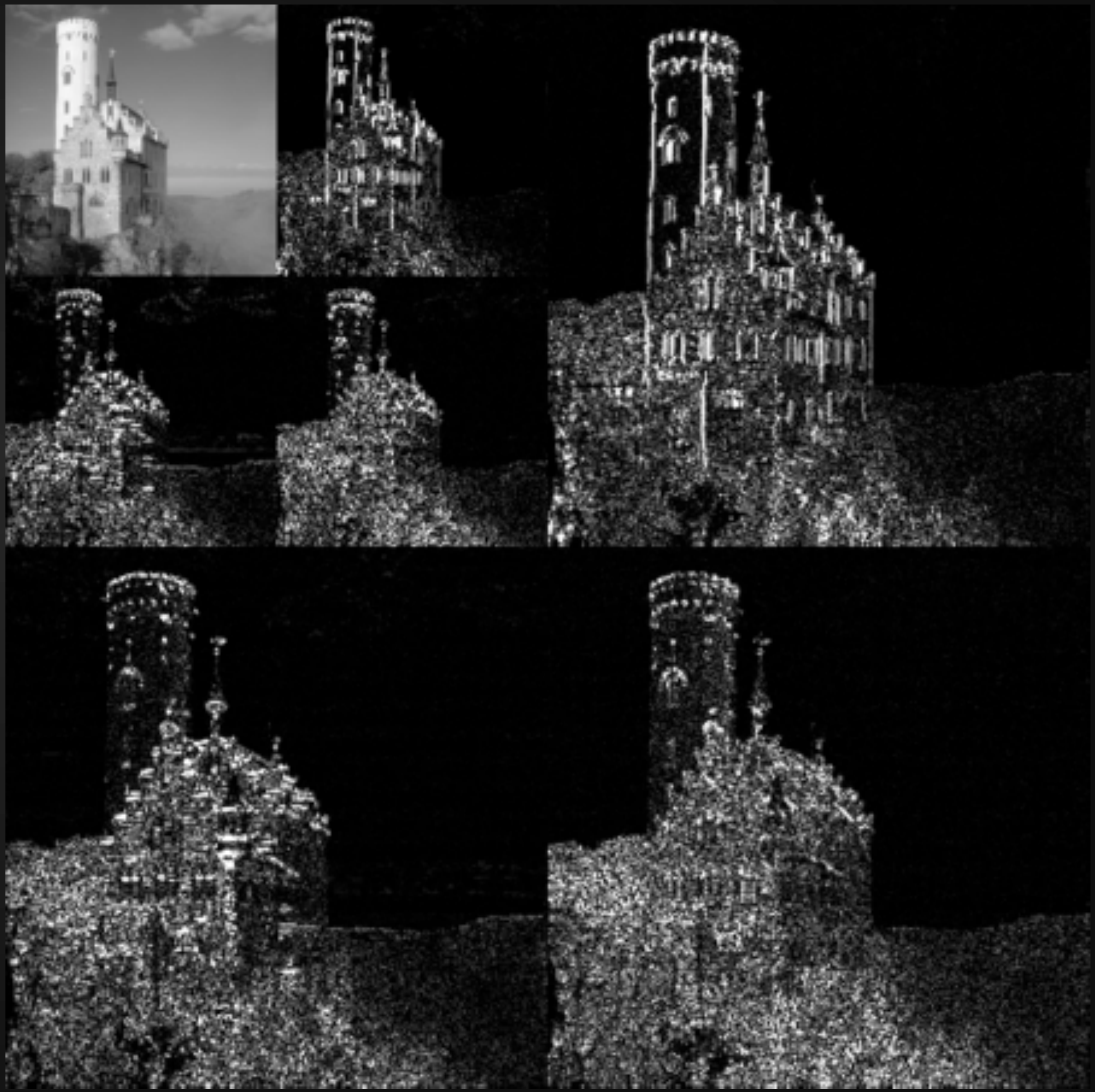


Multi-D wavelet transform

Original

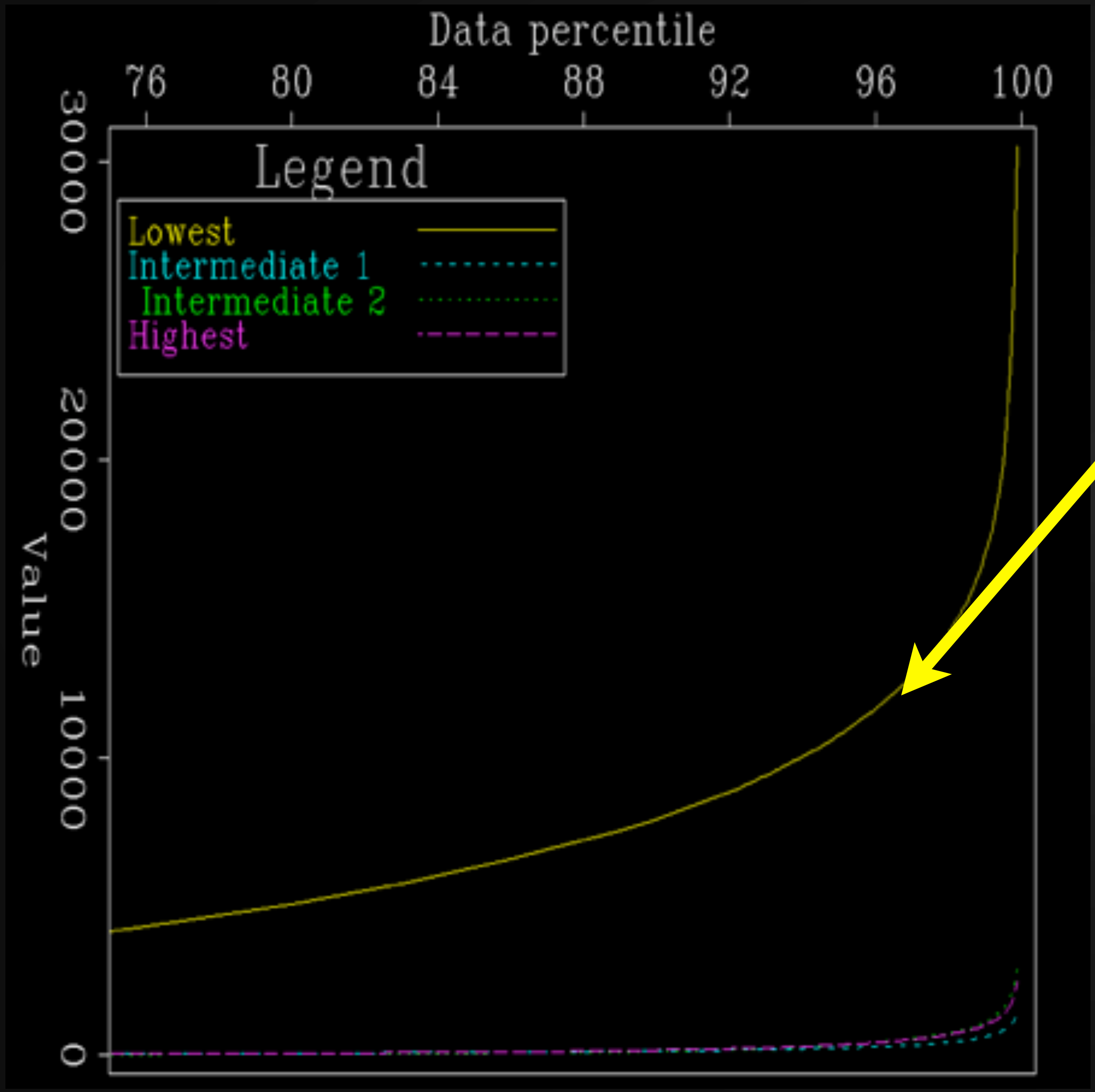


Highest pass

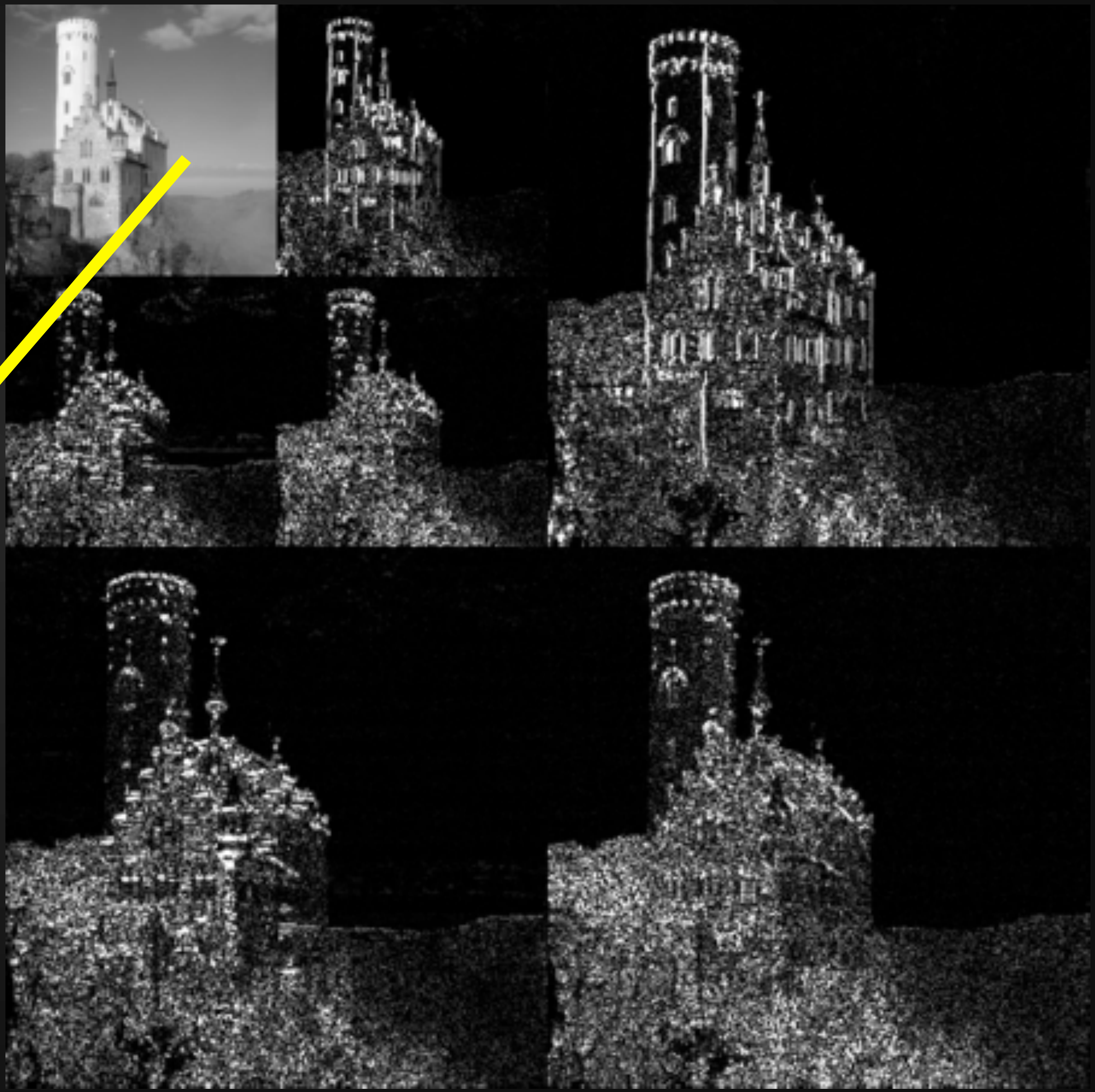


Multi-D wavelet transform

Original

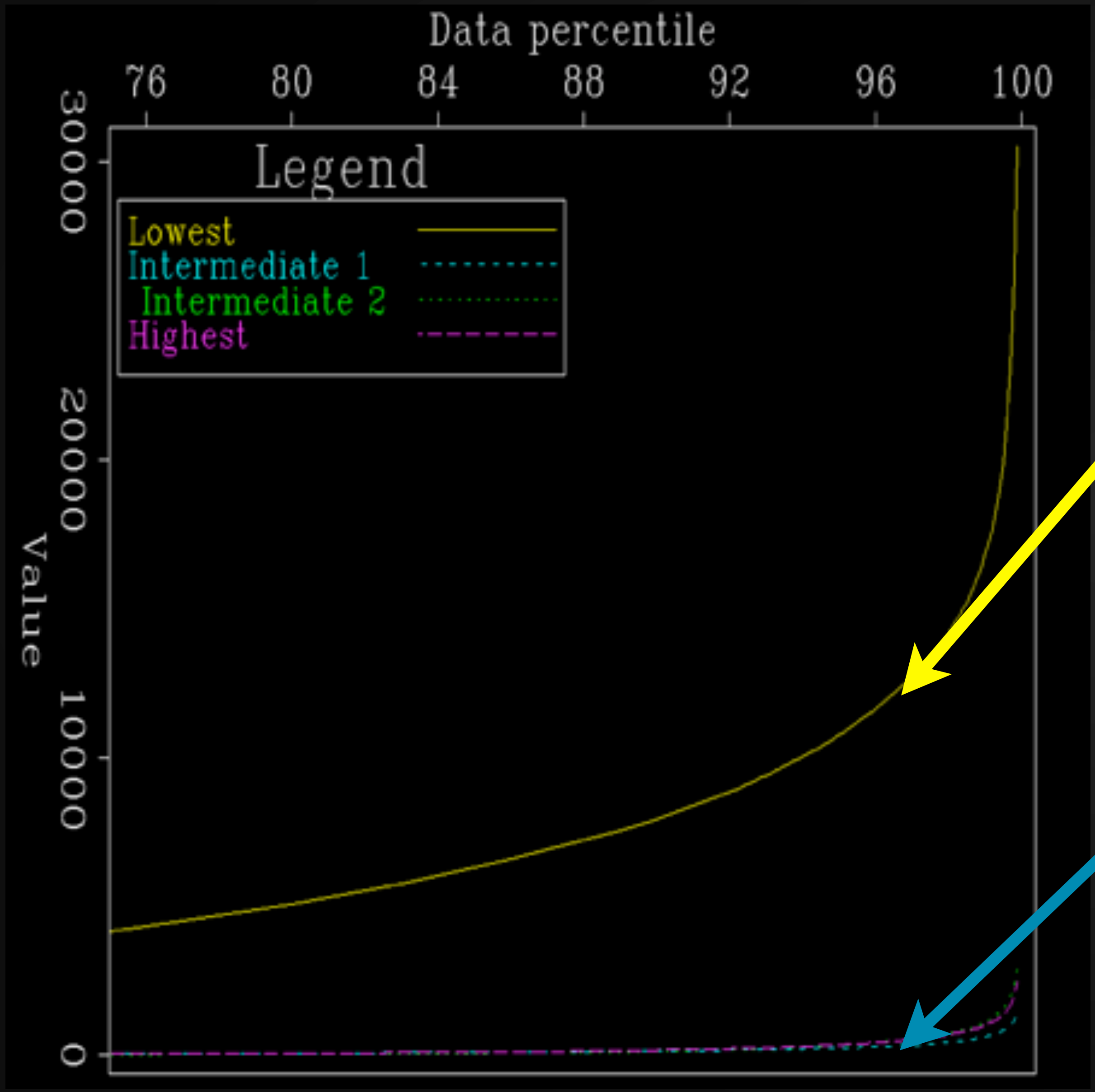


Highest pass

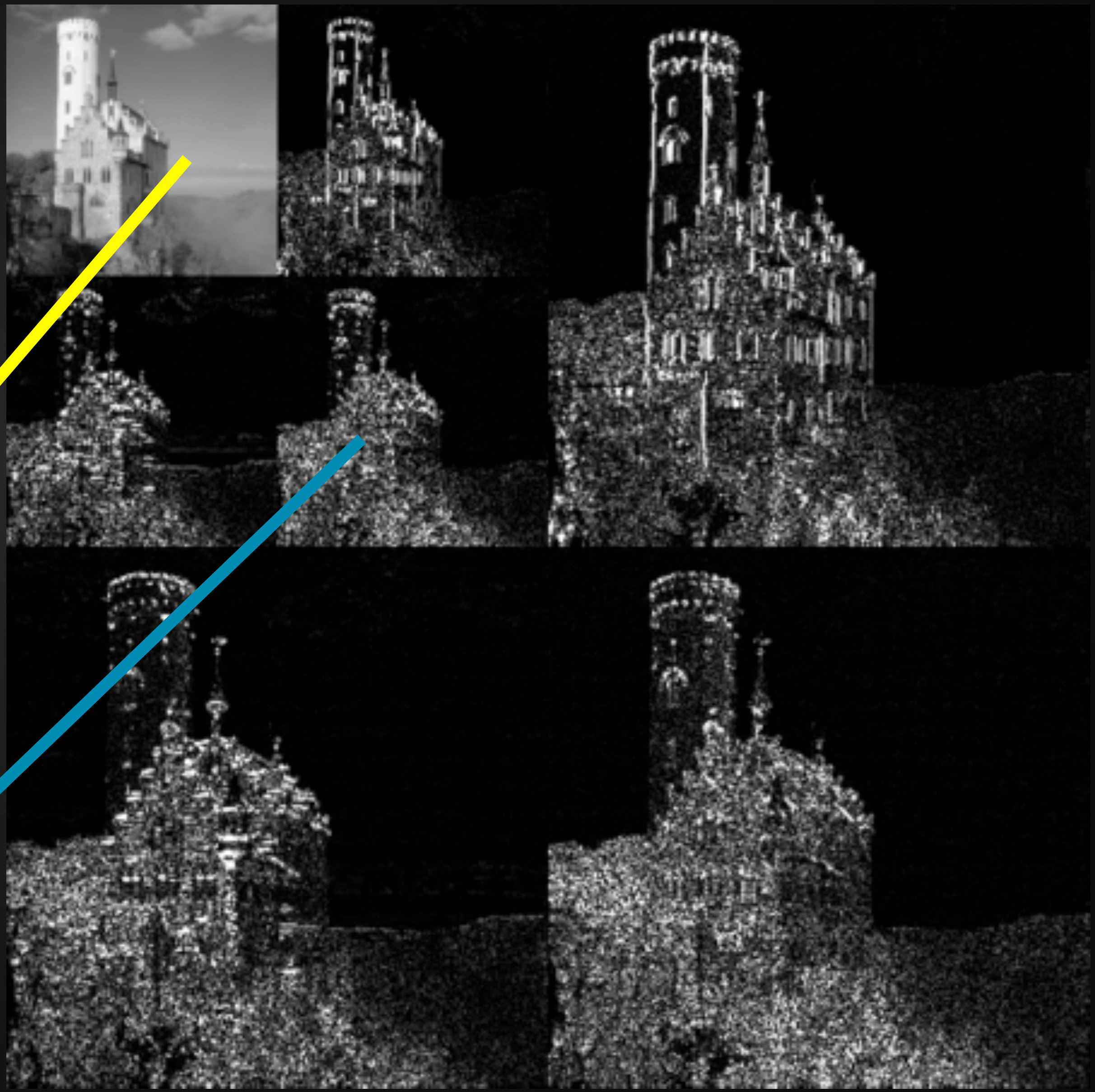


Multi-D wavelet transform

Original



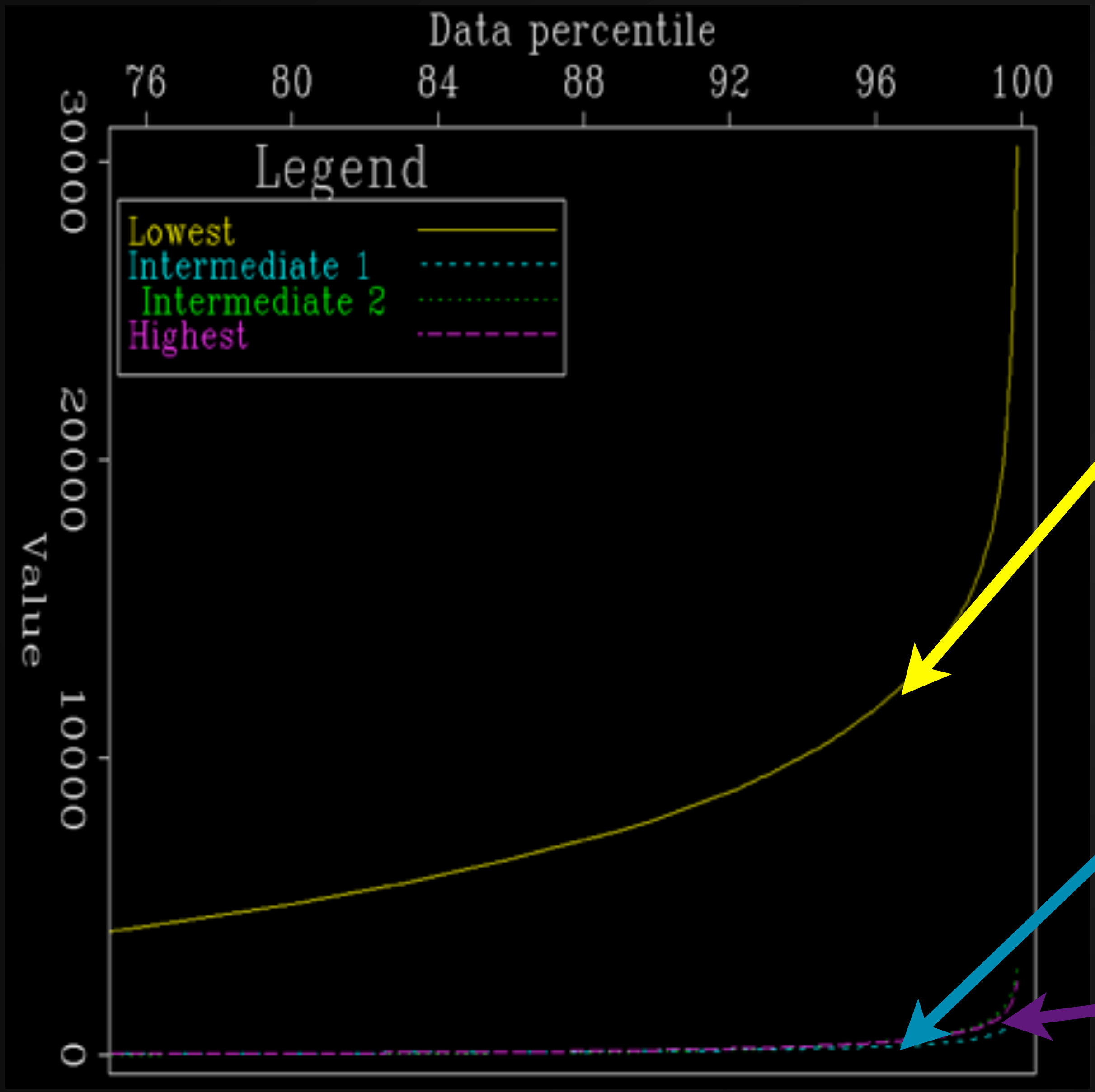
Highest pass

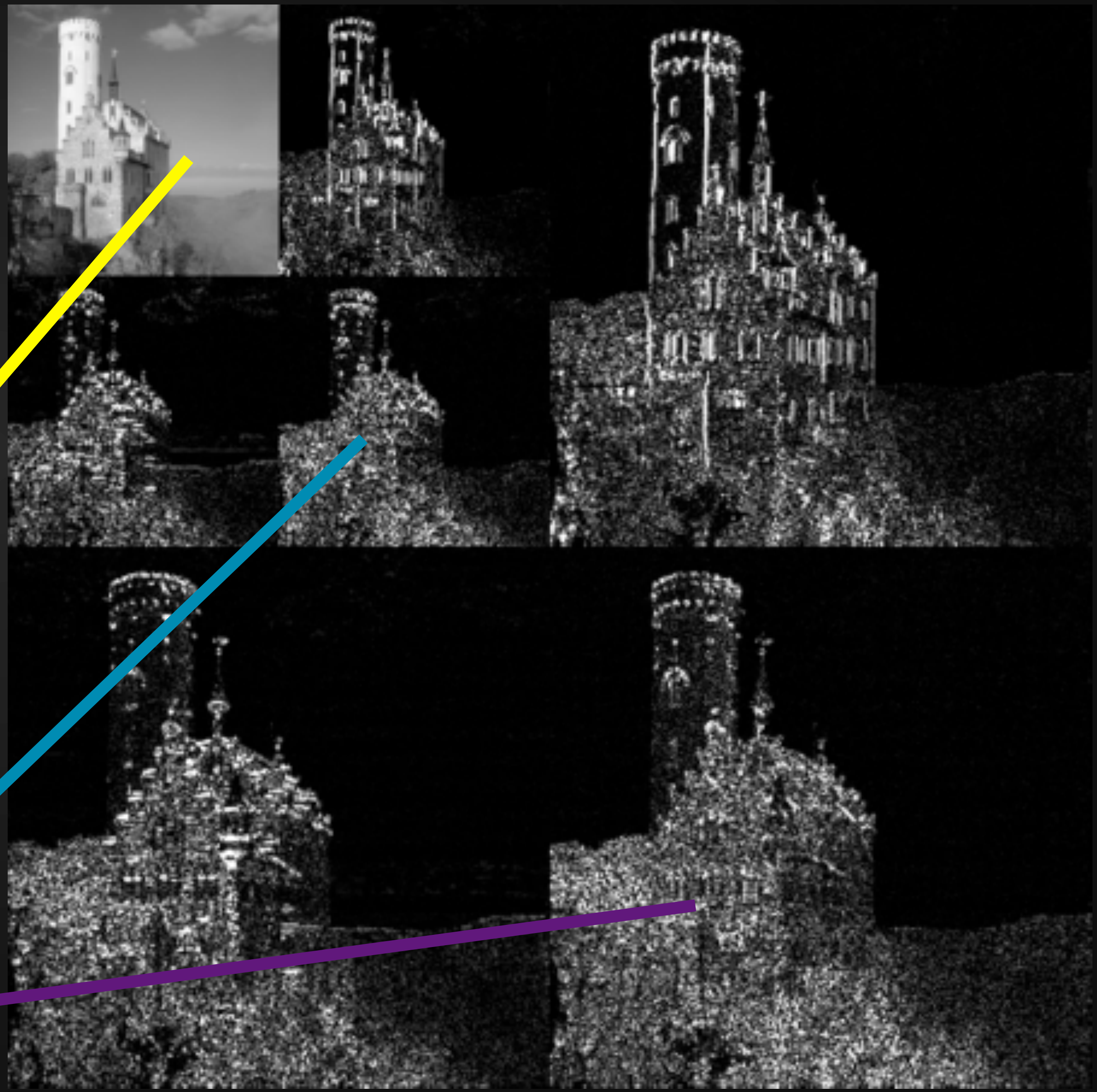


Multi-D wavelet transform

Original

Highest pass





Multi-D wavelet transform

Thresholding scheme

$$\mathbf{x} = \mathbf{L}^{\mathbf{T}}\mathbf{d}$$

$$p = .005$$

k

Thresholding iteration

$$\lambda_k = Q(\mathbf{x}, \mathbf{1.} - \mathbf{p})$$

$$p = p * 1.8$$

$Q(\mathbf{x}, \mathbf{m})$
Return the m value percentile value of $\mathbf{x}$

$$\mathbf{x} = \mathbf{L}^{\mathbf{T}}\mathbf{d}$$

$$p = .003\frac{l_{\text{high,size}}}{l_{\text{size}}}$$

k

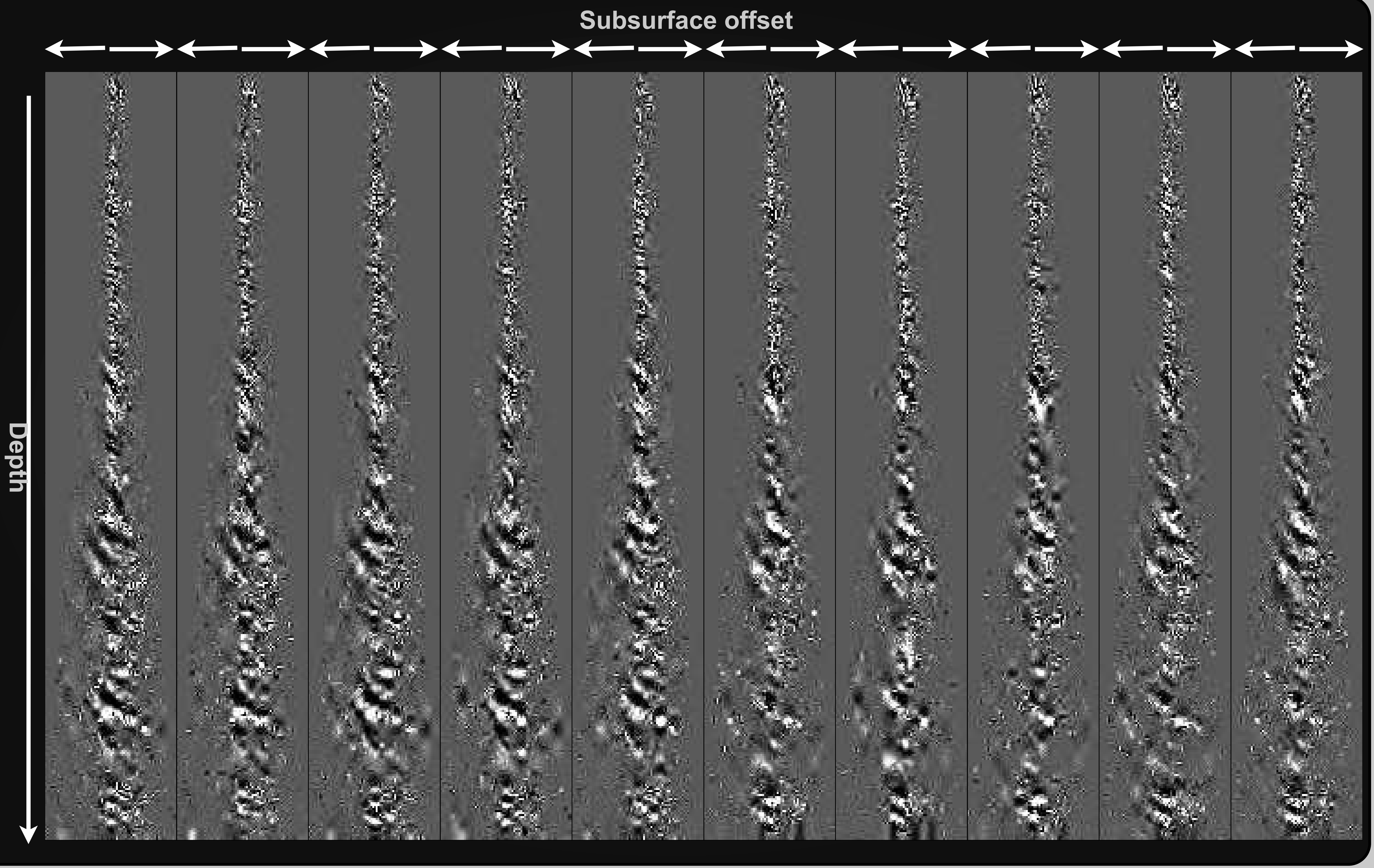
Thresholding iteration

$$\lambda_{k,l} = Q(\mathbf{x}, \mathbf{1.} - \mathbf{p})$$

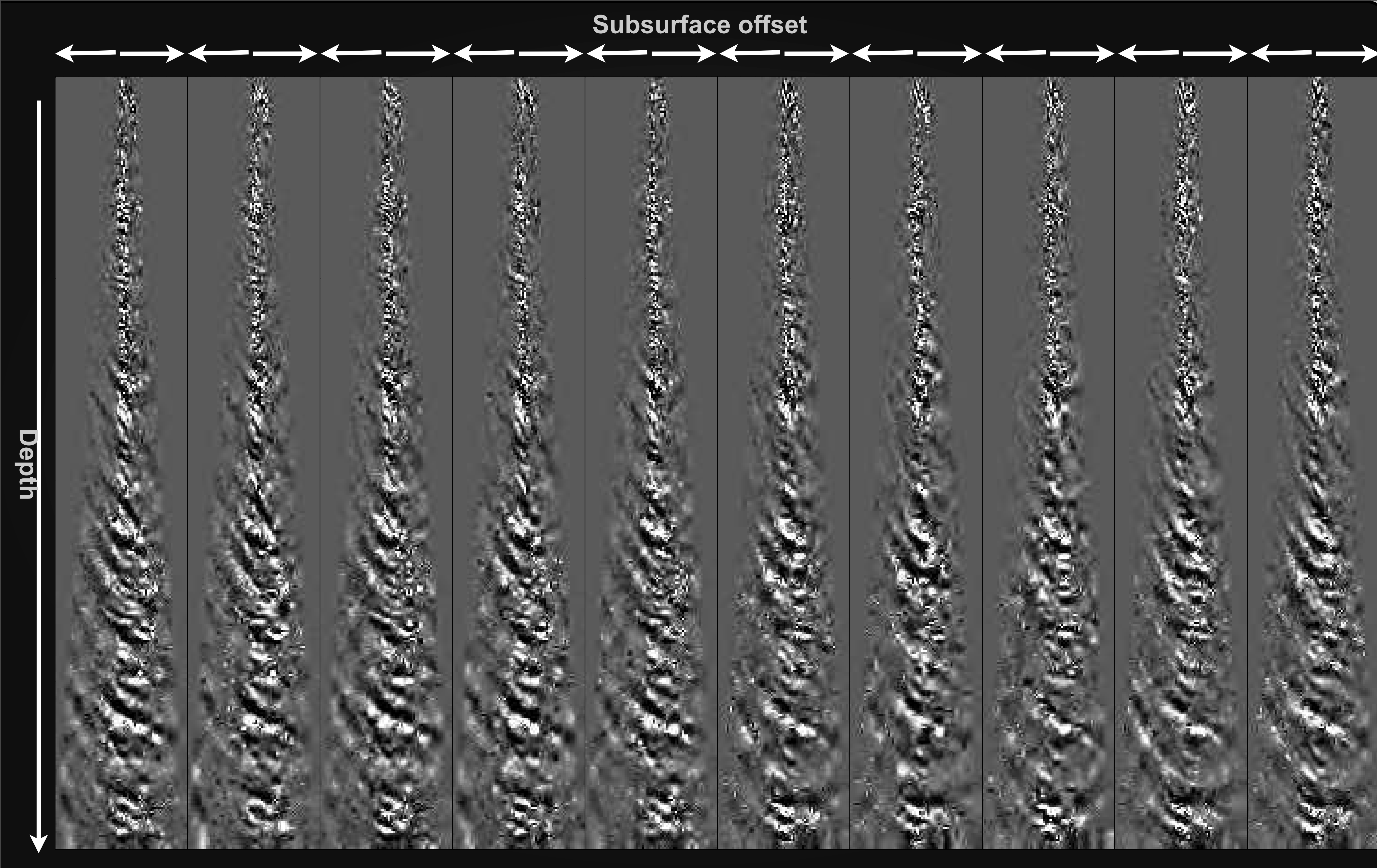
$$p = p * 1.8$$

$Q(\mathbf{x}, \mathbf{m})$
Return the m value percentile value of $\mathbf{x}$

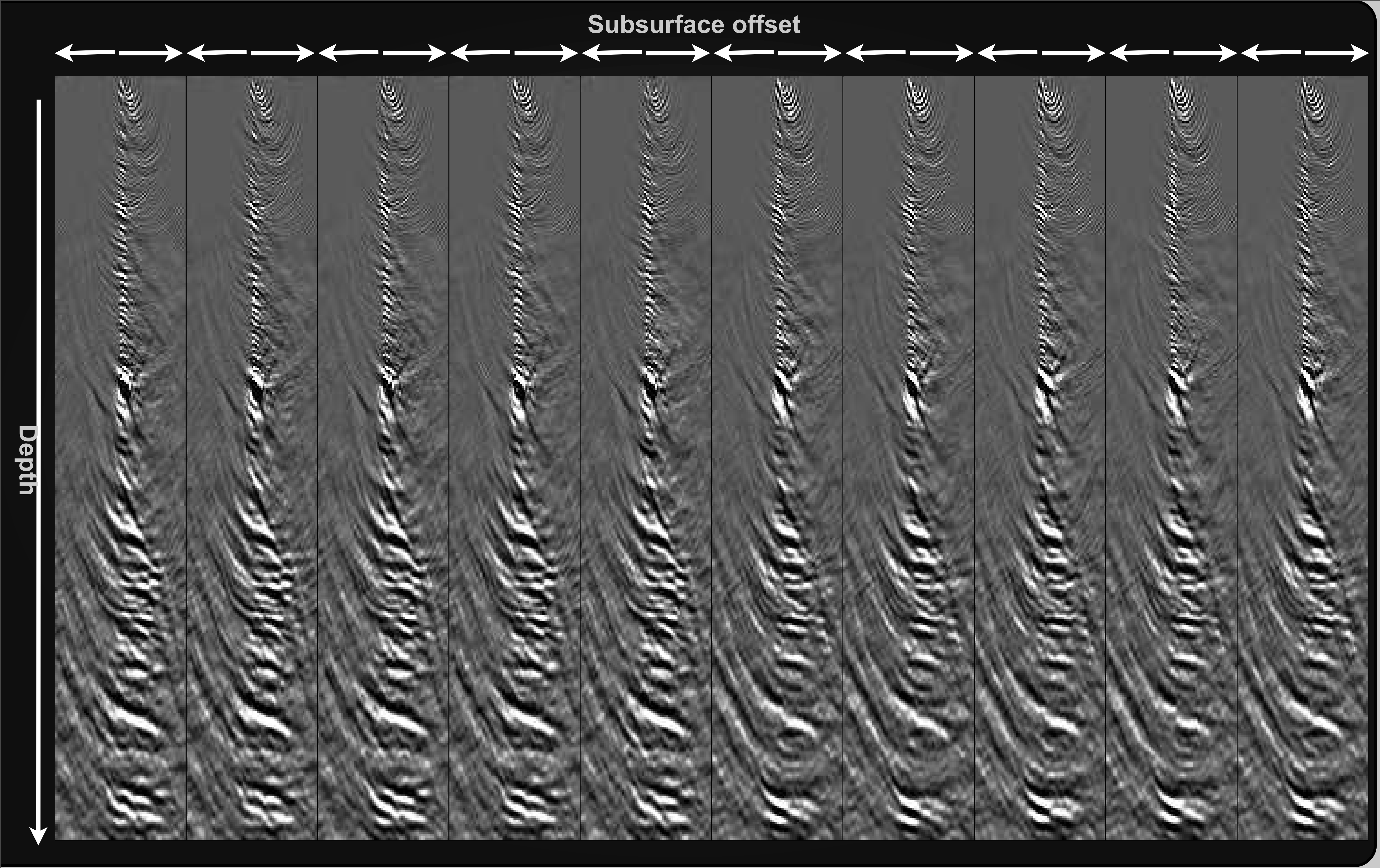
Level based thresholding



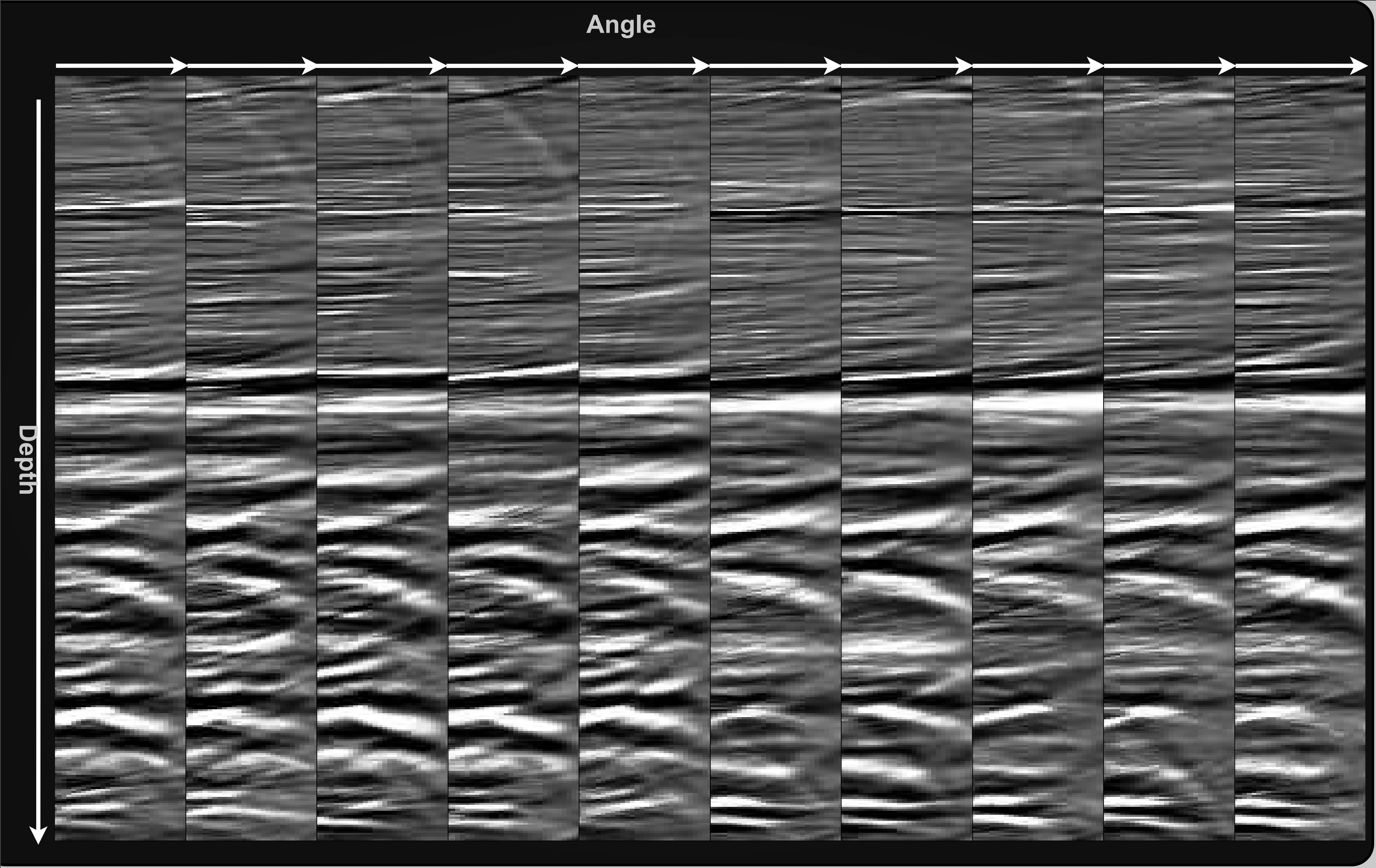
5% cone offsets



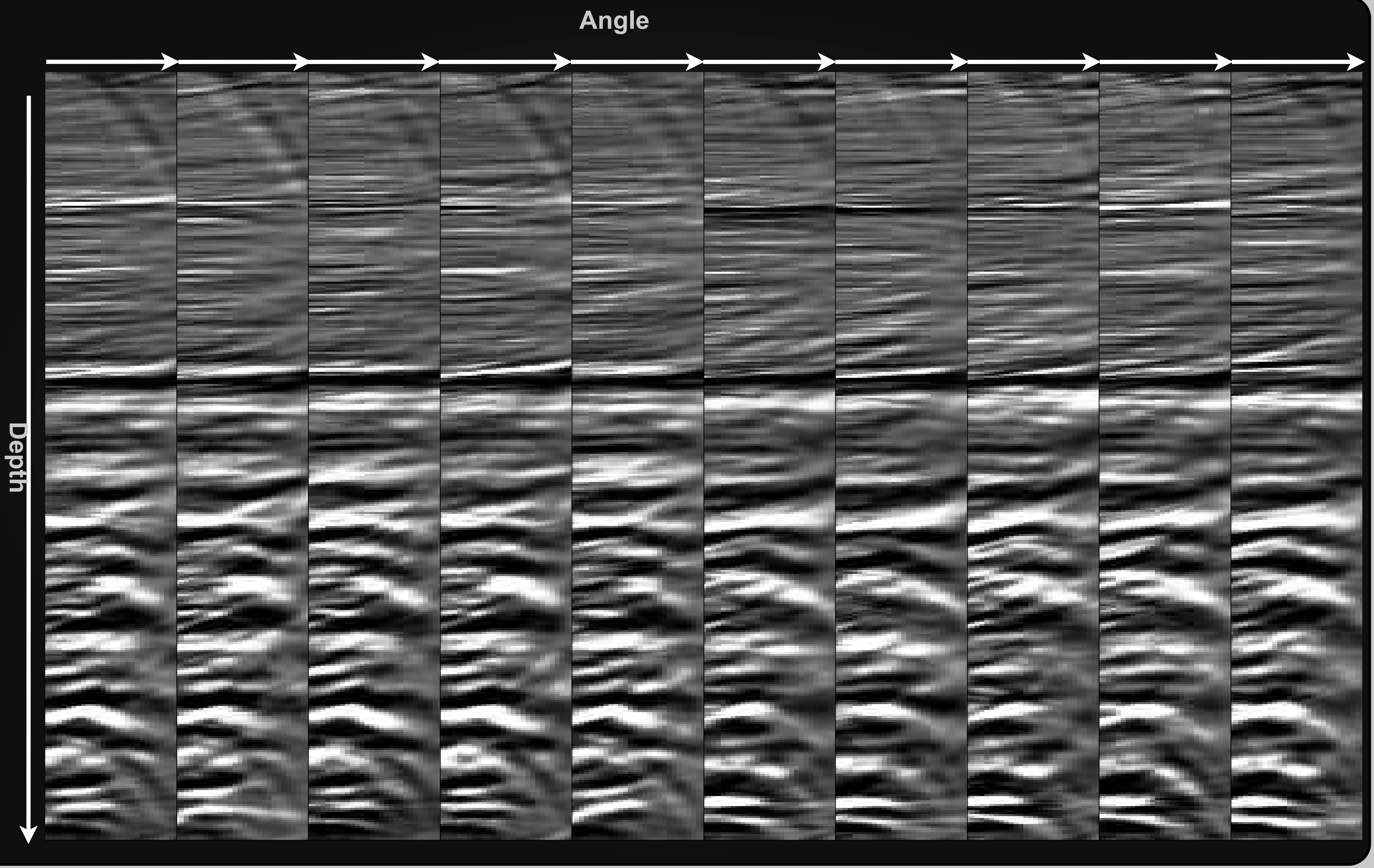
5% multi-level offsets



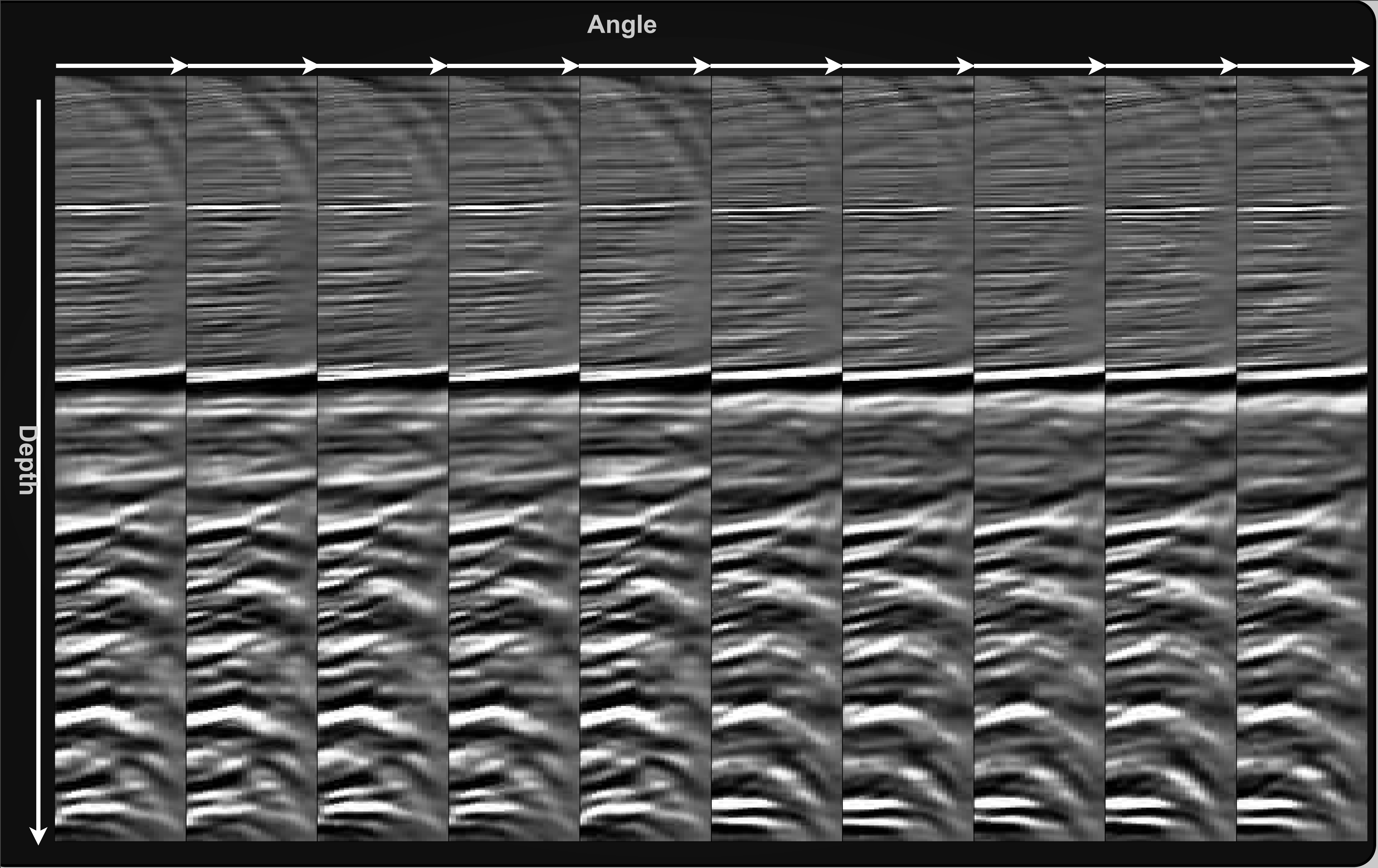
Full offsets



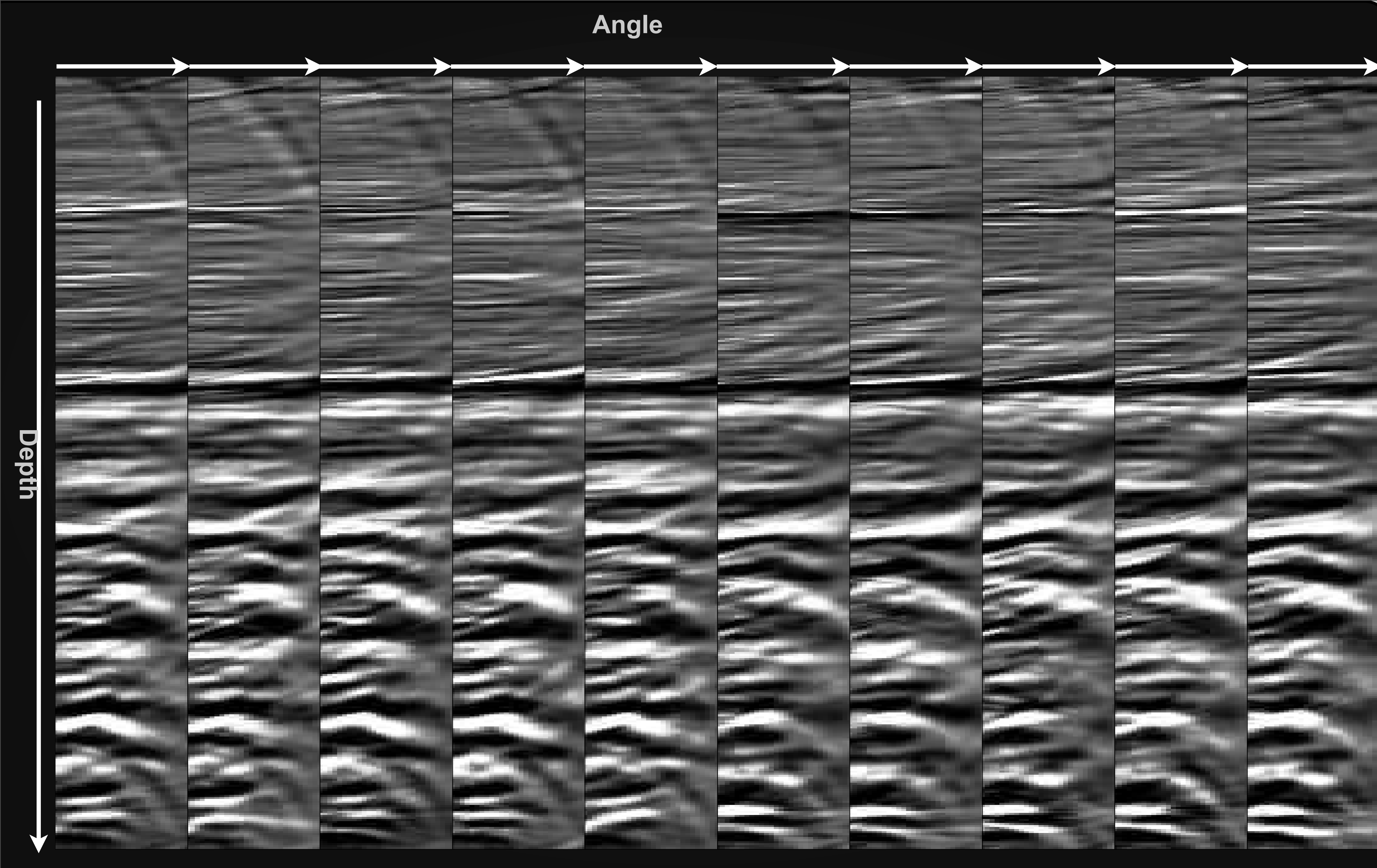
5% cone result



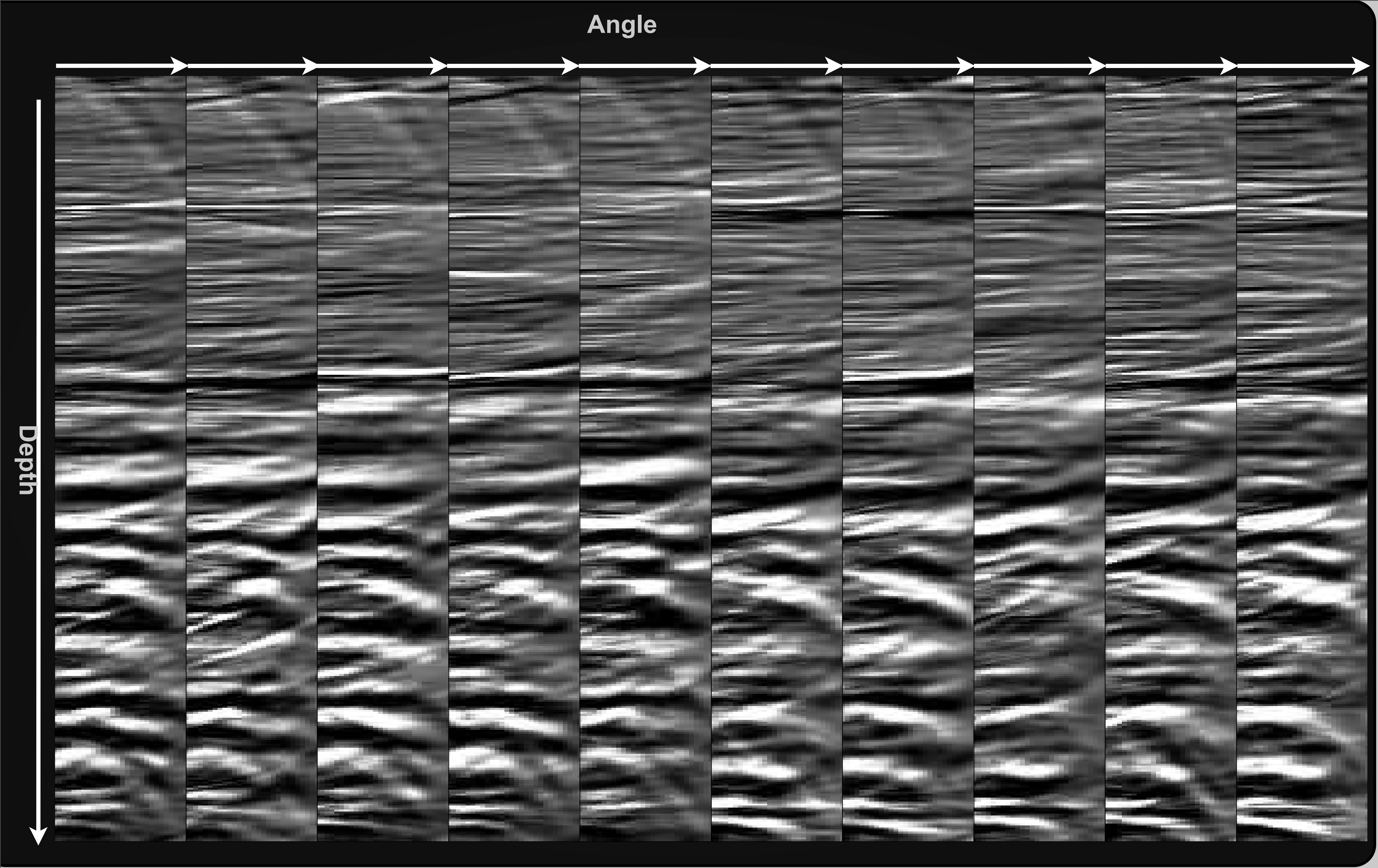
5% multi-level angle



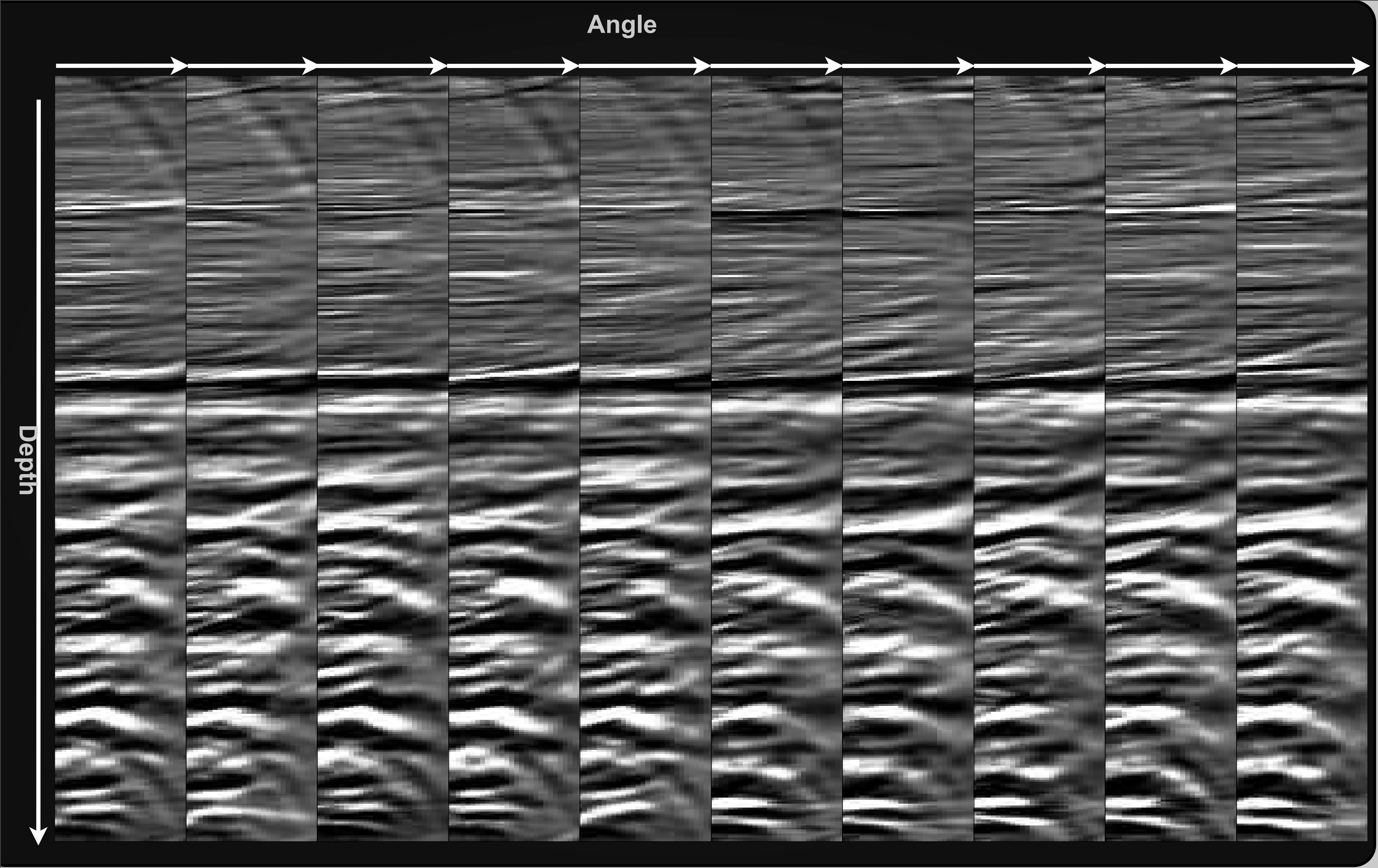
100% angle result



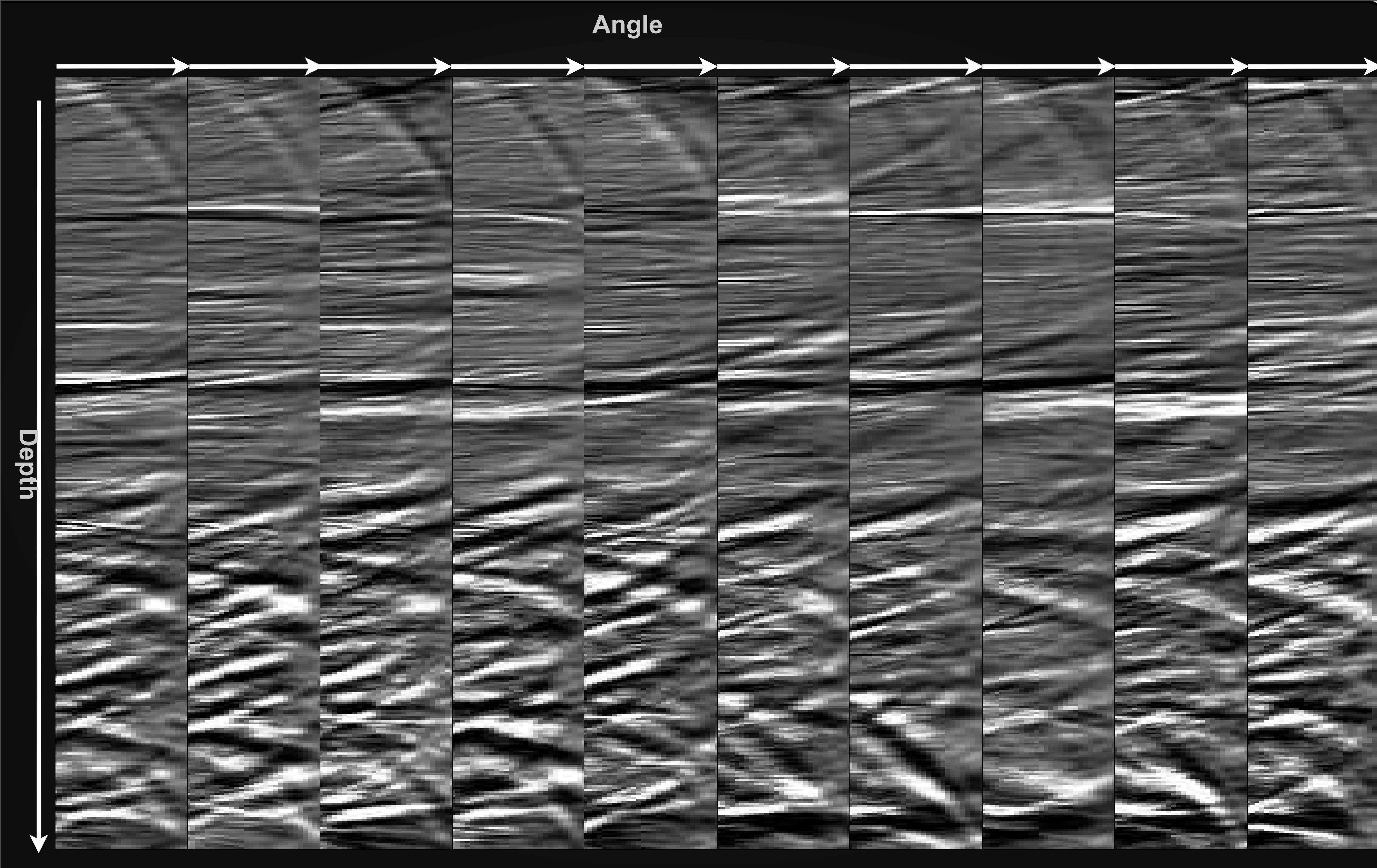
5% multi-level angle



3% multi-level angle



5% multi-level angle



5% multi-level angle

- IST is an effective approach to achieve L_0/L_1 solution to this subsurface offset estimation problem.
- Modifications to the sampling/level based thresholding allows a higher level of compression.

Summary

- John Washbourne's talk last year which led me to retry this method
- Total SA for providing the data

Acknowledgements