Least-squares reverse-time migration with salt-dimming

SEP Sponsor Meeting
Jun 19th, 2013
SEP 149

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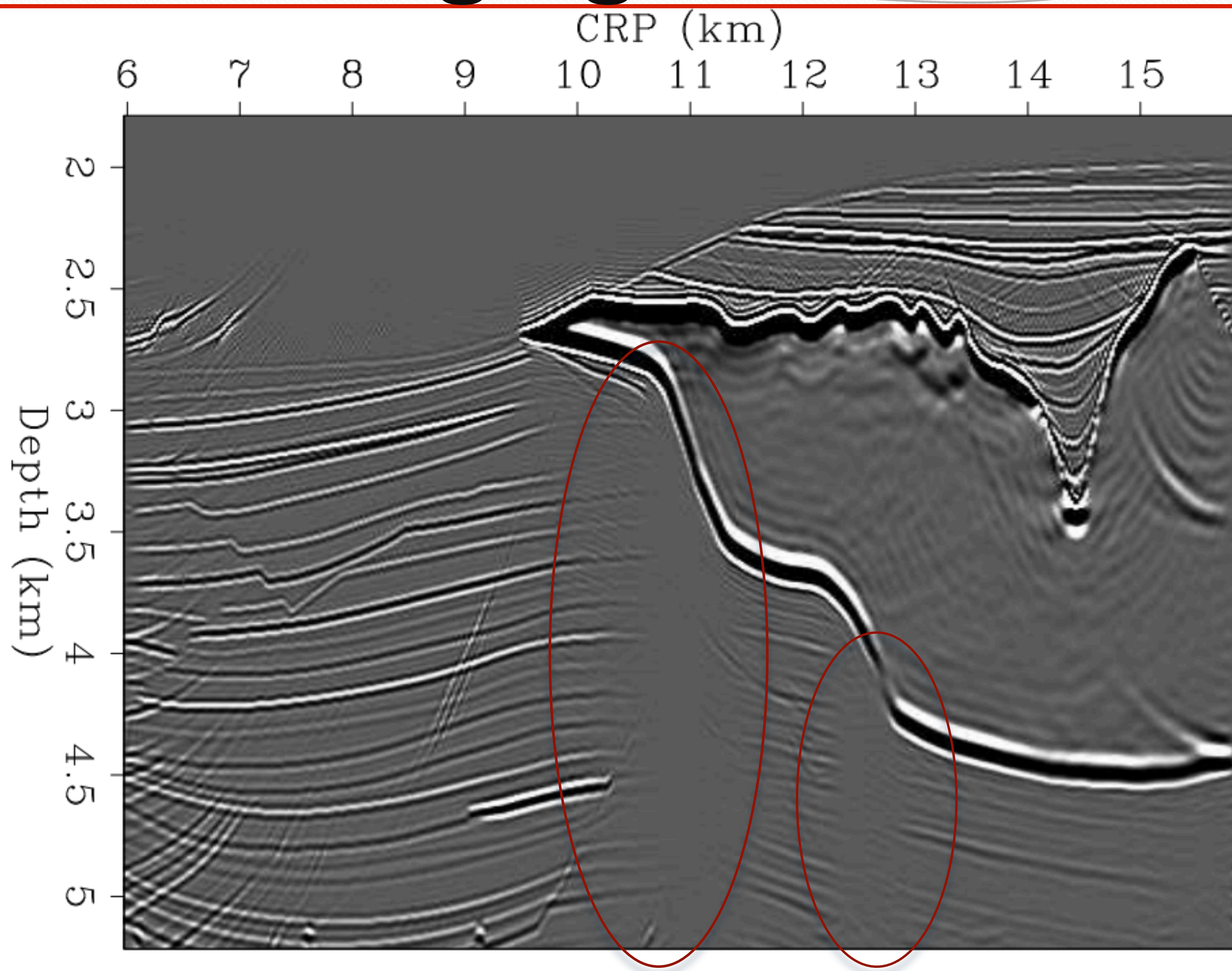
Overview

- Introduction
- Background data
- Theory and examples
- Discussion
- Conclusion

Subsalt Imaging



Clapp (2005)

What is least-squares migration (LSM) ?

$$\boxed{d^{mod} = L\, m}$$

$\mathbf{L}$	Forward modeling operator		
$\mathbf{d^{mod}}$	modeled data	$\mathbf{d}$	field data
$\mathbf{m}$	Image model		

Obtain the best image by minimizing the difference between the modeled data and the field data

$$\boxed{S(m) = \| Lm - d \|^2}$$

Each LSM is associated with a type of migration

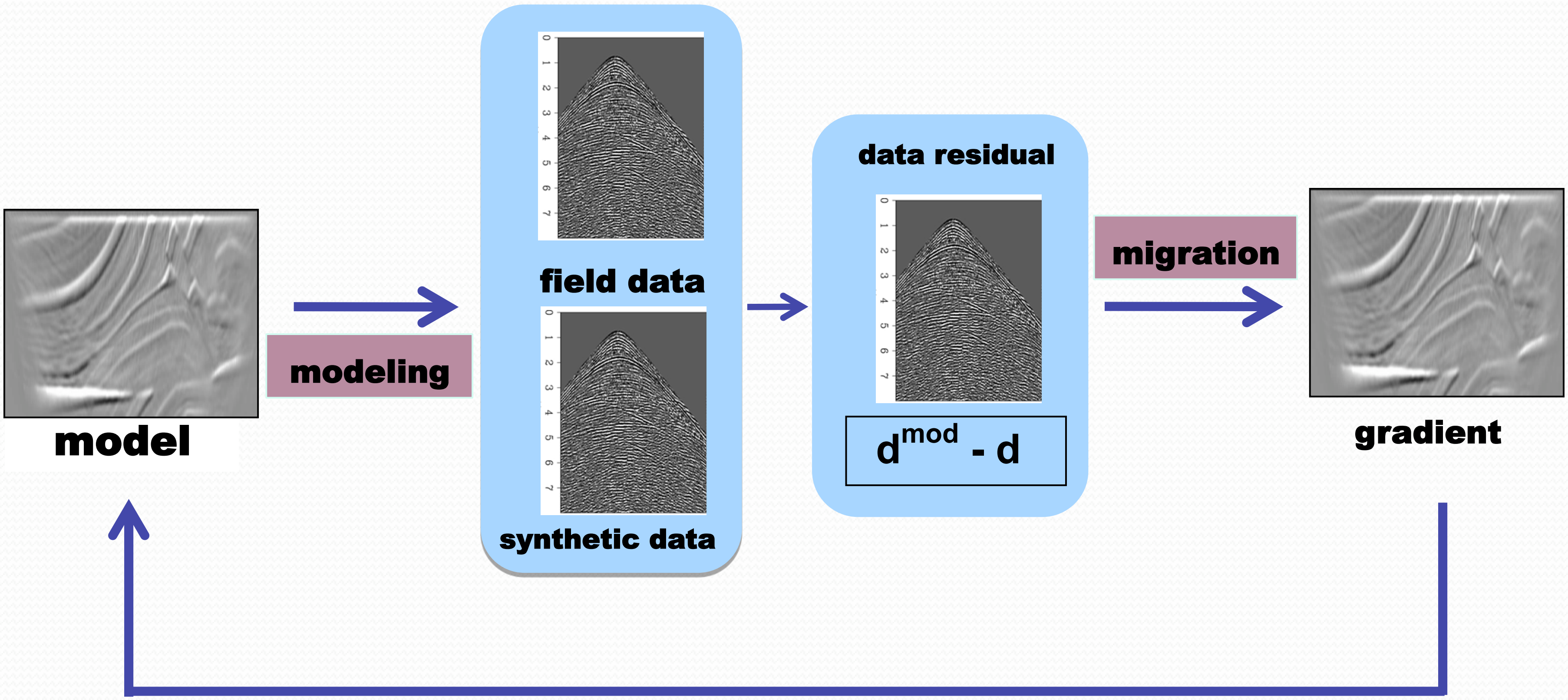
Migration operator

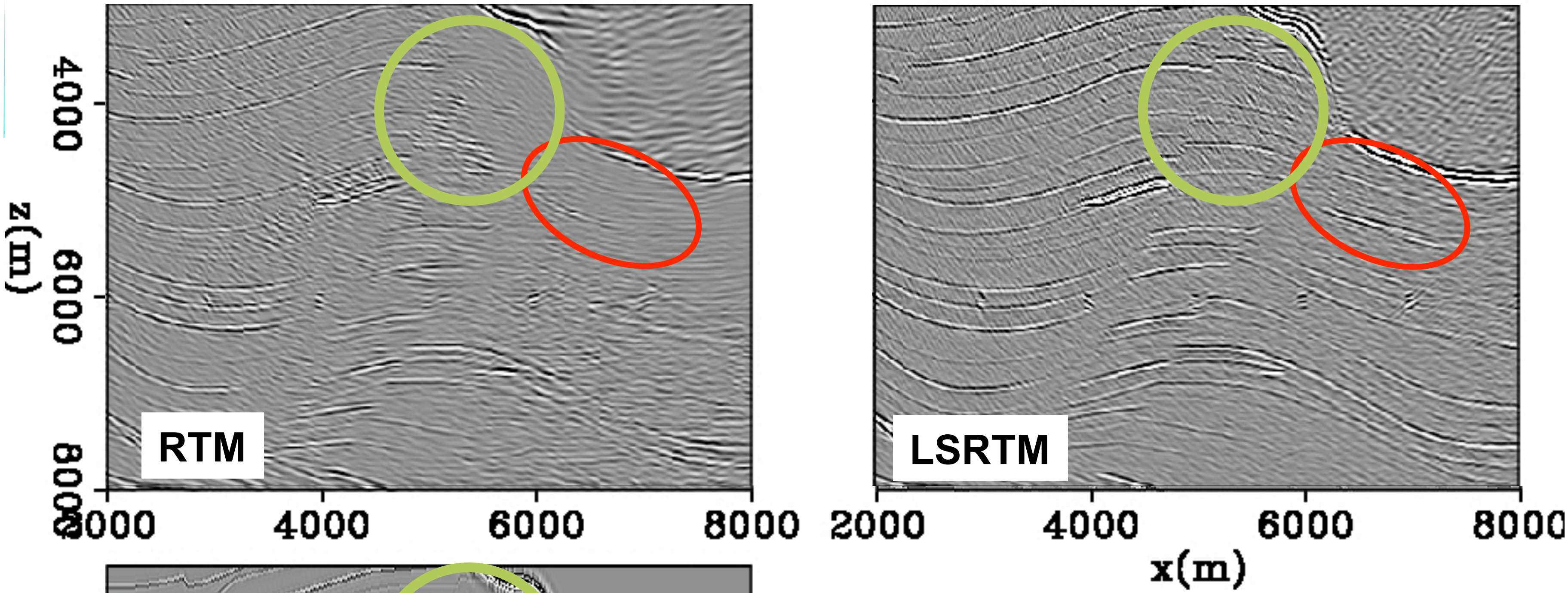
=

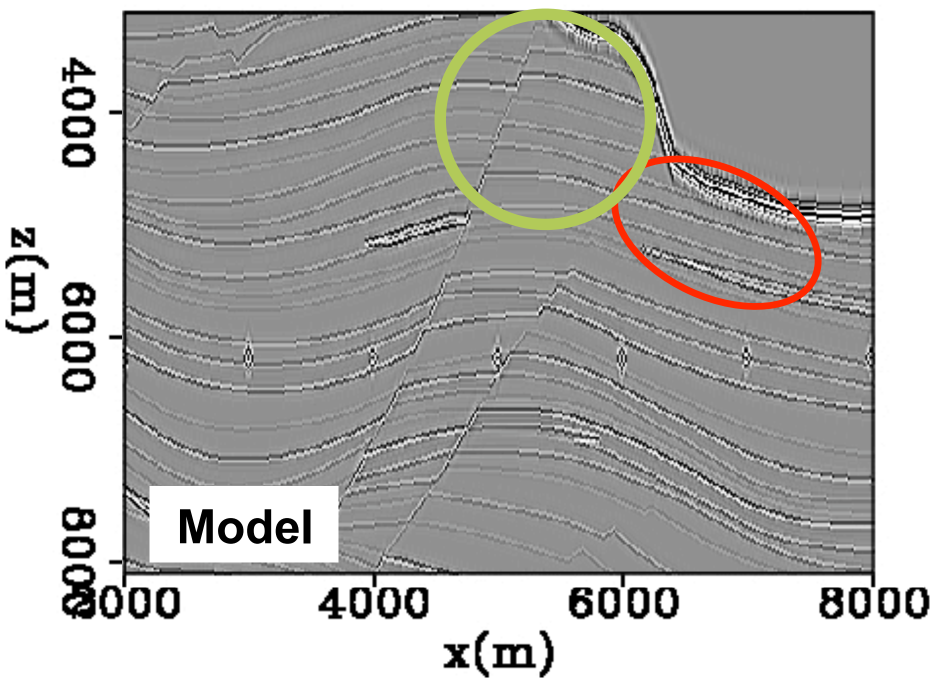
Adjoint of the Forward modeling operator

LSM Workflow

- Iterative inversion by conjugate gradient

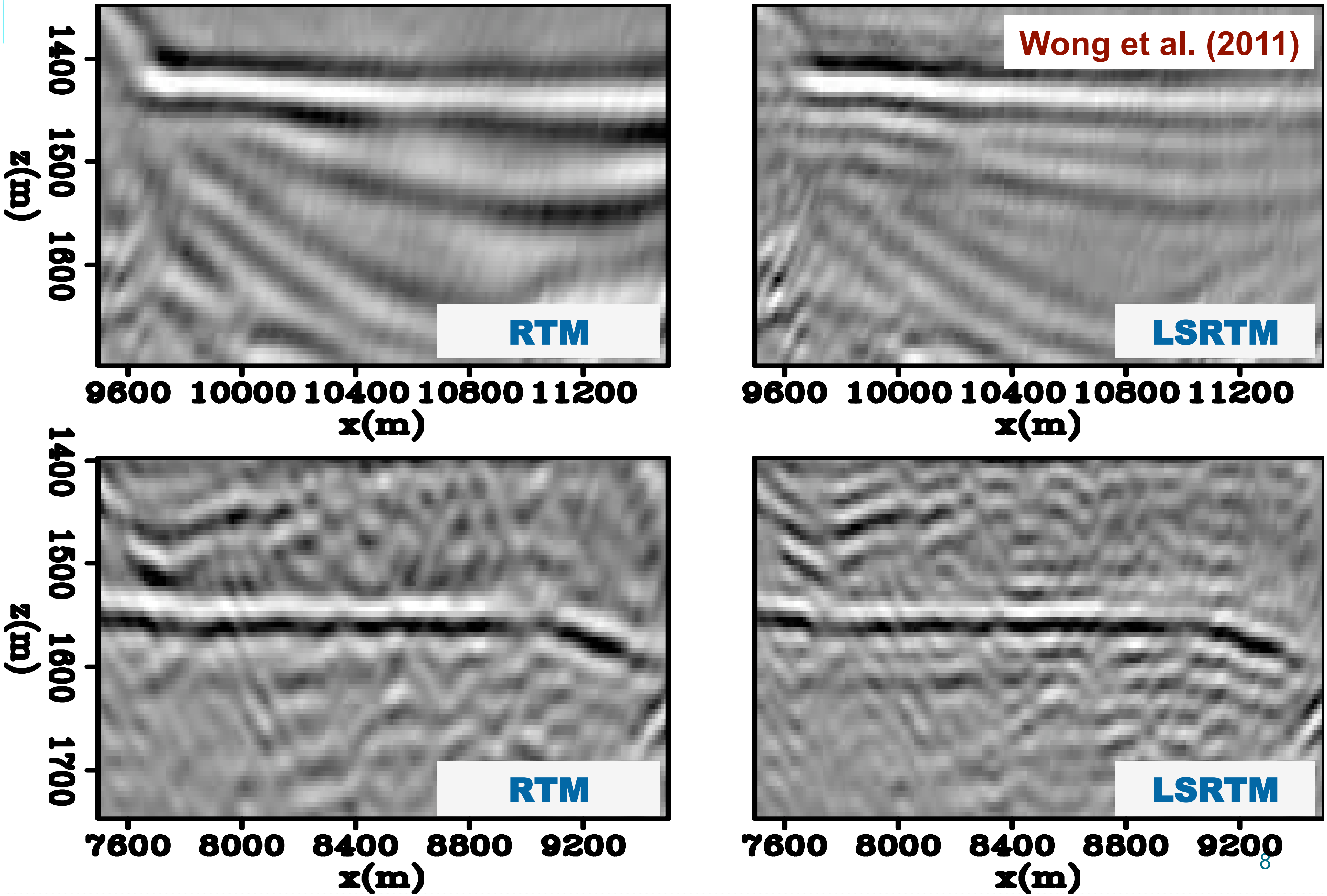




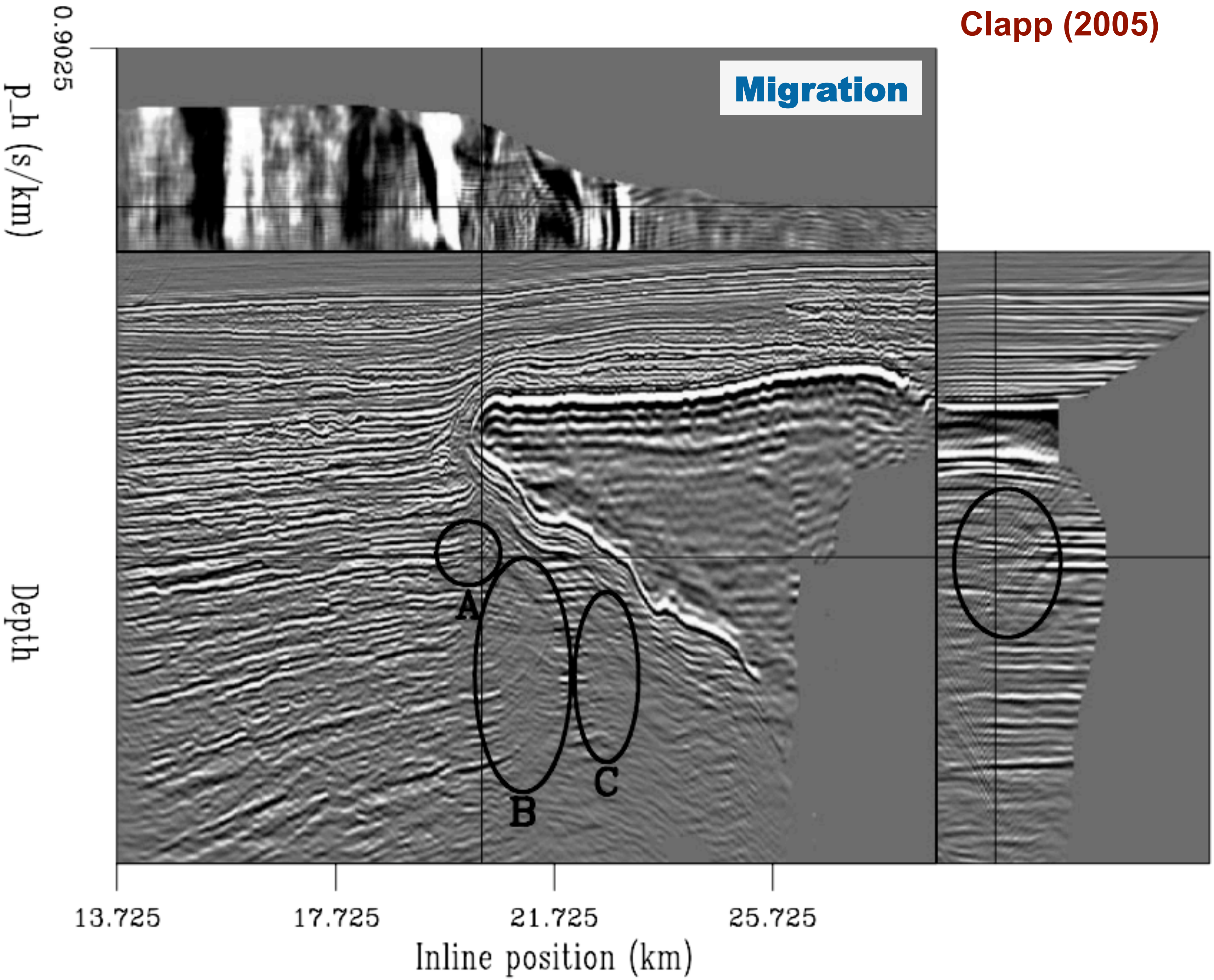


1. **Better relative amplitude information**
2. **Fewer migration artifacts**
3. **Less acquisition footprints**

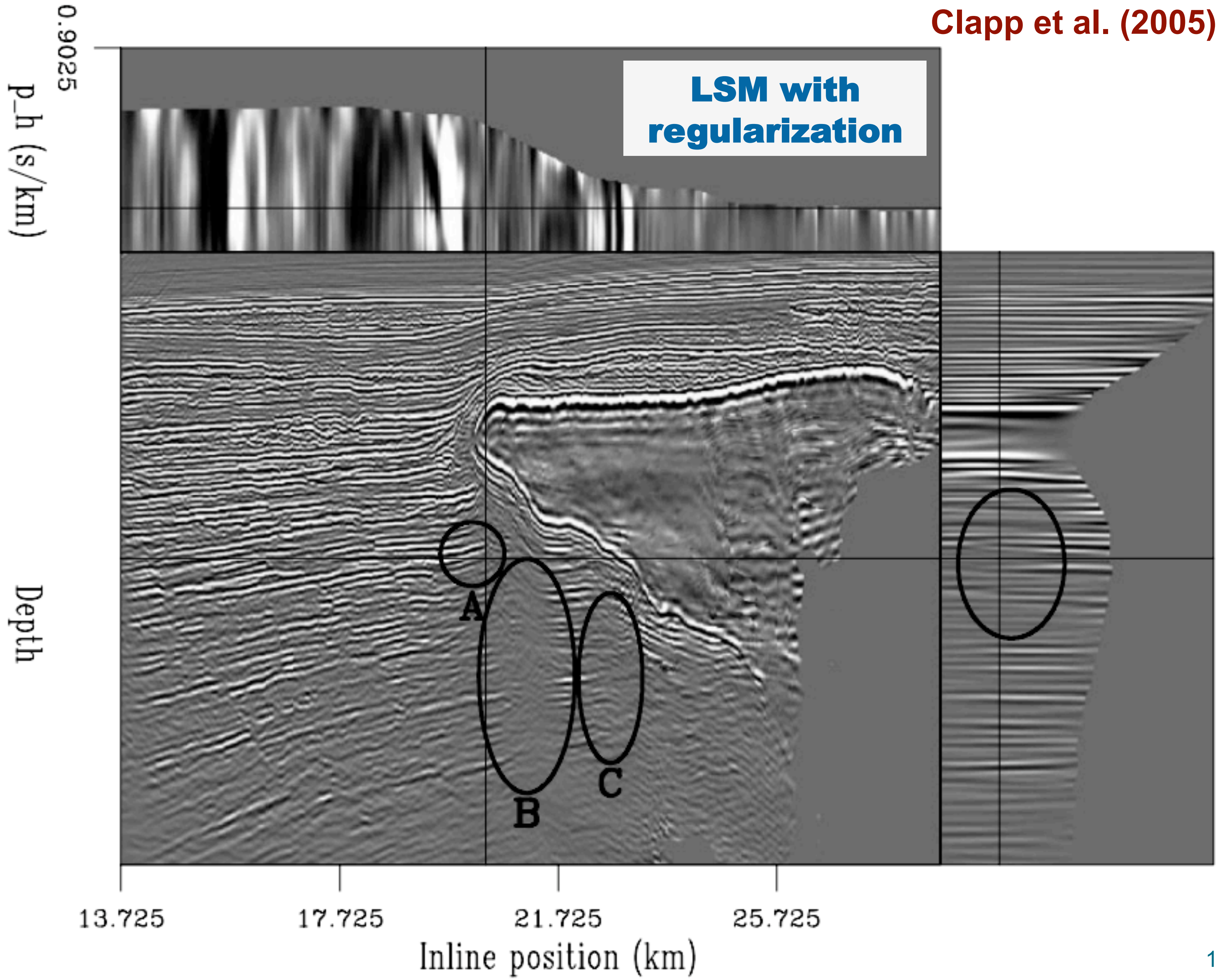
Comparison of RTM and LSRTM



Clapp (2005)


Migration
0.9025
p_h (s/km)
Depth
A
B
C
13.725
17.725
21.725
25.725
Inline position (km)

Clapp et al. (2005)



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The issue with background data

- Applying LSM in regions like Gulf of Mexico
  - Sharp velocity contrast
  - Background data term needs to be subtracted from observed data
  - Affects LSM result, especially for deep reflectors

LSM Parameterization

Linearizing the wave equation with respect to model m

$$s(\mathbf{x}) = s_o(\mathbf{x}) + \Delta s(\mathbf{x})$$

$$m(\mathbf{x}) = \Delta s(\mathbf{x}) s_o(\mathbf{x})$$

$s(\mathbf{x})$ **true slowness**

$s_o(\mathbf{x})$ **migration slowness**

$m(\mathbf{x})$ **model**

$\Delta s(\mathbf{x})$ **slowness perturbation**

Forward modeling in LSM

The forward modeling operator is linearized with respect to $m(\mathbf{x})$

$$d^{mod} = F(s_o^2 + m)$$

$$\approx F(s_o^2) + \mathbf{L}m$$

$\mathbf{L}$ **Linearized forward modeling operator**

d^{mod} **Synthetic data**

$F(s_o^2)$ **Background data**

LSM objective function

The forward modeling operator is linearized with respect to $m(\mathbf{x})$

$$S(\mathbf{m}) = \|d^{mod} - d^{obs}\|^2$$
$$= \|\mathbf{L}m - (d^{obs} - F(s_o^2))\|^2$$

$\mathbf{L}$ **Linearized forward modeling operator**

d^{mod} **Synthetic data**

d^{obs} **Observed data**

$F(s_o^2)$ **Background data**

LSM objective function

The forward modeling operator is linearized with respect to $m(\mathbf{x})$

$$
\begin{aligned}
S(\mathbf{m}) &= \| d^{mod} - d^{obs} \|^2 \\
&= \| \mathbf{L}m - \boxed{\left(d^{obs} - F\left(s_o^2\right)\right)} \|^2
\end{aligned}
$$

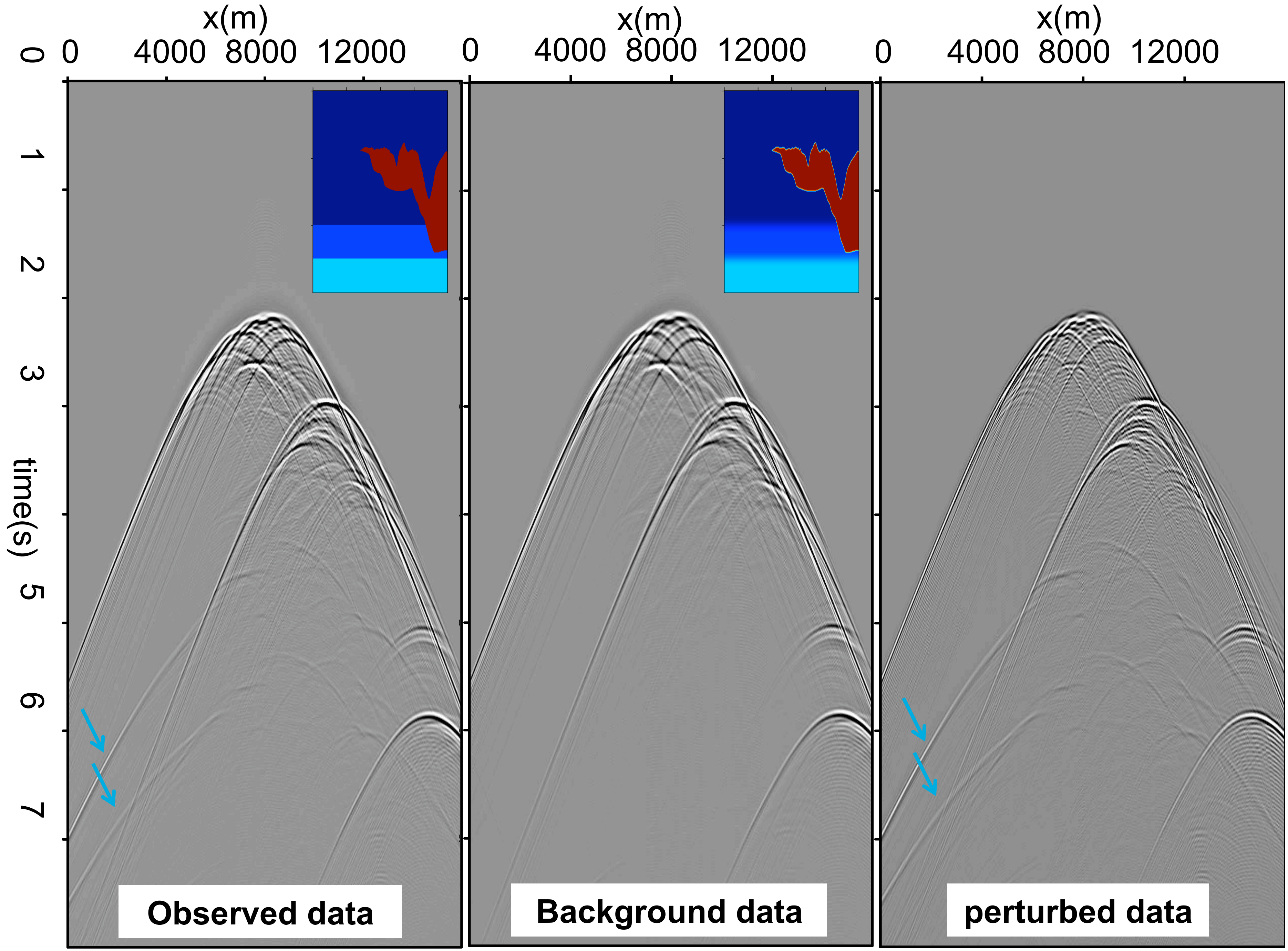
Perturbed data

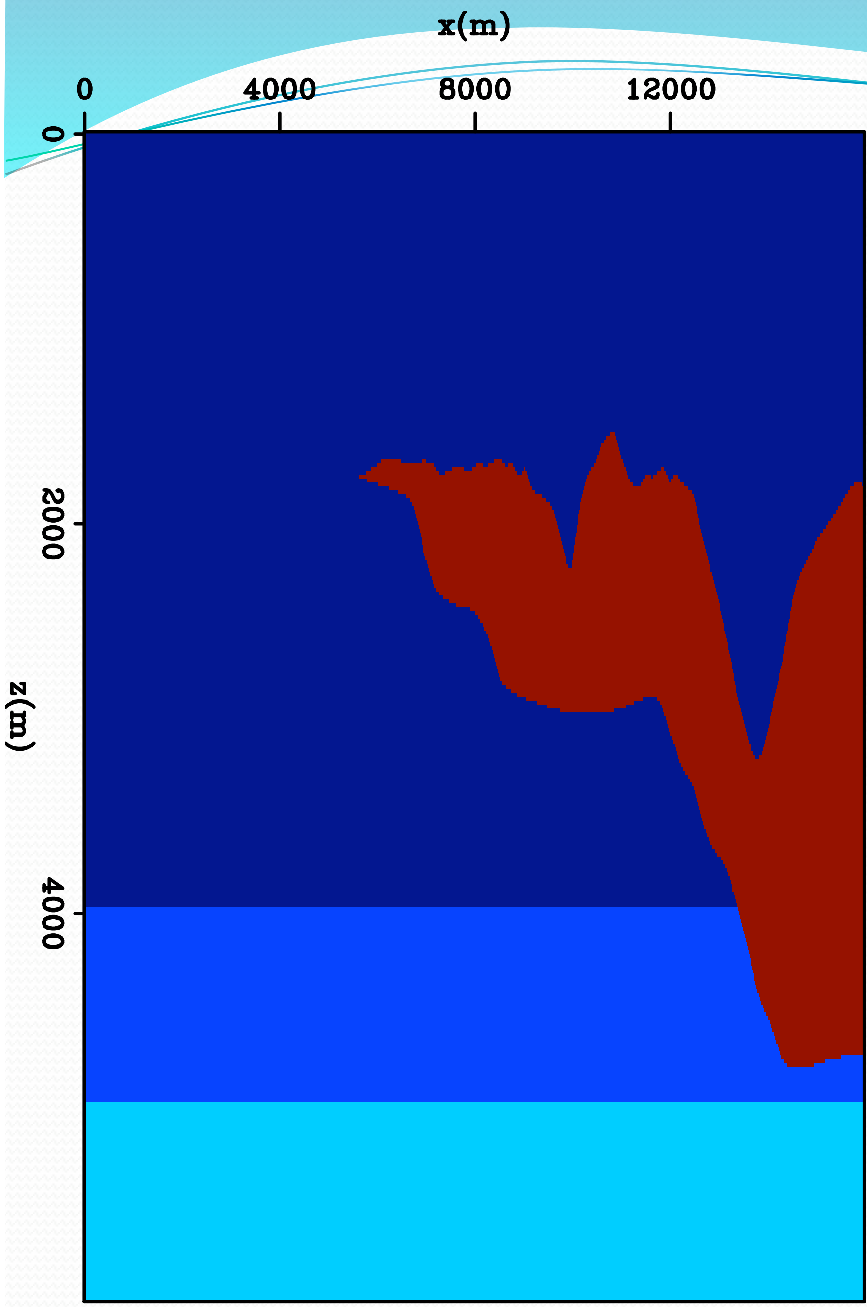
$\mathbf{L}$ **Linearized forward modeling operator**

d^{mod} **Synthetic data**

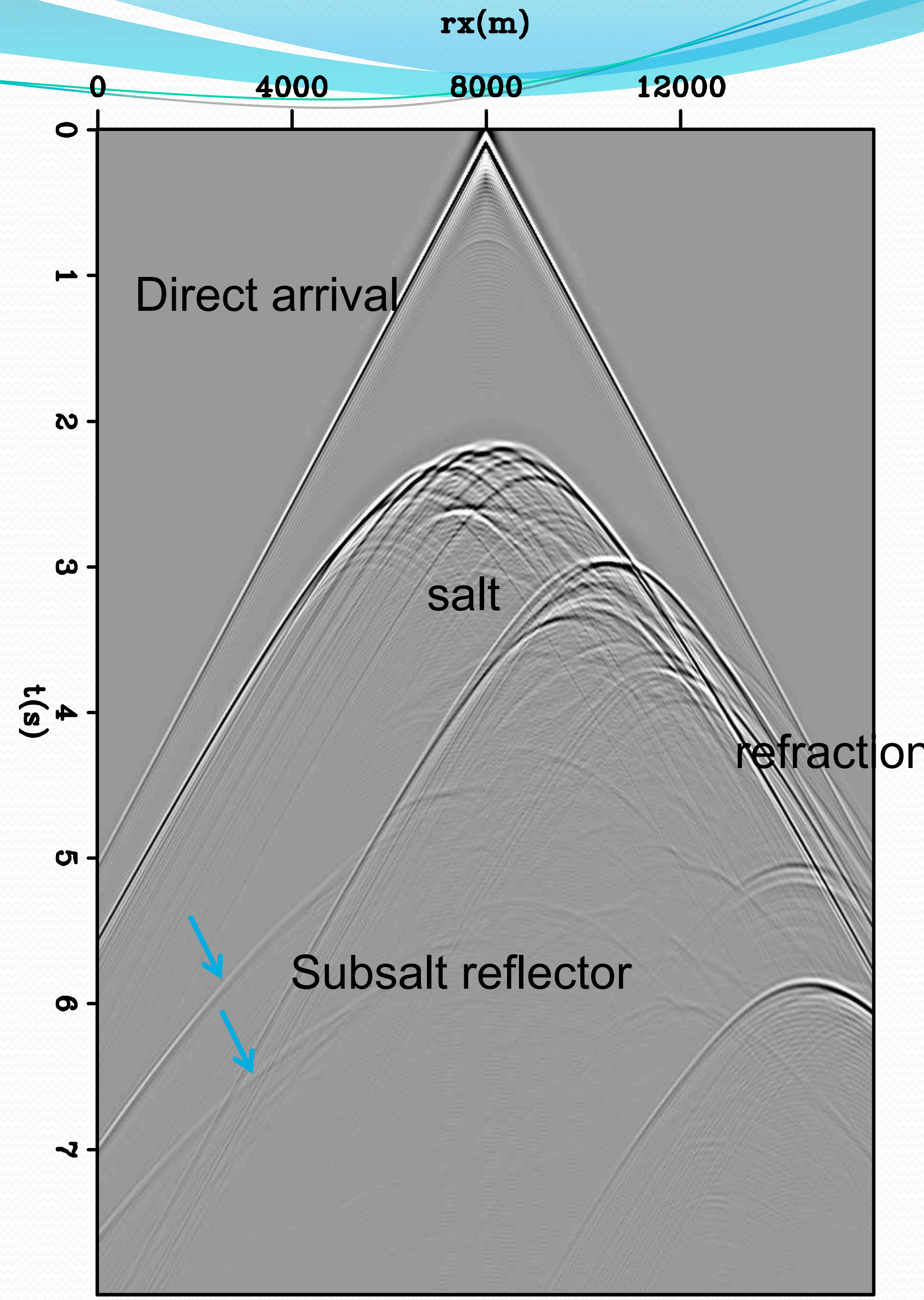
d^{obs} **Observed data**

$F\left(s_o^2\right)$ **Background data**


x(m)
0
4000
8000
12000
time(s)
0
1
2
3
5
6
7
Observed data
Background data
perturbed data

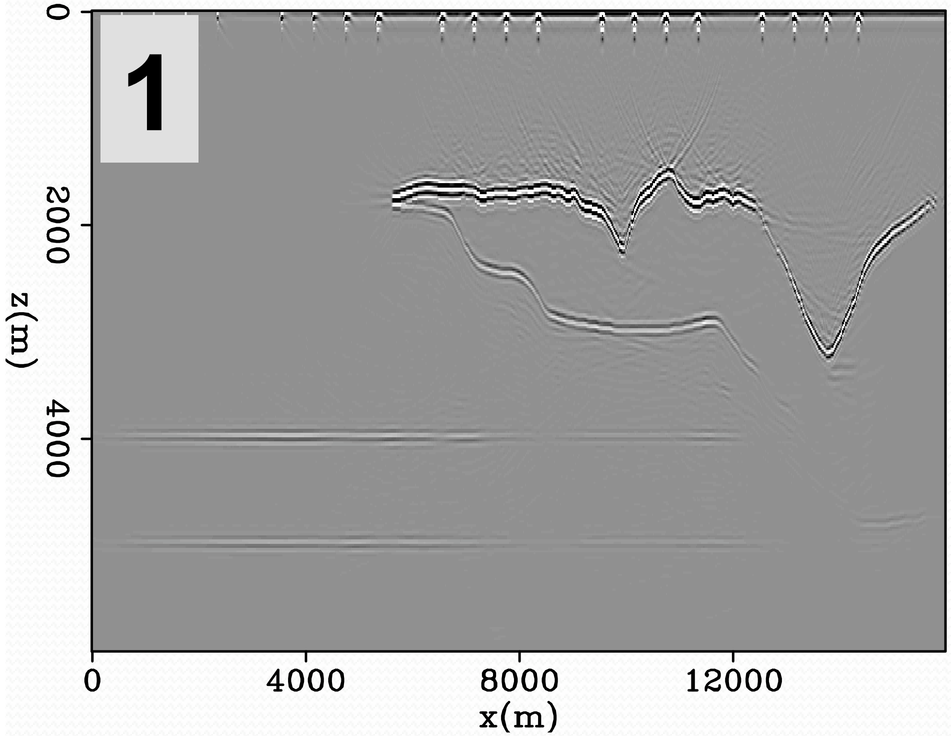


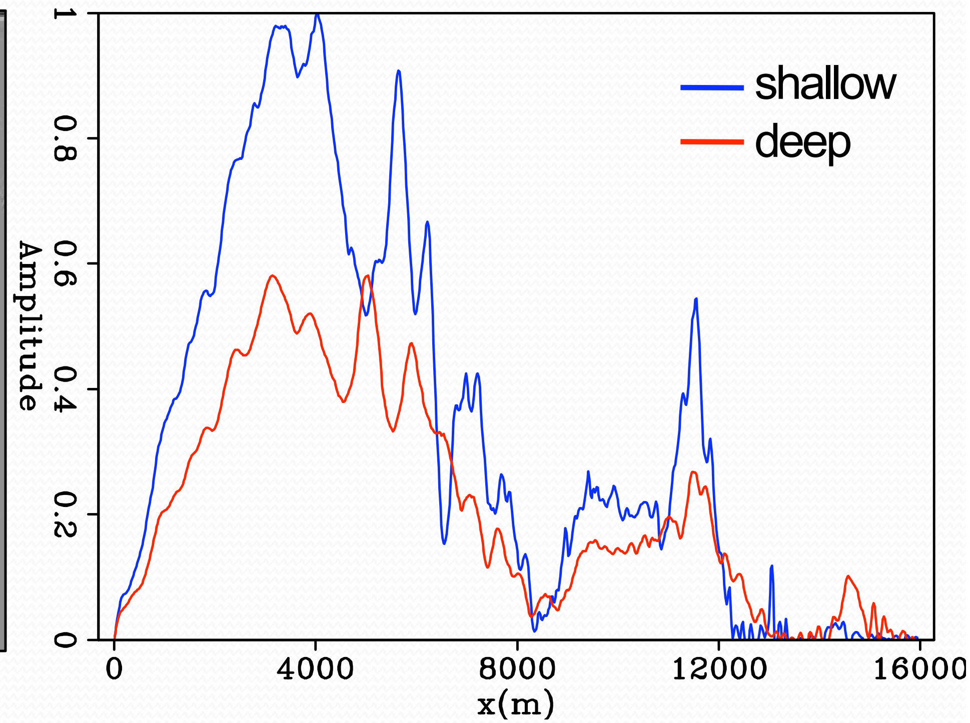
Salt+layer



Salt+layer

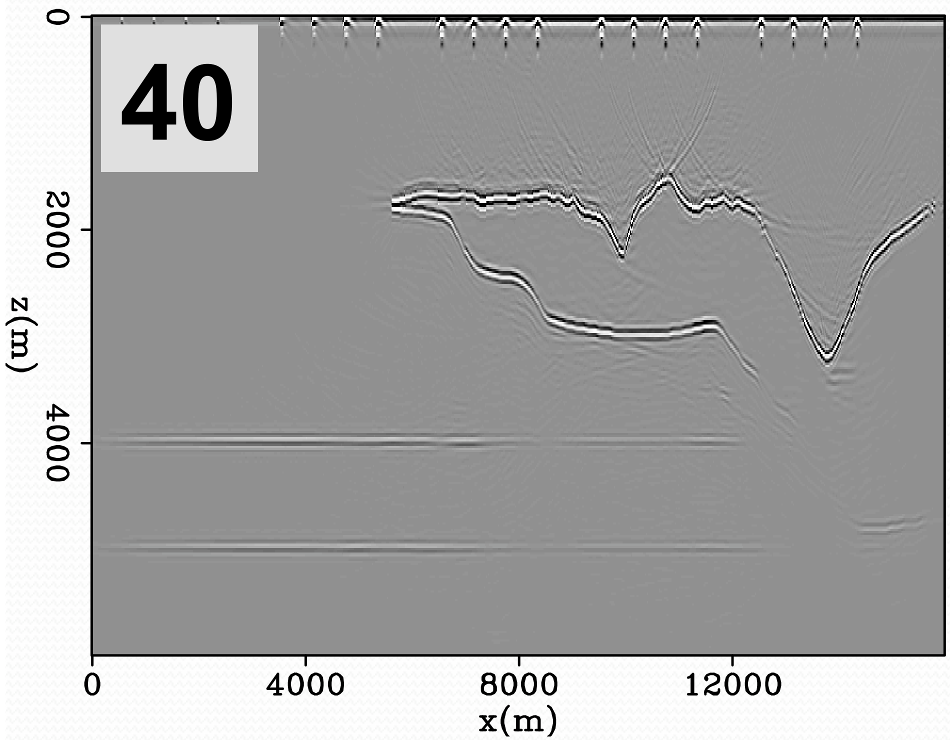
Test 1: LSRTM ignoring background data

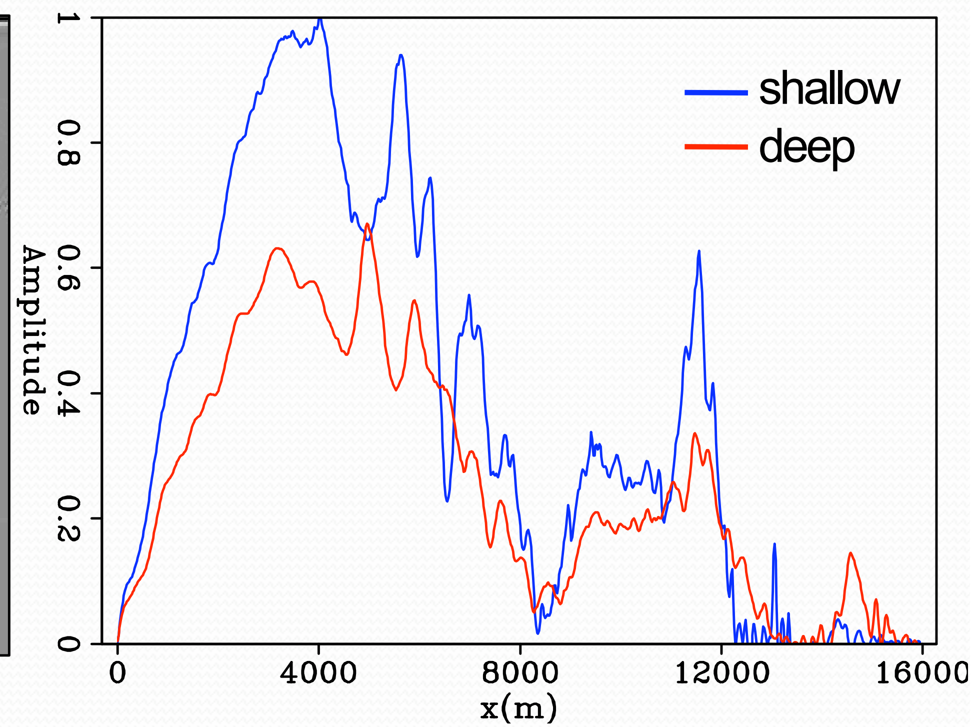




$$S_1(\mathbf{m}) = \|\mathbf{L}m - d^{obs}\|^2$$

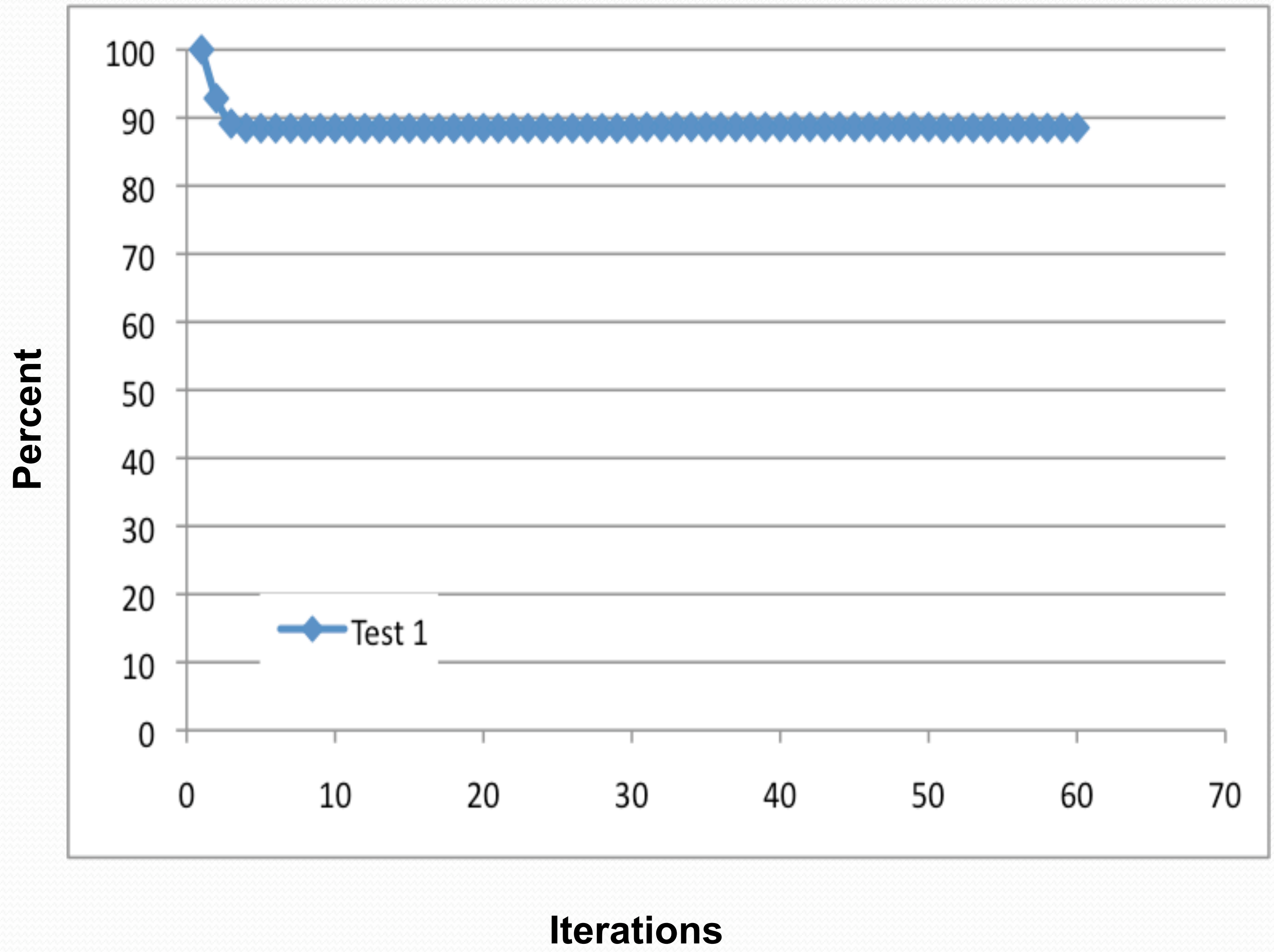
Test 1: LSRTM ignoring background data





$$S_1(\mathbf{m}) \quad = \quad \|\mathbf{L}m - d^{obs}\|^2$$

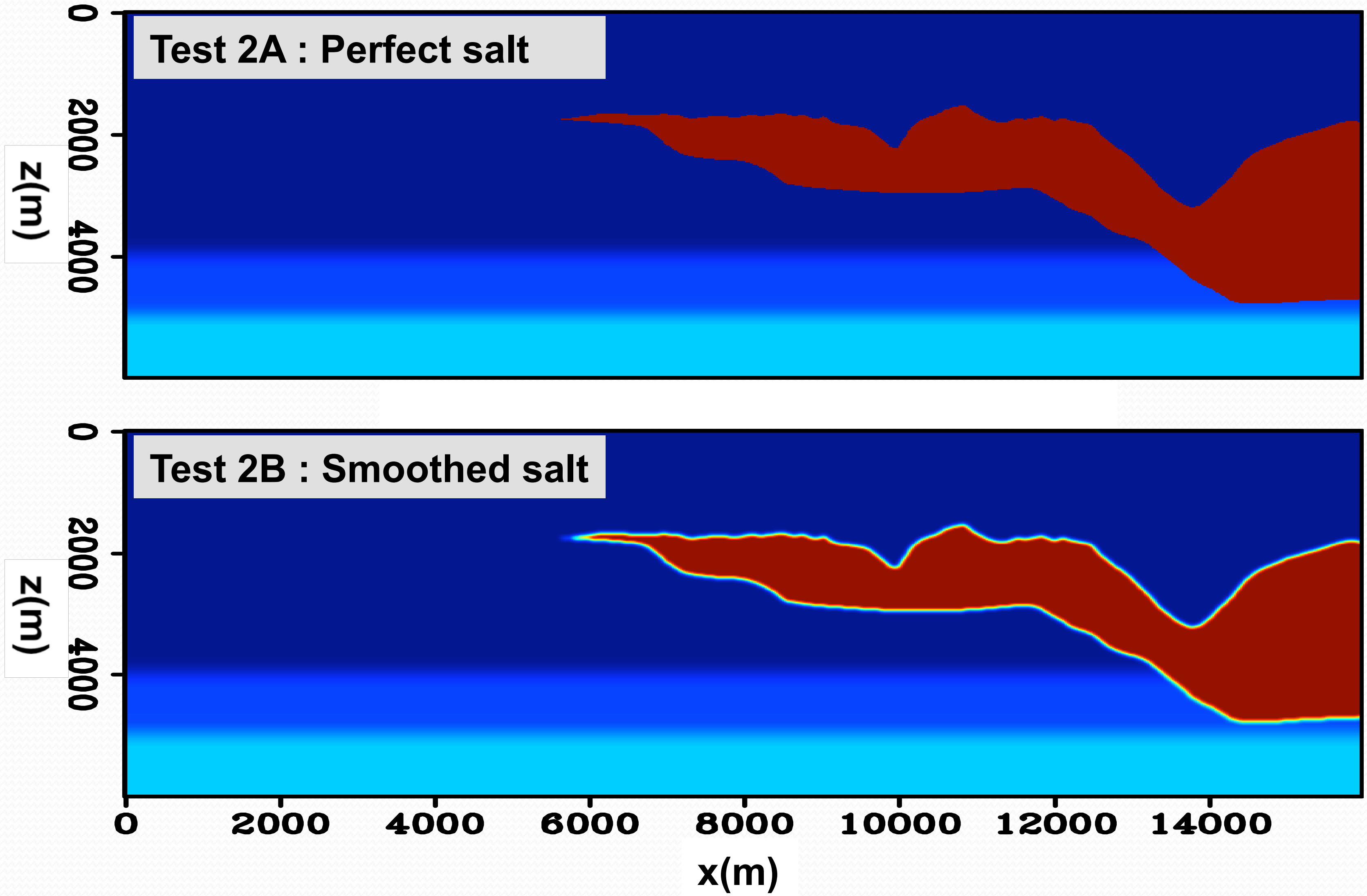
Residual Curve for Test 1



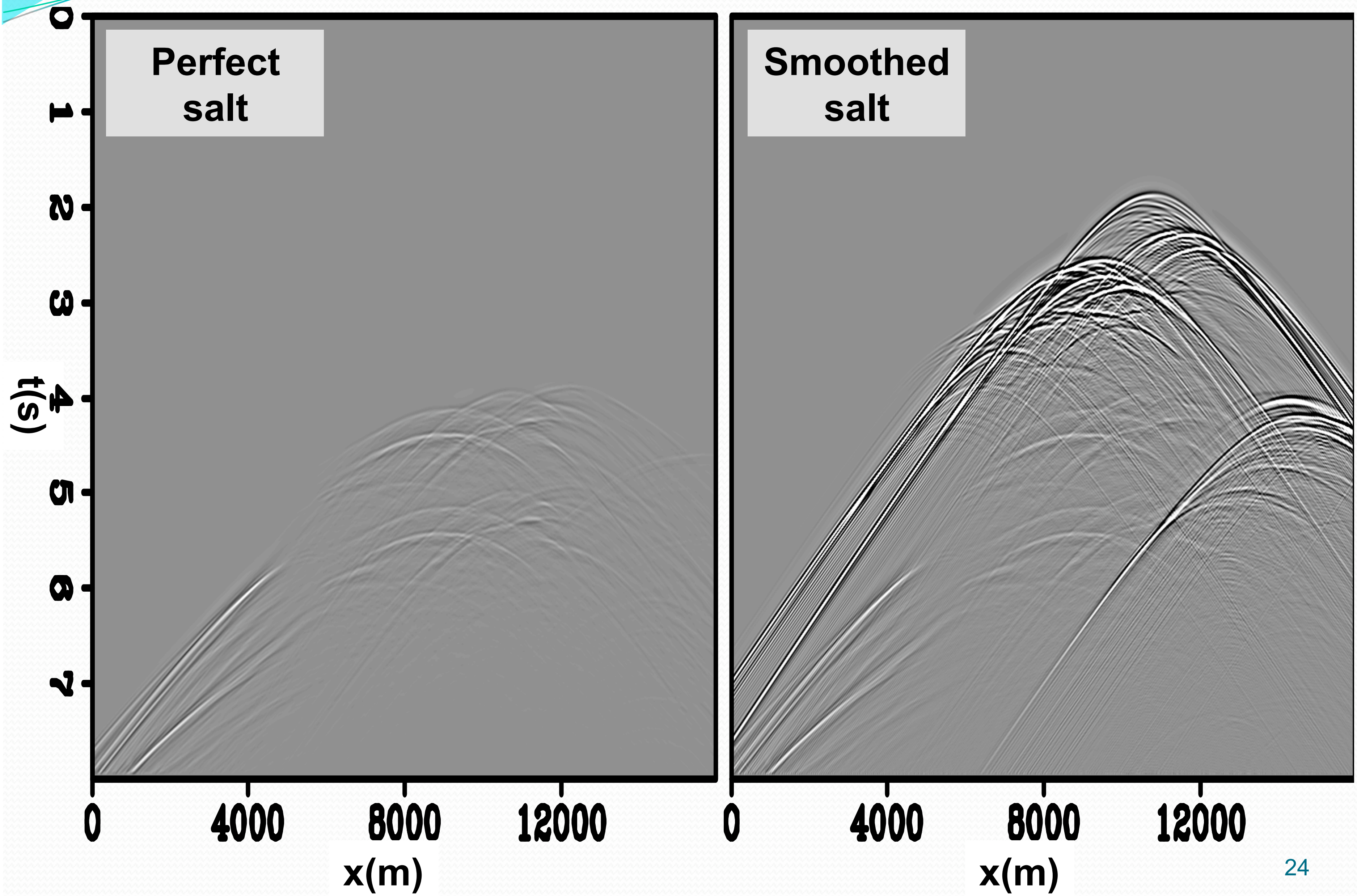
Let use the correct objective funtion

$$S_2(\mathbf{m}) \quad = \quad \|\mathbf{L}m - (d^{obs} - F(s_o^2))\|^2$$

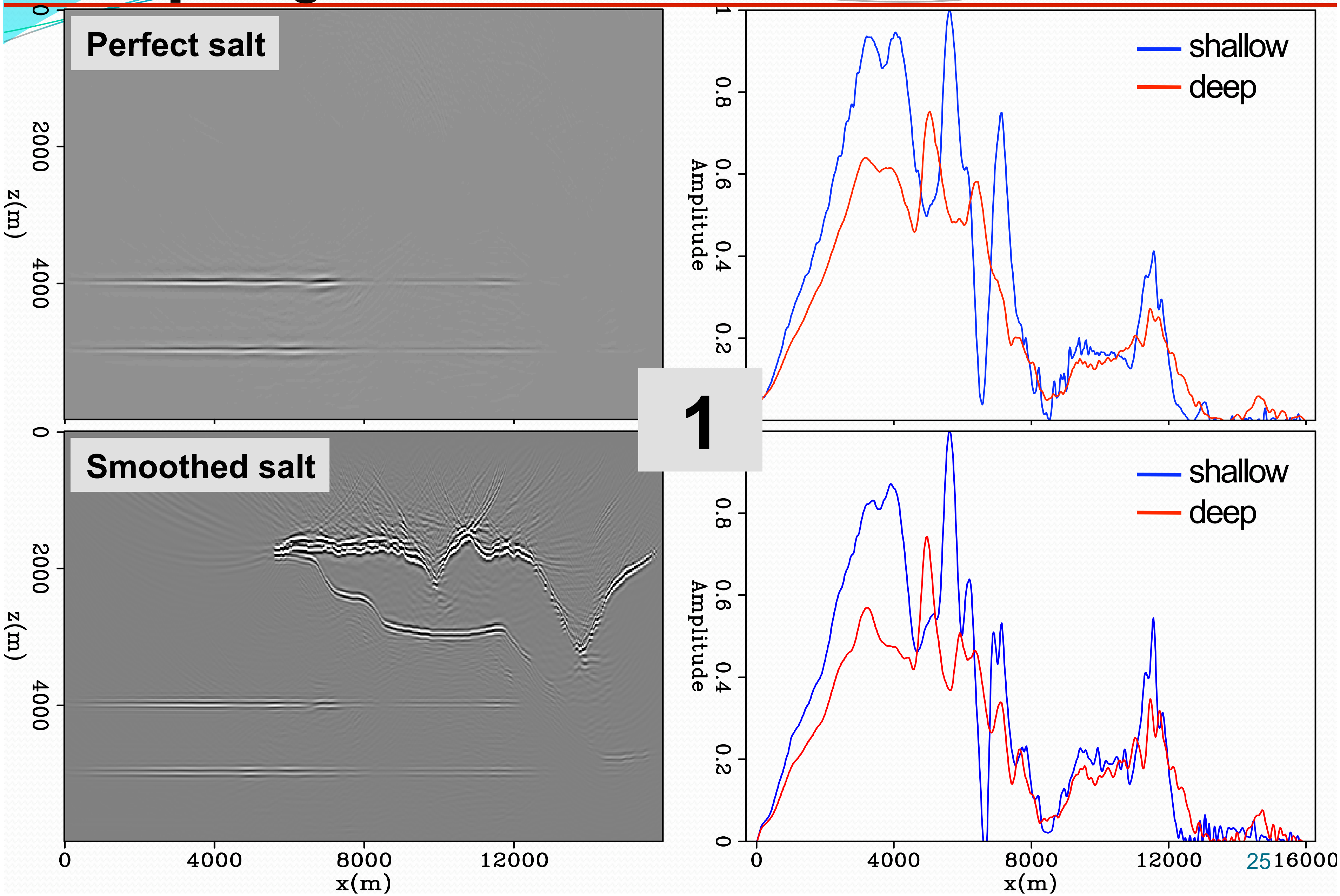
Test 2 – Subtract background data with varying degree of accuracy



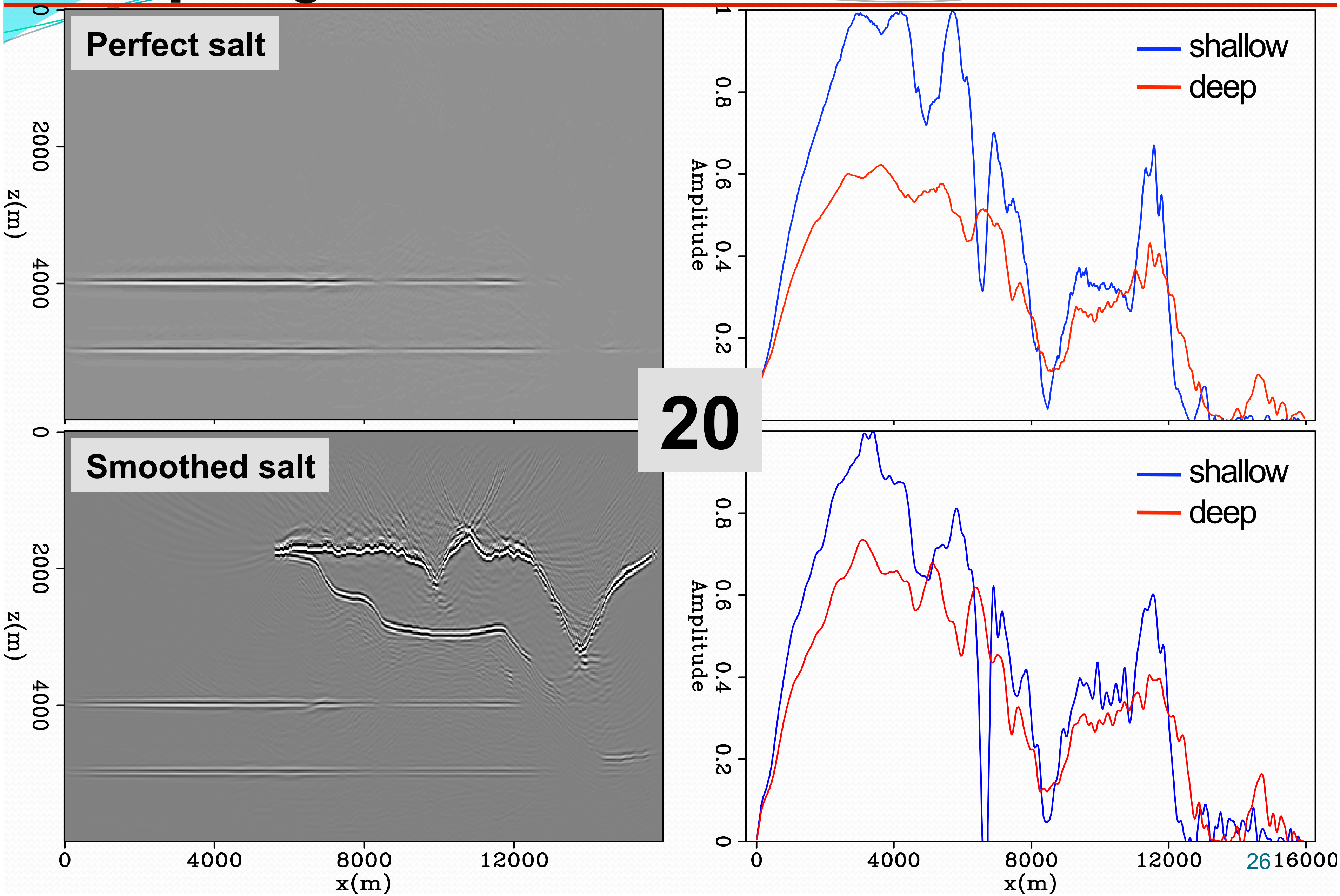
Input for inversion: perturbed data



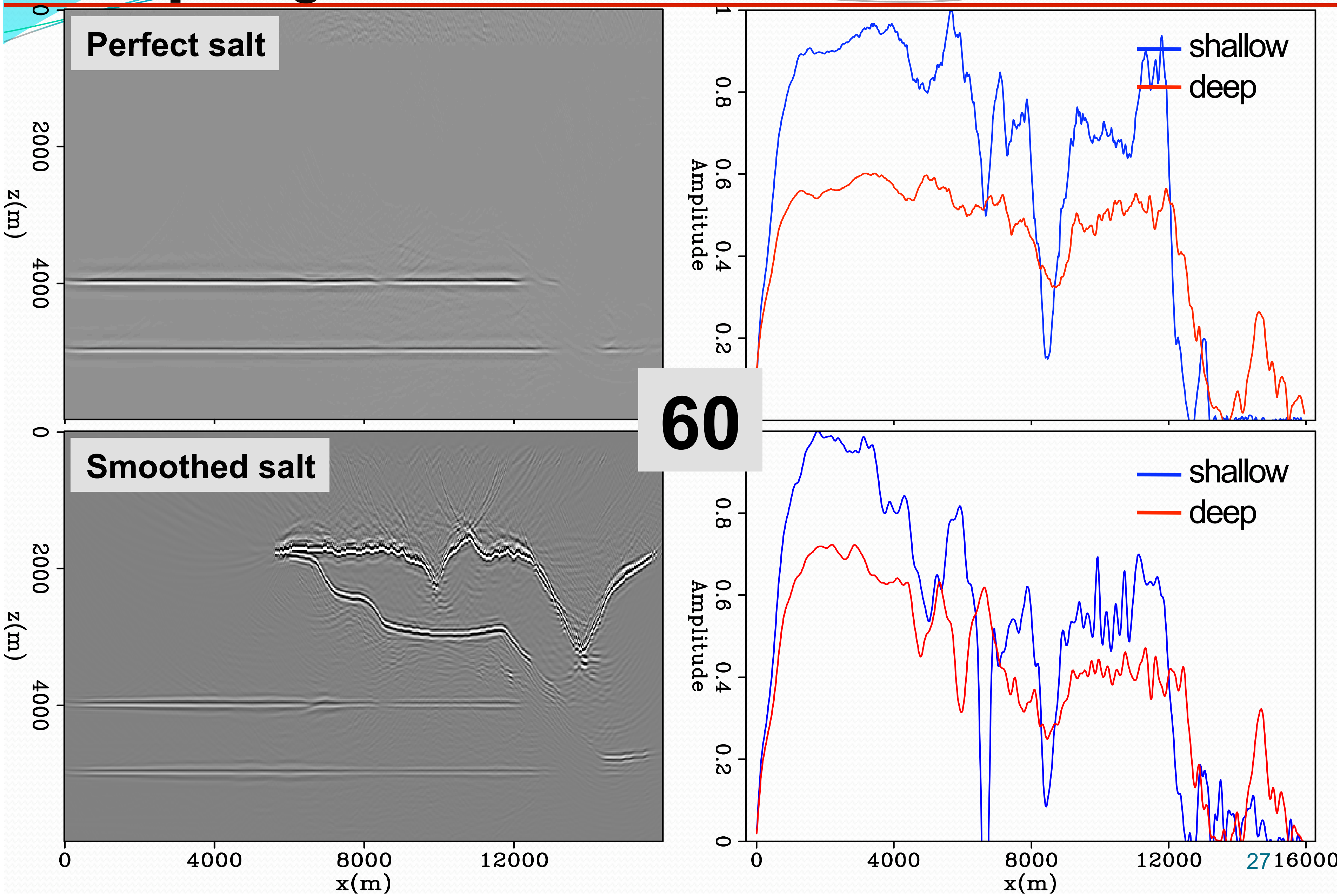
Comparing between two LSRTM



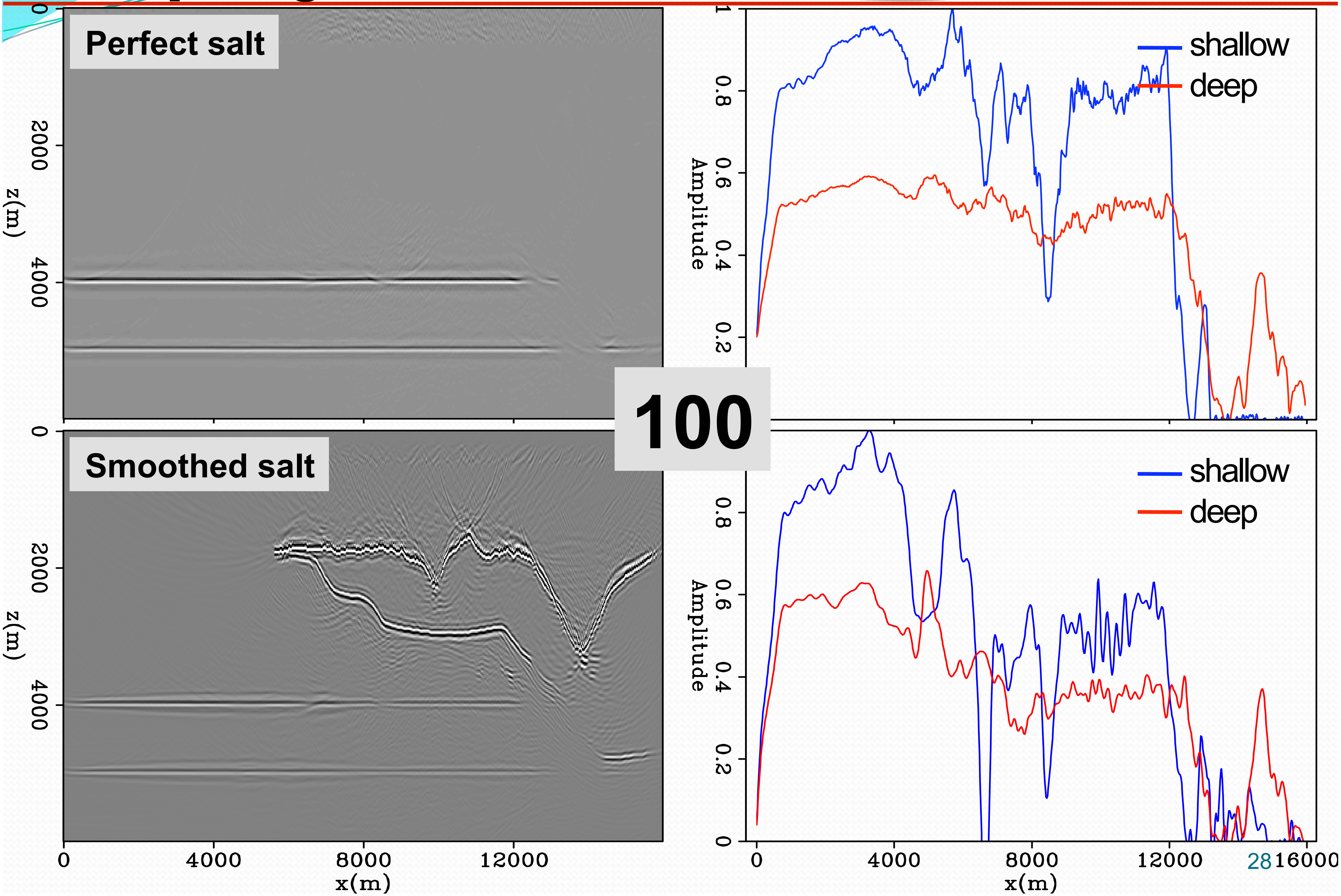
Comparing between two LSRTM



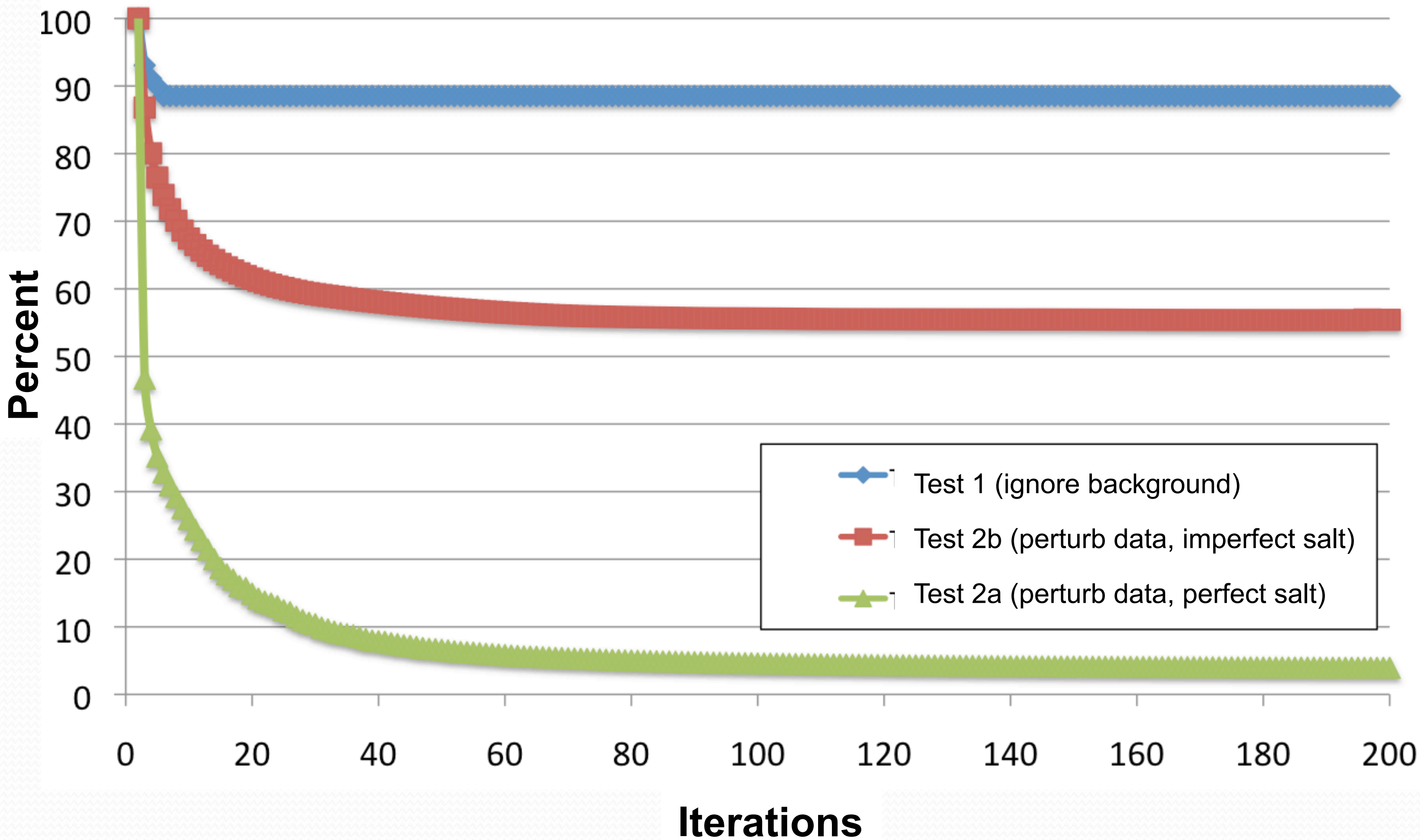
Comparing between two LSRTM



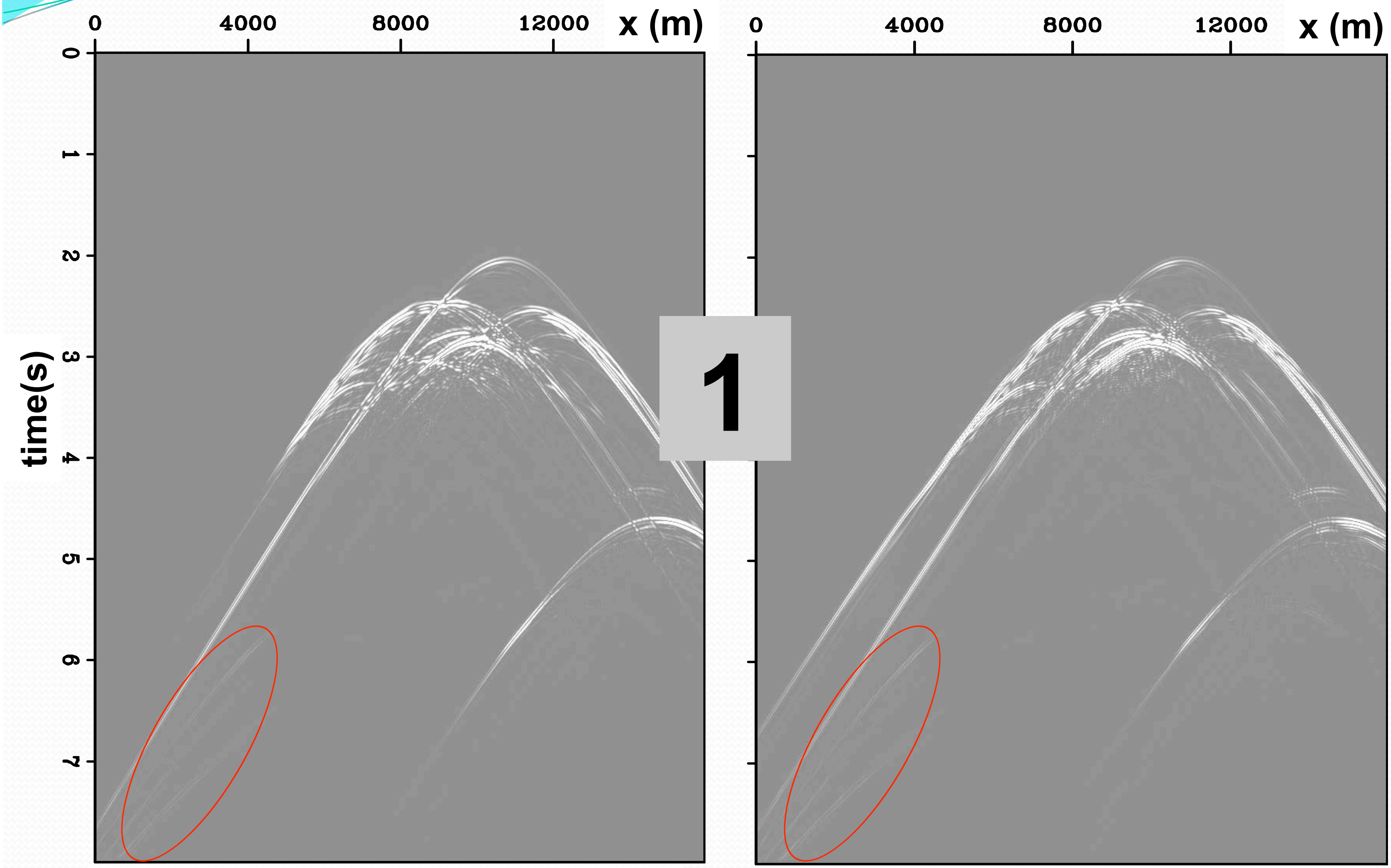
Comparing between two LSRTM



Convergence curve



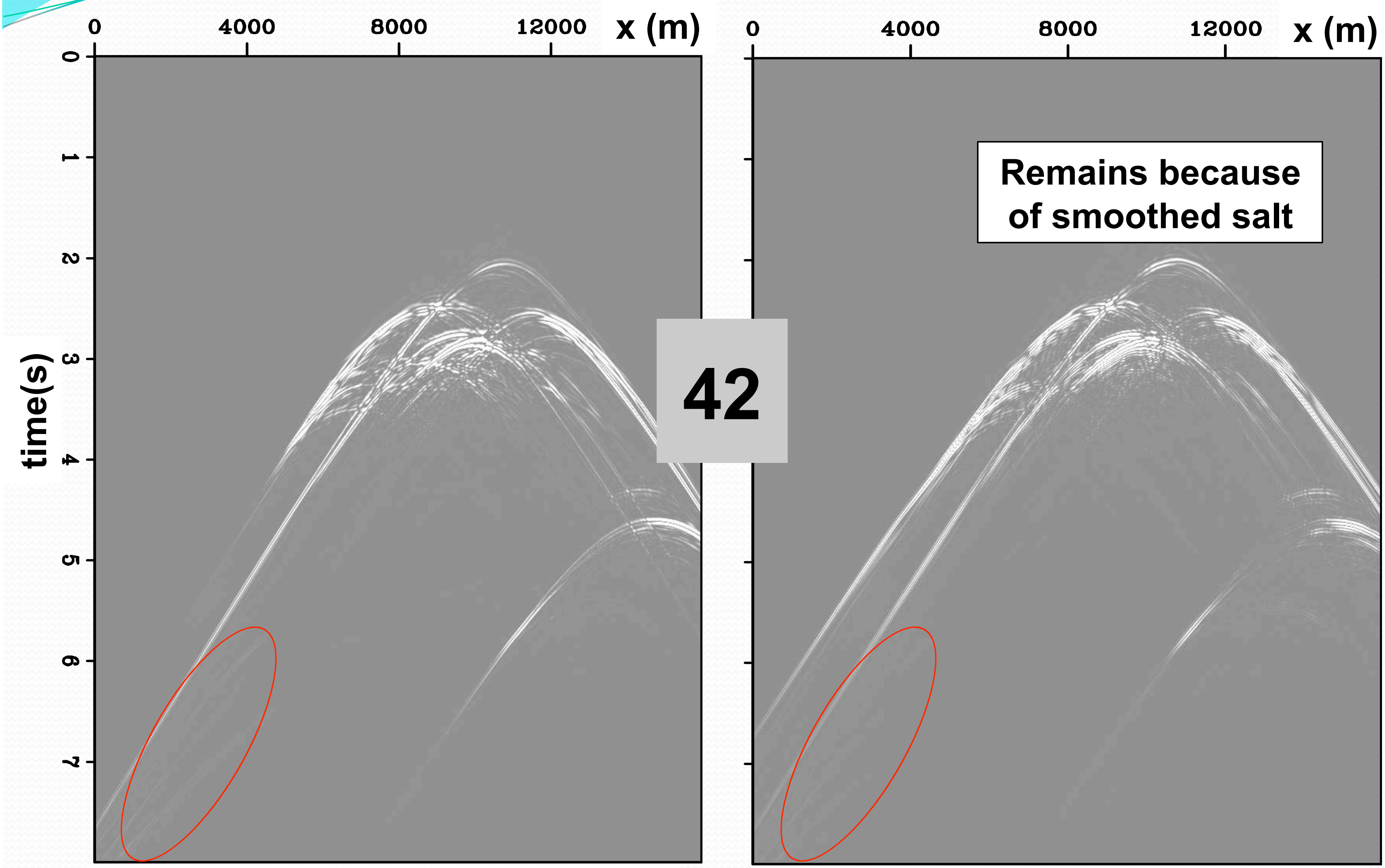
Data residual squared for the smoothed salt test



Test 1: ignore background data

Test 2: subtract background data

Data residual squared for the smoothed salt test



Test 1: ignore background data

Test 2: subtract background data

Summary

- **When the background data is significant**

$$\cancel{S_1(\mathbf{m}) \quad = \quad \|\mathbf{L}m - d^{obs}\|^2}$$

$$S_2(\mathbf{m}) \quad = \quad \|\mathbf{L}m - (d^{obs} - F(s_o^2))\|^2$$

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Challenges when applying to field dataset

- **Applying this to field dataset is non-trivial**
  - **the background data is just an approximation**

$$d^{obs} - F\left(s_o^2\right)$$

- **Perhaps LSM in the image space can handle this problem better**

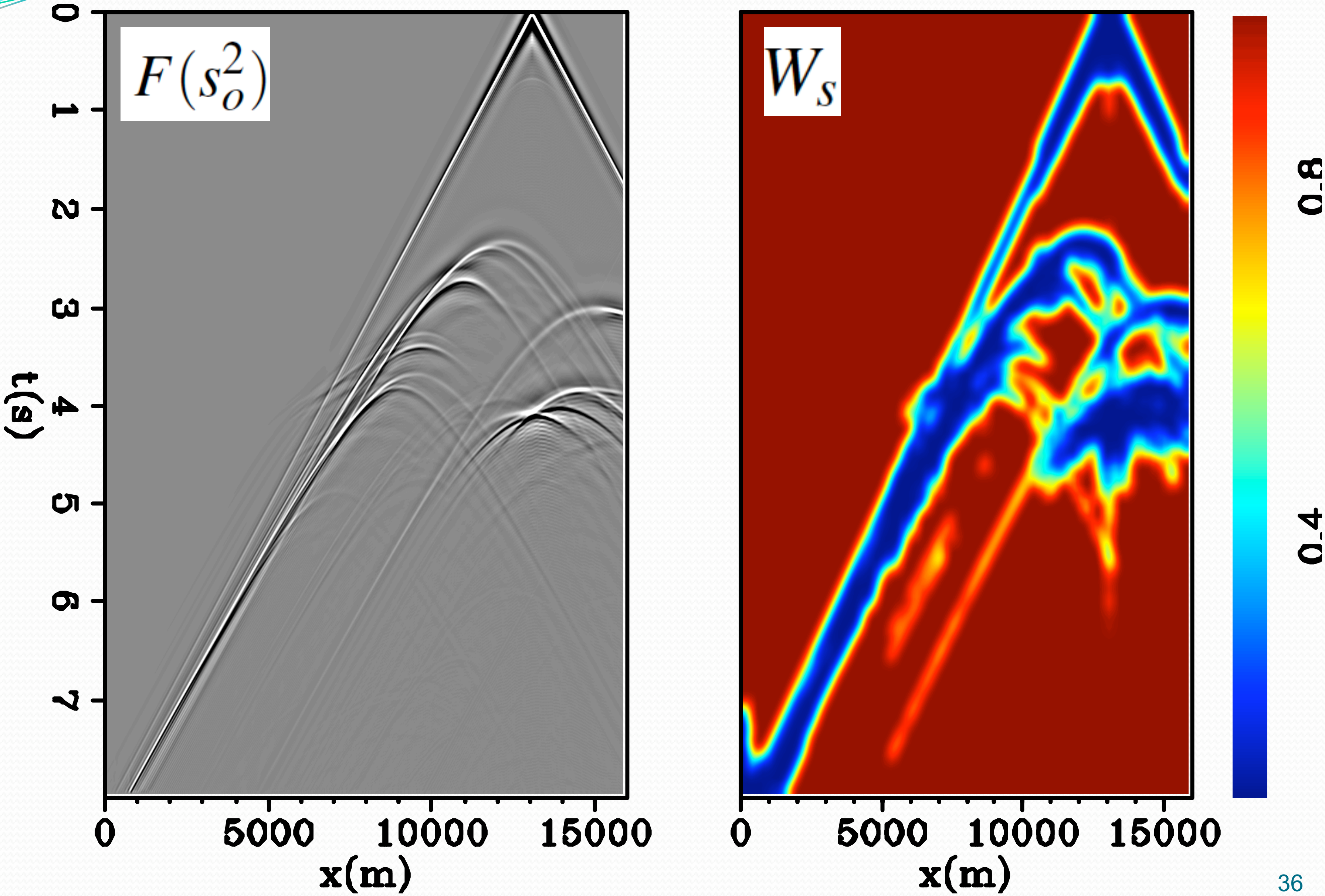
LSRTM with salt dimming

$$S_3(\mathbf{m}) = \|W_s(\mathbf{L}m - d^{obs})\|^2$$

W_s **data weighting function that down-weights the salt reflection energy.**

- **Get the most out of as few iterations as possible while addressing the background data problem**

Calculating W_s



LSRTM result with and without salt dimming

Test 1 : ignore background

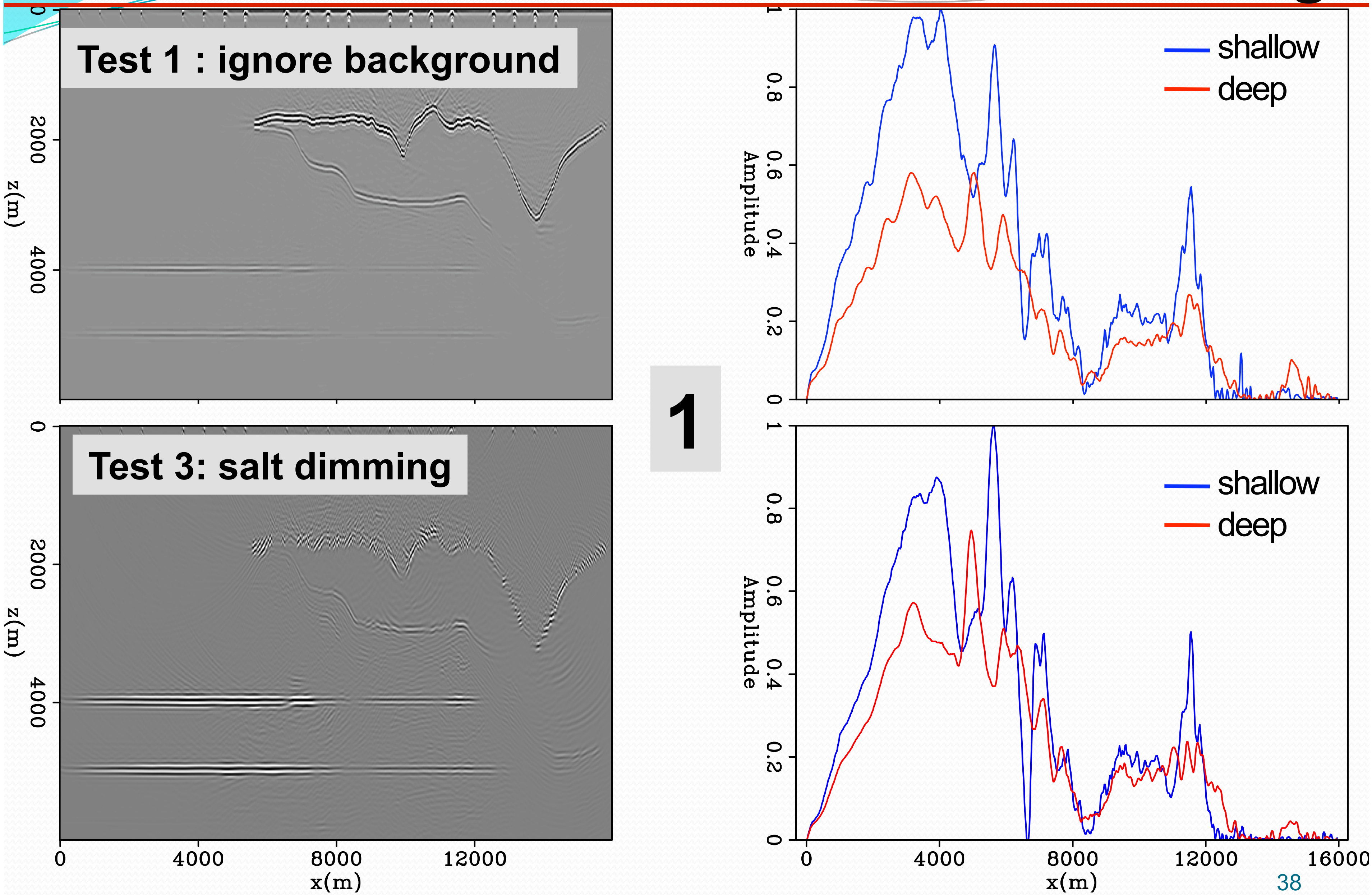
$$S_1(\mathbf{m}) \quad = \quad \|\mathbf{L}m - d^{obs}\|^2$$

using observed data

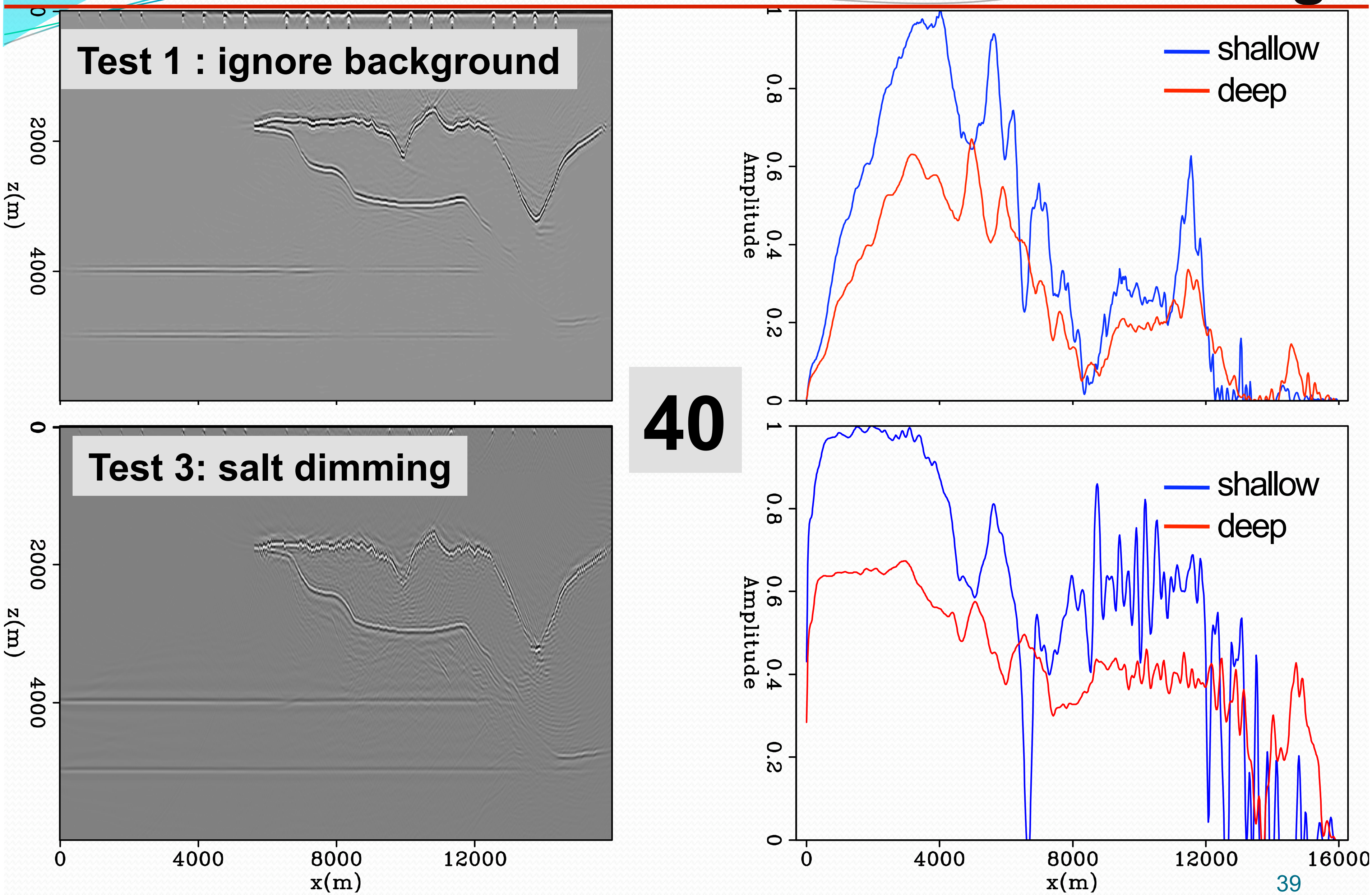
Test 3: salt dimming

$$S_3(\mathbf{m}) = \|W_s(\mathbf{L}m - d^{obs})\|^2$$

LSRTM result with and without salt dimming

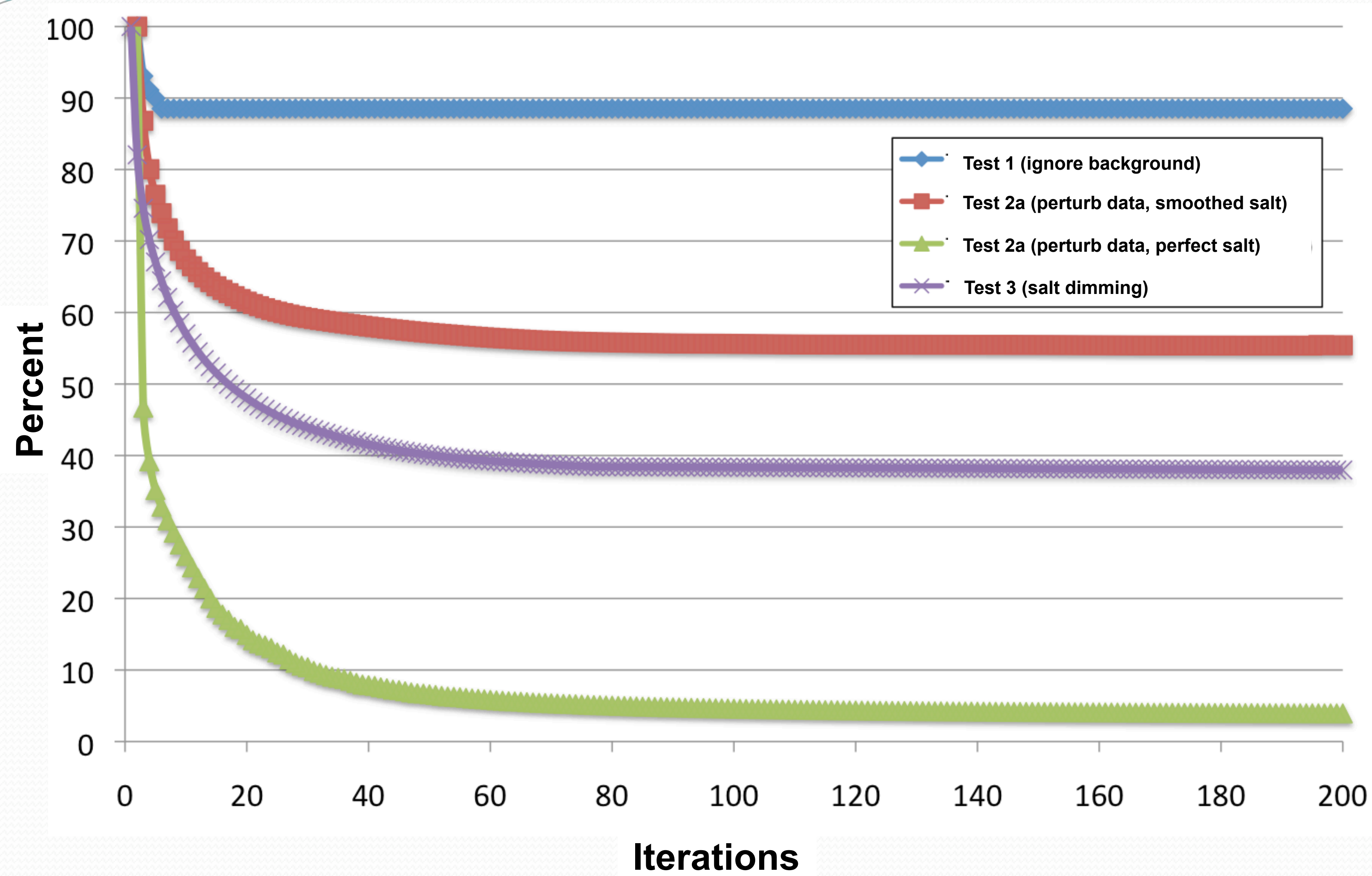


LSRTM result with and without salt dimming



40

Convergence Curves



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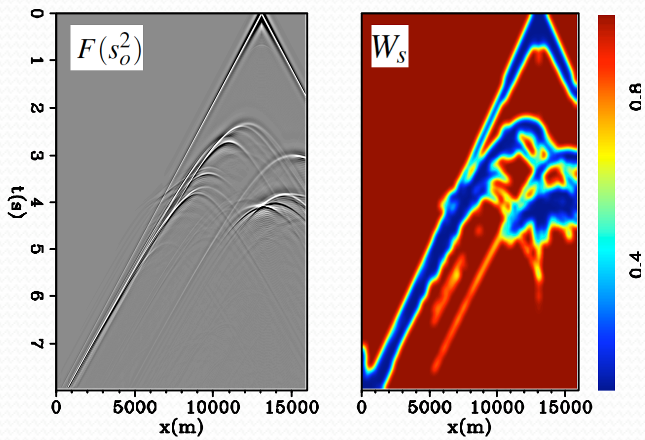
Conclusion

- **Subtracting the background data in LSM is important but non-trivial to apply on field datasets**

$$S_2(\mathbf{m}) \quad = \quad \|\mathbf{L}m - (d^{obs} - F(s_o^2))\|^2$$

- **Salt-dimming is a viable solution to address this issue**



$$S_3(\mathbf{m}) = \|W_s(\mathbf{L}m - d^{obs})\|^2$$

Acknowledgement

- Biondo Biondi and Bob Clapp for helpful discussions

Backup Slides

Some successful examples

- **LSM – using one-way operator**
  - Kuehl, H. and Sacchi M., 2002, Robust AVP estimation using least-squares wave-equation migration: SEG Technical Program Expanded Abstract, 21, 281
  - Clapp, M. R. Clapp, and B. Biondi, 2005, Regularized least-squares inversion for 3-D subsalt imaing: SEG Technical Expanded Abstract, 24, 1814-1817

- **LSRTM**
  - Dai, W., C. Boonyasiriwat, and G. Schuster, 2010 – 3D multi-source least-squares reverse time migration: SEG Technical Expanded Abstract, 29, 3120-3124
  - Wong, M., Biondo B., and Ronen S., 2010, Joint inversion of up- and down-going signal for ocean bottom data, SEG Expanded Abstracts,
  - Dai, W., X. Wang, and G. Schuster, 2011, Least-squares migration of multisource data with a deblurring filter: Geophysics, 76, R135-R146
  - Gang, Y. and Jakubowicz M., 2012 – Least-squares Reverse-Time Migration
  - Dong et al., 2012, Least-squares reverse time migration: towards true amplitude imaging and improving the resolution
  - Wong, M., Biondo B. and Ronen S., 2012, Imaging with multiples using linearized full-wave inversion