Least-squares reverse-time migration using wavefield decomposition

SEP Sponsor Meeting
June 19th, 2013
SEP 149

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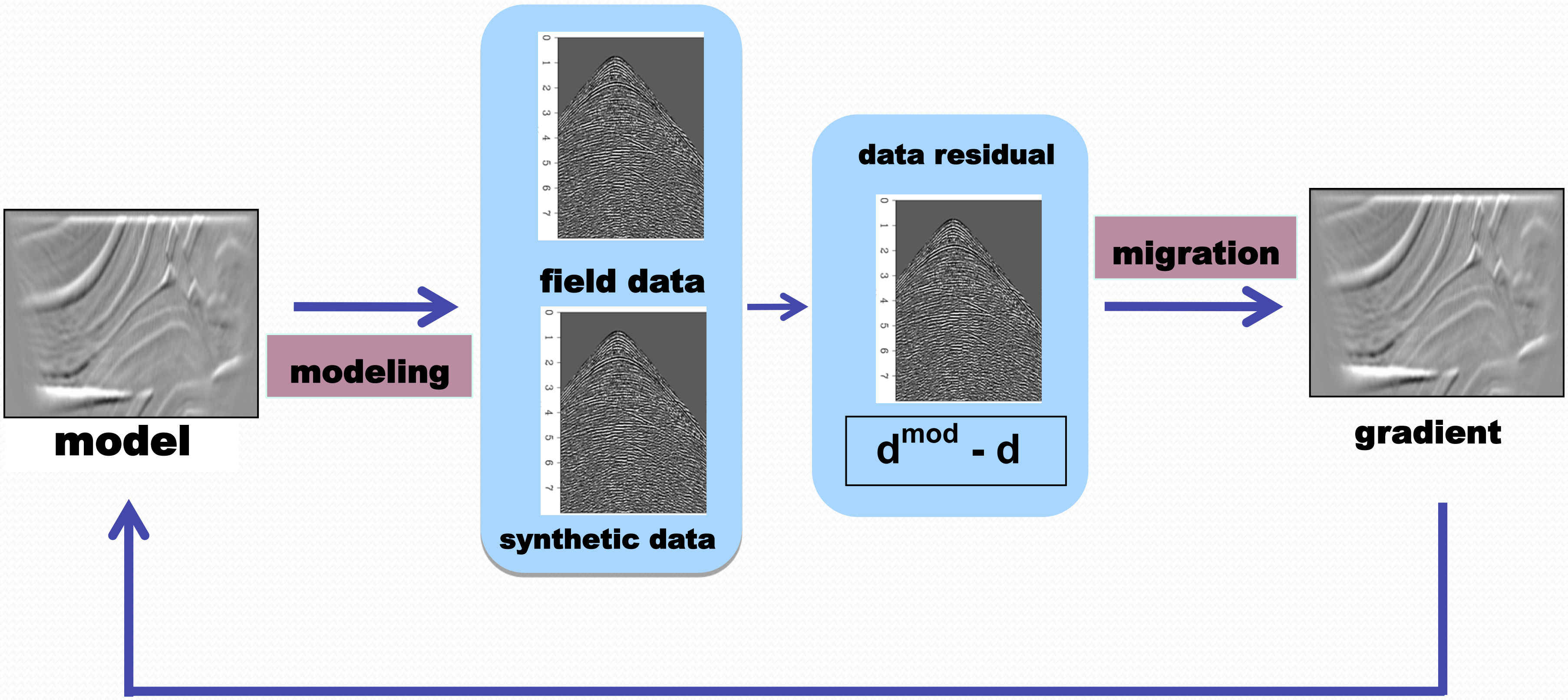
Overview

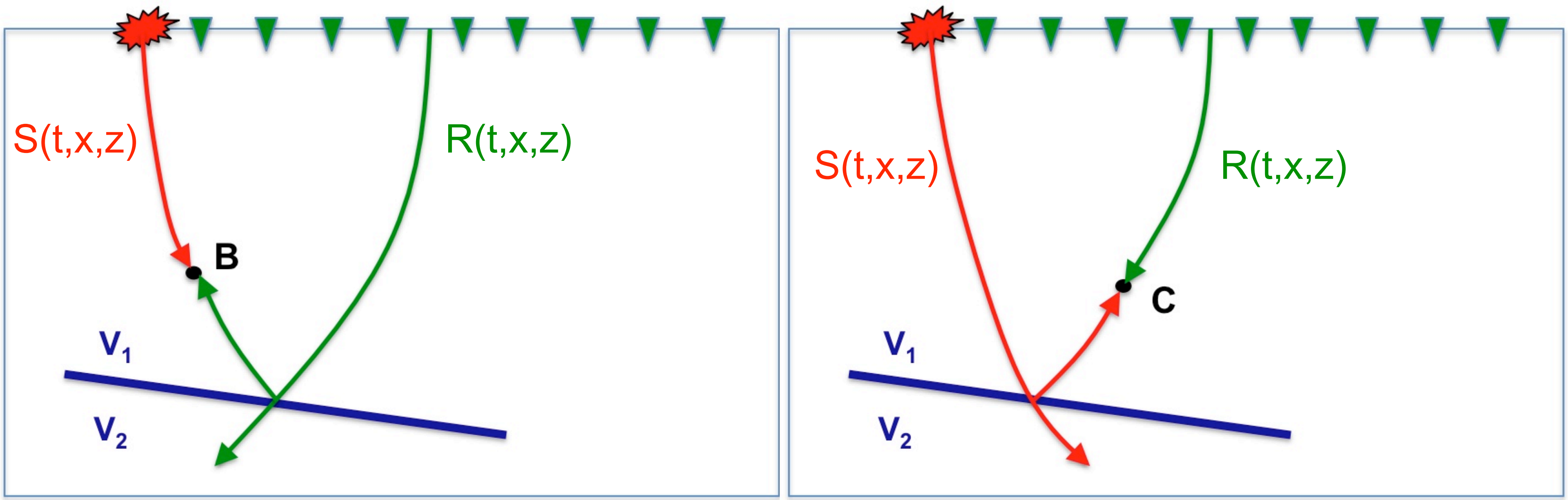
- Introduction
  - RTM artifacts
- Theory
- Synthetic 2D SEAM example
- Discussion
- Conclusion

LSM Workflow

- Iterative inversion by conjugate gradient

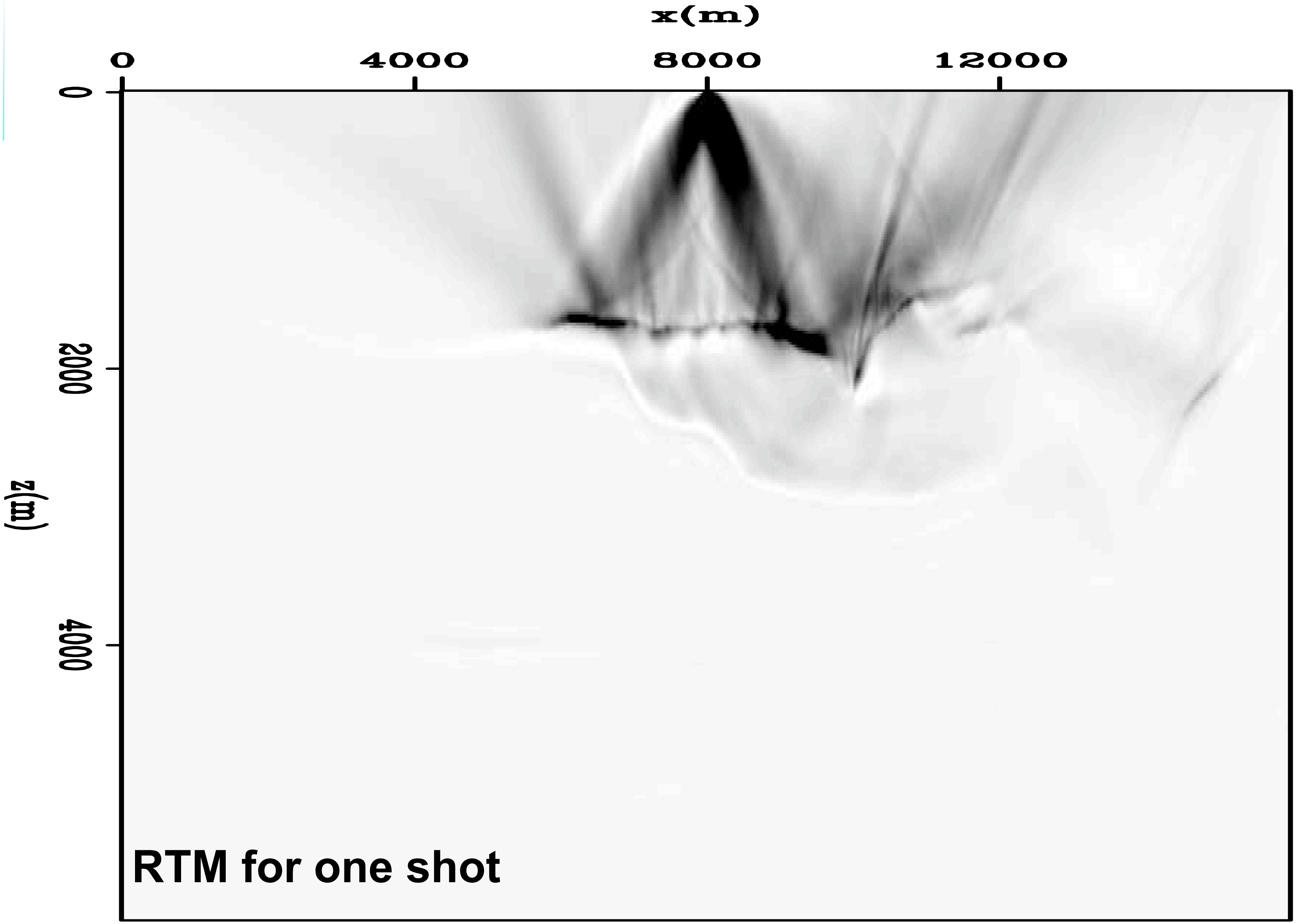


Artifacts in RTM



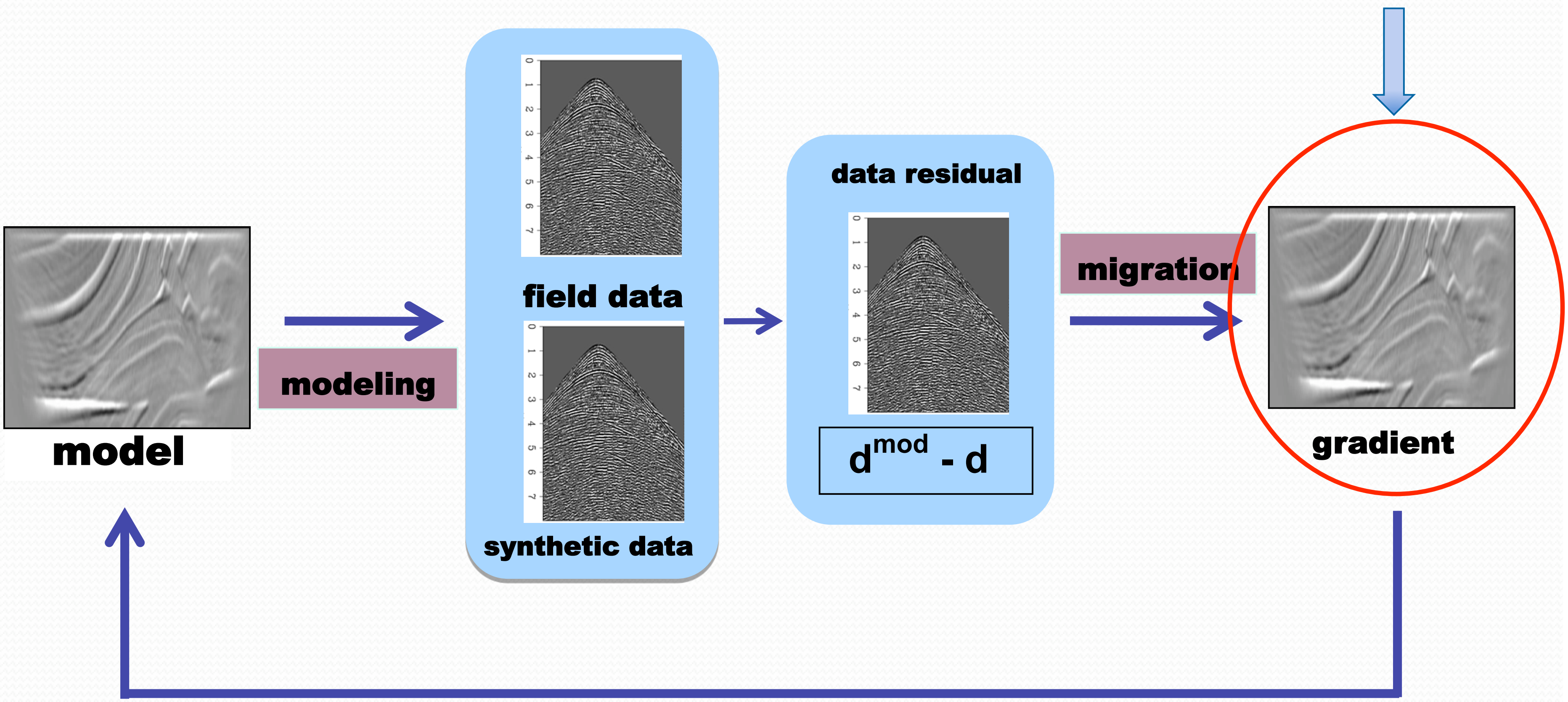
- **Red = source wavefield with time running forward.**
- **Green = receiver wavefield with time running backward.**

- **At both points B and C, the cross-correlation is undesirable, i.e. noise.**

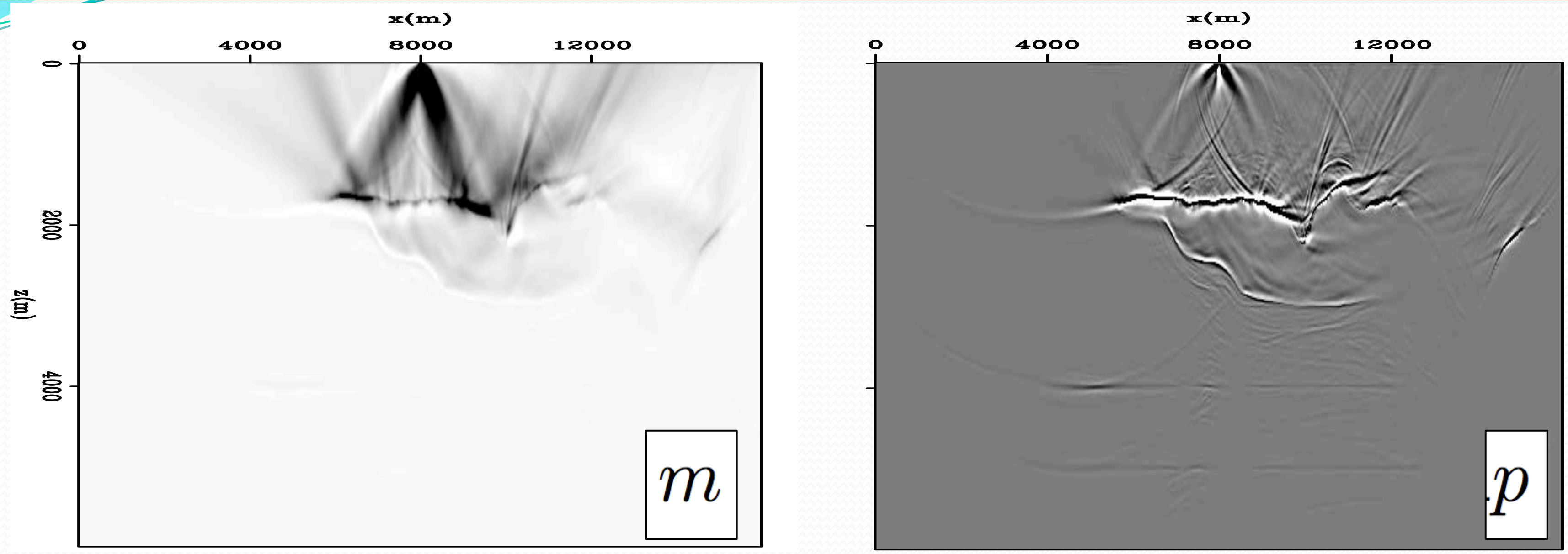

x(m)
0
4000
8000
12000
0
2000
4000
z(m)
RTM for one shot

RTM artifacts slow down convergence

Gradient is corrupted with noise



Suppressing RTM artifacts with a Laplacian



LSRTM with Laplacian preconditioner (LSRTM-Laplace)

$$S(p) = \left\| W_s \left(\mathbf{L}\mathbf{A}p - d_{\mathrm{obs}} \right) \right\|^2$$

$$m = \mathbf{A}p$$

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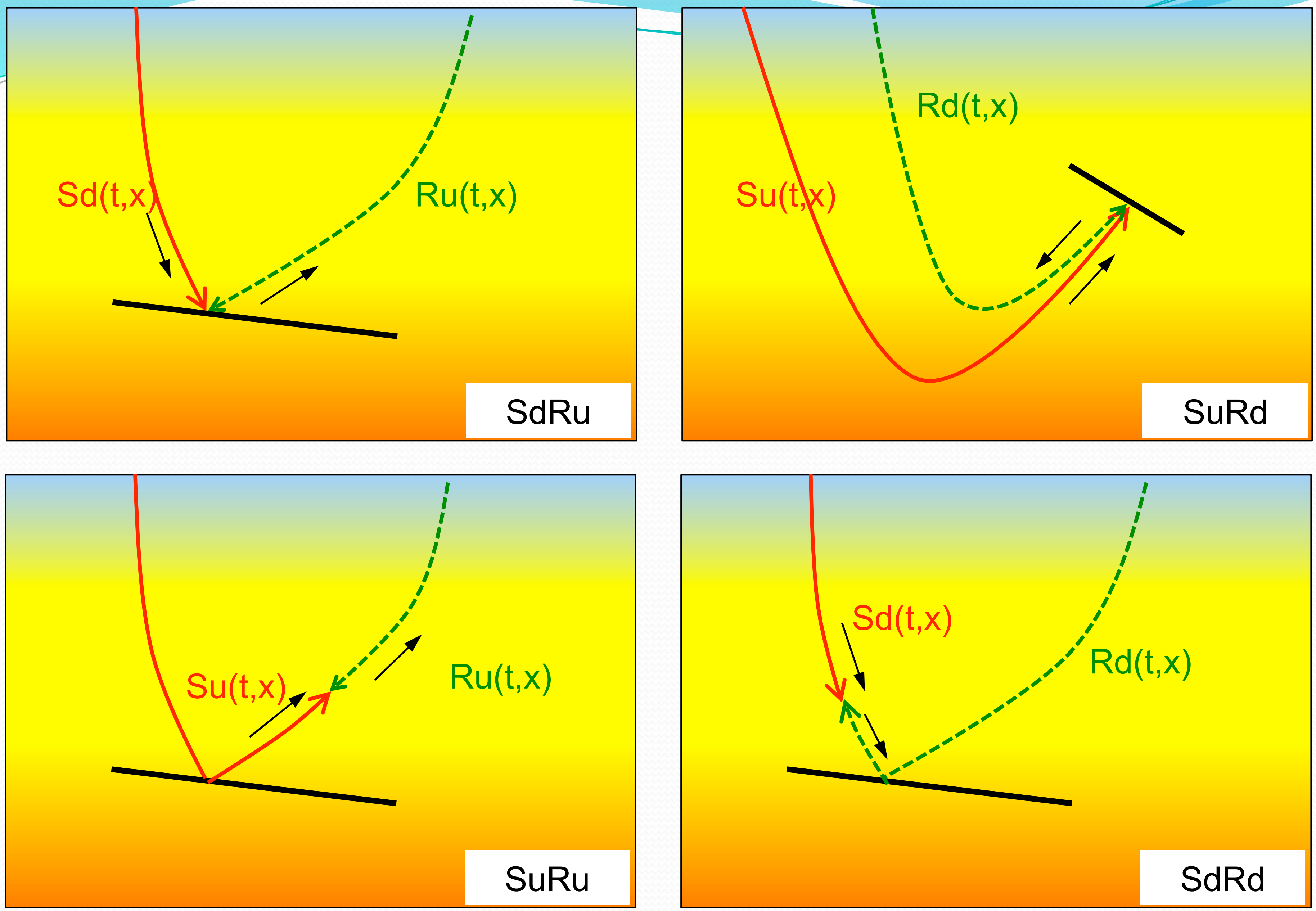
Breaking down the imaging condition

$$
\begin{aligned}
m_{mig}(\mathbf{x}) &= I_1(\mathbf{x}) + I_2(\mathbf{x}) + I_3(\mathbf{x}) + I_4(\mathbf{x}) \\
I_1(\mathbf{x}) &= \sum_{\mathbf{x_s},t} s_d(t,\mathbf{x};\mathbf{x_s}) r_u(T-t,\mathbf{x};\mathbf{x_s}) \\
I_2(\mathbf{x}) &= \sum_{\mathbf{x_s},t} s_u(t,\mathbf{x};\mathbf{x_s}) r_d(T-t,\mathbf{x};\mathbf{x_s}) \\
I_3(\mathbf{x}) &= \sum_{\mathbf{x_s},t} s_u(t,\mathbf{x};\mathbf{x_s}) r_u(T-t,\mathbf{x};\mathbf{x_s}) \\
I_4(\mathbf{x}) &= \sum_{\mathbf{x_s},t} s_d(t,\mathbf{x};\mathbf{x_s}) r_d(T-t,\mathbf{x};\mathbf{x_s})
\end{aligned}
$$

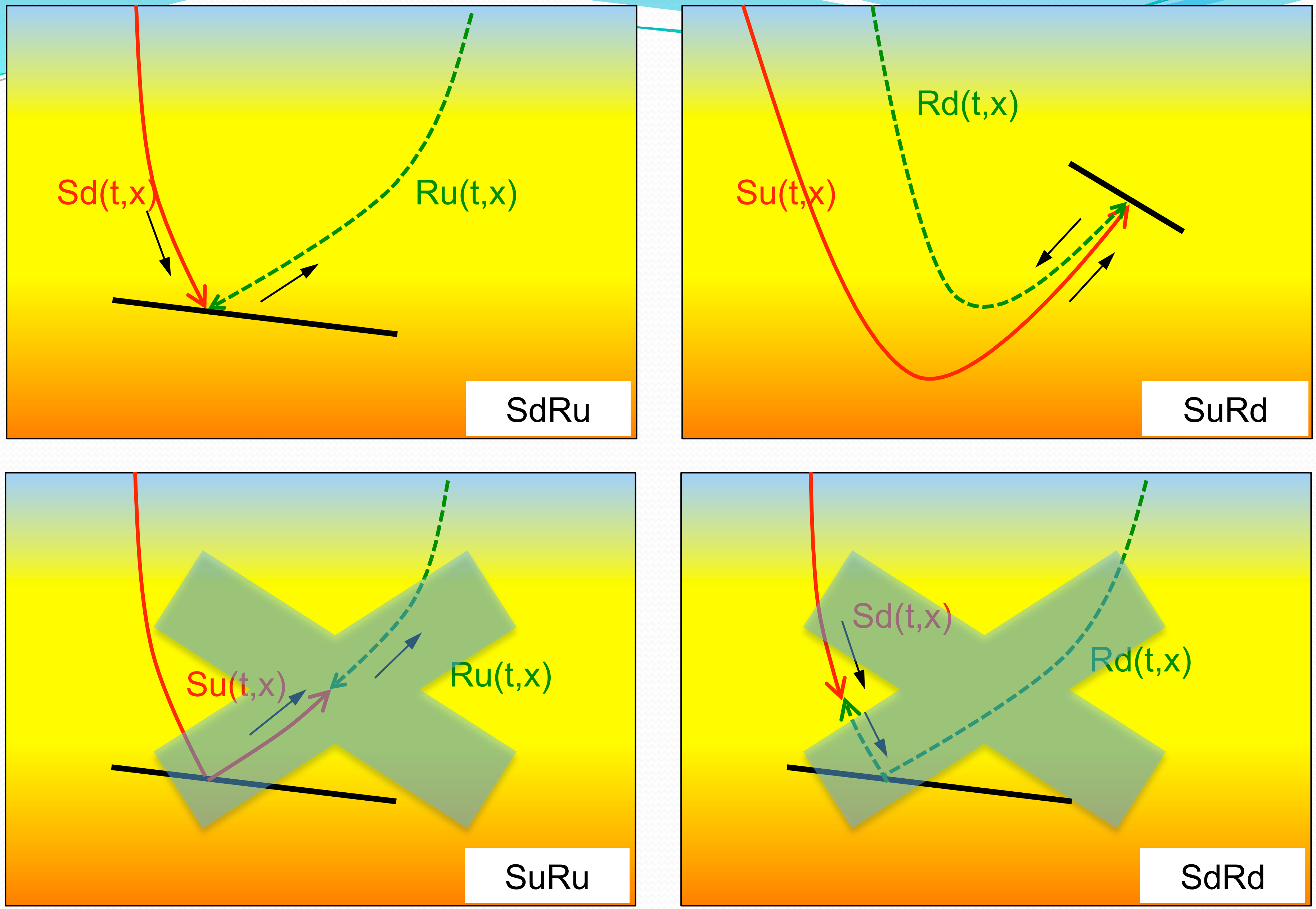

Sd(t,x)
Ru(t,x)
SdRu
Rd(t,x)
Su(t,x)
SuRd
Su(t,x)
Ru(t,x)
SuRu
Sd(t,x)
Rd(t,x)
SdRd

Modify the migration operator

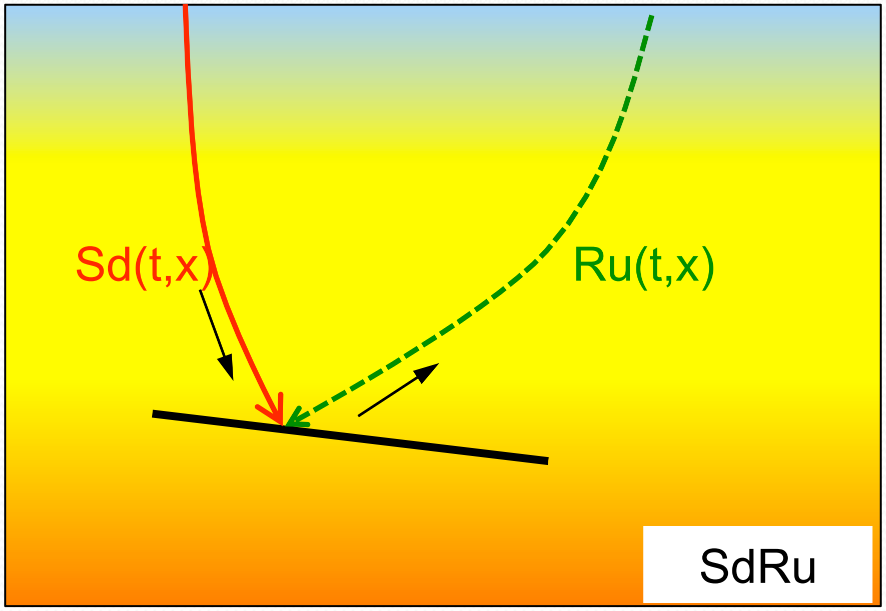
$$\mathbf{L}' \longrightarrow \mathbf{L}'_D$$

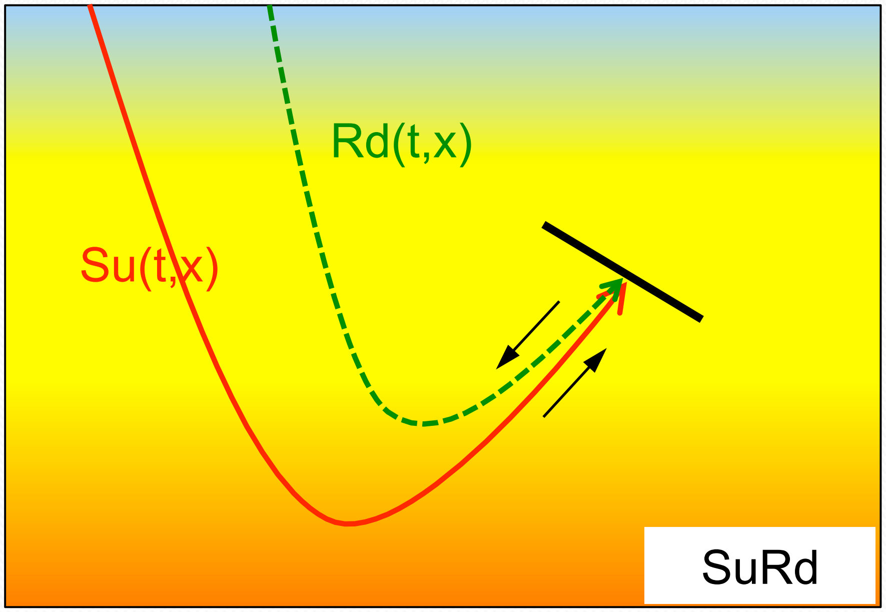
regular RTM operator

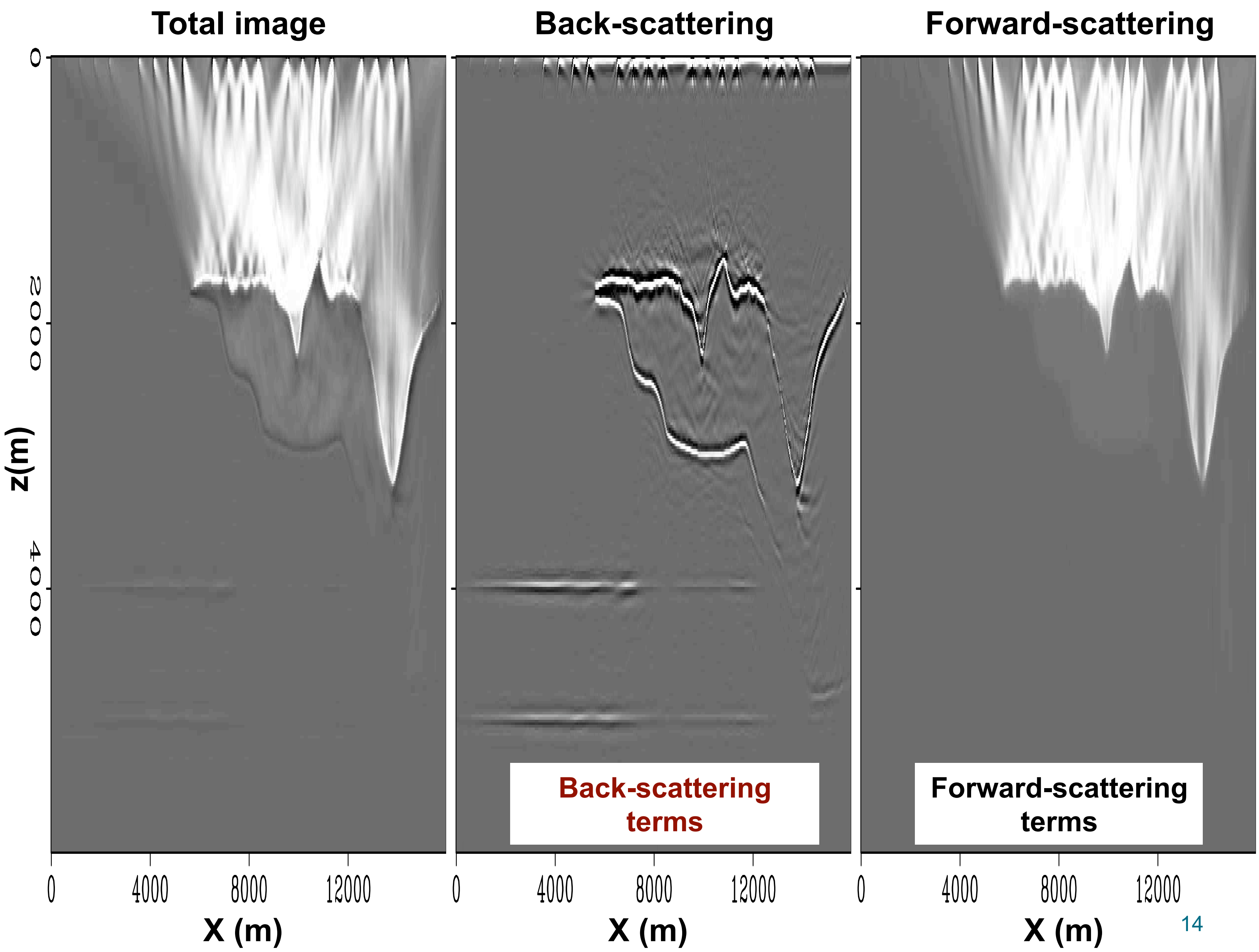
RTM with wavefield decomposition


Sd(t,x)
Ru(t,x)
SdRu
Rd(t,x)
Su(t,x)
SuRd
Su(t,x)
Ru(t,x)
SuRu
Sd(t,x)
Rd(t,x)
SdRd

$$m_{decomp}(\mathbf{x}) = \sum_{\mathbf{x_s},t} s_d(t,\mathbf{x};\mathbf{x_s})r_u(t,\mathbf{x};\mathbf{x_s}) + s_u(t,\mathbf{x};\mathbf{x_s})r_d(t,\mathbf{x};\mathbf{x_s}).$$






Total image
Back-scattering
Forward-scattering
z(m)
0
2000
4000
0
4000
8000
12000
X (m)
Back-scattering terms
Forward-scattering terms

Forward operator as an adjoint pair

$$\mathbf{L}' \longrightarrow \mathbf{L}'_D$$

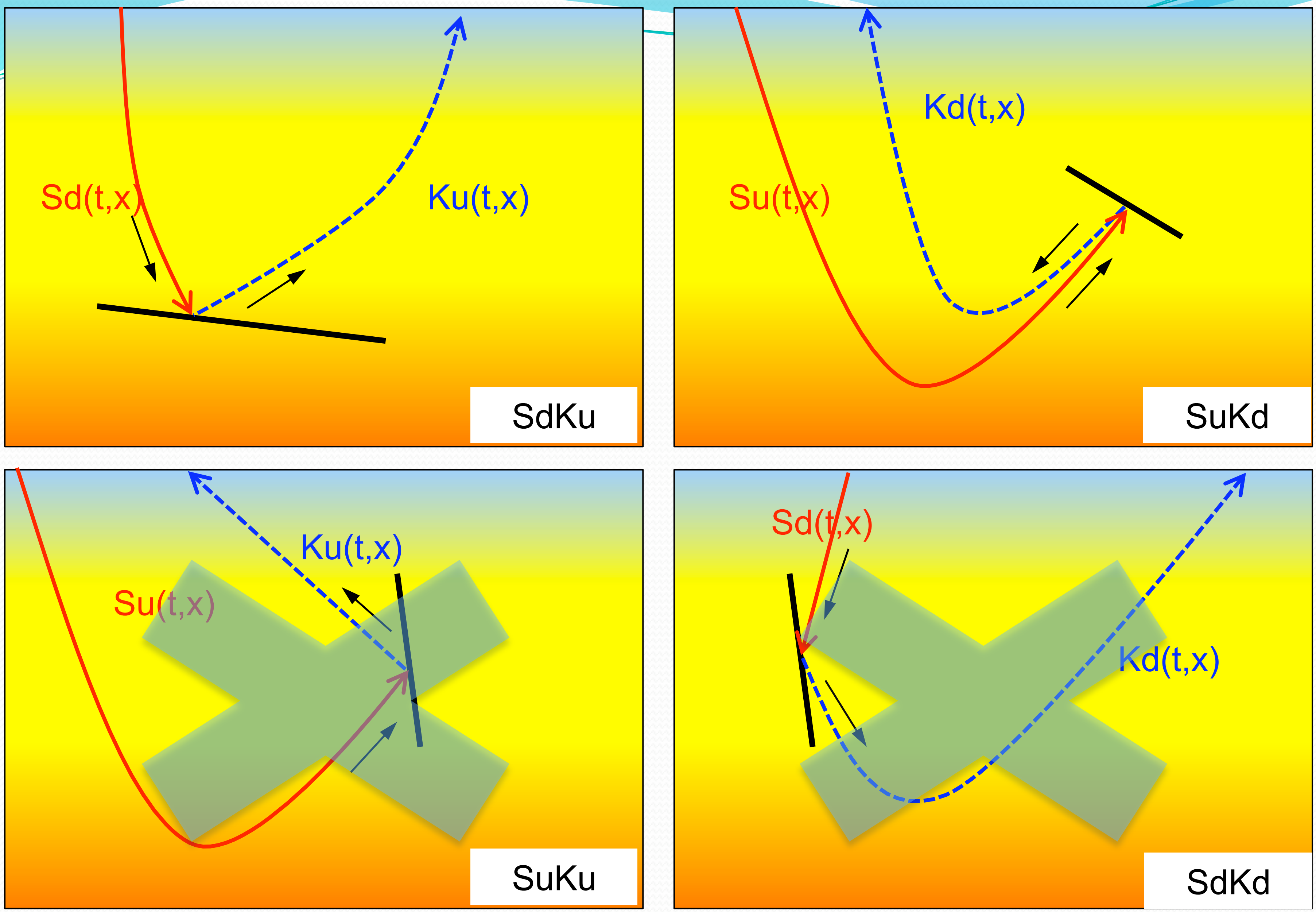
regular RTM operator

RTM with wavefield decomposition

$$\mathbf{L} \longrightarrow \mathbf{L}_D$$

Forward operator

Forward with wavefield decomposition


Sd(t,x)
Ku(t,x)
SdKu
Kd(t,x)
Su(t,x)
SuKd
Ku(t,x)
Su(t,x)
SuKu
Sd(t,x)
Kd(t,x)
SdKd

Wavefield decomposition is expensive.

Wavefield decomposition in the T-K domain

- **The backscatter-based imaging condition can be written as (Liu, 2011)**

$$I_{\mathrm{vert}}(\vec{x}) = \sum_{t=0}^{t_{\max}} S^*_{k_z+}(t,\vec{x}) R_{k_z-}(t,\vec{x}) + \sum_{t=0}^{t_{\max}} S^*_{k_z-}(t,\vec{x}) R_{k_z+}(t,\vec{x})$$

- **where**

$$\tilde{P}_{k_z+}(t,k_z) = \begin{cases} \tilde{P}(t,k_z) & \text{for } k_z \geq 0 \\ 0 & \text{for } k_z < 0 \end{cases},$$

$$\tilde{P}_{k_z-}(t,k_z) = \begin{cases} 0 & \text{for } k_z \geq 0 \\ \tilde{P}(t,k_z) & \text{for } k_z < 0 \end{cases}$$

Computational Cost

- **Regular LSRTM**

$$\propto N_x N_y N_z N_{\text{order}}$$

- **LSRTM with wavefield decomposition**

$$\propto N_x N_y N_z N_{\text{order}} + N_x N_y N_z log(N_z)$$

Wavefield decomposition is expensive.

But there is a cheap way to do it.

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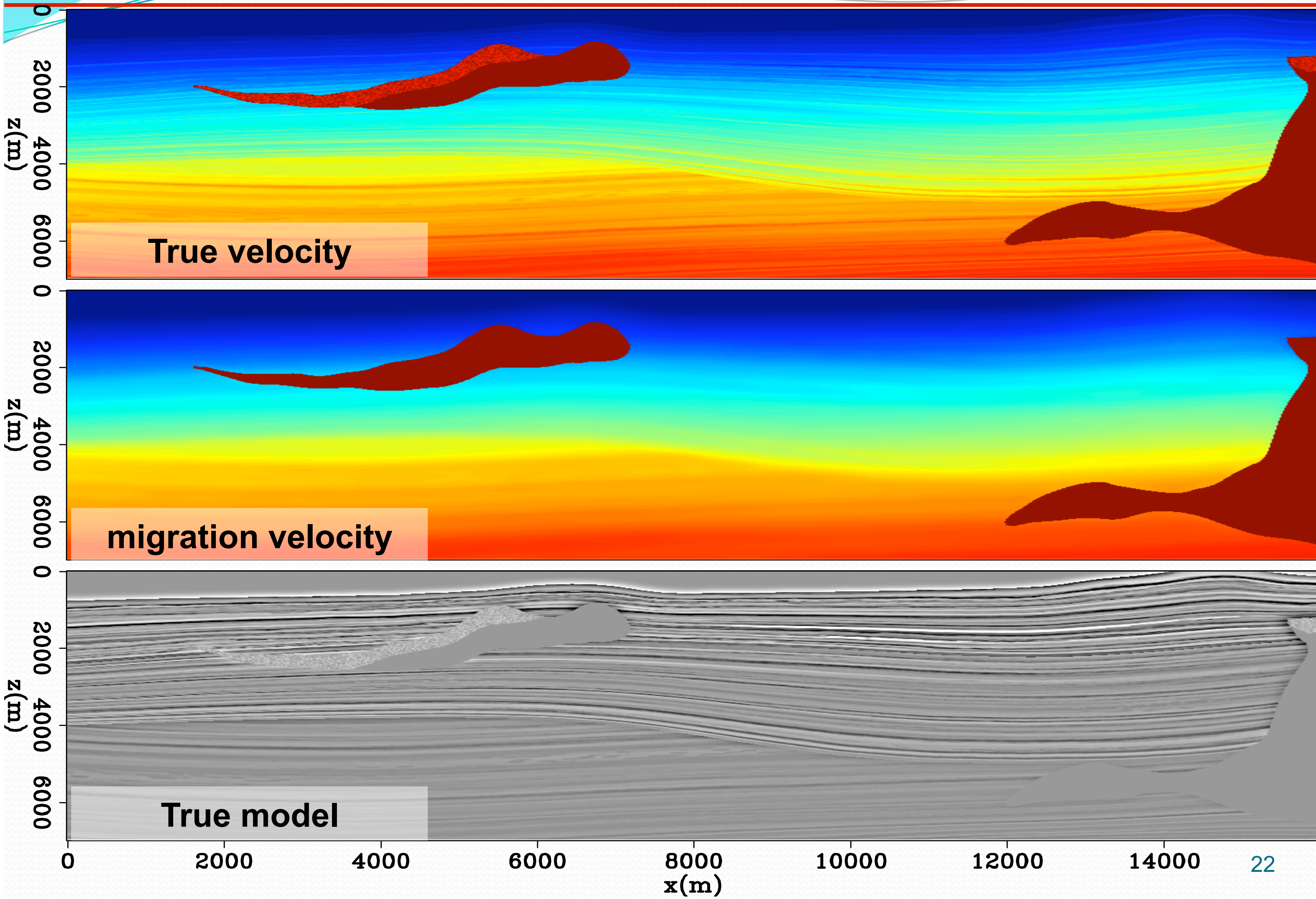
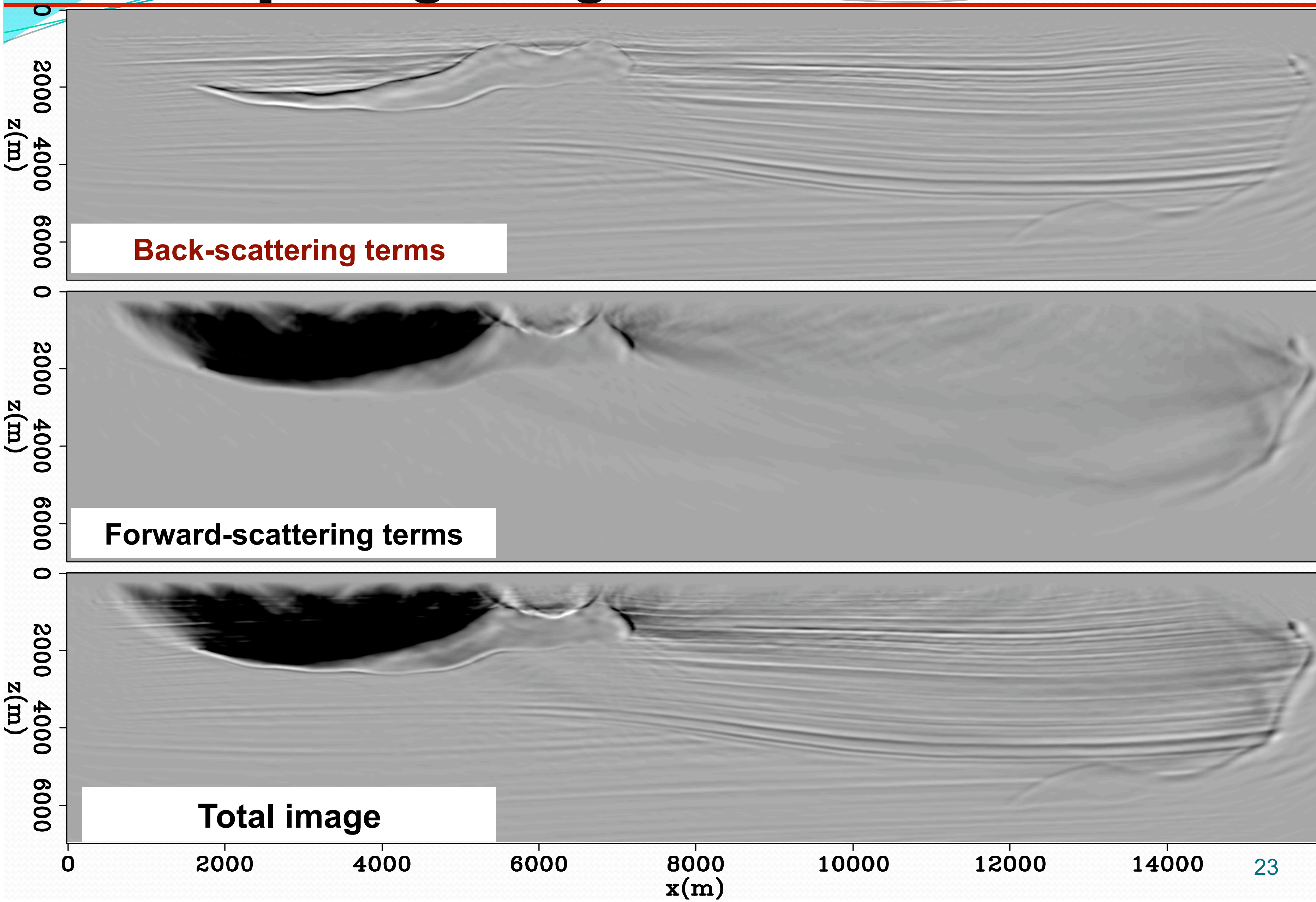
SEAM Model


True velocity
migration velocity
True model
z(m)
0
2000
4000
6000
x(m)
0
2000
4000
6000
8000
10000
12000
14000

Decomposing the gradient



Comparing between two LSRTM method

Test 1: LSRTM with Laplacian preconditioner (LSRTM-Laplace)

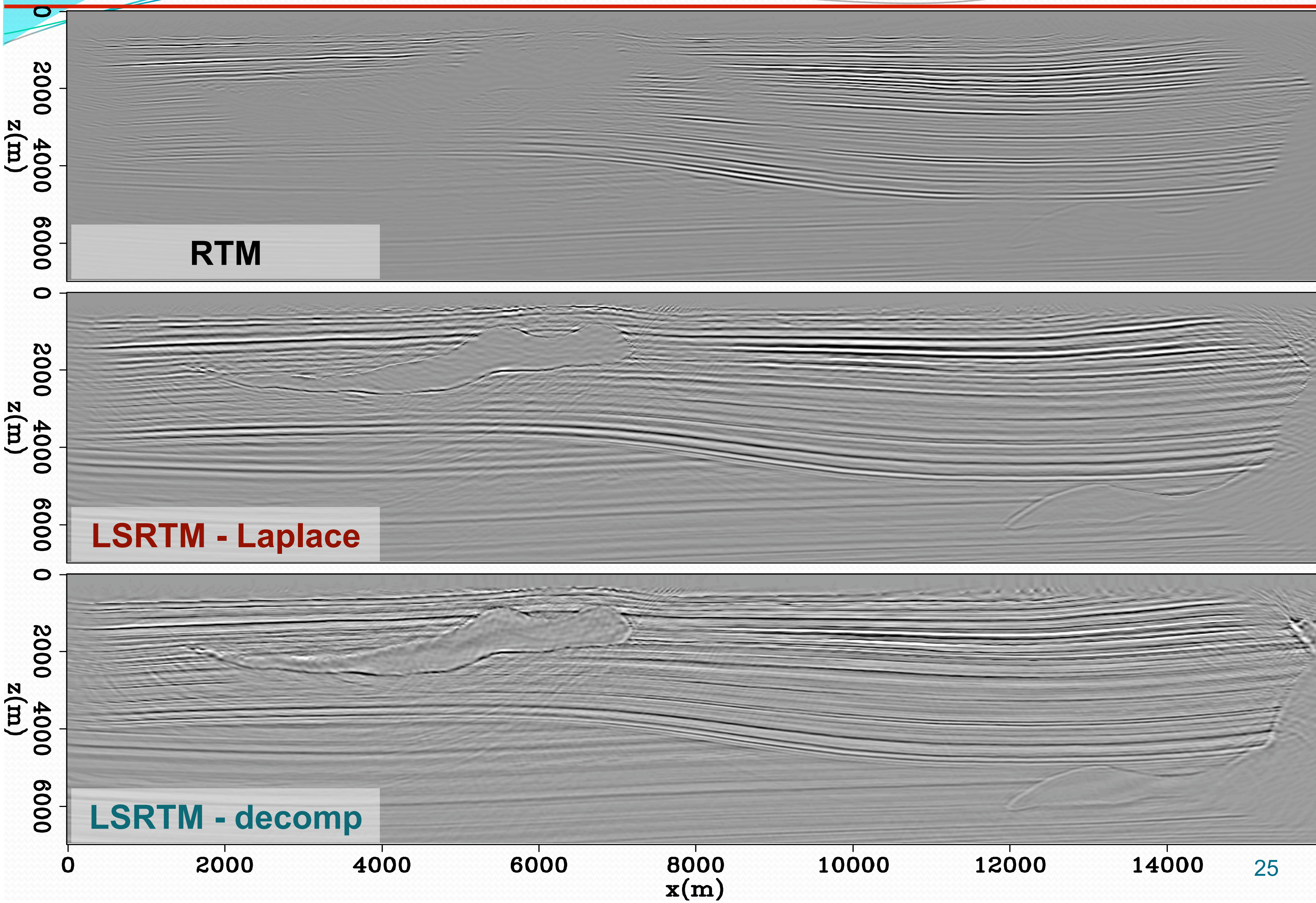
$$S(p) = \|W_s \left(\mathbf{LA}p - d_{\text{obs}}\right)\|^2$$

$$m = \mathbf{A}p$$

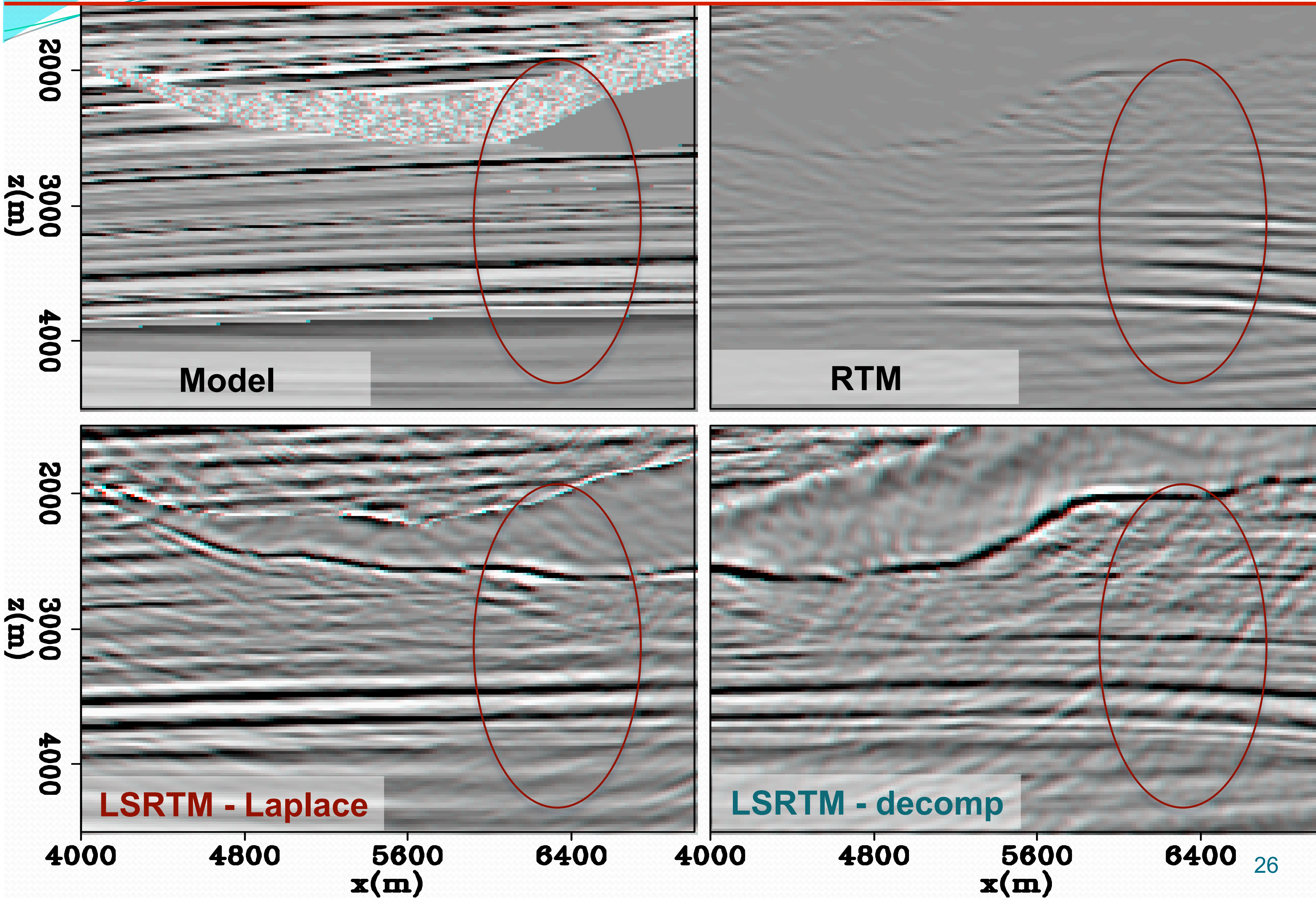
Test 2: LSRTM with wavefield decomposition (LSRTM-decomp)

$$S(m) = \|W_s \left(\mathbf{L}_D m - d_{\text{obs}}\right)\|^2$$

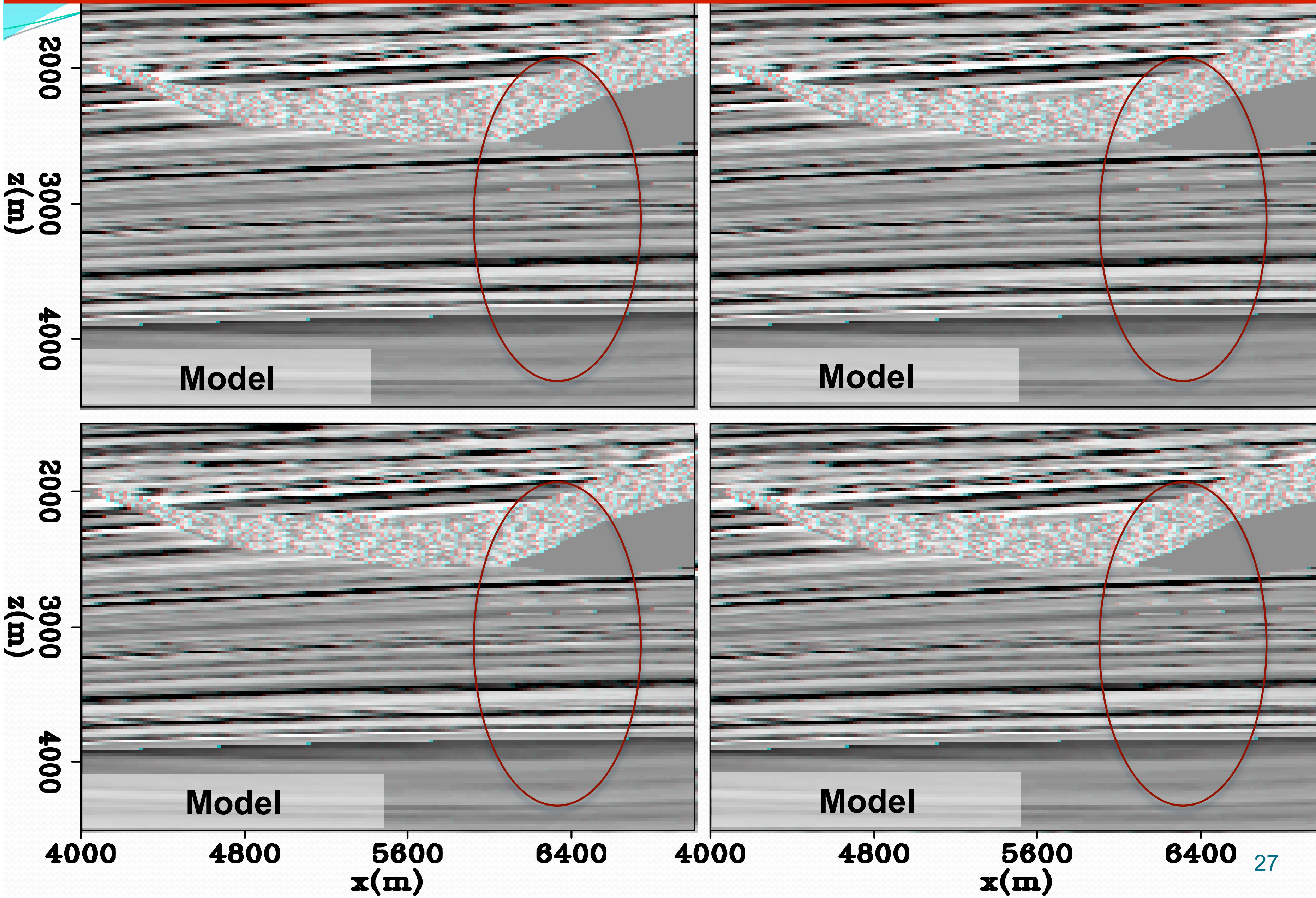
RTM and LSRTM results



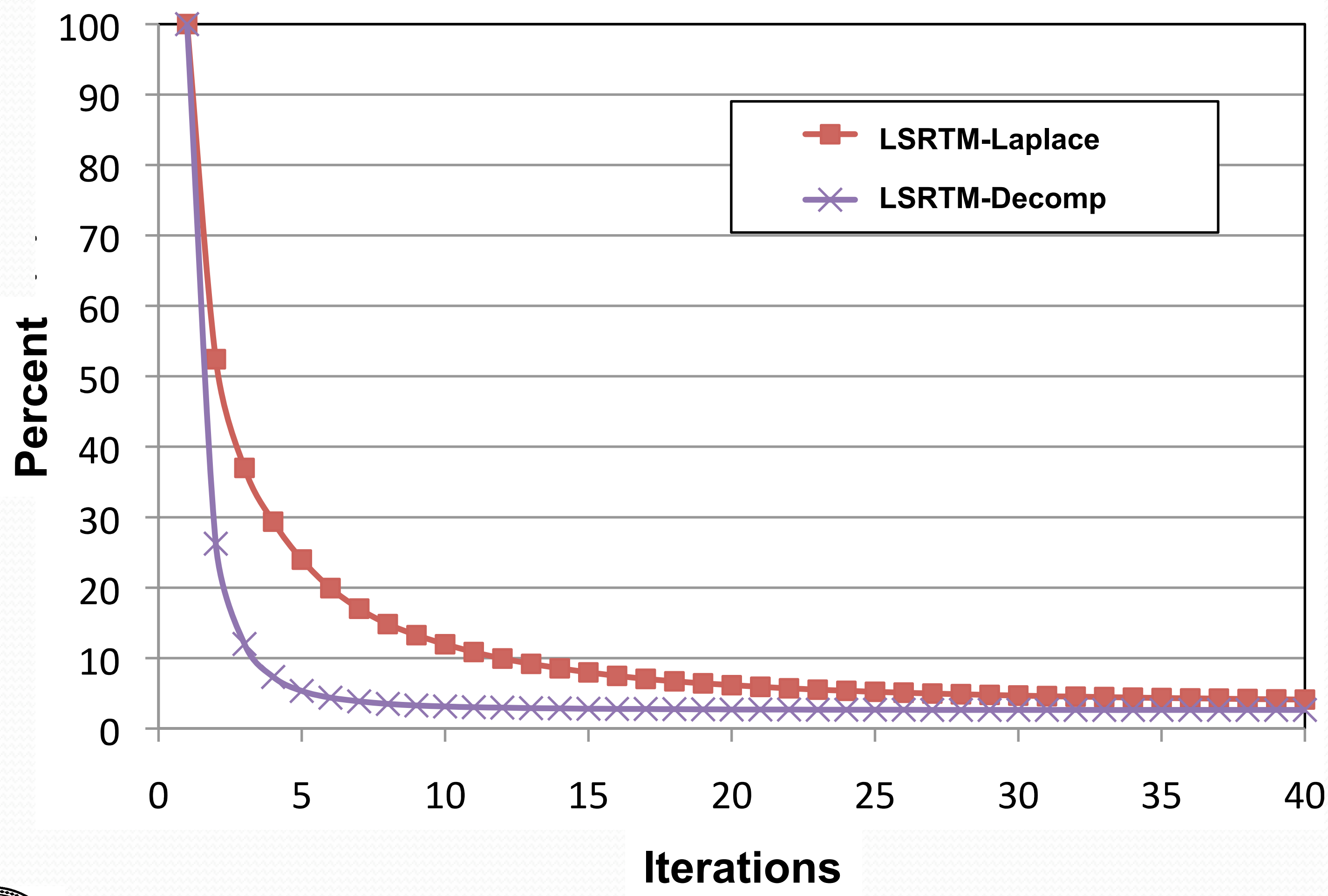
RTM and LSRTM results



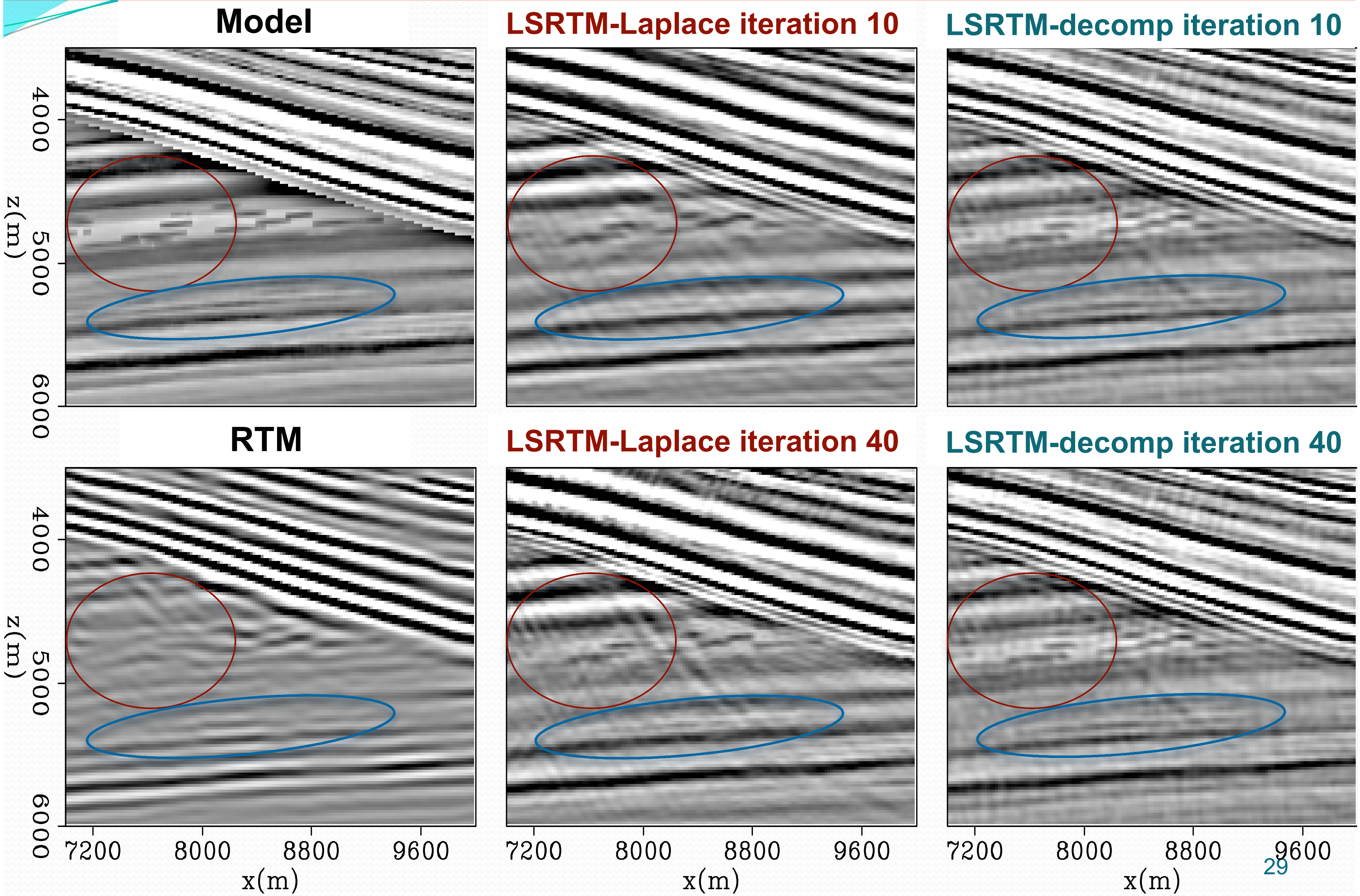
RTM and LSRTM results



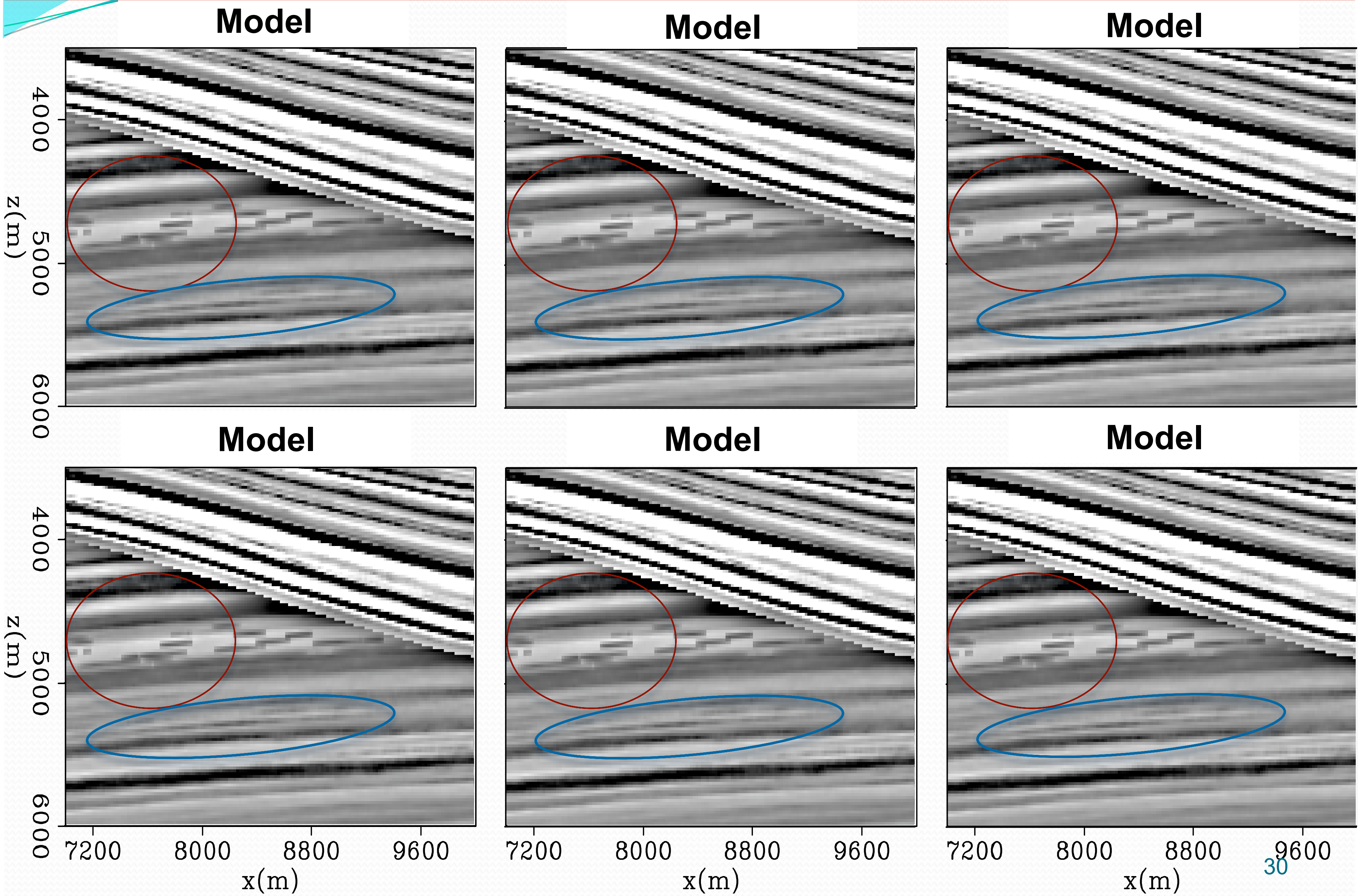
Convergence



LSRTM-decomp converges faster



LSRTM-decomp converges faster



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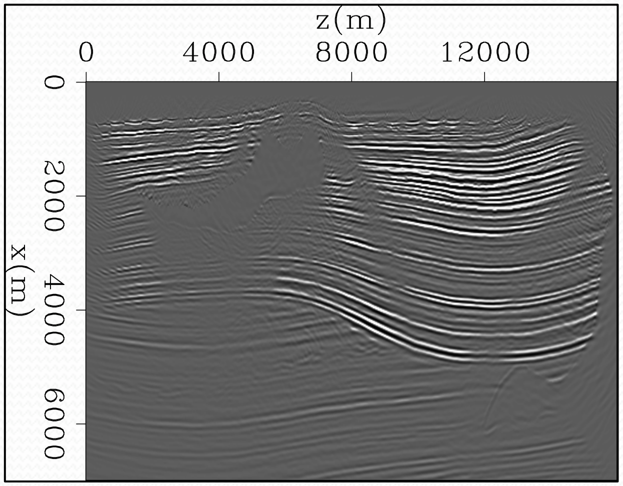
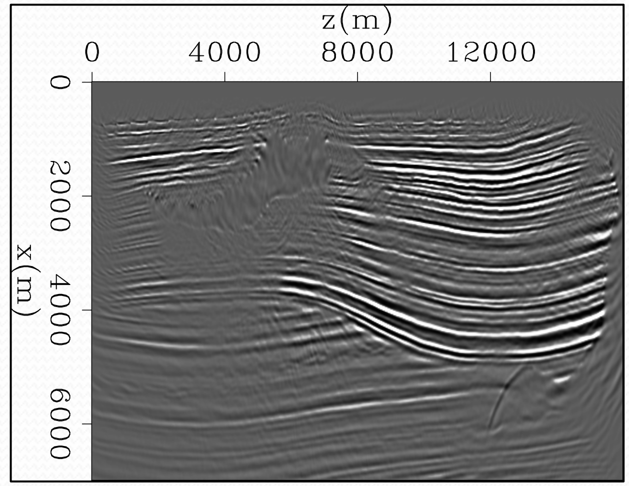
Why does the LSRTM-decomp converge faster than LSRTM-Laplace?

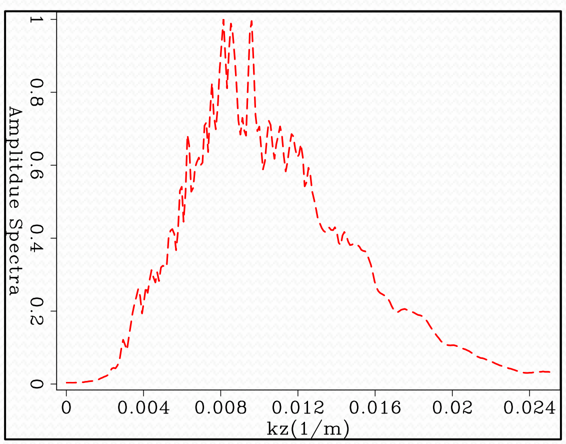
First gradient of LSRTM-precond

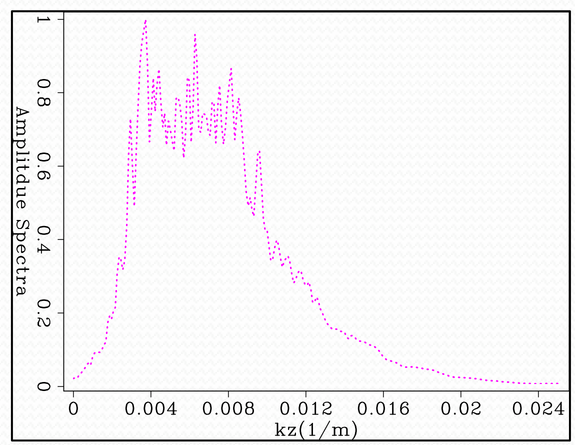


First gradient of LSRTM-decomp



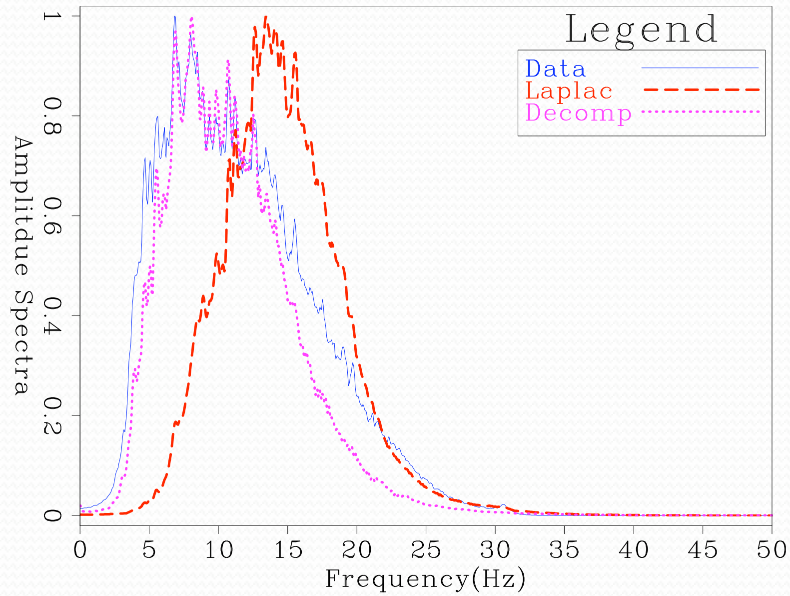
Spectrum in Z





Discussion

Why does the LSRTM-decomp converge faster than LSRTM-Laplace?



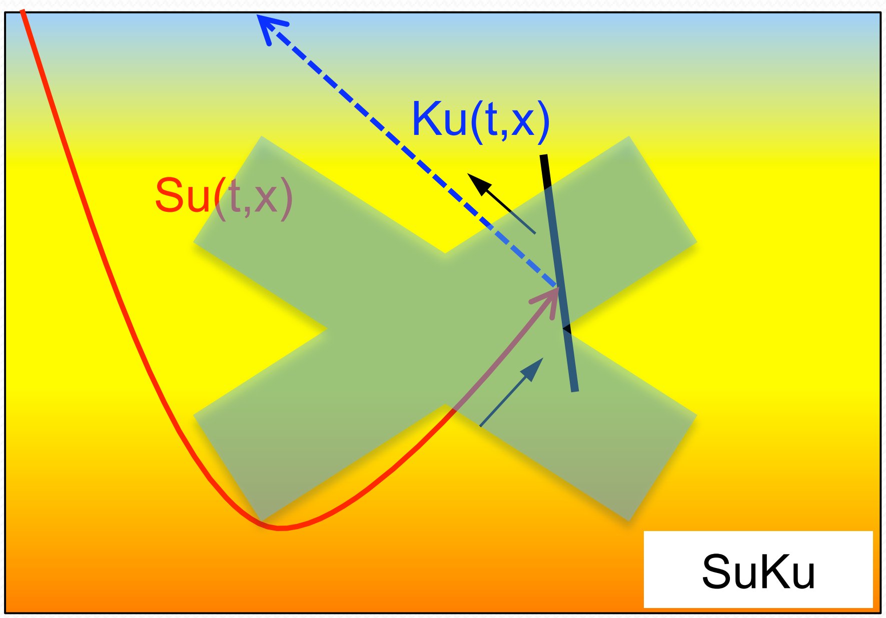
Perhaps incorporating a left preconditioner to balance the frequency

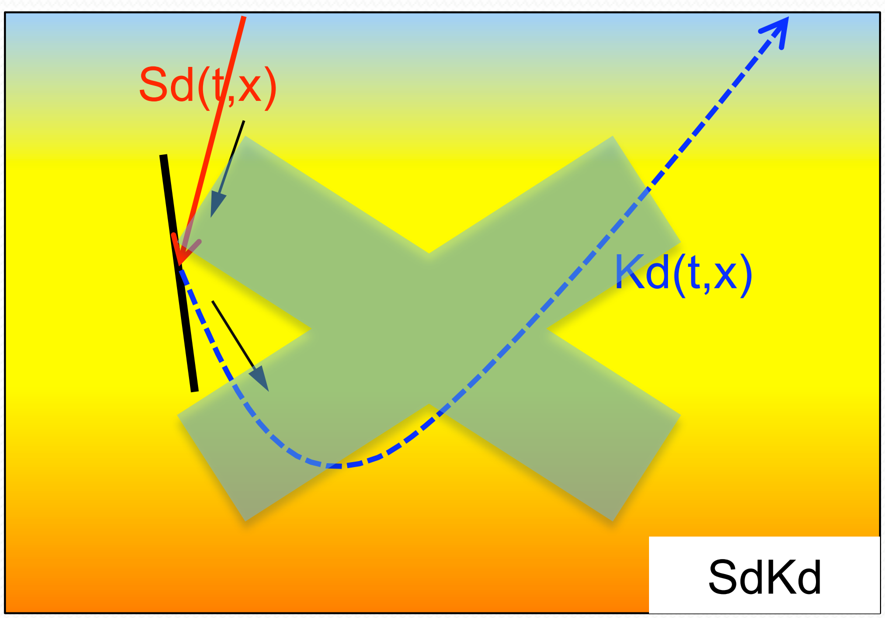
$$S(p) = \|W_s \mathbf{P}\left(\mathbf{L}\mathbf{A}p - d^{obs}\right)\|^2$$

Including the forward-scattering term

To preserve steeply dipping reflector, include the forward-scattering term for deeper region

$$m_{mig}(\mathbf{x}) = I_1(\mathbf{x}) + I_2(\mathbf{x}) + M_{back}(\mathbf{x})(I_3(\mathbf{x}) + I_4(\mathbf{x}))$$





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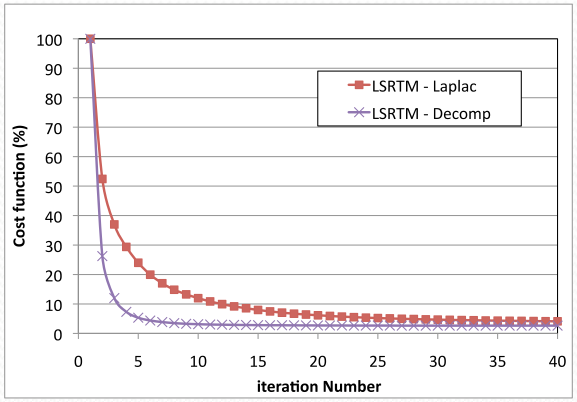
Conclusion

- **LSRTM with wavefield decomposition can effectively suppress RTM artifacts**
- **Results from the SEAM example show that LSRTM-decomp converges faster than LSRTM-Laplace.**



- **Computationally, it is viable too.**

Acknowledgement

- Kittinat Taweesintanon for previous work in SEP
- Biondo Biondi and Shuki Ronen for helpful discussions

Backup Slides

Modifying imaging condition using wavefield decomposition

- **Decomposition of wavefields based on vertical propagation directions:**
  - **upgoing and downgoing components**

$$S(t,\vec{x}) = S_{z+}(t,\vec{x}) + S_{z-}(t,\vec{x})$$

$$R(t,\vec{x}) = R_{z+}(t,\vec{x}) + R_{z-}(t,\vec{x})$$

$$I(\vec{x}) = \boxed{\sum_{t=0}^{t_{\max}} S_{z+}(t,\vec{x})R_{z-}(t,\vec{x}) + \sum_{t=0}^{t_{\max}} S_{z-}(t,\vec{x})R_{z+}(t,\vec{x})}$$

Back-scattering terms

$$+ \sum_{t=0}^{t_{\max}} S_{z+}(t,\vec{x})R_{z+}(t,\vec{x}) + \sum_{t=0}^{t_{\max}} S_{z-}(t,\vec{x})R_{z-}(t,\vec{x})$$

Forward-scattering terms

Wavefield decomposition in the F-K domain

- Decomposition in the F-K domain was first used in VSP data (Hu, 1987):
  - Vertically,

$$\tilde{P}_{z+}(f,k_z) = \begin{cases} \tilde{P}(f,k_z) & \text{for } fk_z \geq 0 \\ 0 & \text{for } fk_z < 0 \end{cases},$$

$$\tilde{P}_{z-}(f,k_z) = \begin{cases} 0 & \text{for } fk_z \geq 0 \\ \tilde{P}(f,k_z) & \text{for } fk_z < 0 \end{cases}$$

  - Horizontally,

$$\tilde{P}_{x+}(f,k_x) = \begin{cases} \tilde{P}(f,k_x) & \text{for } fk_x \geq 0 \\ 0 & \text{for } fk_x < 0 \end{cases},$$

$$\tilde{P}_{x-}(f,k_x) = \begin{cases} 0 & \text{for } fk_x \geq 0 \\ \tilde{P}(f,k_x) & \text{for } fk_x < 0 \end{cases}$$

Wavefield decomposition in the F-K domain

- FFT brings a complex-valued problem:
  - The initial wavefield is a real function, but the decomposed wavefields are complex; for example,

$$
\begin{aligned}
S(t,\vec{x}) &= S_{z+}(t,\vec{x}) + S_{z-}(t,\vec{x}), \\
&= \mathrm{Re}[S_{z+}(t,\vec{x})] + \mathrm{Re}[S_{z-}(t,\vec{x})]
\end{aligned}
$$

  - Previously, only real parts of decomposed wavefields were used in imaging conditions (Liu,2007,2011)

$$
I_{\mathrm{vert}}(\vec{x}) = \sum_{t=0}^{t_{\max}} S_{z+}(t,\vec{x}) R_{z-}(t,\vec{x}) + \sum_{t=0}^{t_{\max}} S_{z-}(t,\vec{x}) R_{z+}(t,\vec{x})
$$

  - This approximately gives the same result as

$$
I_{\mathrm{vert}}(\vec{x}) = \sum_{t=0}^{t_{\max}} S_{z+}^{*}(t,\vec{x}) R_{z-}(t,\vec{x}) + \sum_{t=0}^{t_{\max}} S_{z-}^{*}(t,\vec{x}) R_{z+}(t,\vec{x})
$$

Wavefield decomposition in the T-K domain

- Using Parseval's theorem:

$$\sum_{t=0}^{t_{\max}} S^*(t,\vec{x})R(t,\vec{x}) = \sum_{f=-f_N}^{f_N} \tilde{S}^*(f,\vec{x})\tilde{R}(f,\vec{x})$$

- The backscatter-based imaging condition can be written as (Liu, 2011)

$$I_{\text{vert}}(\vec{x}) = \sum_{t=0}^{t_{\max}} S^*_{k_z+}(t,\vec{x})R_{k_z-}(t,\vec{x}) + \sum_{t=0}^{t_{\max}} S^*_{k_z-}(t,\vec{x})R_{k_z+}(t,\vec{x})$$

- where

$$\tilde{P}_{k_z+}(t,k_z) = \begin{cases} \tilde{P}(t,k_z) & \text{for } k_z \geq 0 \\ 0 & \text{for } k_z < 0 \end{cases},$$

$$\tilde{P}_{k_z-}(t,k_z) = \begin{cases} 0 & \text{for } k_z \geq 0 \\ \tilde{P}(t,k_z) & \text{for } k_z < 0 \end{cases}$$

Some successful examples

- **LSM – using one-way operator**
  - Kuehl, H. and Sacchi M., 2002, Robust AVP estimation using least-squares wave-equation migration: SEG Technical Program Expanded Abstract, 21, 281
  - Clapp, M. R. Clapp, and B. Biondi, 2005, Regularized least-squares inversion for 3-D subsalt imaing: SEG Technical Expanded Abstract, 24, 1814-1817

- **LSRTM**
  - Dai, W., C. Boonyasiriwat, and G. Schuster, 2010 – 3D multi-source least-squares reverse time migration: SEG Technical Expanded Abstract, 29, 3120-3124
  - Wong, M., Biondo B., and Ronen S., 2010, Joint inversion of up- and down-going signal for ocean bottom data, SEG Expanded Abstracts,
  - Dai, W., X. Wang, and G. Schuster, 2011, Least-squares migration of multisource data with a deblurring filter: Geophysics, 76, R135-R146
  - Gang, Y. and Jakubowicz M., 2012 – Least-squares Reverse-Time Migration
  - Dong et al., 2012, Least-squares reverse time migration: towards true amplitude imaging and improving the resolution
  - Wong, M., Biondo B. and Ronen S., 2012, Imaging with multiples using linearized full-wave inversion