

Reflections on  
reflection off a 3D plane  
+  
some useful nuggets

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SEP-148 & 149

Normally, I wouldn't risk boring you with constant velocity ray geometry, even if 3D, but as Sergey Fomel found it worthwhile, I'll take my chances.

## Why reflections?

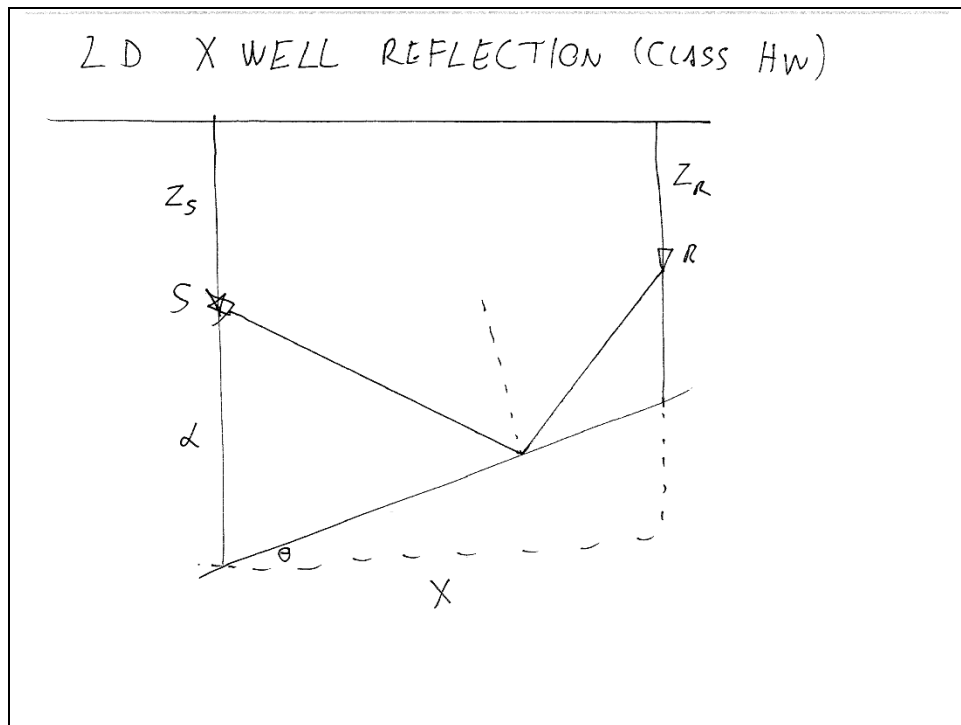
SEP-147: Exploring when low-order interpolation works perfectly well for seismic processes involving large summations, e.g. stacking and migration. (Compression?)

SEP-148: Mathematics behind a couple of SEP-147 experiments

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So where did this all come from? Last year, I explored the opportunities for speeding up summation operators in geophysics by using low order or even “no order” interpolation. The idea is that the worst case behavior is rarely the norm and, in most settings, average case behavior is what is important.

My numerical experiments included NMO stack, CRS, and post-stack and pre-stack 3D Kirchhoff migration. For all of these, I endeavored to simulate the results for reflection off of an arbitrarily-tilted 3D plane and addressed the details of such ray geometry in SEP-148.



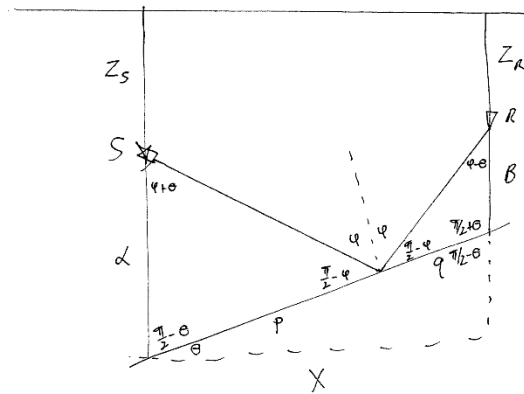
Nonzero offset reflection off a dipping plane is a venerable calculation. The 2D crosswell version of this calculation, depicted on this slide, was given to students in the borehole geophysics class taught by Jerry Harris and took me less time to work out than to look up in textbooks.

### 2D X WELL REFLECTION (CLASS HW)

The diagram illustrates the reflection of a wave in a 2D potential well. The horizontal axis is labeled  $x$  and the vertical axis is labeled  $z$ . The well boundaries are at  $z_S$  (left) and  $z_R$  (right). An incident wave  $S$  is shown with a phase  $\varphi + \theta$  and an angle  $\frac{\pi}{2} - \theta$  with the  $x$ -axis. A reflected wave  $R$  is shown with a phase  $\varphi$  and an angle  $\frac{\pi}{2} - \theta$  with the  $x$ -axis. A transmitted wave  $B$  is shown with a phase  $\frac{\pi}{2} + \theta$  and an angle  $\frac{\pi}{2} - \theta$  with the  $x$ -axis. The angle between the incident and reflected waves is  $\pi - 2\theta$ . The angle between the incident and transmitted waves is  $\pi - \theta$ . The angle between the reflected and transmitted waves is  $\pi - \theta$ . The angle between the incident wave and the  $z$ -axis is  $\frac{\pi}{2} - \theta$ . The angle between the reflected wave and the  $z$ -axis is  $\frac{\pi}{2} - \theta$ . The angle between the transmitted wave and the  $z$ -axis is  $\frac{\pi}{2} - \theta$ . The angle between the incident wave and the  $x$ -axis is  $\frac{\pi}{2} - \theta$ . The angle between the reflected wave and the  $x$ -axis is  $\frac{\pi}{2} - \theta$ . The angle between the transmitted wave and the  $x$ -axis is  $\frac{\pi}{2} - \theta$ .

Like the typical student, I labeled various unknown angles and lengths, imposed equal angles of incidence and reflection, and picked some trigonometric relation to solve.

# 2D X WELL REFLECTION (CLASS HW)



LAW OF SINES:

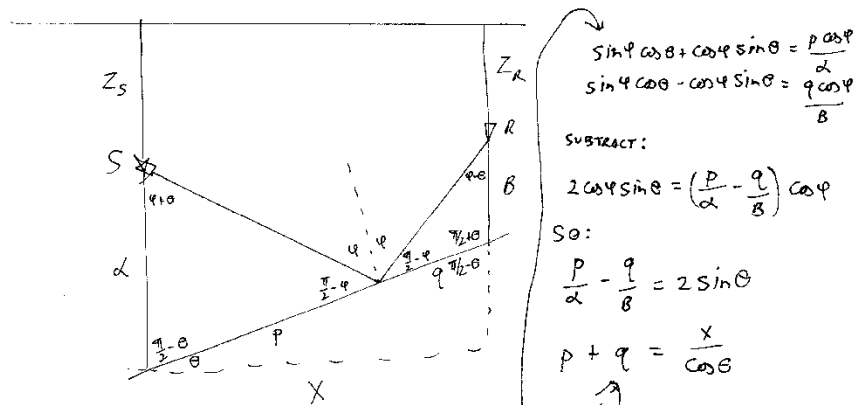
$$\frac{\sin(\phi + \theta)}{p} = \frac{\sin(\frac{\pi}{2} - \phi)}{\alpha}$$

$$\frac{\sin(\phi - \theta)}{q} = \frac{\sin(\frac{\pi}{2} - \phi)}{\beta}$$

$$p + q = x / \cos \theta$$

In this slide, the trigonometric relation I chose was the law of sines.

# 2D X WELL REFLECTION (CLASS HW)



$$\sin \varphi \cos \theta + \cos \varphi \sin \theta = \frac{p \cos \varphi}{\alpha}$$

$$\sin \varphi \cos \theta - \cos \varphi \sin \theta = \frac{q \cos \varphi}{\beta}$$

SUBTRACT:

$$2 \cos \varphi \sin \theta = \left( \frac{p}{\alpha} - \frac{q}{\beta} \right) \cos \varphi$$

SO:

$$\frac{p}{\alpha} - \frac{q}{\beta} = 2 \sin \theta$$

$$p + q = \frac{x}{\cos \theta}$$

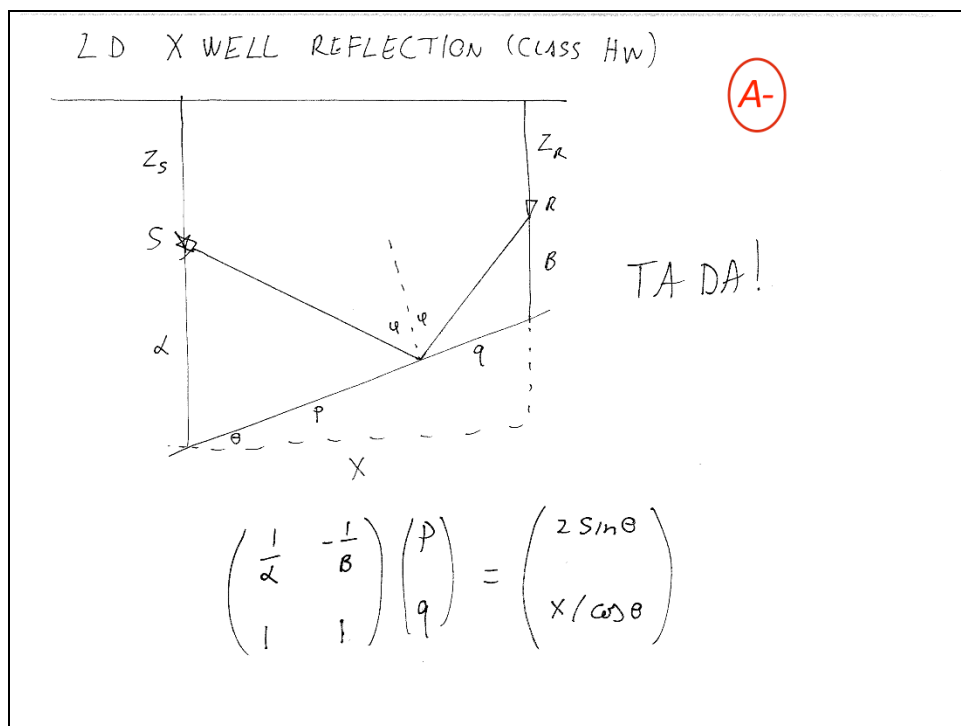
LAW OF SINES:

$$\frac{\sin(\varphi + \theta)}{p} = \frac{\sin(\frac{\pi}{2} - \varphi)}{\alpha}$$

$$\frac{\sin(\varphi - \theta)}{q} = \frac{\sin(\frac{\pi}{2} - \varphi)}{\beta}$$

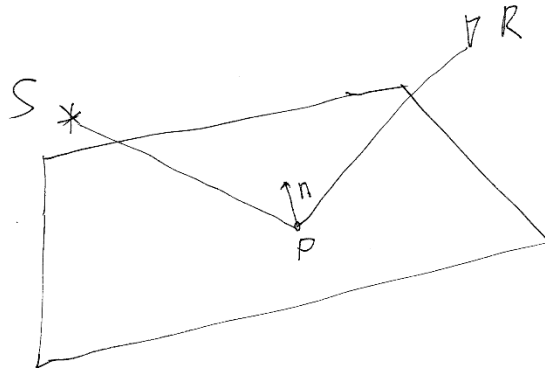
$$(p + q = x / \cos \theta)$$

Expanding and simplifying serendipitously exposed a way to eliminate the unknown angle of reflection and yield two simultaneous linear equations in two unknowns.



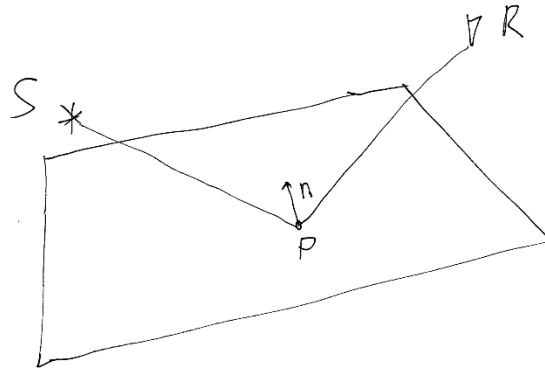
Voila. Or Ta Da. I'd give this solution an A-, technically competent and clean enough, but somewhat more lucky than elegant, ignoring, for example, the method of images.

### 3D X WELL REFLECTION (EXTRA CREDIT)



Of course, in my case, I needed to solve the 3D problem, which I've termed extra credit.

### 3D X WELL REFLECTION (EXTRA CREDIT)



EXTRA COMPLEXITY

AZIMUTH OF PLANE DIP

$\triangle$  SRP  $\perp$  TO PLANE

The 3D version has, of course, extra complexity, i. e. extra angles and equations. My initial attempt at solution was to peruse the classic Slotnick tome and search the internet where I found a calculation by Fomel with a bunch of direction cosines. Due to the nearly 50 combinations of axis, angle, and sing permutations, I found both confusing to implement in my code, so I attempted to rederive it following the lines of the 2D model calculation.

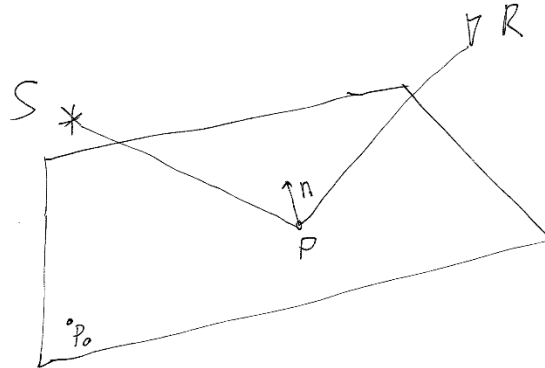
After a dozen pages of algebra and  
trigonometry ...

**OMG**

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Didn't get very far with that approach except to fill up our paper recycling basket. So I put that aside and coded up results for a plane dipping only along the X axis.

### 3D X WEL REFLECTION (EXTRA CREDIT)



A PLANE CAN BE DESCRIBED BY A NORMAL VECTOR  
AND ANY FIXED POINT ON THE PLANE:

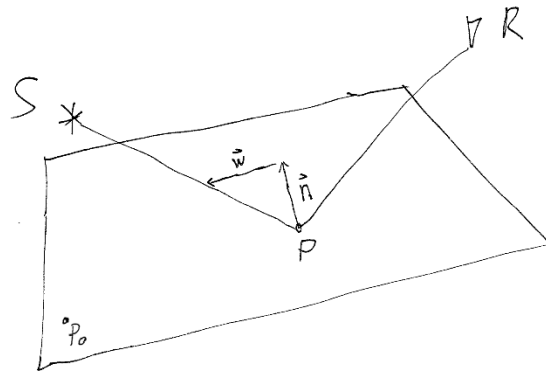
$$\vec{n} \cdot (P - P_0) = 0$$

Later on, I came back to the full 3D calculations and started from scratch, determined not to use actual coordinates and/or angles until I could figure out exactly where they were needed. Turned out they weren't needed at all.

To show you how simple the derivation became, I am going to break one of my cardinal rules: Don't do math in public!

So here goes: A plane may be described by an arbitrary point in the plane and a unit normal to the plane.

### 3D X WELL REFLECTION (EXTRA CREDIT)

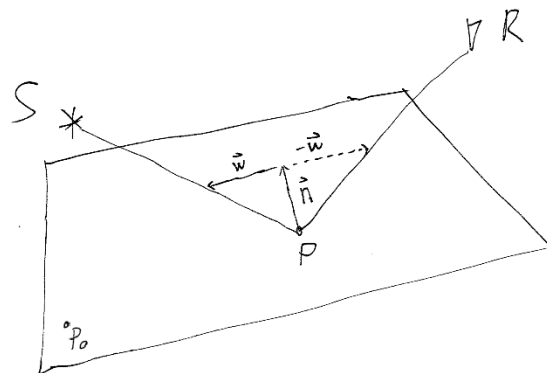


ELEGANT CONSTRUCTION:

DROP  $\perp \vec{w}$  FROM  $\vec{n}$  TO LINE  $S-P$

The key step is to drop a perpendicular from the normal at the to-be-determined reflection point to the line connecting the source to the reflection point.

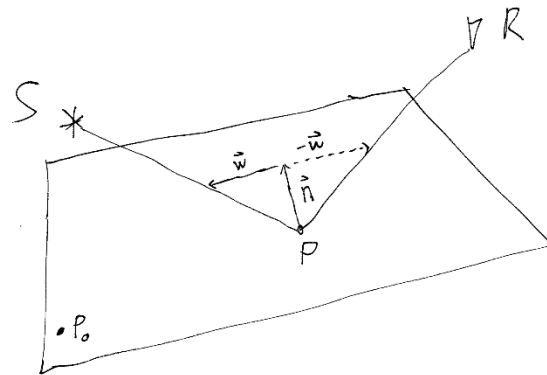
### 3 D X WELL REFLECTION (EXTRA CREDIT)



SNELL'S LAW SAYS  $-\vec{w}$  CONNECTS  $\vec{n}$  TO  $R-P$

Snell's Law immediately says that flipping  $w$  in the other direction connects to the line from the receiver to the reflection point.

# 3D X WELL REFLECTION (EXTRA CREDIT)

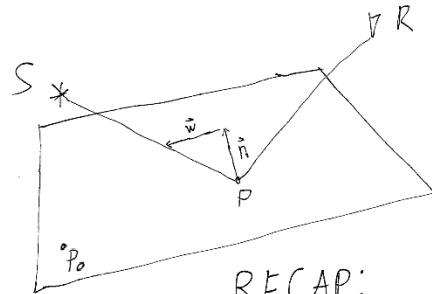


SNELL'S LAW SAYS  $-\vec{w}$  CONNECTS  $\vec{n}$  TO  $R-P$

SO:  $S-P = \alpha(\vec{n} + \vec{w})$      $R-P = \beta(\vec{n} - \vec{w})$   
 FOR SOME  $\alpha$  &  $\beta$  TBD

Hence  $\vec{n} + \vec{w}$  and  $\vec{n} - \vec{w}$  are parallel to  $S-P$  and  $R-P$  respectively and so related by unknown scalar multipliers, here denoted by alpha and beta.

### 3D XWELL REFLECTION (EXTRA CREDIT)



RECAP:

$$S - P = \alpha (\vec{n} + \vec{w})$$

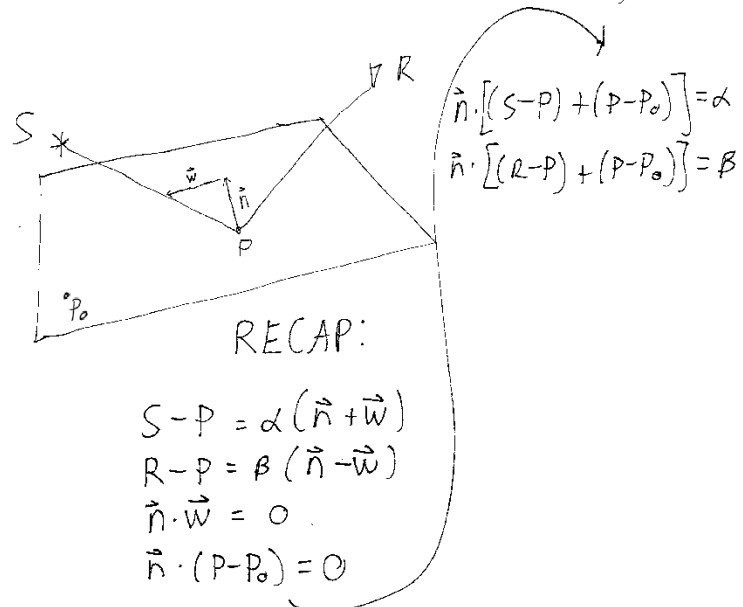
$$R - P = \beta (\vec{n} - \vec{w})$$

$$\vec{n} \cdot \vec{w} = 0$$

$$\vec{n} \cdot (P - P_0) = 0$$

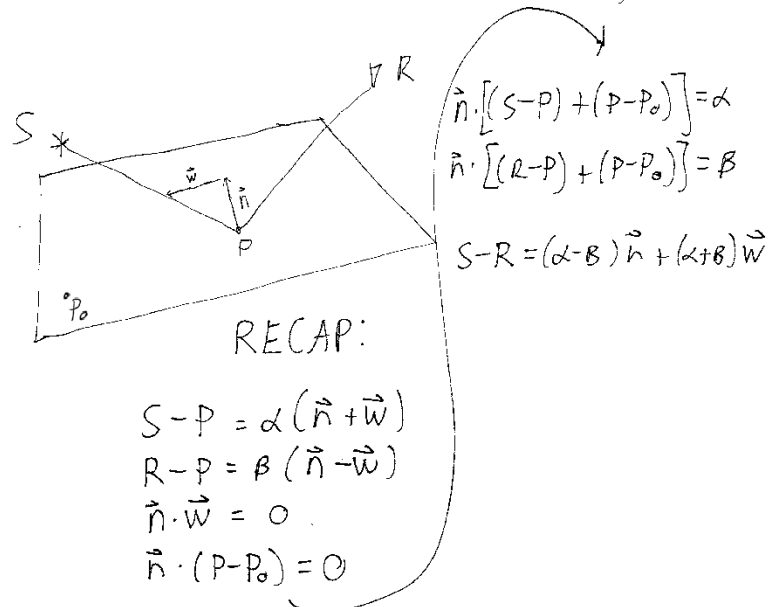
Recapping, we now have four relations, albeit in five apparent unknowns.

### 3D X WELL REFLECTION (EXTRA CREDIT)



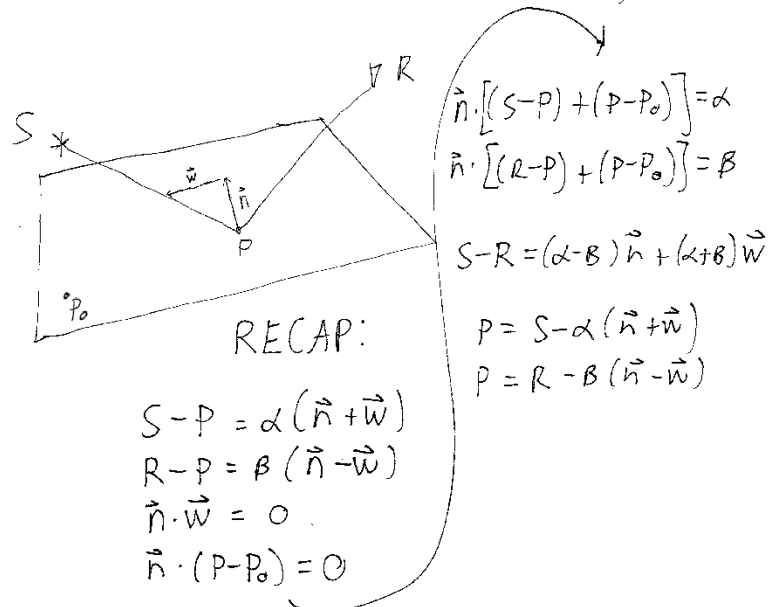
Fortunately, dotting the normal  $\vec{n}$  with each of the first two and adding the last to each gives us  $\alpha$  and  $\beta$  purely in terms of known input parameters.

### 3D X WELL REFLECTION (EXTRA CREDIT)



Subtracting the first two equations now gives us  $w$  in terms of  $\alpha$ ,  $\beta$ , and known input parameters.

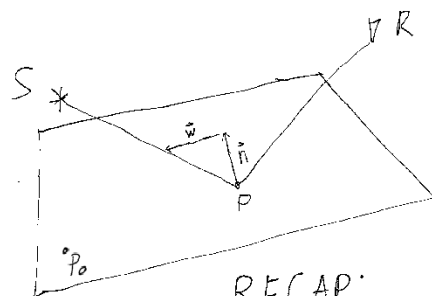
### 3D X WELL REFLECTION (EXTRA CREDIT)



Finally, the reflection point can be read off from either of the two first relations.

# 3D X WELL REFLECTION (EXTRA CREDIT)

A+



RECAP:

$$S - P = \alpha (\vec{n} + \vec{w})$$

$$R - P = \beta (\vec{n} - \vec{w})$$

$$\vec{n} \cdot \vec{w} = 0$$

$$\vec{n} \cdot (P - P_0) = 0$$

$$\vec{n} \cdot [(S - P) + (P - P_0)] = \alpha$$

$$\vec{n} \cdot [(R - P) + (P - P_0)] = \beta$$

$$S - R = (\alpha - \beta) \vec{n} + (\alpha + \beta) \vec{w}$$

$$P = S - \alpha (\vec{n} + \vec{w})$$

$$P = R - \beta (\vec{n} - \vec{w})$$

$$P = \underbrace{\frac{S+R}{2}}_{\text{MIDPOINT}} - \frac{\alpha+\beta}{2} \vec{n} - \frac{\alpha-\beta}{2} \vec{w}$$

Or, if you like, their average produces that location in terms of the source-receiver midpoint.

Look ma, no coordinates, no angles!

I'd give this extra credit an A, well, perhaps, since I'm grading my own work, and A+.

## SEP-148 Extensions

- Converted waves
- Map migration/demigration

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You may look in the SEP 148 article for extensions to converted wave reflection and map migration/demigration.

## Conclusions

- Vector notation shines in 3D
- May yield coordinate-free algorithms

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Overall, I trust I've demonstrated that vector notation provides exceptional clarity in higher dimensional problems and may, at times, obviate the need to impose any *a priori* coordinate system.

## But wait, there's more!

- ProMAX<sup>®</sup> SEP3D Output
  - Exports ProMAX data volume to SEP/Madagascar format
- A Slotnick gem

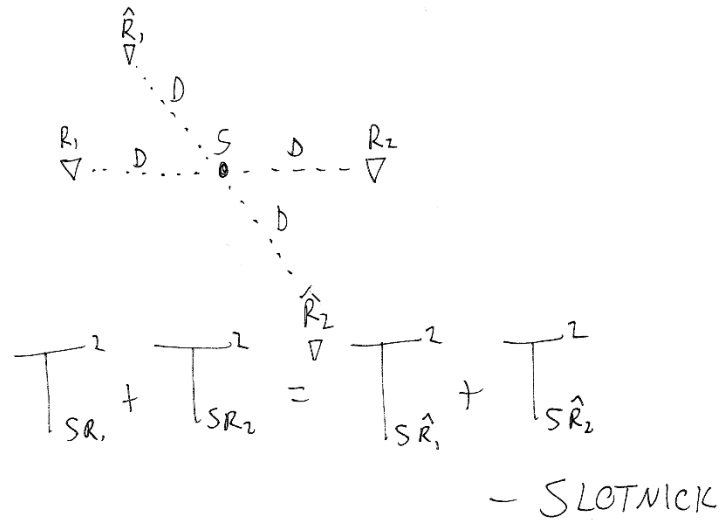
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Before I leave you, I want to call your attention to two other items. The first, appearing in the SEP-149 volume you have in your hands, is a utility for exporting ProMAX/SeisSpace data to modern SEP format. This has been provided to Landmark for inclusion in an upcoming release, replacing the broken and obsolete SEP Output module in the current release.

Finally, while digging through Slotnick, I came across a slick result that yields velocity from 2D seismic in an azimuth-independent way. With the development of CDP-based method and migration velocity analysis, this is most probably not familiar to working geophysicists today.

BEFORE THERE WAS DMO:

AZIMUTH & DIP INDEPENDENT VELOCITY

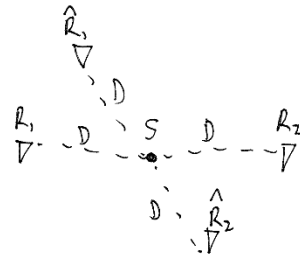
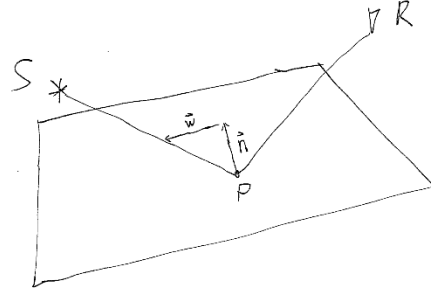
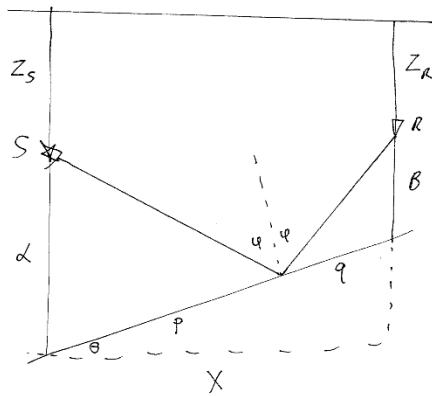


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The Slotnick result, easily proven using Apollonius' Theorem, says that the sum of the squared reflection traveltimes to receivers at equal distance and diametrically opposite the source is azimuth independent for any dipping plane in a constant velocity medium.

While the real earth is not constant velocity, this result also applies locally after downward continuation and may serve to reduce computation or accelerate convergence by separating structure determination from velocity estimation. Worth thinking about, I'd say.

Thank you for your attention – Feedback?



ProMAX® SEP3D Output

So, thank your for your attention and sticking it out through the very last talk of the day. I'll be happy now to entertain questions.