Anti-noise Wave-equation Traveltime Inversion and Application to Salt Estimation

Jun 18th, 2013

SEP149-69

Xukai Shen

Acknowledgement

1. Dr. Yi Luo from Saudi Aramco for discussions that lead to this project
2. Sponsors of SEP for the financial support

Outline

- Motivation
- Theory
- Salt estimation
- Conclusion

Troubleshooting philosophy

“When you have eliminated the impossible, whatever remains, however improbable, must be the truth”

- Sherlock Holmes

Ray based tomography

"When you have eliminated the impossible, whatever remains, however improbable, must be the truth"

- Sherlock Holmes

Repeat:

1. **Tracing rays from all sources to receivers**
2. **Measure traveltime differences**
3. **Distribute traveltime differences evenly on the rays as gradient.**
4. **Calculate step-length and update velocity model**

Ray based tomography troubleshooting

"When you have **eliminated the impossible, whatever remains, however improbable, must be the truth**"

- Sherlock Holmes

Repeat:

1. **Tracing rays from all sources to receivers**
2. **Measure traveltime differences**
3. **Distribute traveltime differences evenly on the rays as gradient.**
4. **Calculate step-length and update velocity model**

Wave-equation Traveltime Inversion(WTI)

"When you have eliminated the impossible, whatever remains, however improbable, must be the truth"

- Sherlock Holmes

Repeat:

1. **Forward modeling of synthetic data**
2. **Measure traveltime differences**
3. **Reverse Time Migration of virtual source to form gradient.**
4. **Calculate step-length and update velocity model**

WTI trouble shooting

"When you have **eliminated the impossible, whatever remains, however improbable, must be the truth**"

- Sherlock Holmes

Repeat:

1. **Forward modeling of synthetic data**
2. **Measure traveltime differences**
3. **Reverse Time Migration of virtual source to form gradient.**
4. **Calculate step-length and update velocity model**

Preferred WTI trouble shooting

"When you have **eliminated the impossible, whatever remains, however improbable, must be the truth**"

- Sherlock Holmes

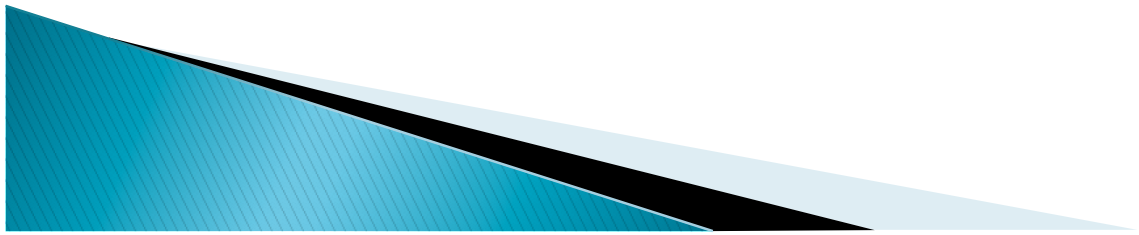
Repeat:

1. **Forward modeling of synthetic data**
2. **Measure traveltime differences**
3. **Reverse Time Migration of virtual source to form gradient.**
4. **Calculate step-length and update velocity model**

Motivation

Explore the possibility of WTI that's :

- **Given a reasonable starting model, the only failure of real data application comes from errors in traveltime difference measurements.**



Outline

- Motivation
- Theory
- Salt estimation
- Conclusion

Objective function

$$\min f(\mathbf{m}) = \sum_{\mathbf{x_s},\mathbf{x_r}} \Delta\tau^2(\mathbf{m}, \mathbf{x_s}, \mathbf{x_r})$$

(Luo and Schuster, 1991)

$\Delta\tau$	First break traveltime difference between observed data and modeled data
$\mathbf{m}$	Near surface model
$\mathbf{x_s}, \mathbf{x_r}$	Source, receiver location

Back propagation term

$$\delta d_{vsrc} = \Delta\tau \frac{\dot{d}_{obs}(t+\Delta\tau)}{\dot{d}_{obs}(t+\Delta\tau) * \dot{d}_{cal}(t)}$$

(Luo and Schuster, 1991)

δd_{vsrc}	Back propagation term
$\dot{d}$	Time derivative
$*$	Cross-correlation

Back propagation term

$$\delta d_{vsrc} = \Delta\tau \frac{\dot{d}_{obs}(t+\Delta\tau)}{\dot{d}_{obs}(t+\Delta\tau) * \dot{d}_{cal}(t)}$$

(Luo and Schuster, 1991)

Noise source #1

δd_{vsrc}	Back propagation term
$\dot{d}$	Time derivative
$*$	Cross-correlation

Back propagation term

$$\delta d_{vsrc} = \Delta\tau \frac{\dot{d}_{obs}(t+\Delta\tau)}{\dot{d}_{obs}(t+\Delta\tau) * \dot{d}_{cal}(t)}$$

(Luo and Schuster, 1991)

Noise source #2

δd_{vsrc}	Back propagation term
$\dot{d}$	Time derivative
$*$	Cross-correlation

Modified back propagation term

$$\dot{d}_{obs}(t+\Delta\tau) \approx \dot{d}_{cal}(t)$$

$$\delta d'_{vsrc} = \Delta\tau \frac{\dot{d}_{cal}(t)}{\dot{d}_{cal}(t) * \dot{d}_{cal}(t)}$$

δd_{vsrc}	Back propagation term
$\dot{d}$	Time derivative
$*$	Cross-correlation

Corresponding gradient

$$G = \Delta\tau G_w$$

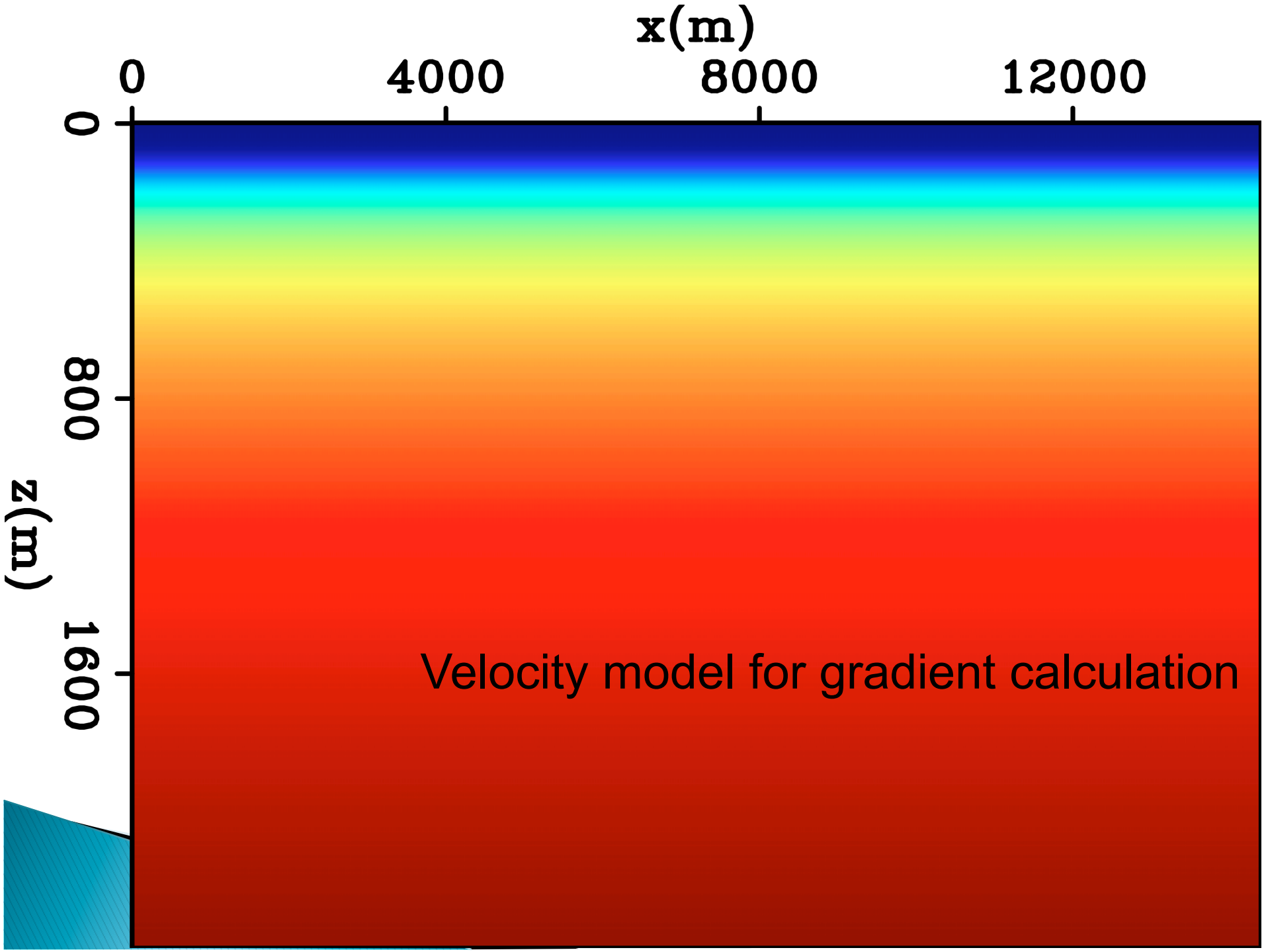
Reverse Time Migration of $\dfrac{\dot{d}_{cal}(t)}{\dot{d}_{cal}(t) * \dot{d}_{cal}(t)}$

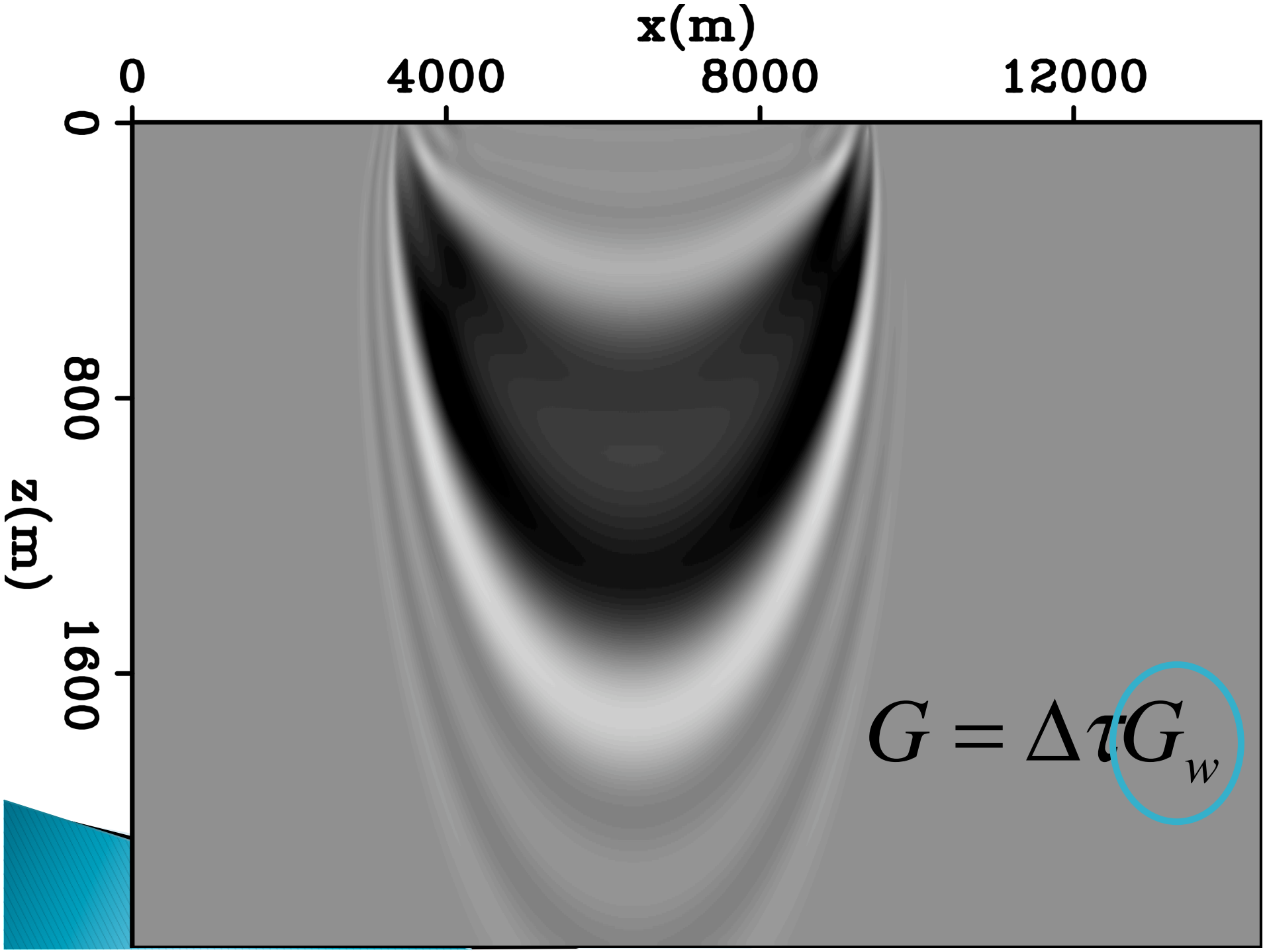
Corresponding gradient

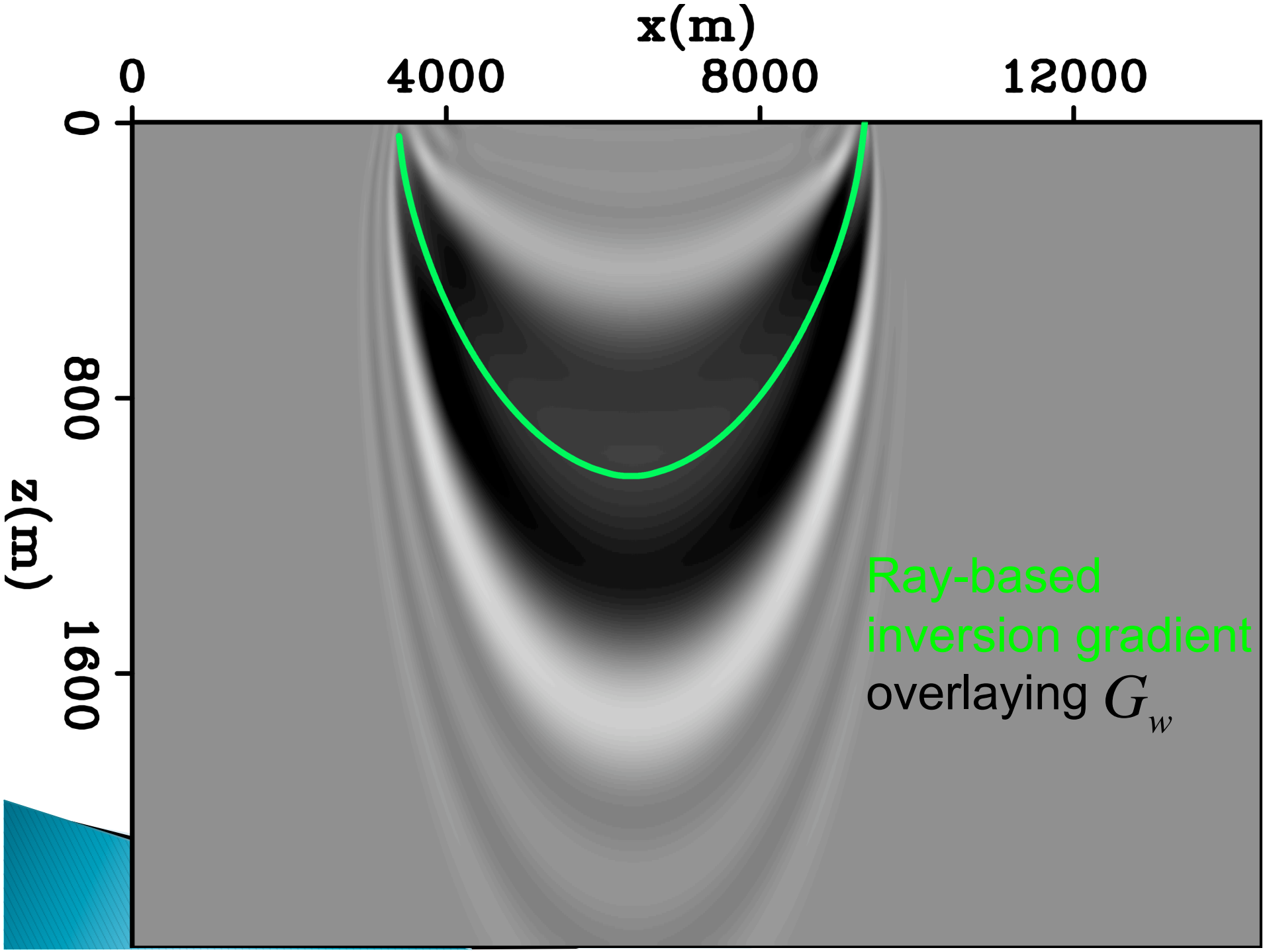
$$G = \Delta\tau G_w$$

Reverse Time Migration of $\dfrac{\dot{d}_{cal}(t)}{\dot{d}_{cal}(t) * \dot{d}_{cal}(t)}$

Completely synthetic kernel

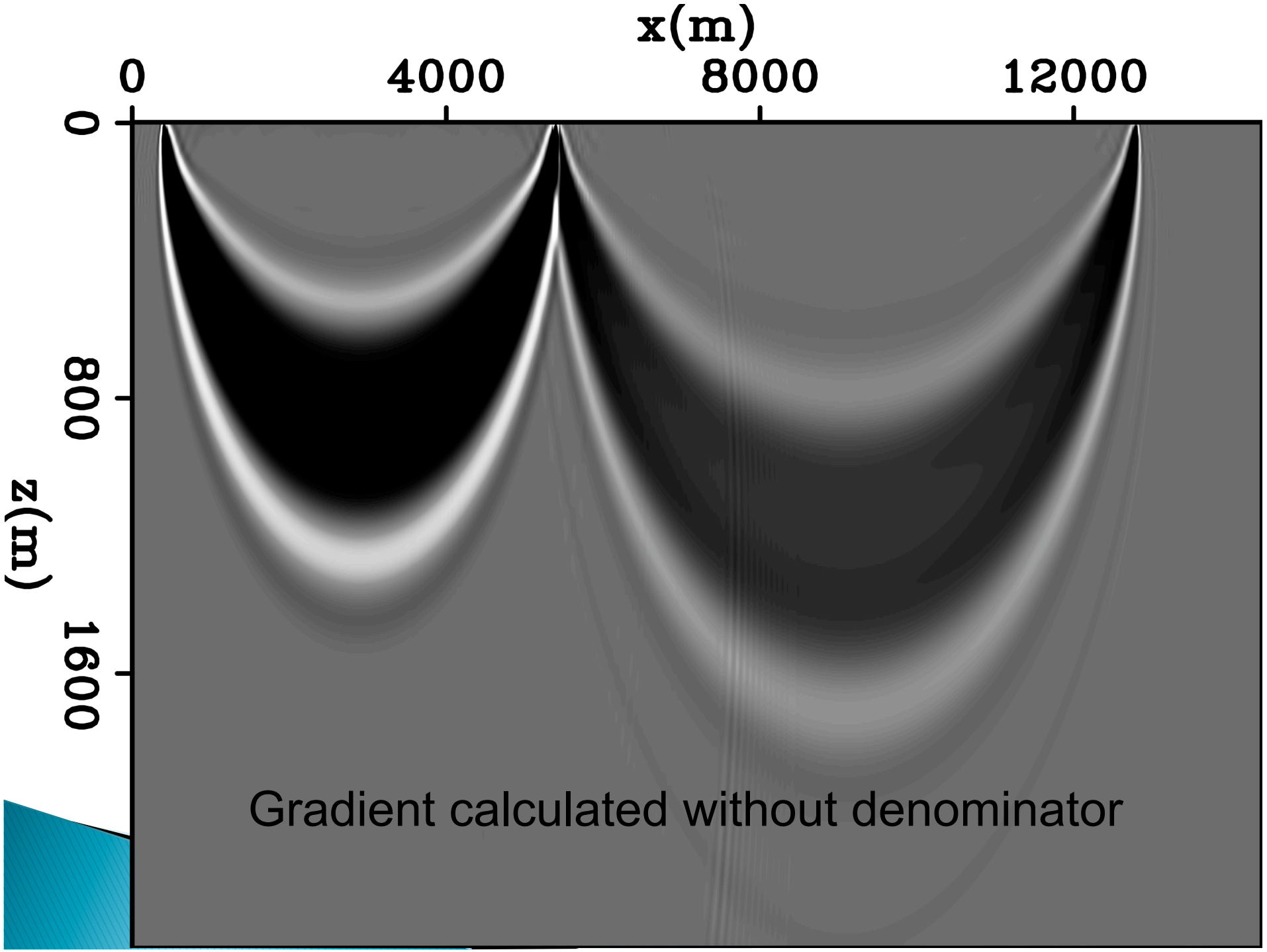

x(m)
0
4000
8000
12000
0
800
1600
z(m)
Velocity model for gradient calculation

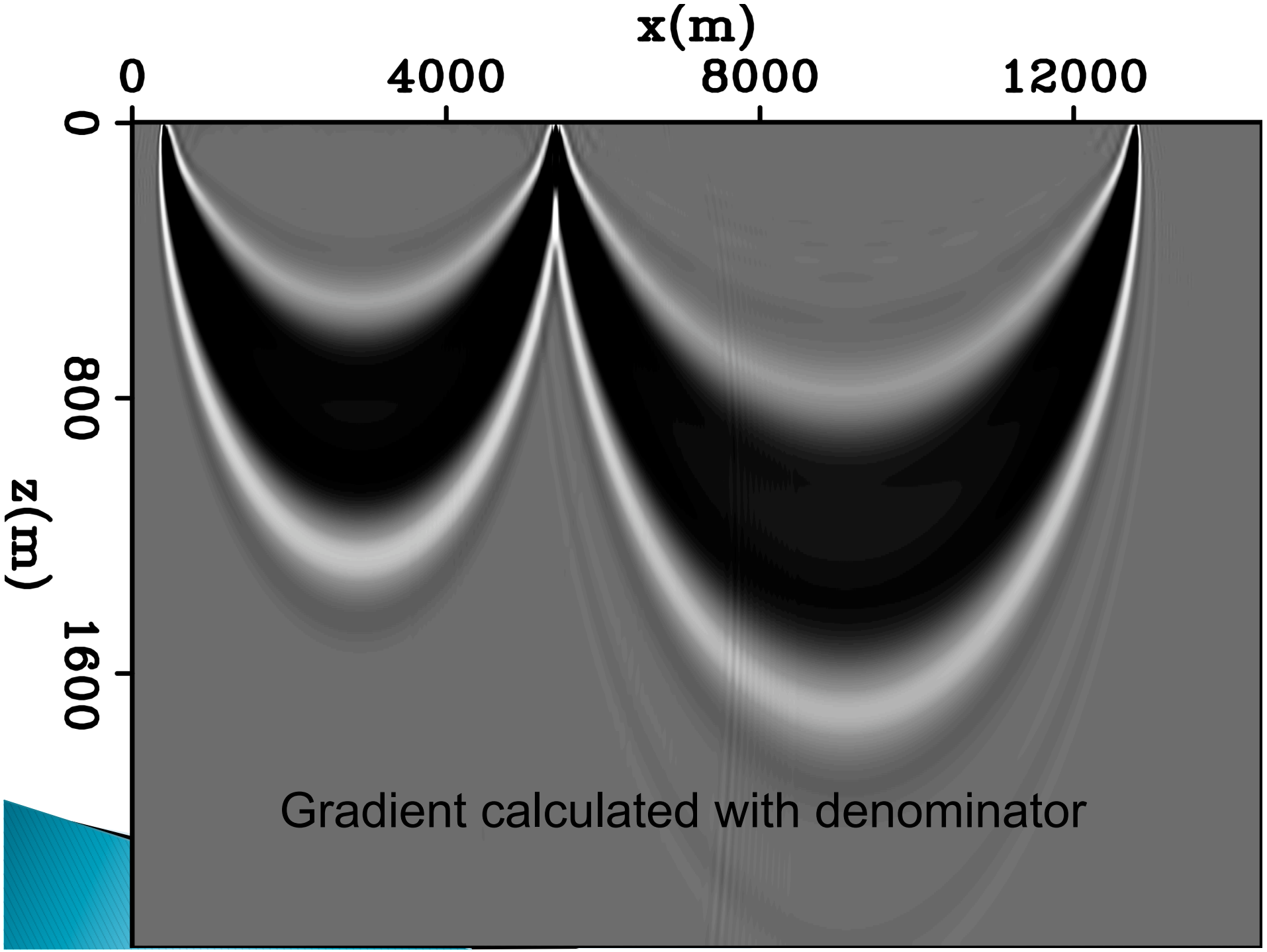

x(m)
0
4000
8000
12000
0
800
1600
z(m)
$G = \Delta\tau G_w$


x(m)
0
4000
8000
12000
0
800
1600
z(m)
Ray-based
inversion gradient
overlaying G_w

Purpose of the denominator

$$G_w = \text{Reverse Time Migration of } \frac{\dot{d}_{cal}(t)}{\dot{d}_{cal}(t) * \dot{d}_{cal}(t)}$$


x(m)
0
4000
8000
12000
0
800
1600
z(m)
Gradient calculated without denominator


x(m)
0
4000
8000
12000
0
800
1600
z(m)
Gradient calculated with denominator

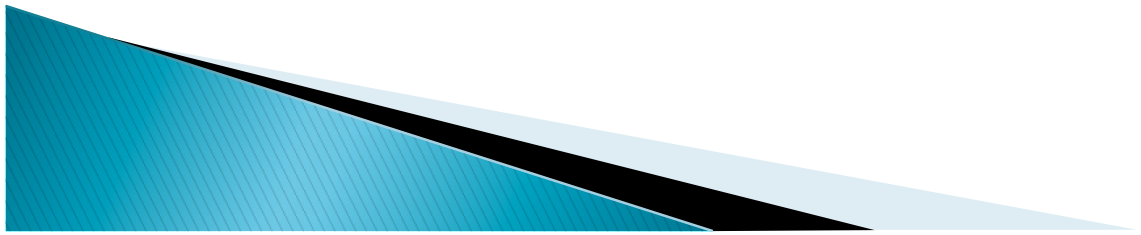
Outline

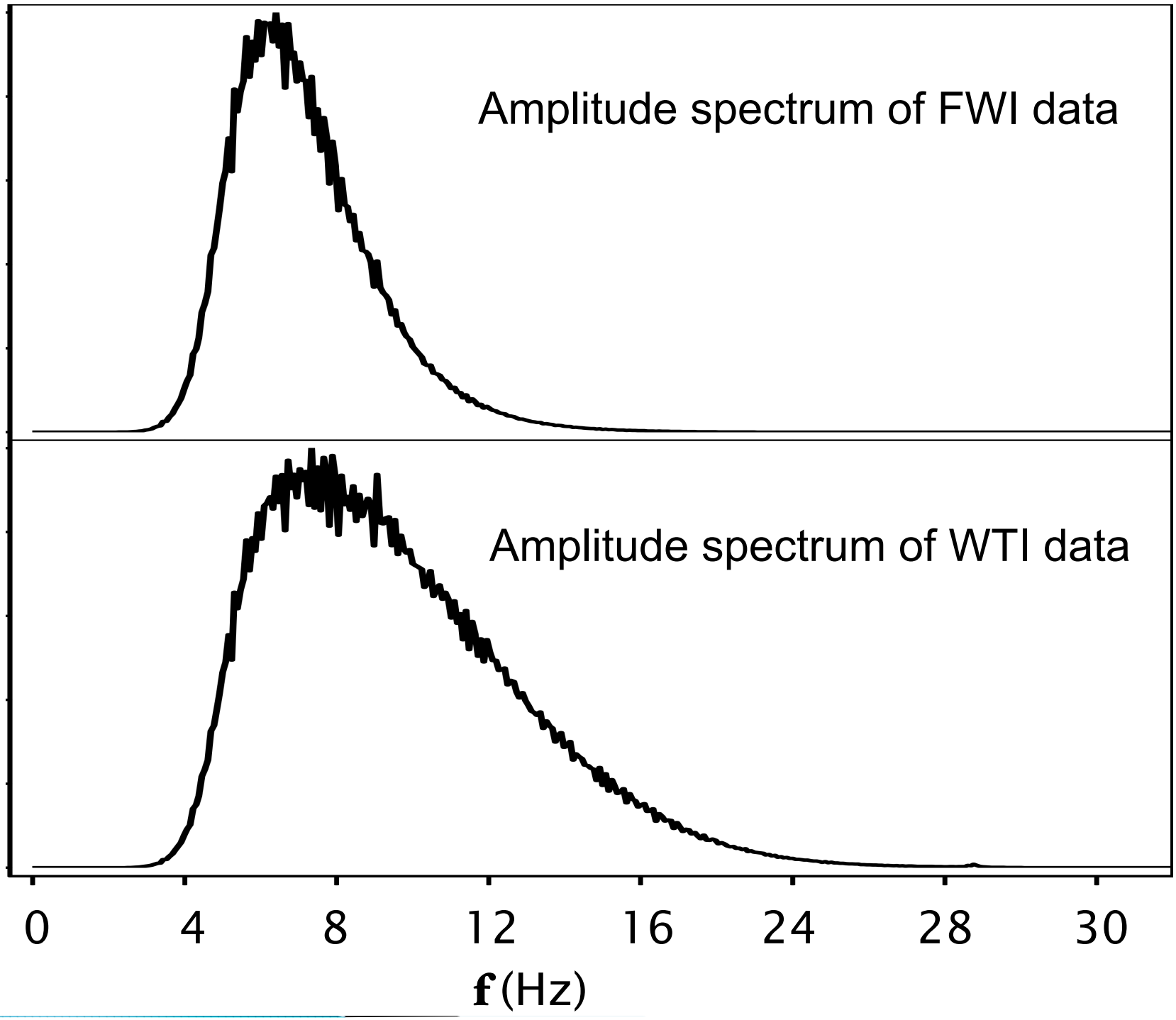
- Motivation
- Theory
- Salt estimation
- Conclusion

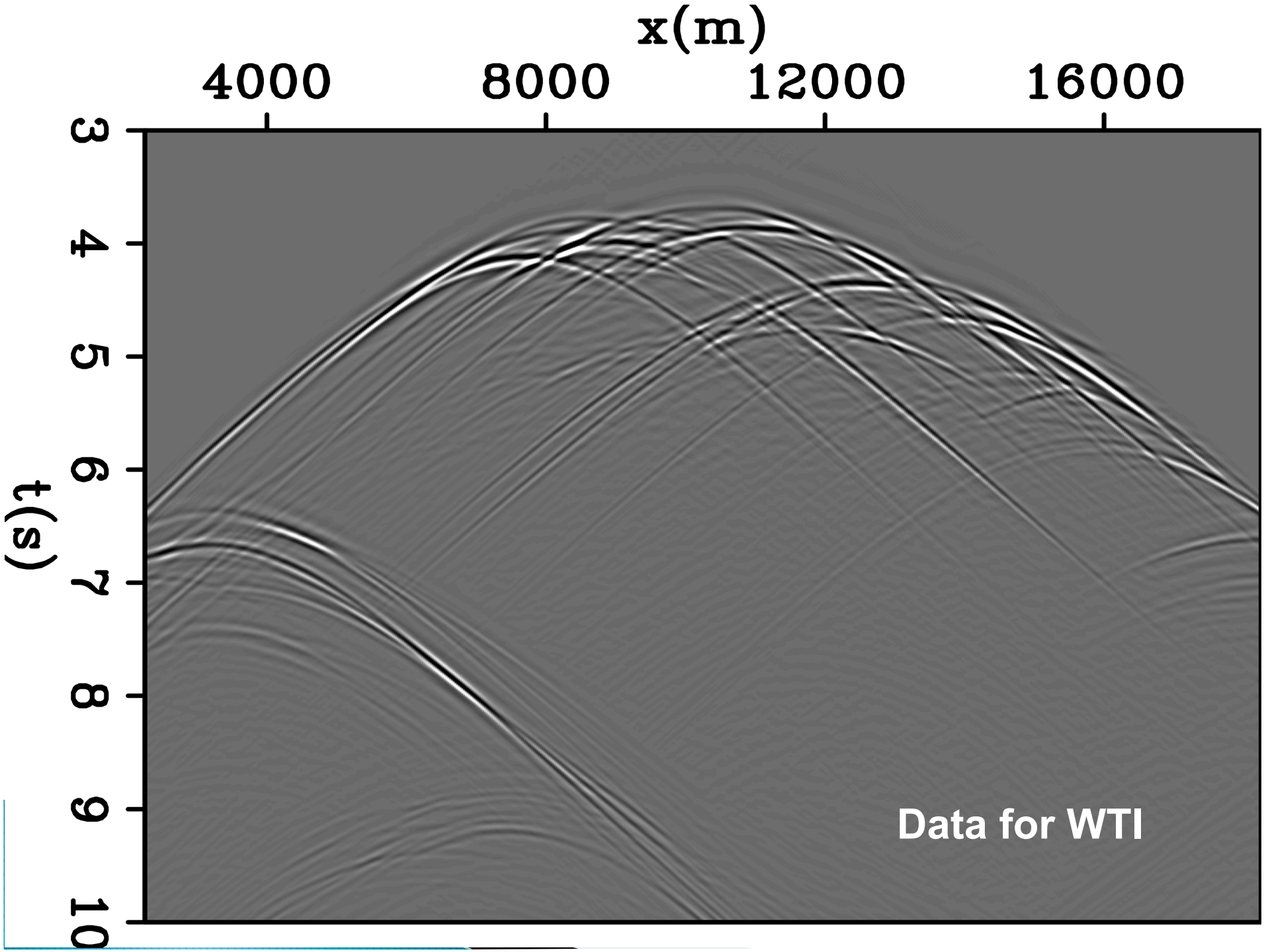
x(m)
0
8000
16000
24000
z(m)
0
4000
8000
km/s
2
3
4
True Model

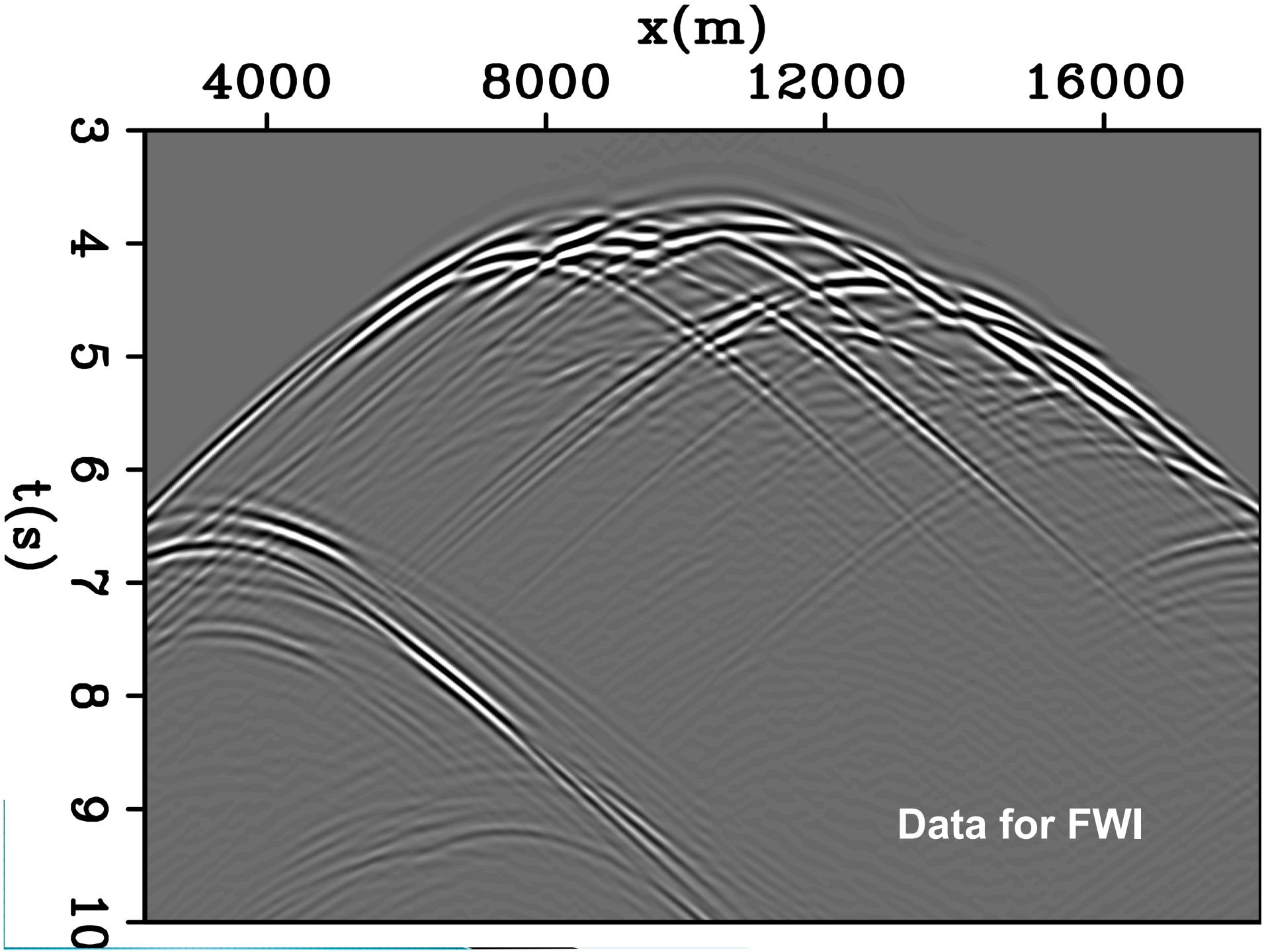
Survey Geometry

Shot number: **96**
Shot spacing: **250m**
Offset for each shot: **-8km~8km**



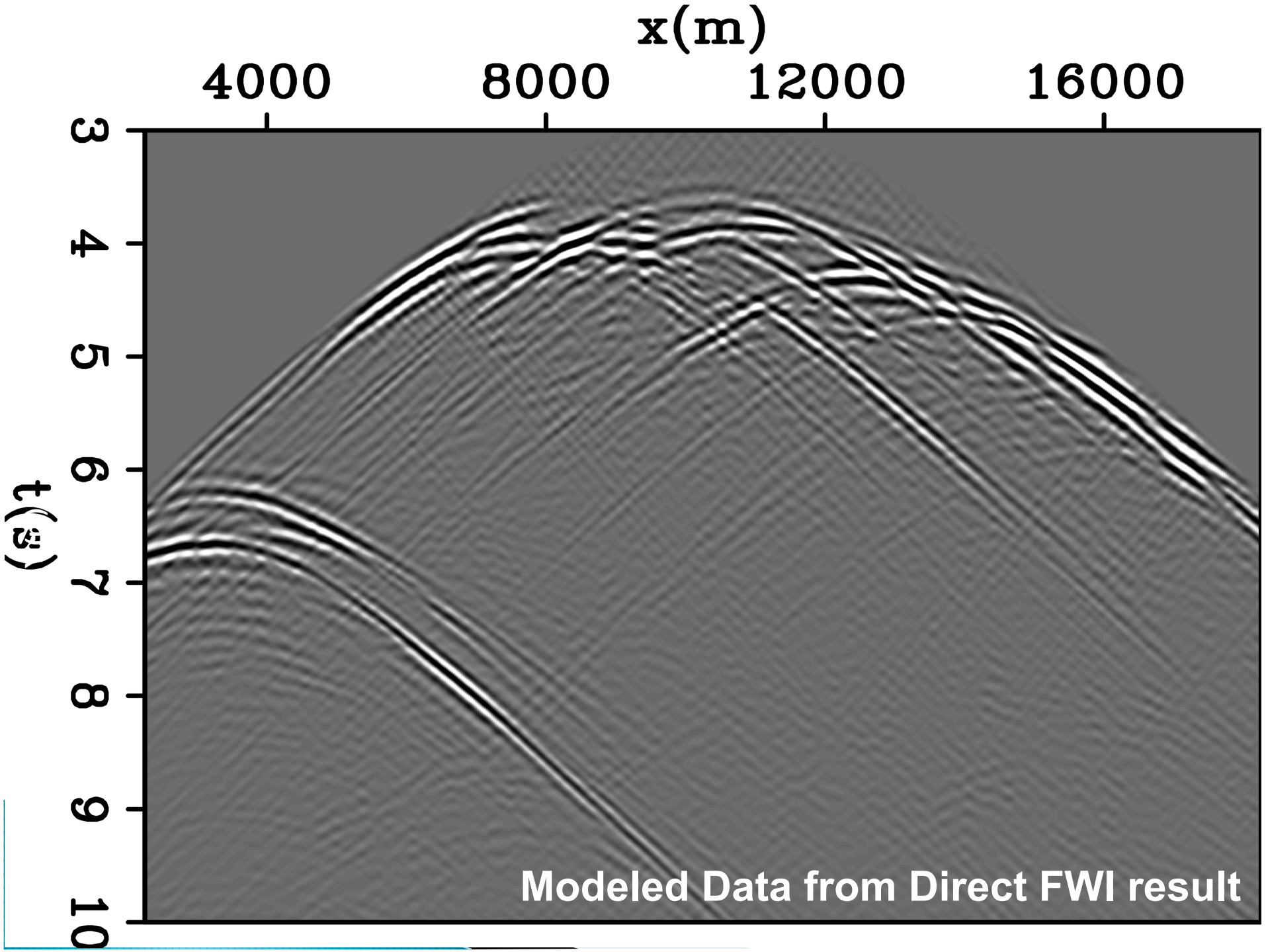

Amplitude spectrum of FWI data
Amplitude spectrum of WTI data
0
4
8
12
16
24
28
30
f(Hz)

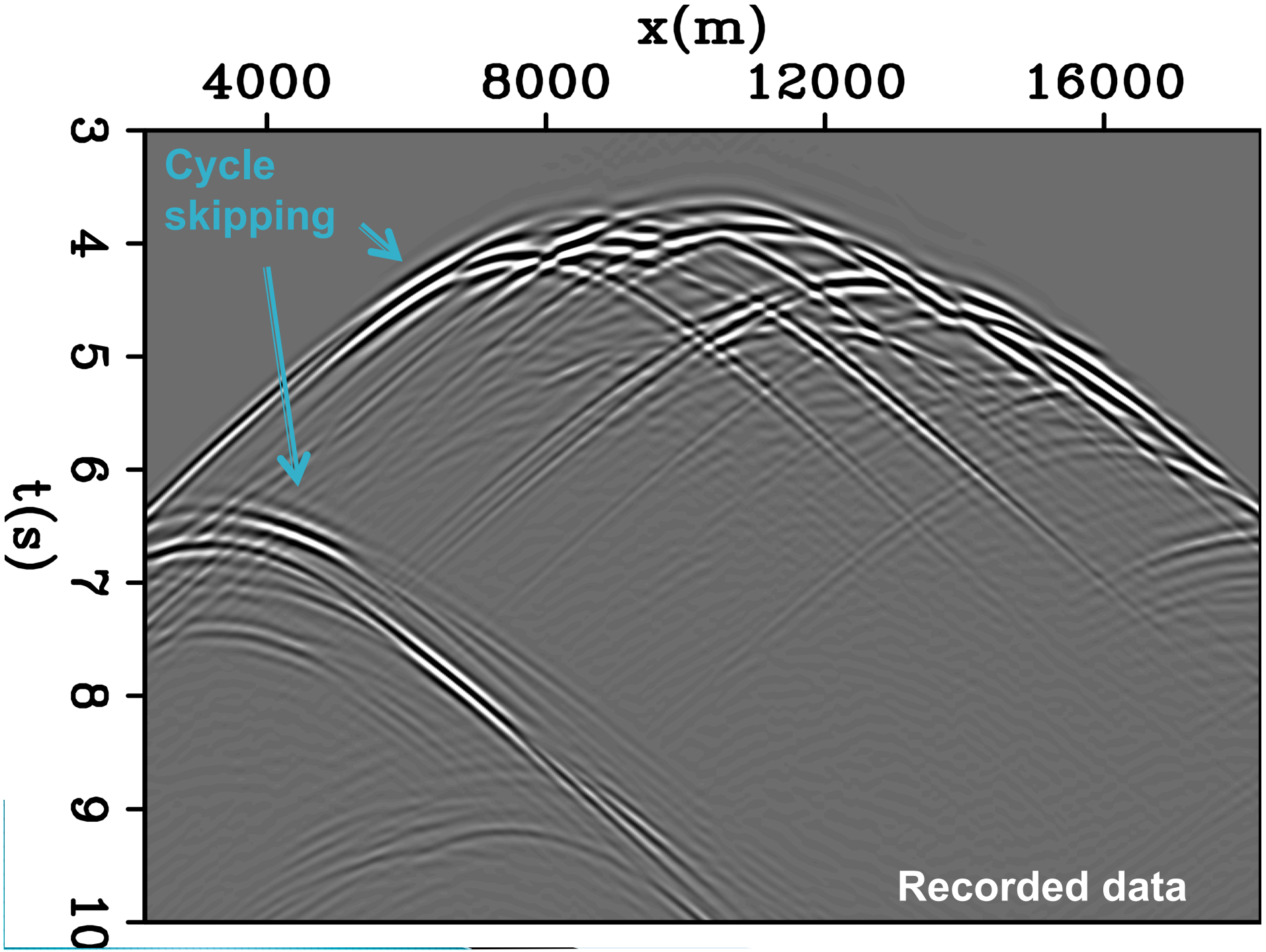

x(m)
4000
8000
12000
16000
t(s)
3
4
5
6
7
8
9
10
Data for WTI

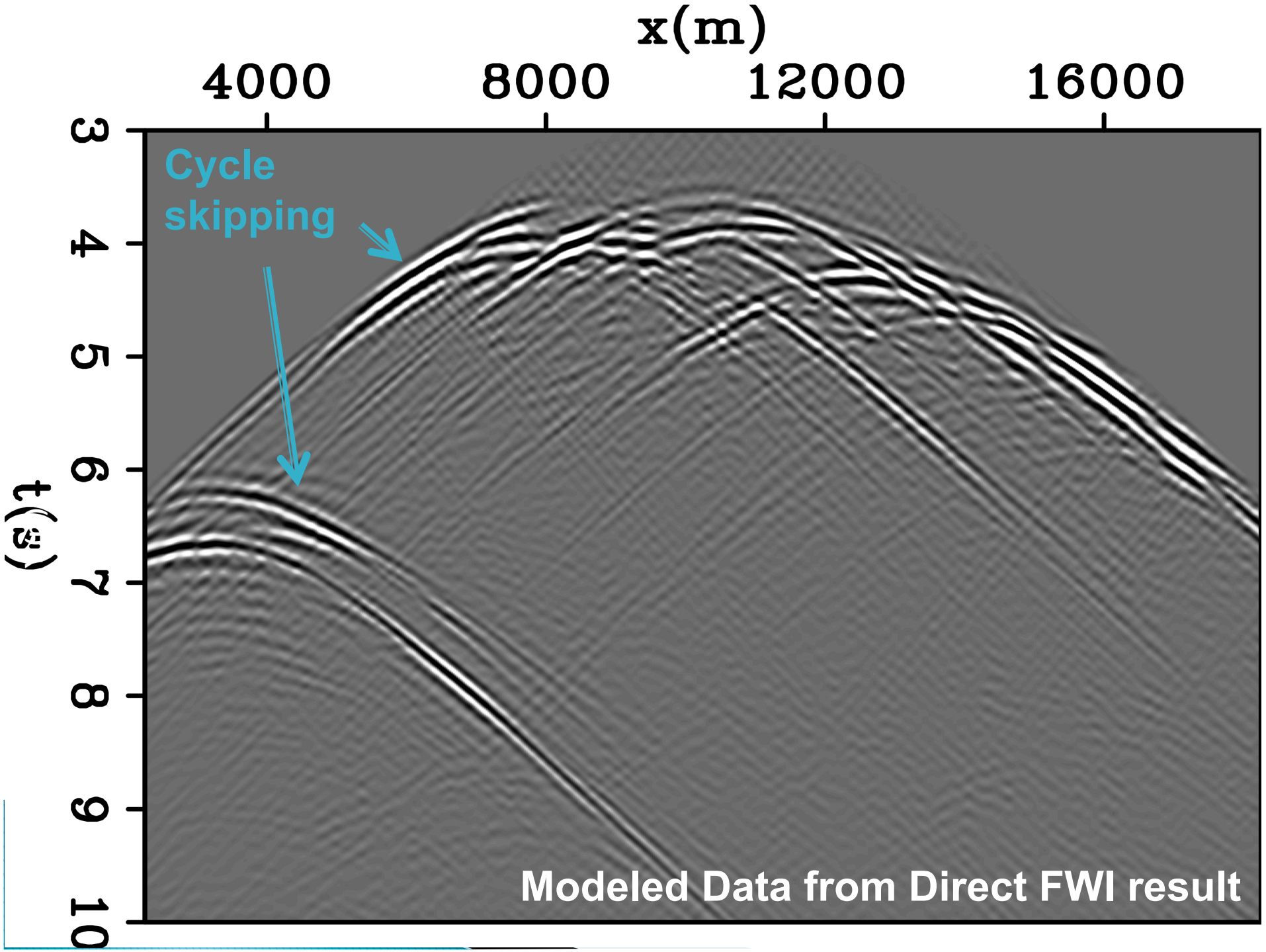

x(m)
4000
8000
12000
16000
t(s)
3
4
5
6
7
8
9
10
Data for FWI

x(m)
0
8000
16000
24000
z(m)
0
4000
8000
km/s
2
3
4
Starting Model

x(m)
0
8000
16000
24000
z(m)
0
4000
8000
km/s
2
3
4
Direct FWI from Starting Model

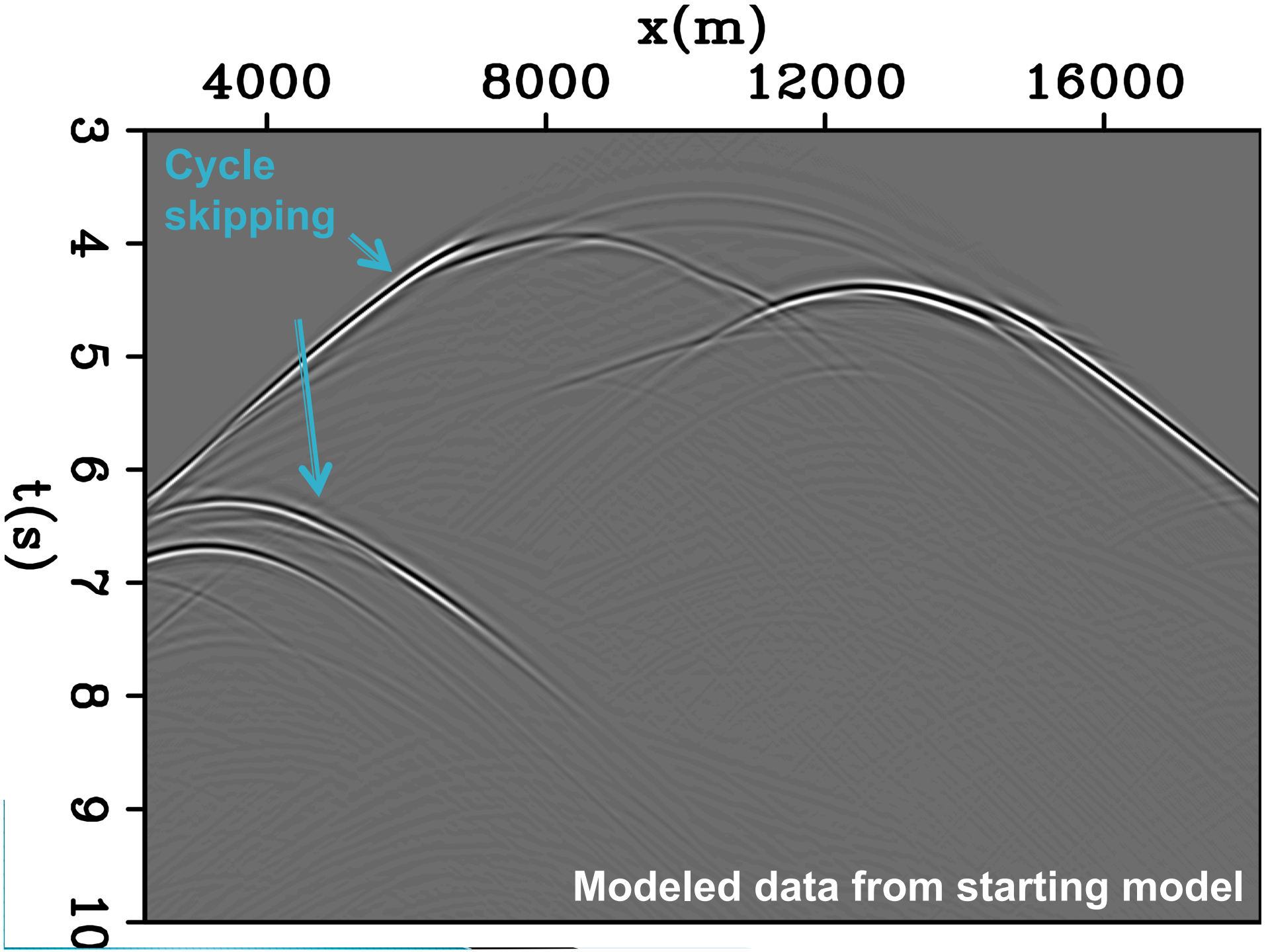

x(m)
4000
8000
12000
16000
t(s)
3
4
5
6
7
8
9
10
Modeled Data from Direct FWI result

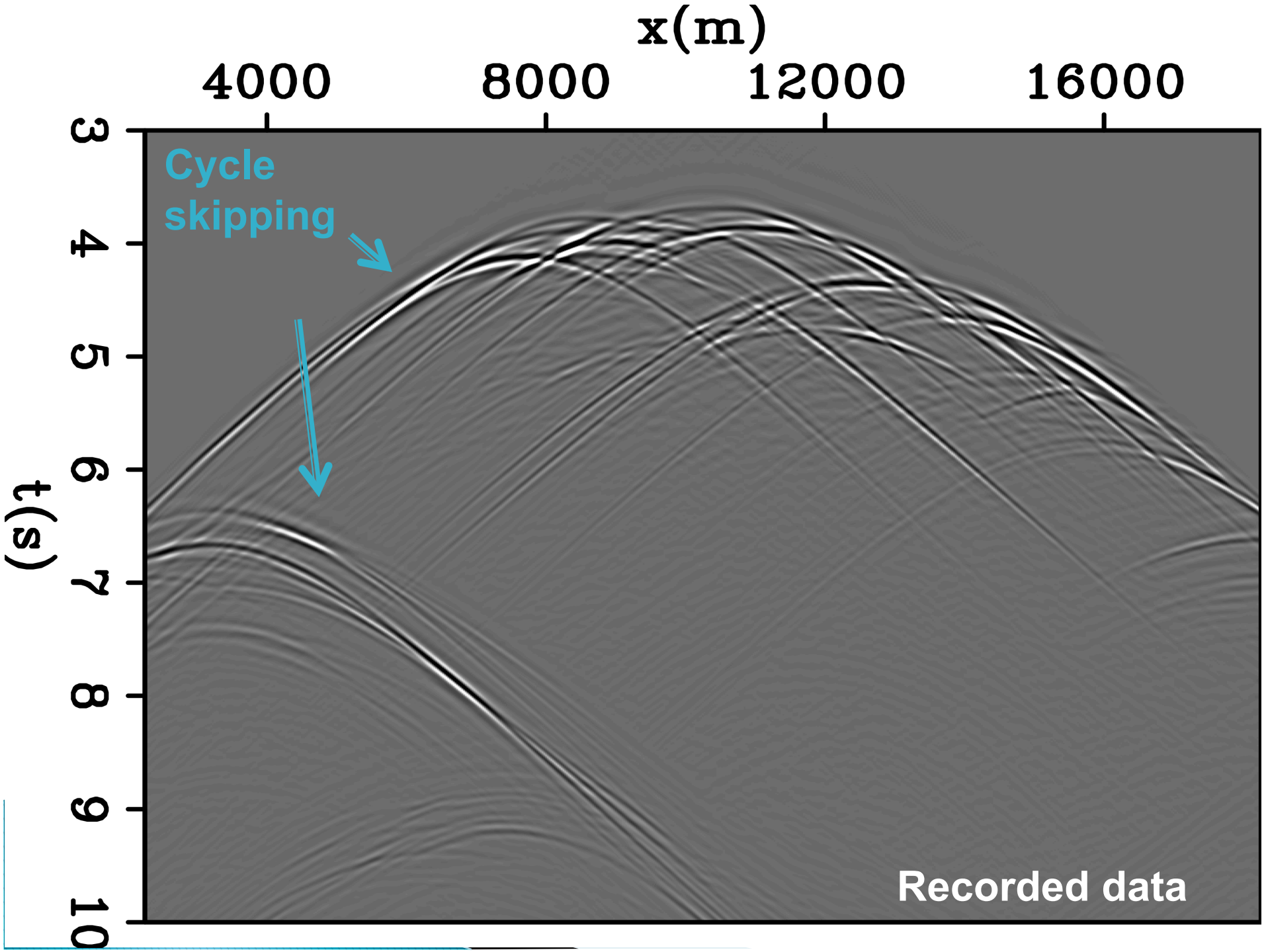

x(m)
4000
8000
12000
16000
t(s)
3
4
5
6
7
8
9
10
Cycle
skipping
Recorded data

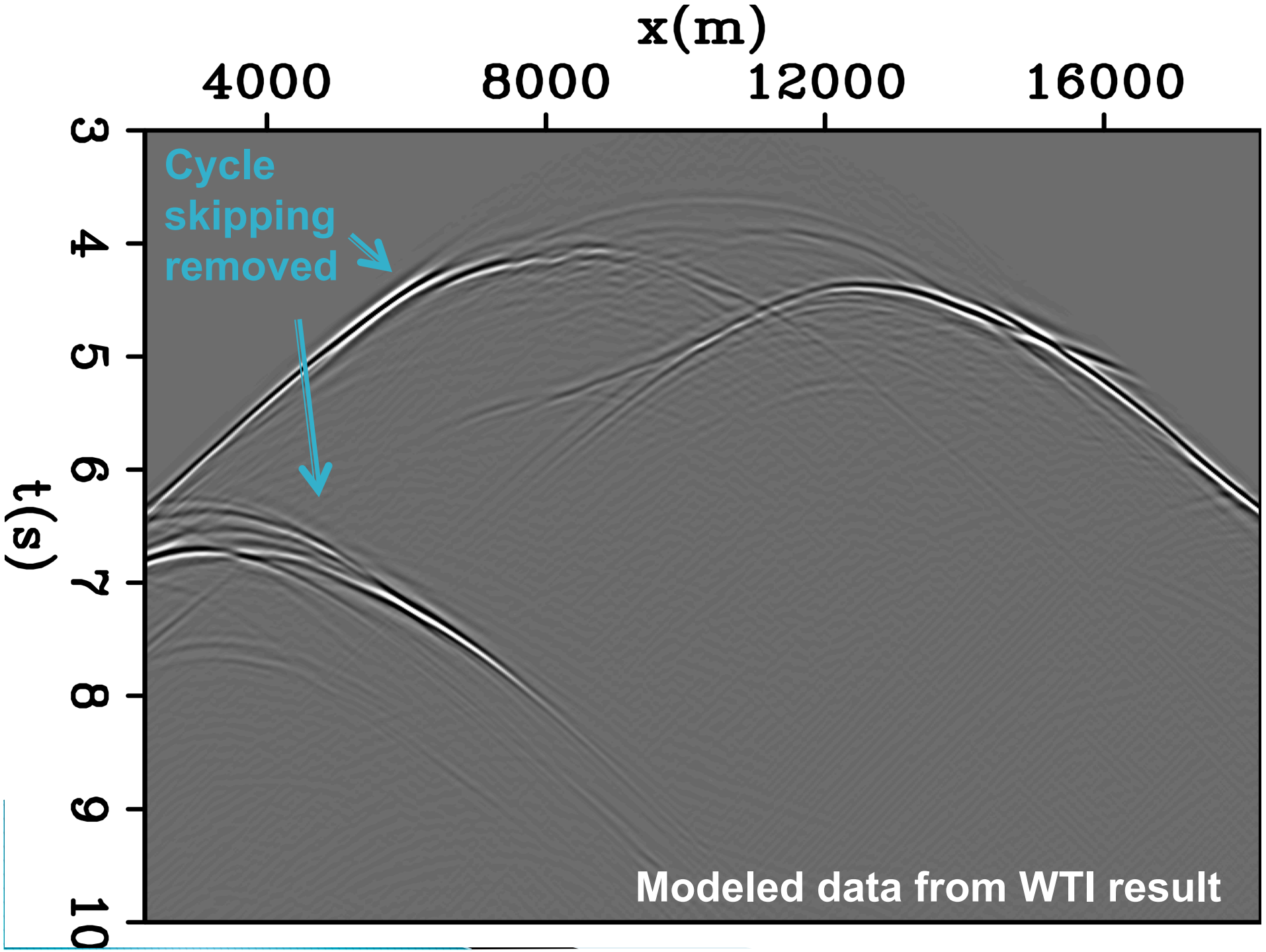

x(m)
4000
8000
12000
16000
3
4
5
6
7
8
9
10
t(s)
Cycle
skipping
Modeled Data from Direct FWI result

x(m)
0
8000
16000
24000
z(m)
0
4000
8000
km/s
2
3
4
Starting Model

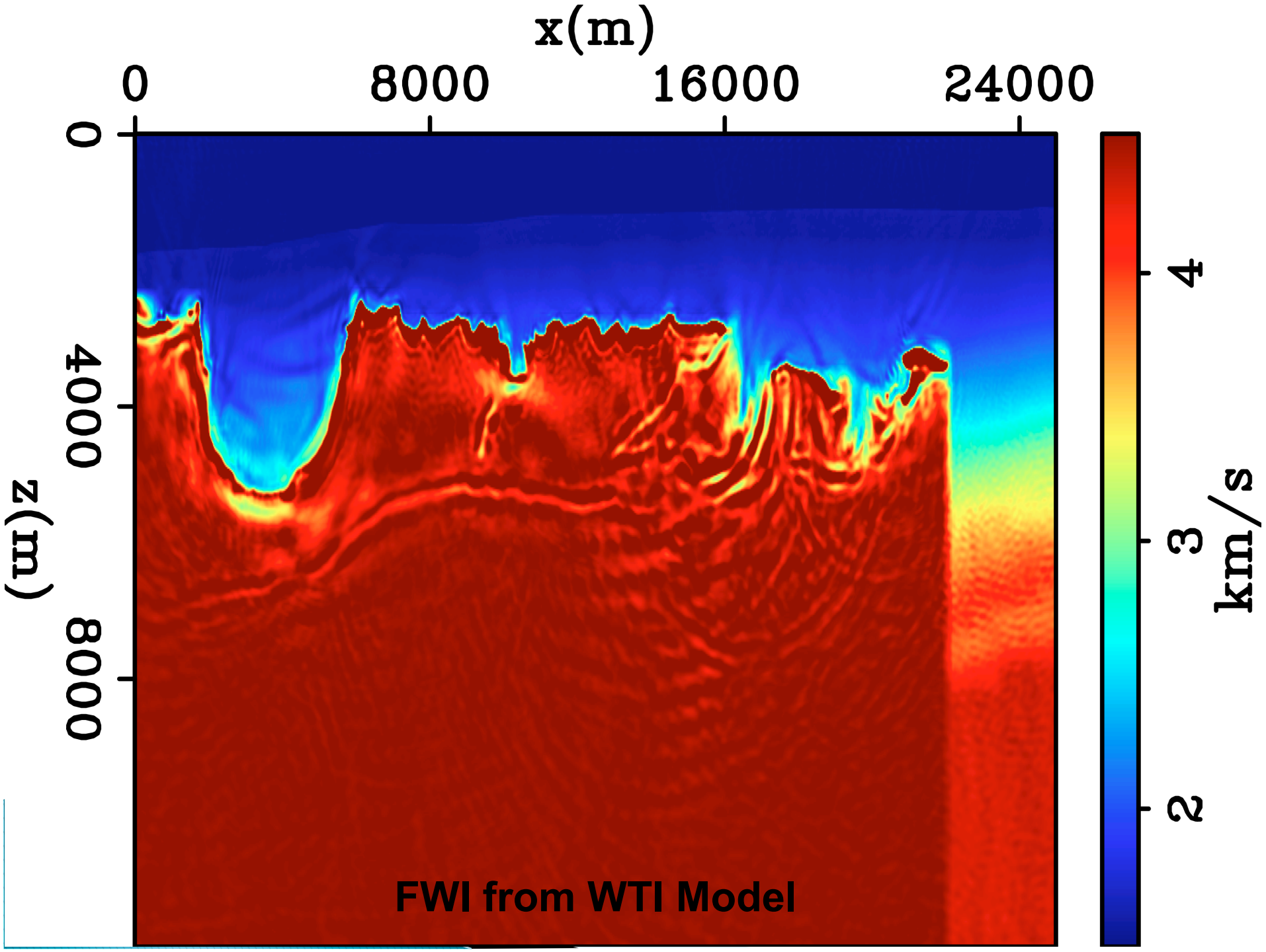
x(m)
0
8000
16000
24000
z(m)
0
4000
8000
km/s
2
3
4
WTI from Starting Model

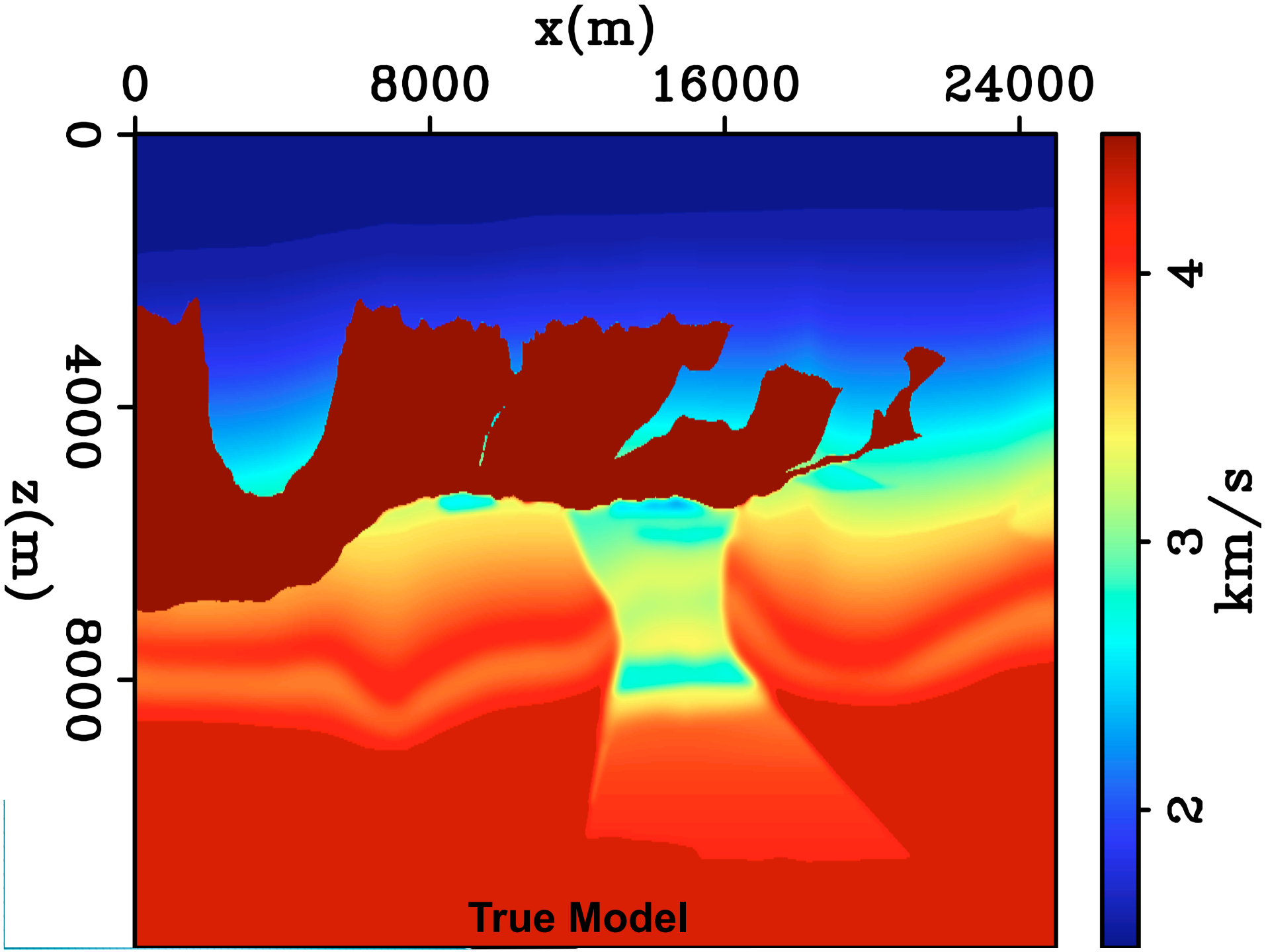

x(m)
4000
8000
12000
16000
t(s)
3
4
5
6
7
8
9
10
Cycle
skipping
Modeled data from starting model

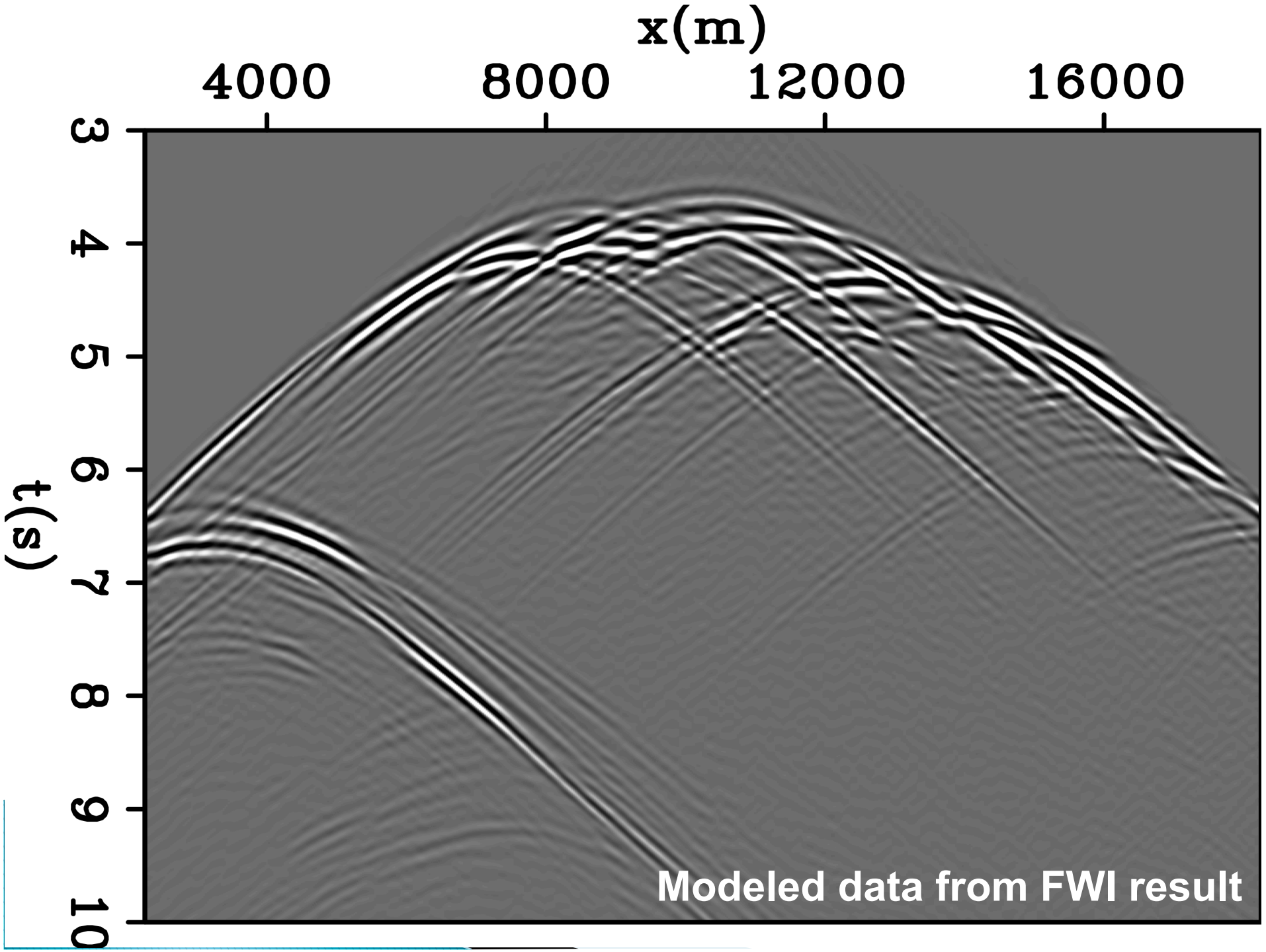

x(m)
4000
8000
12000
16000
t(s)
3
4
5
6
7
8
9
10
Cycle
skipping
Recorded data

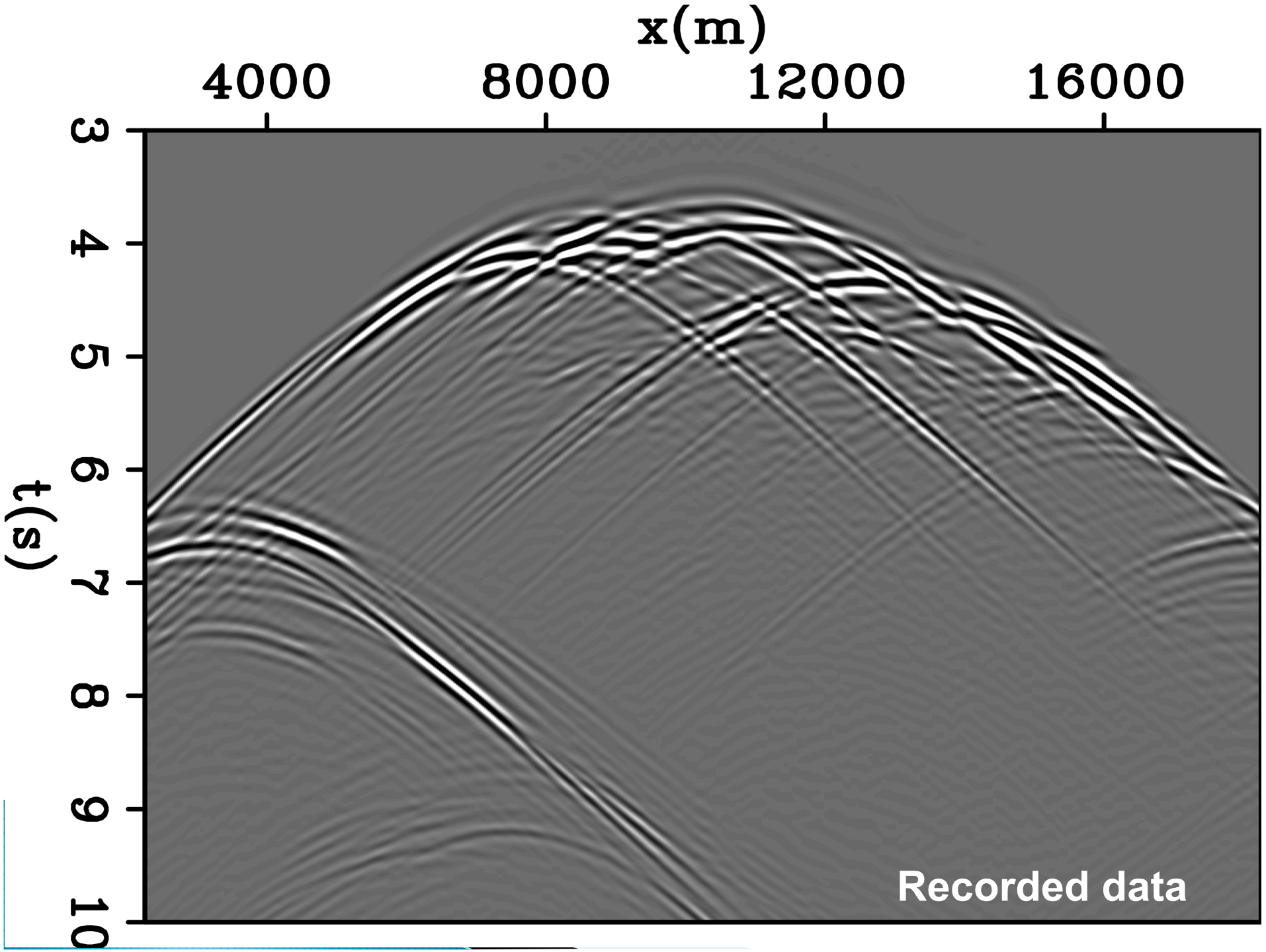

x(m)
4000
8000
12000
16000
t(s)
3
4
5
6
7
8
9
10
Cycle
skipping
removed
Modeled data from WTI result

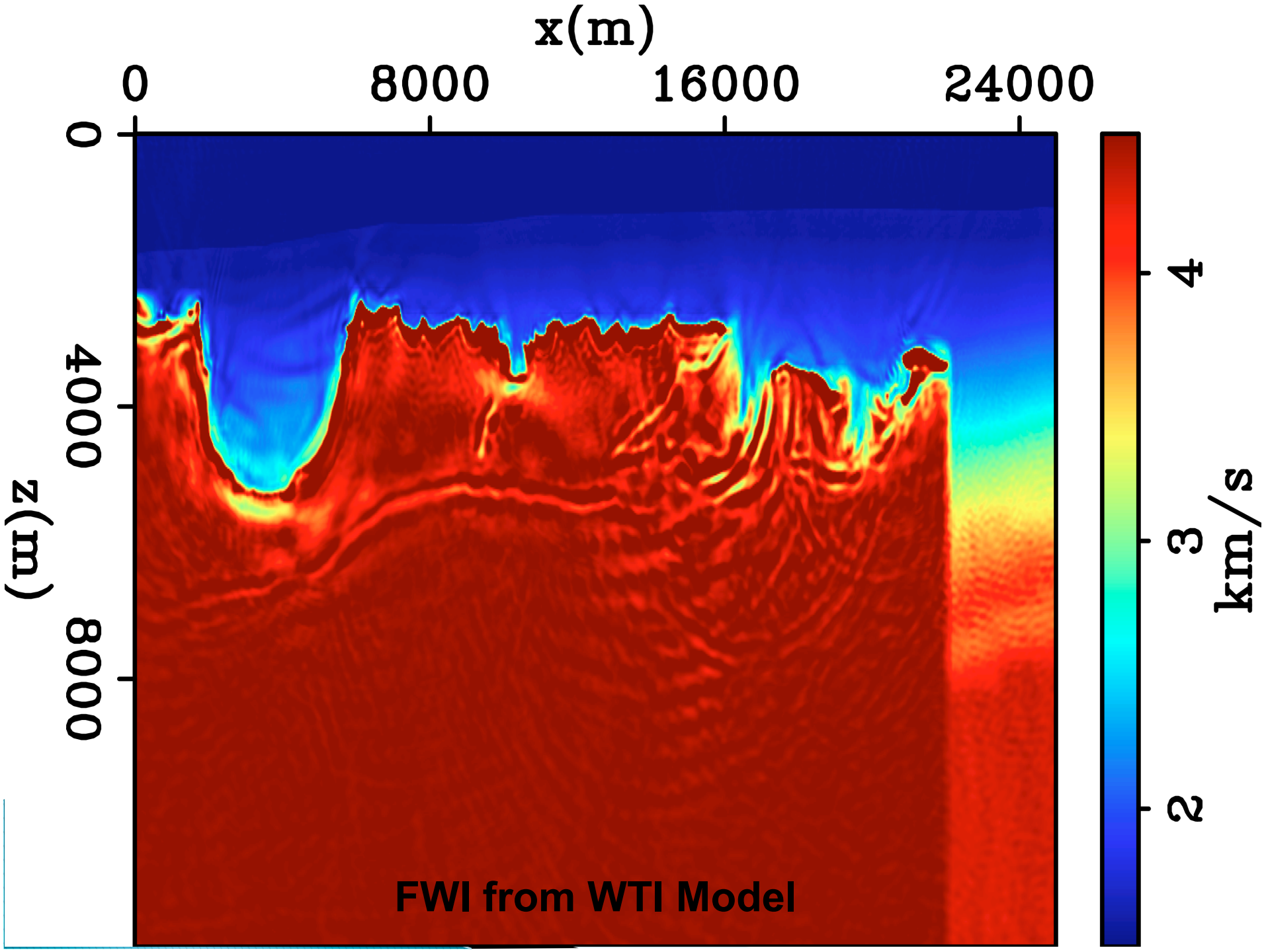
x(m)
0
8000
16000
24000
z(m)
0
4000
8000
km/s
2
3
4
WTI from Starting Model


x(m)
0
8000
16000
24000
z(m)
0
4000
8000
2
3
4
km/s
FWI from WTI Model

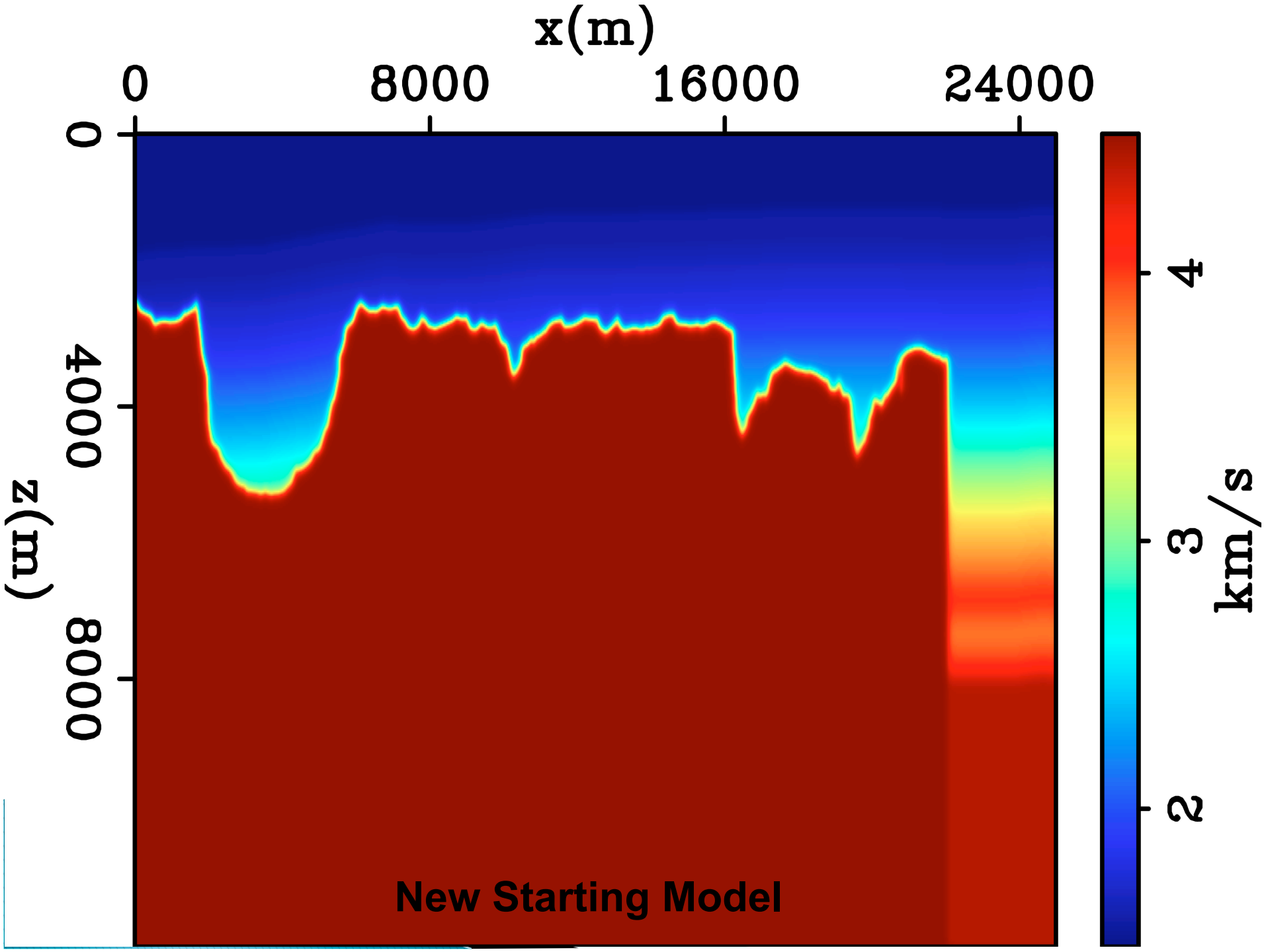

x(m)
0
8000
16000
24000
z(m)
0
4000
8000
km/s
2
3
4
True Model

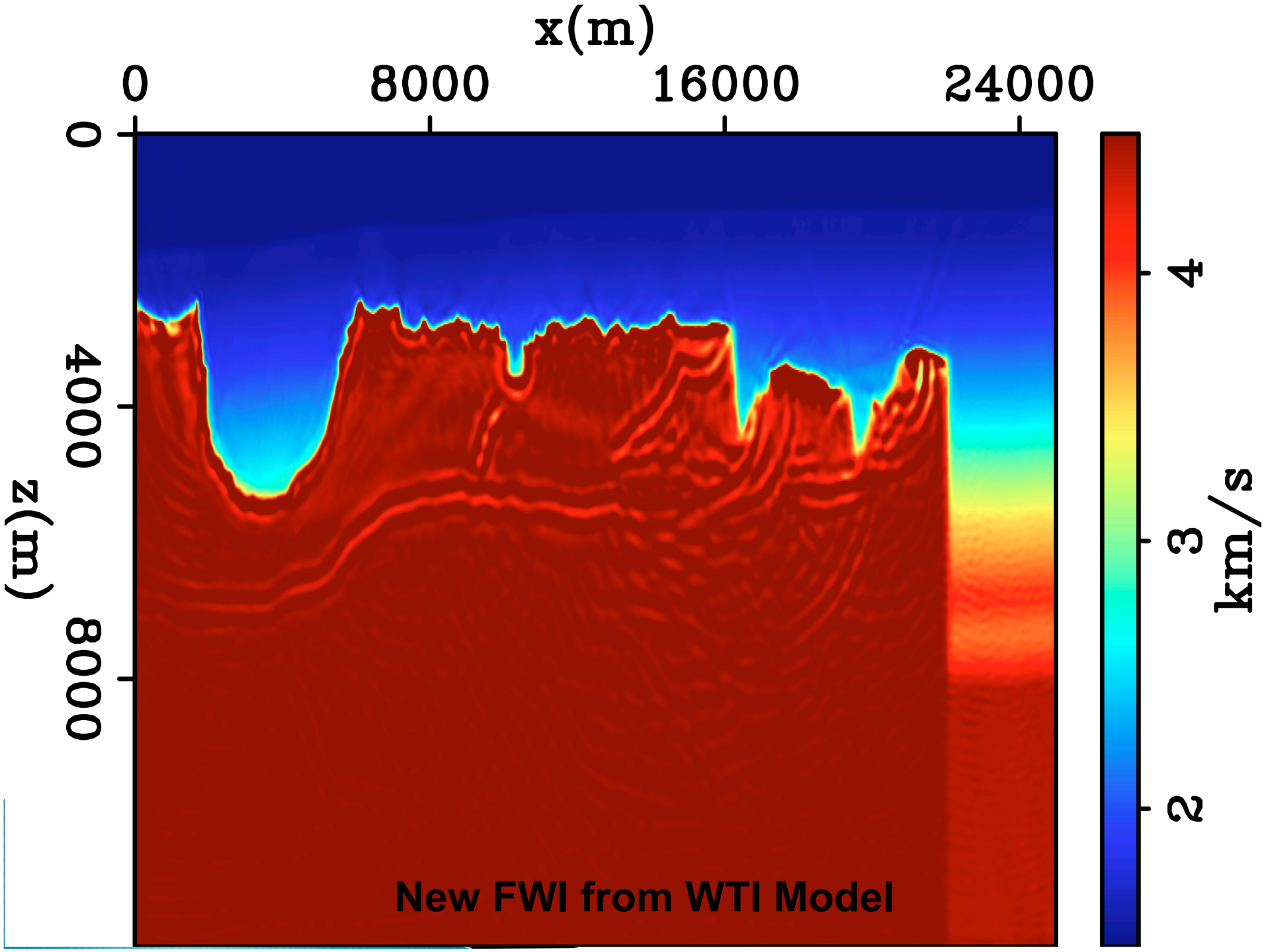

x(m)
4000
8000
12000
16000
t(s)
3
4
5
6
7
8
9
10
Modeled data from FWI result

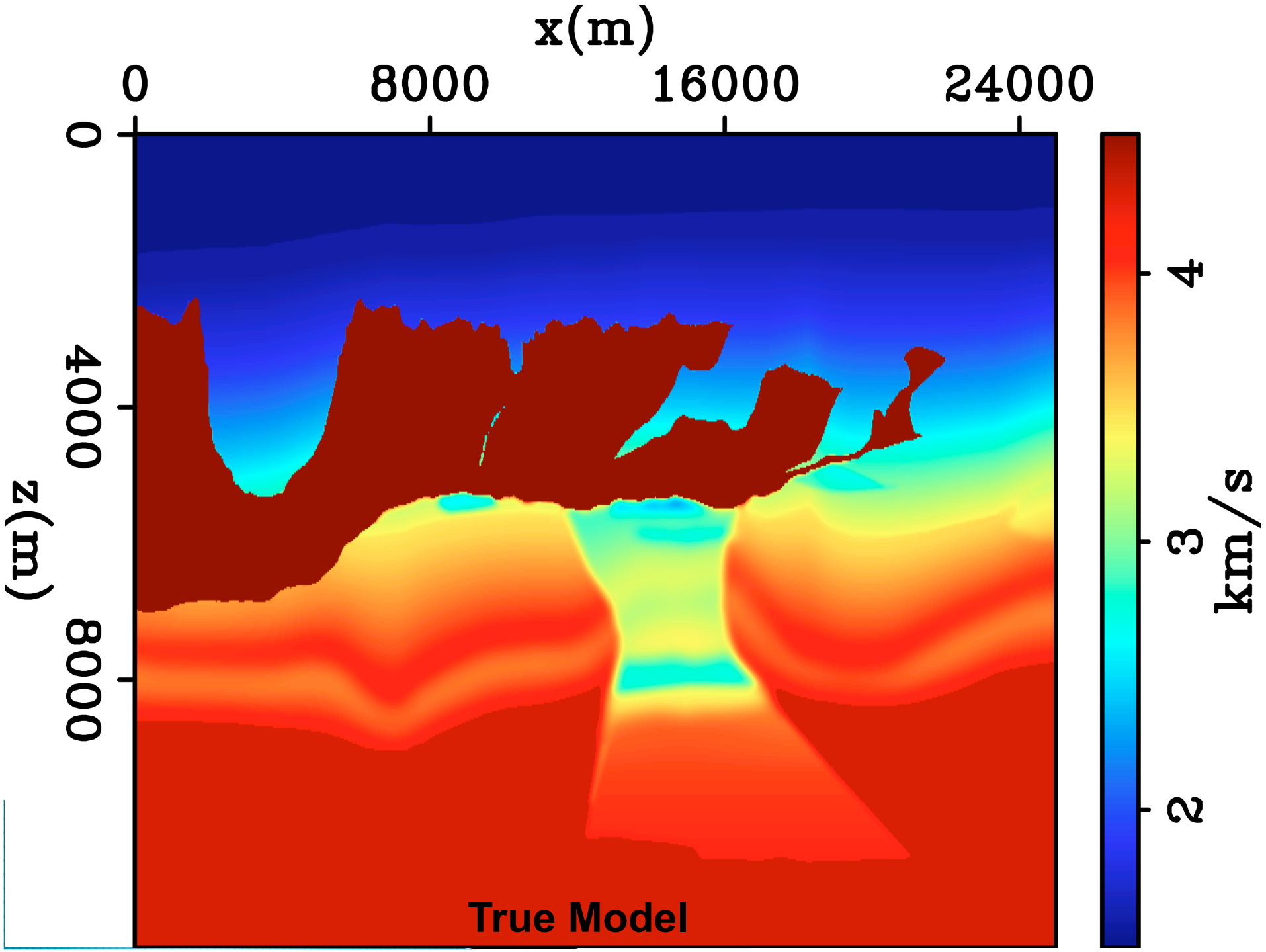

x(m)
4000
8000
12000
16000
t(s)
3
4
5
6
7
8
9
10
Recorded data


x(m)
0
8000
16000
24000
z(m)
0
4000
8000
2
3
4
km/s
FWI from WTI Model

x(m)
0
8000
16000
24000
z(m)
0
4000
8000
Picked Top-of-Salt
2
3
4
km/s


x(m)
0
8000
16000
24000
z(m)
0
4000
8000
New Starting Model
km/s
2
3
4


x(m)
0
8000
16000
24000
z(m)
0
4000
8000
km/s
2
3
4
New FWI from WTI Model


x(m)
0
8000
16000
24000
z(m)
0
4000
8000
km/s
2
3
4
True Model

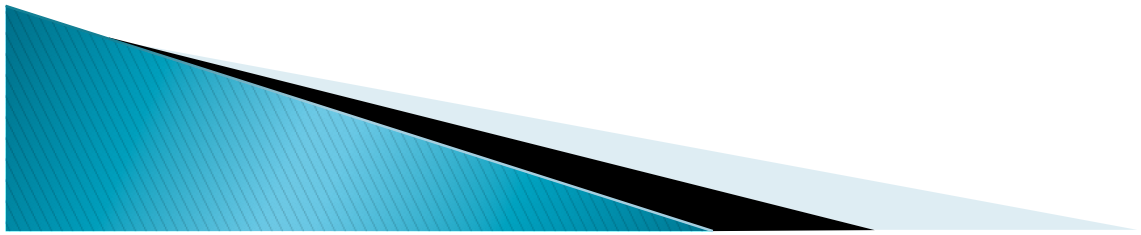
Comparison with WTI using original term

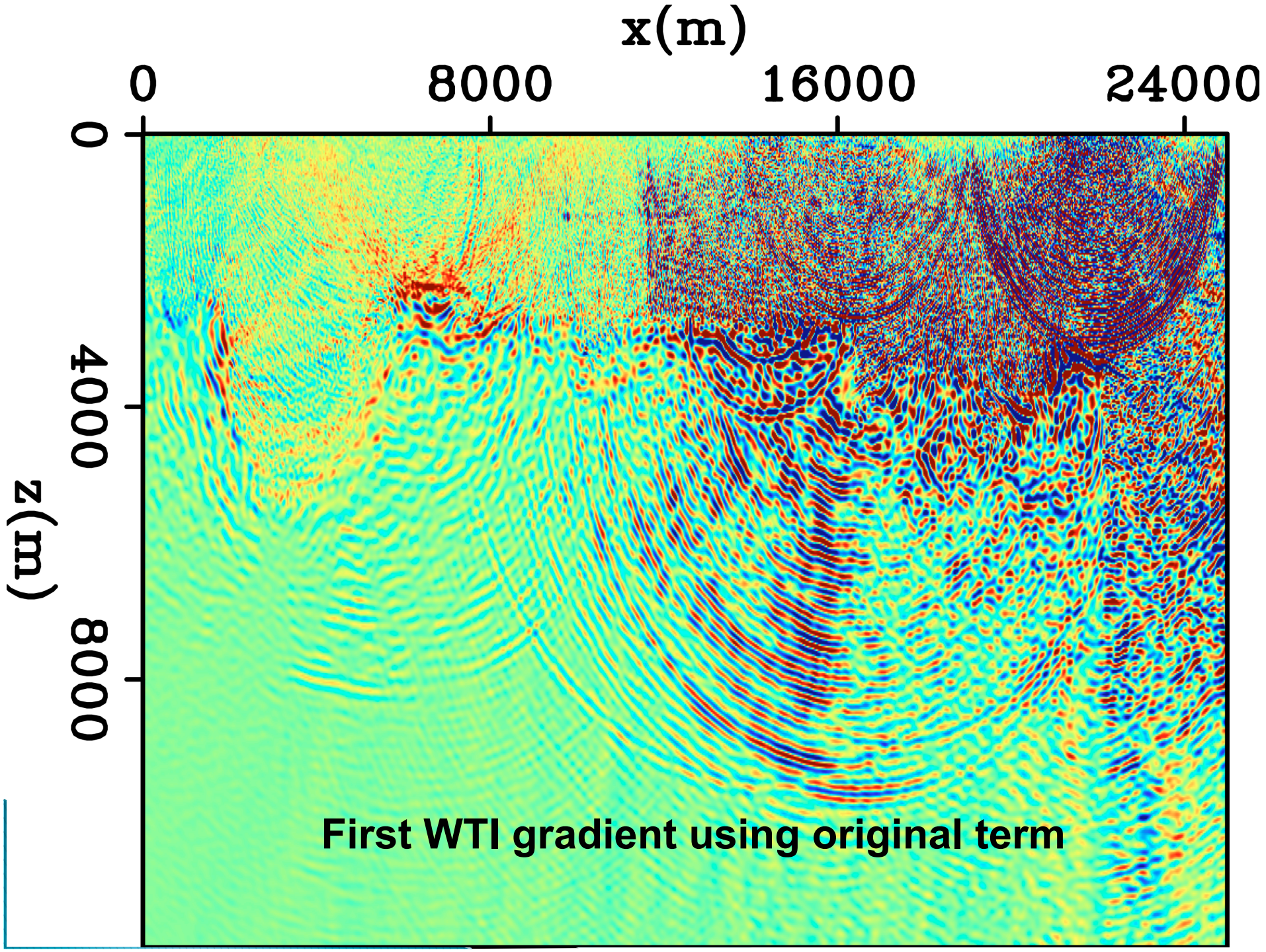
Original:

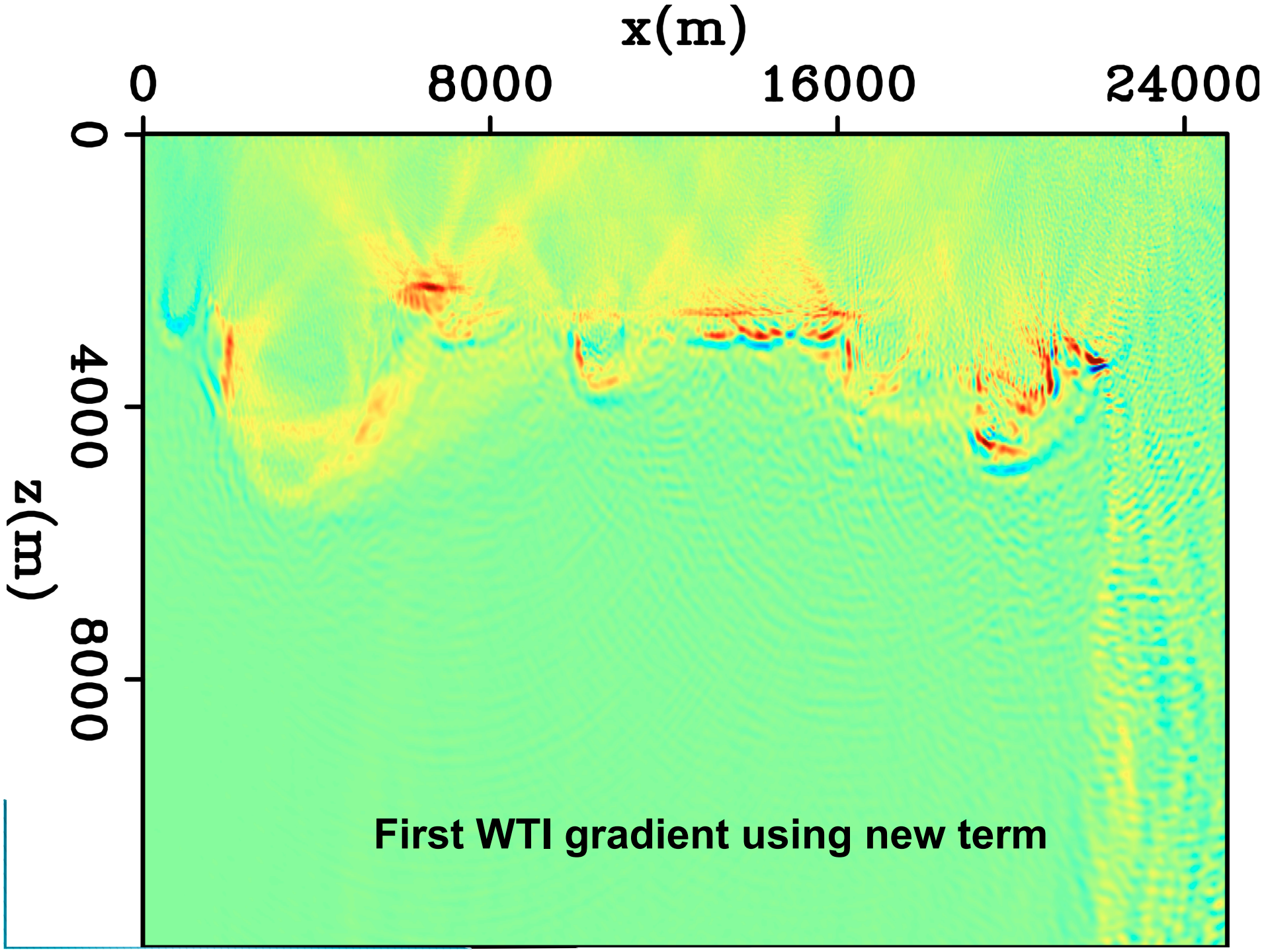
$$\delta d_{vsrc} = \Delta\tau \frac{\dot{d}_{obs}(t+\Delta\tau)}{\dot{d}_{obs}(t+\Delta\tau) * \dot{d}_{cal}(t)}$$

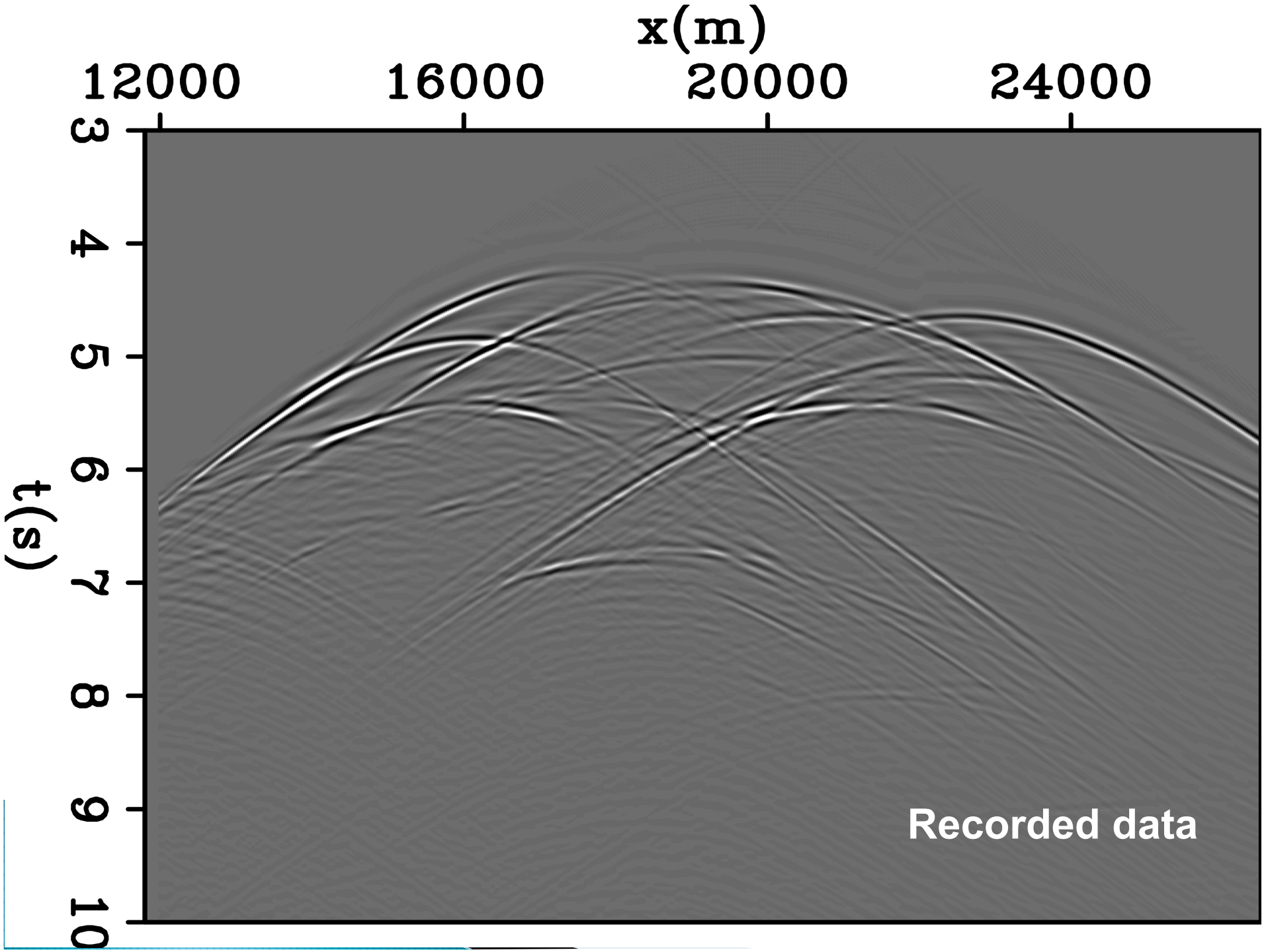
New:

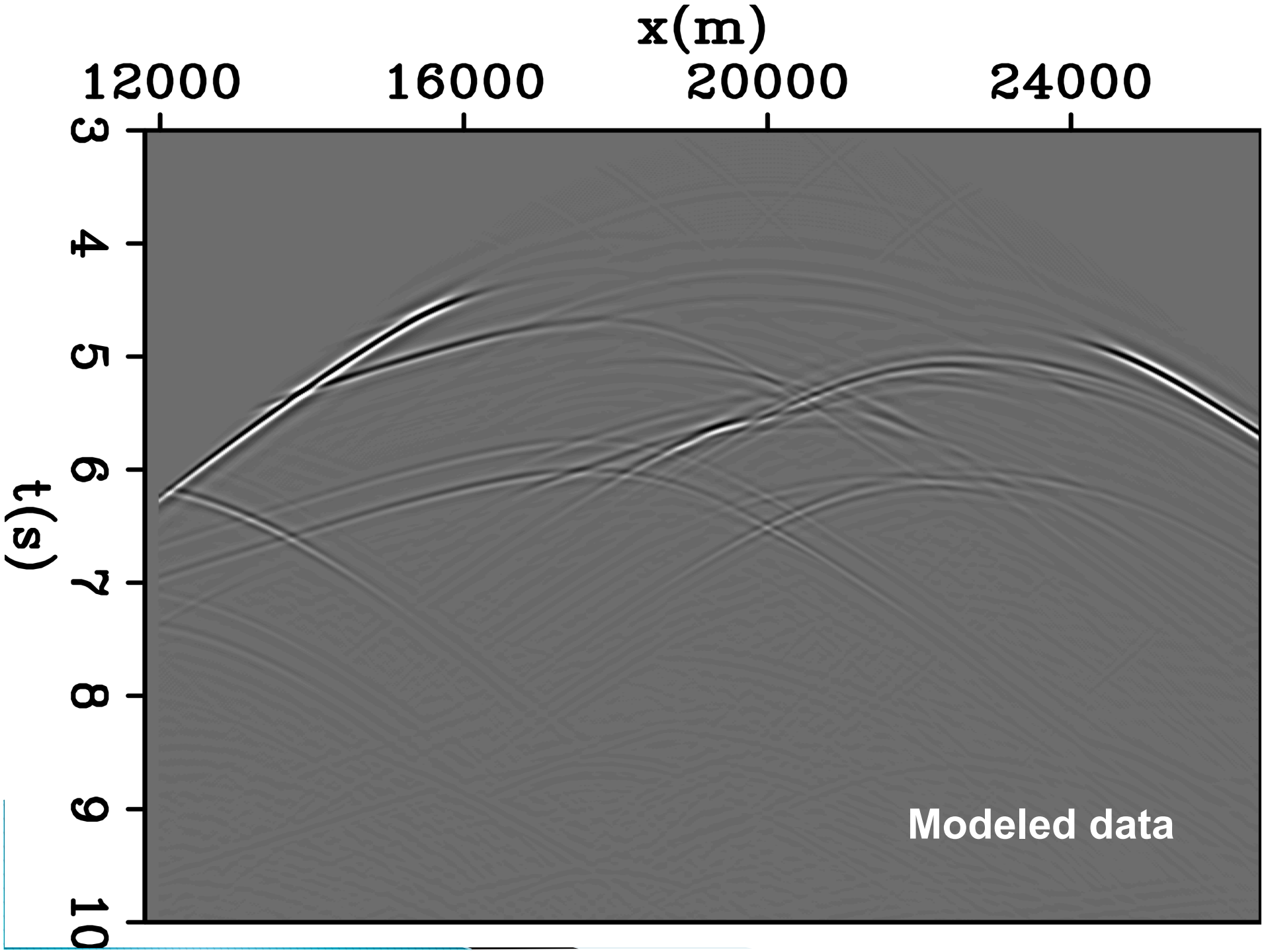
$$\delta d'_{vsrc} = \Delta\tau \frac{\dot{d}_{cal}(t)}{\dot{d}_{cal}(t) * \dot{d}_{cal}(t)}$$

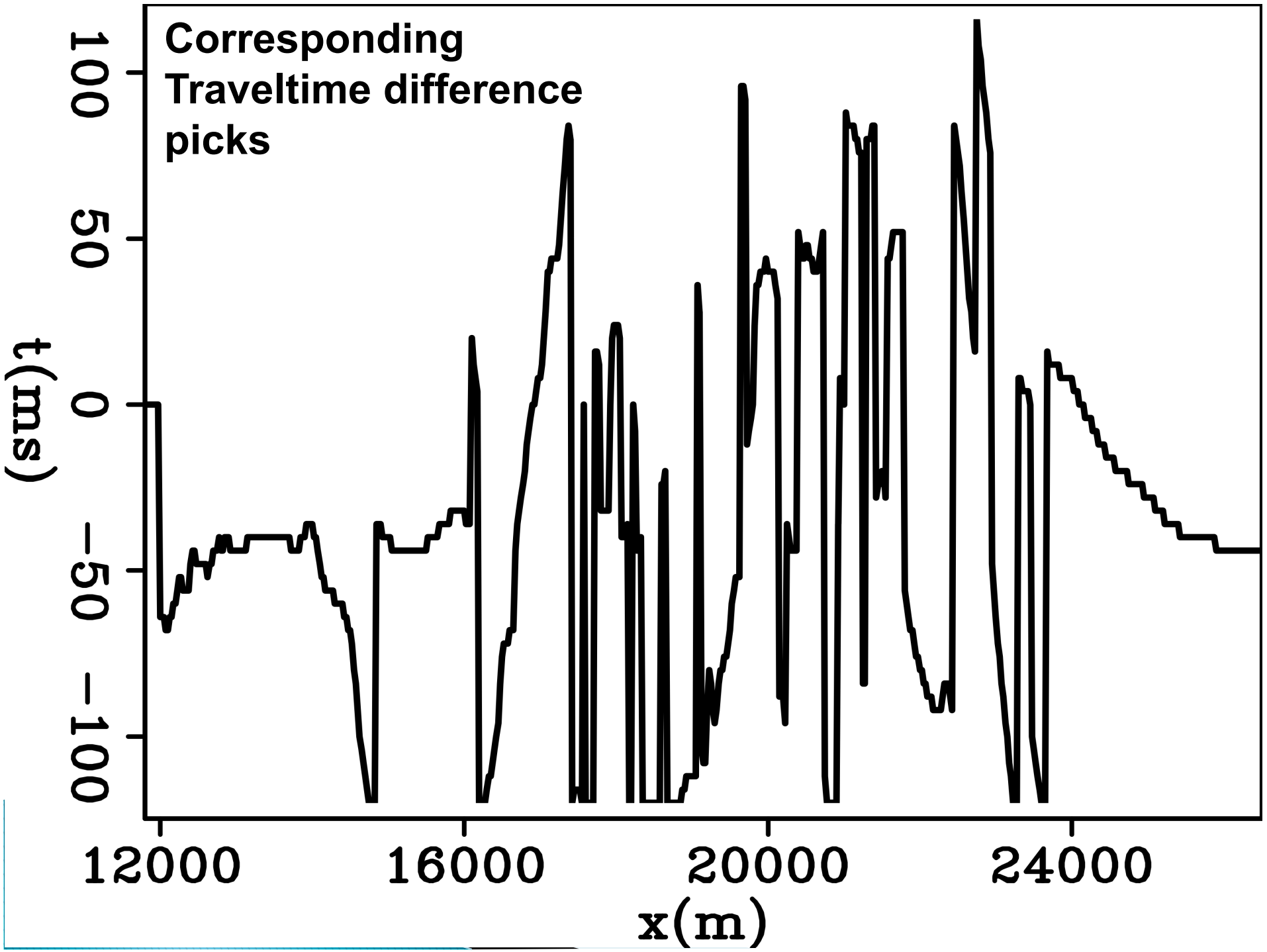


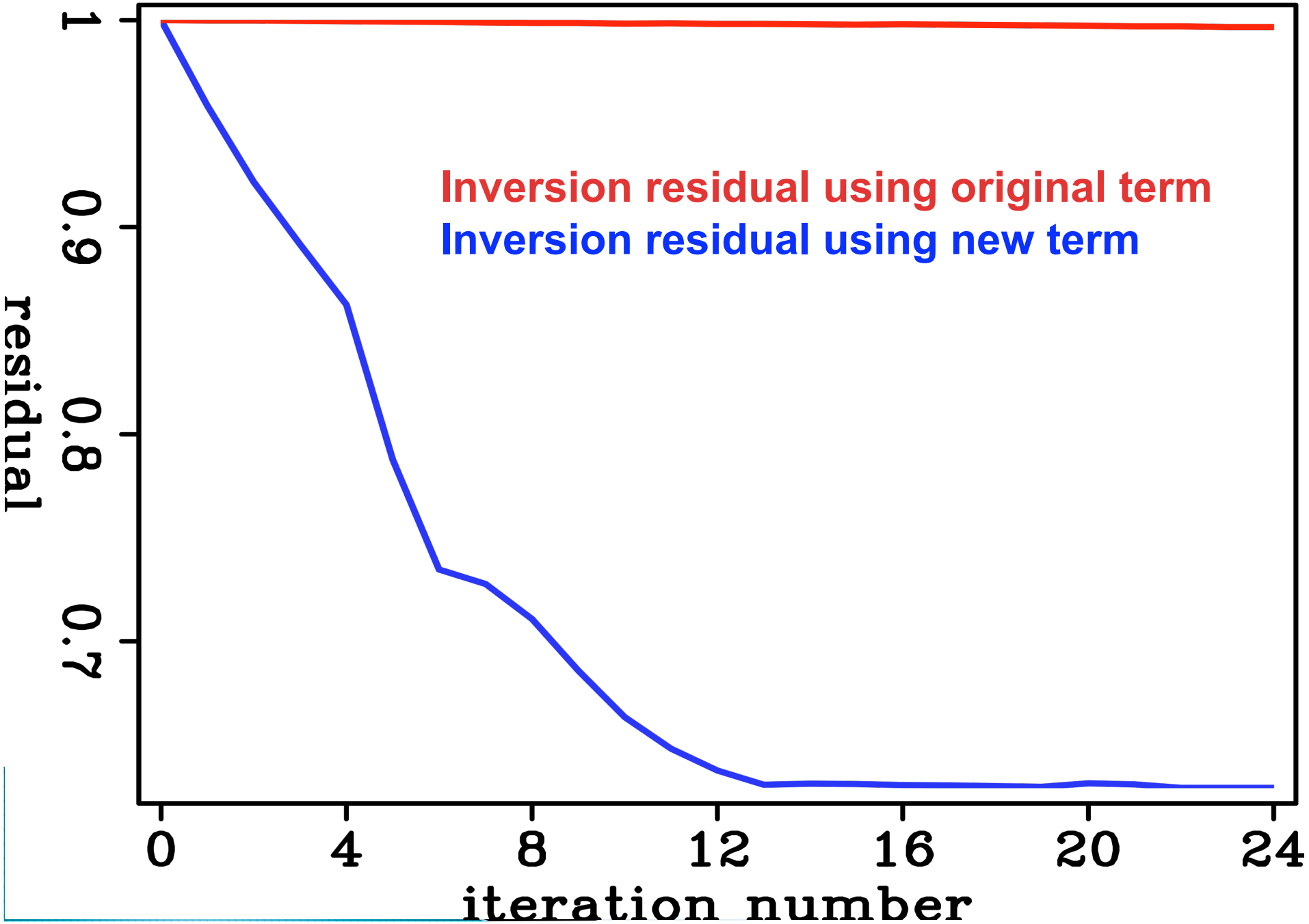

x(m)
0
8000
16000
24000
0
4000
8000
z(m)
First WTI gradient using original term


x(m)
0
8000
16000
24000
0
4000
8000
z(m)
First WTI gradient using new term


x(m)
12000
16000
20000
24000
t(s)
3
4
5
6
7
8
9
10
Recorded data


x(m)
12000
16000
20000
24000
t(s)
3
4
5
6
7
8
9
10
Modeled data

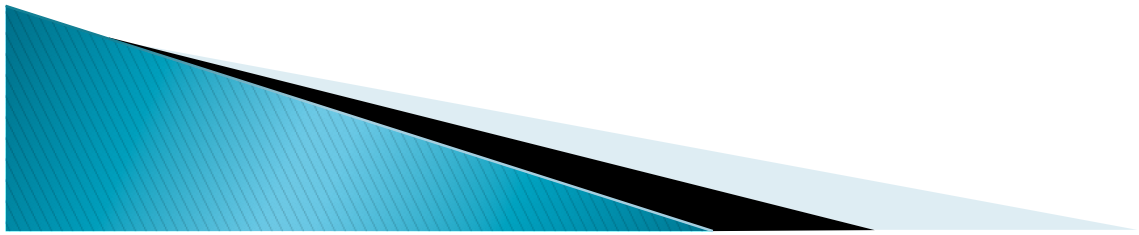

Corresponding
Traveltime difference
picks
100
50
0
−50
−100
t(ms)
12000
16000
20000
24000
x(m)


Inversion residual using original term
Inversion residual using new term
residual
1
0.9
0.8
0.7
0
4
8
12
16
20
24
iteration number

Conclusion

Proposed modification:

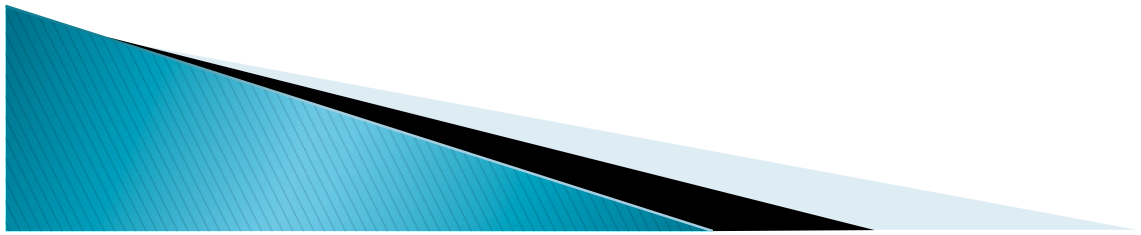
1. **Makes trouble shooting easier than original formulation.**



Conclusion

Proposed modification:

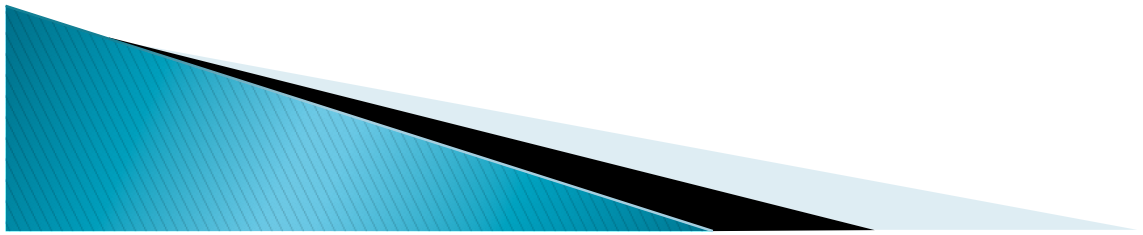
1. **Makes trouble shooting easier than original formulation.**
2. **Has a very clear analogy to ray based method.**



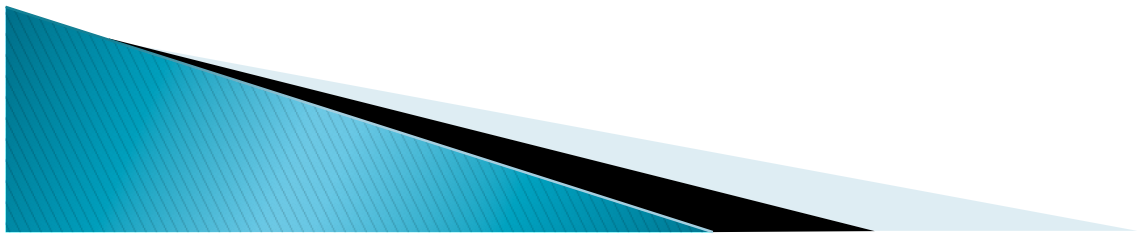
Conclusion

Proposed modification:

1. **Makes trouble shooting easier than original formulation.**
2. **Has a very clear analogy to ray based method.**
3. **More tolerant of errors in traveltime difference picks.**



Conclusion

Proposed modification:

1. **Makes trouble shooting easier than original formulation.**
2. **Has a very clear analogy to ray based method.**
3. **More tolerant of errors in traveltime difference picks.**
4. **Helps improving salt estimation significantly.**