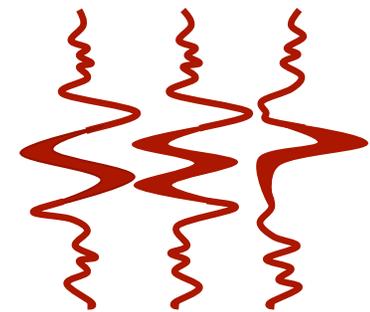
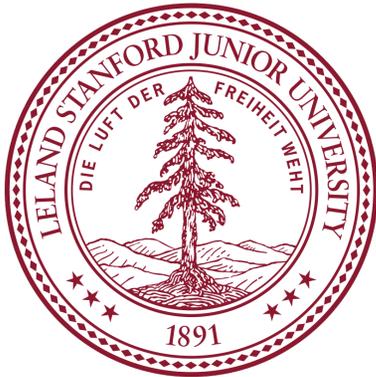


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# Accelerating RMO-based wave-equation MVA with compressed-sensing

*Y. Zhang, B. Biondi and R. Clapp*  
*June, 2013*  
*SEP—149*

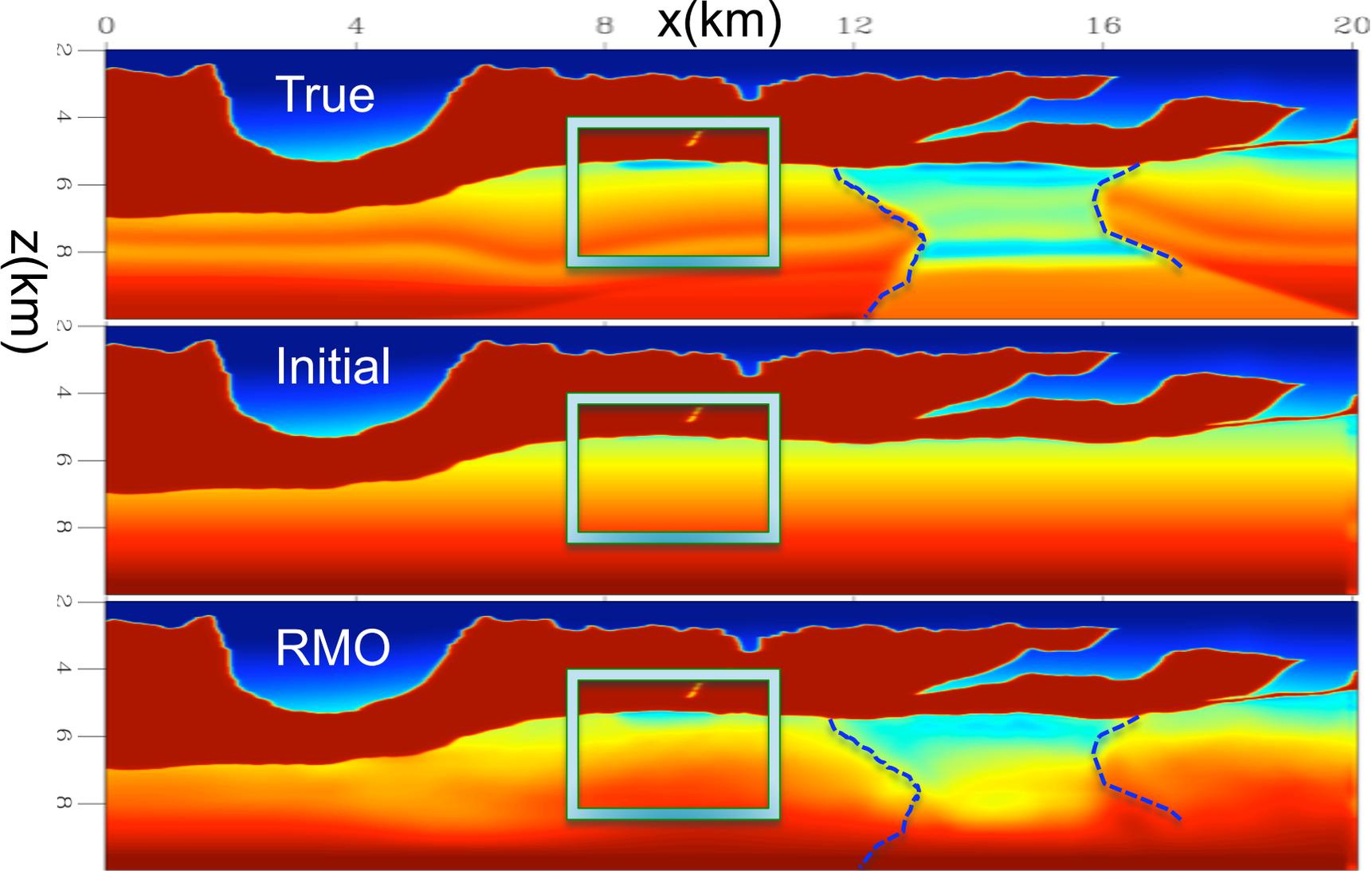


**Stanford  
Exploration  
Project**

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**Stanford Exploration Project**

# BP model results



# Outline

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- Overview of RMO-based wave-equation migration velocity analysis (WEMVA)
- Our approach to speed up the computation
  - ADCIG reconstruction by compressed sensing
  - Approximation of image perturbation back-projection

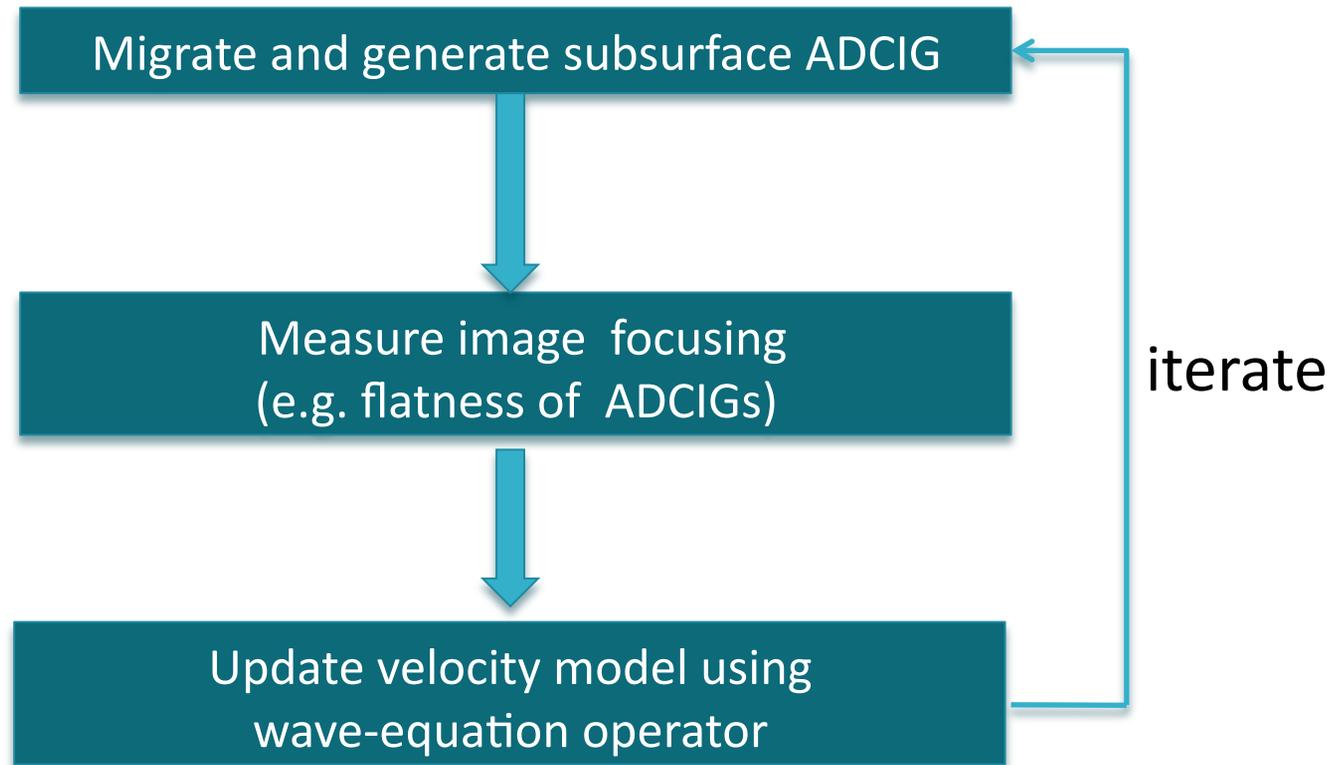
# WEMVA characteristics

---

- Wave-equation migration velocity analysis
  - is a reflection tomography method.
  - optimizes objective function defined in image domain.
  - uses wave-equation operator to compute images & velocity updates

# General WEMVA flow

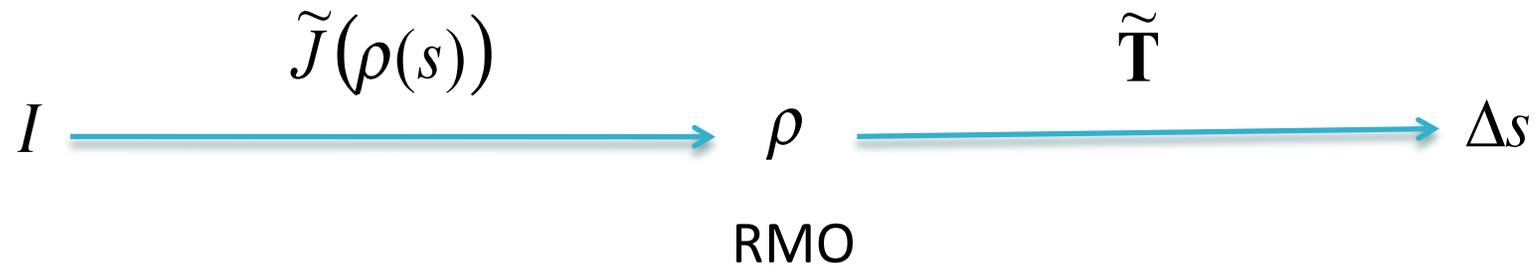
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# RMO-based WEMVA: concept

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Combining the strength of ray tomography  
and wave-equation operators



$\tilde{J}$  : Cost function associates velocity model through moveout

$\tilde{\mathbf{T}}$  : Modified image space wave-equation tomographic operator

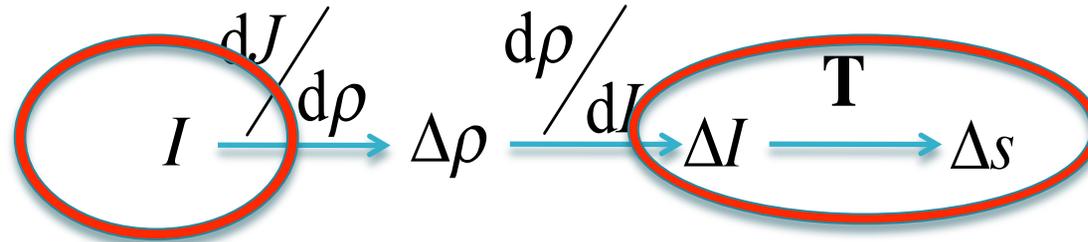
# RMO-based WEMVA: benefits

---

- More robust to the cycle-skipping issue
  - RMO parameters scales linearly with the velocity error
- Produce less artifacts by focusing on kinematic
- Produce the wave-equation based sensitivity kernel

# RMO WEMVA slowness update

---



$I$  : Migrated image (with gathers)

$J$  : RMO WEMVA objective function

$\Delta I$  : Image perturbation

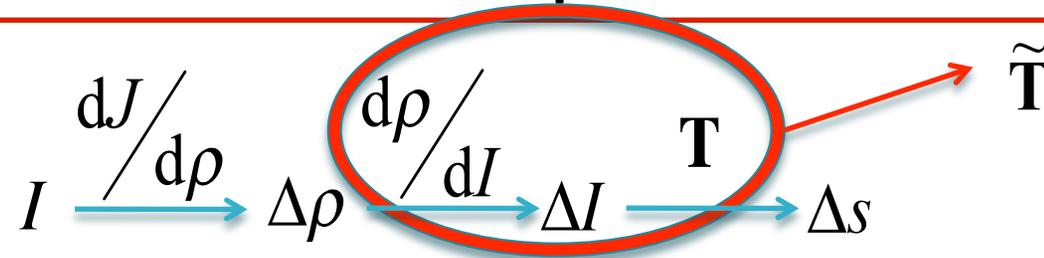
$\rho$  : Residual moveout parameter

$\Delta\rho$  : Residual moveout parameter perturbations

$\Delta s$  : slowness model update

# RMO WEMVA slowness update

---

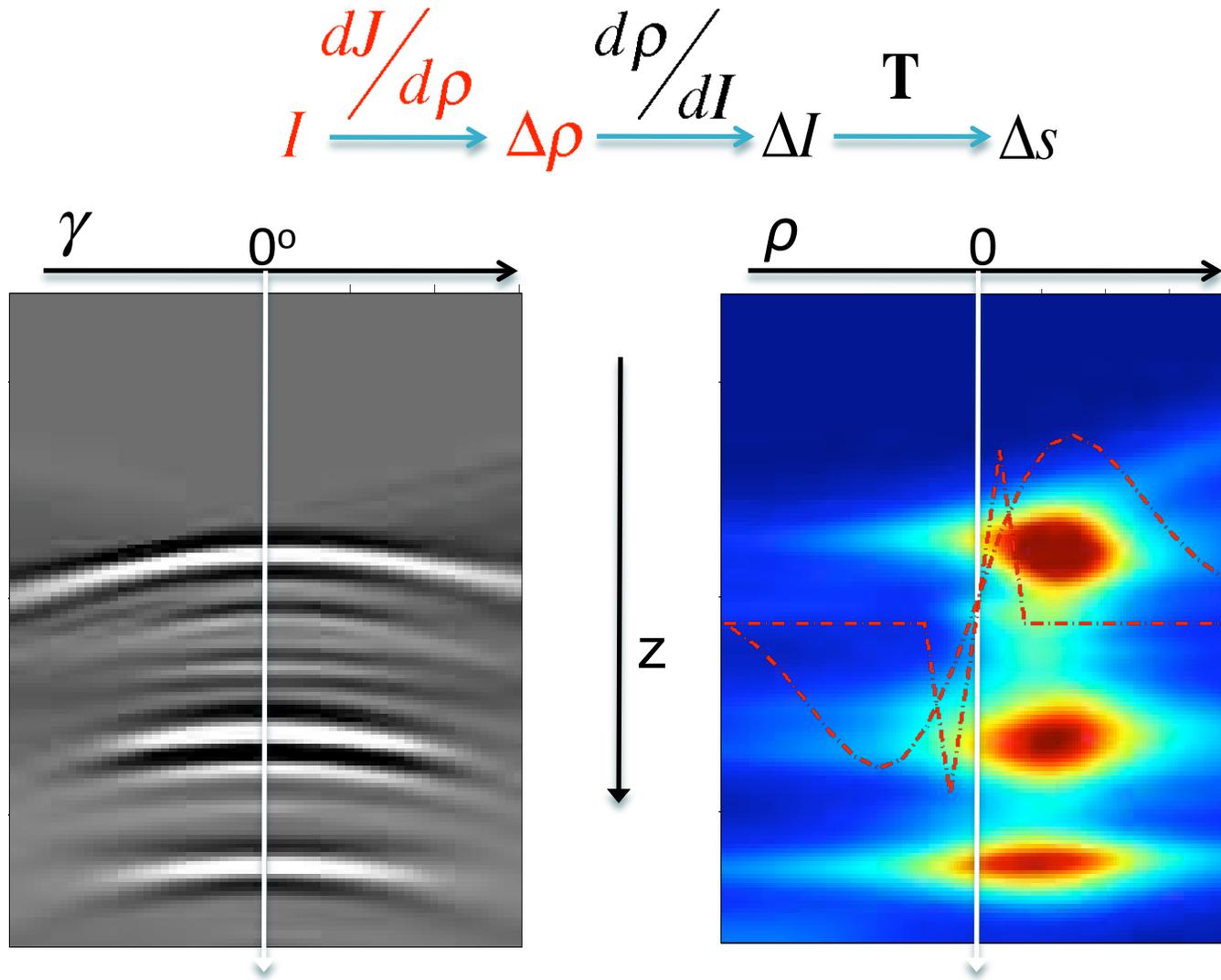


$\rho$  :Residual moveout parameter

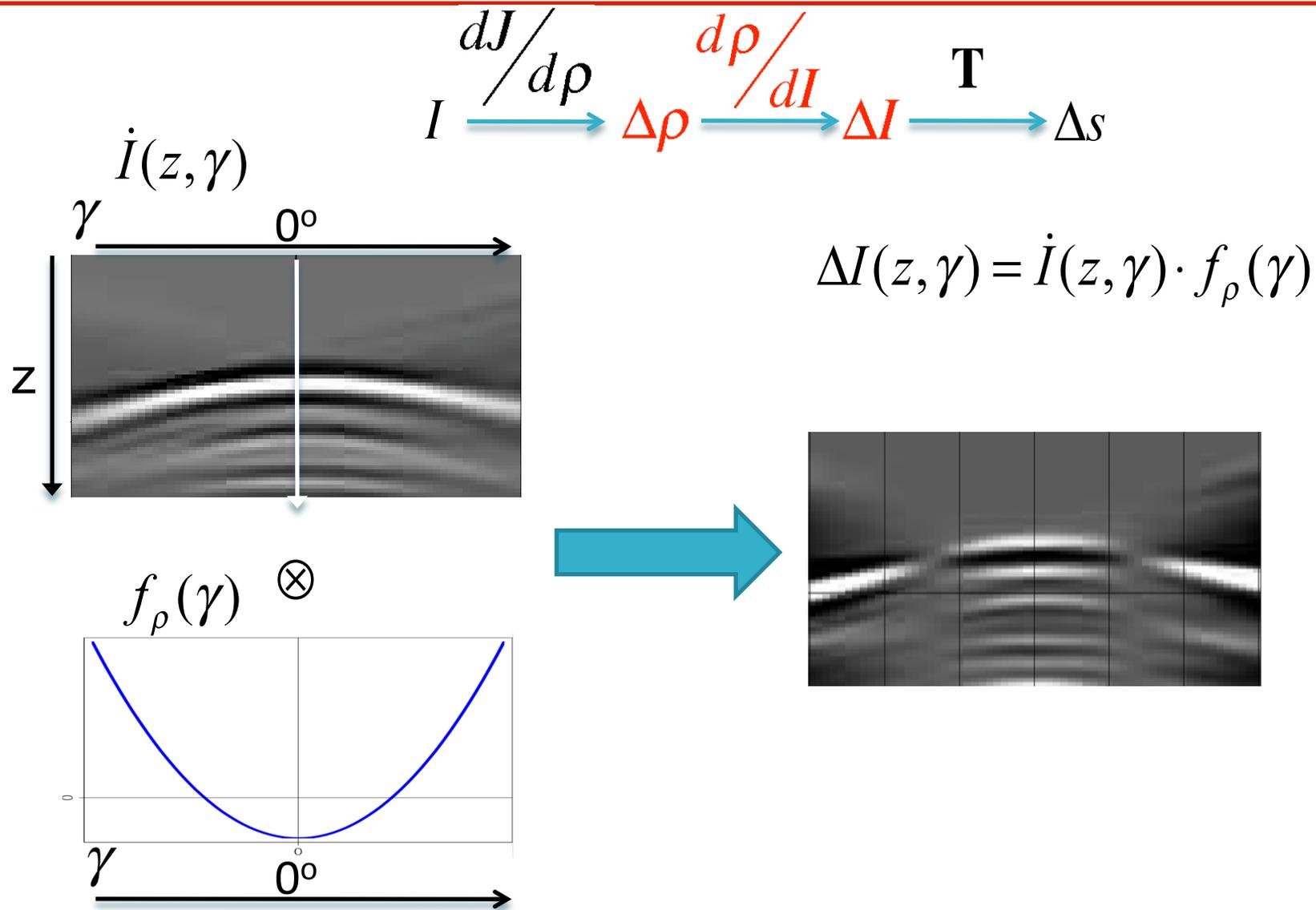
$\Delta\rho$  :Residual moveout parameter perturbations

# Gradient: 1) measure RMO from image

---



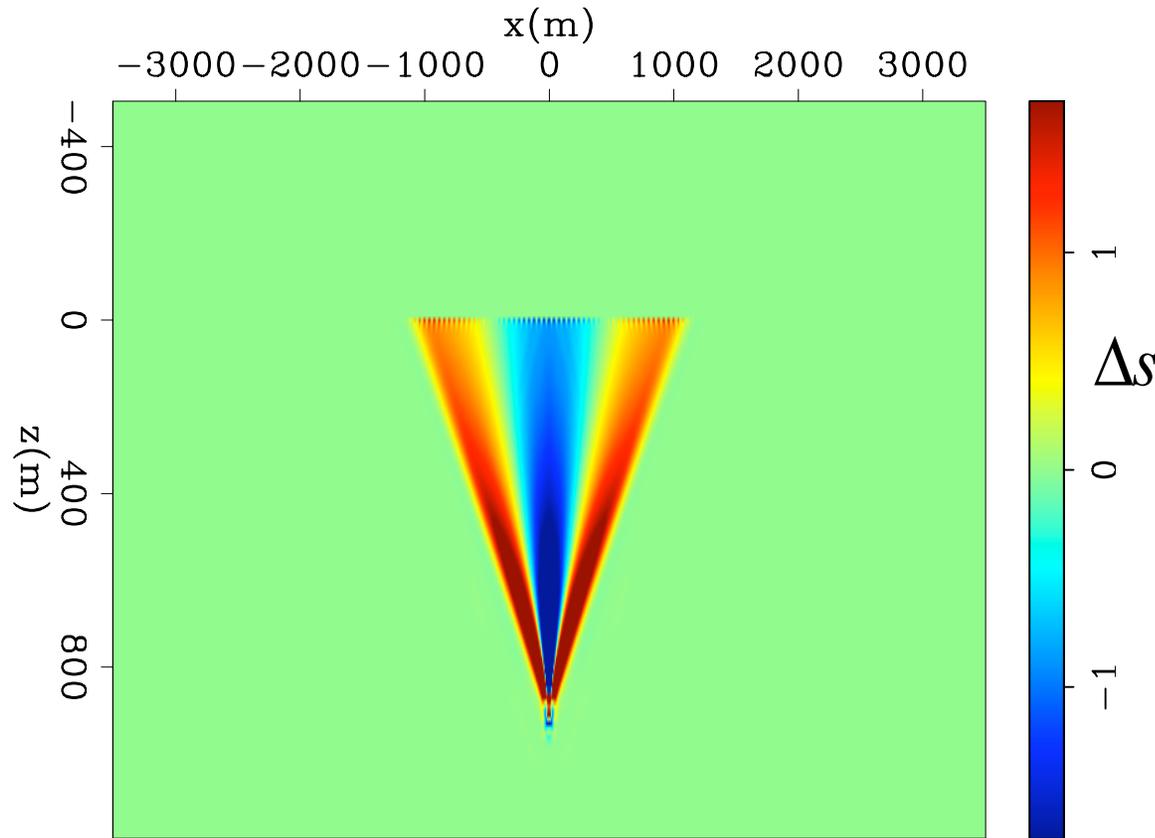
# Gradient: 2) modulate the ADCIG with RMO

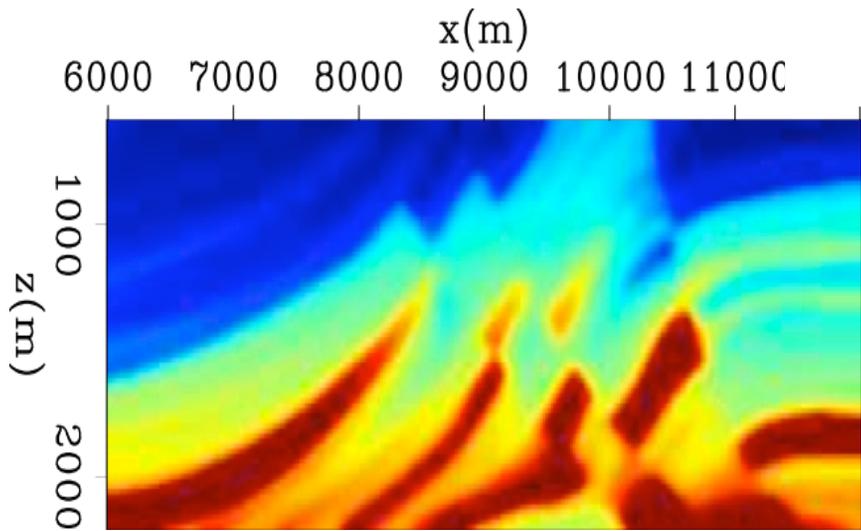


# Gradient: 3) back project $\Delta I$ into model

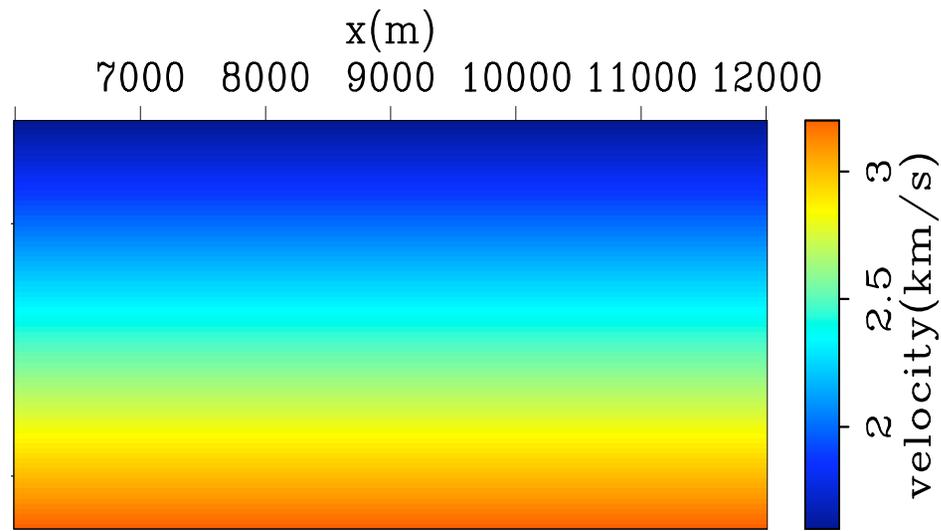
---

$$I \xrightarrow{\frac{dJ}{d\rho}} \Delta\rho \xrightarrow{\frac{d\rho}{dI}} \Delta I \xrightarrow{\mathbf{T}} \Delta s$$

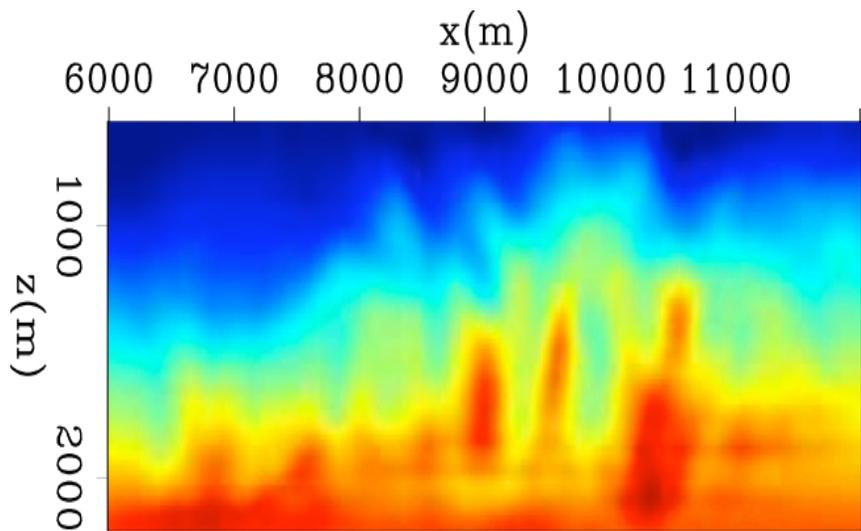




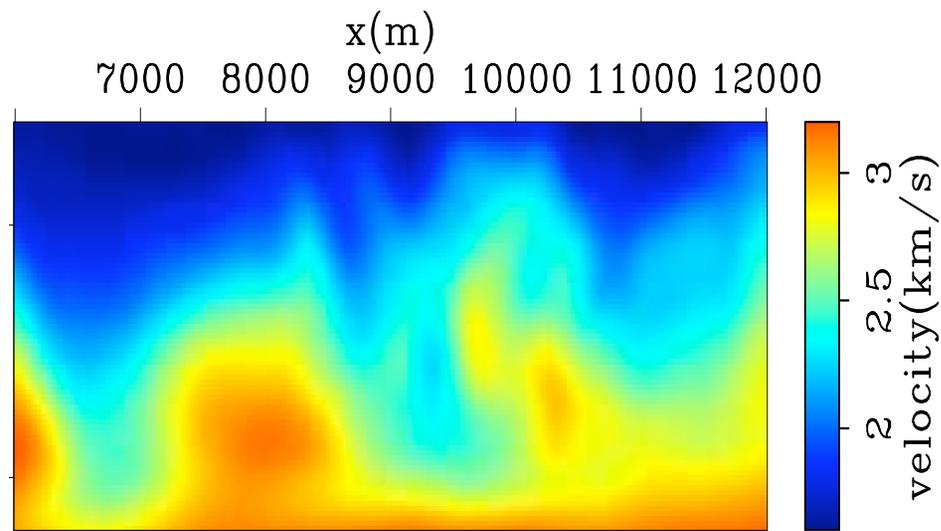
True model



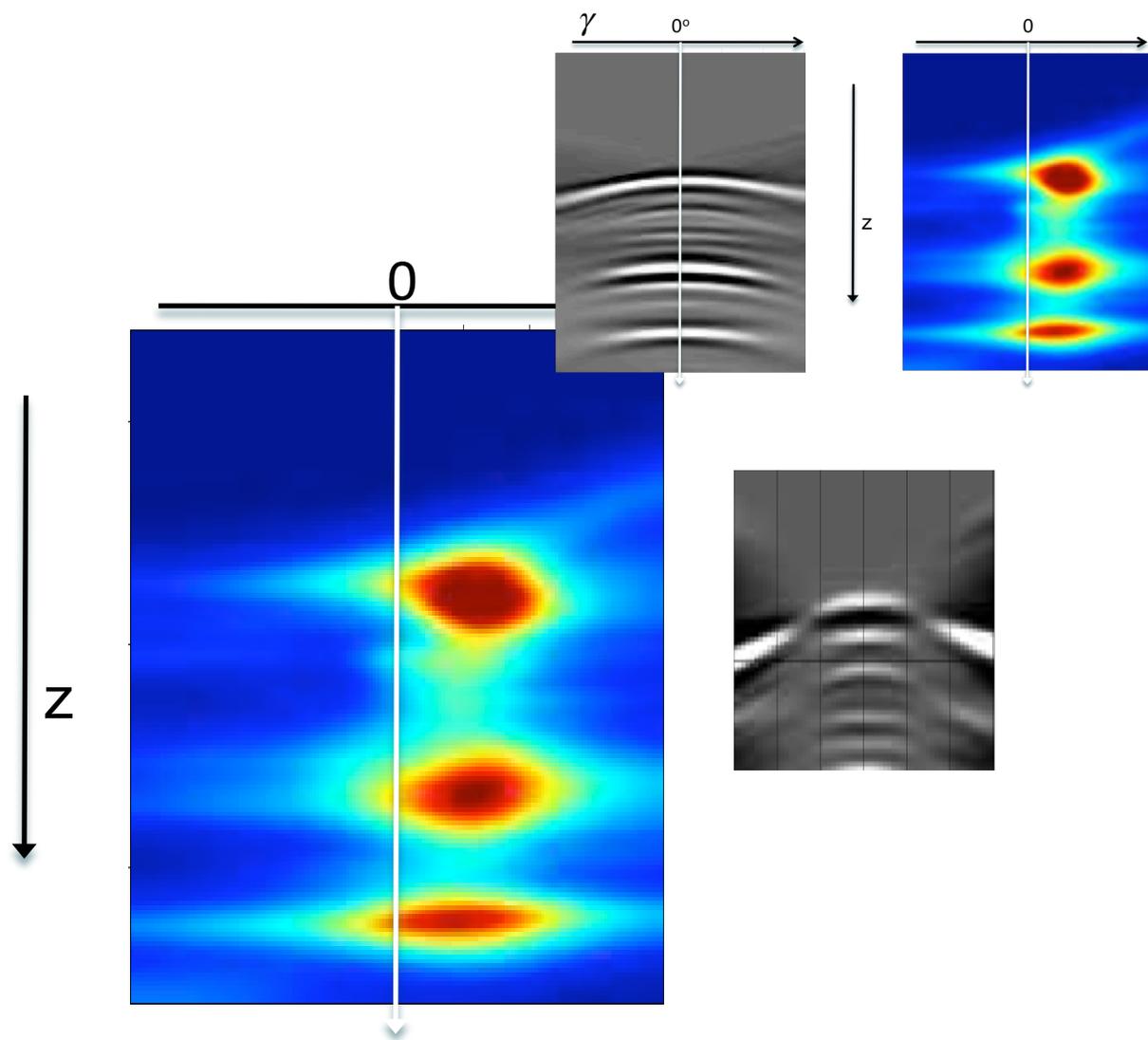
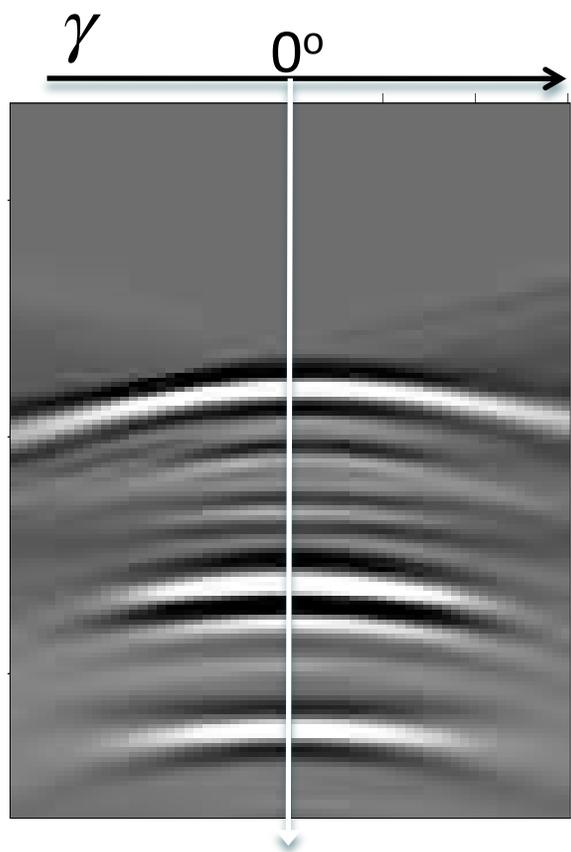
Starting model  $v(z) = v_0 + \alpha z$



Model from WEMVA with RMO



Textbook offset DSO



# Computational expense

---

- Measuring the moveout information, requires one migration with full subsurface offset,  
 $O(n_x * n_y * n_z * n_{h_x} * n_{h_y})$
- Back-projection requires image space wave-equation tomography operator with full subsurface offset,  
 $O(n_x * n_y * n_z * n_{h_x} * n_{h_y})$

# Speed up (I)

---

- Issue: Measuring the moveout information requires one migration with full subsurface offset
- Solution: Reconstruct the full ADCIG without computing all subsurface offset, using **compressed-sensing (CS)** (Clapp, SEP-147 & SEP-149),  $O(n_x * n_y * n_z * n_{h_x} * n_{h_y})$

# Equations mathType

---

$d$ : ODCIG;  $m$ : ADCIG in wavelet domain

$\mathbf{M}_d$ : Random subsampling of the ODCIG

$\mathbf{R}$ : Wavelet transform + angle to offset transform

$$J(\mathbf{m}) = \|\mathbf{M}_d(\mathbf{d} - \mathbf{Rm})\|_2^2 + \epsilon \|\mathbf{Am}\|_2^2$$

$\mathbf{R}$ : Transform operator;  $\mathbf{A}$ : regularization operator

$\mathbf{d}$ : ODCIG;  $\mathbf{m}$ : ADCIG in wavelet domain

$\mathbf{M}_d$ : Random subsampling of the ODCIG

$\mathbf{R}$ : Wavelet transform + angle to offset transform

$d$ : ODCIG;  $m$ : ODCIG in wavelet domain

$\mathbf{M}_d$ : Random subsampling of the ODCIG

$\mathbf{R}$ : Wavelet transform

---

# ADCIG reconstruction by compressed sensing

---

$$J_{cs}(m) = \|\mathbf{M}_d(\mathbf{d} - \mathbf{Rm})\|_2^2 + \epsilon \|\mathbf{m}\|_1$$

$\mathbf{d}$ : ODCIG;  $\mathbf{m}$ : ODCIG in wavelet domain

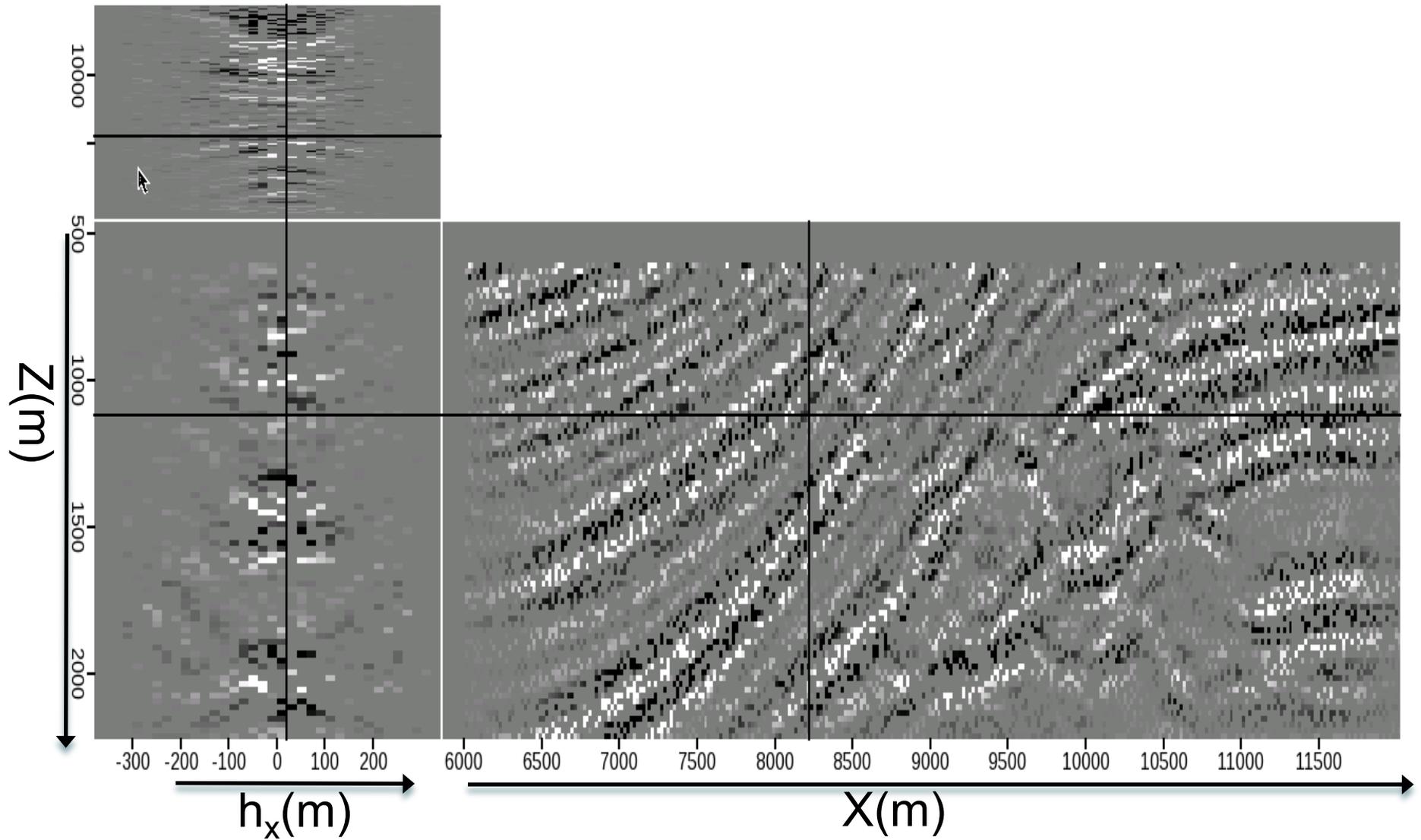
$\mathbf{M}_d$ : Random subsampling of the ODCIG

$\mathbf{R}$ : Wavelet transform

- The subsampling ratio we chose
  - 5 to 1 in my example, 2-D
  - 10 to 1 in Clapp's example (SEP147,149), 3-D inline offset
  - Should do even better in full 3-D, high dimension gives higher compression ratio

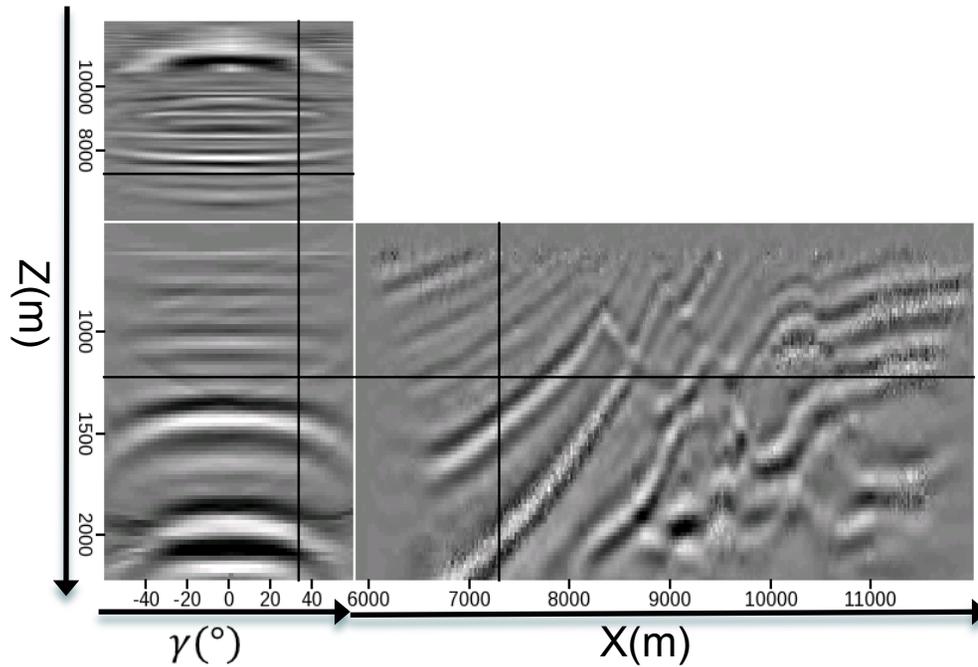
# Example

---



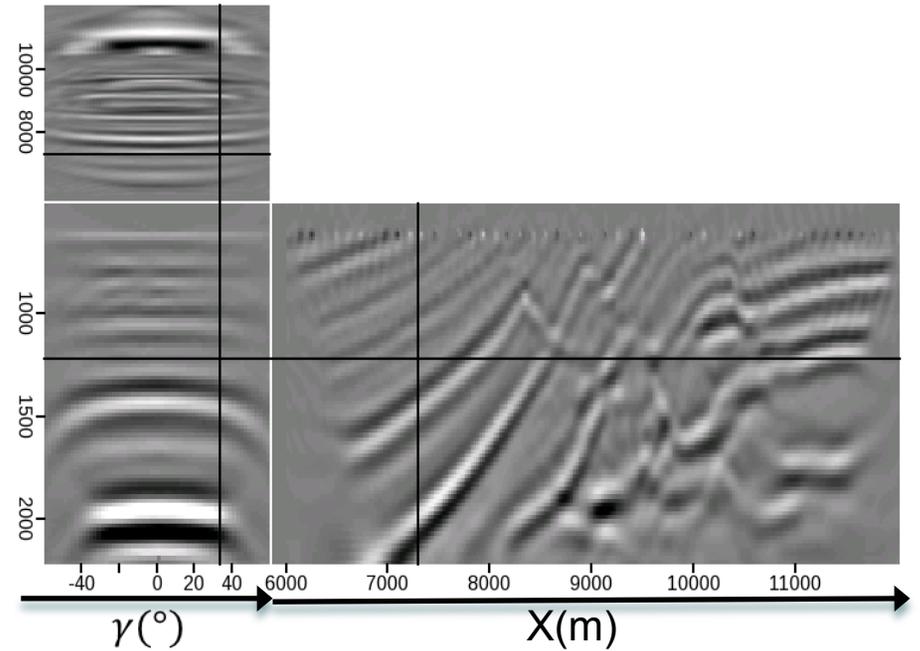
# Reconstructed ADCIG

---



(a)

Reconstructed ADCIG



(b)

Full ADCIG

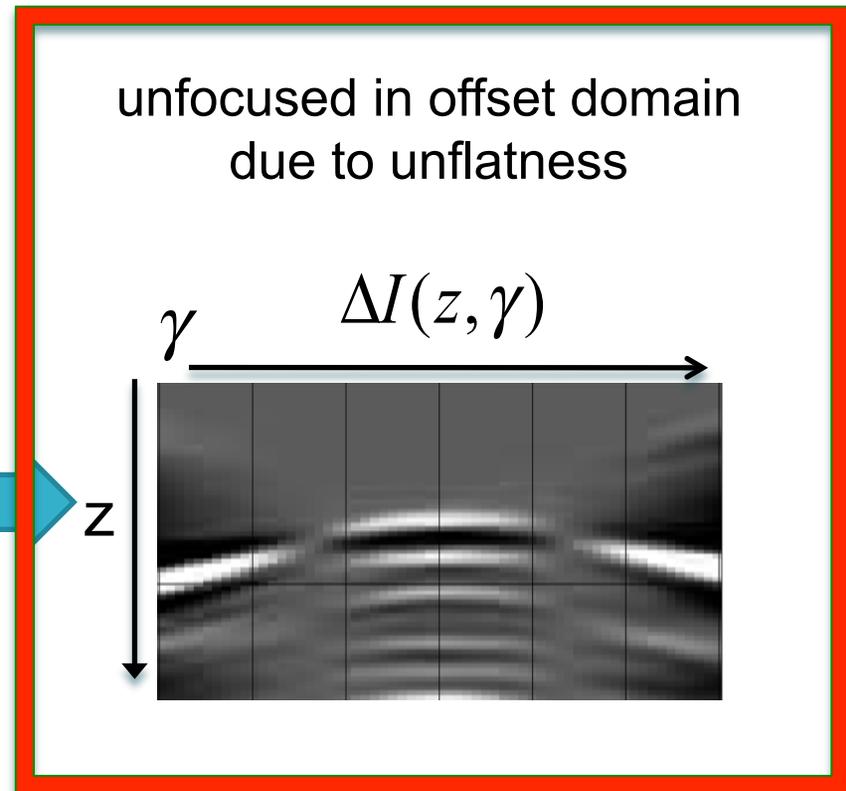
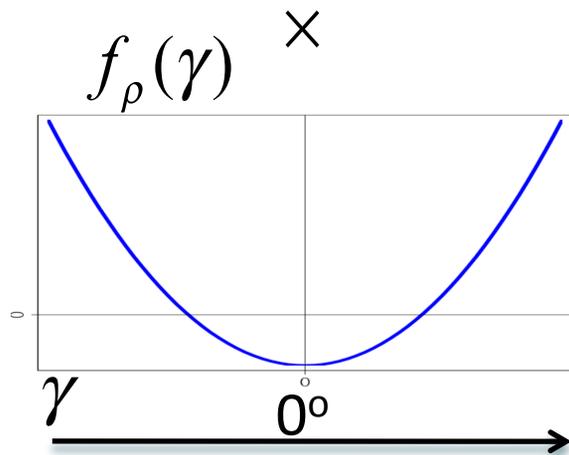
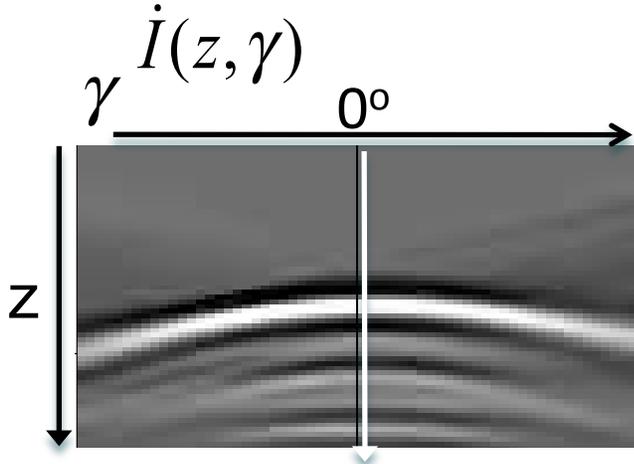
# Speed up (II)

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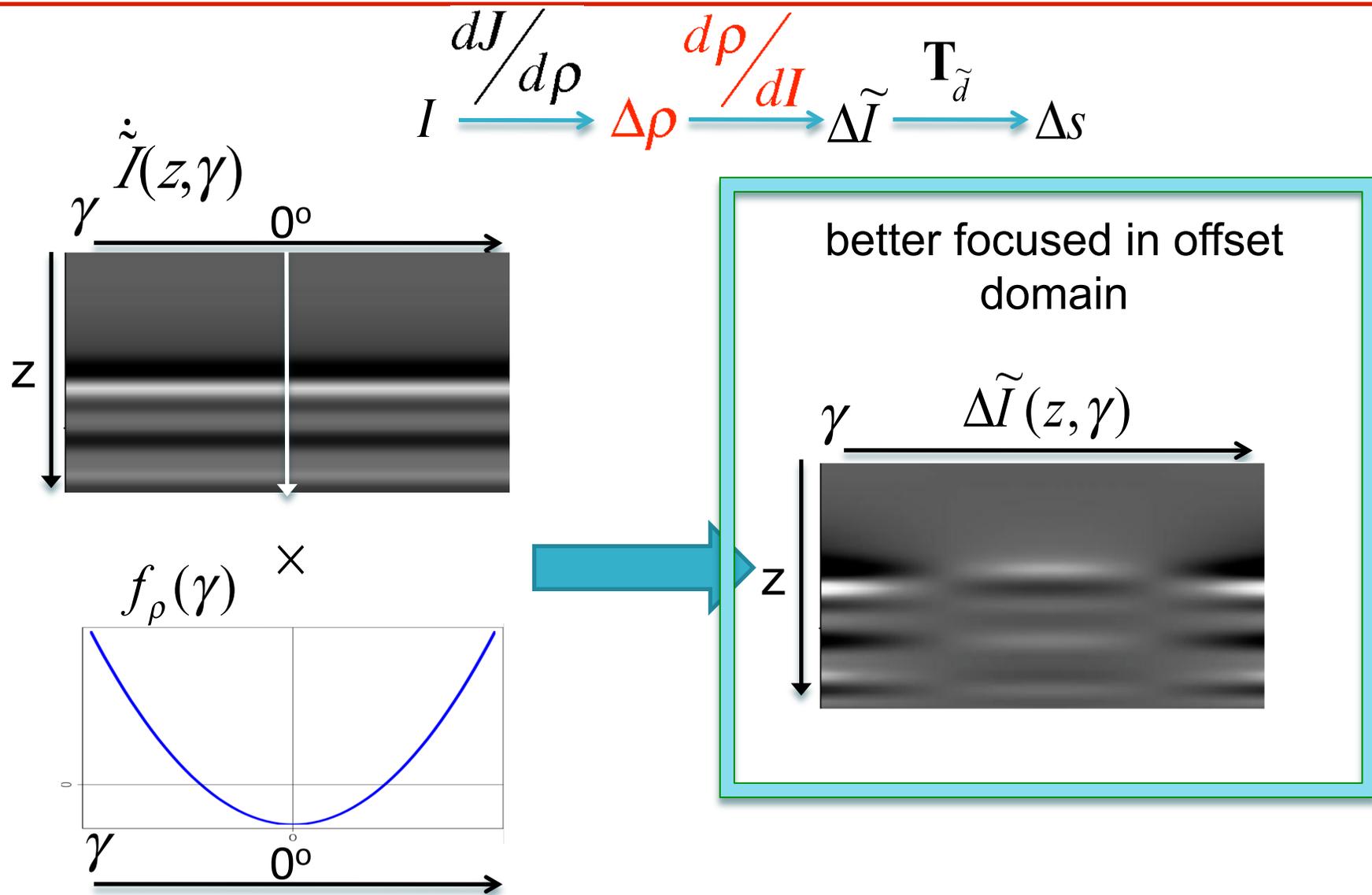
- Issue: Back projection requires image space wave-equation tomographic operator with full subsurface offset.
- Solution: Approximate the back-projection operator such that it requires fewer subsurface offsets,  
 $O(n_x * n_y * n_z * n_{h_x} * n_{h_y})$

# Gradient back-projection, approximation

$$I \xrightarrow{\frac{dJ}{d\rho}} \Delta\rho \xrightarrow{\frac{d\rho}{dI}} \Delta I \xrightarrow{\mathbf{T}_d} \Delta s$$

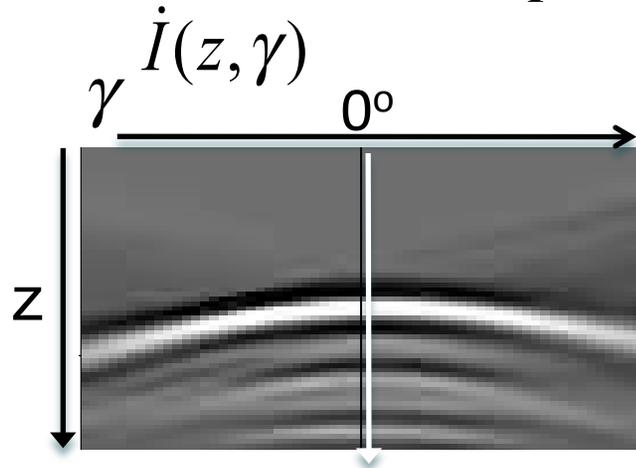


# Gradient back-projection, approximation

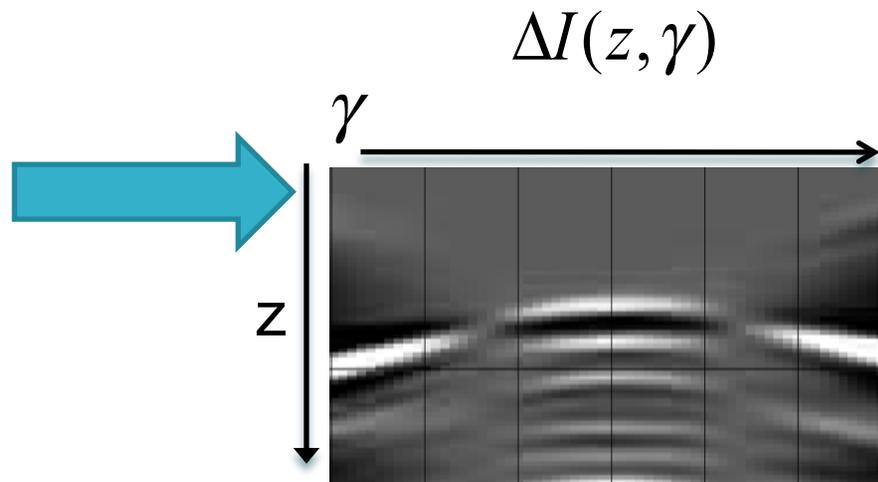
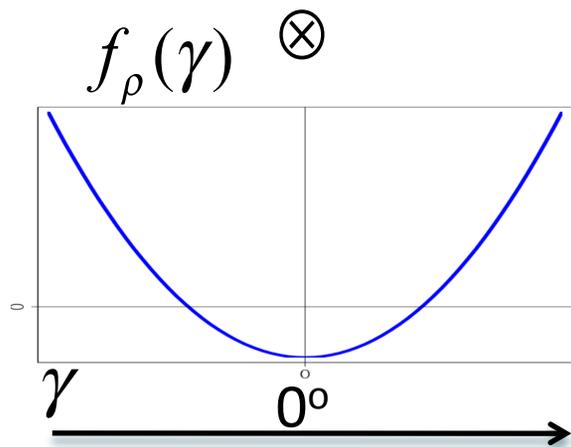


# Gradient back-projection, approximation

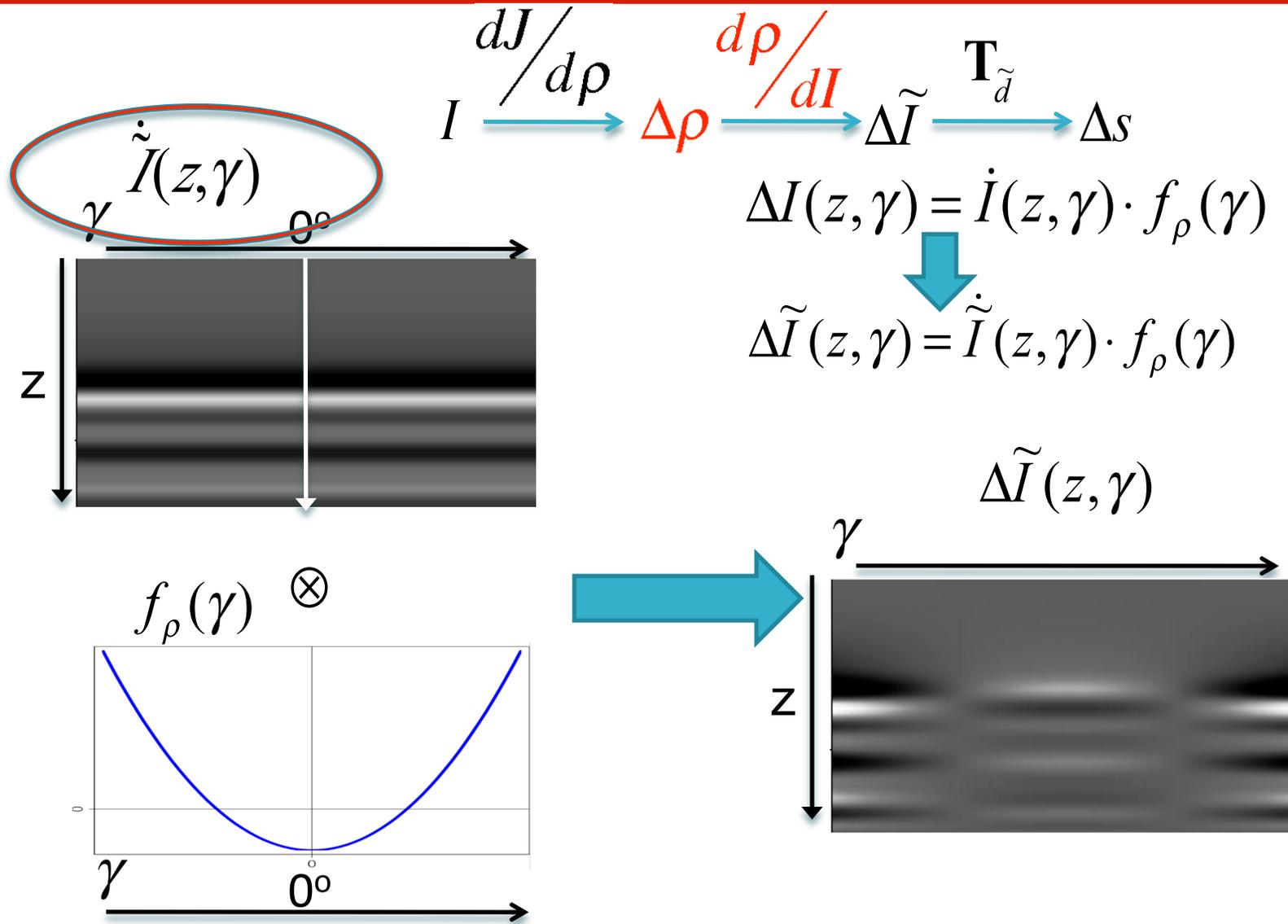
$$I \xrightarrow{\frac{dJ}{d\rho}} \Delta\rho \xrightarrow{\frac{d\rho}{dI}} \Delta I \xrightarrow{\mathbf{T}_d} \Delta s$$



$$\Delta I(z, \gamma) = \dot{I}(z, \gamma) \cdot f_\rho(\gamma)$$



# Gradient back-projection, approximation

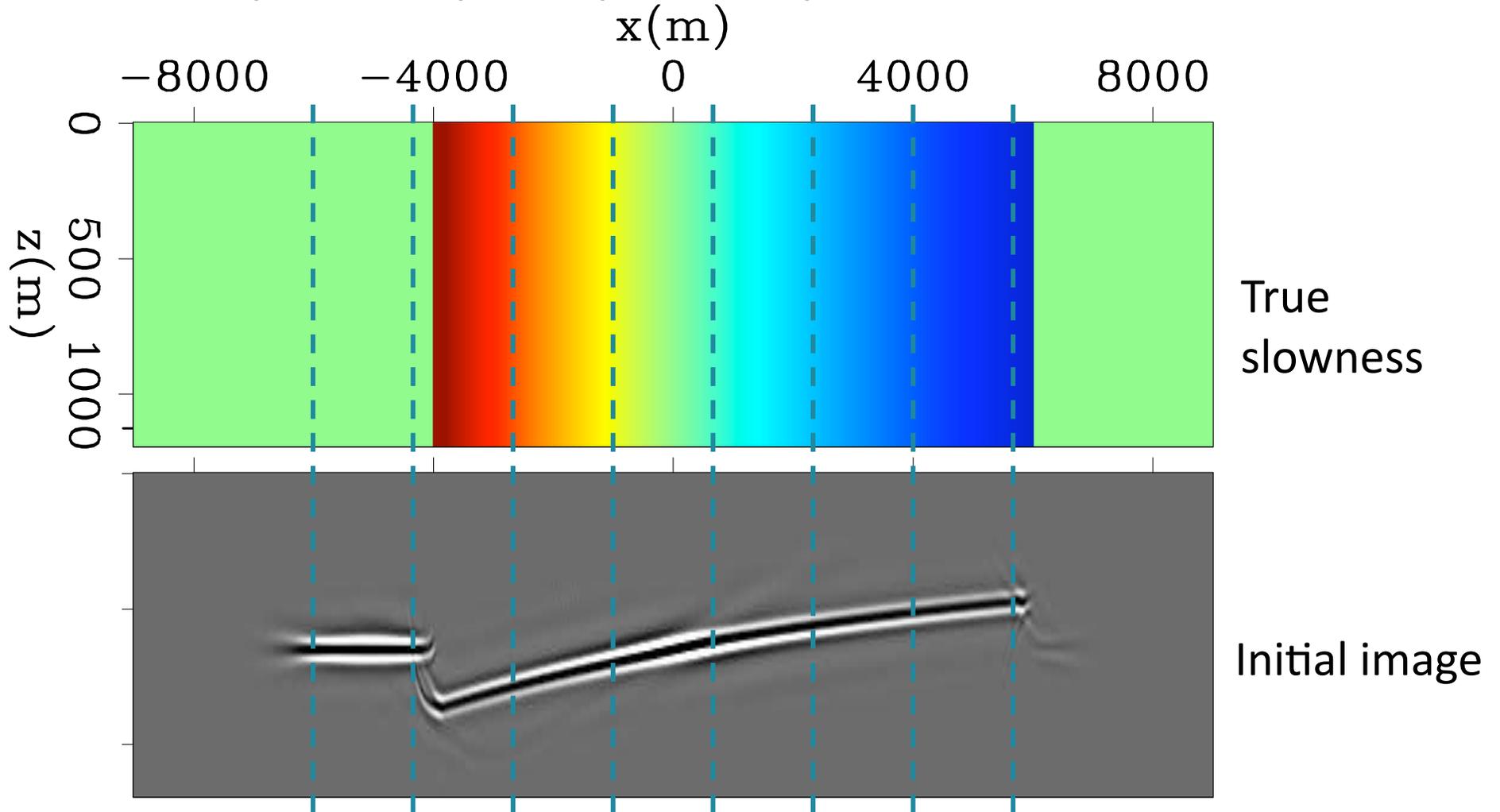


# Analogy to ray-based tomography

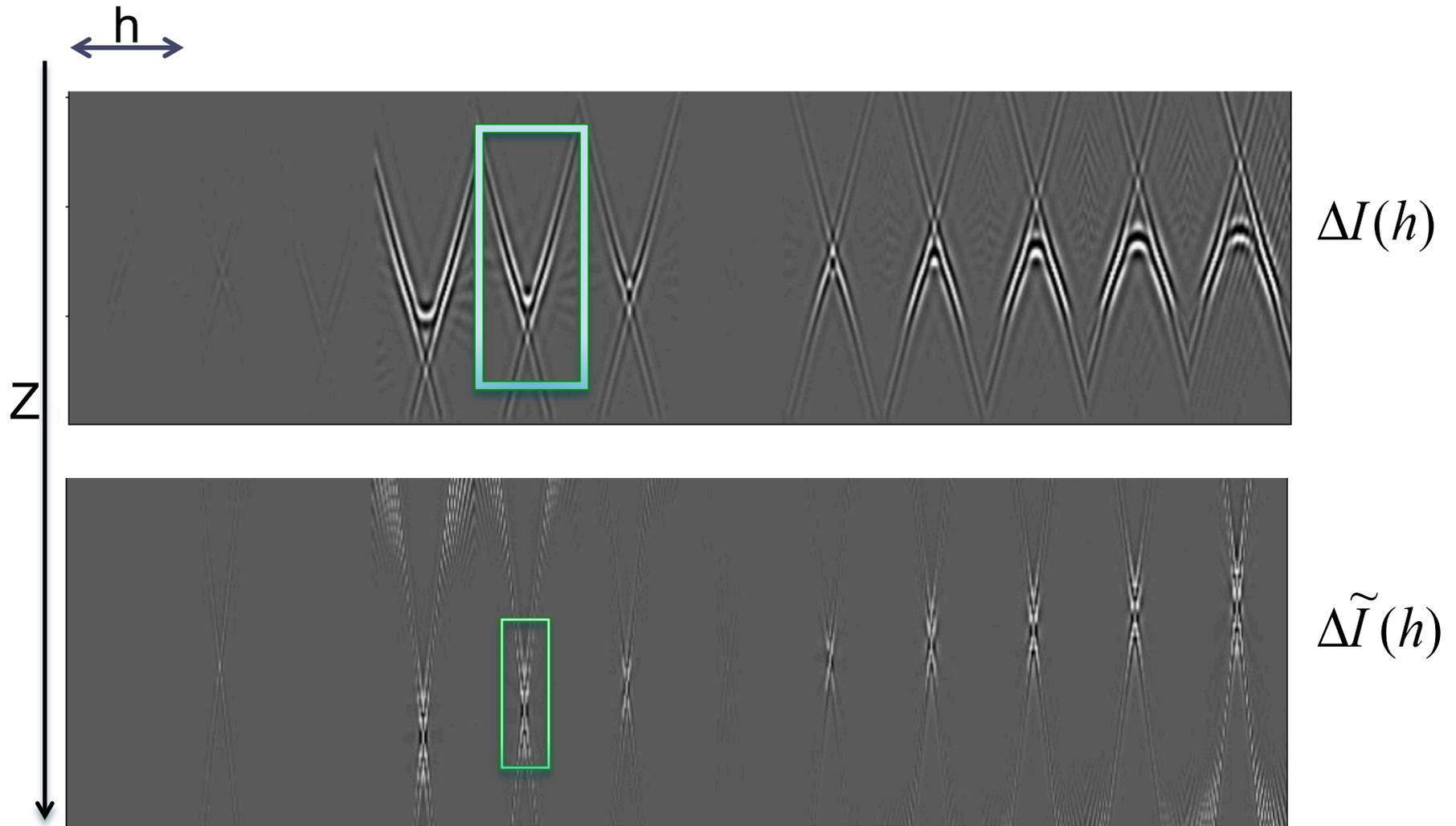
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# Flat reflector & horizontal velocity gradient

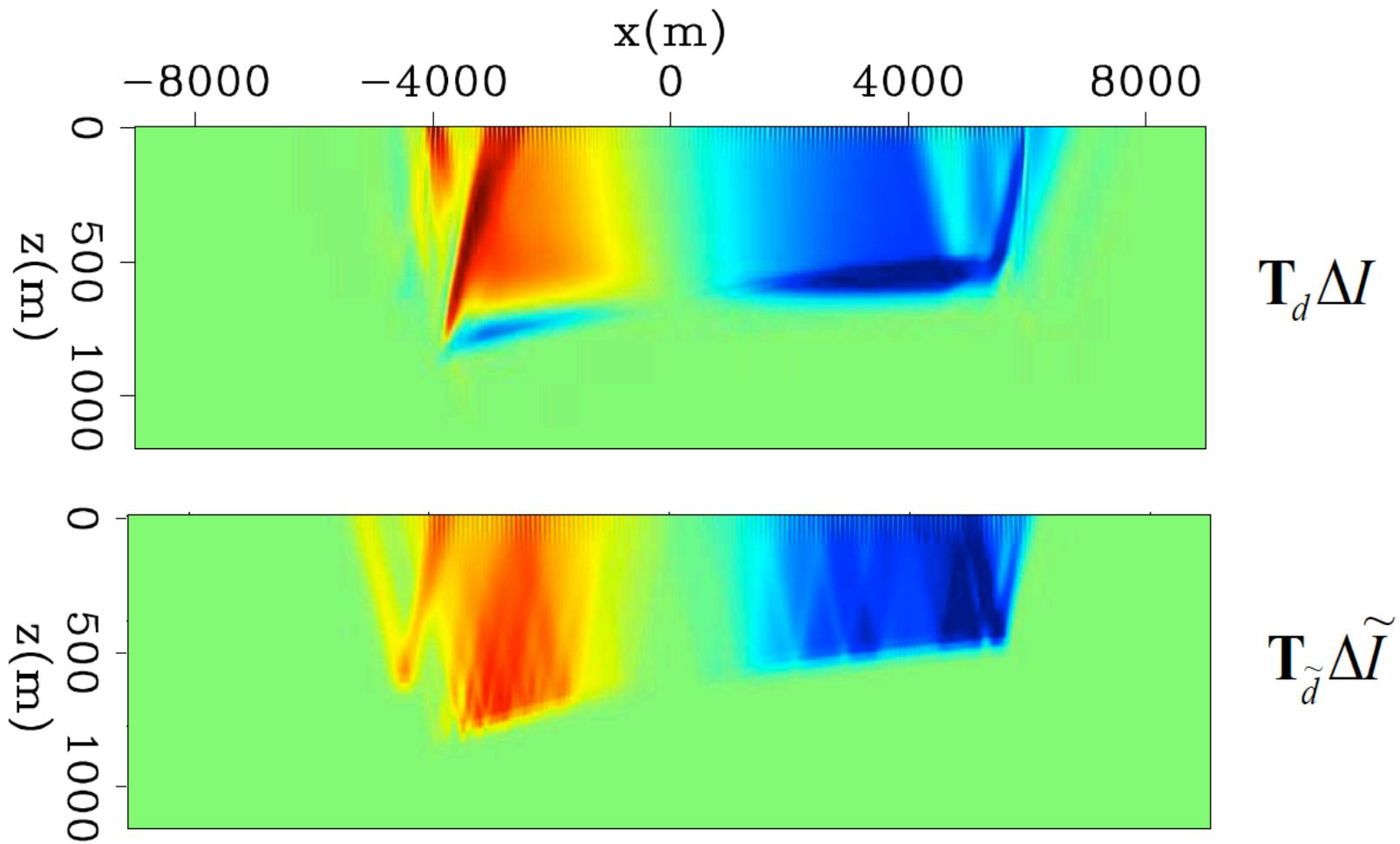
from  $1/(1500\text{m/s})$  to  $1/(2700\text{m/s})$



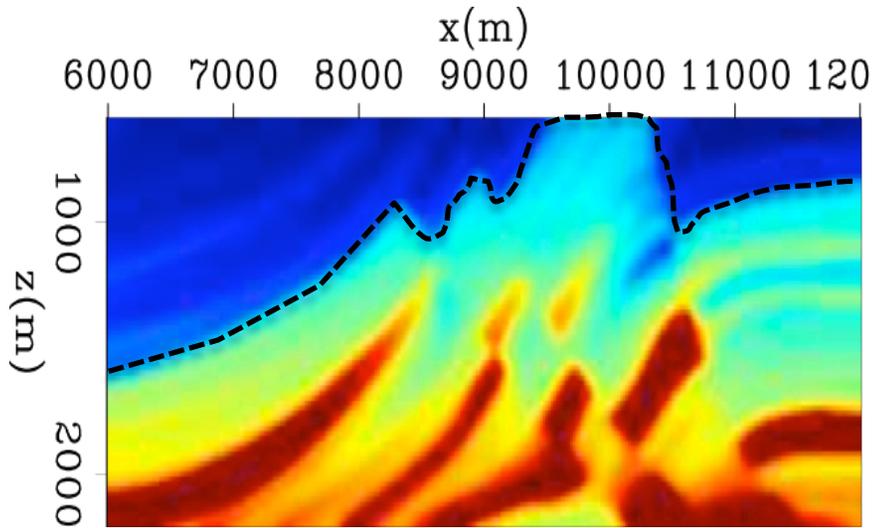
# Comparisons of perturbed images



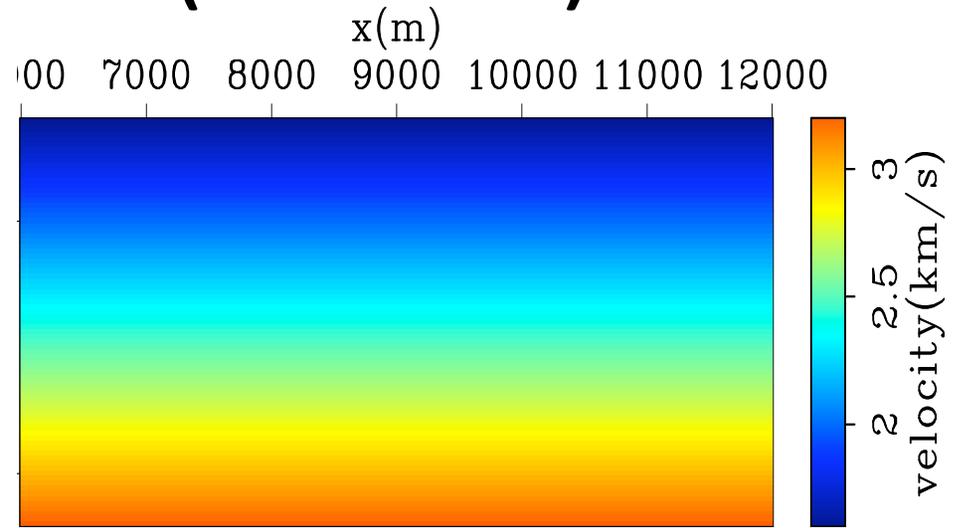
# Comparisons of model gradient



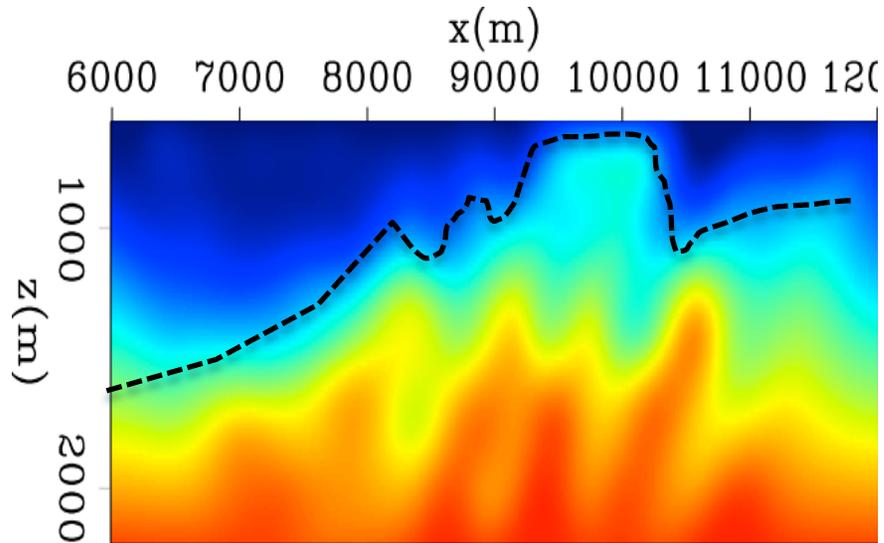
# Inversion result (20 iters)



True model

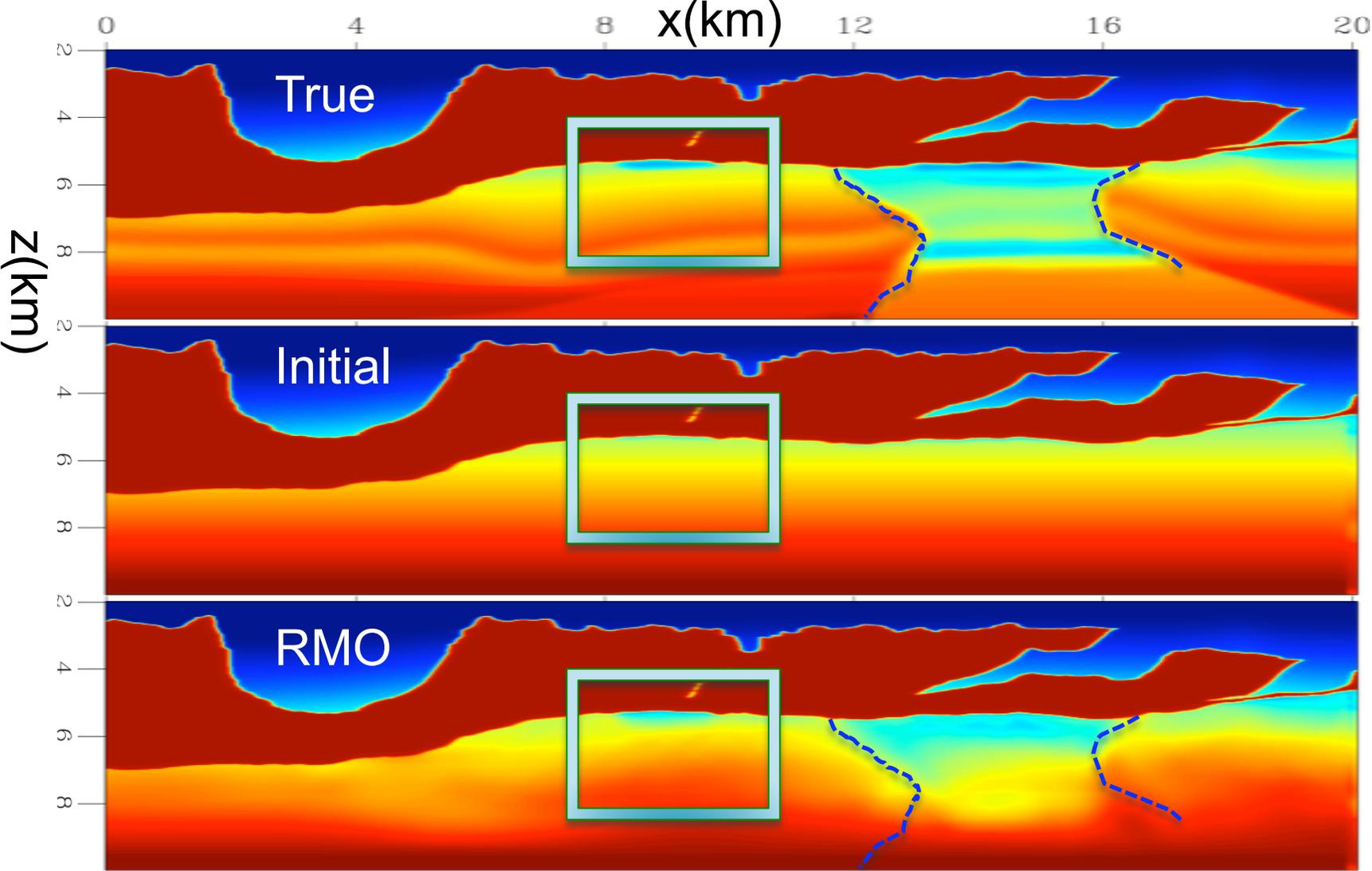


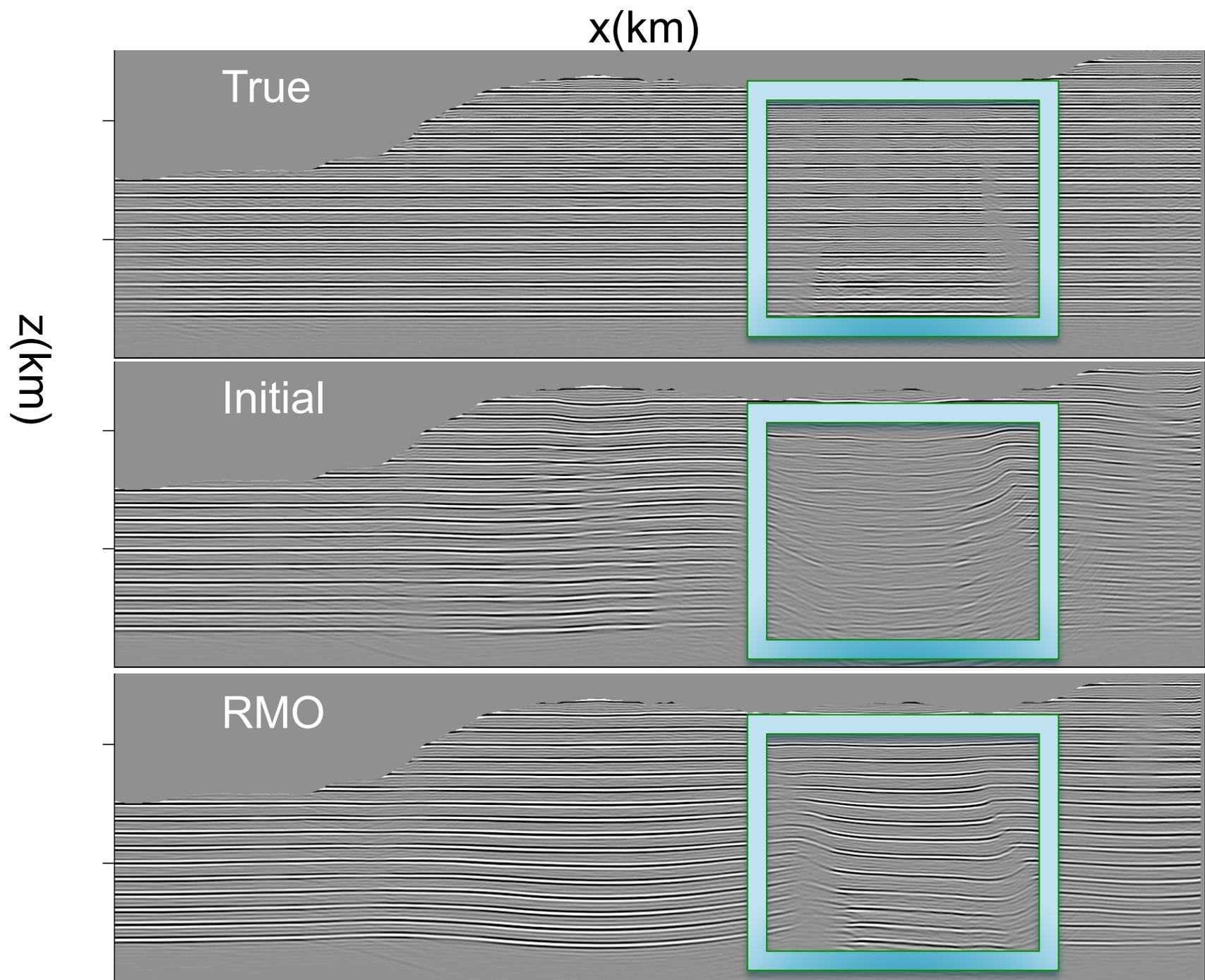
Starting model  $v(z) = v_0 + \alpha z$

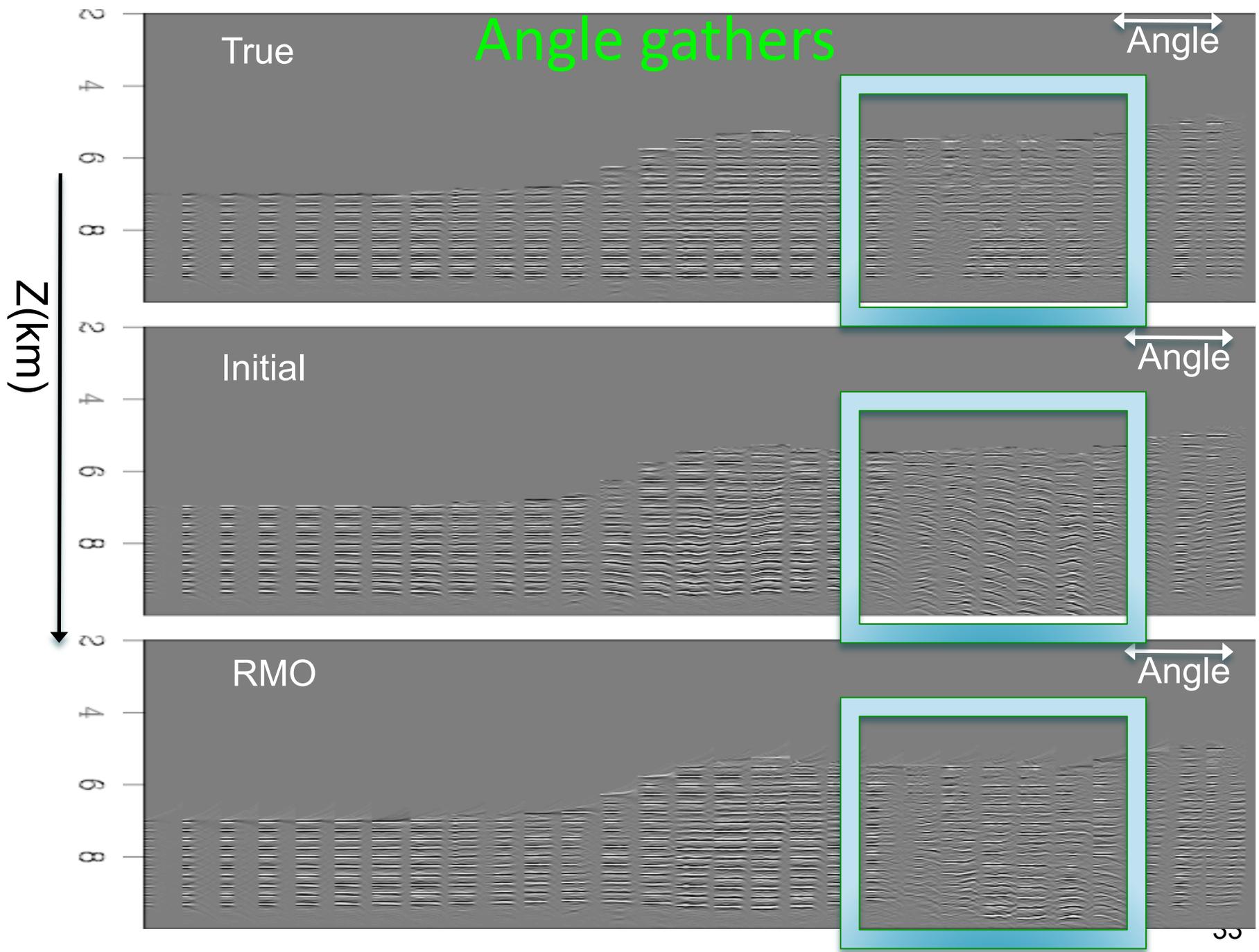


Accelerated RMO-WEMVA

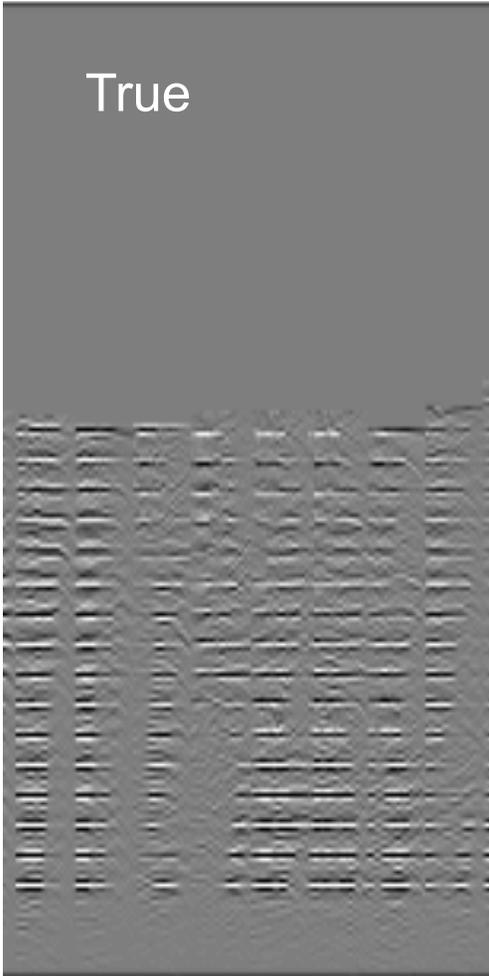
# BP model results



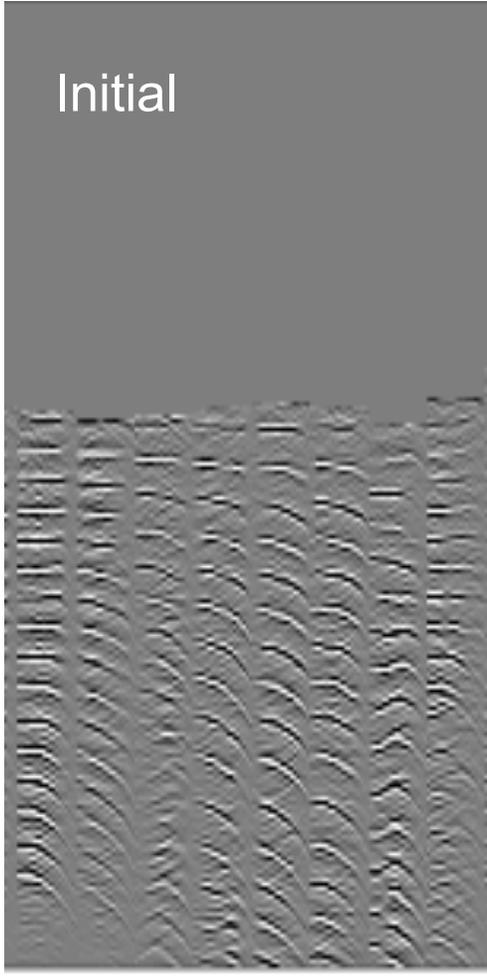




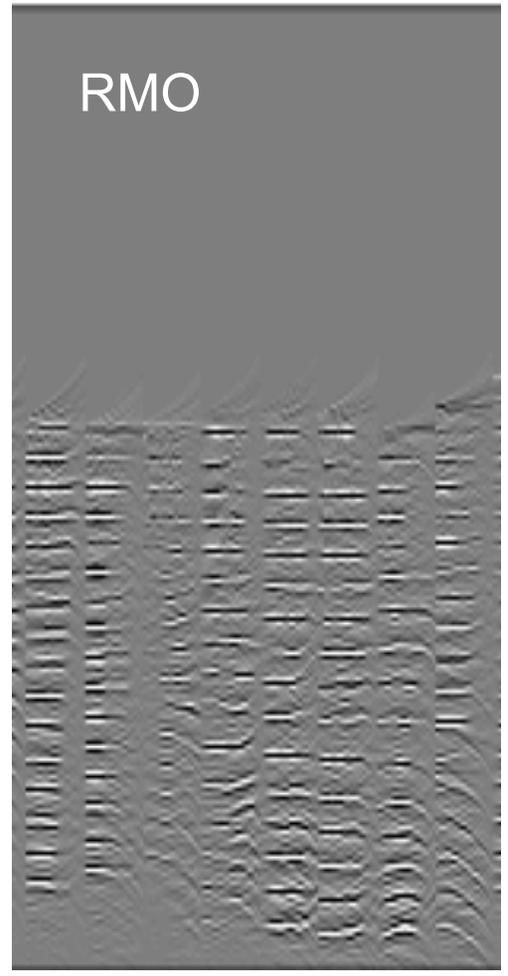
True



Initial



RMO



# Computation savings

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- For my 2-D Marmousi & BP example
  - Migration cost reduced to 20% of original
  - Applying back projection operator also reduce to 20% (subsurface offset reduce from 35 to 7)
  - Total saving is approximately 5-fold.
- In 3-D case, the speed up should be significantly better (>50x)

# Conclusion

---

- We propose acceleration approaches to speed up the RMO-based WEMVA
  - Using compressed sensing, we can reduce the imaging cost by calculating only a portion of the full CIGs
  - By back-projecting a synthesized data/image pair, we can reduce the cost of tomographic operator by having better focusing
    - Such approximation further decouples the RMO calculation phase and the back-projection phase, making it more robust

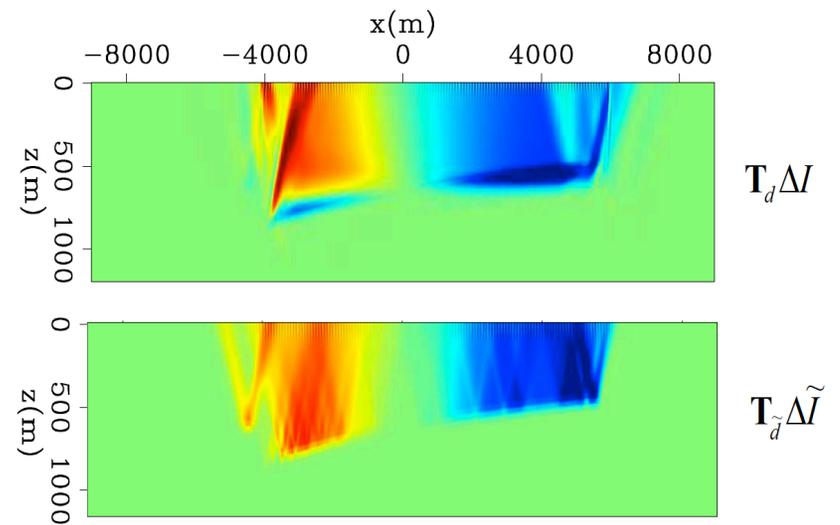
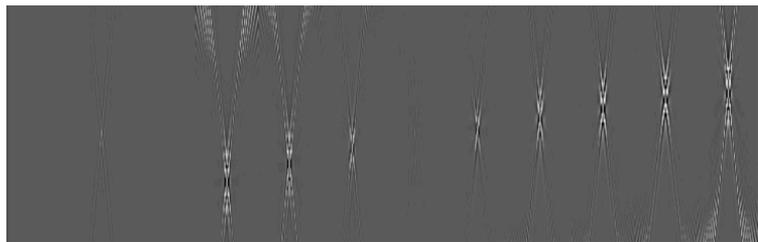
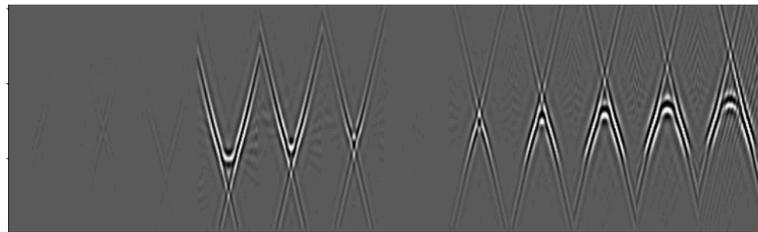
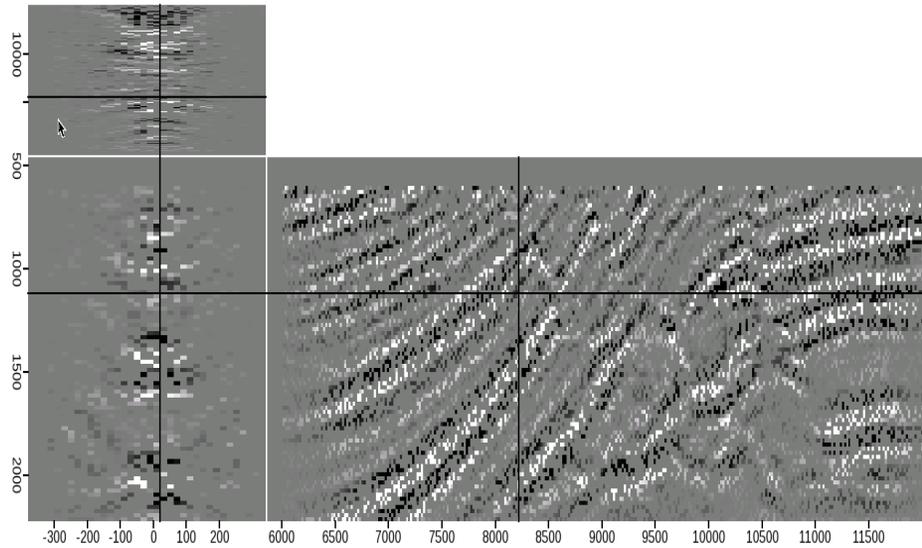
# Acknowledgement

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- Dave Nichols

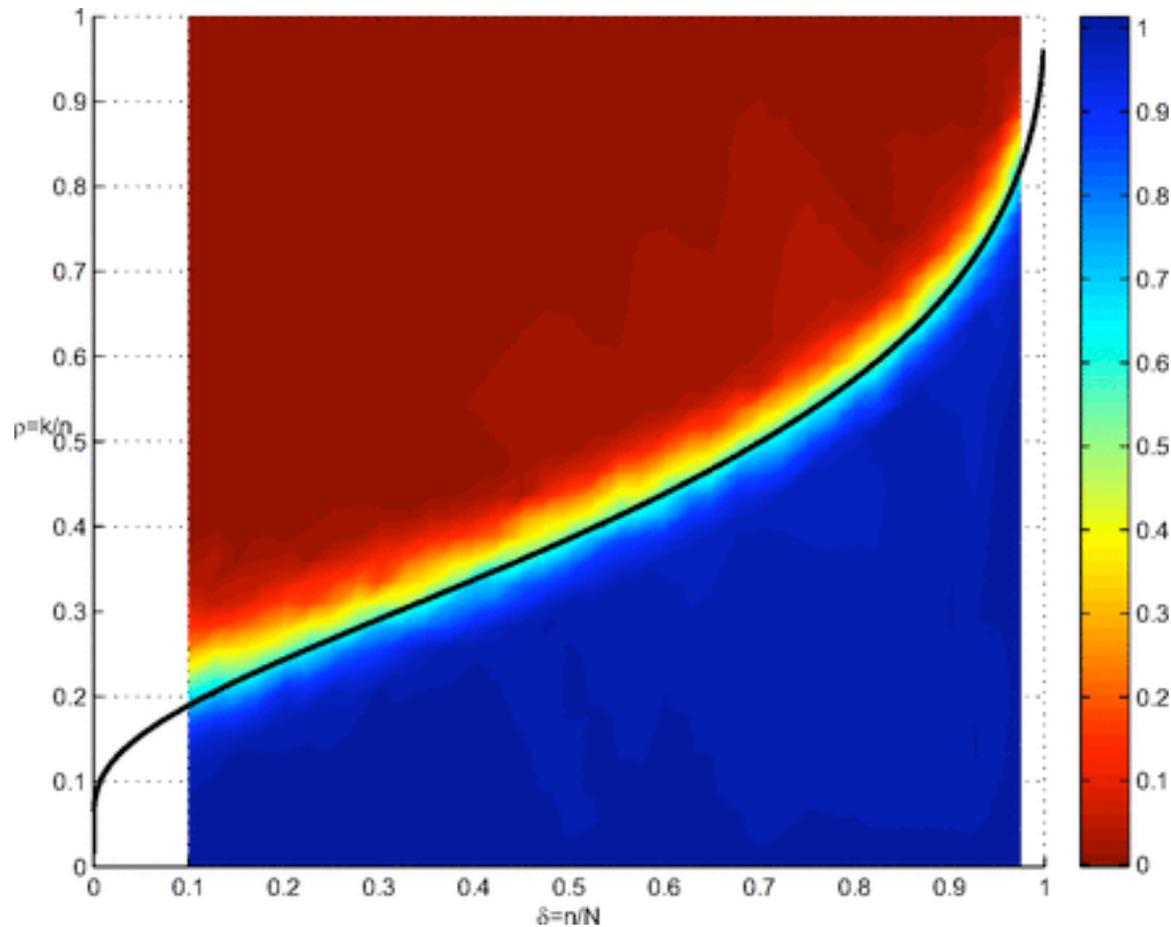
# Questions?

yang@sep.stanford.edu



# The phase transition curve of CS

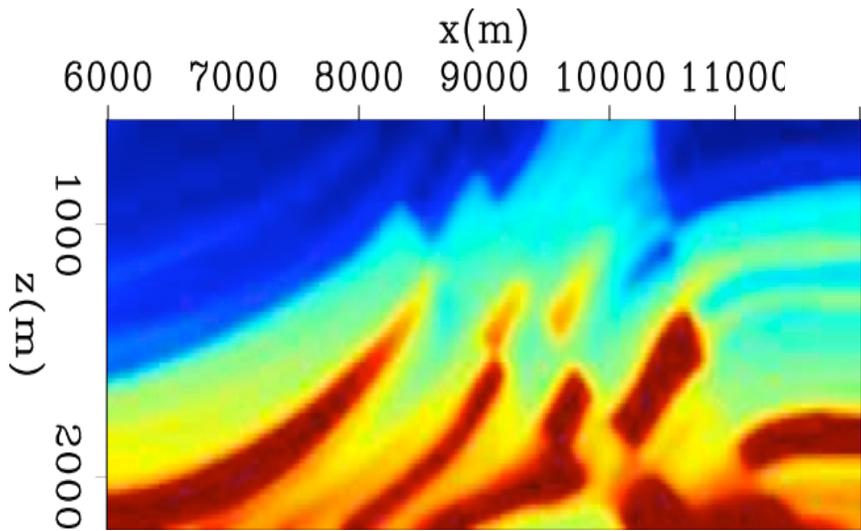
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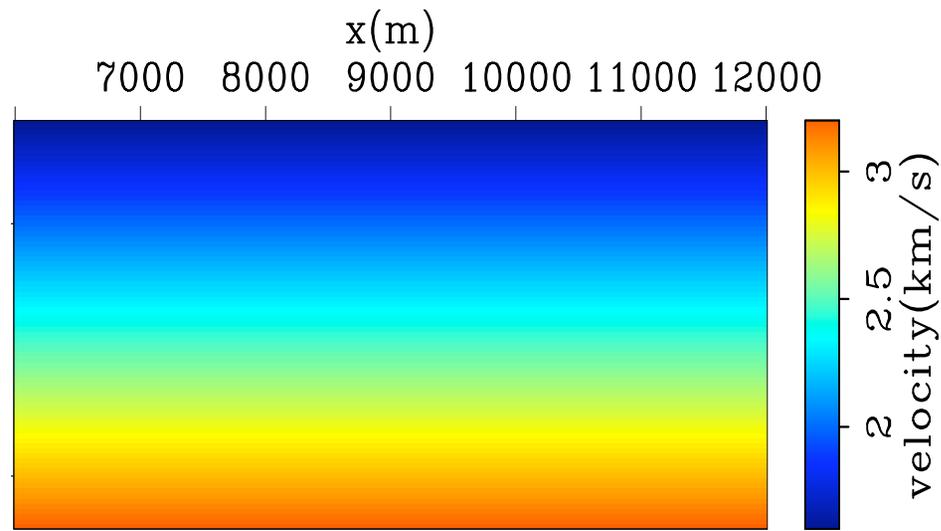
# Synthetic-data example

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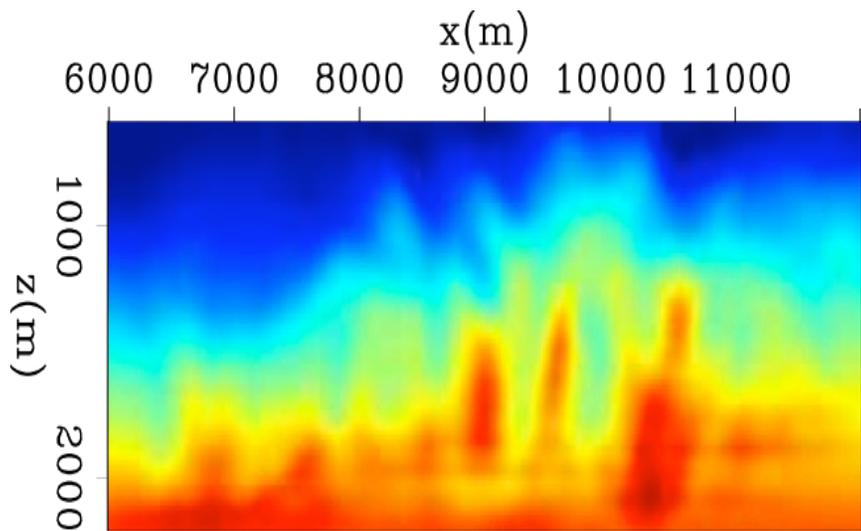
- Model 16km X 1.2km
- Sampling 20m X 10m
- Marine acquisition
  - 150 shots from -6km to +6km, spacing 80m
  - 201 receivers, spacing 20m, i.e. 4km cable length
- One way propagator is used, frequency from 5Hz to 40Hz, 106 frequencies
- Initial slowness model is constant,  $1/(2000\text{m/s})$
- Gradient smoothing using B-splines
- Line search by parabolic approximation
- 5 non-linear iterations



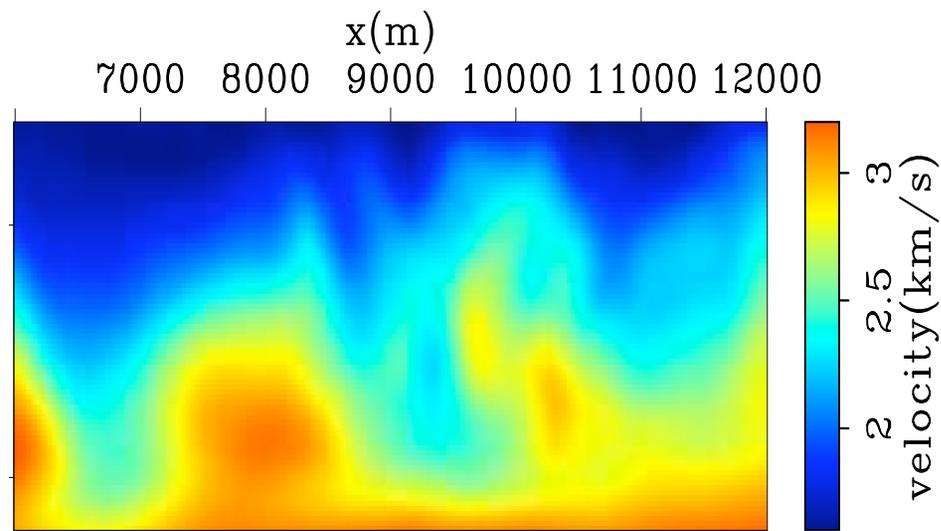
True model



Starting model  $v(z) = v_0 + \alpha z$



Model from WEMVA with RMO



Textbook offset DSO