Wave-equation MVA using partial stack-power-maximization

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SEP—149, p51

Outline

- Wave equation MVA (WEMVA) background
  - Common Image Gather in angle and offset domain
  - WEMVA workflow
- Partial image-stack power objective function

WEMVA characteristics

- Wave-equation migration velocity analysis
  - Is a reflection tomography method
  - Optimizes objective function defined in image domain (Common image gathers)
  - Uses wave-equation operator to compute images & velocity updates (wave path **not** ray path).

Common-image gather in reflection angle domain

- Imaging the same reflector with different incident angles (same in reflection ray-tomography)
- In ideal case, the more correct the velocity model, the more the gathers are **flat**
- More expensive to compute using wave-equation

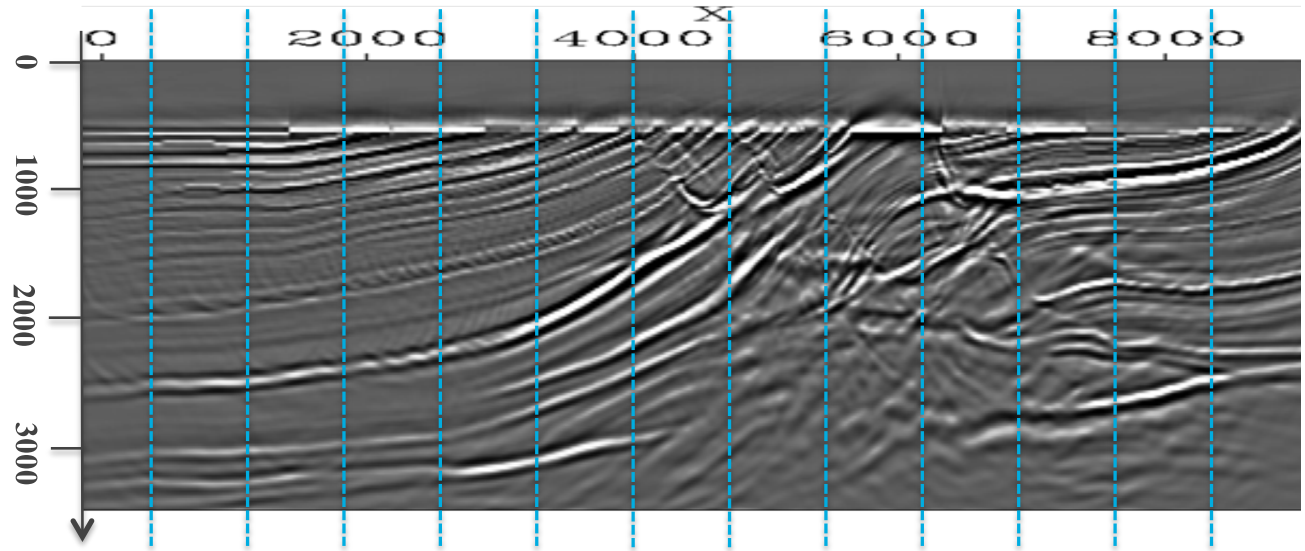
Common-image gather in subsurface offset domain

- Different from surface offset-gathers used frequently in Kirchhoff migration
- In ideal case, the more correct the velocity model, the gathers are more **focused on h=0** (rather than being flat)
- Computationally easy to implement

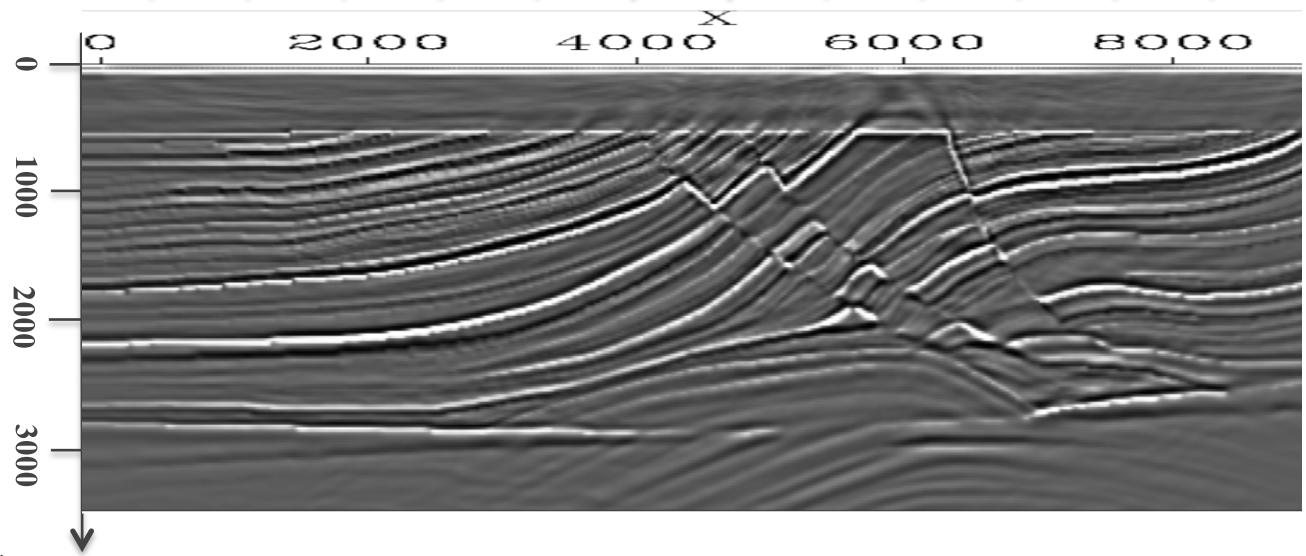
$$I(z,x,h) = \sum_{t} U(z,x+h,t) D(z,x-h,t)$$

Zero-subsurface offset domain

v(z) image



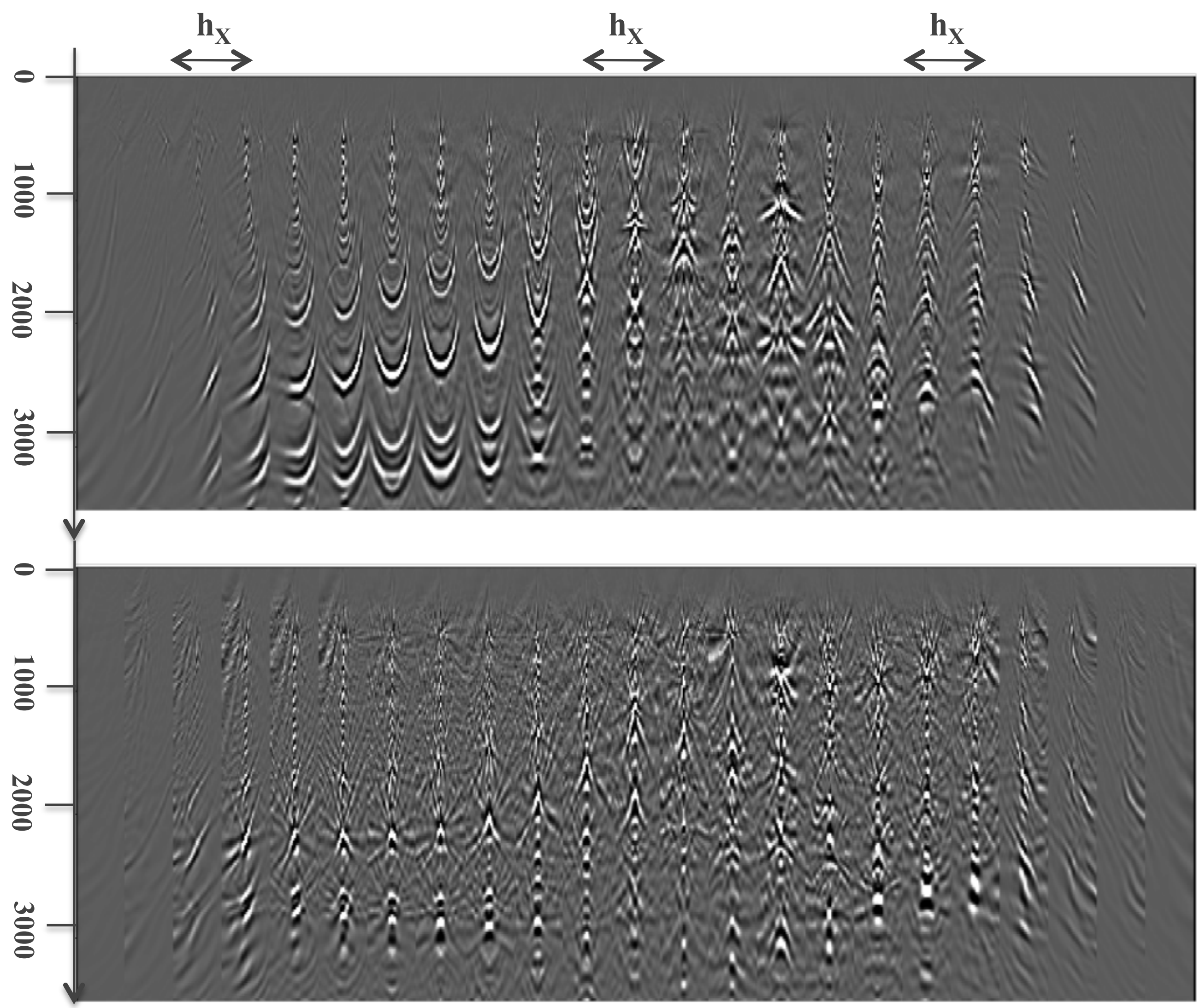
true image



Subsurface **offset** gathers

v(z) gather

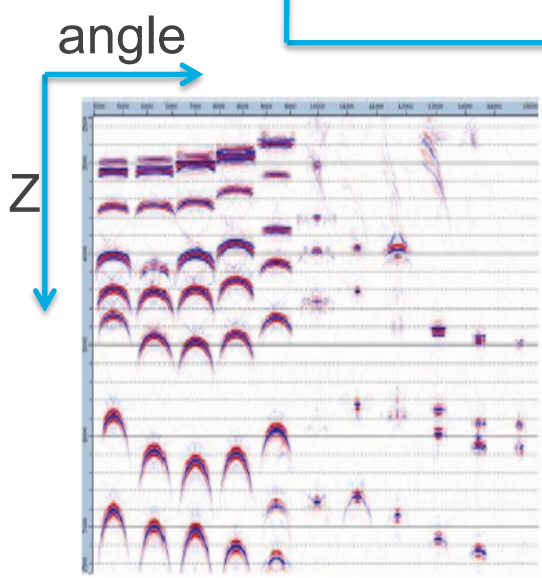
true gathers

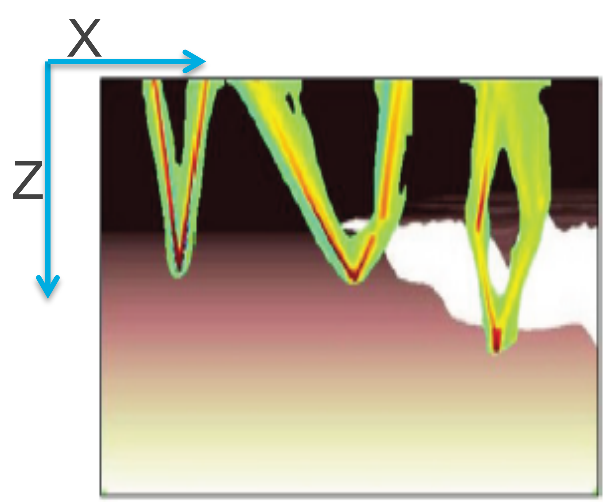


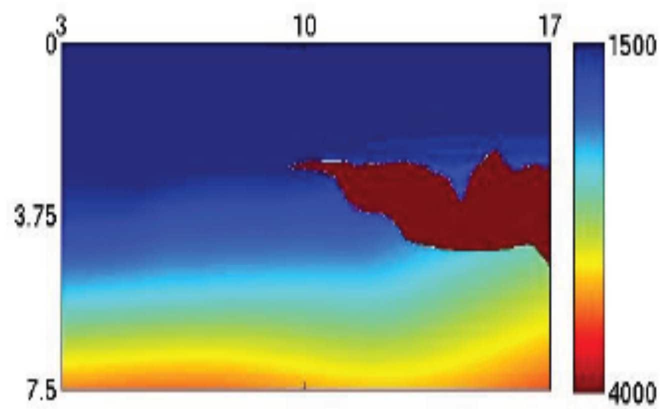
General WEMVA flow

Migrate and generate subsurface ADCIG → Measure image focusing (e.g. flatness of ADCIGs) → Update velocity model using wave-equation operators

iterate







Gerritsen et.al, SEG 2011

WEMVA objective functions

- Stack-power-maximization (SPM)

$$\max_{s} J(s) \quad \text{with } J(s) = \sum_{z,x} \left[\sum_{\gamma} I(z,x,\gamma;s) \right]^2$$

- Differential semblance optimization (DSO)

$$\min_{s} J(s) \quad \text{with } J(s) = \sum_{z,x} \sum_{\gamma} \left[\frac{\partial I(z,x,\gamma;s)}{\partial \gamma} \right]^2$$

z,x : spatial coordinate, γ : reflection angle, s : slowness fiel

WEMVA objective functions

- Stack-power-maximization (SPM) (Gratacous,2005)

$$\max_{s} J(s) \quad \text{with } J(s) = \sum_{z,x} \left[\sum_{\gamma} I(z,x,\gamma;s) \right]^2$$

• Global convergence problems

- Differential semblance optimization (DSO), (Shen&Symes, 2008)

$$\min_{s} J(s) \quad \text{with } J(s) = \sum_{z,x} \sum_{\gamma} \left[\frac{\partial I(z,x,\gamma;s)}{\partial \gamma} \right]^2$$

• Sensitive to image amplitudes

• Sensitive to CIGs artifacts
➔ Problems in 3D

z,x : spatial coordinate, γ : reflection angle, s : slowness field

Outline

- Wave equation MVA background
- Partial Image-stack power objective function

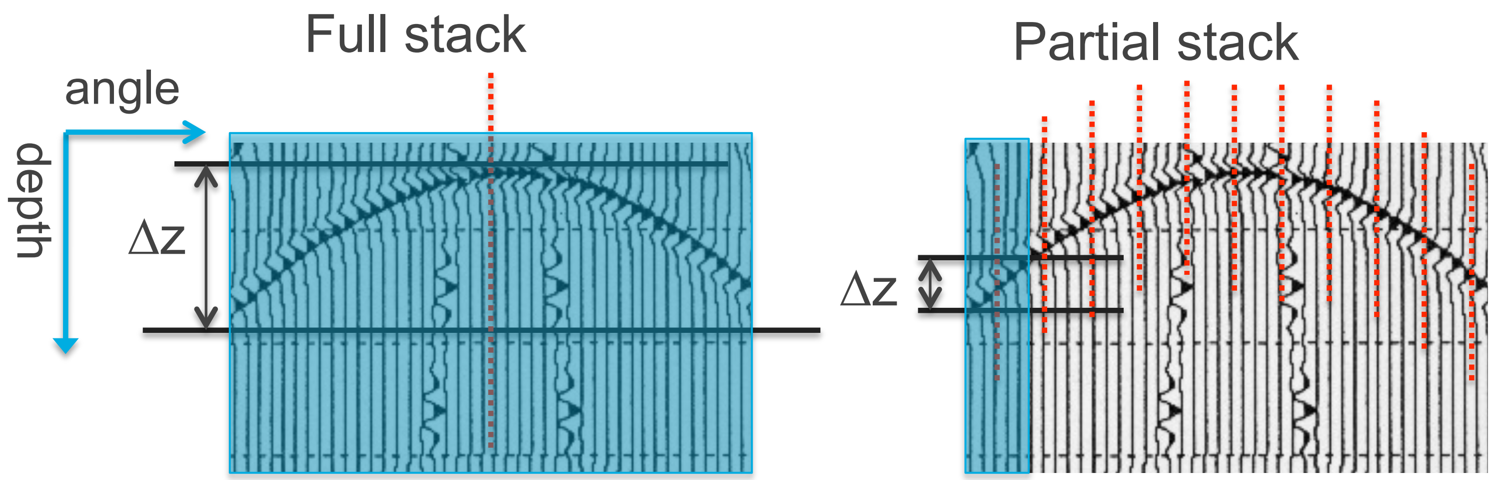
Partial stack power maximization

- Existing stack power maximization $$J = \| \sum_{\gamma} I(z,x,\gamma) \|_2^2$$

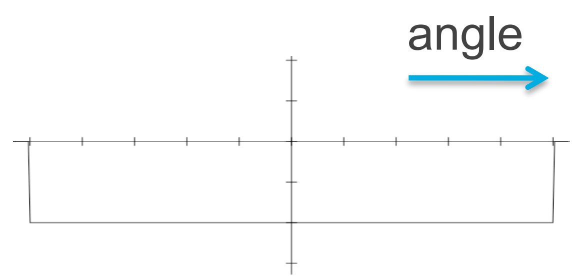
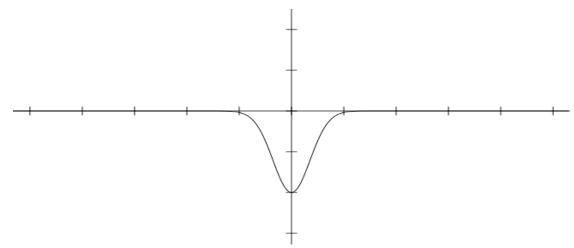
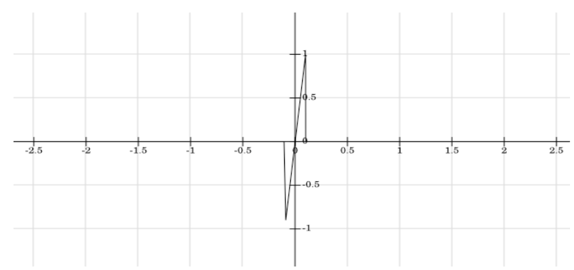
Partial stack power maximization

- Existing stack power maximization $J = \| \sum_{\gamma} I(z,x,\gamma) \|_2^2$
- Using partial stack $J = \| \sum_{\gamma'} I(z,x,\gamma-\gamma')g(\gamma') \|_2^2$
  - $g(\gamma')$ is a low-pass filtering kernel
    - Box function / triangular function / Gaussian function
- Properties of partial stack
  - More robust against cycle-skipping problem as in previous stack-power maximization method
  - Is a generalization of the previous method

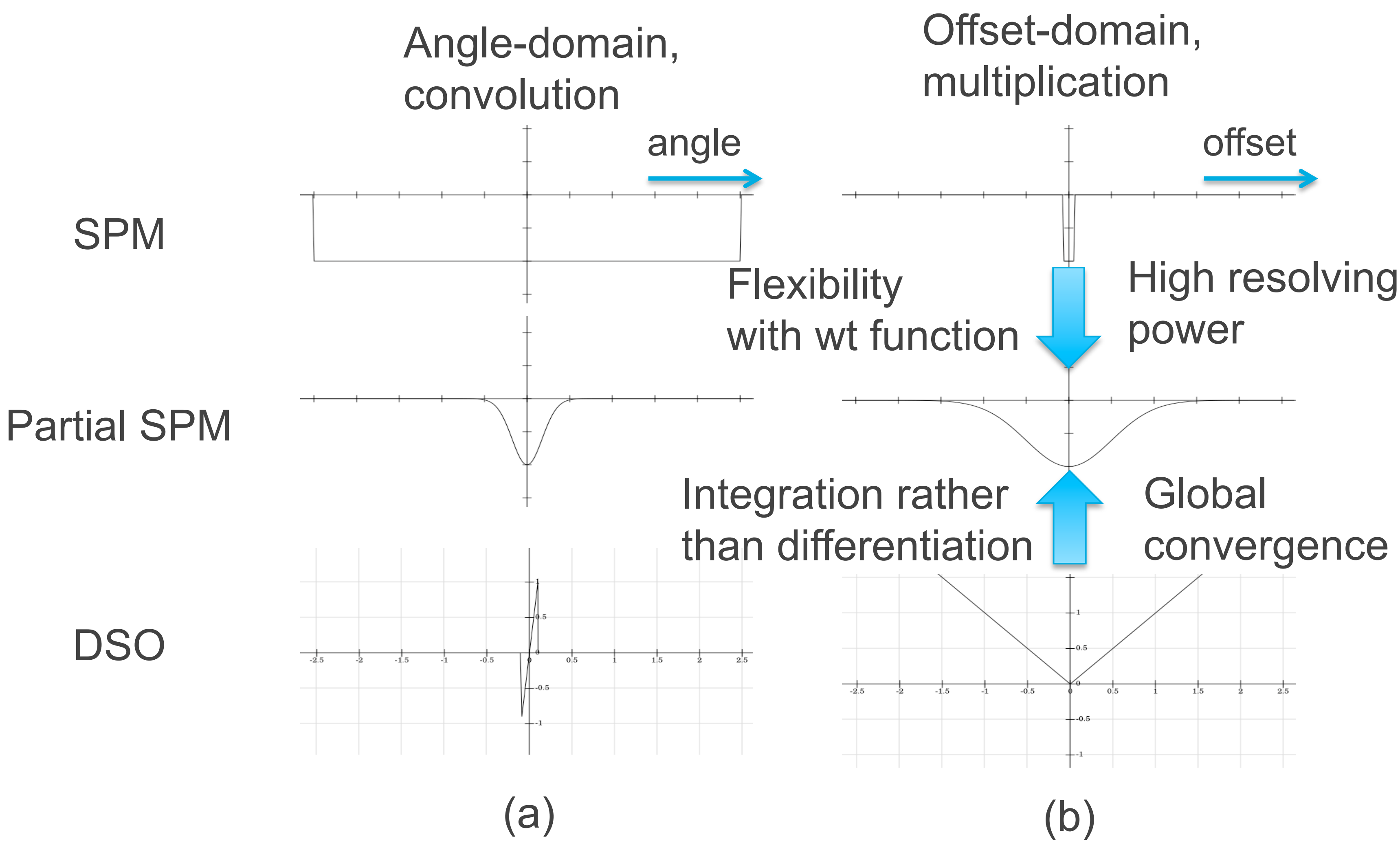
Partial stacking avoids cycle-skipping



WEMVA objective functions in angle domain

	Angle-domain	Convolution kernel
Stack Power Maximization (SPM)	$\|\| \sum_{\gamma} I(z,x,\gamma) \|\|_2^2$	
Partial SPM	$\|\| \sum_{\gamma'} I(z,x,\gamma-\gamma')g(\gamma') \|\|_2^2$	
DSO	$\|\| \frac{\partial}{\partial \gamma} I(z,x,\gamma) \|\|_2^2$	

Comparison of WEMVA objective functions (graphic)



Comparison of WEMVA objective functions (equation)

	Angle-domain	Offset-domain
Stack Power Maximization (SPM)	$\| \sum_{\gamma} I(z,x,\gamma) \|_2^2$	$\| I(z,x,h=0) \|_2^2$
		Flexibility with G(h) — High resolving power
Partial SPM	$\| \sum_{\gamma'} I(z,x,\gamma-\gamma')g(\gamma') \|_2^2$	$\| G(h)I(z,x,h) \|_2^2$
		Integration rather than differentiation — Global convergence
DSO	$\| \frac{\partial}{\partial\gamma} I(z,x,\gamma) \|_2^2$	$\| hI(z,x,h) \|_2^2$

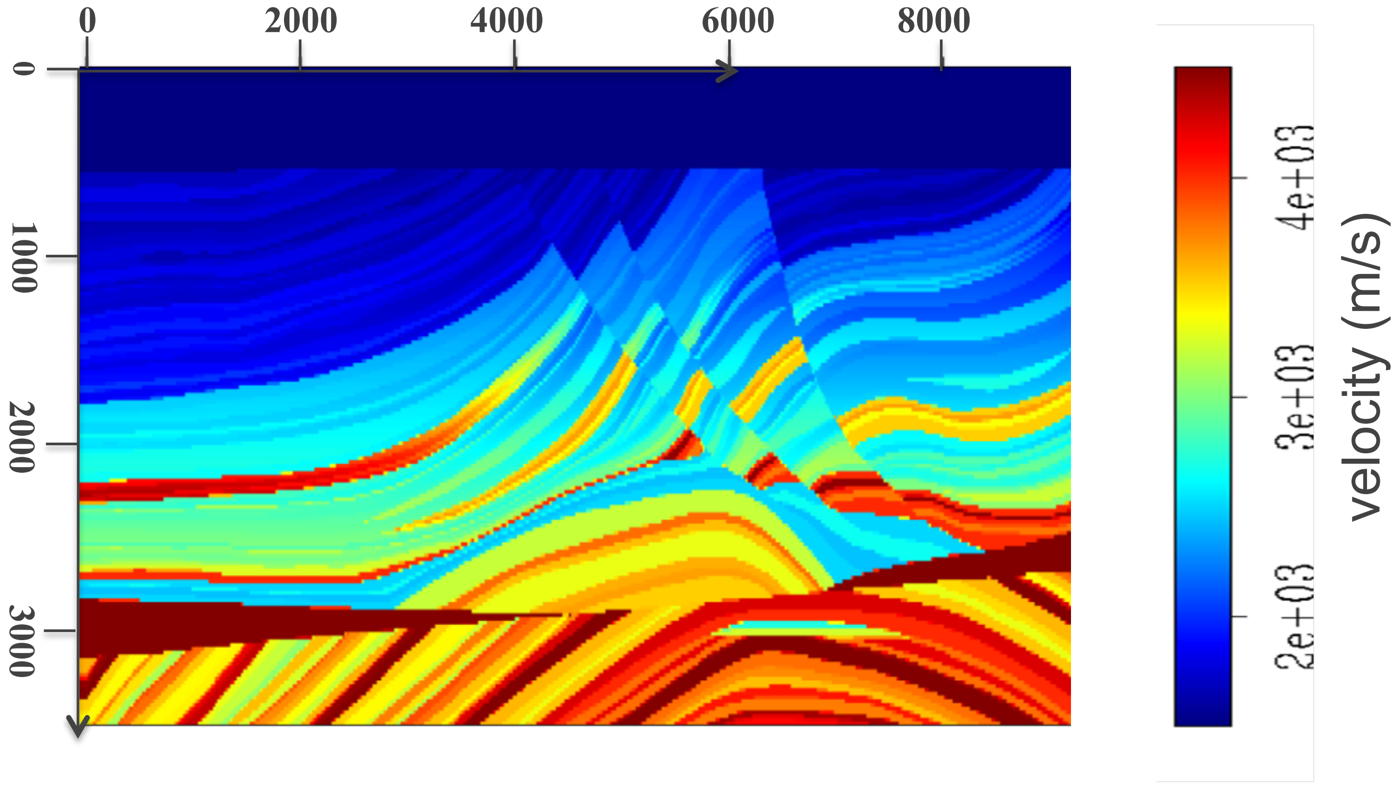
Energy Normalized WEMVA objective functions

Partial SPM $\| G(h) I(z,x,h) \|_2^2$ $\Rightarrow$ $\sum_x \frac{\sum_{z,h} [G(h) I(z,x,h)]^2}{\sum_{z,h} [I(z,x,h)]^2}$

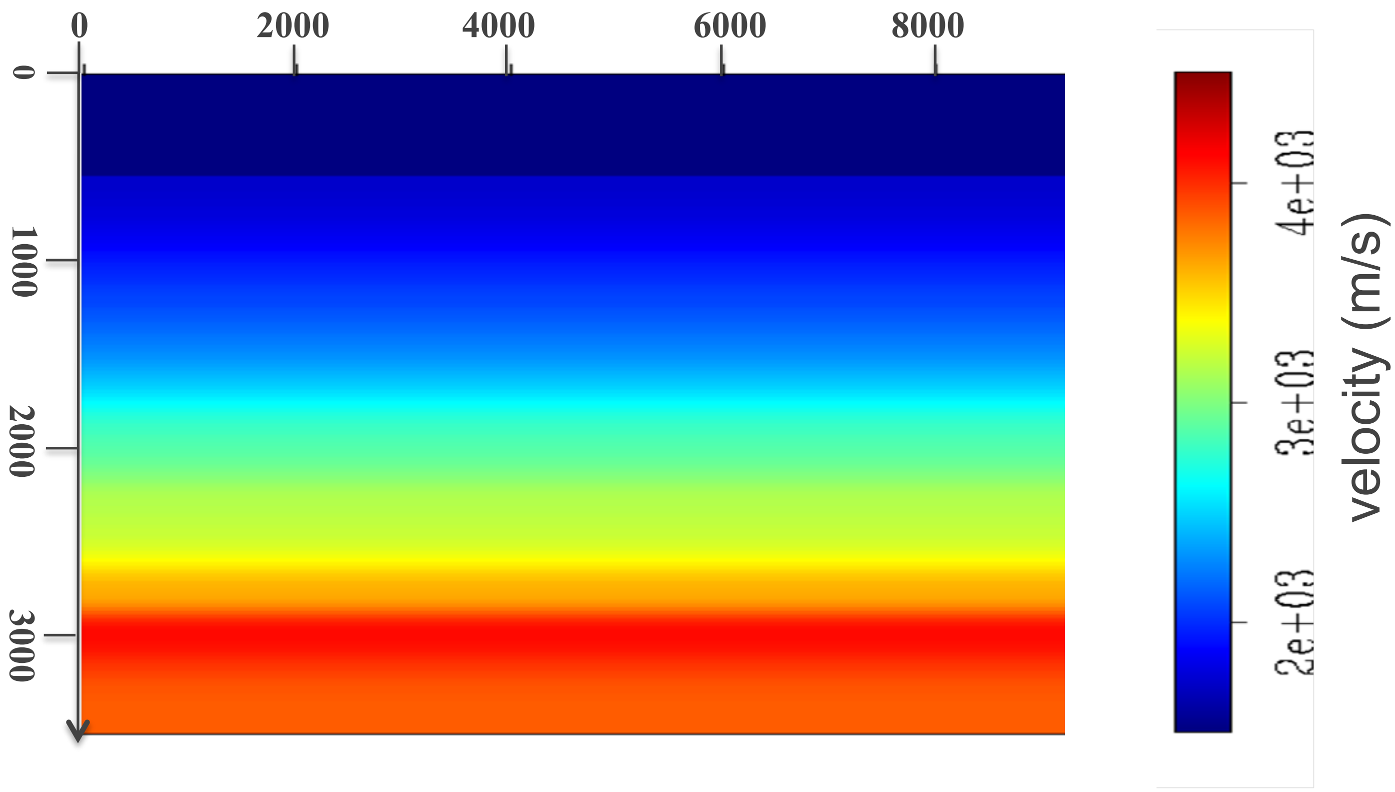
DSO $\| h I(z,x,h) \|_2^2$ $\Rightarrow$ $\sum_x \frac{\sum_{z,h} [h I(z,x,h)]^2}{\sum_{z,h} [I(z,x,h)]^2}$

From Tang, Ph.D Thesis, 2011

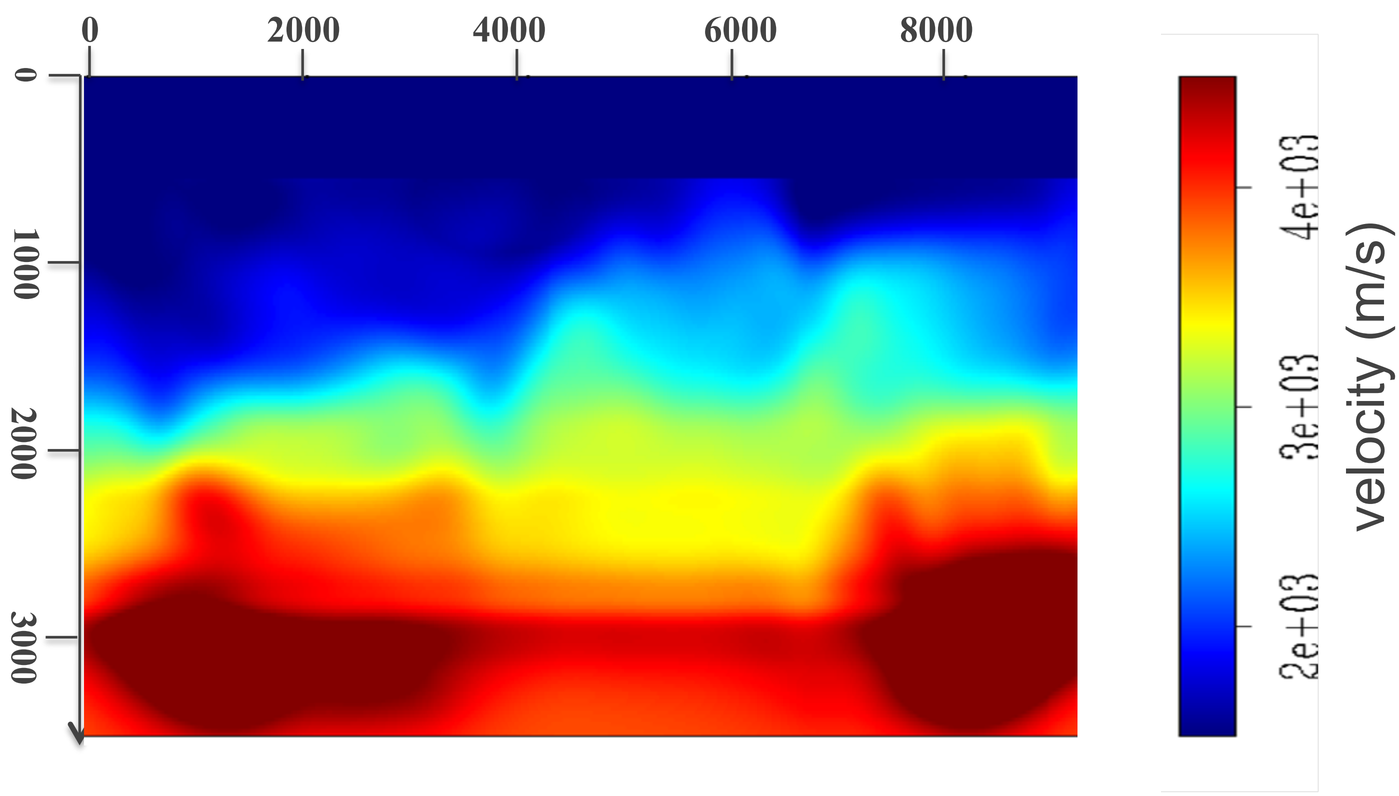
Marmousi model: true velocity



Starting velocity

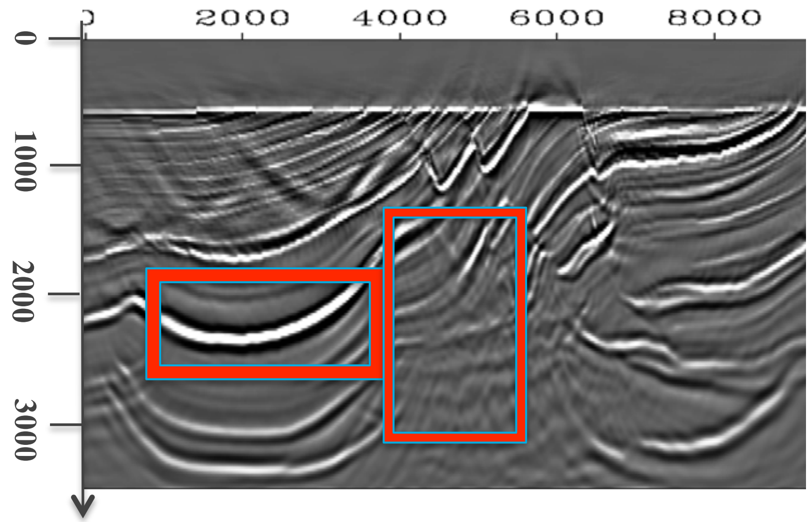


Partial-SPM without normalization (20 iterations)

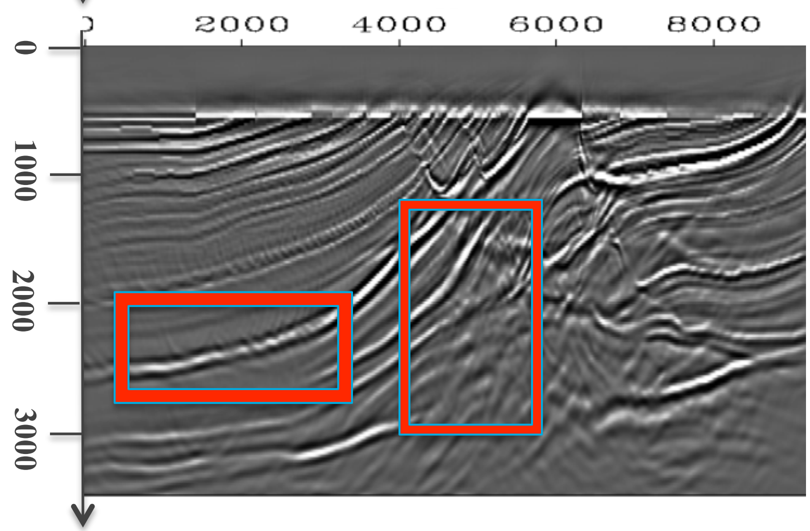


Partial-SPM without normalization

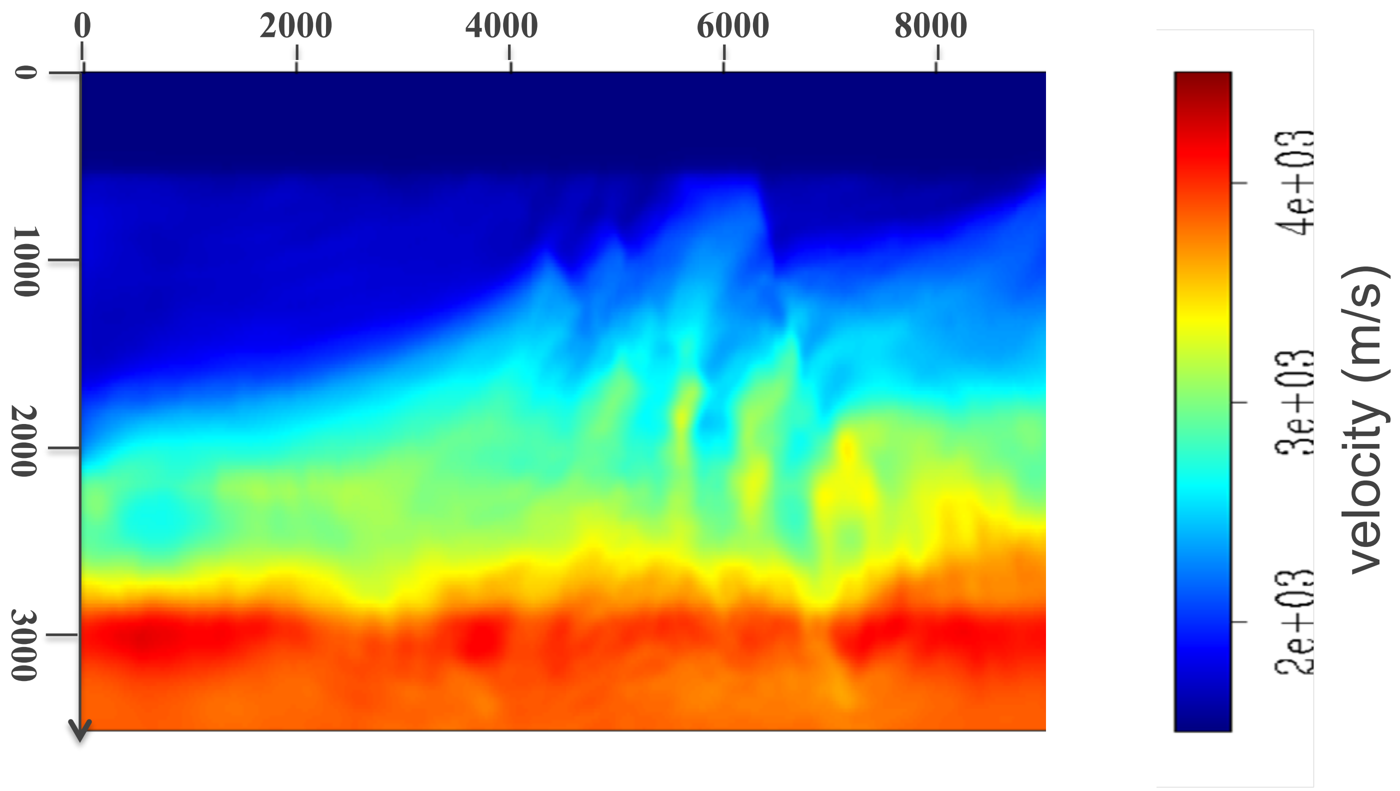
Final image
Display clip 4x



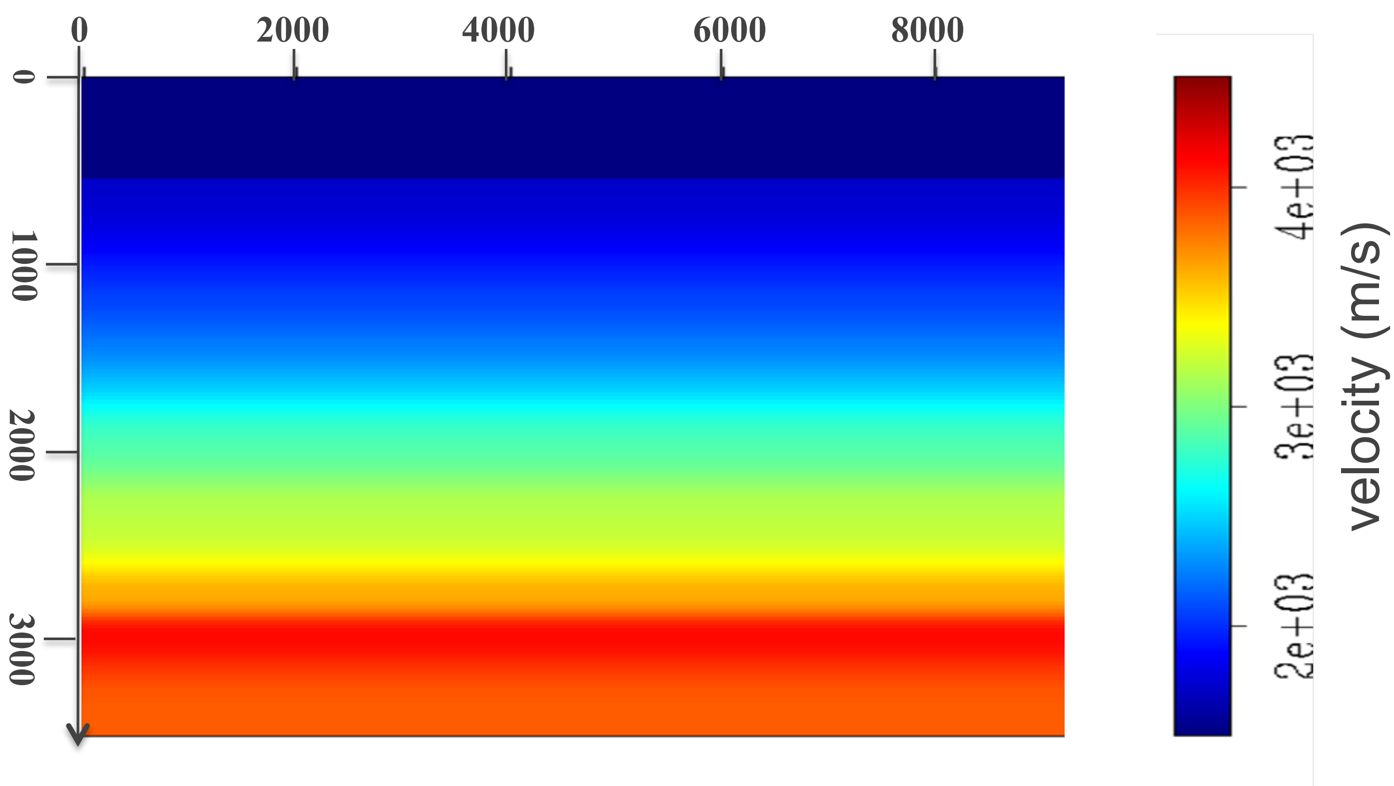
Initial image



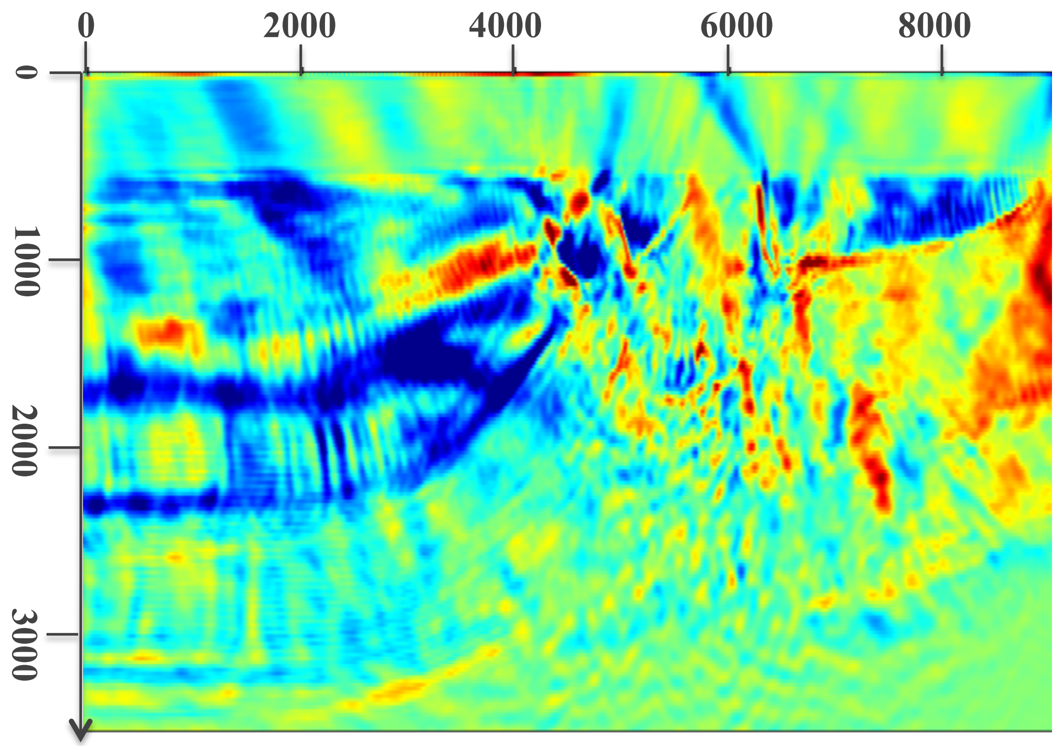
Normalized Partial stack power (40 iterations)



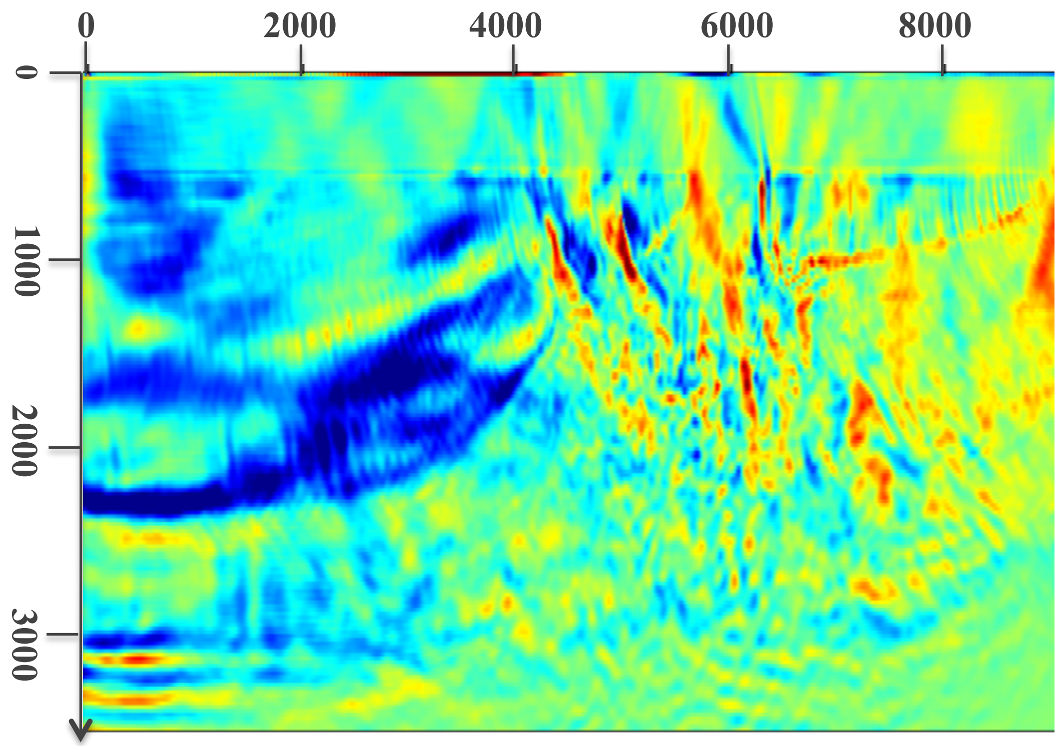
Marmousi test, normalized partial SPM method, starting model



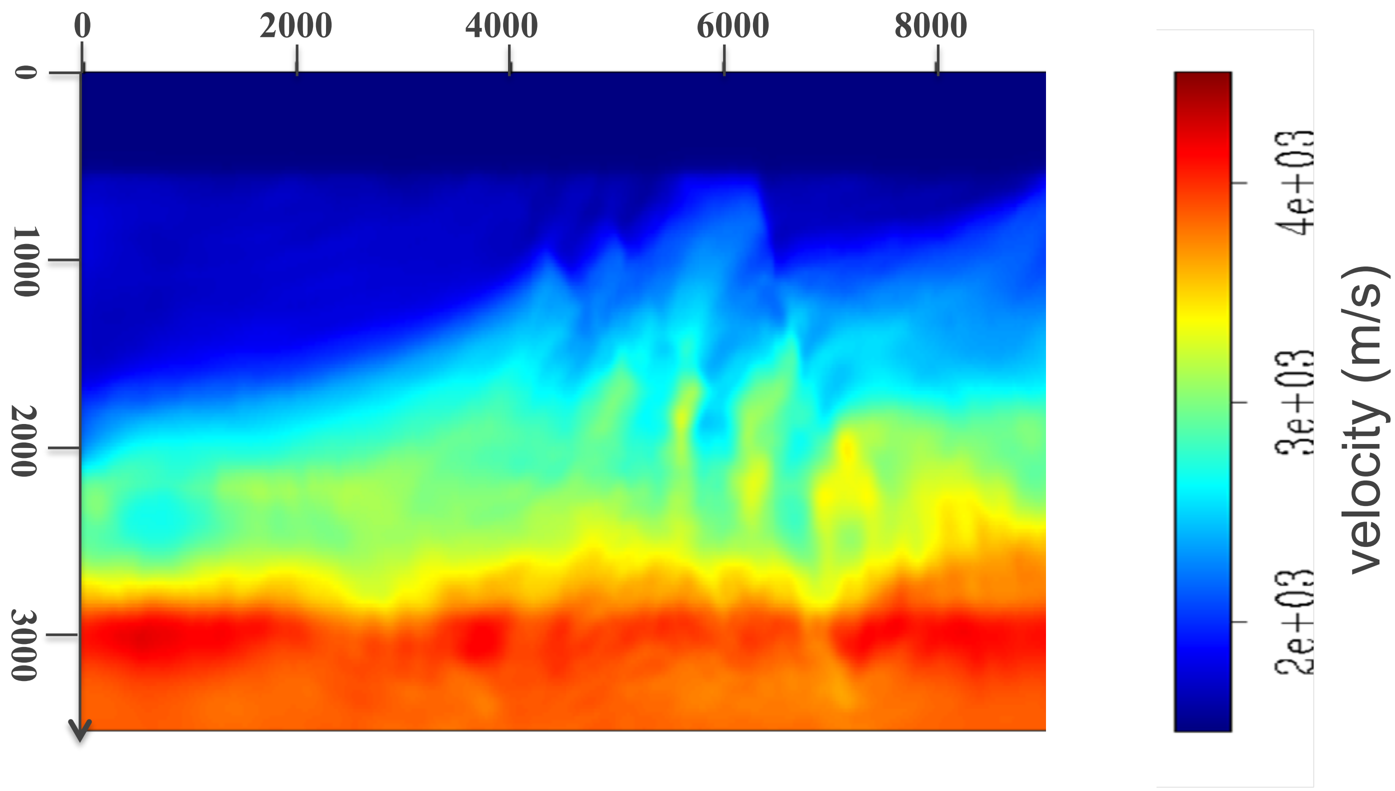
First update, Normalized partial stack power



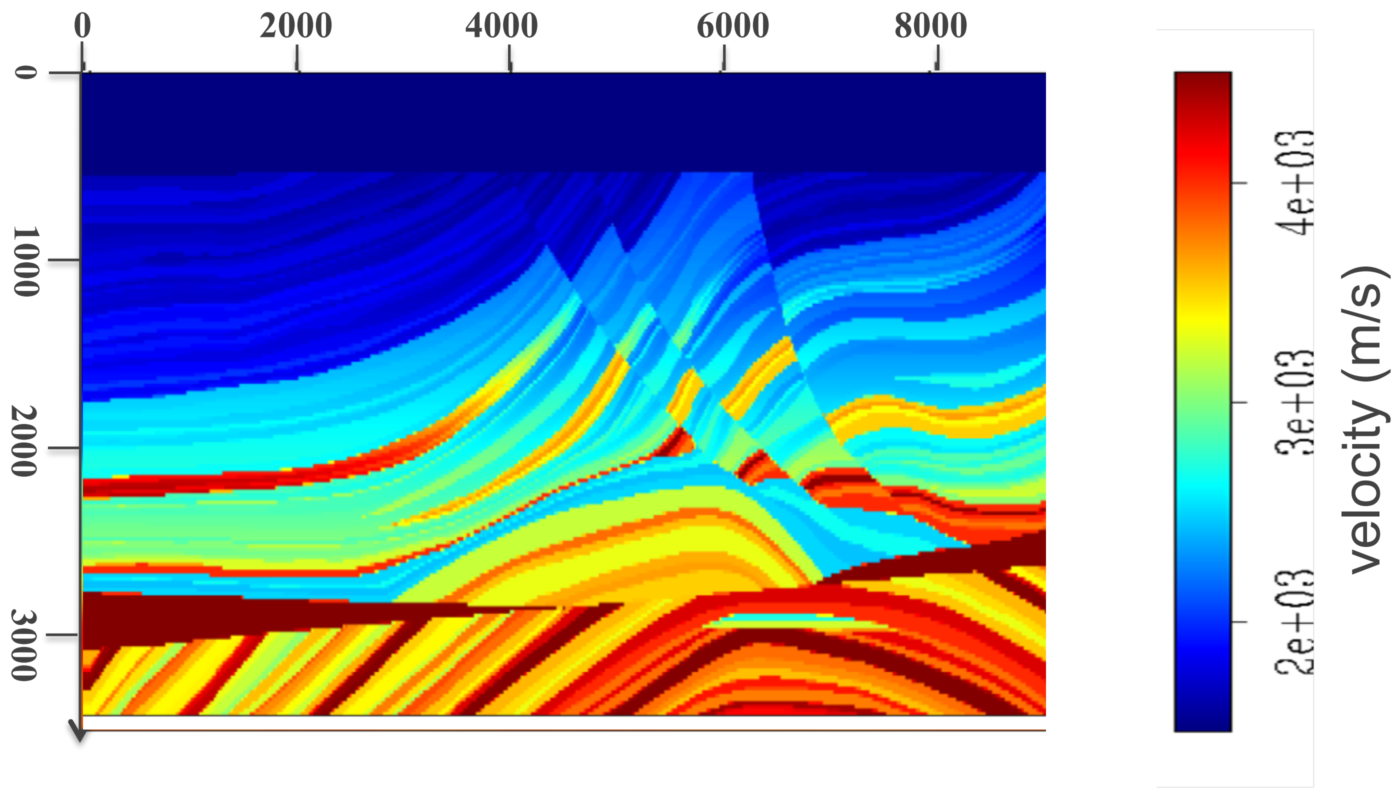
First update, Normalized DSO



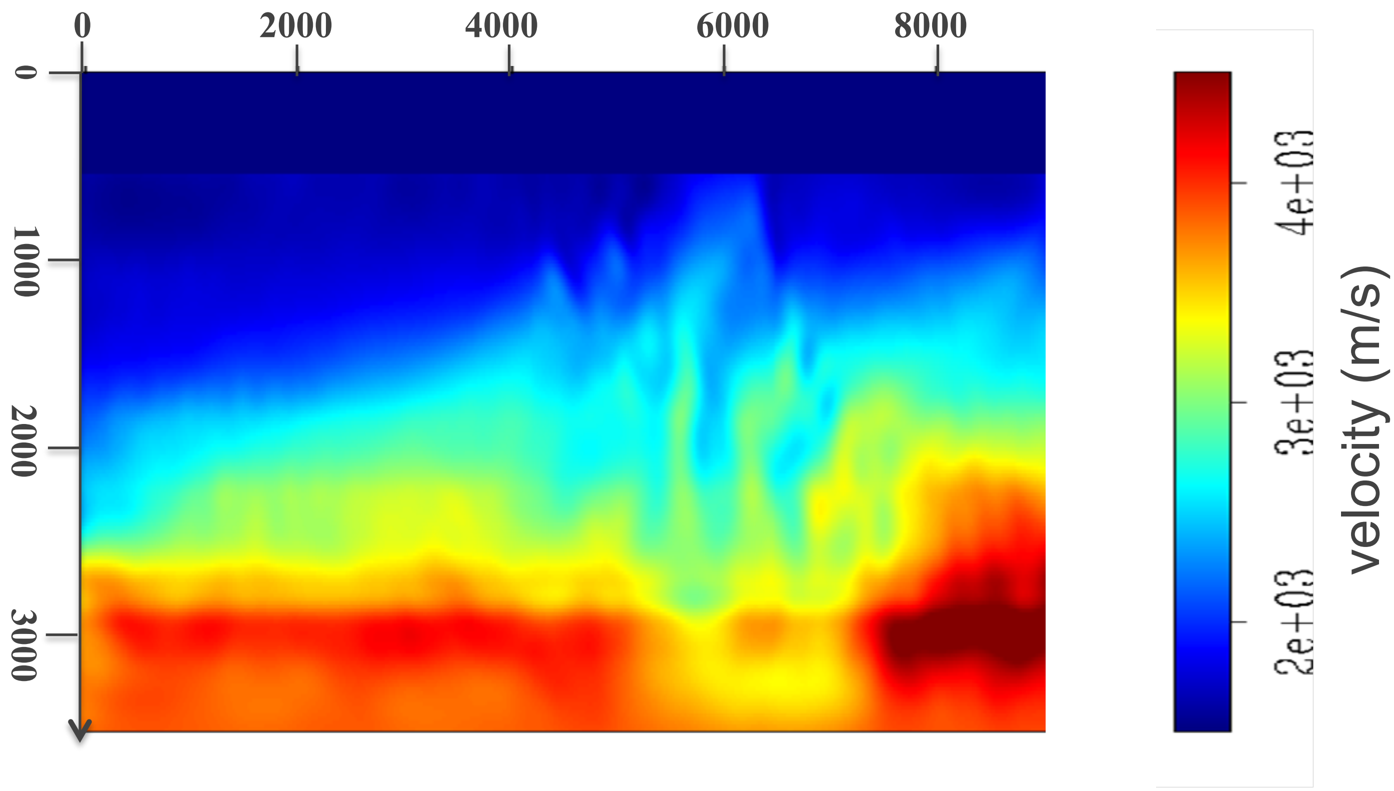
Normalized Partial stack power (40 iterations)



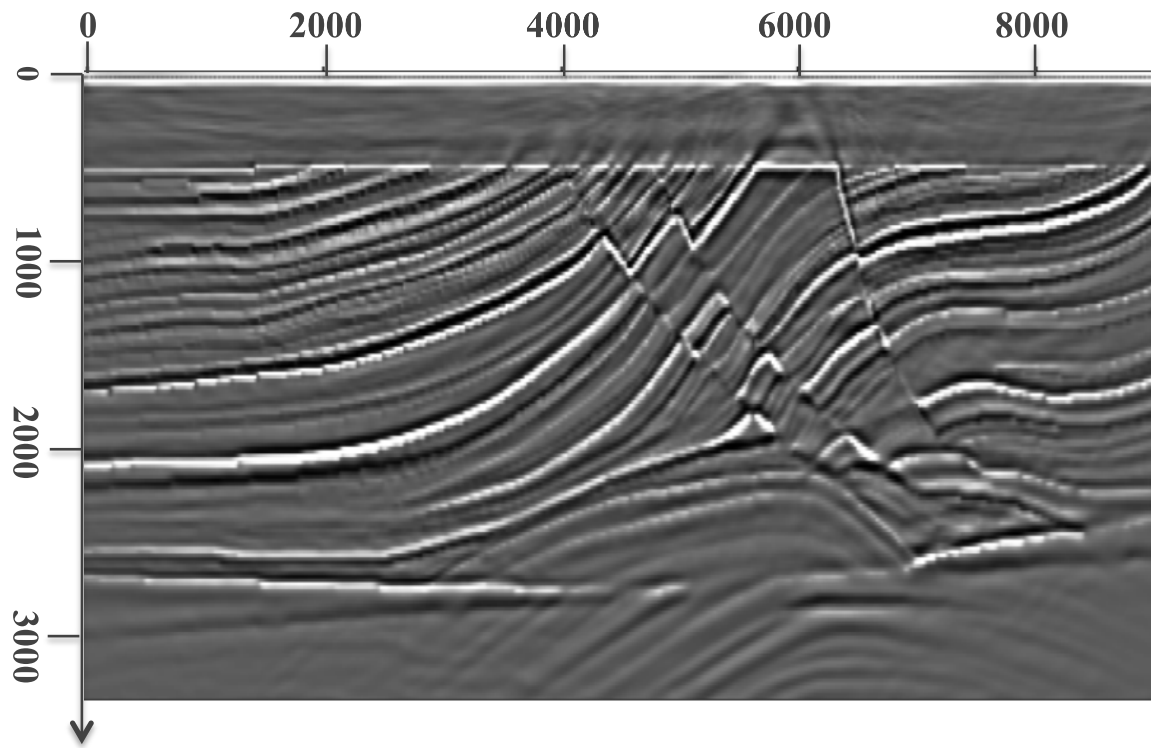
Normalized Partial stack power (40 iterations)



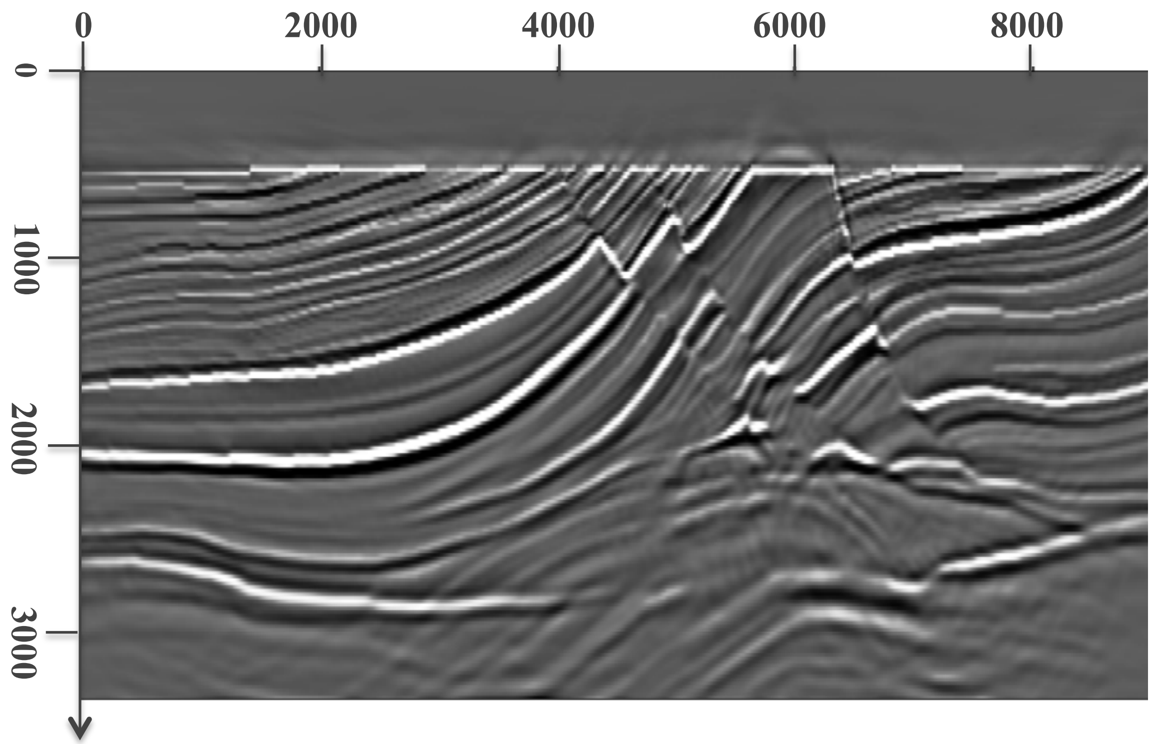
Normalized DSO (40 iterations)



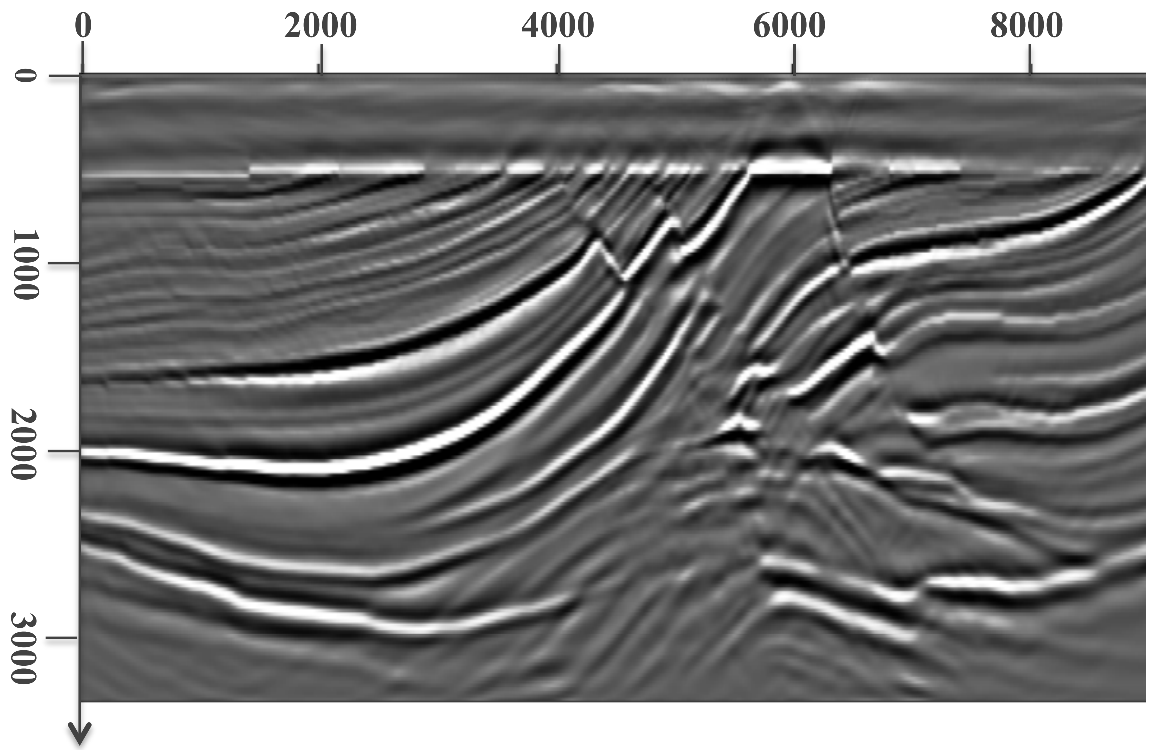
True image



Normalized Partial SPM



Normalized DSO



Conclusion & Discussion

- Partial SPM merges the merits of both SPM and DSO objective functions, also more flexible
- Normalization for reflector amplitude is not only preferred but necessary
- Remaining question:
  - Compared to data-domain, image-domain is easier to look at, but we are still missing the **ultimate correct** objective function

Conclusion & Discussion

- Partial SPM merges the merits of both SPM and DSO objective functions, also more flexible
  - Normalization for reflector amplitude is not only preferred but necessary
- Steering filters helps improves convergence speed and reduces the null space of the inverse problem
  - Most effective if the model is poorly constrained along the dipping direction
- Question Remained:
  - Compared to data-domain, image-domain is easier to look at, but we are still missing the **ultimate correct** objective function.
  - Is the steering filter always desirable? What if the geometry of the velocity model does not follow the reflectors', like BP 2004 model?

Acknowledgement

- Chevron ETC, Imaging & Velocity model building group