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Salt body segmentation using level set methods

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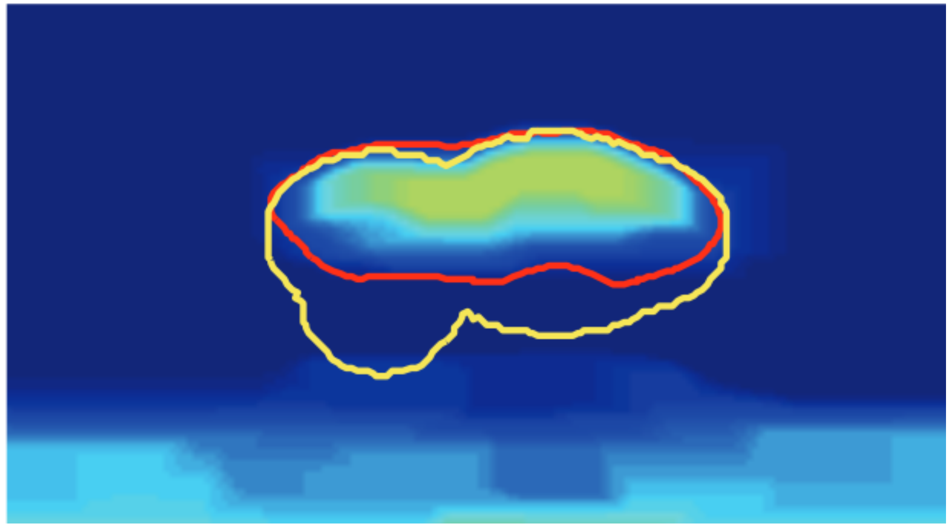
Outline

1. **Problem description**
2. **Level set overview**
3. **Gradient calculation**
4. **Gradient application**
5. **Preliminary results**
6. **Future work**

1.0 Problem description

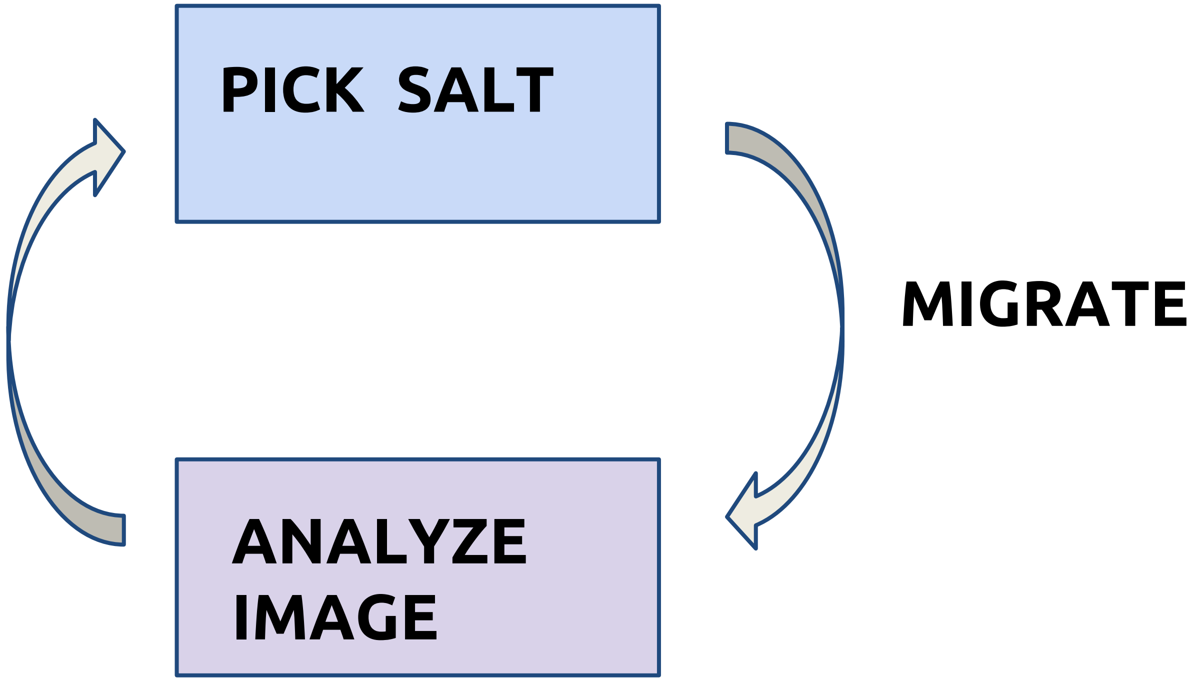
- Migration results can be very sensitive to salt body boundaries
  - High velocity contrast means salt acts as a lense

- Tomographic methods cannot always make sharp salt body delineations
  - Not often enough reliable high frequency information
  - Doesn't intrinsically deal with discrete boundaries



1.1 Current methods

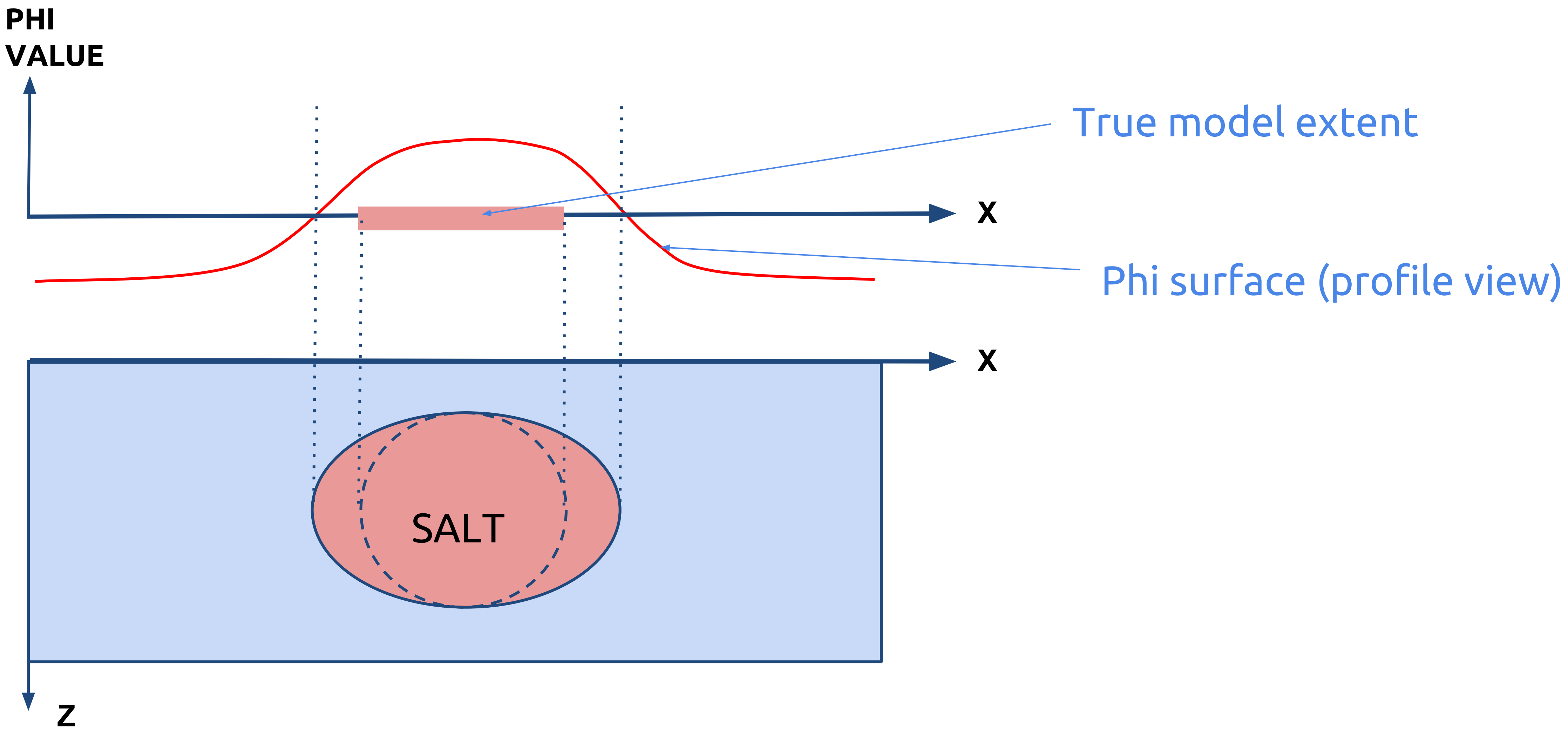
- Manual & Automated picking
  - Needs to be iteratively picked / re-migrated
  - Oversight / inputting picks is time consuming



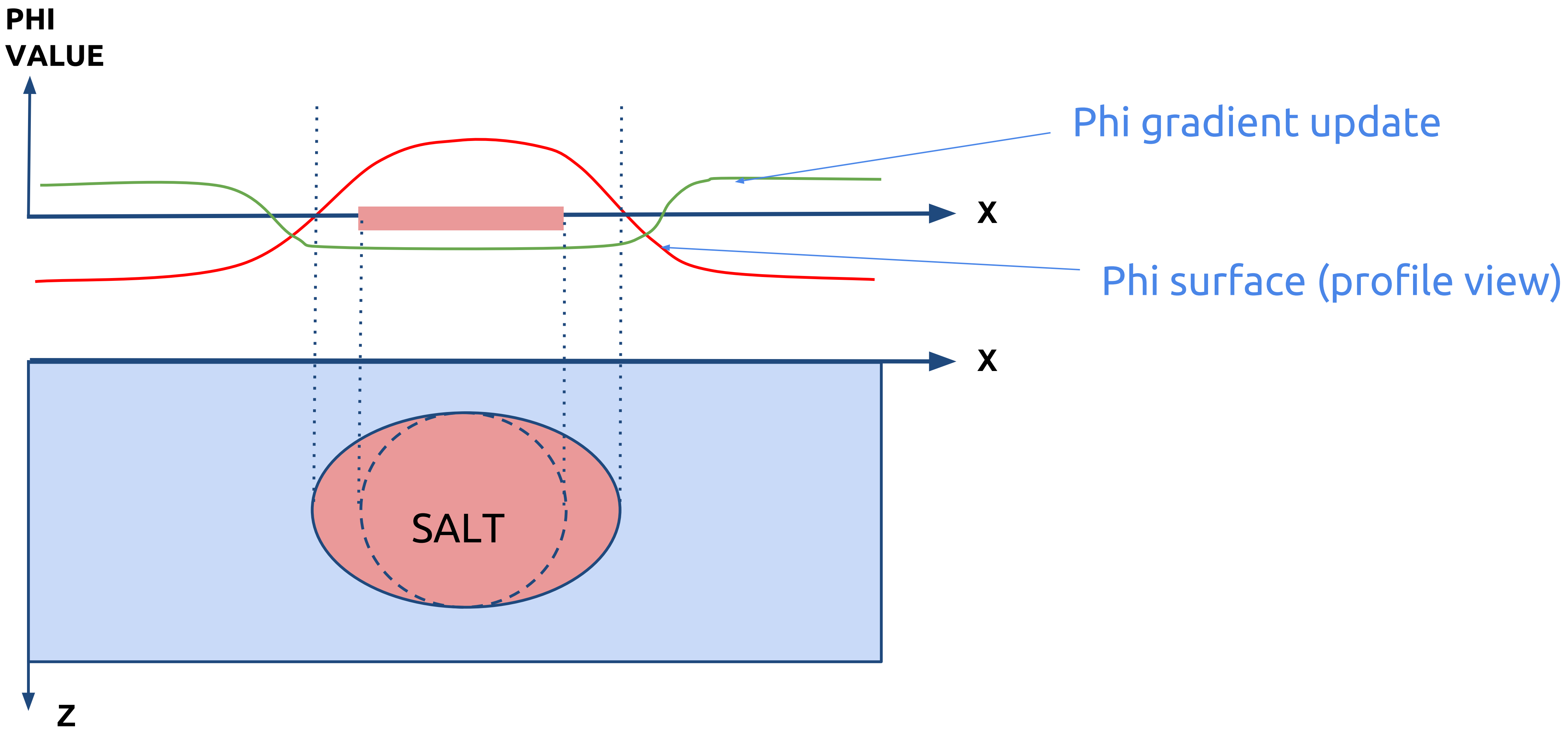
1.2 Proposed method

- Model salt boundaries as a contour of an implicit surface (level set).
- Build velocity model based on these boundaries.
- Evolve the implicit surface (and subsequently the 2D boundary) to minimize the FWI objective function.

2.0 Level set overview



2.0 Level set overview



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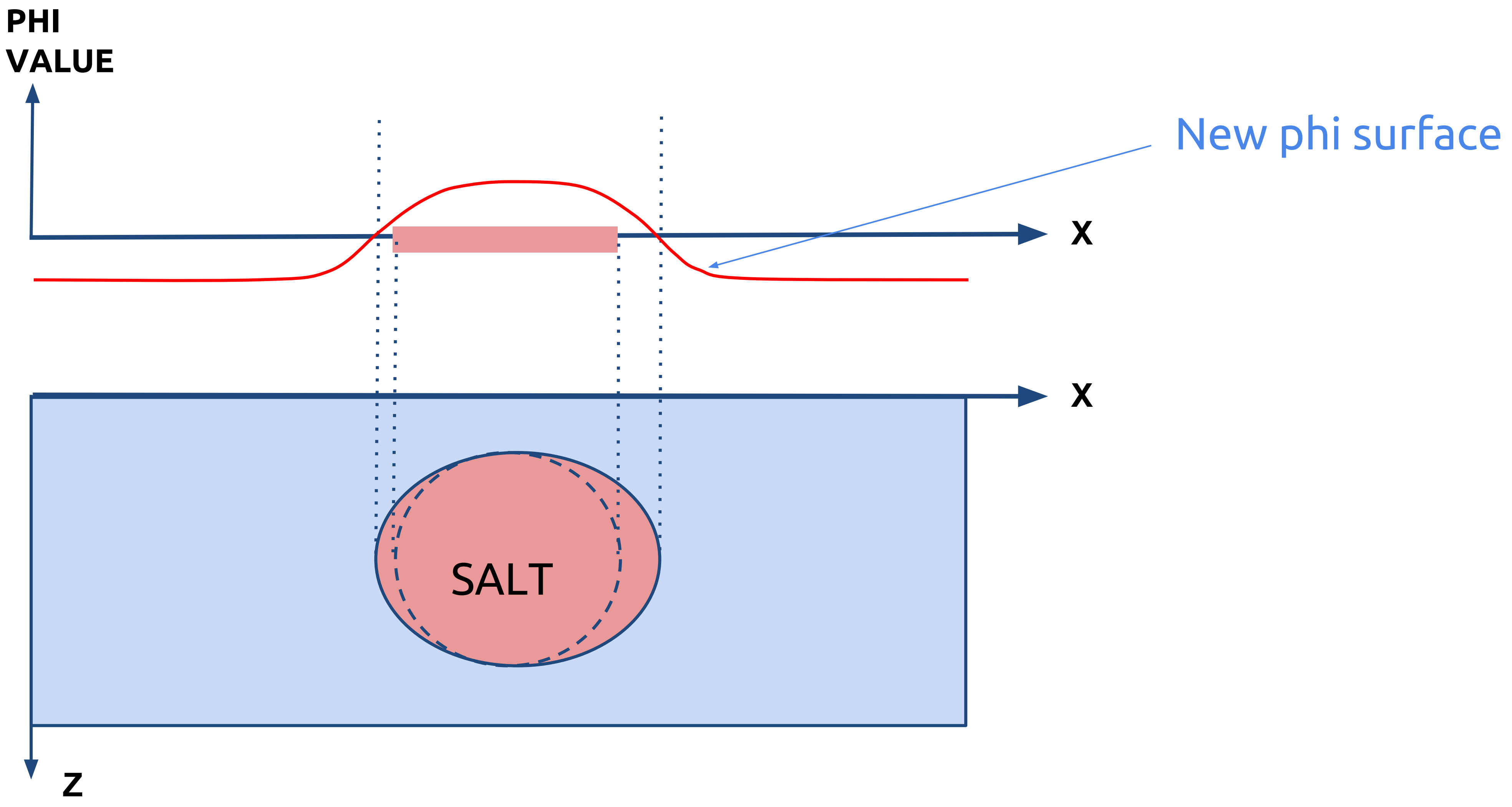

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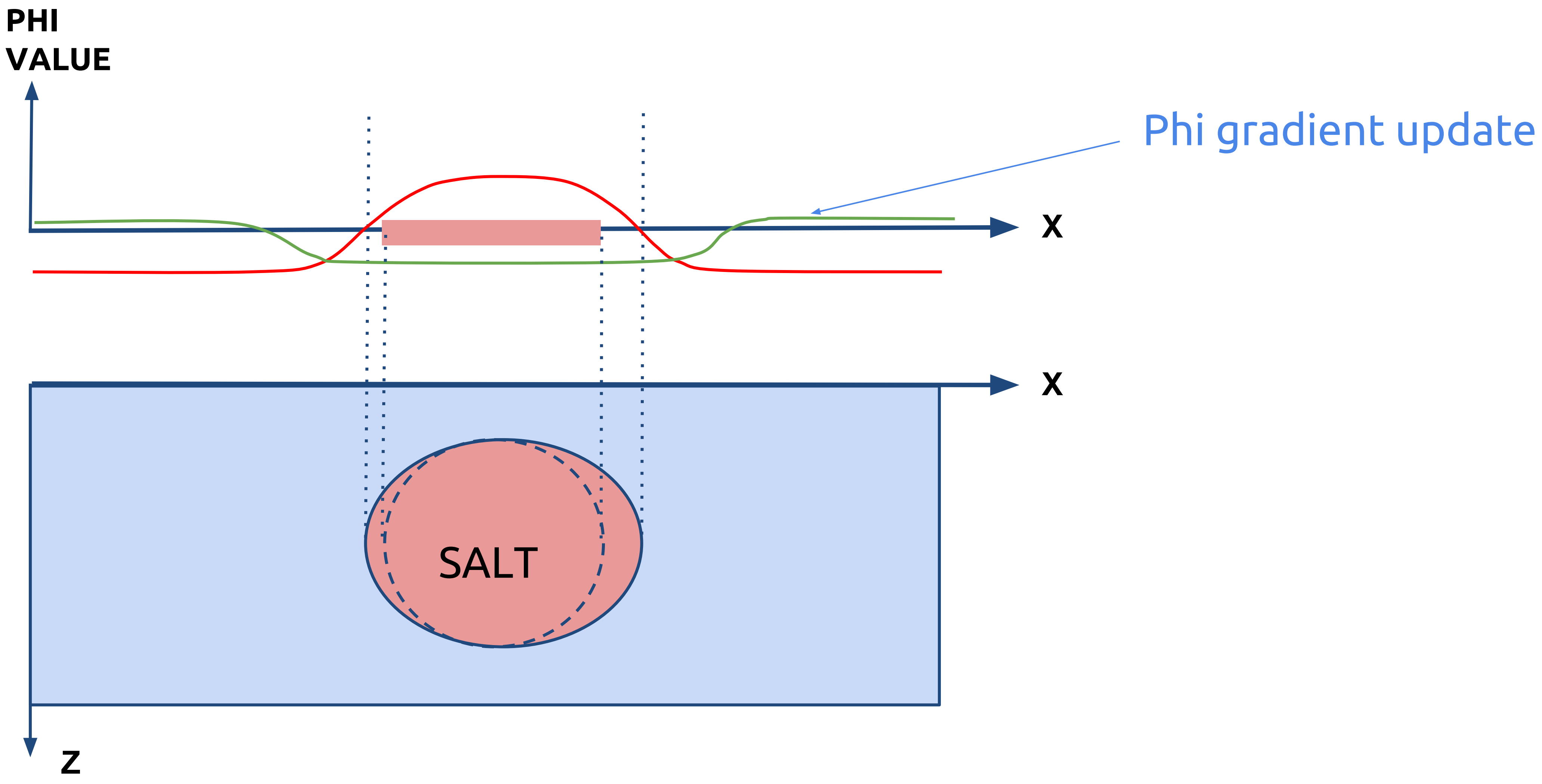
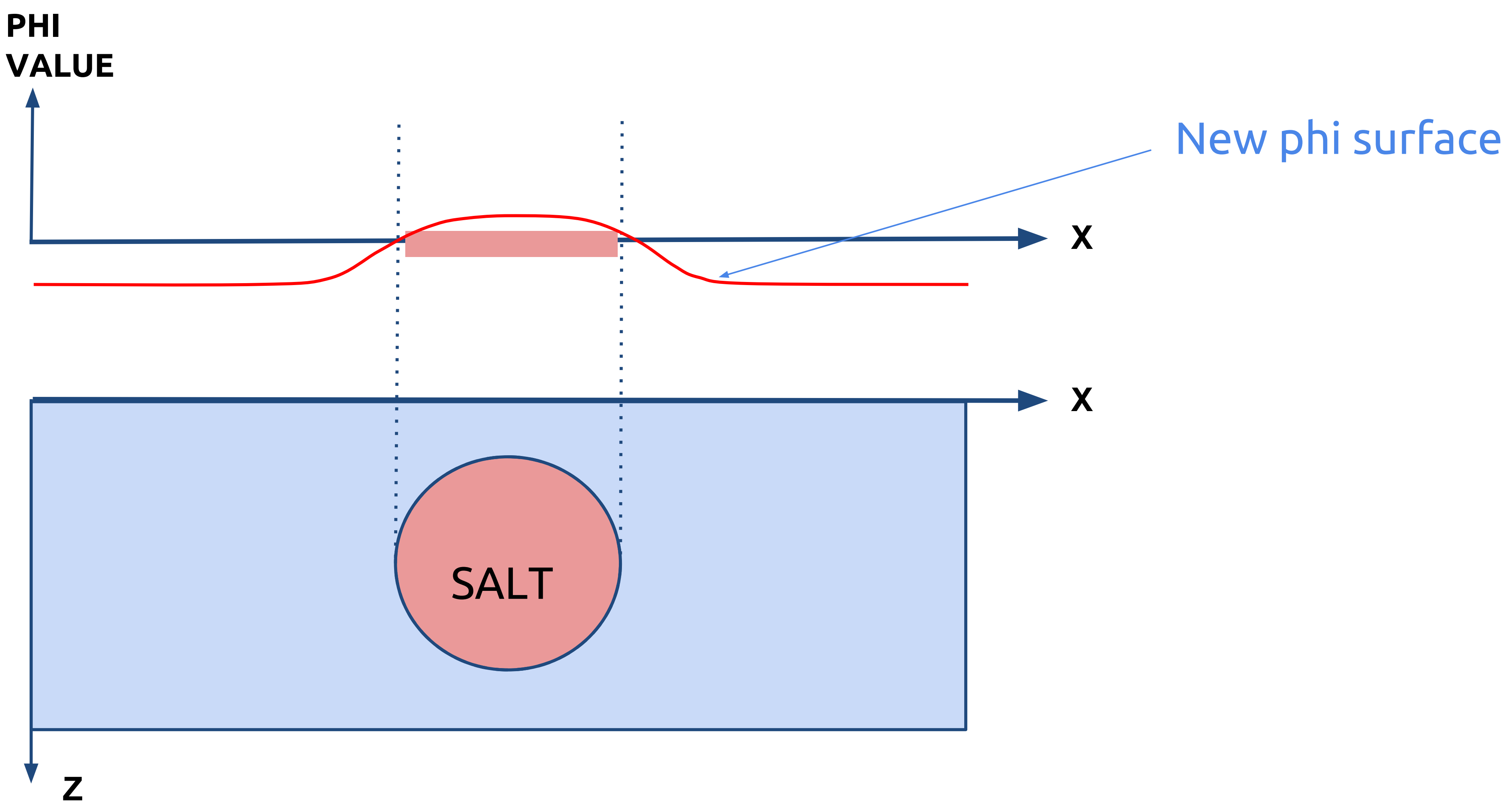
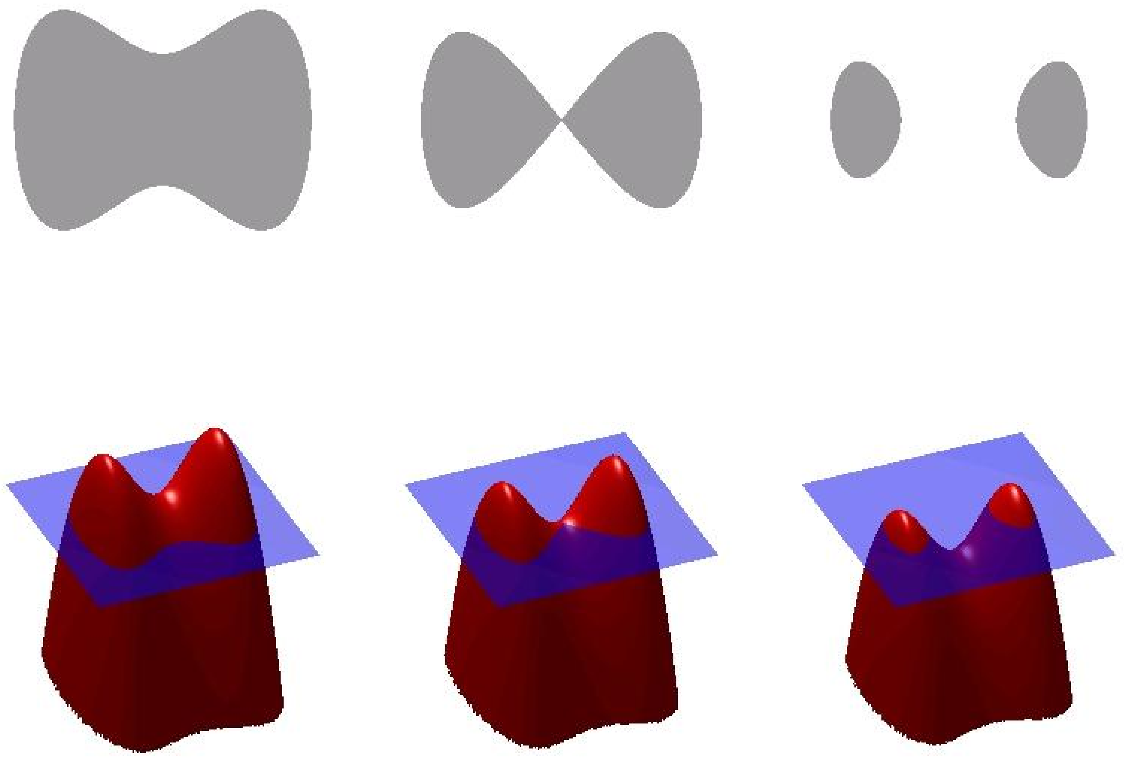


2.0 Level set overview



2.1 Level set advantages



- Can handle merging and separation of segmented bodies
- Implicit approach can handle sharp corners and cusps without going unstable
- Easily extendable to 3D

2.2 Level set disadvantages

- Can be expensive
- Most applicable on bodies that we can approximate as homogeneous
  - Salt is not always best approximated that way
- No guarantees of convergence (not monotonically decreasing)

2.3 Previous work

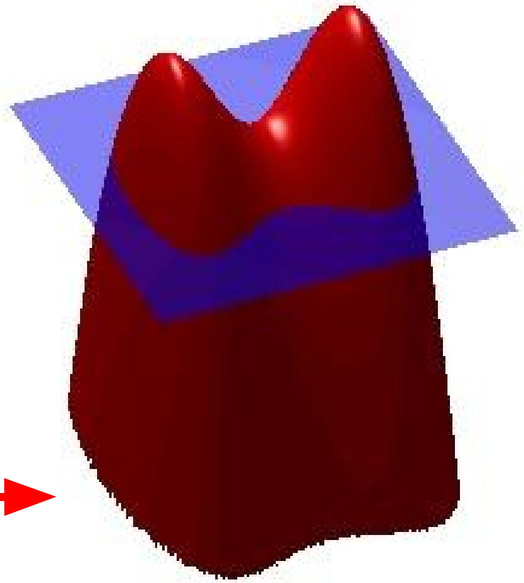
- Osher & Sethian (1988):
  - Introduced general level set method
- Santosa (1996):
  - Evolved level sets using FWI objective function on 2D bodies
- Guo and de Hoop (2013):
  - Expand on Santosa's work
  - Use frequency domain forward operator
  - Also apply alternating tomographic updating

2.4 New approach

- Builds on Guo & de Hoop
  - Experimenting with concurrent tomographic updating
  - Using time domain forward operator
  - Investigating the inclusion of expert input boundary confidence intervals

3.0 Level set derivation

$$\phi(x_T) = 0$$

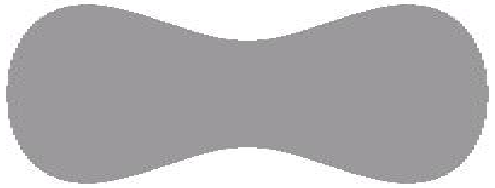


BOUNDARY

$$\phi(x_T)$$

IMPLICIT SURFACE

$$\phi(x_T) > 0$$

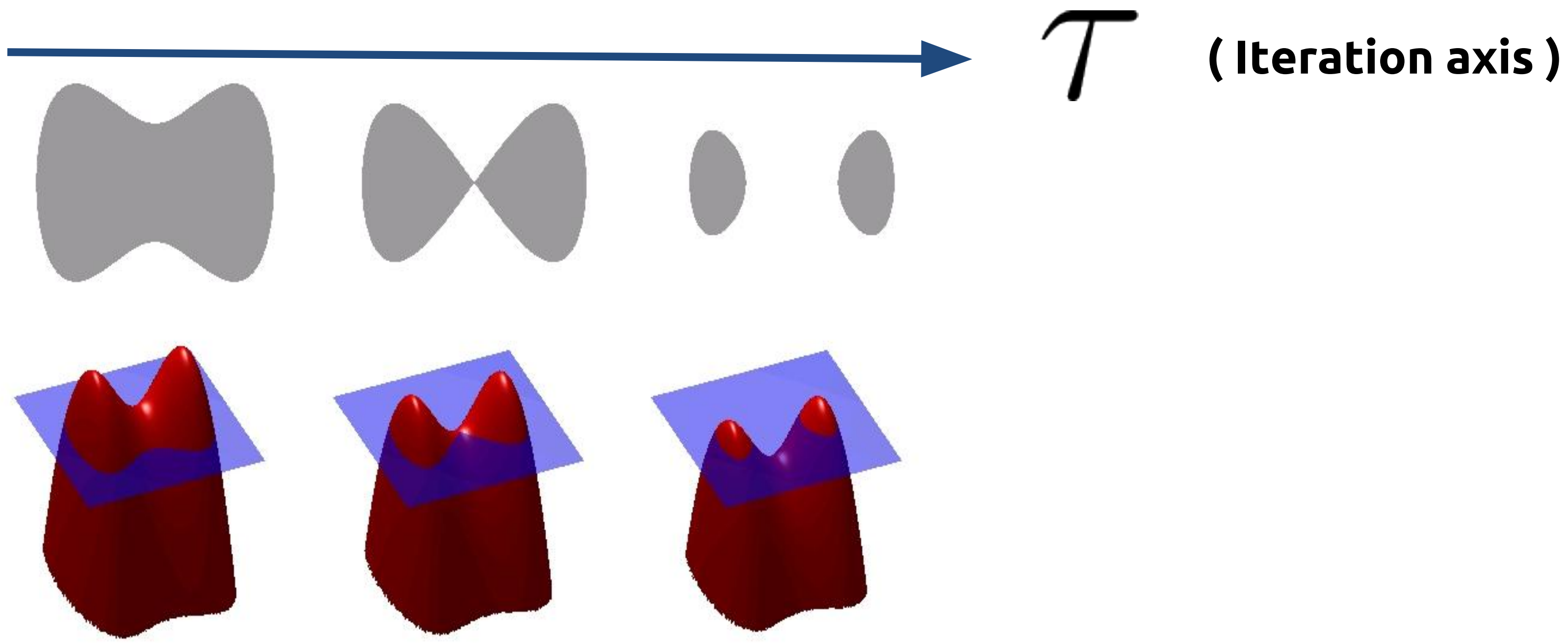


BODY

3.0 Level set derivation

$$\phi(x_\mathcal{T}) = 0$$

Boundary represented by level set



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3.0 Level set derivation

$$\phi(x_\tau) = 0$$

Boundary represented by level set

$$\frac{\partial \phi(x_\tau)}{\partial \tau} + \frac{\partial \phi(x_\tau)}{\partial x_\tau}\frac{\partial x_\tau}{\partial \tau} = 0$$

CHAIN RULE

3.0 Level set derivation

$$\phi(x_\tau) = 0$$

Boundary represented by level set

$$\frac{\partial \phi(x_\tau)}{\partial \tau} + \frac{\partial \phi(x_\tau)}{\partial x_\tau}\frac{\partial x_\tau}{\partial \tau} = 0$$

CHAIN RULE

$$\frac{\partial \phi(x_\tau)}{\partial \tau} = -\frac{\partial \phi(x_\tau)}{\partial x_\tau}\frac{\partial x_\tau}{\partial \tau}$$

RE-ARRANGE TERMS

3.0 Level set derivation

$$\phi^{\tau+1} = \phi^{\tau} + \frac{\partial \phi(x_\tau)}{\partial \tau} \delta \tau$$

EVOLUTION UPDATE EQUATION

$$\frac{\partial \phi(x_\tau)}{\partial \tau} = -\frac{\partial \phi(x_\tau)}{\partial x_\tau} \frac{\partial x_\tau}{\partial \tau}$$

RE-ARRANGE TERMS

3.0 Level set derivation

$$\frac{\partial \phi}{\partial \tau} = (m_s - m_b) \sum_k \int_0^T \lambda_k(x,z,t) \frac{\partial^2 u_k(x,z,t)}{\partial t^2} dt \, |\nabla \phi|$$

SPATIAL GRADIENT OF PHI

$$\frac{\partial \phi(x_\tau)}{\partial \tau} = -\frac{\partial \phi(x_\tau)}{\partial x_\tau} \frac{\partial x_\tau}{\partial \tau}$$

3.0 Level set derivation

$$\frac{\partial \phi}{\partial \tau} = (m_s - m_b) \sum_k \int_0^T \lambda_k(x,z,t) \frac{\partial^2 u_k(x,z,t)}{\partial t^2} dt \, |\nabla \phi|$$

SCALAR "SPEED" TERM

$$\frac{\partial \phi(x_\tau)}{\partial \tau} = - \frac{\partial \phi(x_\tau)}{\partial x_\tau} \frac{\partial x_\tau}{\partial \tau}$$

3.0 Level set derivation

$$\frac{\partial \phi}{\partial \tau} = (m_s - m_b) \sum_k \int_0^T \lambda_k(x,z,t) \frac{\partial^2 u_k(x,z,t)}{\partial t^2} dt \, |\nabla \phi|$$

Back-propagated residual

3.0 Level set derivation

BOUNDARY GRADIENT

$$\frac{\partial \phi}{\partial \tau} = (m_s - m_b) \sum_k \int_0^T \lambda_k(x,z,t) \frac{\partial^2 u_k(x,z,t)}{\partial t^2} dt \, |\nabla \phi|$$

$$\frac{\partial V_{back}}{\partial \tau} = \sum_k \int_0^T \lambda_k(x,z,t) \frac{\partial^2 u_k(x,z,t)}{\partial t^2} dt$$

TOMOGRAPHY GRADIENT

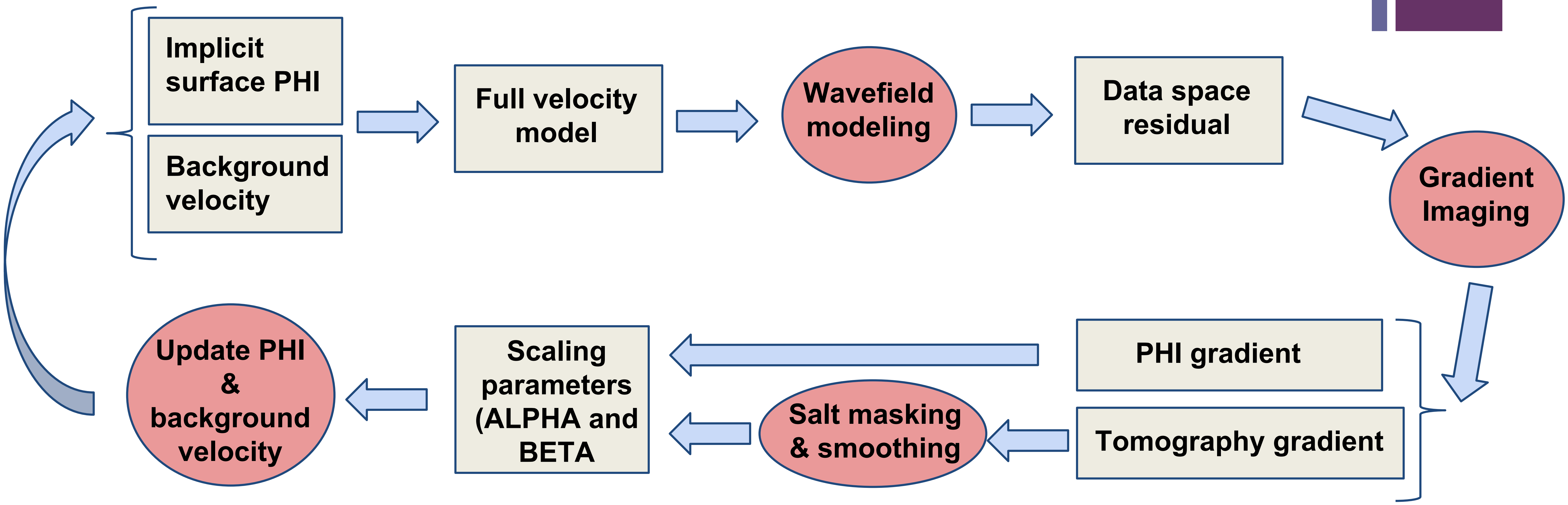
4.0 Alternating velocity update

	i = 1		i = 2
TOMOGRAPHY UPDATE	• Freeze background velocity	• **Smooth full velocity model** • **Update background velocity**	• Freeze background velocity
SALT BOUNDARY UPDATE	• **Update PHI surface**	• Freeze PHI surface	• **Update PHI surface**

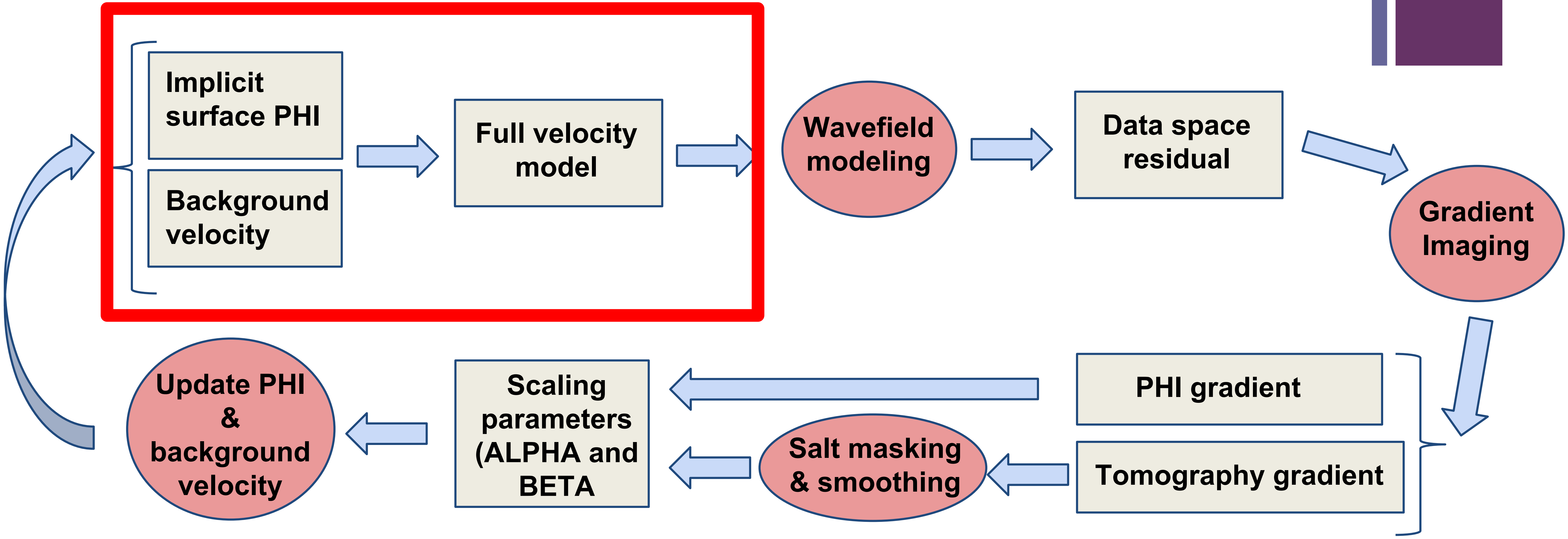
4.1 Concurrent velocity update

	i = 1	i = 2	i = 3
TOMOGRAPHY UPDATE	• **Update background velocity** →	• **Update background velocity** →	• **Update background velocity**
SALT BOUNDARY UPDATE	• **Update PHI surface** →	• **Update PHI surface** →	• **Update PHI surface**

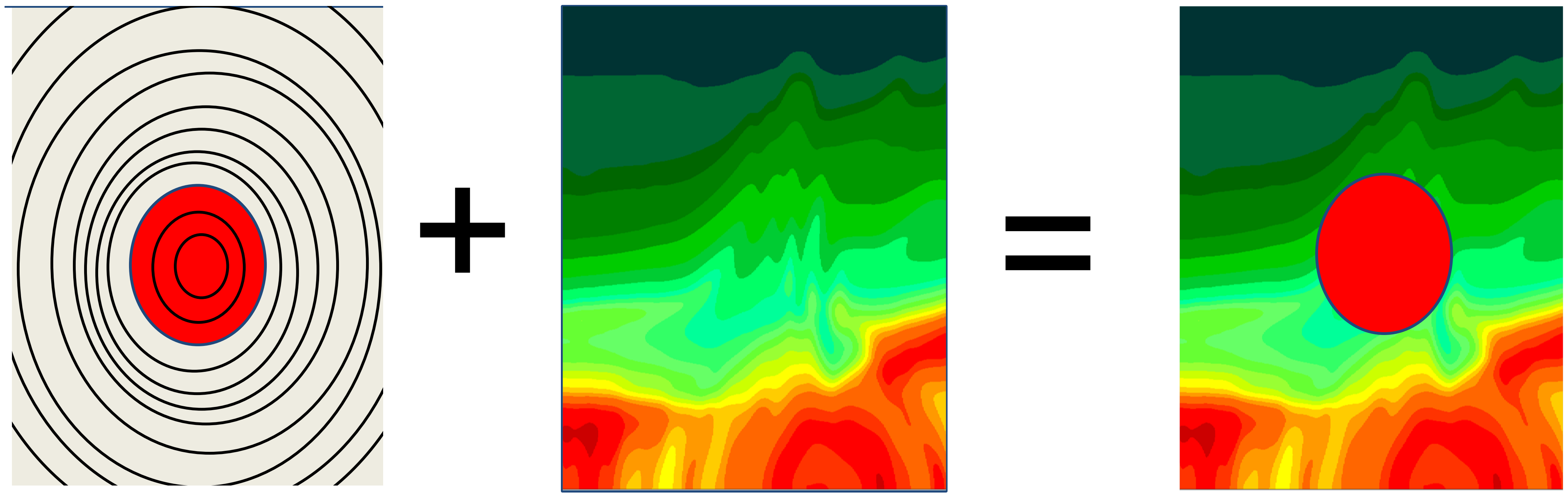
4.2 Update workflow



4.2.1 Build full velocity model



4.2.1 Build full velocity model

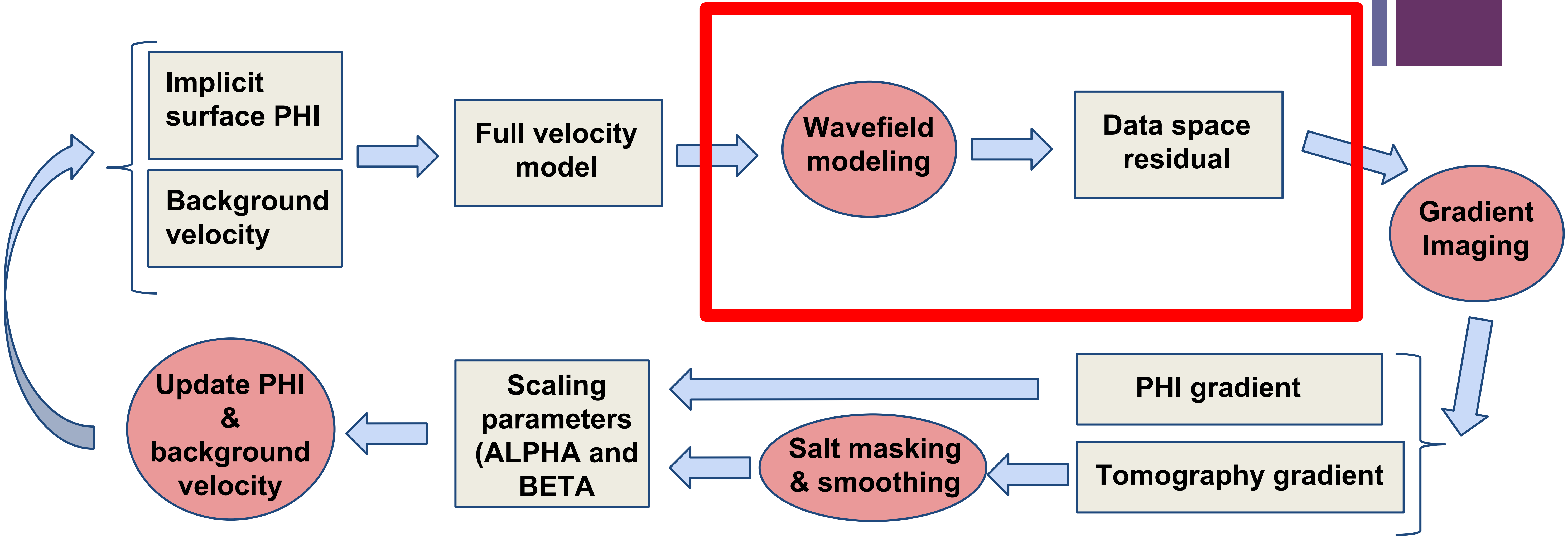


Salt boundary as zero level contour

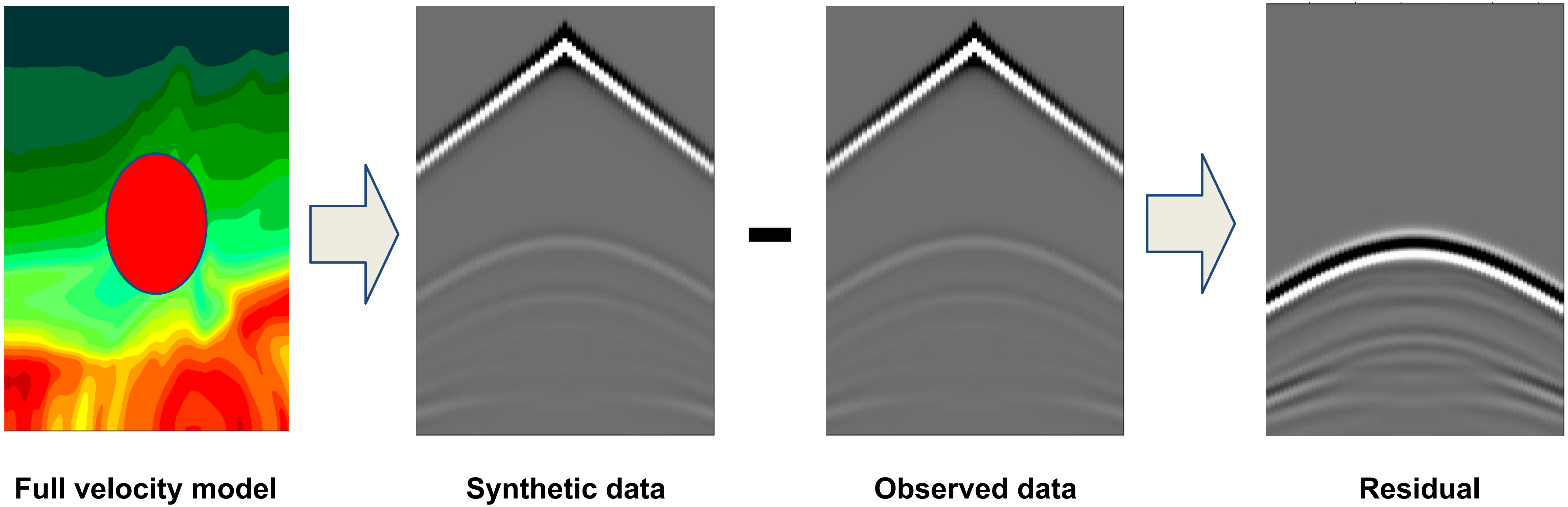
Background velocity

Full velocity model

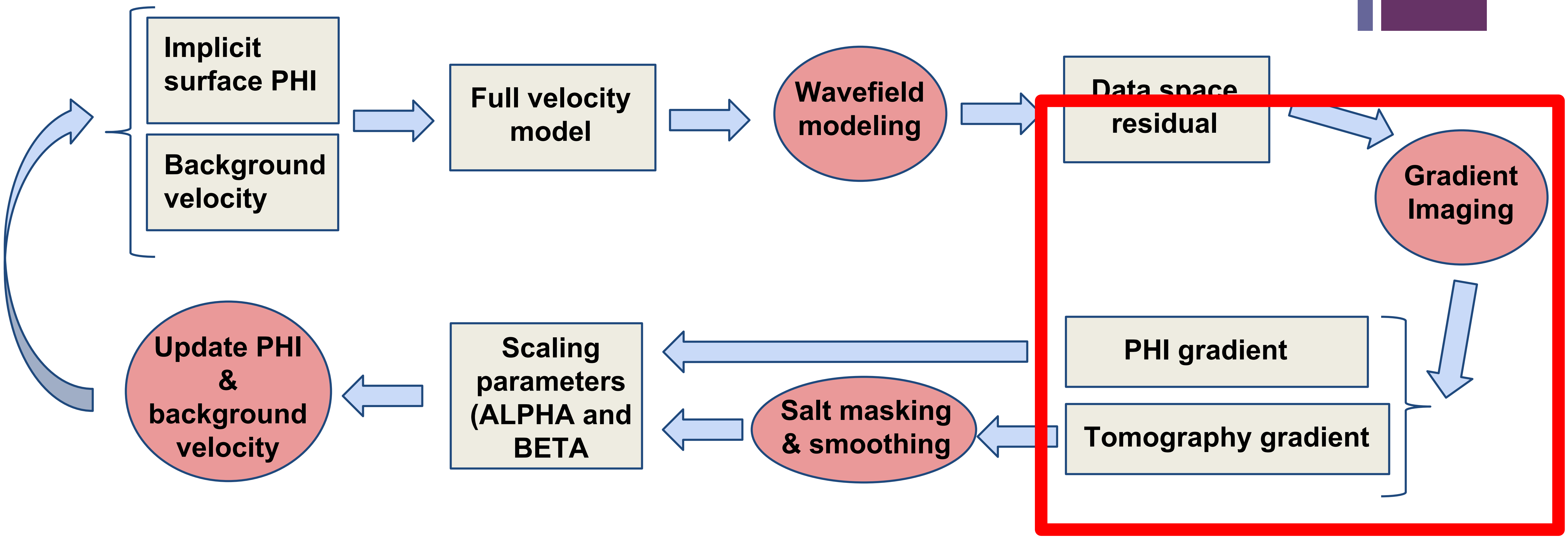
4.2.2 Forward model



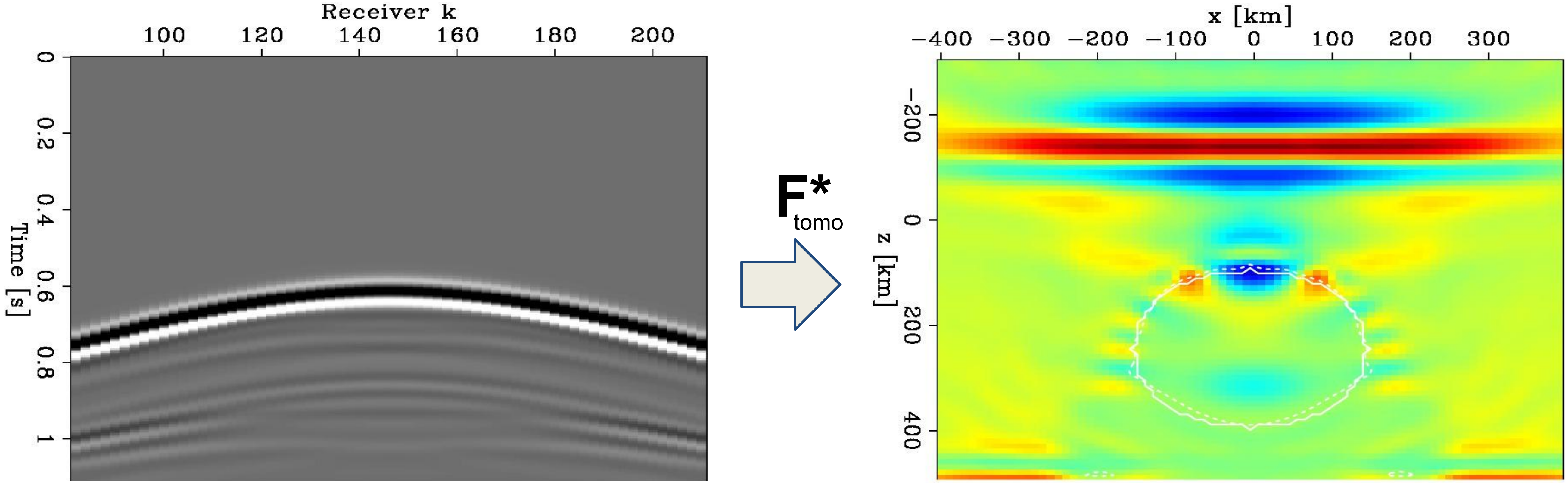
4.2.2 Forward model



4.2.3 Linearized adjoint gradients

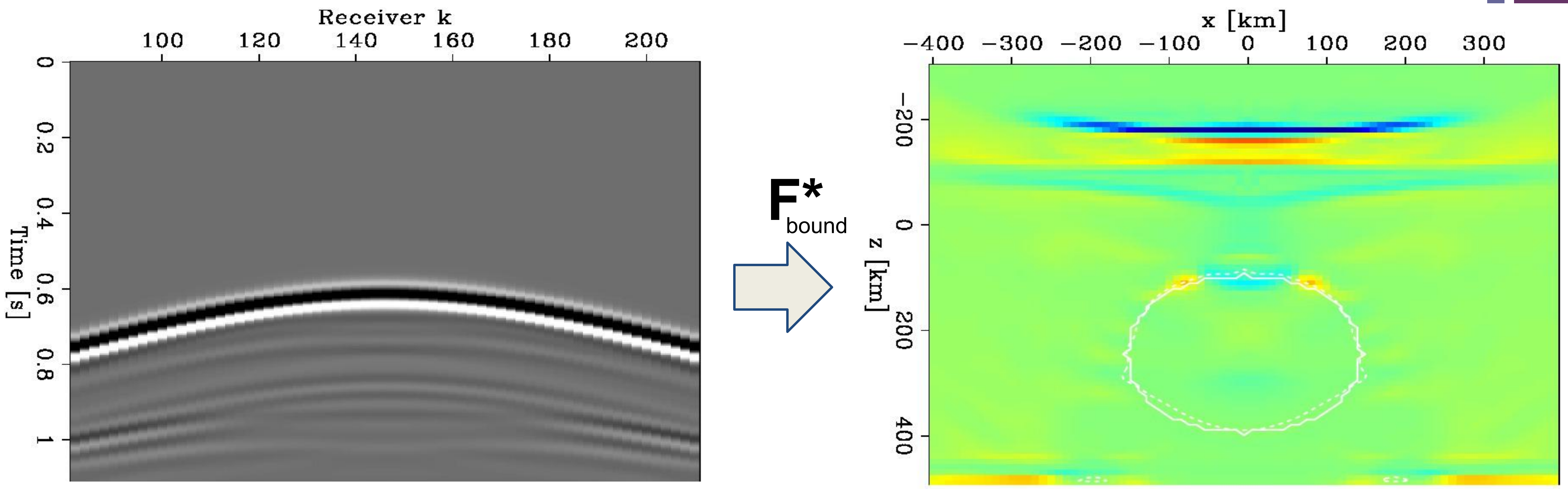


4.2.3 Linearized adjoint gradients



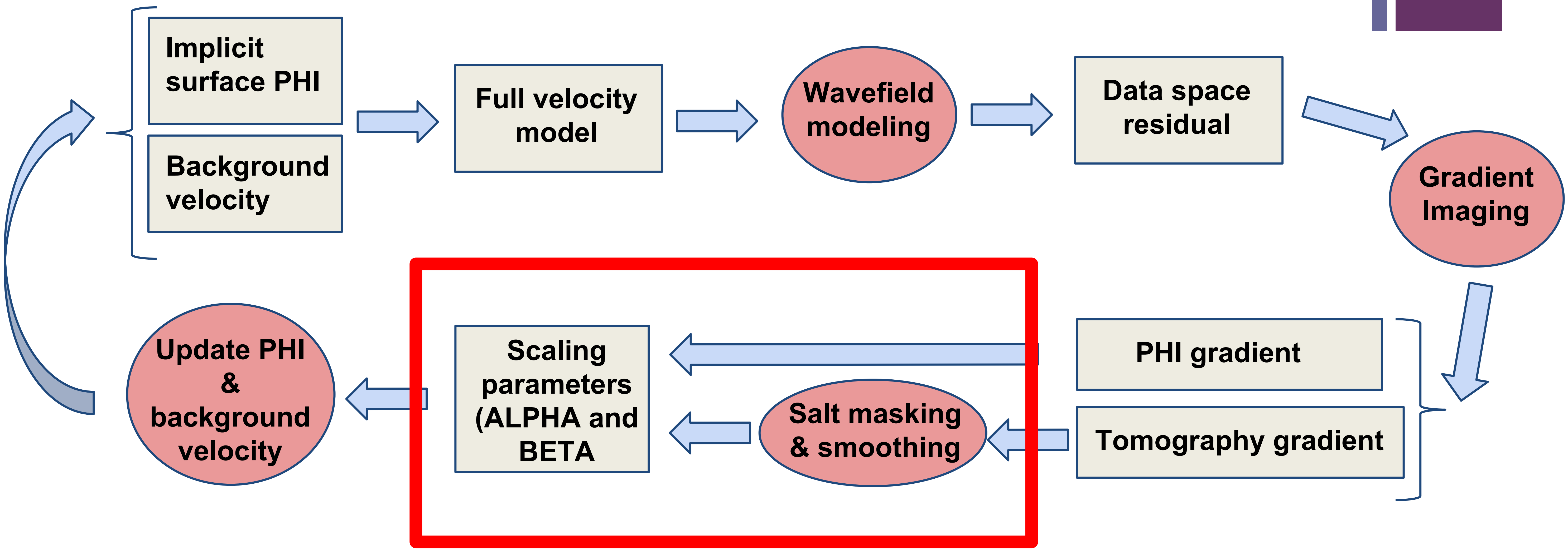
G_{tomo} **(Tomographic update gradient)**

4.2.3 Linearized adjoint gradients



G_{bound} **(Salt boundary update gradient)**

4.2.4 Scaling parameters



Scaling parameter search

EXPLICIT STEPPING FUNCTIONS :

$$\phi^{\tau+1} = \phi^{\tau} + \frac{\partial \phi}{\partial \tau}\beta$$

$$V_{\text{back}}^{\tau+1} = V_{\text{back}}^{\tau} + \frac{\partial V_{\text{back}}}{\partial \tau}\alpha$$

Scaling parameter search

$$\min_{\alpha_L, \beta_L} ||F_{\text{tomo}} G_{\text{tomo}} \alpha_L + F_{\text{bound}} G_{\text{bound}} \beta_L - \text{residual}||$$

LINEAR PLANE SEARCH OBJECTIVE FUNCTION

MINIMIZE FOR ALPHA AND BETA

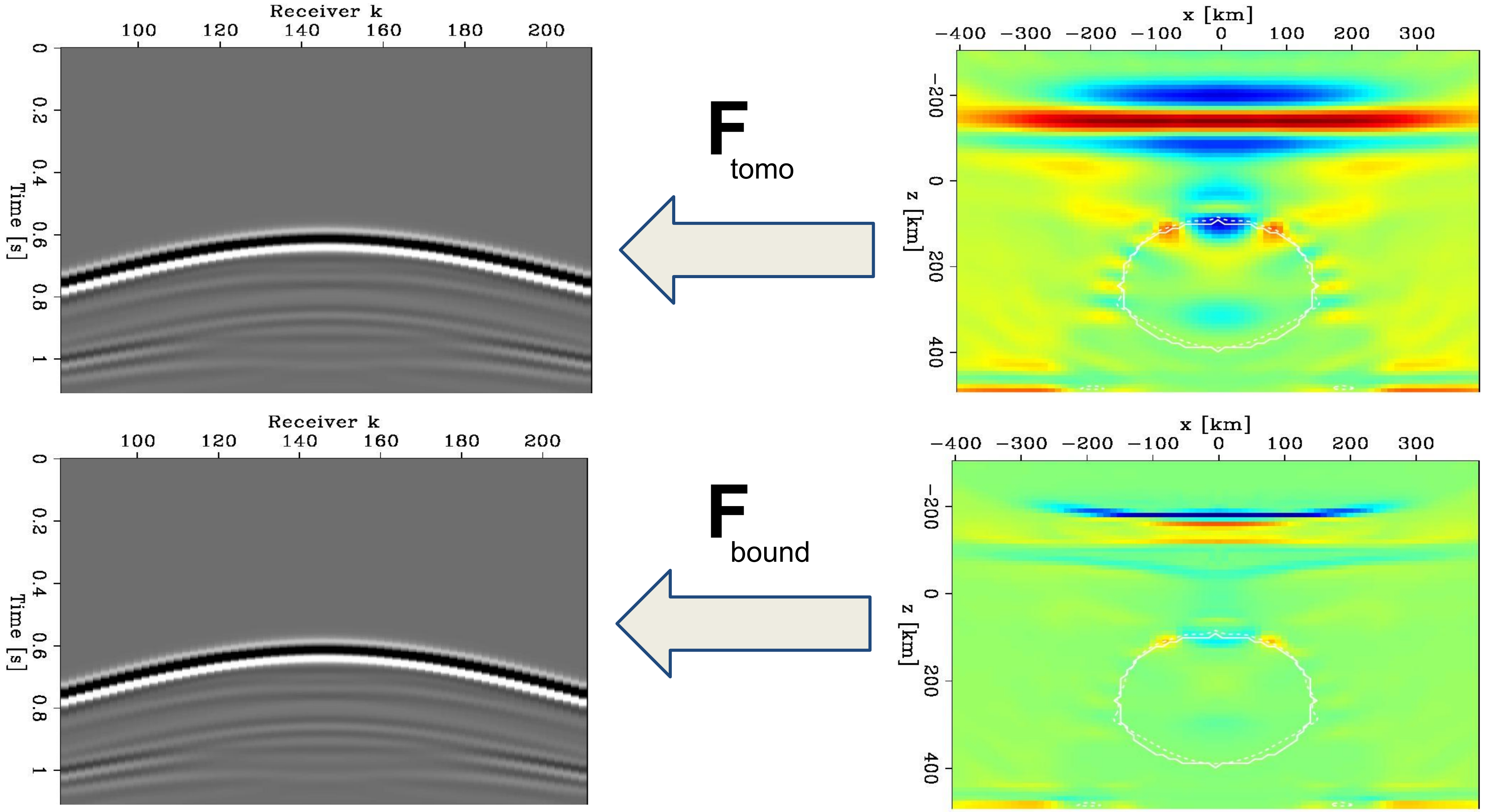
Scaling parameter search

$$\min_{\alpha_L, \beta_L} ||F_{\text{tomo}} G_{\text{tomo}} \alpha_L + F_{\text{bound}} G_{\text{bound}} \beta_L - \text{residual}||$$

LINEAR PLANE SEARCH OBJECTIVE FUNCTION

RETURN TO RESIDUAL SPACE USING LINEARIZED FORWARD OPERATORS

4.2.4 Scaling parameters



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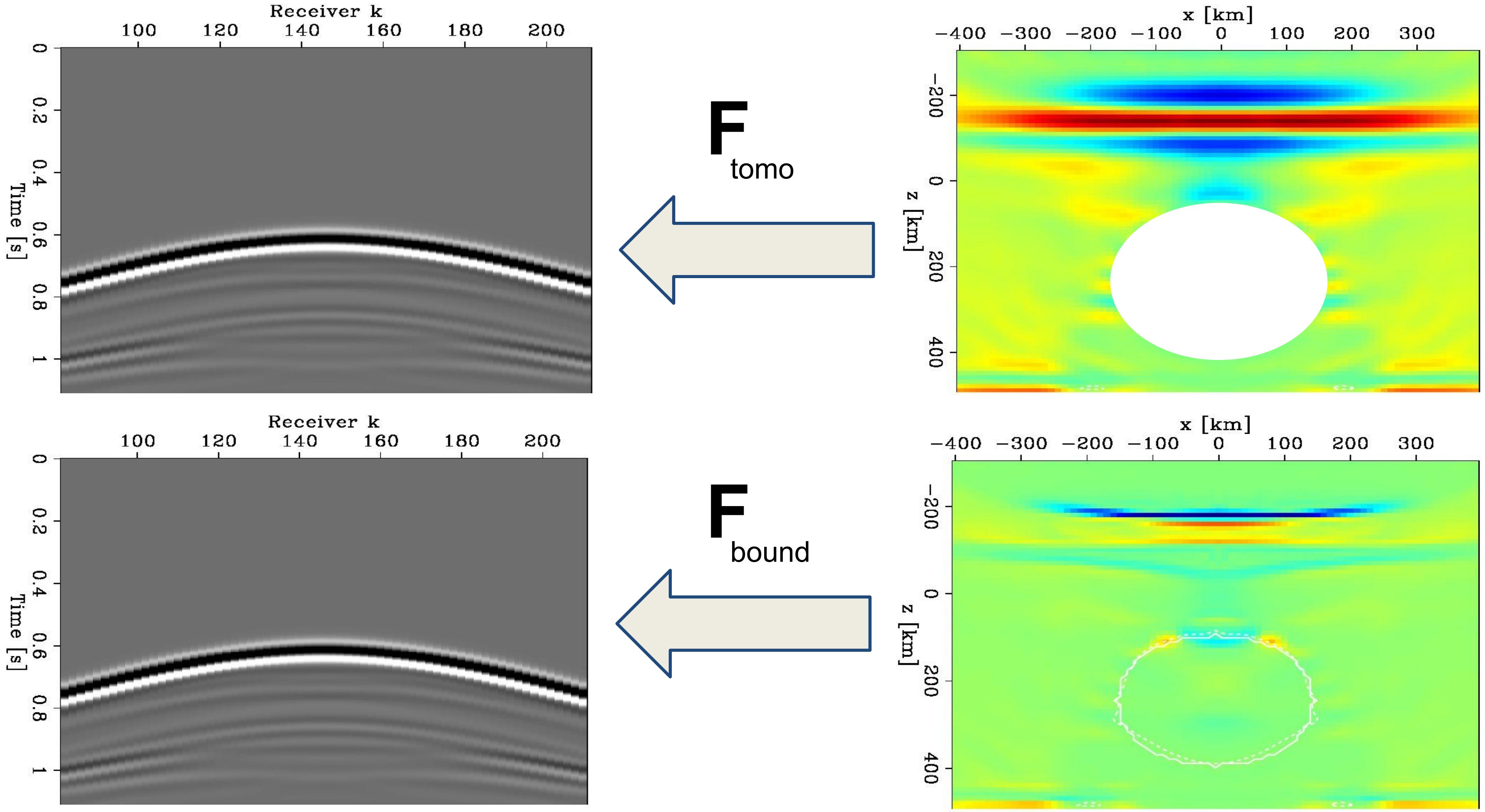
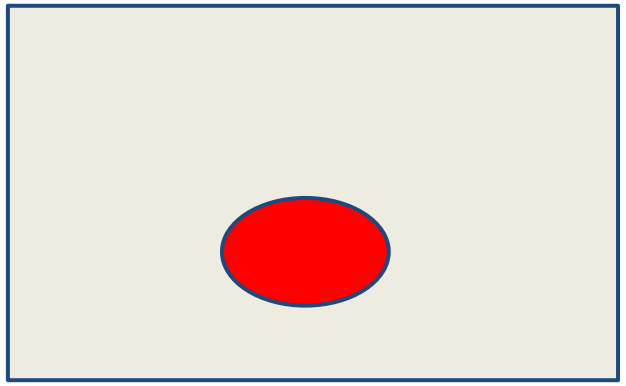


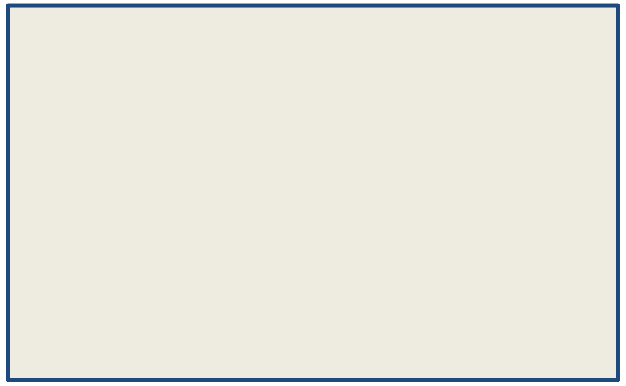
- Tomographic update is never applied to salt area
- We mask out this portion of the update before forward operation
- We base our masking on the previous salt boundary

4.2.4 Scaling parameters



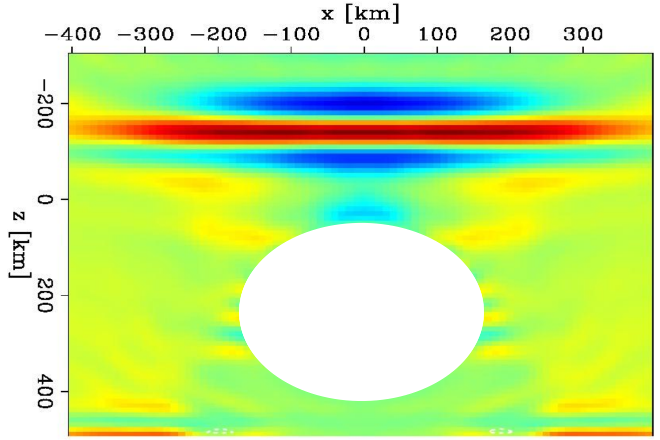
New salt boundary

+

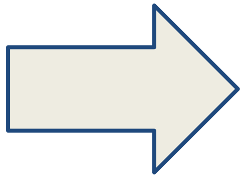


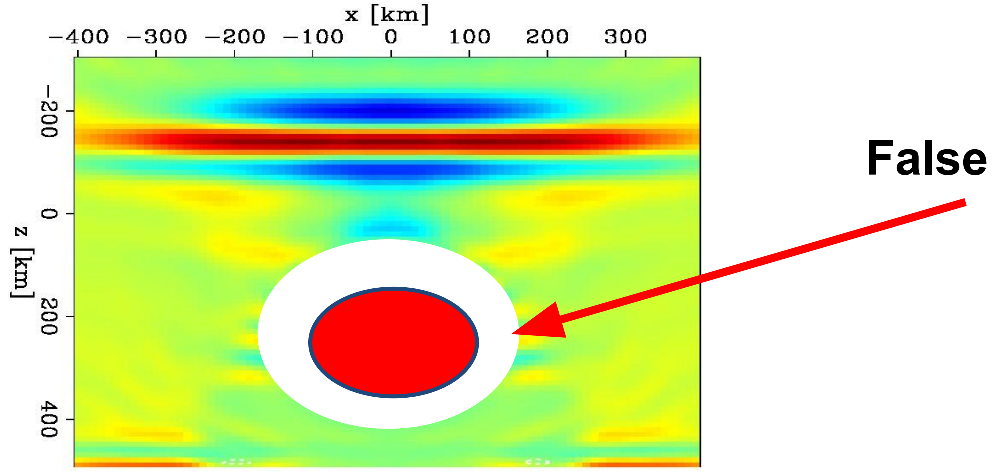
Constant background velocity

+



Tomographic update gradient





New full velocity model

False reflector

- What if salt update shrinks?
- Then false reflectors show up
- To prevent this, we smooth after masking
- Also prevents other reflection data from being applied too strongly

Scaling parameter search

$$\min_{\alpha_L,\beta_L} ||F_{\text{tomo}}G_{\text{tomo}}\alpha_L + F_{\text{bound}}G_{\text{bound}}\beta_L - \text{residual}||$$

LINEAR PLANE SEARCH OBJECTIVE FUNCTION

RE-DEFINE ALPHA AND BETA IN TERMS OF GAMMA

$$\alpha_\gamma = \gamma\alpha_L \qquad \beta_\gamma = \gamma\beta_L$$

Scaling parameter search

$$\min_{\alpha_L,\beta_L} ||F_{\text{tomo}}G_{\text{tomo}}\alpha_L + F_{\text{bound}}G_{\text{bound}}\beta_L - \text{residual}||$$

LINEAR PLANE SEARCH OBJECTIVE FUNCTION

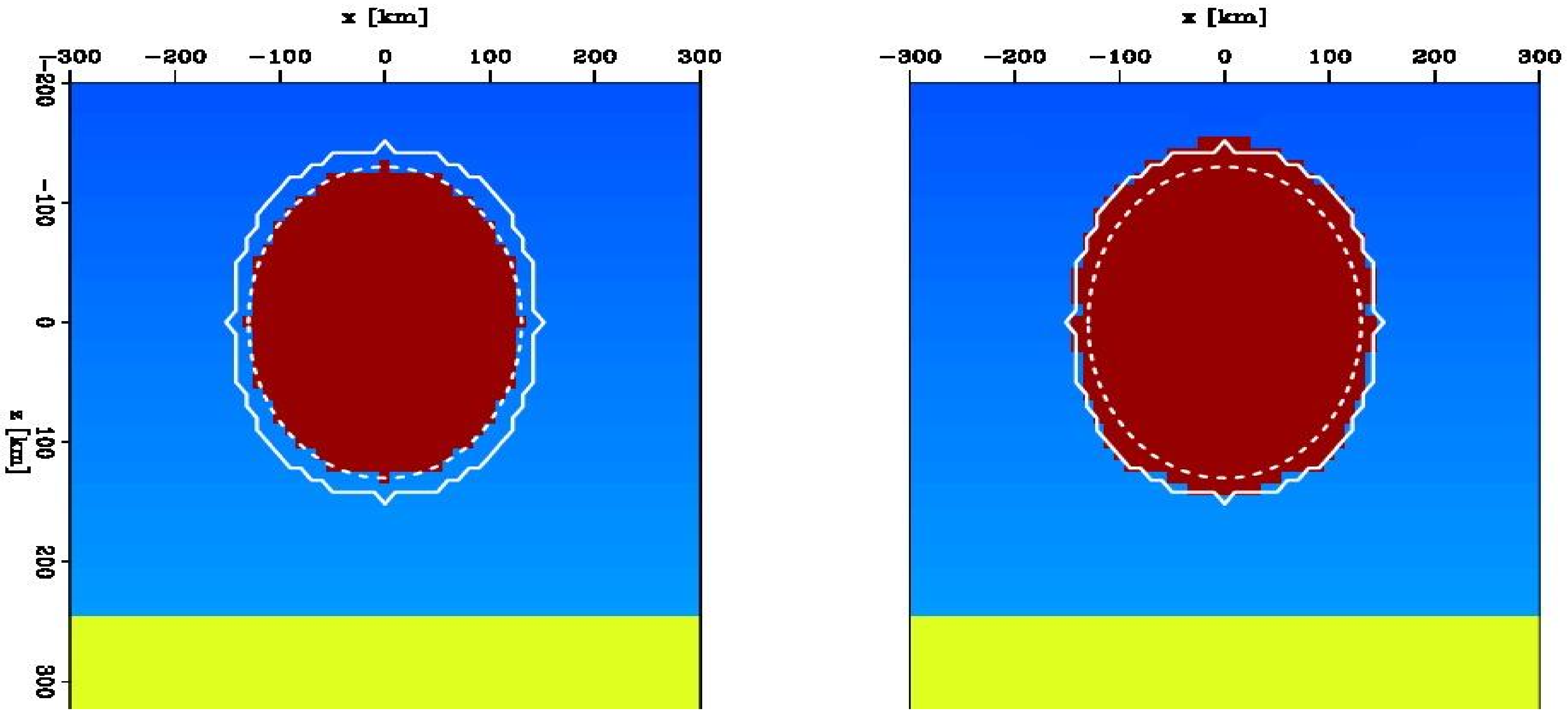
$$\alpha_\gamma = \gamma\alpha_L \qquad \beta_\gamma = \gamma\beta_L$$

$$\min_{\gamma} ||F(m(\alpha_\gamma, \beta_\gamma)) - d||$$

SOLVE FOR GAMMA TO RE-SCALE ALPHA AND BETA

NON-LINEAR LINE SEARCH OBJECTIVE FUNCTION

5.1 Convergence demonstrations



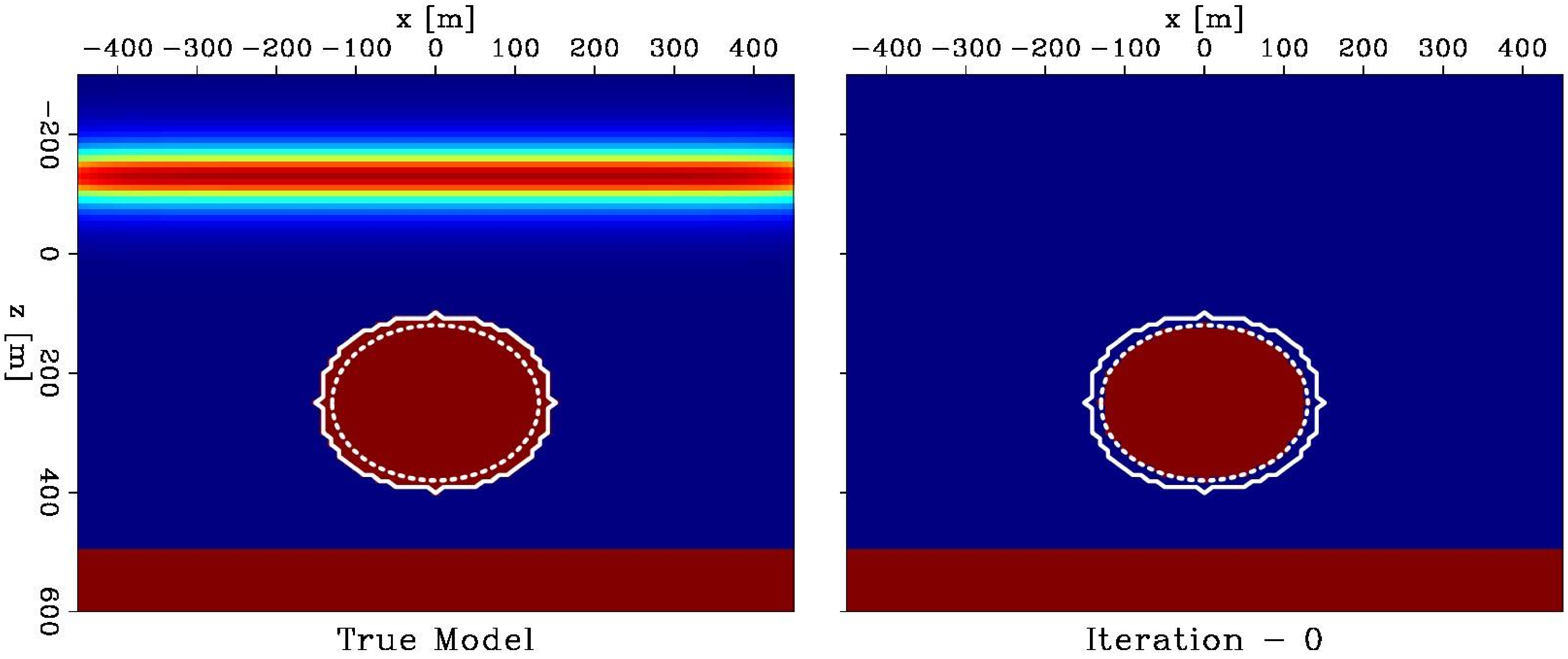
Iteration = 0

Iteration = 49

Dashed = Guess

Solid = True model

5.2 Convergence demonstrations



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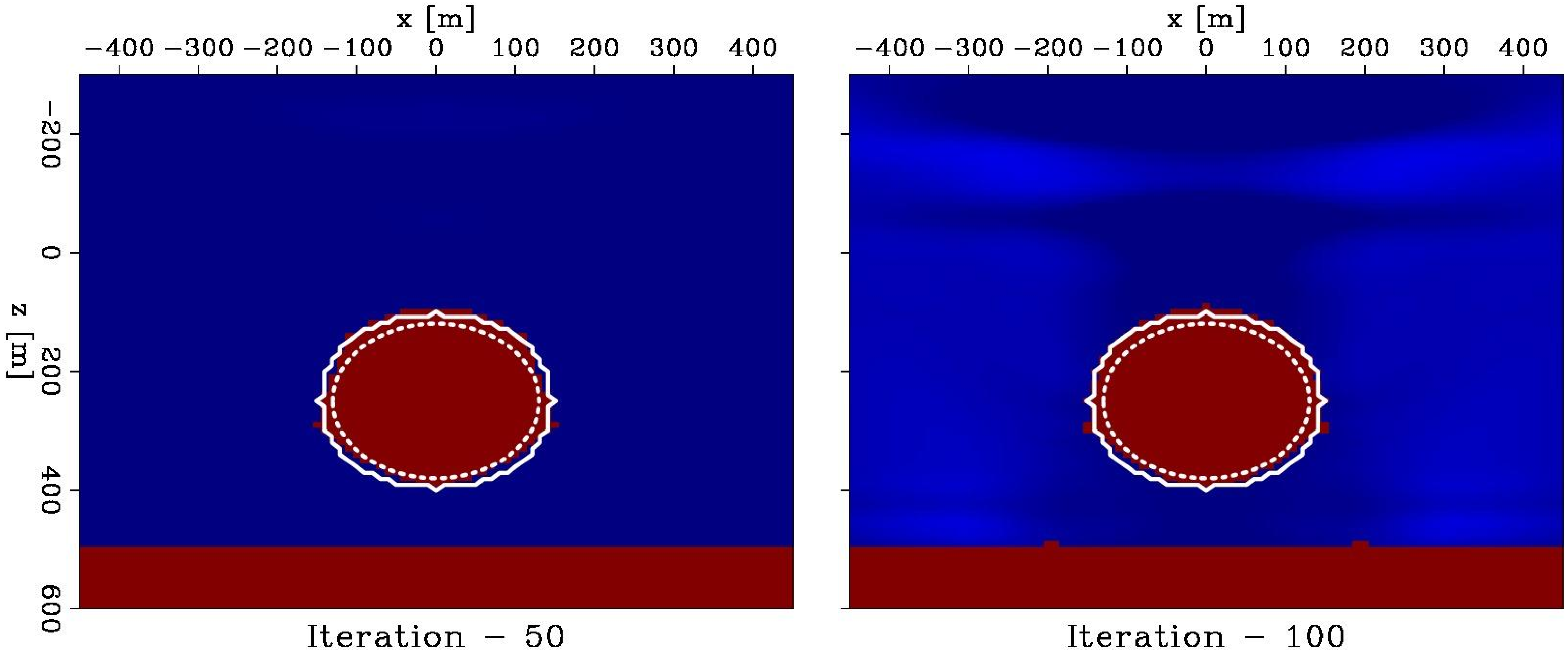
5.2 Convergence demonstrations



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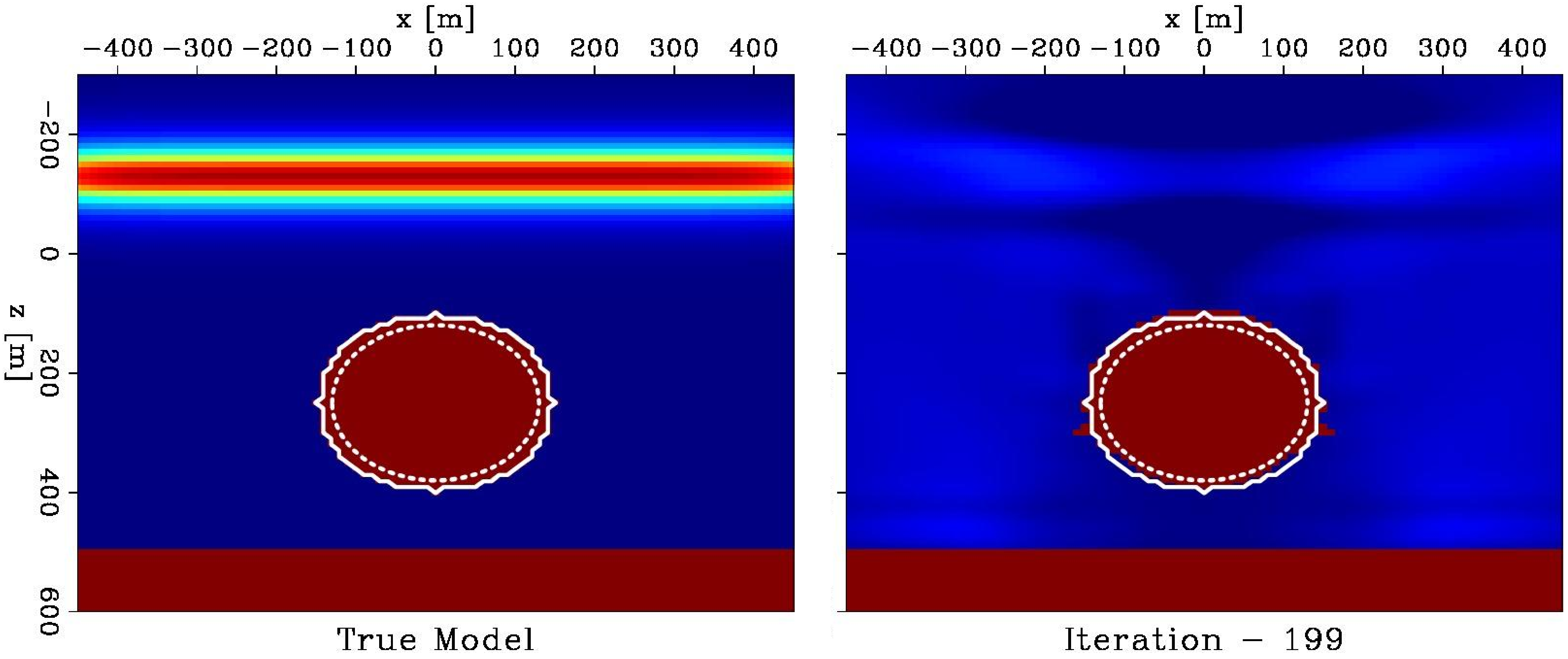
5.2 Convergence demonstrations



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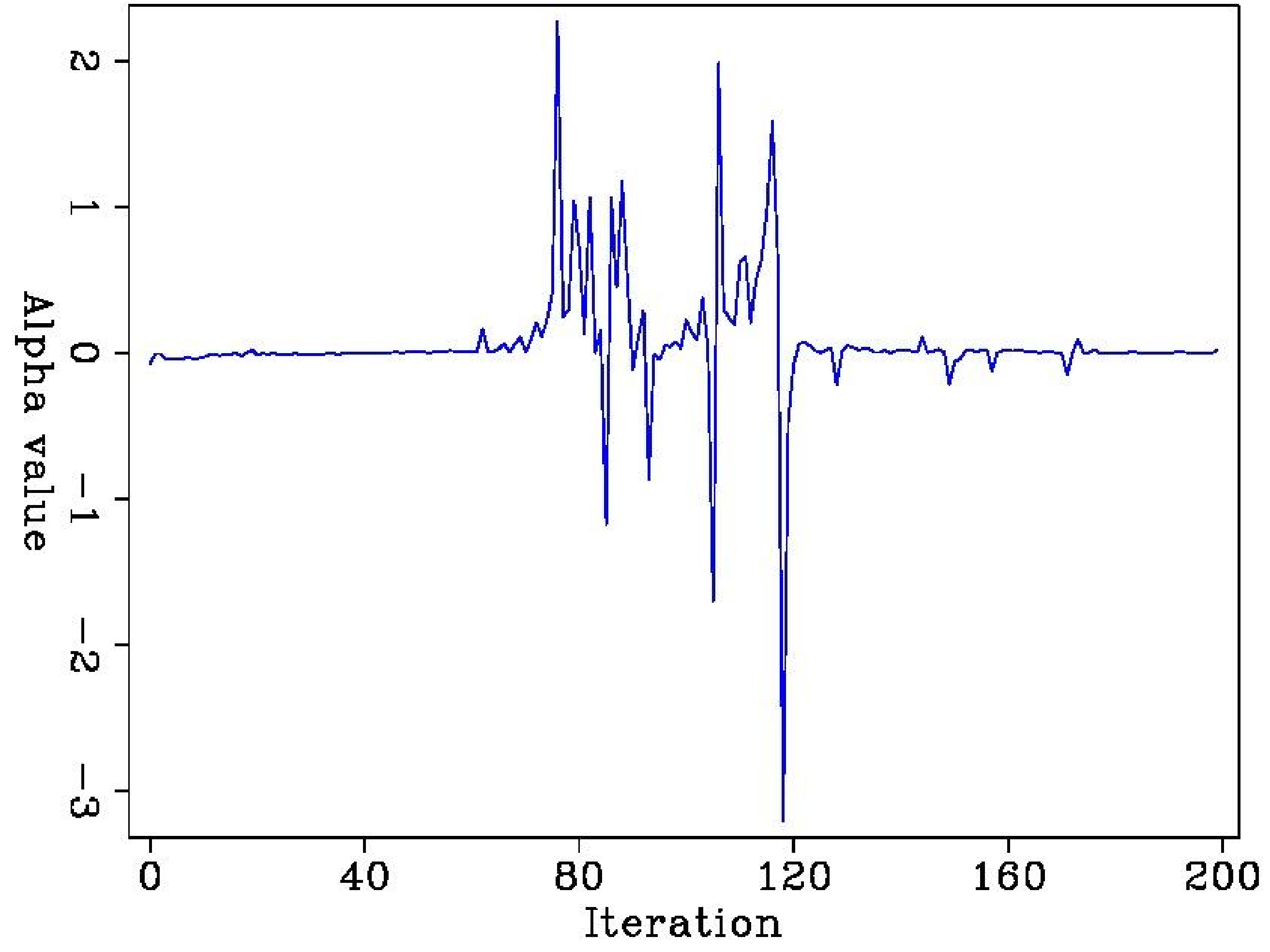
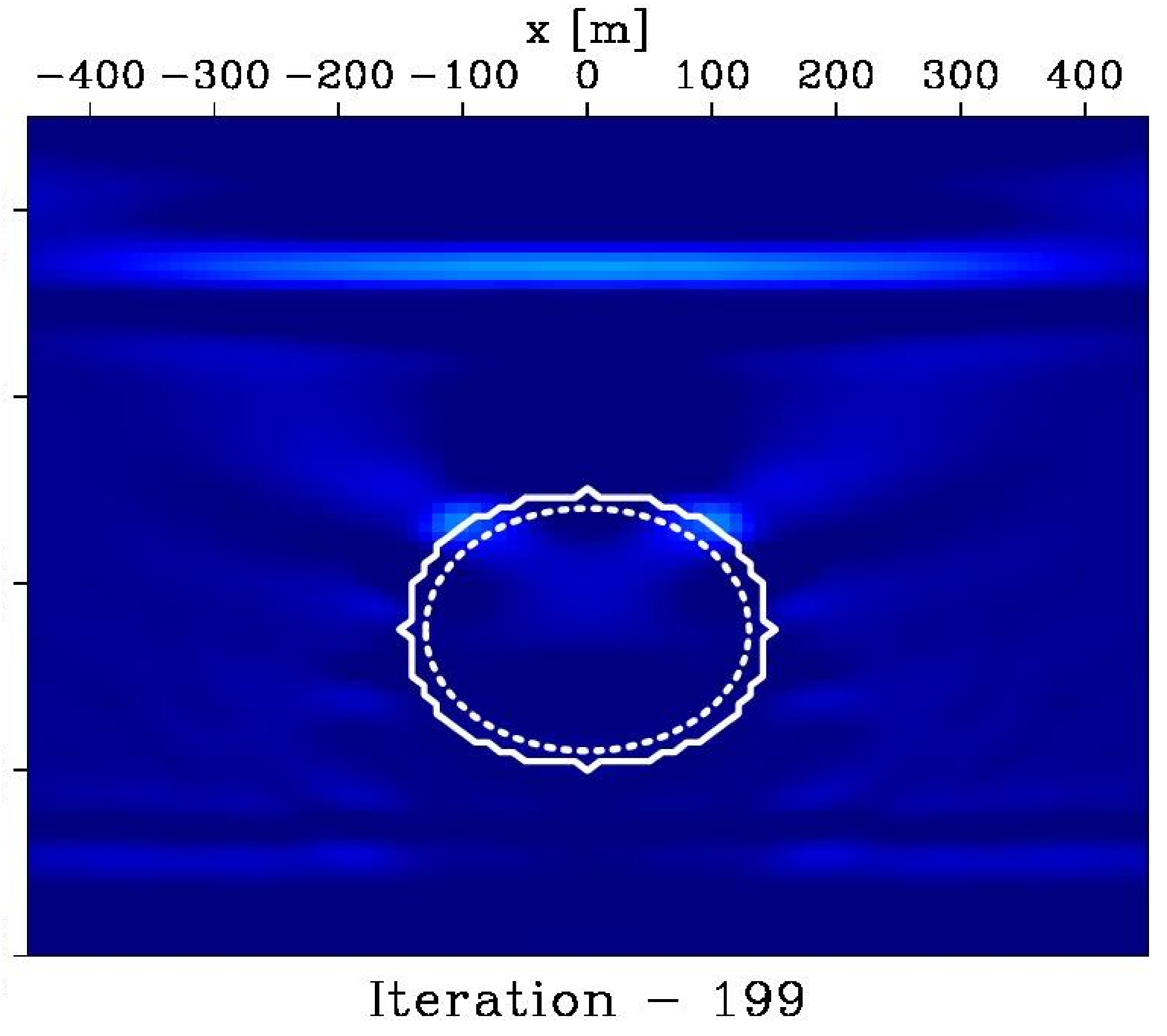
5.2 Convergence demonstrations





Dashed = Guess

Solid = True model

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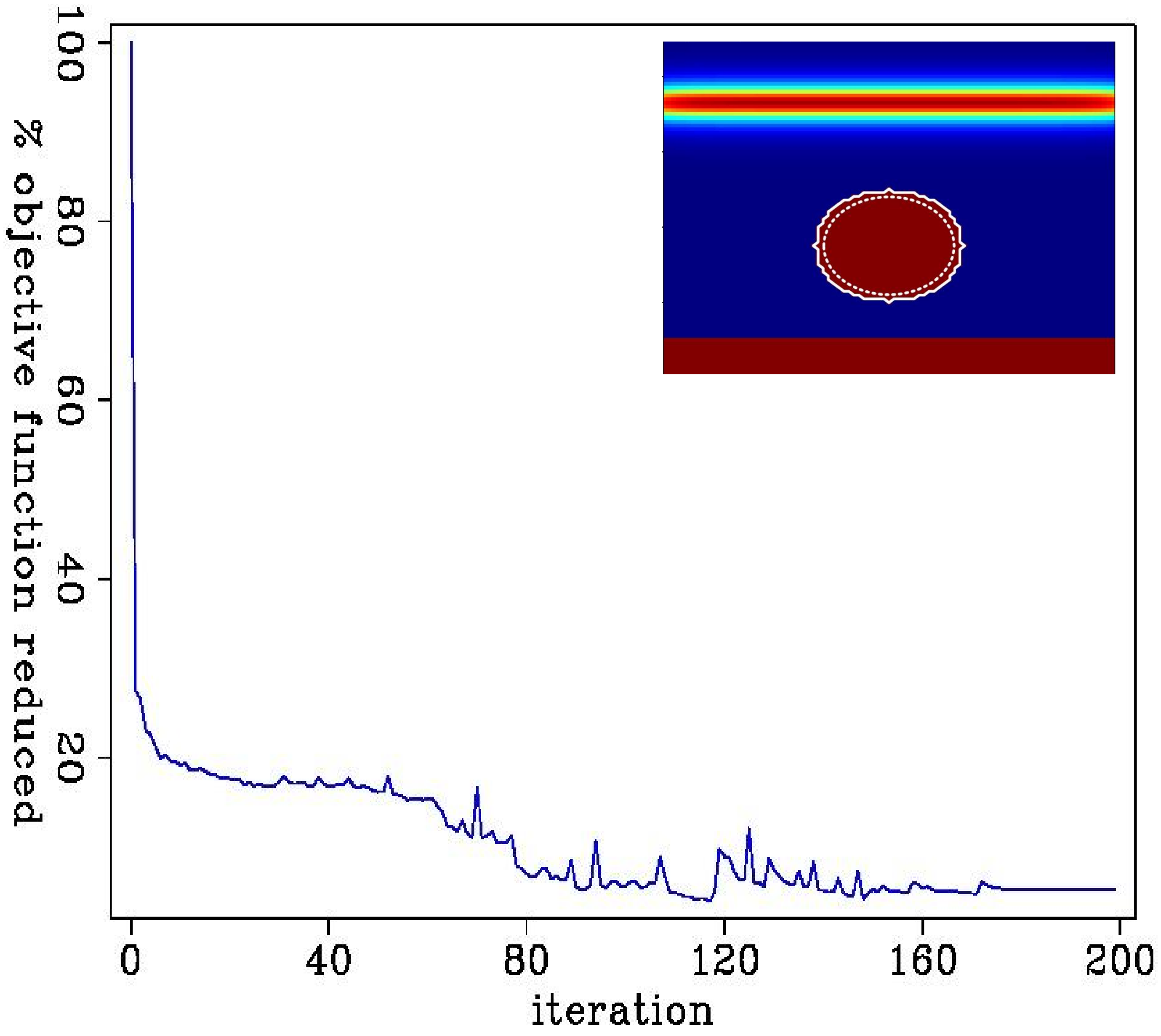
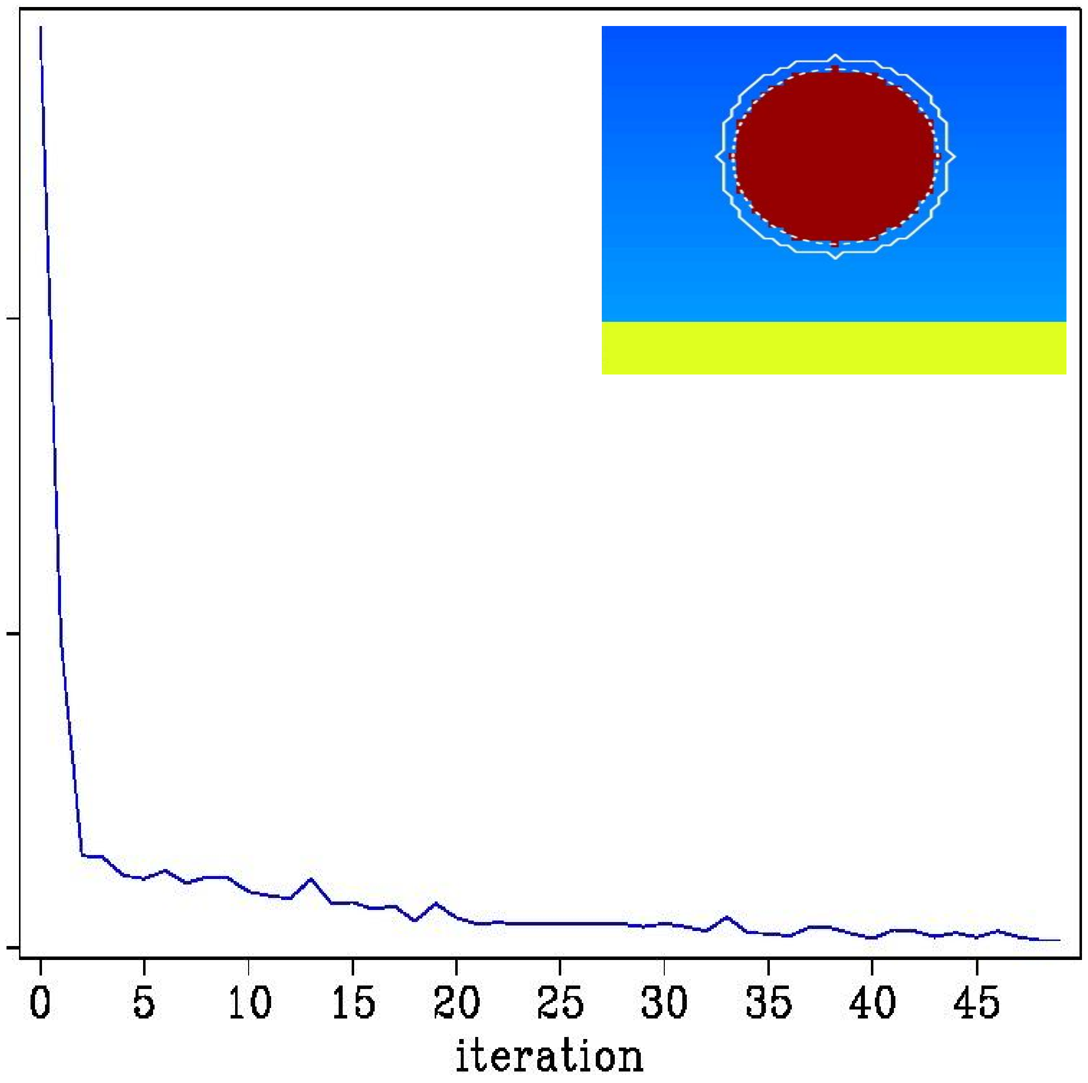
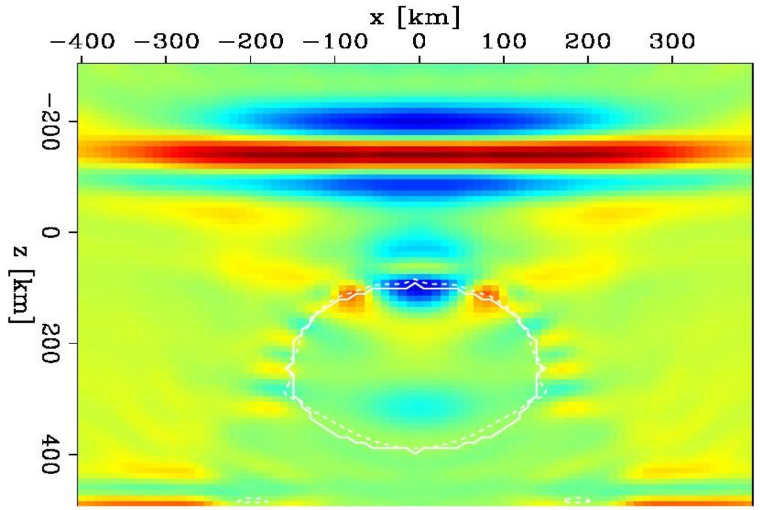
5.3 Convergence demonstrations



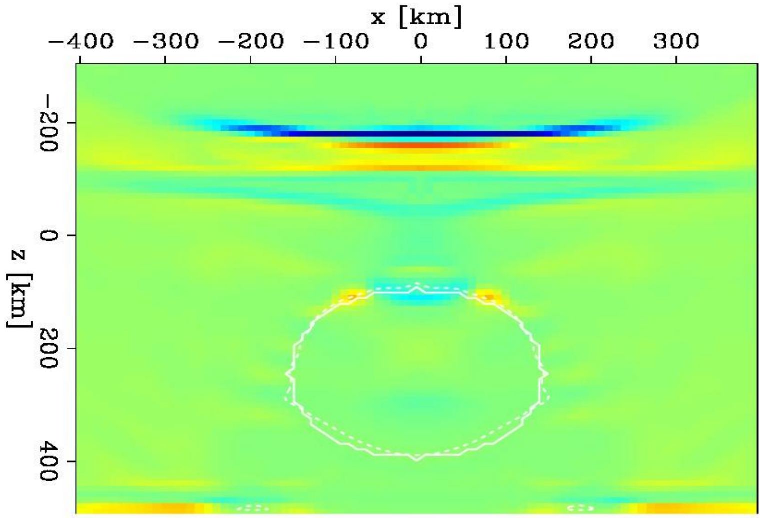


5.4 Convergence demonstrations

G_{tomo}

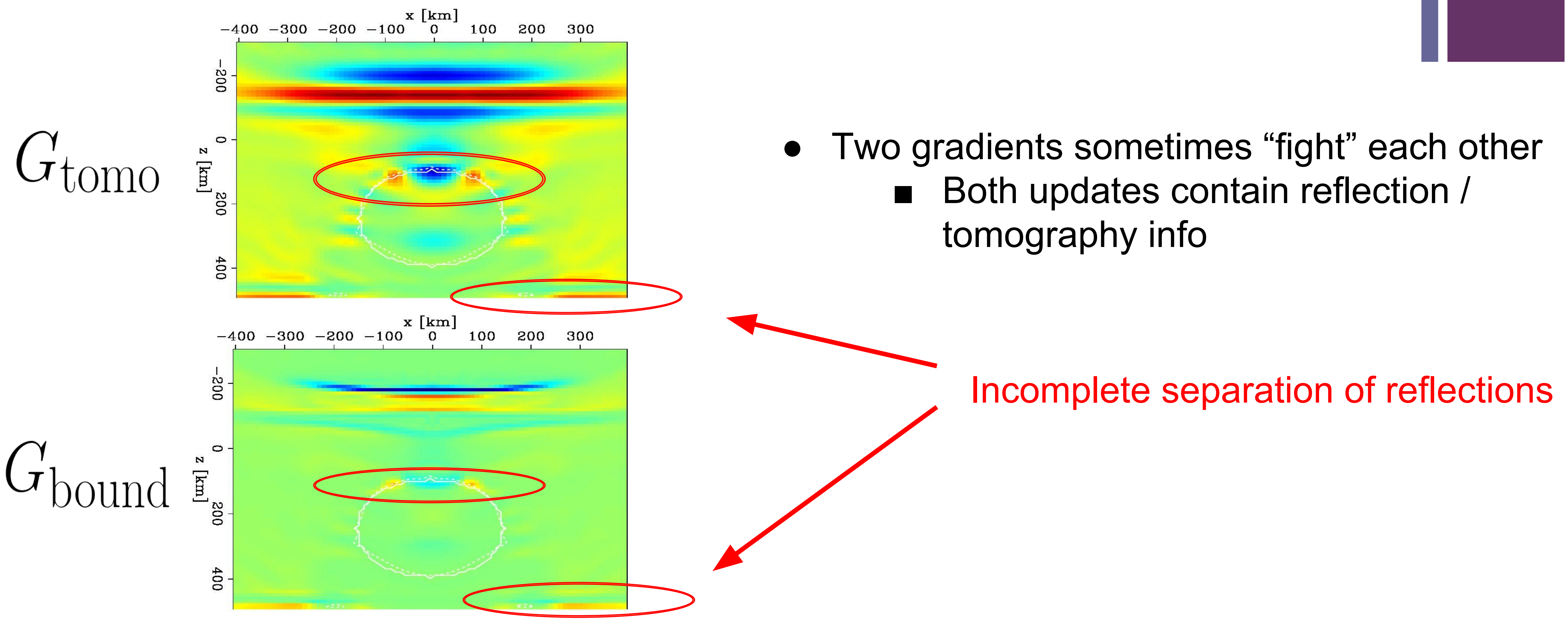


G_{bound}



- Two gradients sometimes “fight” each other
  - Both updates contain reflection / tomography info

5.4 Convergence demonstrations



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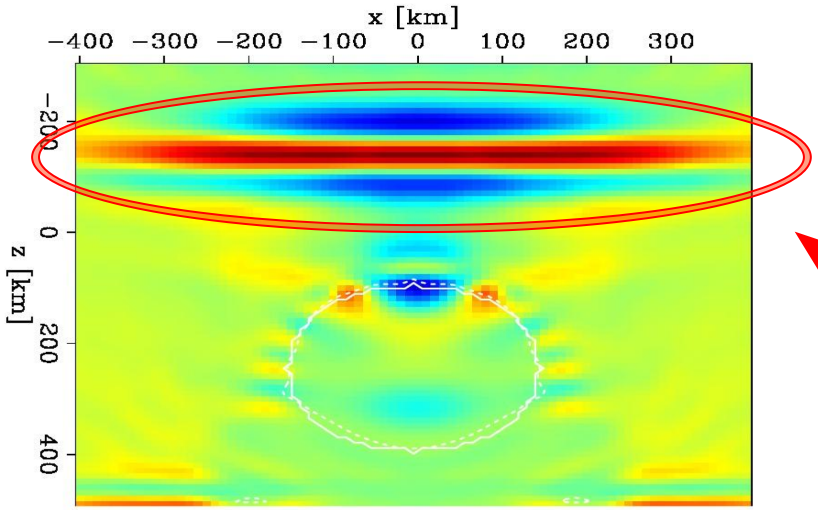
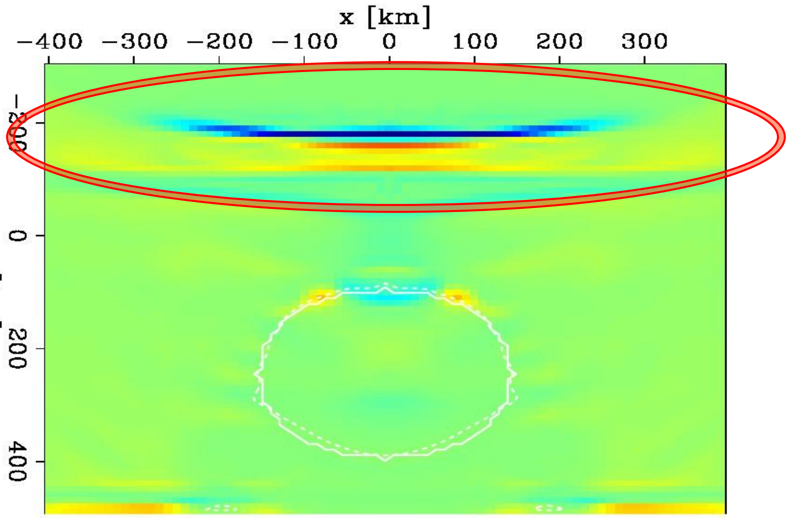
5.4 Convergence demonstrations

G_{tomo}



G_{bound}

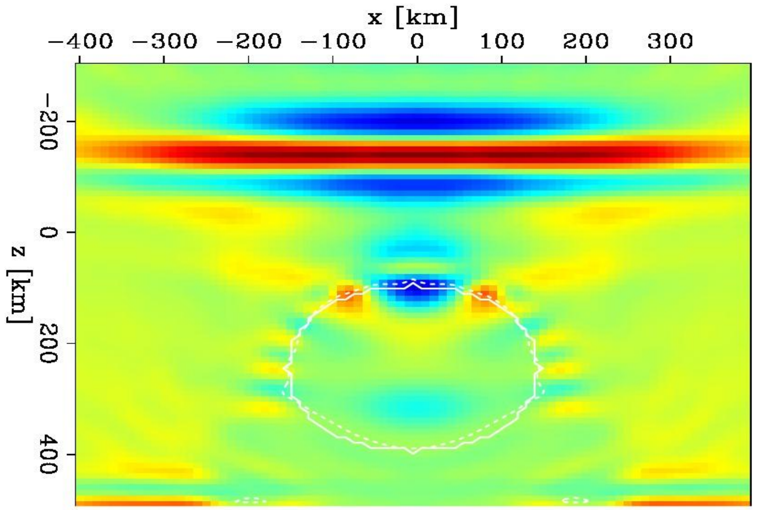


- Two gradients sometimes "fight" each other
  - Both updates contain reflection / tomography info

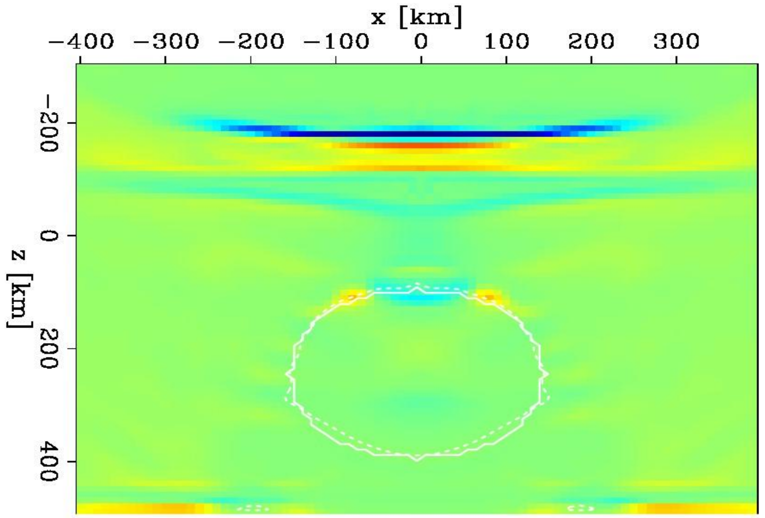
Incomplete separation of tomography

5.4 Convergence demonstrations

G_{tomo}



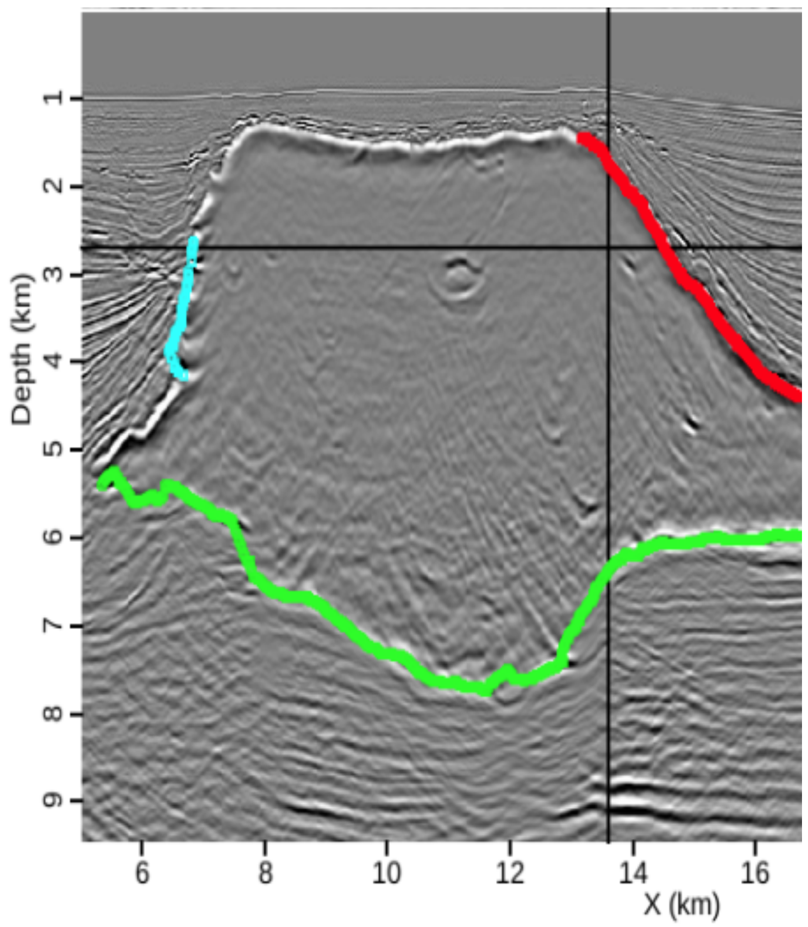
G_{bound}

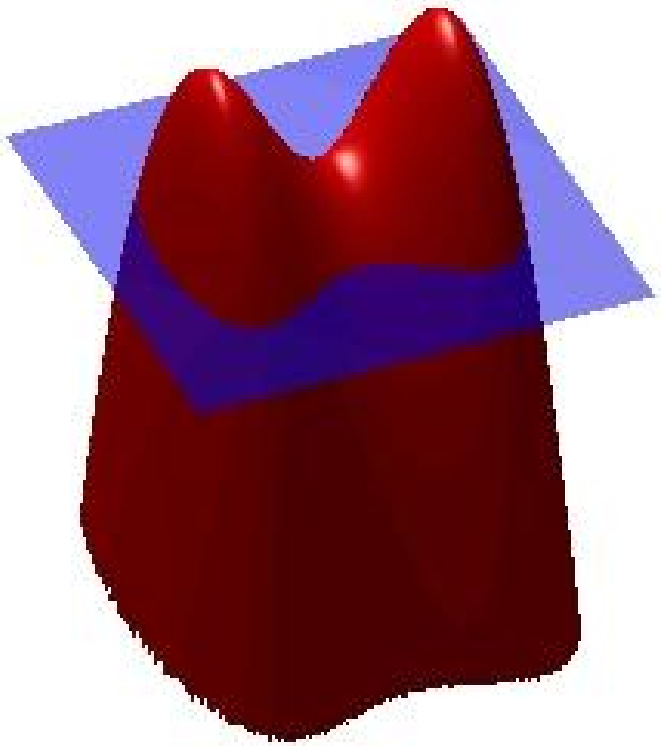


- More error introduced to gradients from:
  - Masking using previous salt boundary
  - Smoothing of tomographic gradient
  - Regularization applied to boundary gradient

6.1 Future work

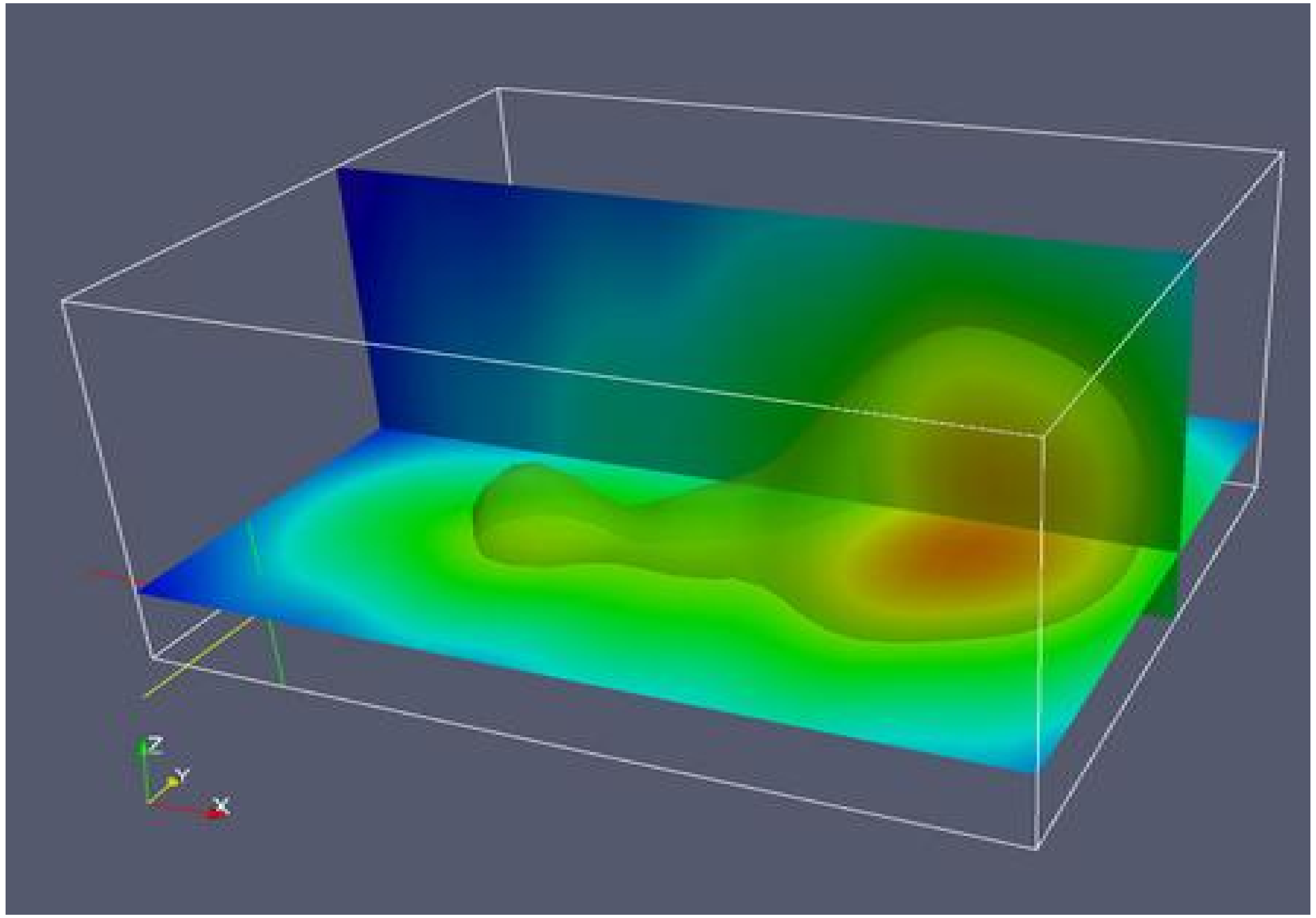
Halpert, SEP-149





- Incorporate expert confidence input
  - Currently the only expert input is from initial boundary guess
  - Evolution could be influenced by confidence mapping of boundary

6.2 Future work



- Implement convergence criteria
    - Cooling method
- Extend to 3D
    - No major theoretical differences
    - Level set is a surface, not a contour

Summary

- Level set approach can be well posed to defining salt bodies
- Concurrent tomographic updating shows promise, but must overcome the inherent limitations of mixed tomographic/boundary information
- Potential to integrate expert boundary confidence input as constraint on evolution

Acknowledgements

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- the sponsors of SEP for their generous financial support,
- Robert Clapp and Biondo Biondi for their advising,
- my colleagues in SEP for their helpful discussion and suggestions.