Removing Shear Artifacts in Acoustic Anisotropic Modeling

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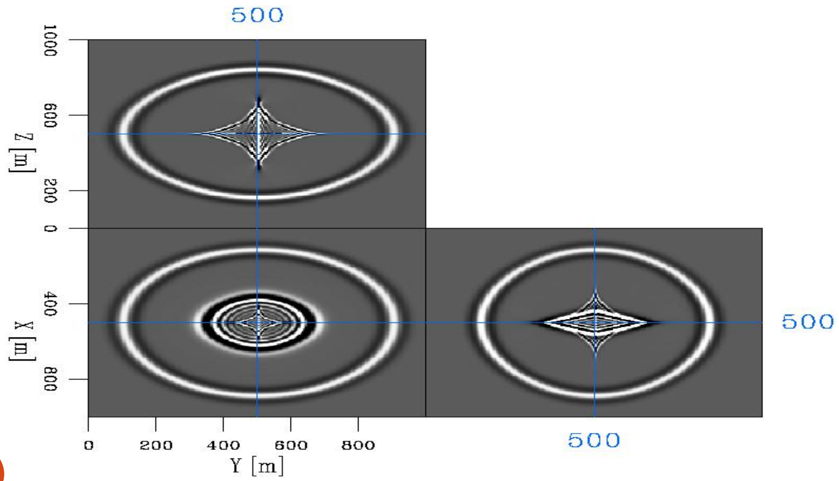
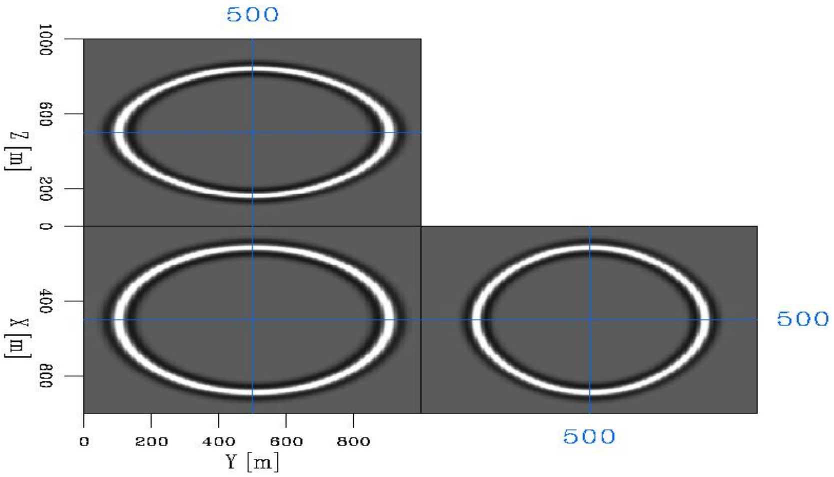
SEP 152, p. 129-140





Outline

1. Acoustic approximation and shear artifacts
2. Existing methods
3. Proposed method
4. Accuracy and cost estimates
5. Application to inhomogeneous media
6. Conclusions

Shear artifacts

- Exact wave equations in anisotropic media couple P and S.
- Acoustic approximation: setting S-velocity **along symmetry axes** to 0 (Alkhalifah, 1998)
- S-velocity is **not 0 everywhere** (Grechka et al., 2004).

Existing methods

1. Put the source in isotropic region (Alkhalifah, 1998)
    - Limited application
    - Converted shear artifacts
2. In VTI, use finite shear velocity (Fletcher et al., 2004)
    - What shear velocity to choose?
    - Real shear wave
3. Factorize the dispersion relation (majority)
    - Applicable with source in any media
    - No shear velocity choice

Proposed method

- Factorize dispersion relation to decouple P and S
- Dispersion relation in VTI:

  cubic = quadratic of P and SV + linear of SH

$\Rightarrow$ factorization can be done analytically.

- Similar to elastic wave mode separation (Dellinger and Etgen, 1990; Yan and Sava, 2009)
- In orthorhombic: **eigenvalue decomposition**

Acoustic wave equations in orthorhombic media

$$\frac{\partial^2}{\partial t^2}\boldsymbol{\sigma} = M\mathbf{D}\boldsymbol{\sigma}$$

$$\boldsymbol{\sigma} = \begin{bmatrix}\sigma_{11} & \sigma_{22} & \sigma_{33}\end{bmatrix}^T$$

$$M = \frac{1}{\rho}\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} & c_{23} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}, \mathbf{D} = \begin{bmatrix} \frac{\partial^2}{\partial x^2} & & \\ & \frac{\partial^2}{\partial y^2} & \\ & & \frac{\partial^2}{\partial z^2} \end{bmatrix}$$

Decoupling orthorhombic wave equations

$$\frac{\partial^2}{\partial t^2}\boldsymbol{\sigma} = M\mathbf{D}\boldsymbol{\sigma}$$

Decoupling orthorhombic wave equations

$$\frac{\partial^2}{\partial t^2}\boldsymbol{\sigma} = M\mathbf{D}\boldsymbol{\sigma}$$

$$M\mathbf{D} = Q\boldsymbol{\Lambda}Q^{-1}, \boldsymbol{\Lambda} = diag[\boldsymbol{\lambda}_i]$$

Decoupling orthorhombic wave equations

$$\frac{\partial^2}{\partial t^2}\boldsymbol{\sigma} = M\mathbf{D}\boldsymbol{\sigma}$$

$$M\mathbf{D} = Q\boldsymbol{\Lambda}Q^{-1}, \boldsymbol{\Lambda} = diag\left[\boldsymbol{\lambda}_i\right]$$

$\boldsymbol{\sigma}^q = Q^{-1}\boldsymbol{\sigma}$: projection on row vectors

Decoupling orthorhombic wave equations

$$\frac{\partial^2}{\partial t^2}\boldsymbol{\sigma} = M\mathbf{D}\boldsymbol{\sigma}$$

$$M\mathbf{D} = Q\boldsymbol{\Lambda}Q^{-1}, \boldsymbol{\Lambda} = diag[\boldsymbol{\lambda}_i]$$

$\boldsymbol{\sigma}^q = Q^{-1}\boldsymbol{\sigma}$: projection on row vectors

$$\frac{\partial^2}{\partial t^2}\sigma_i^q = \boldsymbol{\lambda}_i\sigma_i^q$$

$$i = 1,2,3$$

Decoupling orthorhombic wave equations

$$\frac{\partial^2}{\partial t^2}\boldsymbol{\sigma} = M\mathbf{D}\boldsymbol{\sigma}$$

system of 3 coupled equations

$$M\mathbf{D} = Q\boldsymbol{\Lambda}Q^{-1}, \boldsymbol{\Lambda} = diag[\boldsymbol{\lambda}_i]$$

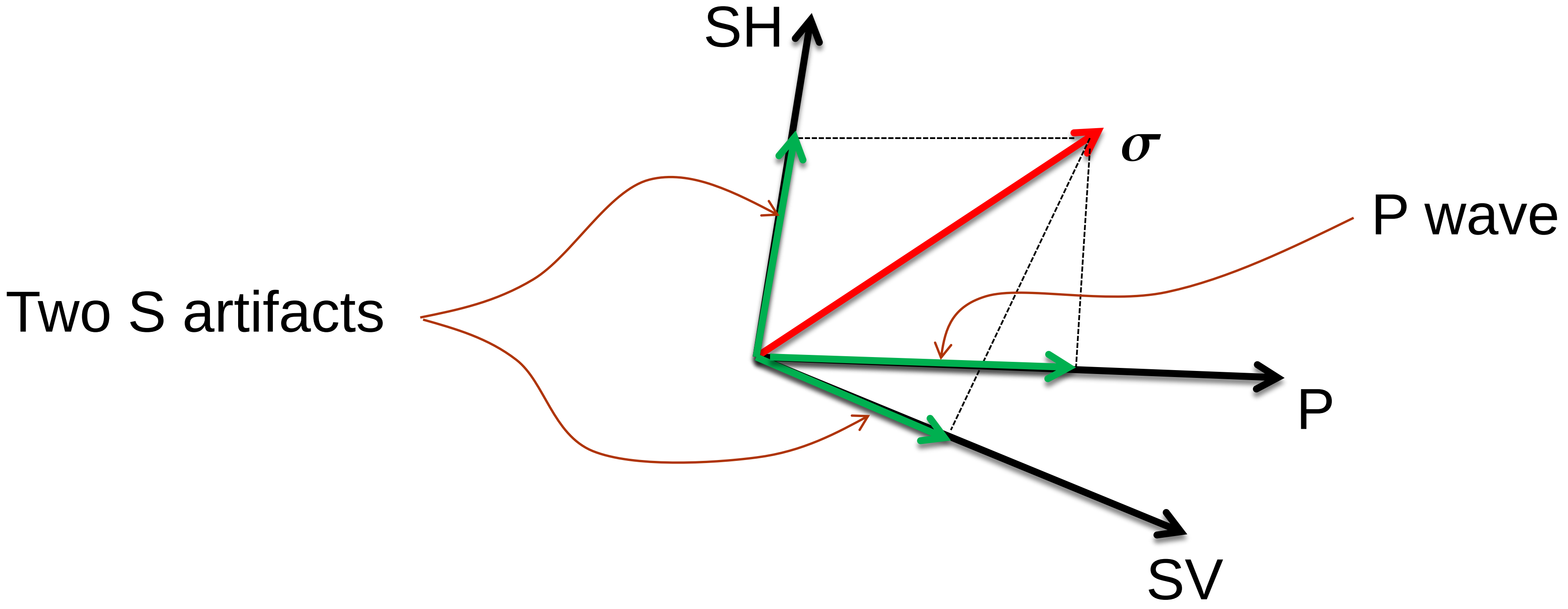
$\boldsymbol{\sigma}^q = Q^{-1}\boldsymbol{\sigma}$: projection on row vectors

$$\frac{\partial^2}{\partial t^2}\sigma_i^q = \boldsymbol{\lambda}_i\sigma_i^q$$

3 decoupled equations

$$i = 1,2,3$$

Projections on row vectors



Degenerate case: Isotropic

$$\boldsymbol{\lambda}_1 = -k_x^2 - k_y^2 - k_z^2$$: Laplacian

$$\boldsymbol{\lambda}_2 = \boldsymbol{\lambda}_3 = 0$$

$$\boldsymbol{\sigma}^q = \begin{bmatrix} p \\ 0 \\ 0 \end{bmatrix}$$: pressure

Proposed method: use **eigenvalues** as propagators
(elastic wave mode separation: projection on **eigenvectors**)

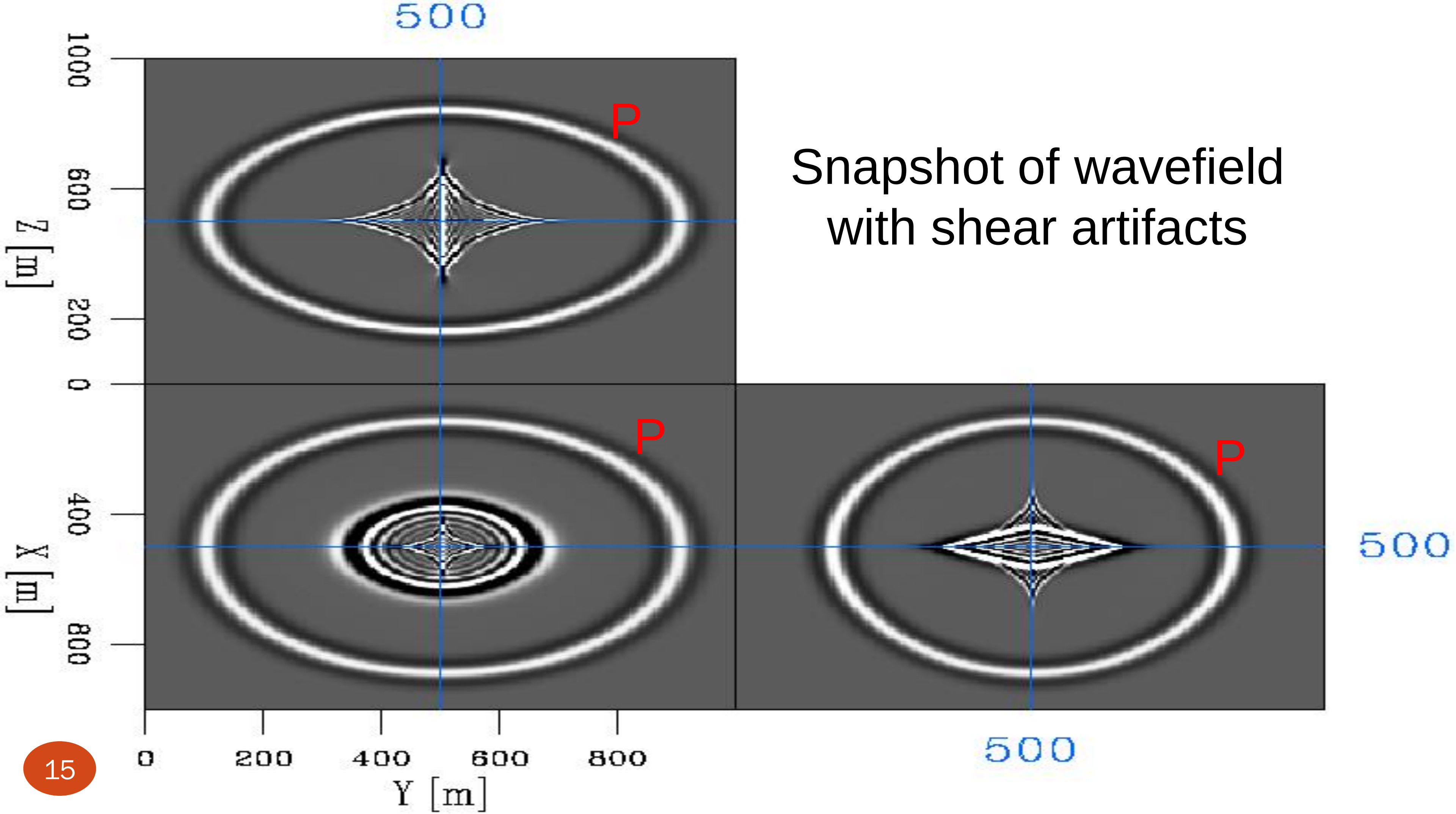
Wavenumber domain method

For all *k*'s: $M\tilde{\mathbf{D}} = Q\tilde{\boldsymbol{\Lambda}}Q^{-1}, \tilde{\boldsymbol{\Lambda}} = diag\left[\tilde{\boldsymbol{\lambda}}_i(k)\right]$

Choose eigenvalue corresponding to P-wave

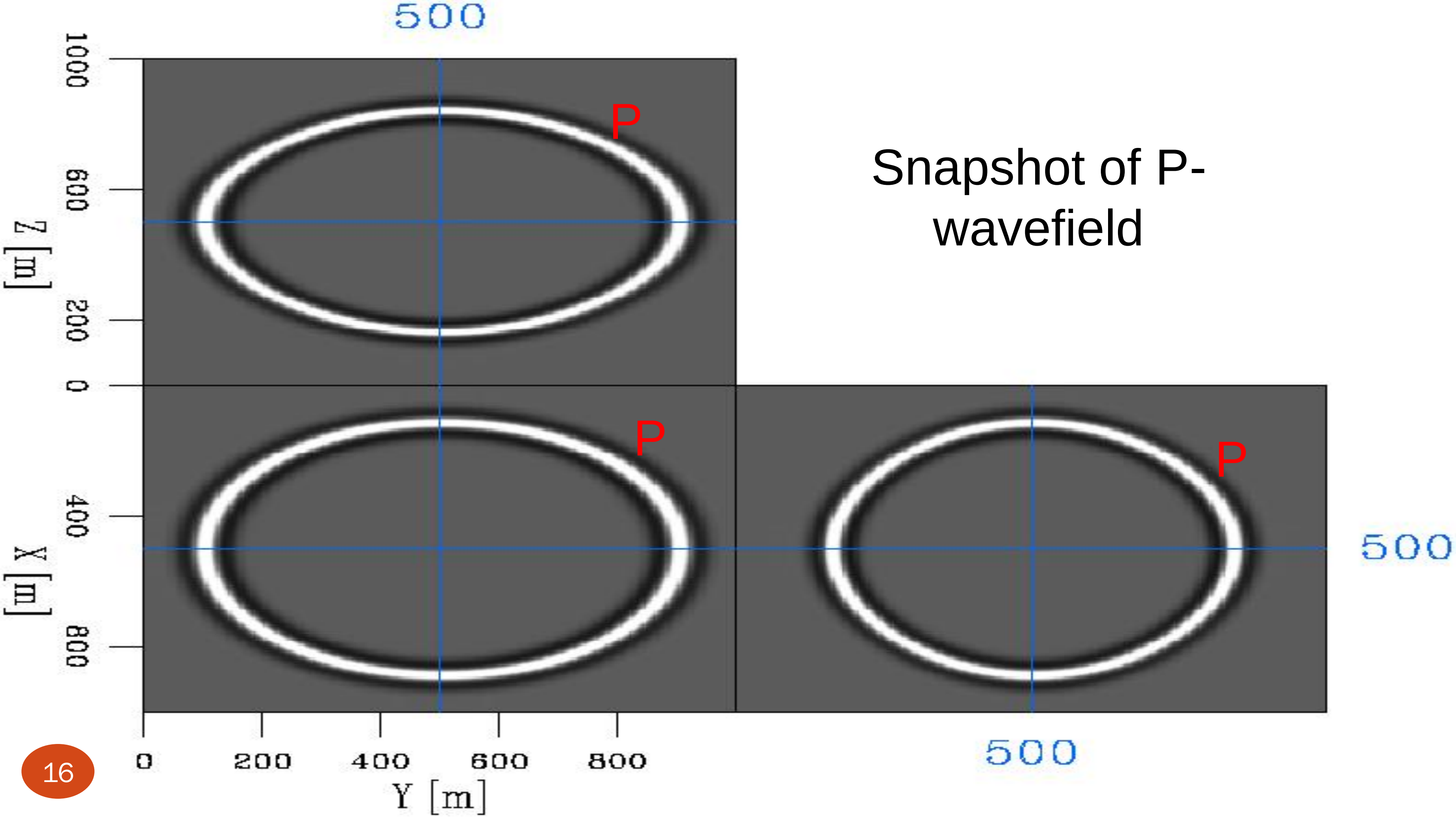
Option 1: $p(x) \rightarrow \tilde{p}(k)$

$$\frac{\partial^2 \tilde{p}}{\partial t^2} = \tilde{\boldsymbol{\lambda}}_i(k)\tilde{p}(k)$$

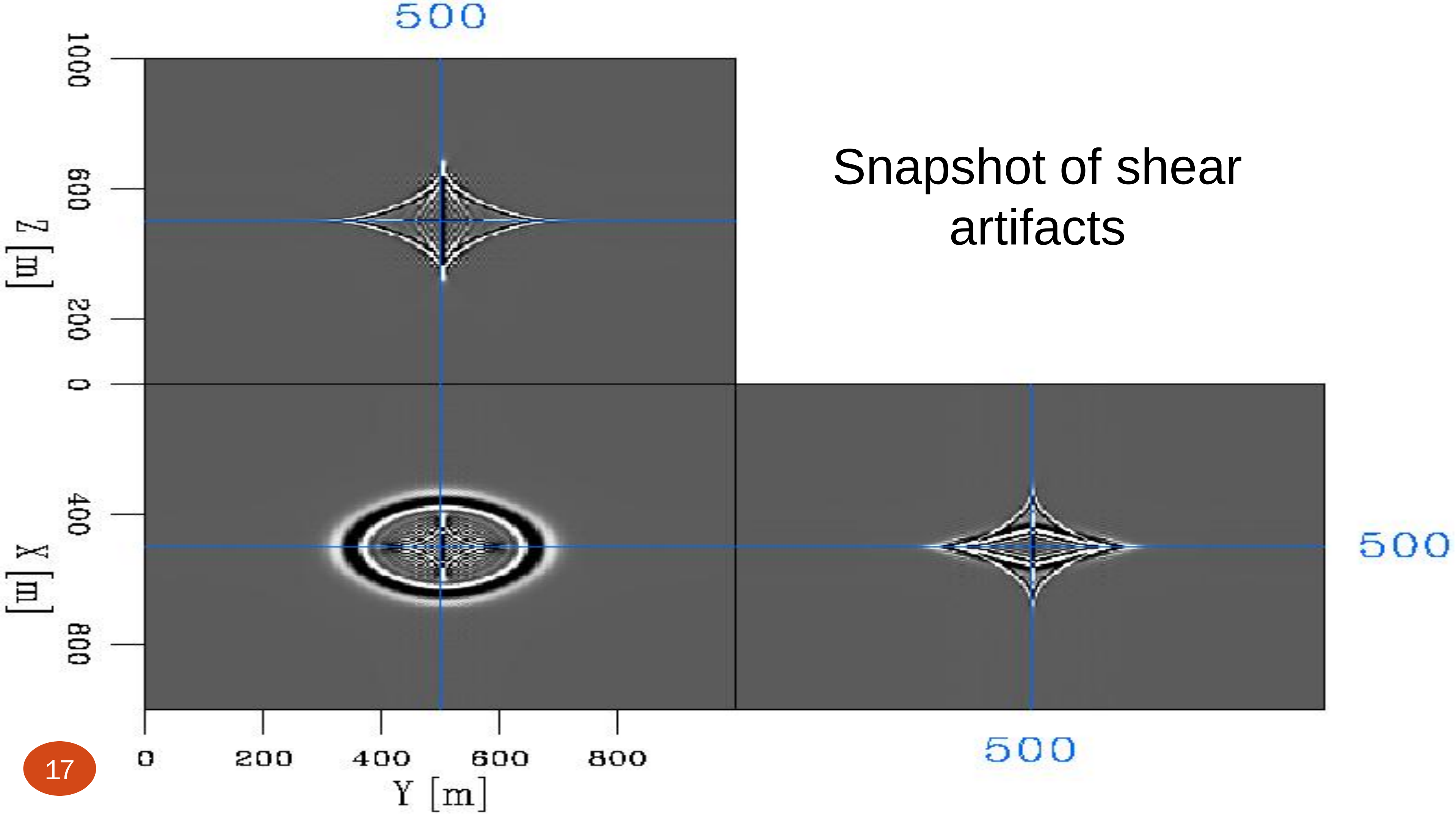


Snapshot of wavefield
with shear artifacts

Snapshot of P-wavefield



Snapshot of shear artifacts



Space-time domain method

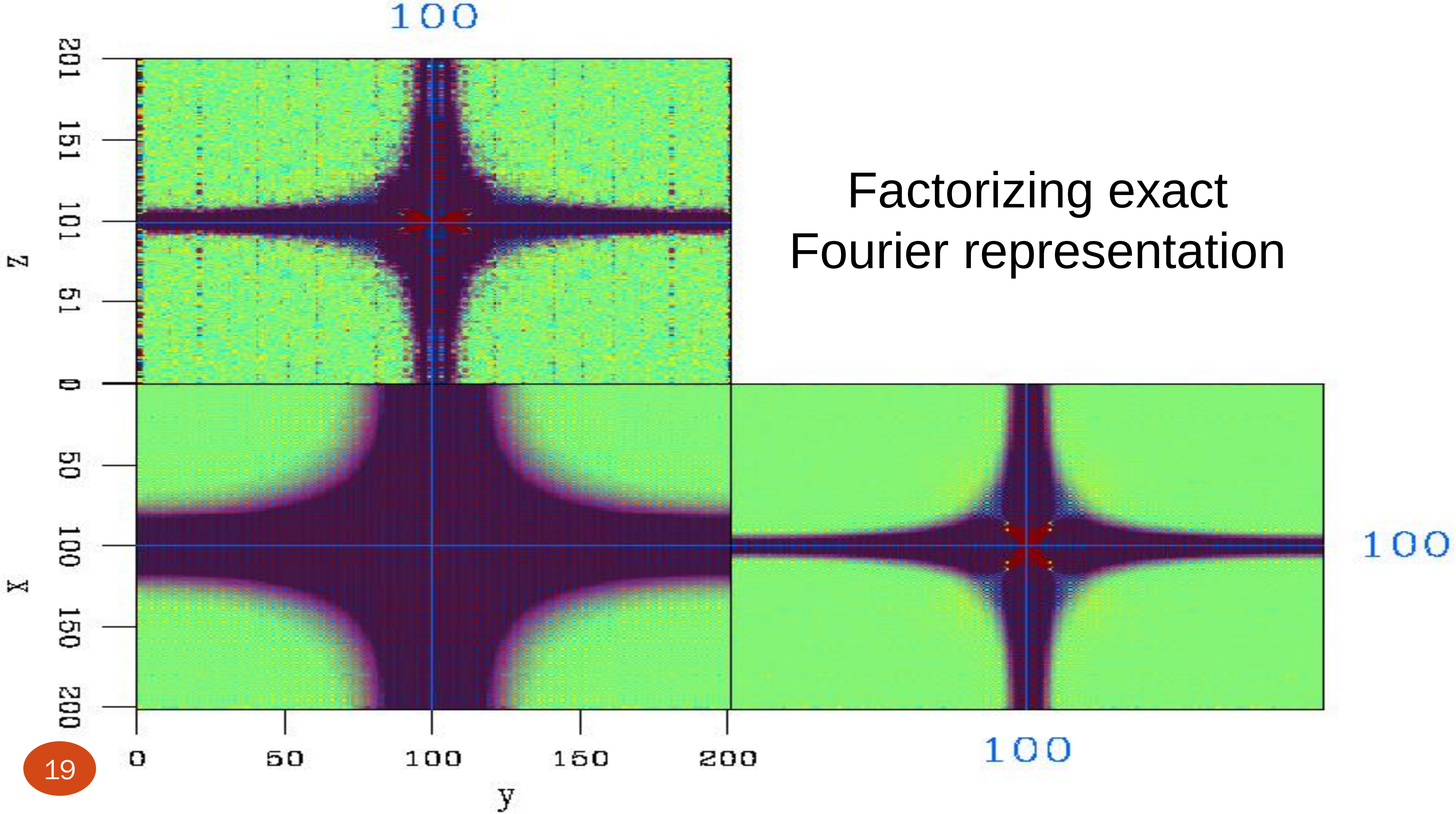
For all *k*'s: $M\tilde{\mathbf{D}} = Q\tilde{\mathbf{\Lambda}}Q^{-1}, \tilde{\mathbf{\Lambda}} = diag\left[\tilde{\lambda}_i(k)\right]$

Choose eigenvalue corresponding to P-wave

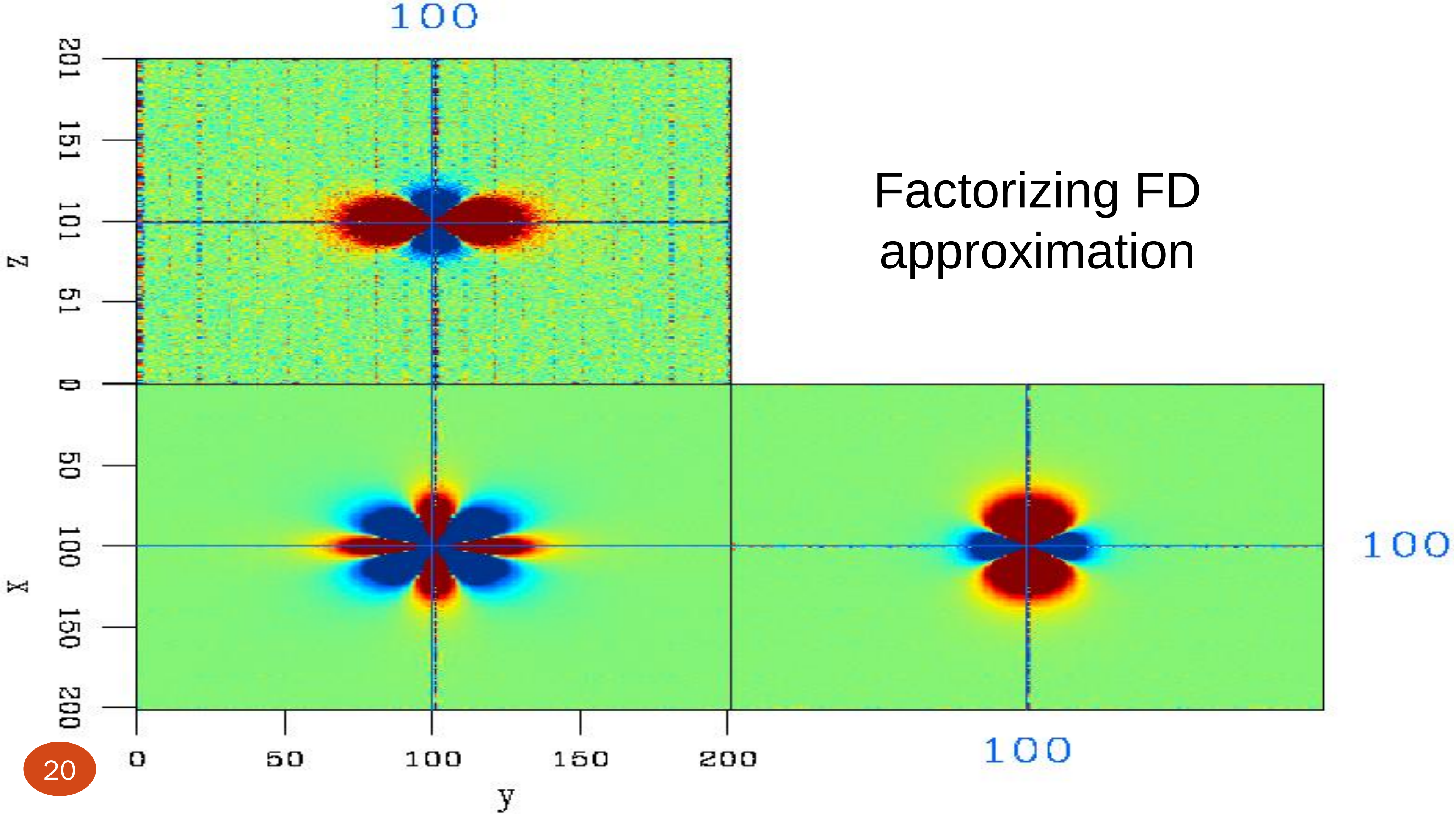
Option 2: $\tilde{\lambda}_i(k) \to \lambda_i(x)$

$$\frac{\partial^2 p}{\partial t^2} = \lambda_i(x) * p(x)$$

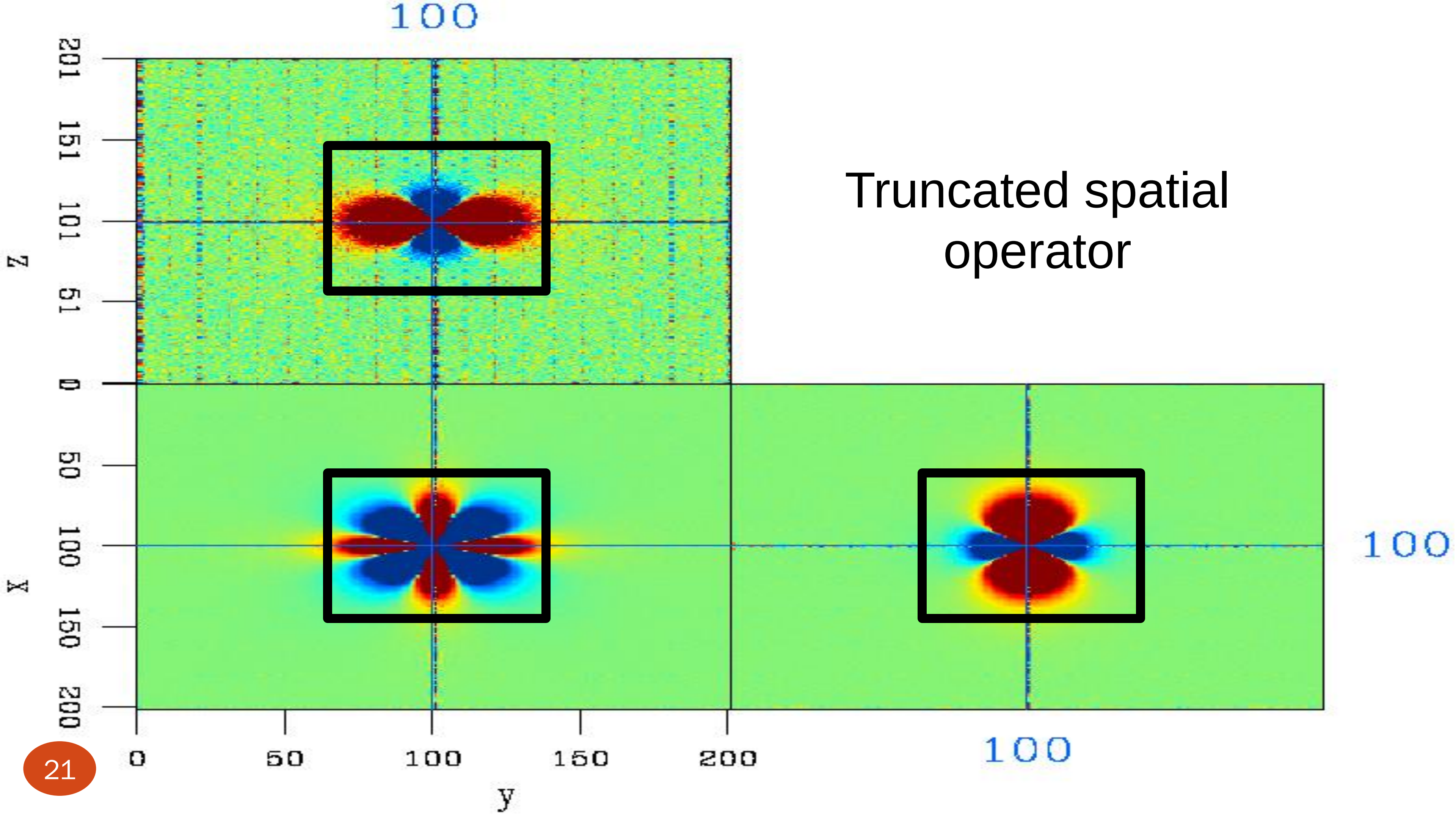
Factorizing exact Fourier representation



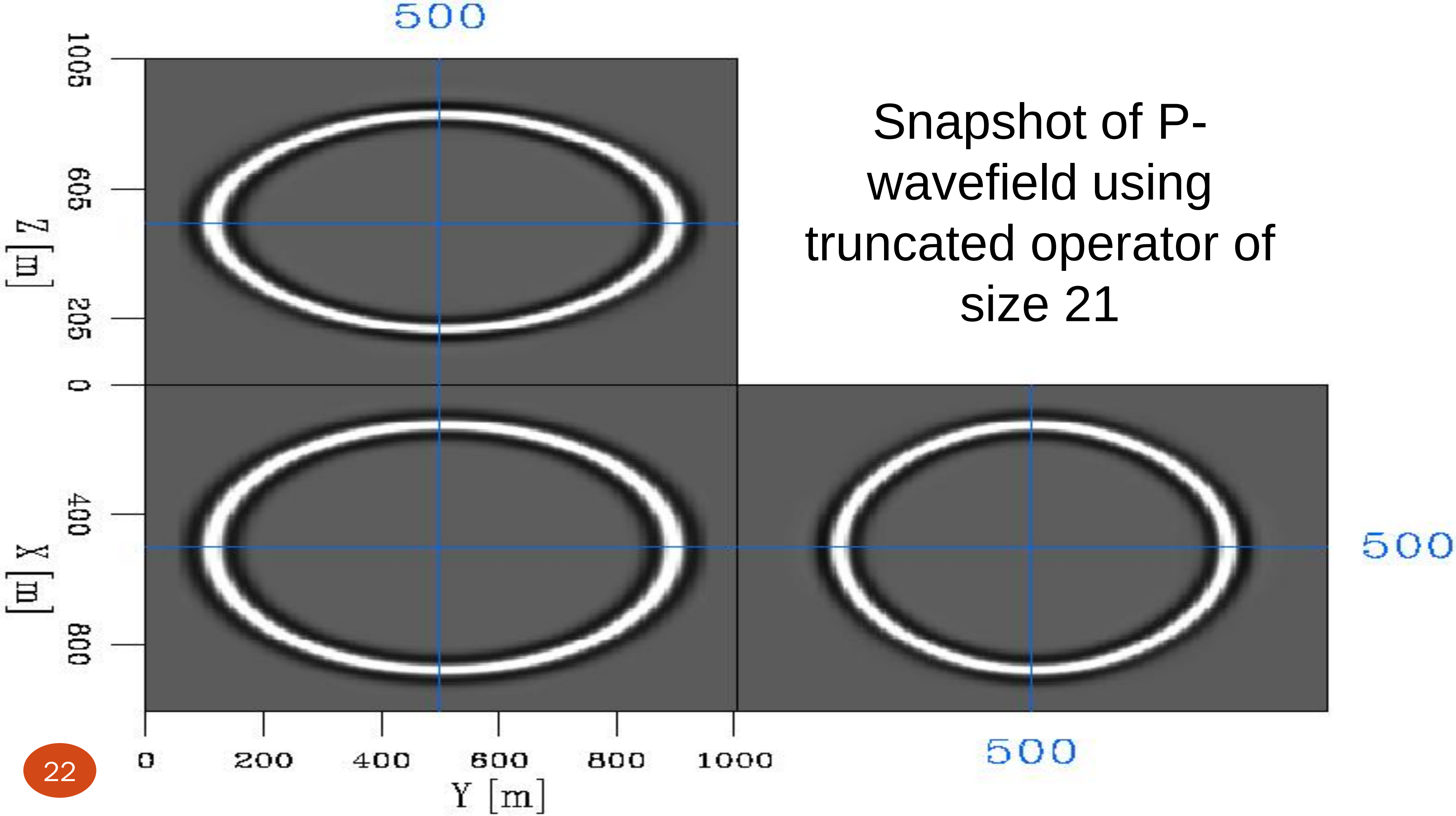
Factorizing FD approximation

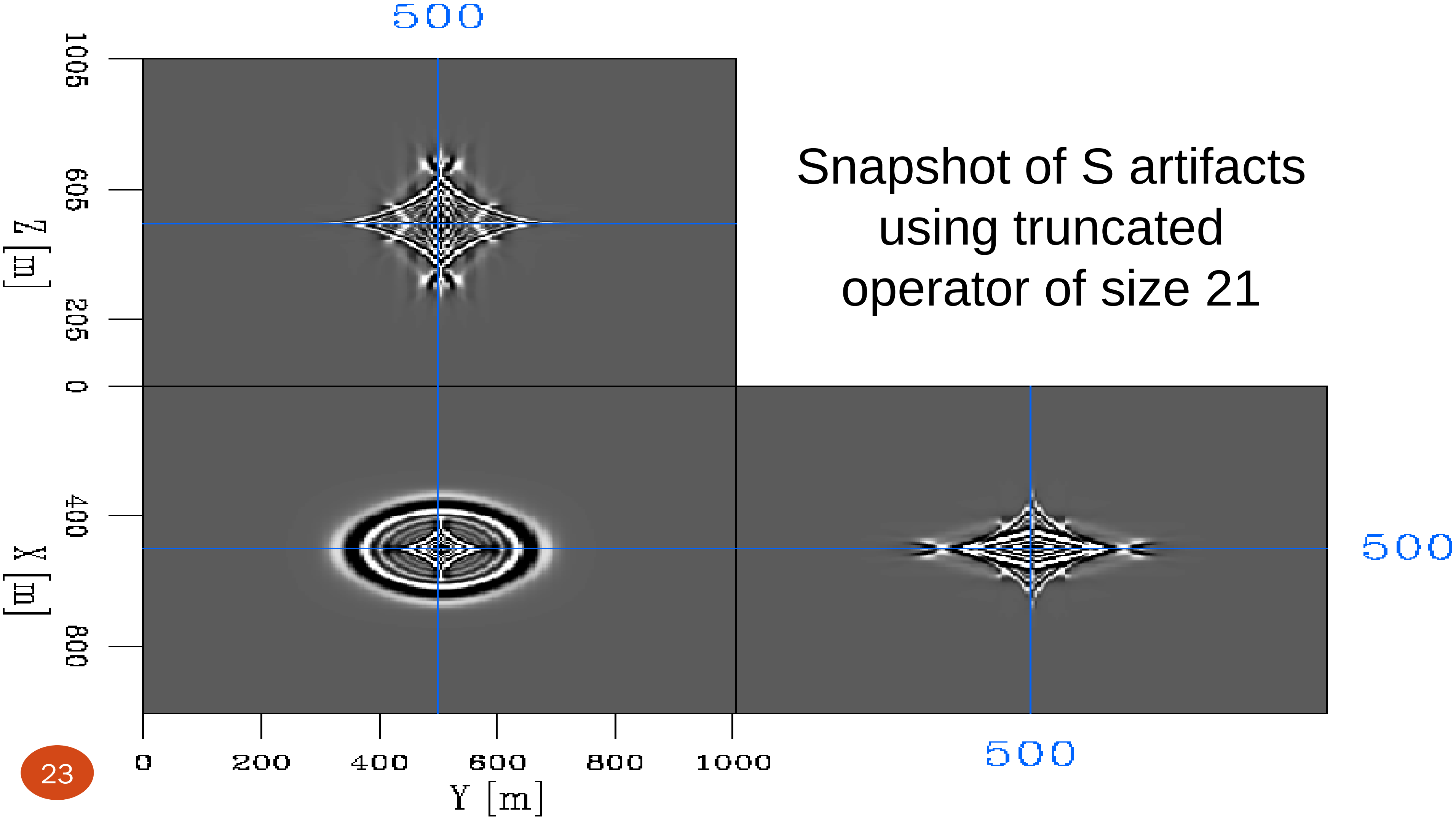


Truncated spatial operator



Snapshot of P-wavefield using truncated operator of size 21

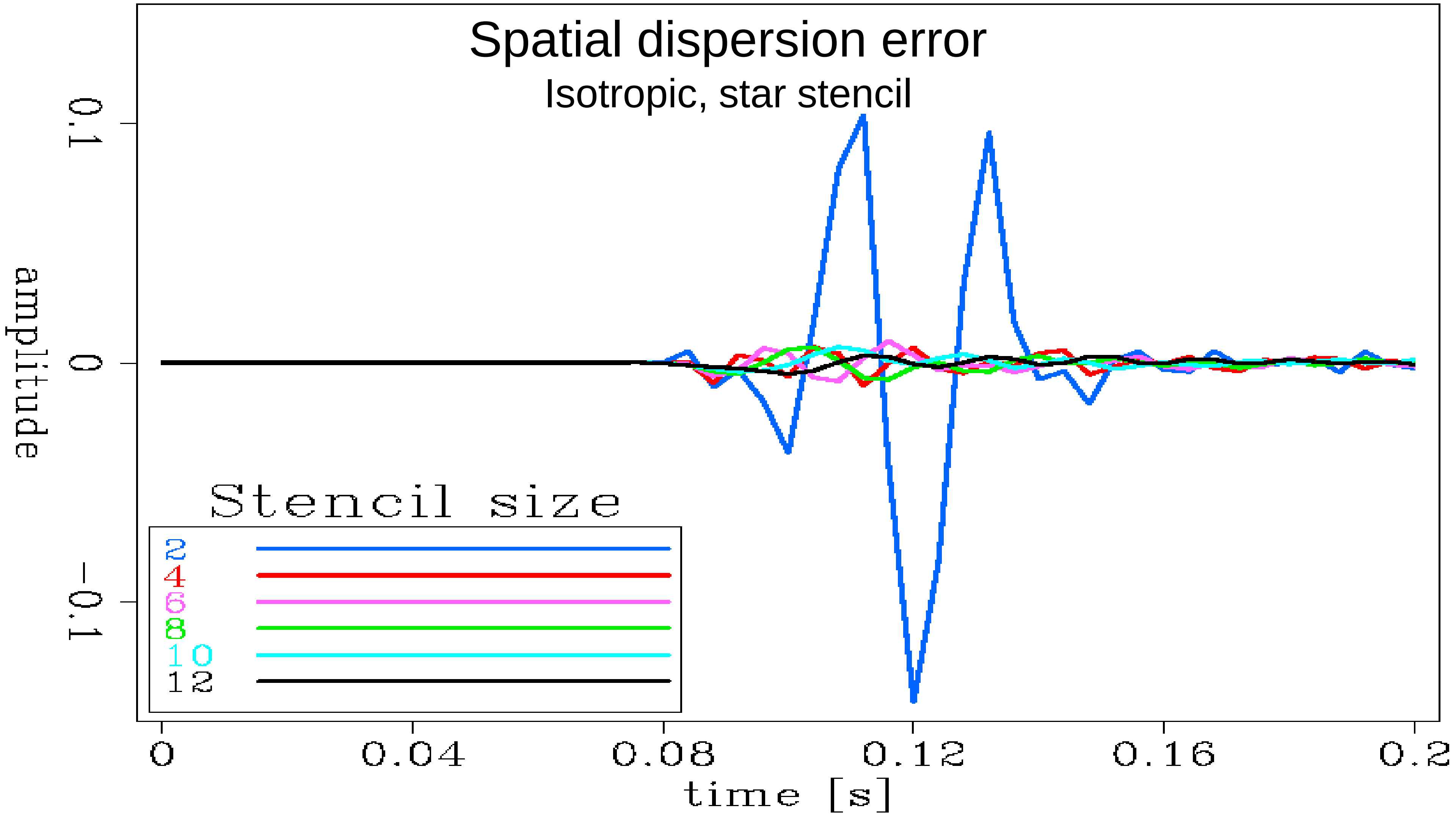




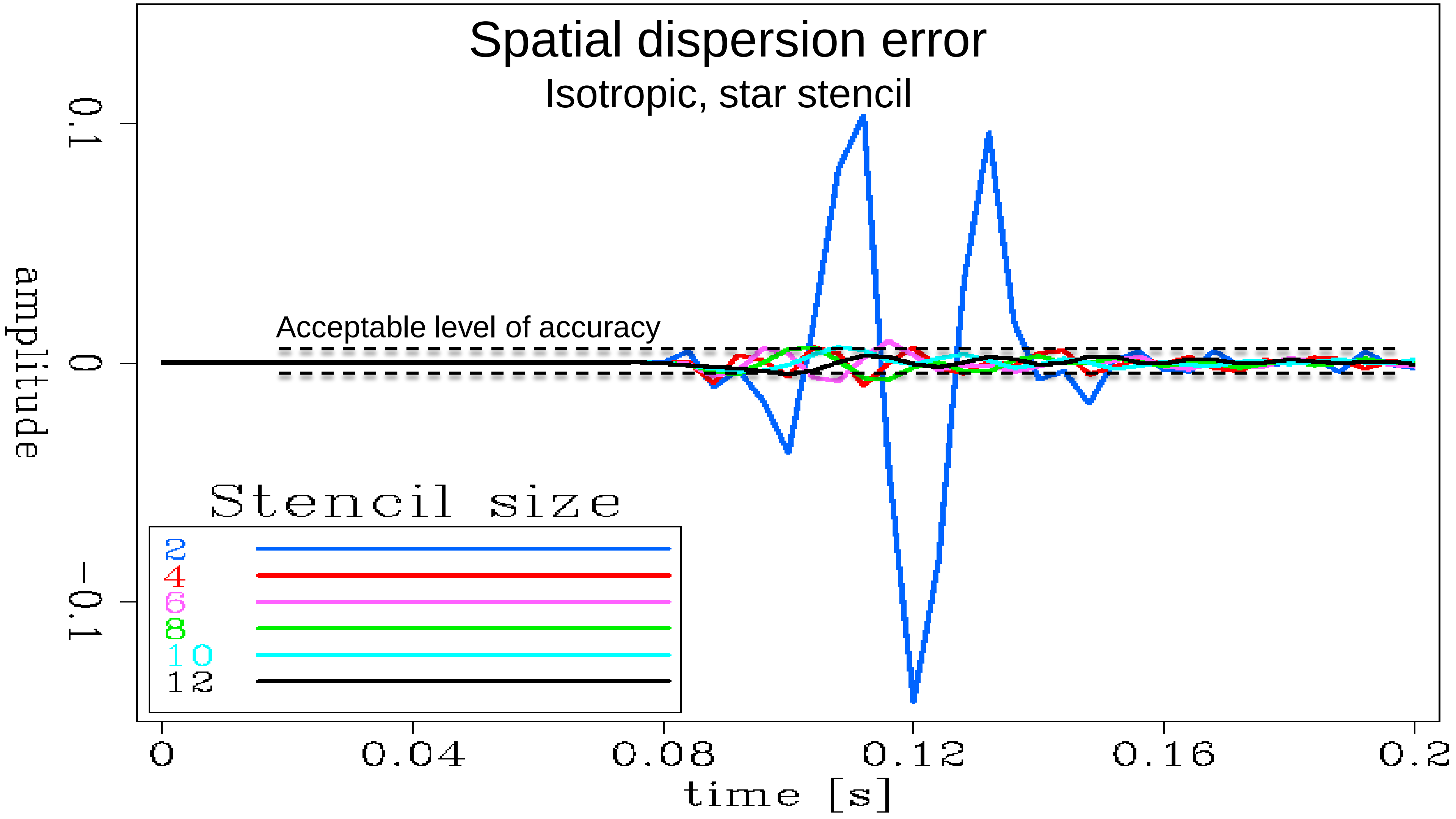
Snapshot of S artifacts using truncated operator of size 21

Operator's accuracy after truncation

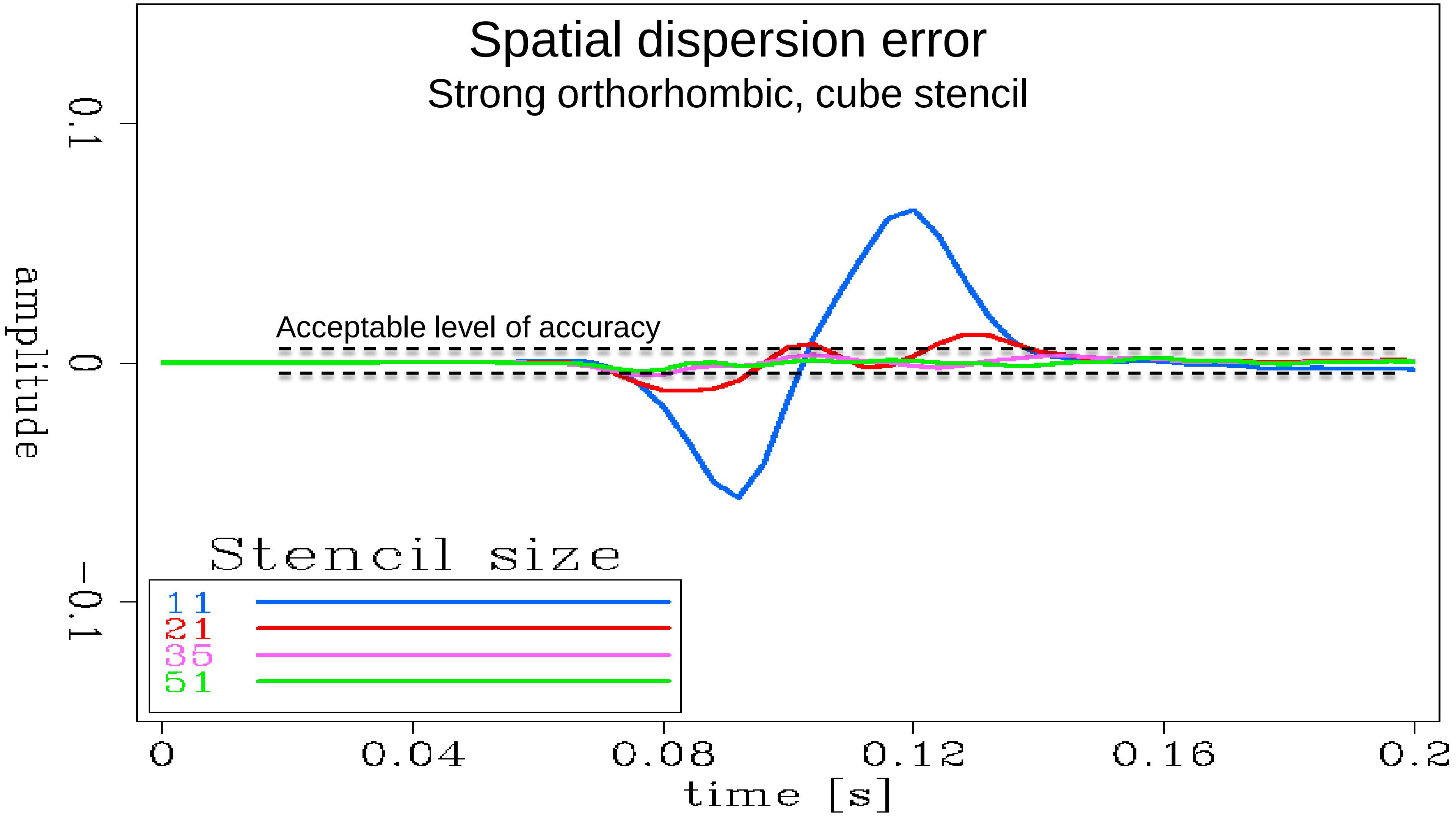
- Wavenumber domain: accurate but slow
- Spatial domain: less accurate but fast
- Both use 2nd-order temporal FD, same time steps, and spatial discretizations
  => same temporal dispersion error
- Spatial subtracts wavenumber
  => spatial dispersion error

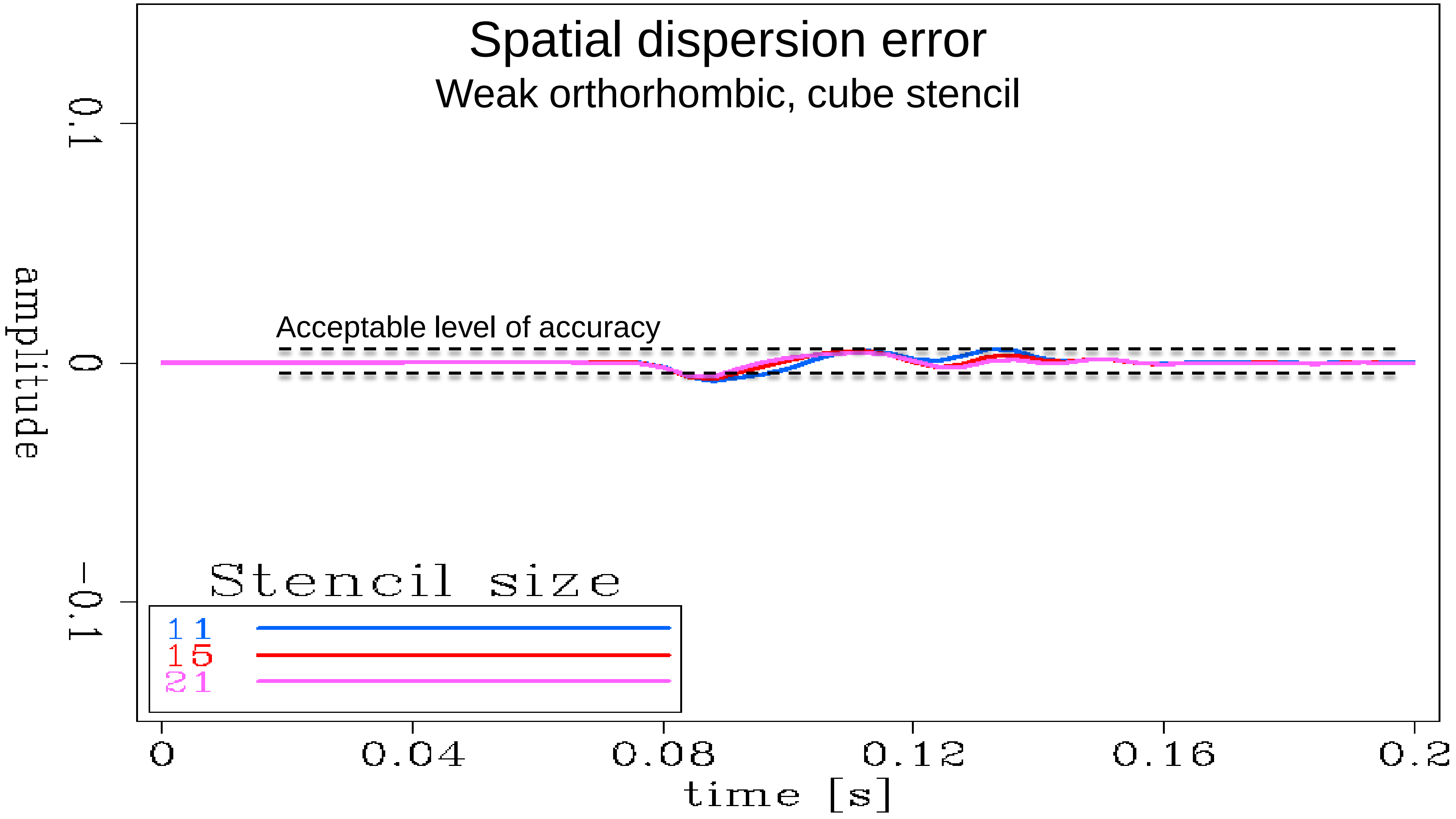


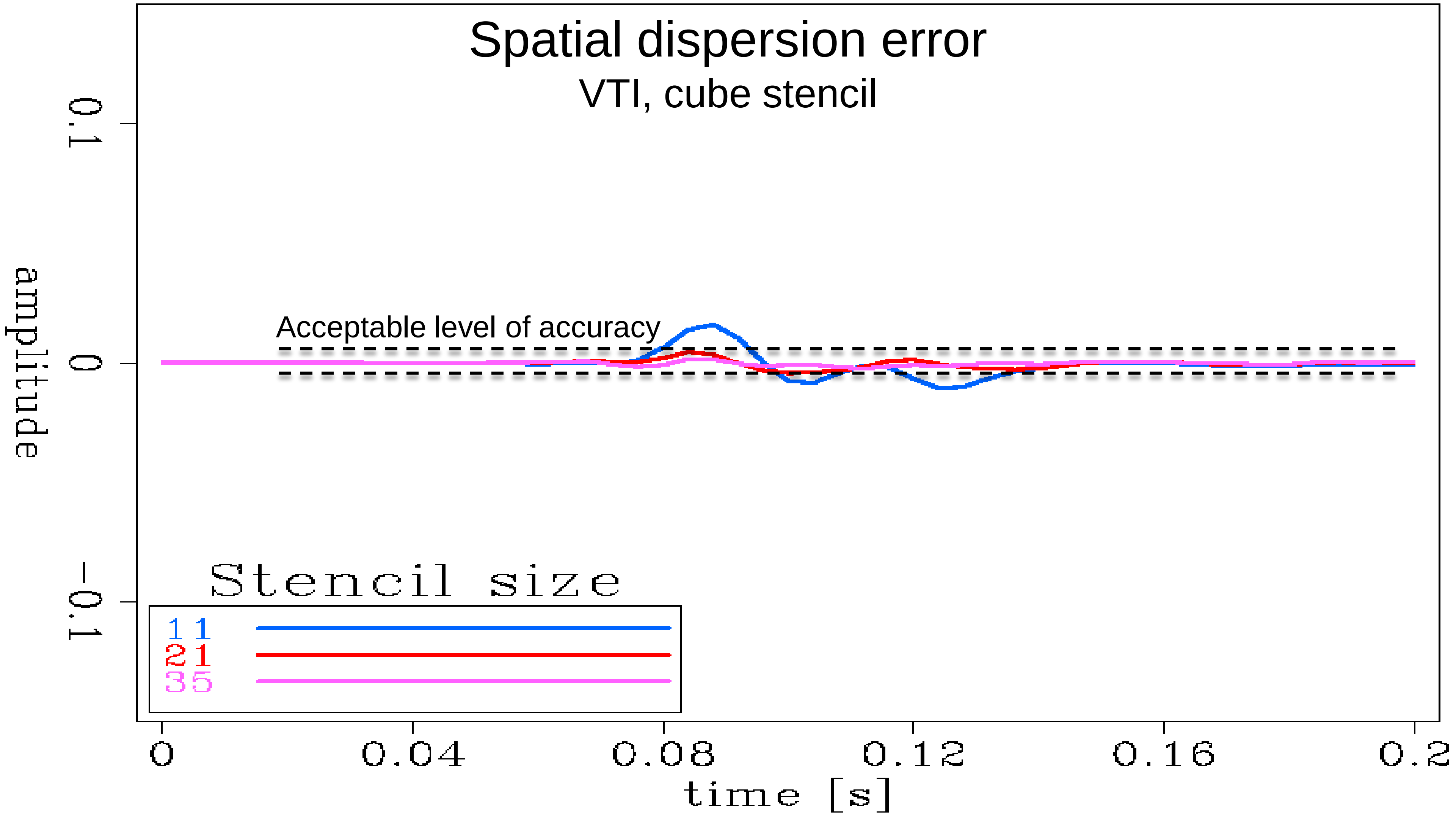
Spatial dispersion error
Isotropic, star stencil
amplitude
0.1
0
−0.1
Stencil size
2
4
6
8
10
12
0
0.04
0.08
0.12
0.16
0.2
time [s]



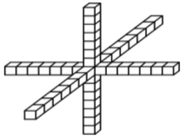
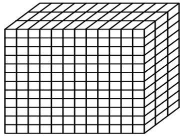
Spatial dispersion error
Isotropic, star stencil
Acceptable level of accuracy
Stencil size
2
4
6
8
10
12
amplitude
0.1
0
−0.1
0
0.04
0.08
0.12
0.16
0.2
time [s]


Spatial dispersion error
Strong orthorhombic, cube stencil
Acceptable level of accuracy
Stencil size
11
21
35
51
amplitude
0.1
0
−0.1
0
0.04
0.08
0.12
0.16
0.2
time [s]

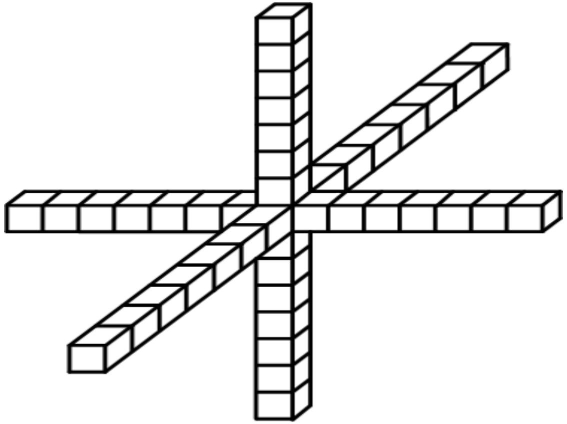

Spatial dispersion error
Weak orthorhombic, cube stencil
Acceptable level of accuracy
Stencil size
11
15
21
amplitude
0.1
0
-0.1
0
0.04
0.08
0.12
0.16
0.2
time [s]

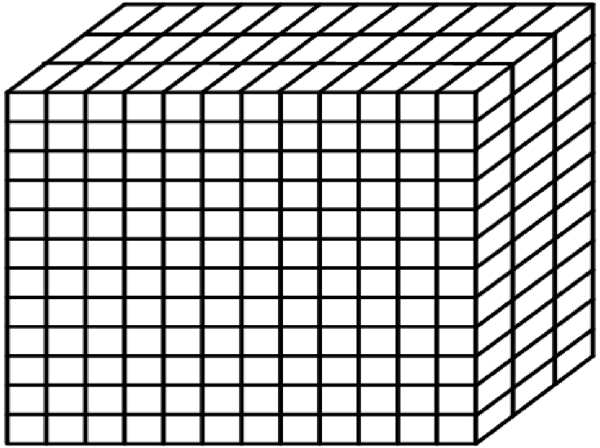

Spatial dispersion error
VTI, cube stencil
Acceptable level of accuracy
Stencil size
11
21
35
amplitude
0.1
0
-0.1
0
0.04
0.08
0.12
0.16
0.2
time [s]

Cost comparison

	$\frac{\partial^2}{\partial t^2}\boldsymbol{\sigma} = M\mathbf{D}\boldsymbol{\sigma}$	$\frac{\partial^2 p}{\partial t^2} = \boldsymbol{\lambda}_i(x) * p(x)$
Stored wavefields	3	1
Computation (+ and ×)	1 (10th-order FD) 	140 (21-sized cube) 

Cost comparison

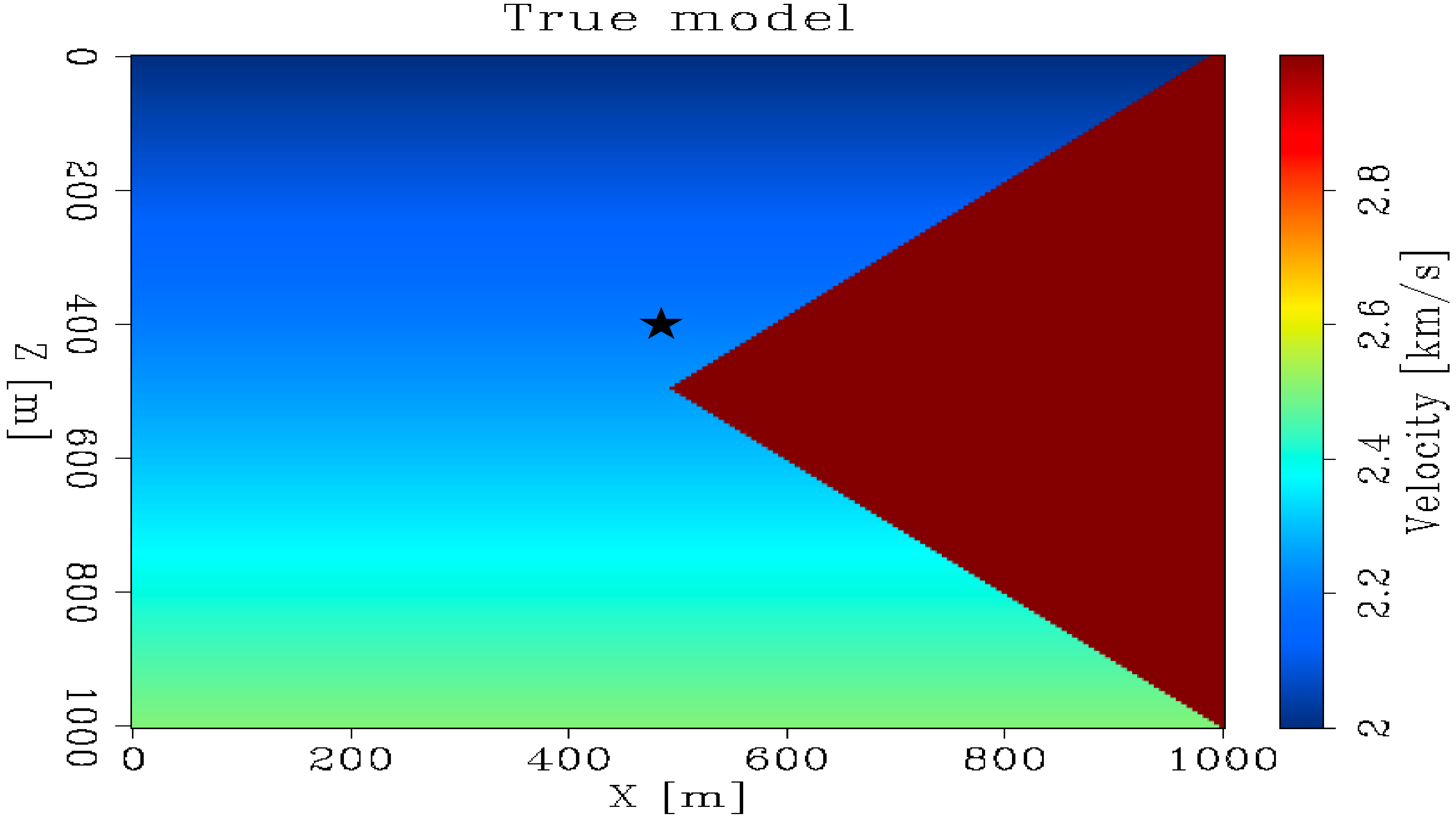


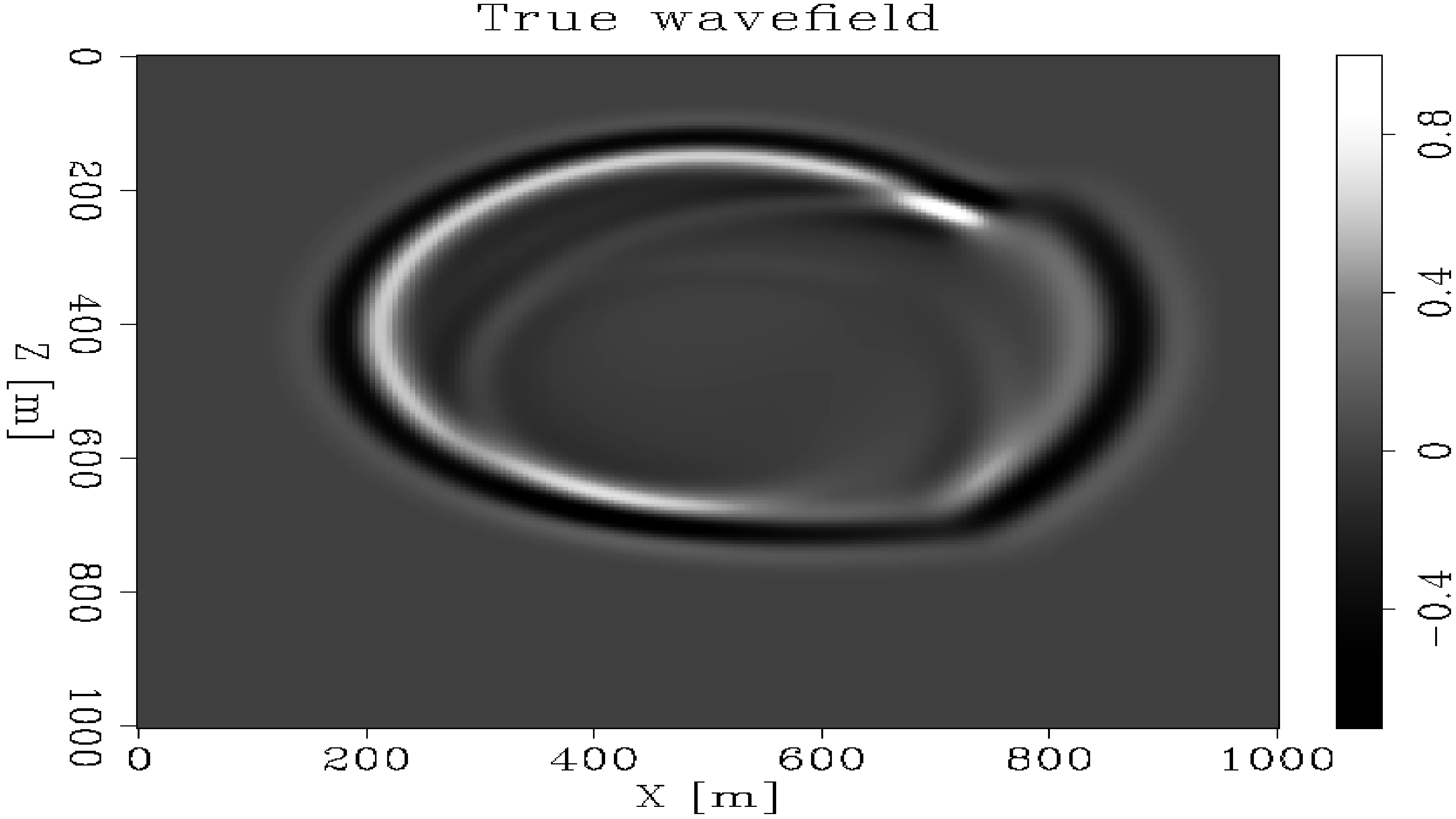


Computation hides latency
Highly parallelizable
Better cache reuse

Application to inhomogeneous media

- Spatially varying operator
- Exact solution requires calculation of operator for **every** present model parameters.


True model
Z [m]
0
200
400
600
800
1000
X [m]
0
200
400
600
800
1000
Velocity [km/s]
2
2.2
2.4
2.6
2.8

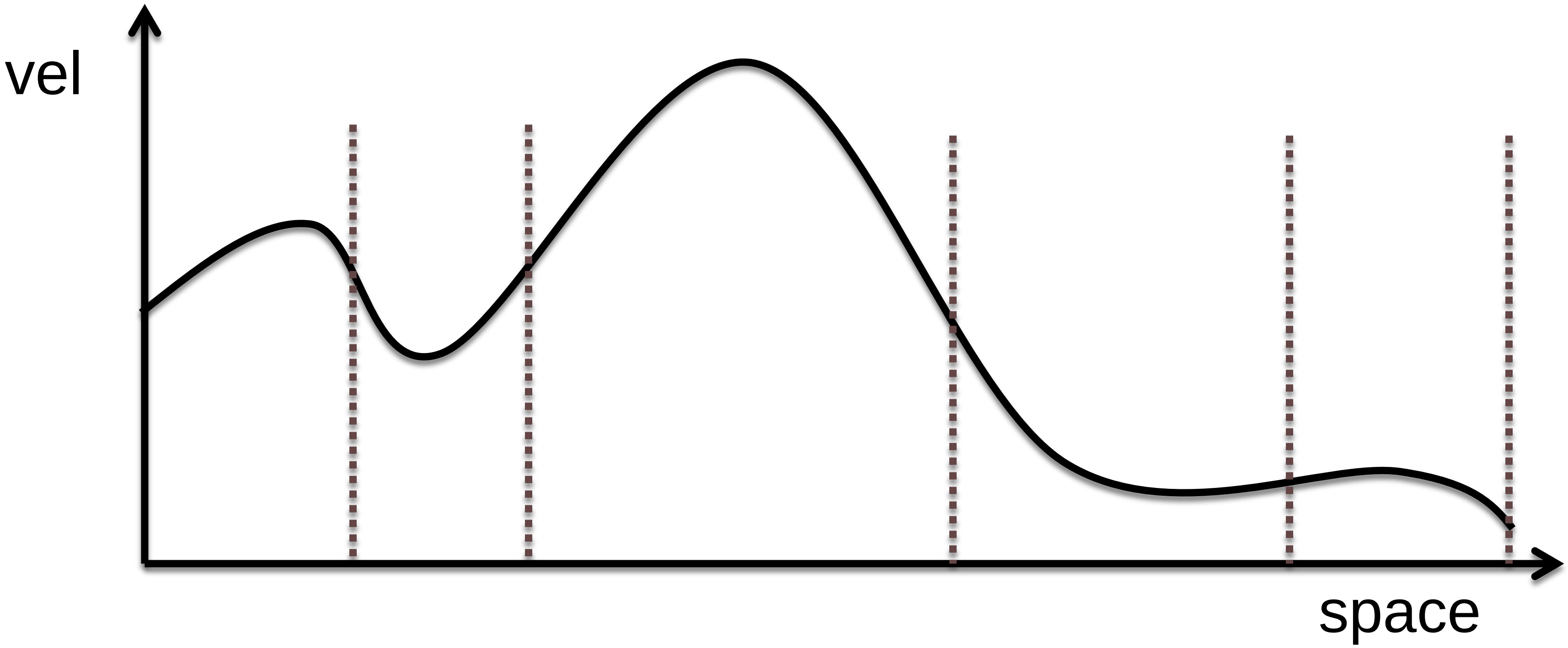


True wavefield
Z [m]
X [m]
0
200
400
600
800
1000
0
200
400
600
800
1000
0.8
0.4
0
−0.4

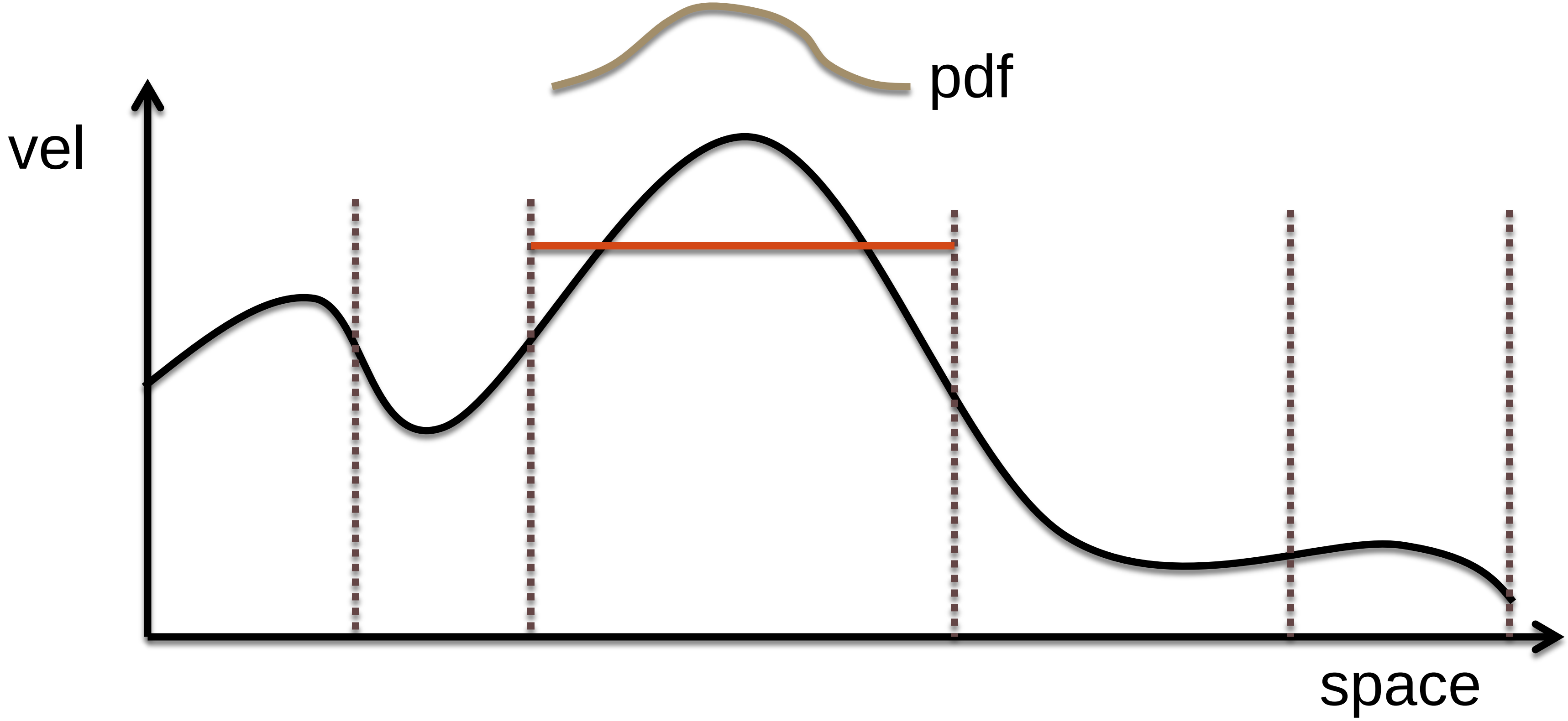
Application to inhomogeneous media

- Spatially varying operator
- Exact solution requires calculation of operator for **every** present model parameters
  => expensive
- Approximation: compute operators for a number of reference model parameters
  - Reference selection: Lloyd algorithm (Clapp, 2006)
- Interpolation of operators

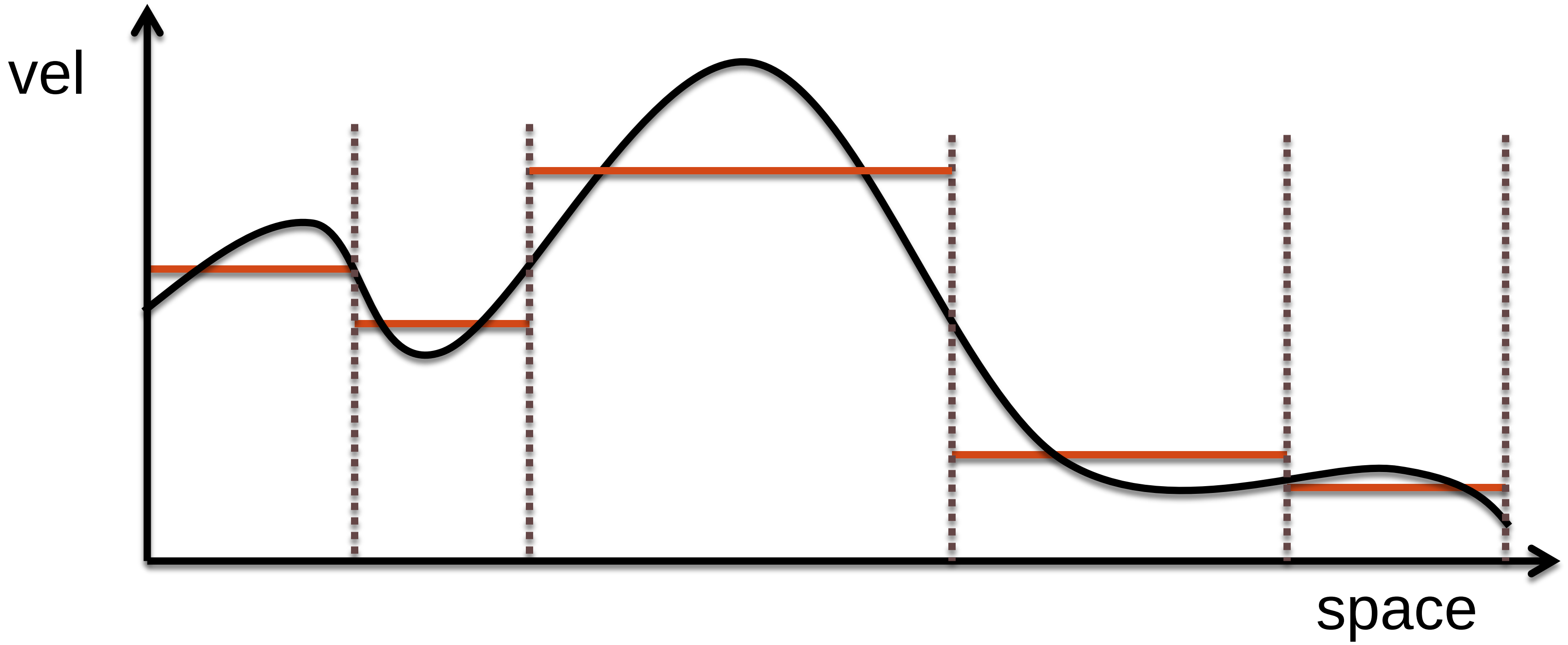
Lloyd algorithm for reference selection

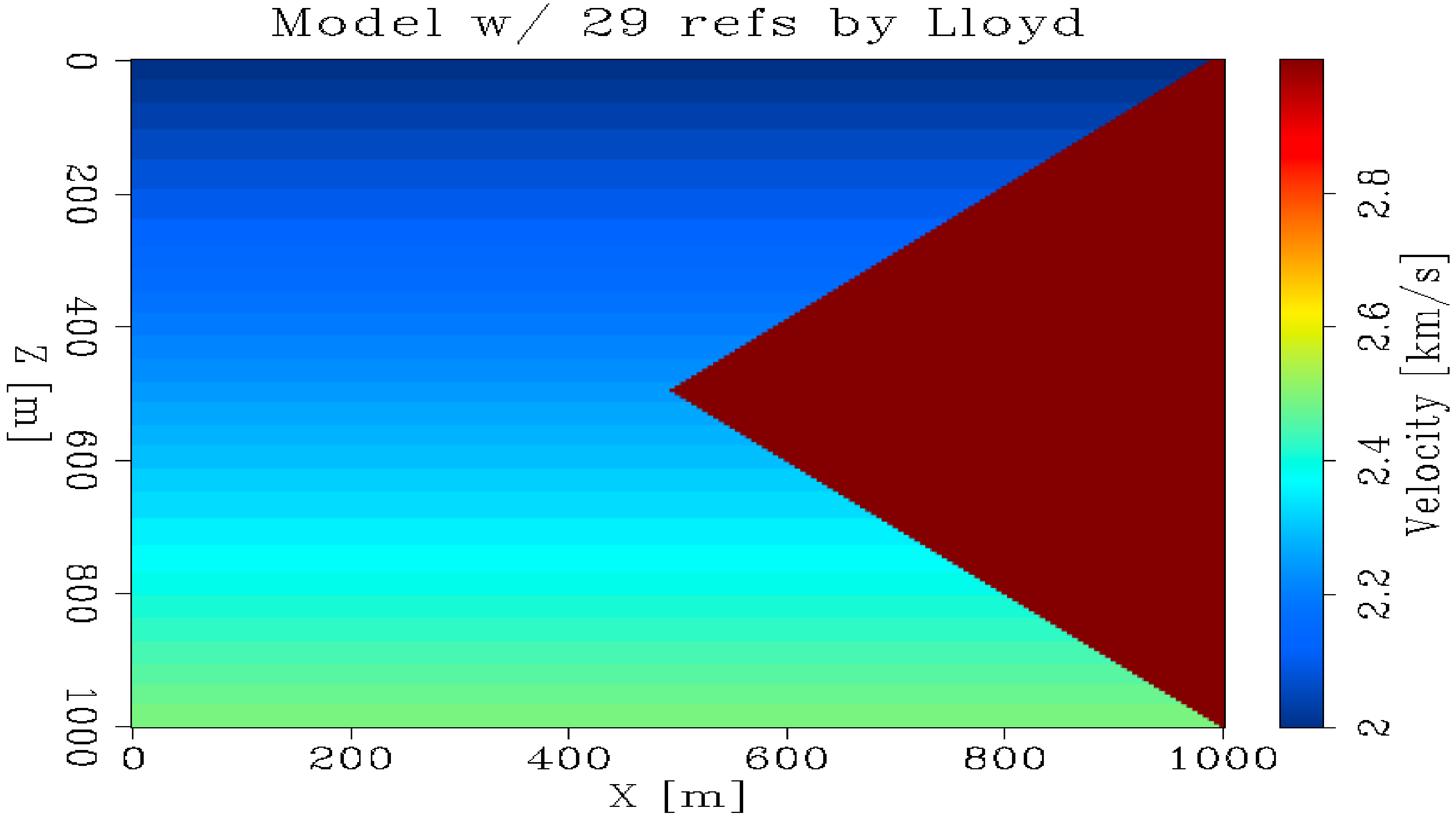


Lloyd algorithm for reference selection

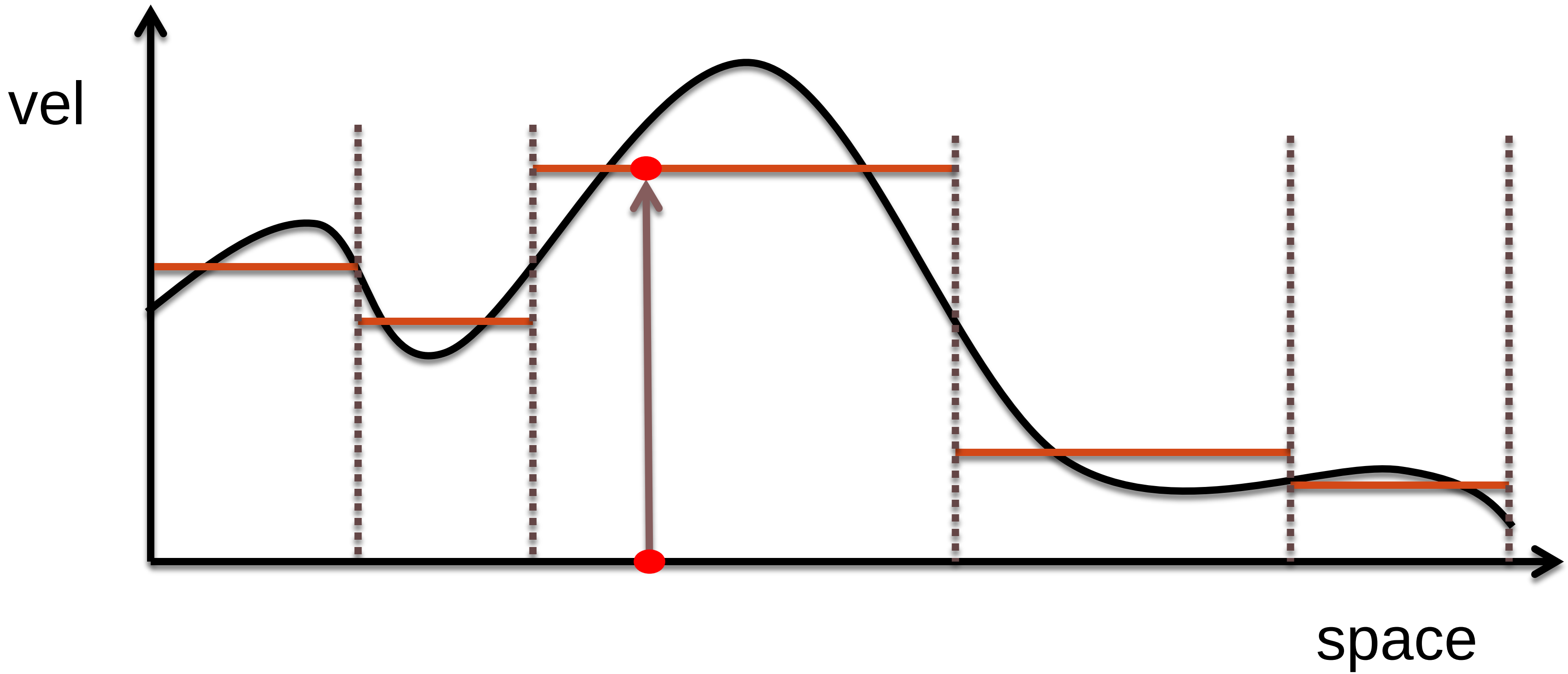


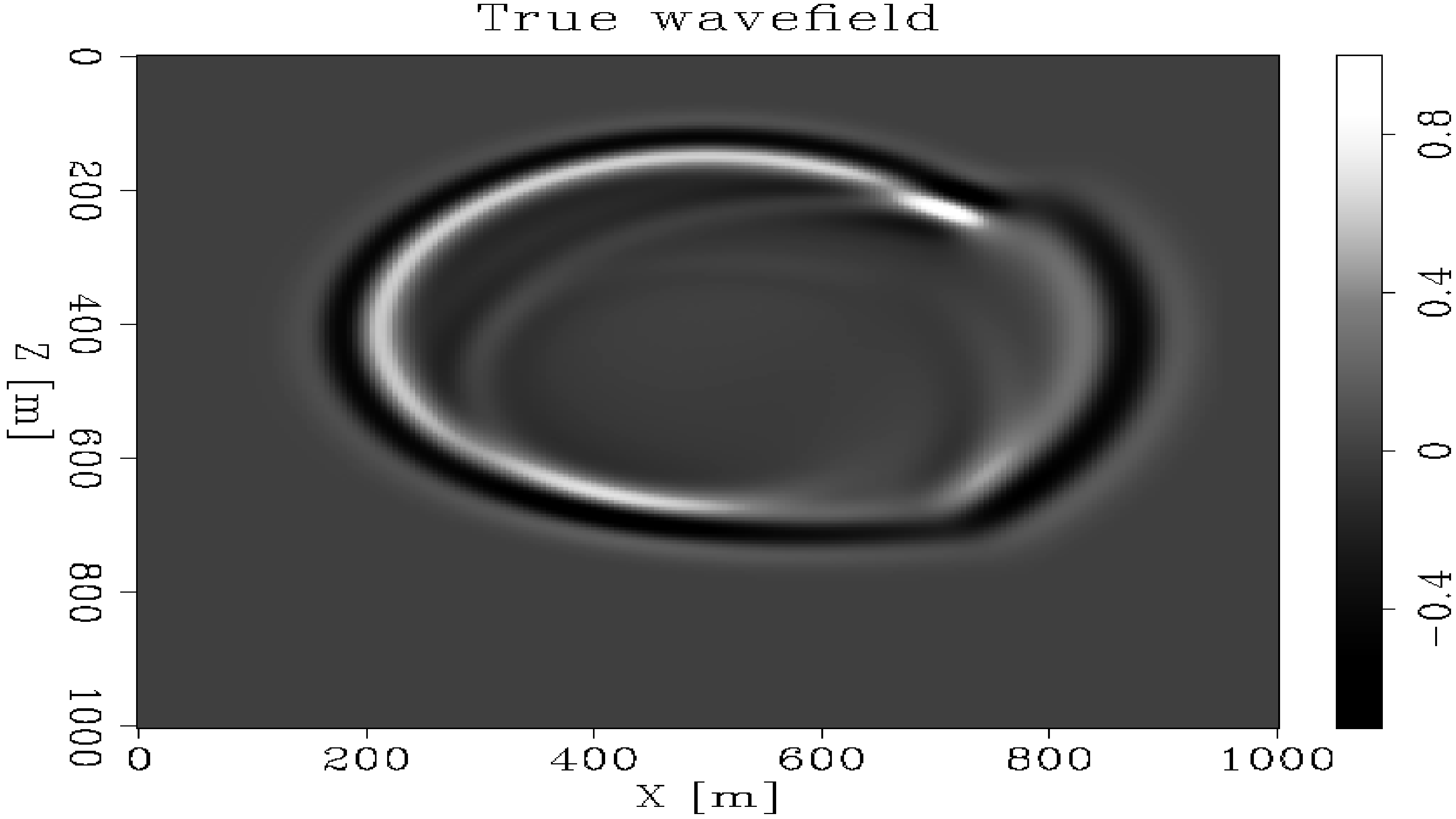
Lloyd algorithm for reference selection

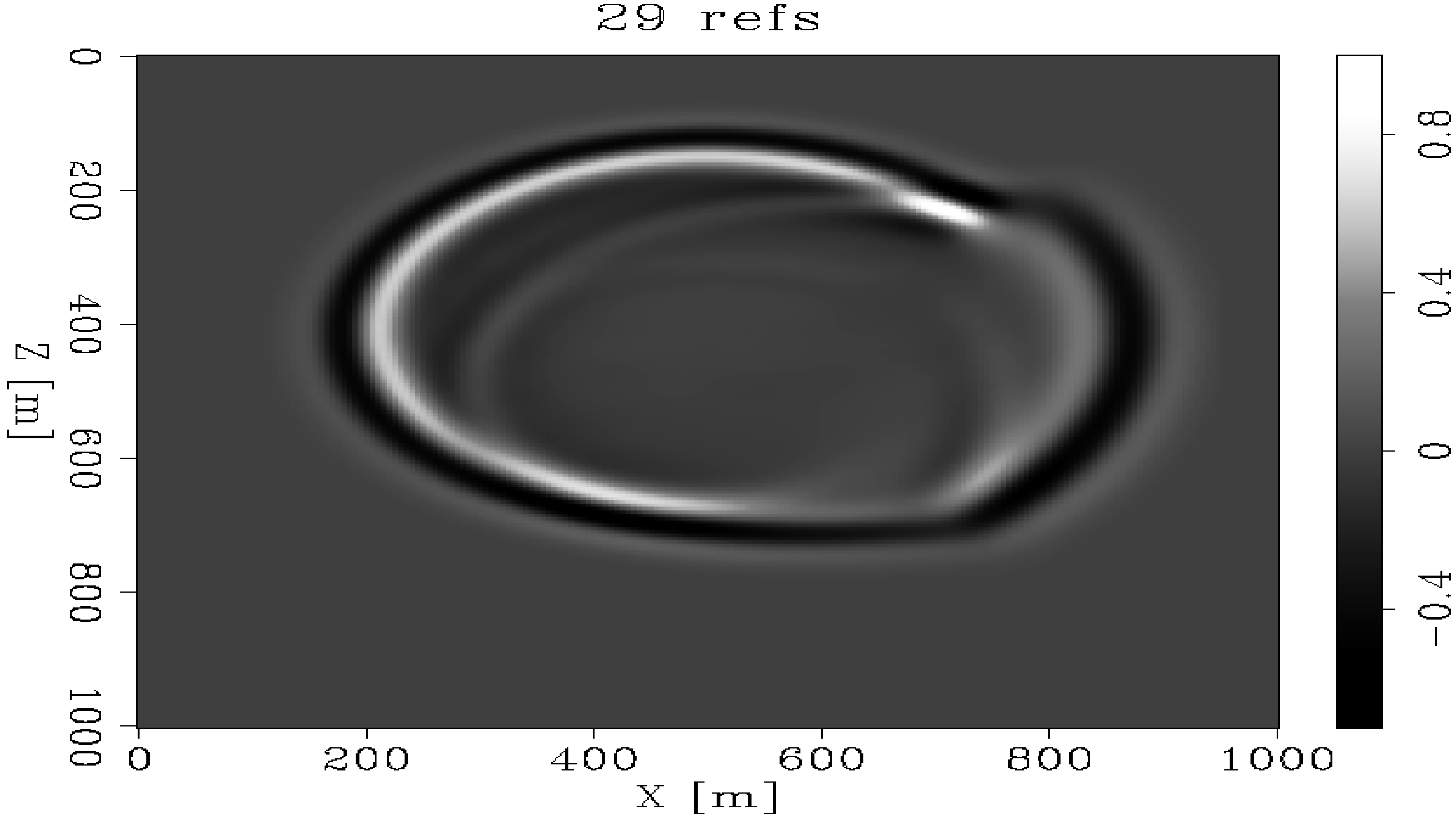


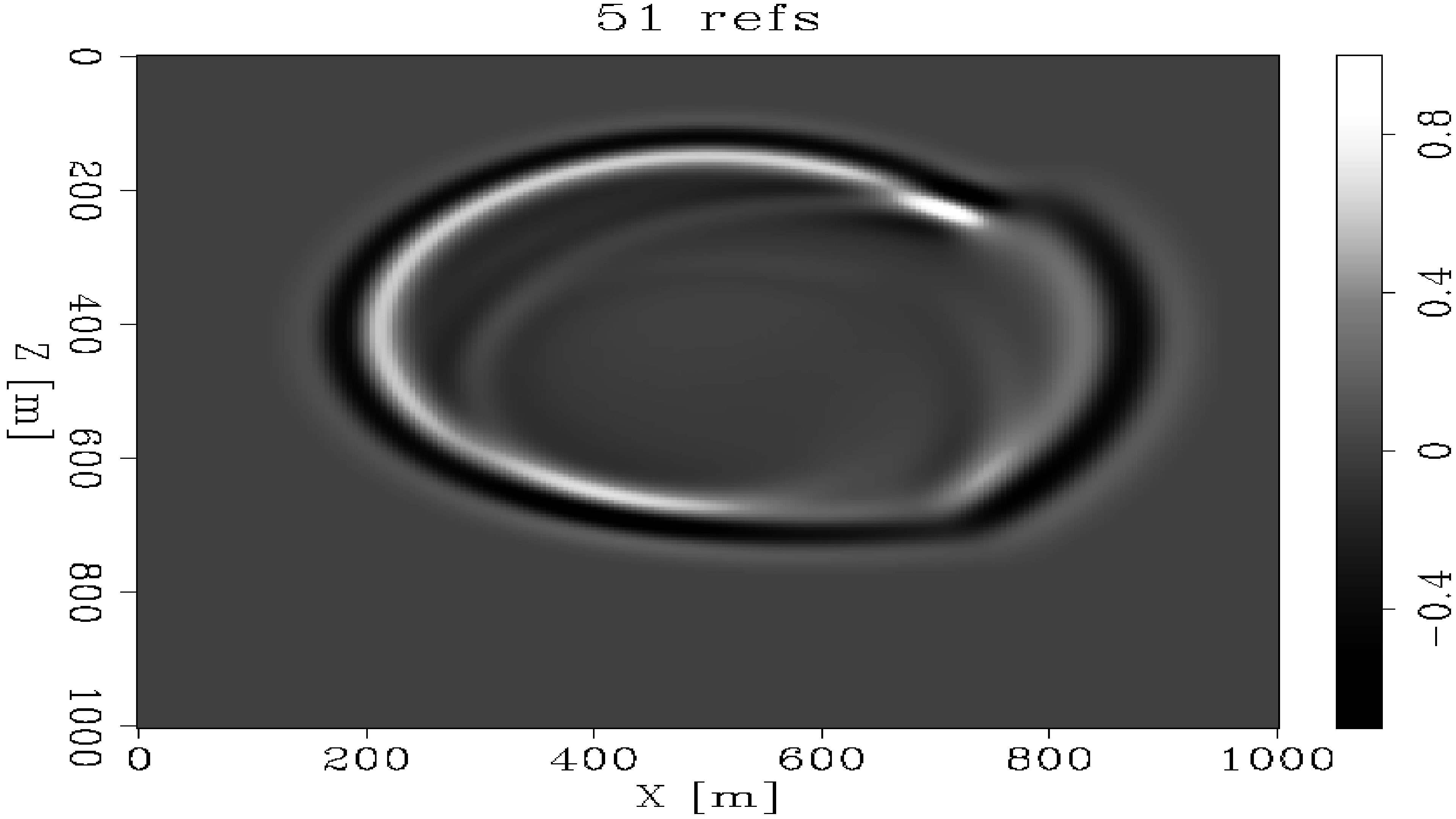

Model w/ 29 refs by Lloyd
Z [m]
0
200
400
600
800
1000
X [m]
0
200
400
600
800
1000
Velocity [km/s]
2
2.2
2.4
2.6
2.8

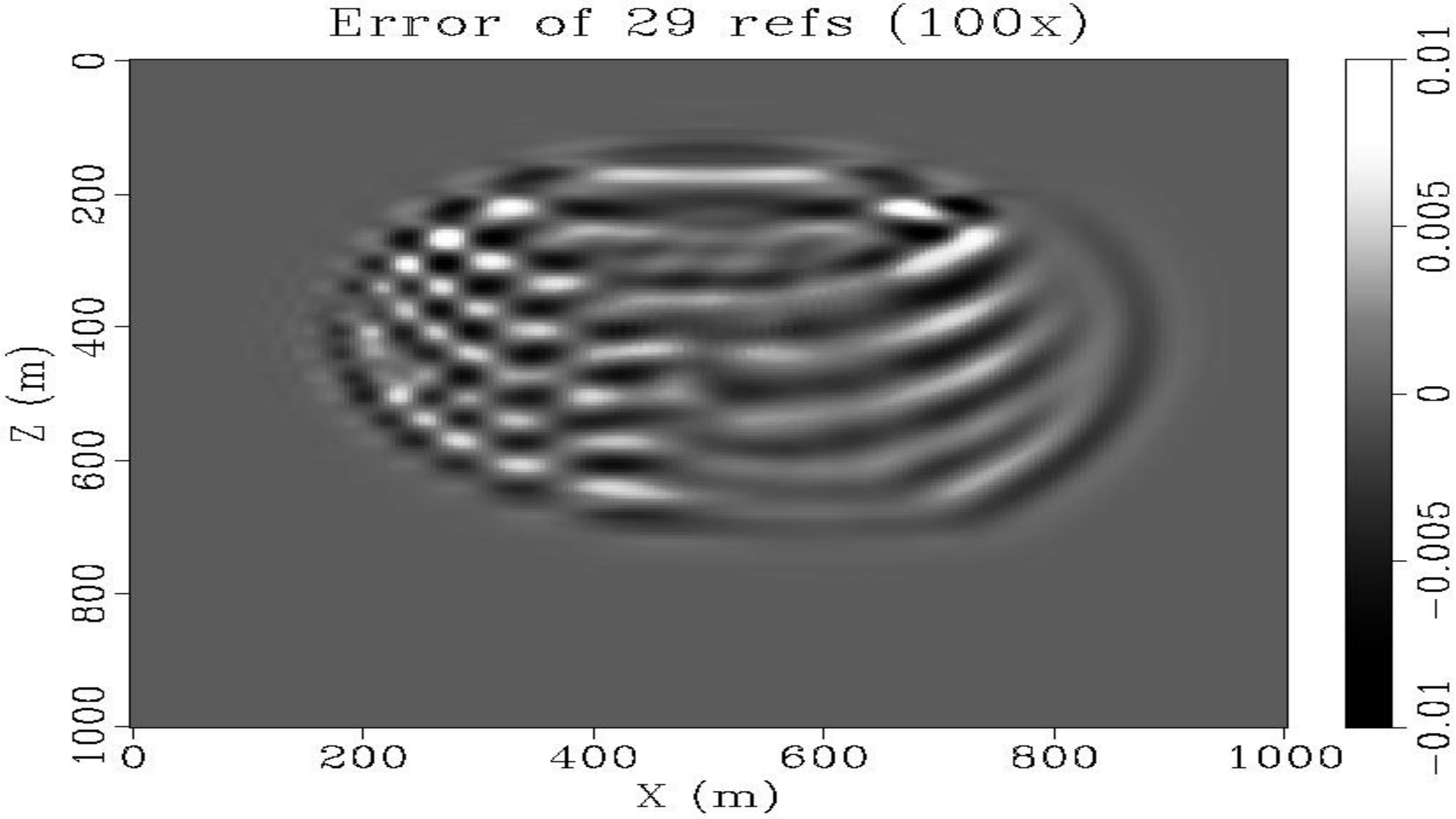
No interpolation

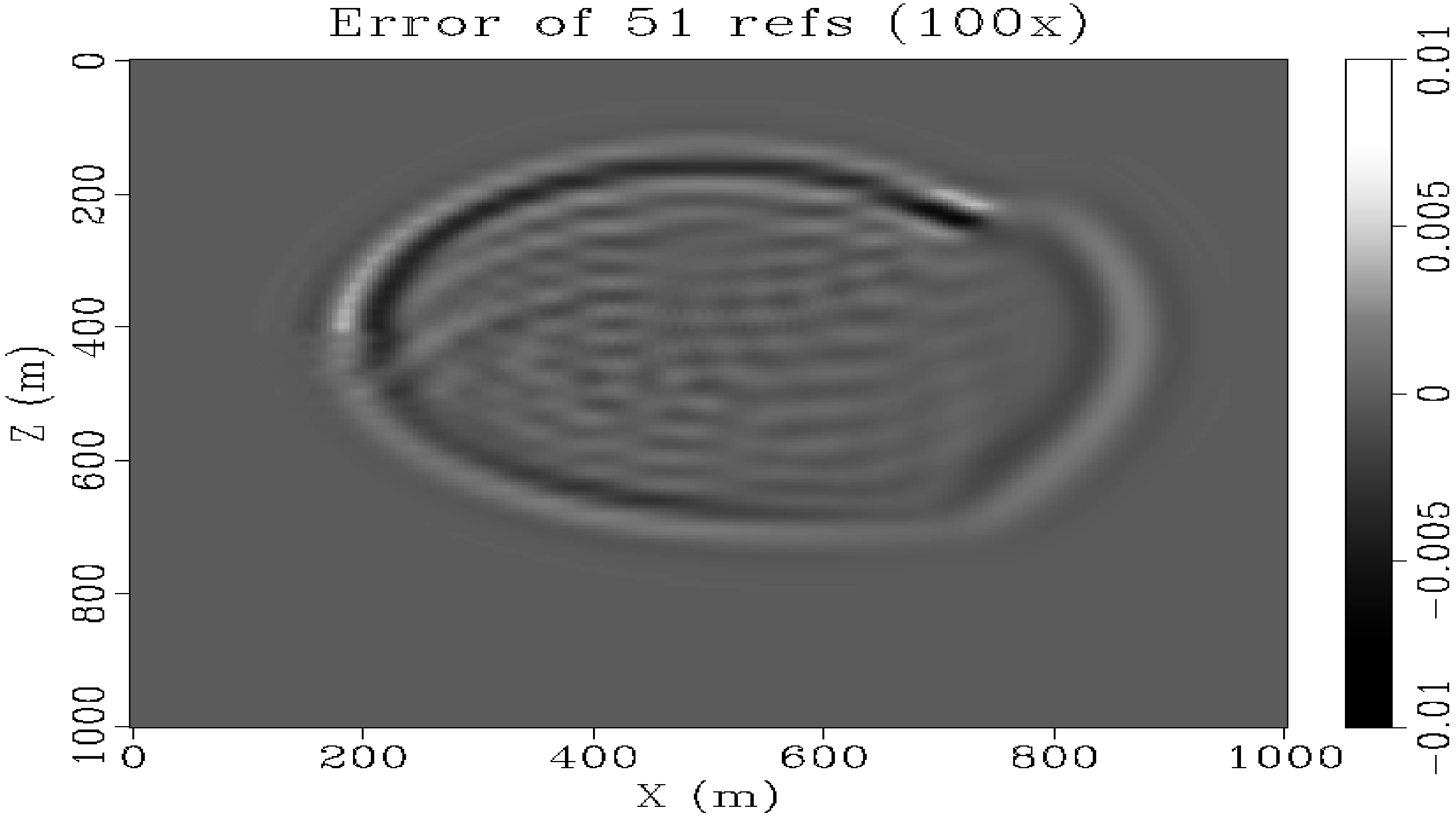



True wavefield
Z [m]
X [m]
0
200
400
600
800
1000
0
200
400
600
800
1000
0.8
0.4
0
−0.4


29 refs
0
200
400
600
800
1000
Z [m]
X [m]
0.8
0.4
0
−0.4

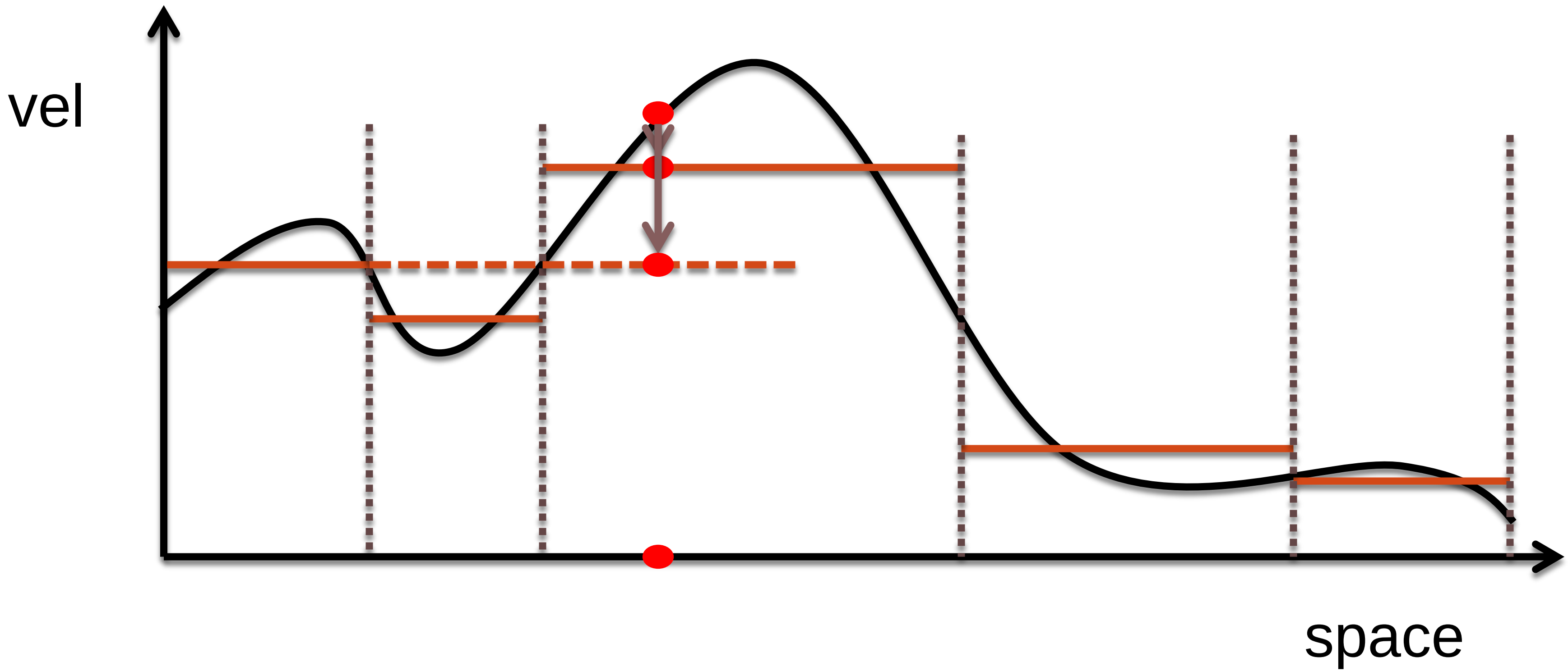

51 refs
Z [m]
0
200
400
600
800
1000
X [m]
0
200
400
600
800
1000
0.8
0.4
0
−0.4

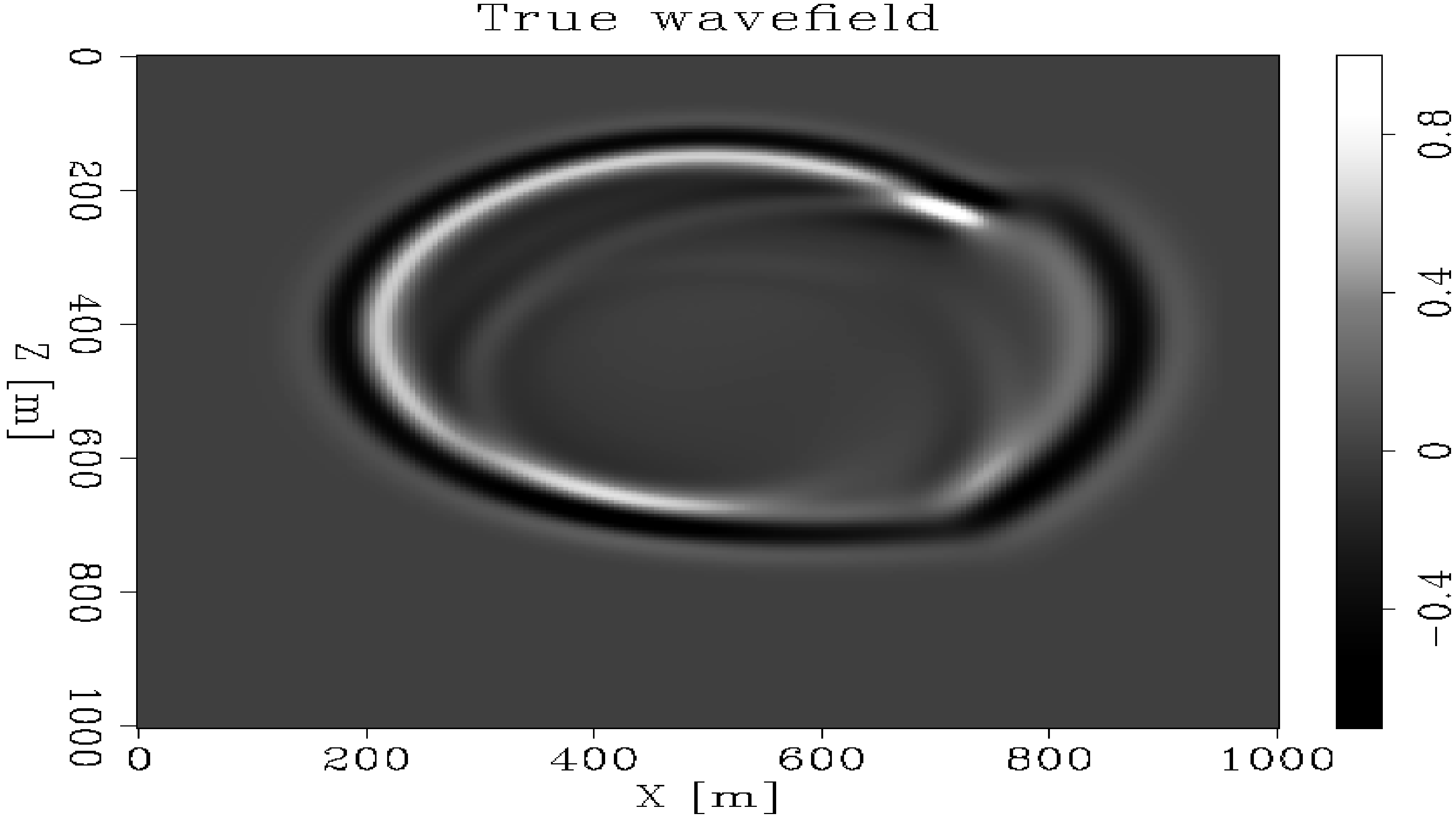

Error of 29 refs (100x)
Z (m)
X (m)
0
200
400
600
800
1000
0
200
400
600
800
1000
0.01
0.005
0
−0.005
−0.01

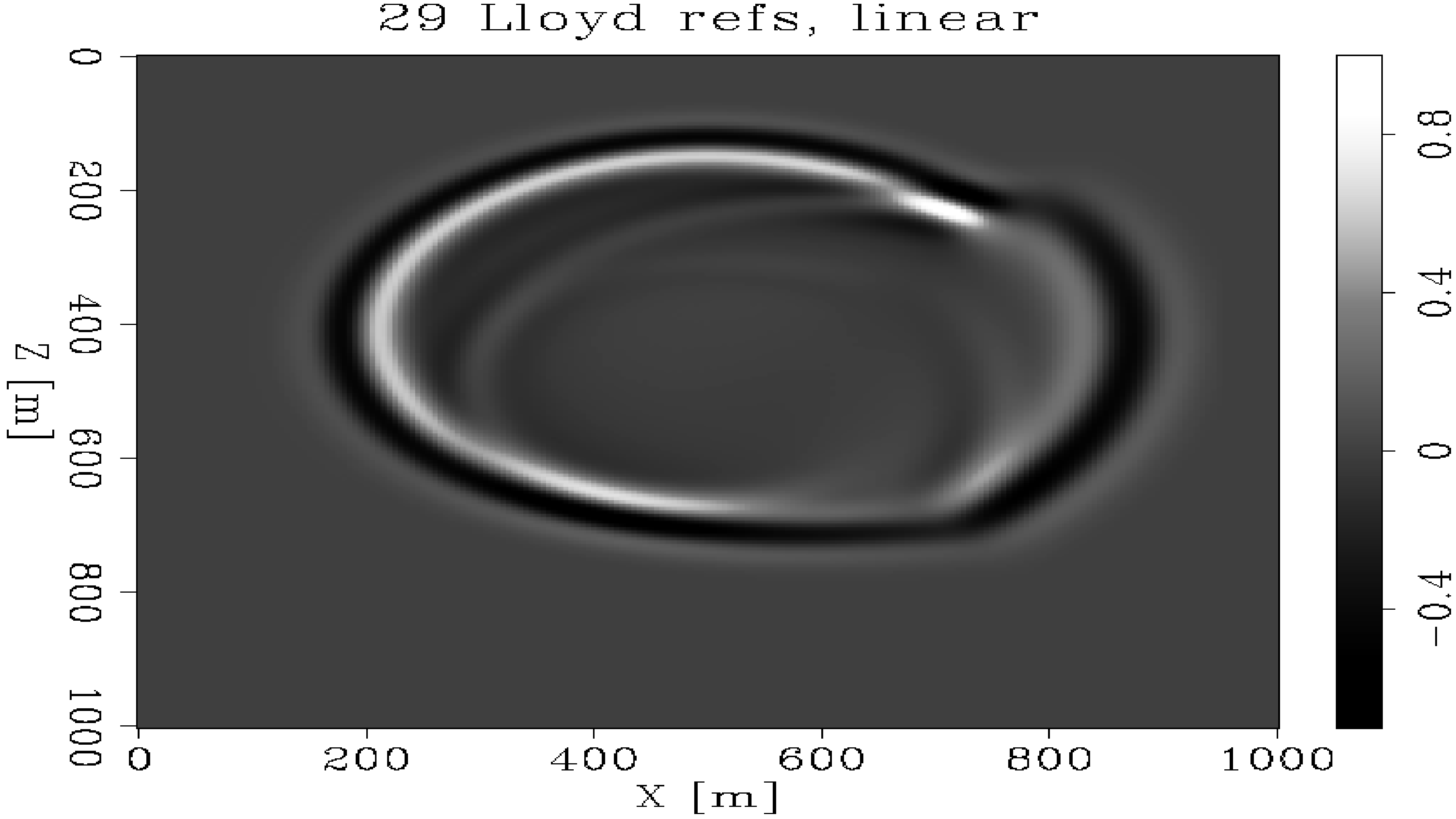


Error of 51 refs (100x)
Z (m)
X (m)
0
200
400
600
800
1000
0
200
400
600
800
1000
0.01
0.005
0
−0.005
−0.01

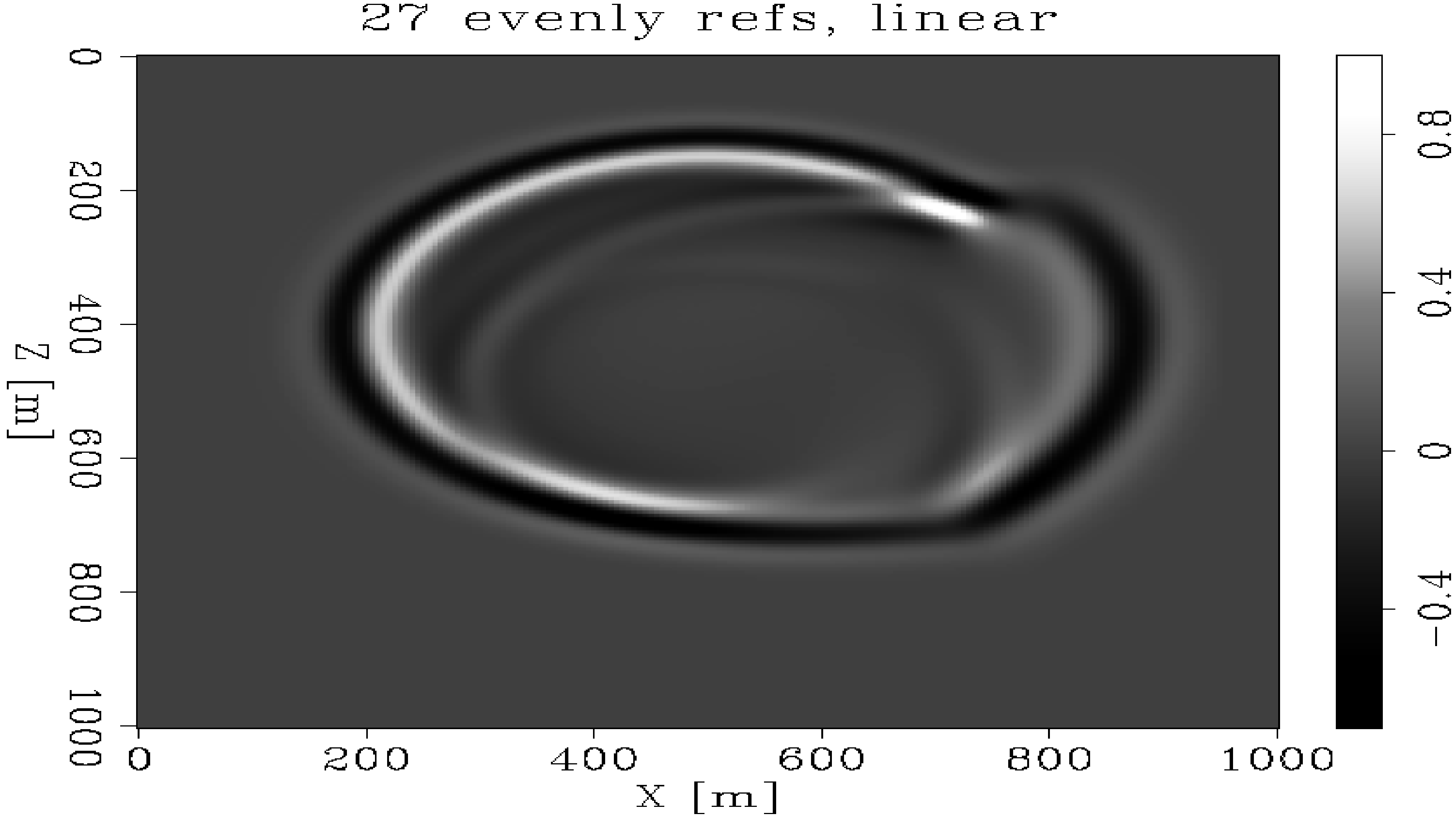
Operator interpolation: linear

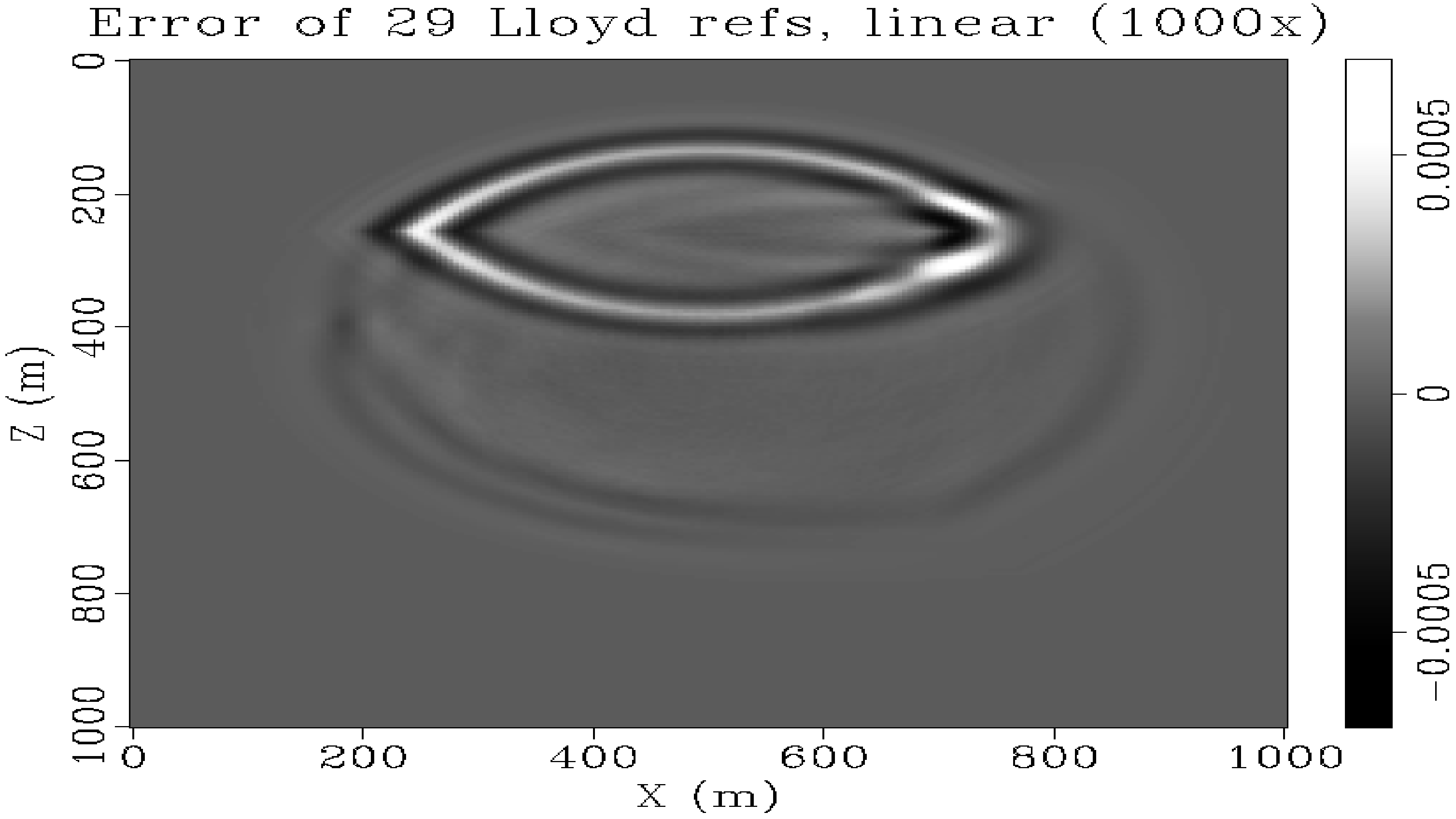


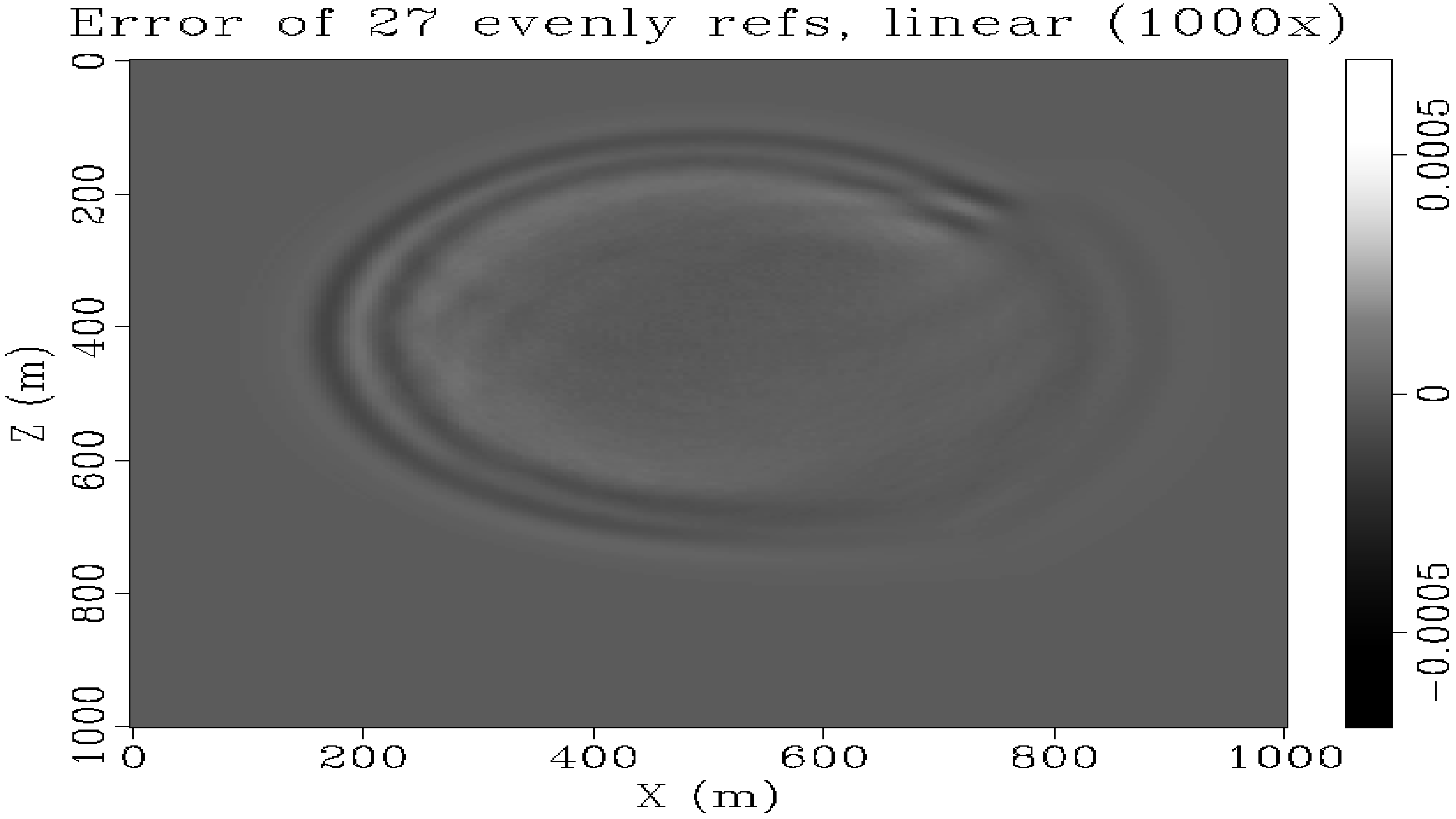

True wavefield
Z [m]
X [m]
0
200
400
600
800
1000
0
200
400
600
800
1000
0.8
0.4
0
−0.4



29 Lloyd refs, linear
Z [m]
X [m]
0
200
400
600
800
1000
0.8
0.4
0
−0.4


27 evenly refs, linear
Z [m]
X [m]
0
200
400
600
800
1000
0.8
0.4
0
−0.4


Error of 29 Lloyd refs, linear (1000x)
Z (m)
X (m)
0
200
400
600
800
1000
0
200
400
600
800
1000
0.0005
0
-0.0005


Error of 27 evenly refs, linear (1000x)
Z (m)
0
200
400
600
800
1000
X (m)
0
200
400
600
800
1000
0.0005
0
−0.0005

Conclusions

- Shear artifacts can be removed completely by wavenumber-domain eigenvalue decomposition.
- Proposed method can be applied in wavenumber domain or space-time domain.
- Lower degree of anisotropy, smaller operator => possible hybrid scheme
- Dense operator is computationally expensive, but highly parallelizable and has better cache reuse.
- Application in inhomogeneous media by operator interpolation is stable and acceptably accurate.

Acknowledgements

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- Stanford School of Earth Sciences
- SEP colleagues, especially Ohad Barak and Elita (Yunyue) Li