Artifact reduction in pseudo-acoustic modeling by pseudo-source injection

SEP 152, pages 95-104

Musa Maharramov

Stanford Exploration Project

June 4, 2013

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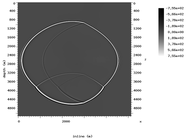

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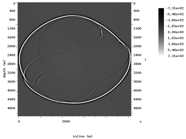

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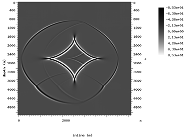


Outline

- Motivation
- Overview of existing pseudo-acoustic methods
- Evolutionary pseudo-differential equation
- NEW: *pseudo-sources* and finite-difference systems
- Numerical tests
- Conclusions and perspectives
- Acknowledgements

Existing methods and motivation

- Computationally efficient pseudo-acoustic modeling is of significant value for applications (Fowler et al, 2010)

- FD methods are fast but suffer from *pseudo-shear* artifacts (Alkhalifah, 2000; Zhang and Zhang, 2008; Fletcher et al., 2009)

- Artifact-free spectral methods require multiple FFTs per time step (Etgen and Brandsberg-Dahl, 2009)

- Goal: a fast FD method with a significant reduction of pseudo-shear artifacts

Evolutionary pseudo-differential operator

Convert the azimuthal pressure velocity curve $V(\theta)$ (Tsvankin 2001) into a pseudo-differential equation

$$\frac{V^2(\theta)}{V_P^2} = 1 + \epsilon \sin^2\theta - \frac{f}{2} \pm \pm \left\{ \left(1 + \frac{2\epsilon \sin^2\theta}{f}\right)^2 - \frac{2(\epsilon - \delta)\sin^2 2\theta}{f} \right\}^{1/2}, \quad (1)$$

$$f = 1 - \frac{V_S^2}{V_P^2}, \; \sin\theta = \frac{V(\theta)\left[\frac{\partial}{\partial x}\right]}{\left[\frac{\partial}{\partial t}\right]}, \; \cos\theta = \frac{V(\theta)\left[\frac{\partial}{\partial z}\right]}{\left[\frac{\partial}{\partial t}\right]}$$

and solve it using a spectral method plus interpolation.

Evolutionary pseudo-differential operator

The full pseudo-differential operator equation (1) can be approximated with a trigonometric polynomial:

$$V^2(\theta) \approx V_P^2 \sum_{n=0}^{N} a_n \sin^{2n}(\theta), \tag{2}$$

leading to separable pseudo-differential operators used in practice:

$$\frac{\partial^2}{\partial t^2} = V_P^2 \sum_{n=0}^{N} a_n \frac{\partial^{2n}}{\partial x^{2n}} \Delta^{1-n}, \tag{3}$$

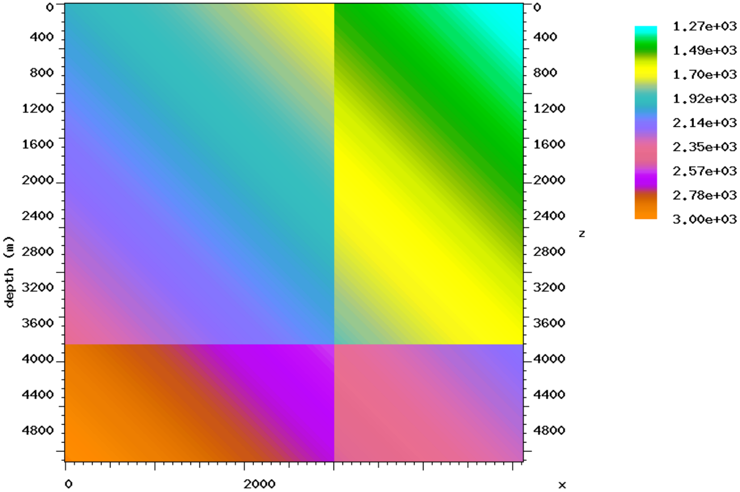
Examples of (2), the *weak anisotropy* approximation:

$$V^2(\theta) \approx V_P^2 \left(1 + \delta \sin^2\theta + \frac{\epsilon - \delta}{1 + 2\delta} \sin^4\theta\right), \tag{4}$$

and Harlan and Lazear relation (Etgen and Brandsberg-Dahl, 2009):

$$V^2(\theta) = V_P^2 \cos^2\theta + \left(V_{P\mathrm{NMO}}^2 - V_{P\mathrm{Hor}}^2\right) \cos^2\theta \sin^2\theta + V_{P\mathrm{Hor}}^2 \sin^2\theta, \tag{5}$$

Model 1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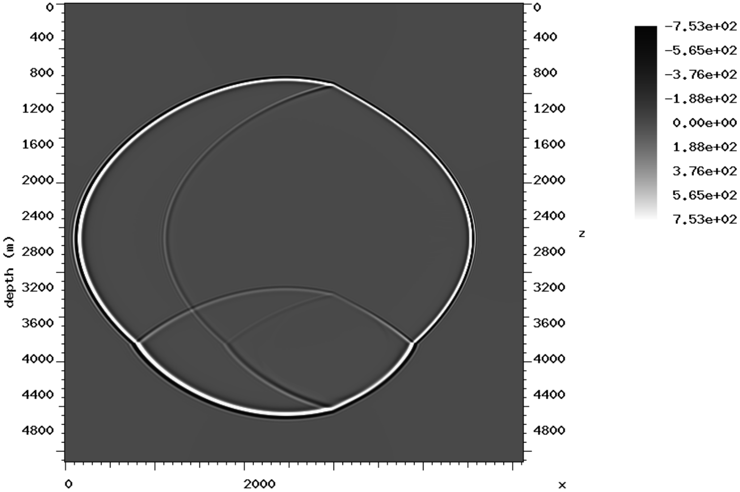
Pseudo-differential operator result



A finite-difference method

Convert (1) into a DO by squaring the square root (Alkhalifah, 2000):

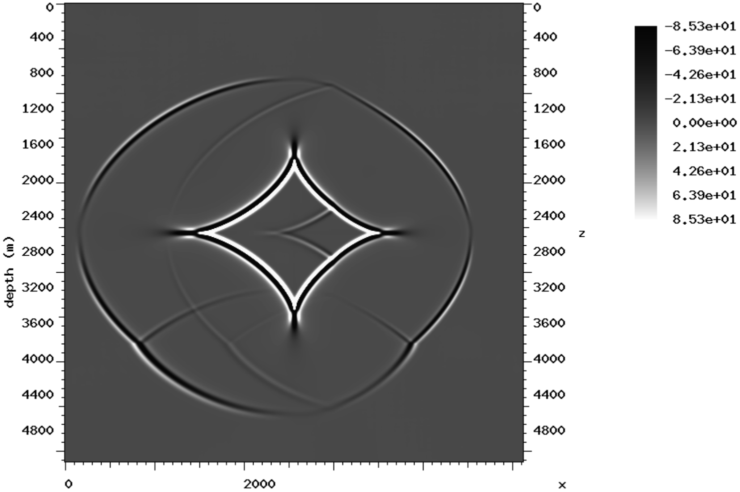
$$\begin{aligned} \frac{\partial^2 q}{\partial t^2} &= V^2_{P\text{Hor}} \frac{\partial^2 q}{\partial x^2} + V^2_P \frac{\partial^2 q}{\partial z^2} + V^2_P \left(V^2_{P\text{Hor}} - V^2_{P\text{NMO}}\right) \frac{\partial^4 r}{\partial x^2 \partial z^2}, \\ \frac{\partial^2 r}{\partial t^2} &= q, \end{aligned} \tag{6}$$

where

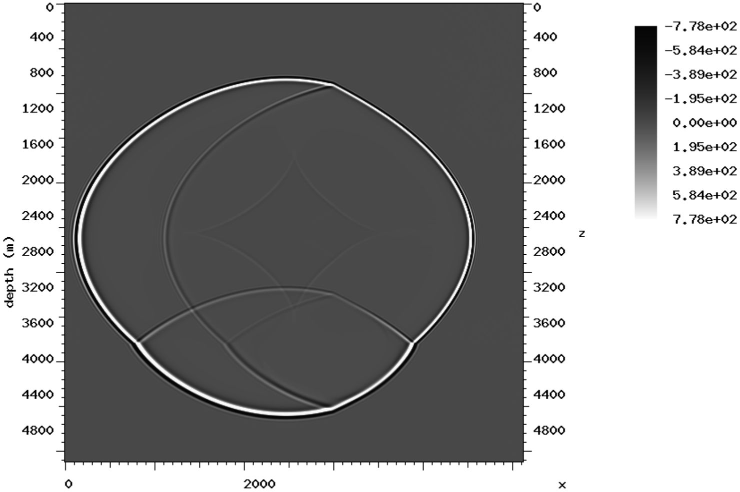
$$V^2_{P\text{Hor}} = V^2_P \left(1 + 2\epsilon\right), \; V^2_{P\text{NMO}} = V^2_P \left(1 + 2\delta\right). \tag{7}$$

- r corresponds to solution of (1)
- In *conventional implementations* source is injected in q
- Equivalent systems exist with components as linear combinations of r and q

Pseudo-shear artifacts



Source injected into q



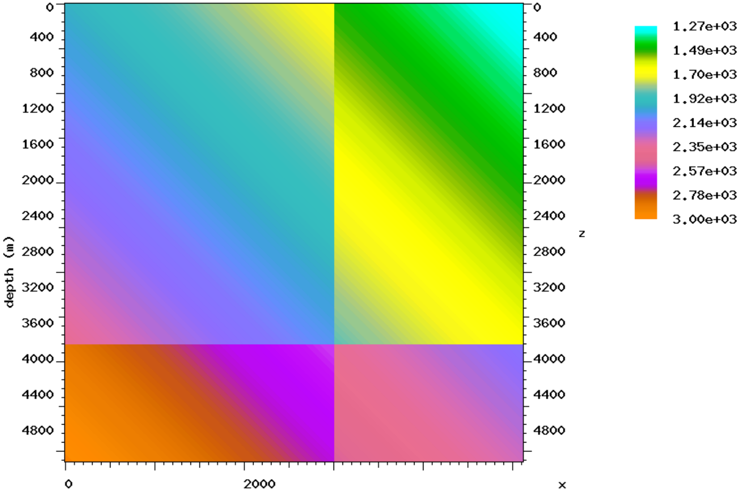
Pseudo-source injection

At each time step inject the actual source in r and the *pseudo-source* into q:

$$\begin{aligned} r(z,x,t_n) &= r(z,x,t_n) + \phi(z,x,t_n), \\ q(z,x,t_n) &= q(z,x,t_n) + V_P^2 \left\{ (z,x)\frac{\Delta}{2} + \epsilon(z,x)\frac{\partial^2}{\partial x^2} + \right. \\ &\left. + \frac{\Delta}{2}\sqrt{\left[1 + 2\epsilon(z,z)\frac{\partial^2}{\partial x^2}\frac{1}{\Delta}\right]^2 - 8\left(\epsilon(z,x) - \delta(z,x)\right)\frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial z^2}\frac{1}{\Delta^2}} \right\} \phi, \end{aligned} \tag{8}$$

Apply the finite-difference time step of (6).

Model 1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1.70e+03
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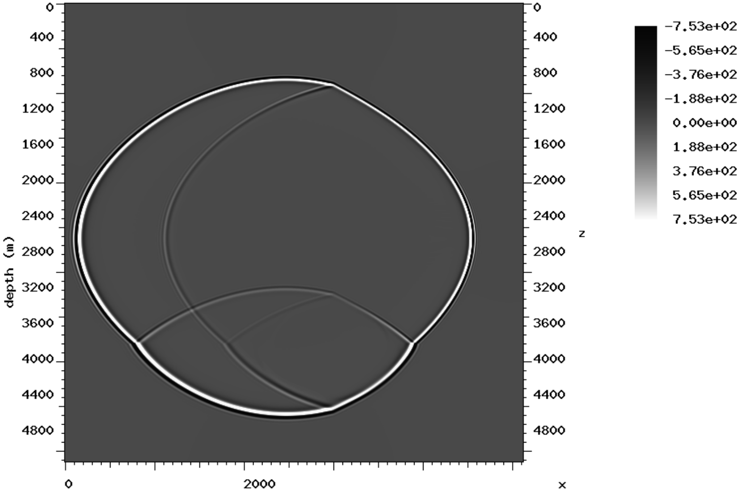
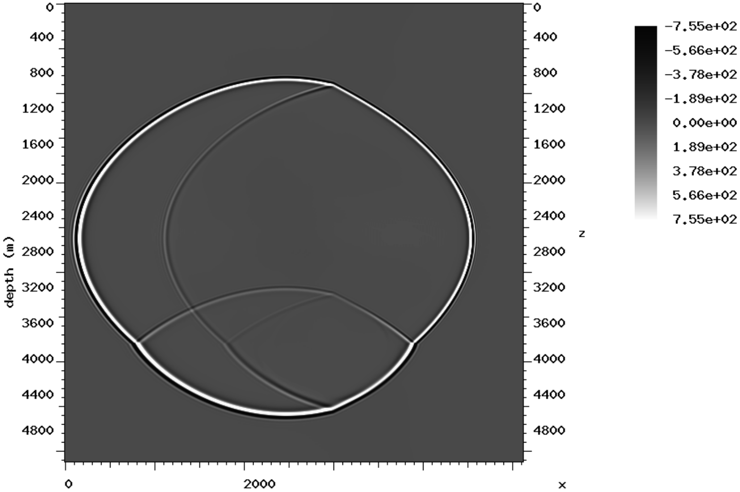
2.57e+03
2.78e+03
3.00e+03

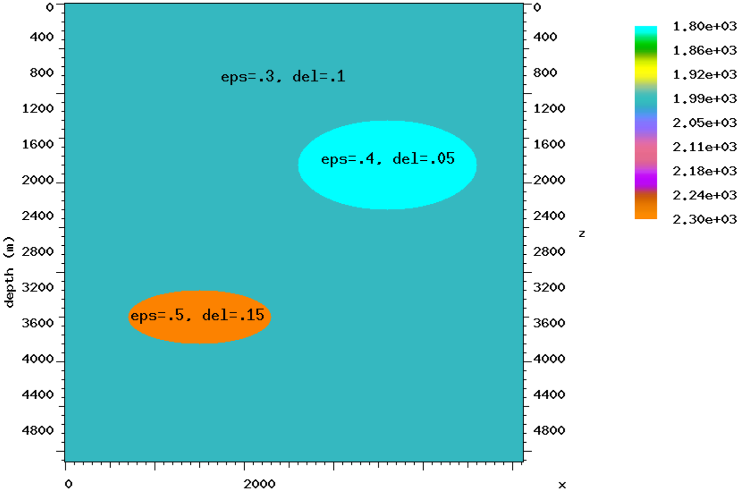
Pseudo-differential operator



Finite-difference with pseudo-sources



Model 2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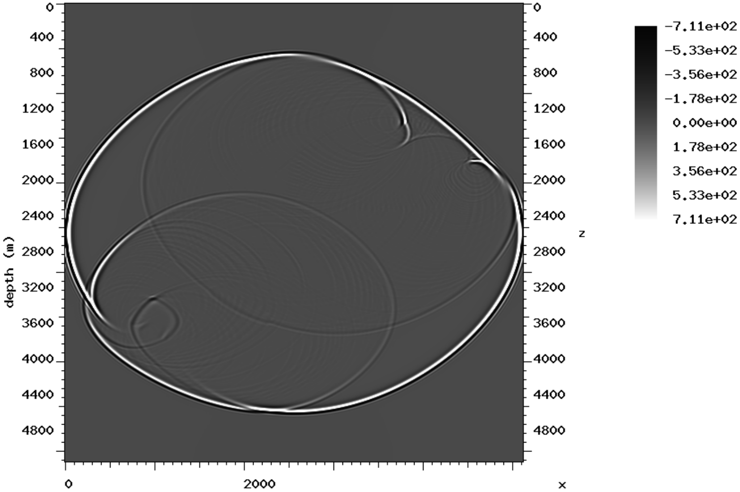
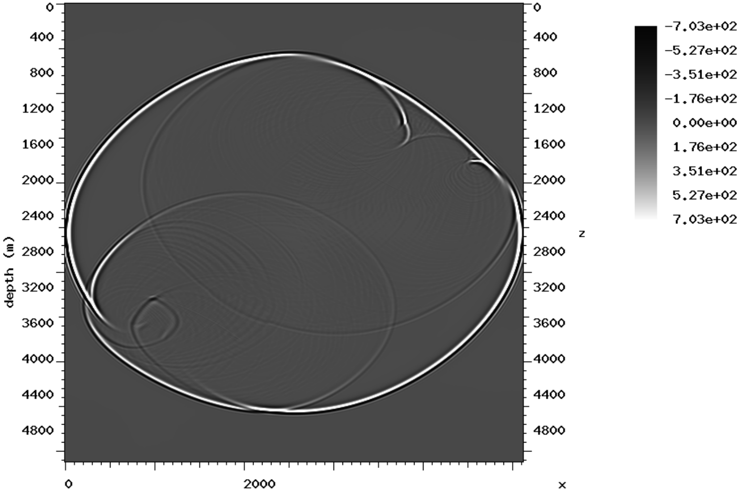
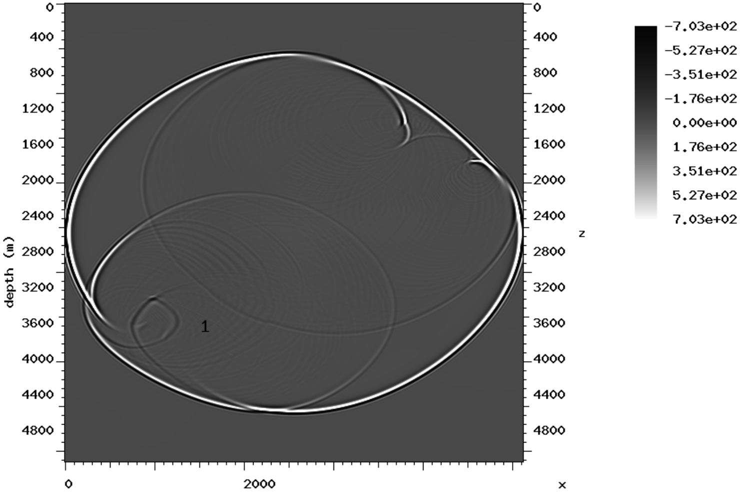
Pseudo-differential operator



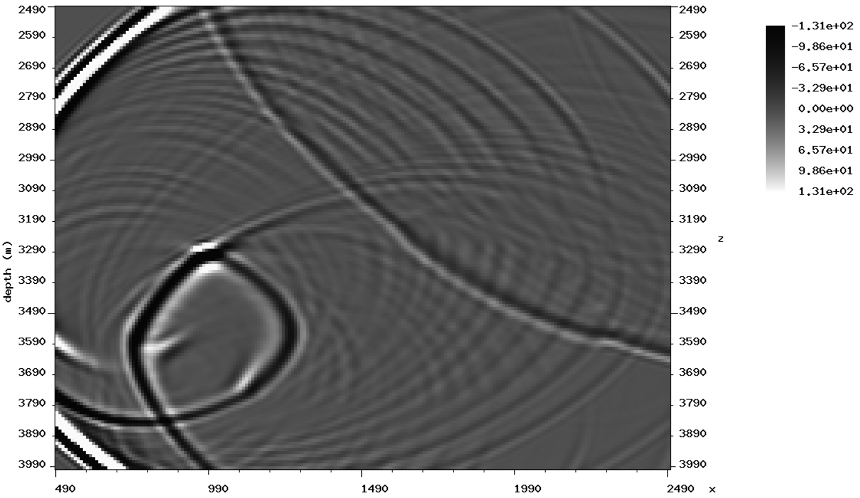
Finite-difference with pseudo-sources



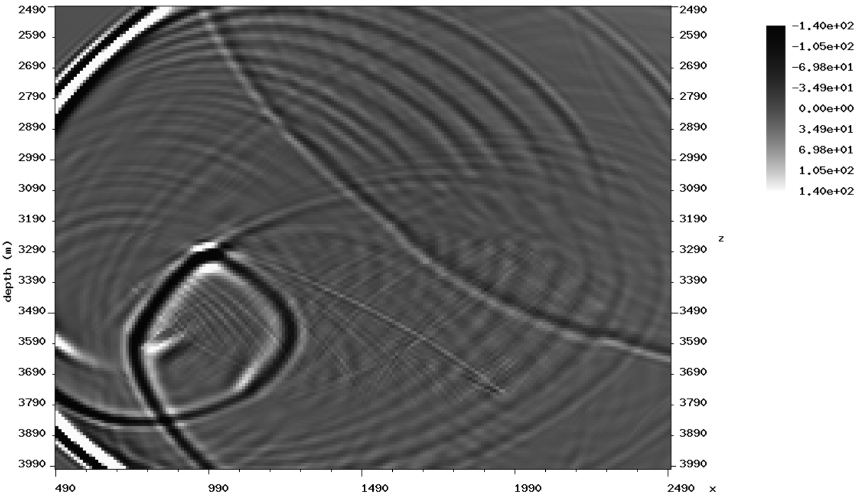
Finite-difference with pseudo-sources



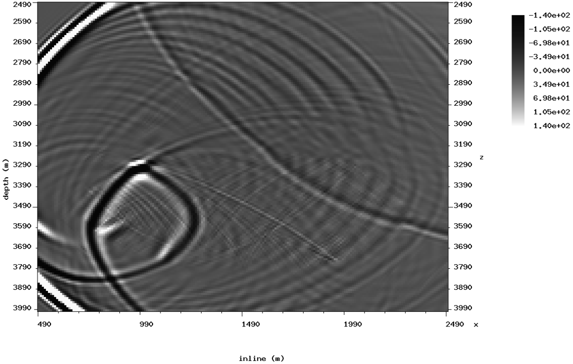
Pseudo-differential operator – ϵ, δ contrasts

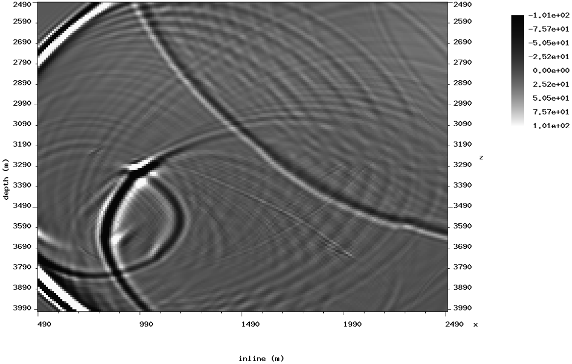


Finite-difference – ϵ, δ contrasts



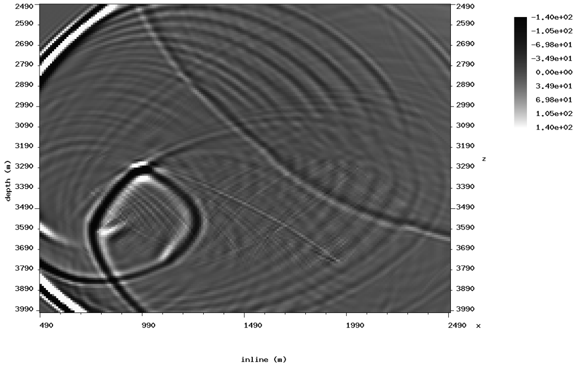
Non-smoothed Thomsen vs smoothed Thomsen

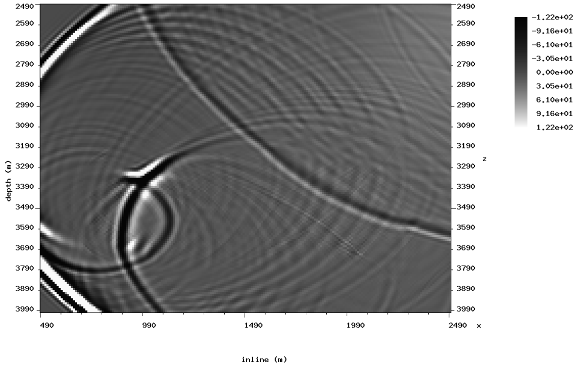




Weak high-frequency artifacts appear where FD approximation has largest errors.

Non-smoothed Thomsen vs constant Thomsen





Reduction in FD error (and error anisotropy) reduces the artifacts.

Conclusions and perspectives

- Computationally cheap FD technique with artifacts suppressed by injecting a "pseudo-source"
- Savings come from spatial boundedness of pseudo-source support
- Achieves a significant reduction of shear artifacts
- Can be used with a wide range of PA systems (Fowler et al., 2010)
- Next steps: apply to energy-conserving tilted anisotropy systems (Zhang and Zhang, 2009; Macesanu, 2011)
- Combine with Lax-Wendroff to reduce dispersion

Acknowledgements

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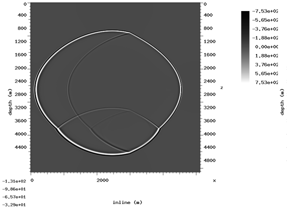
Q&A



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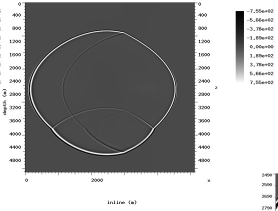

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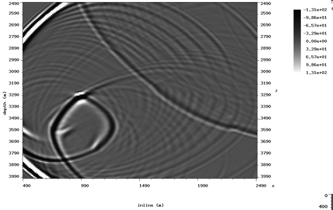

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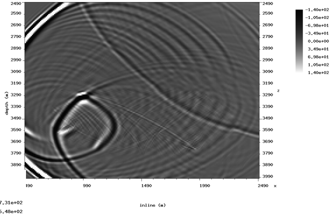
Q&A



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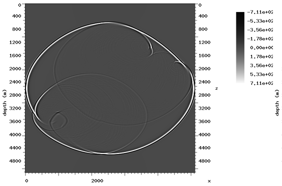

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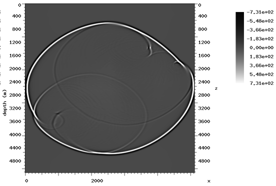

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