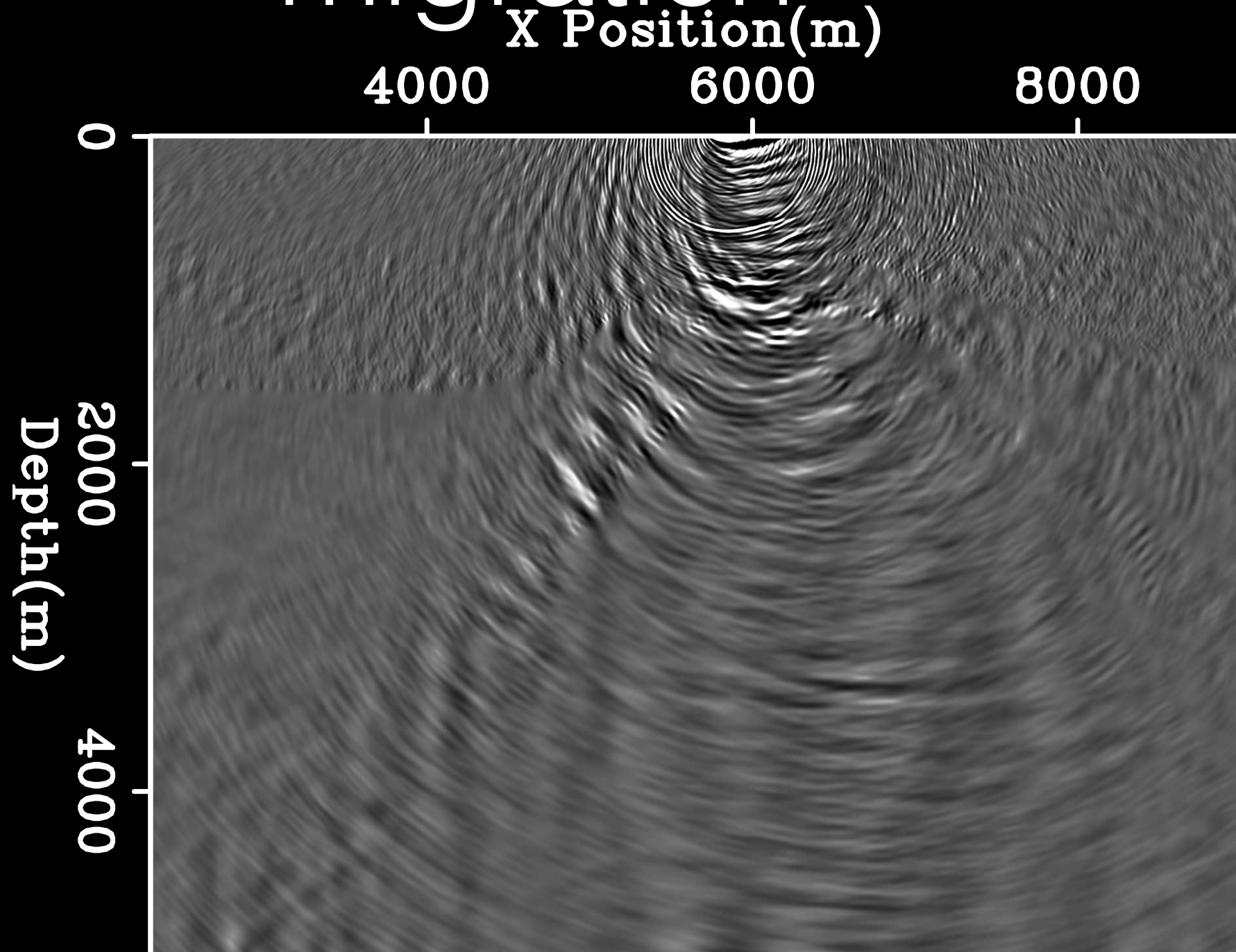


# Approximating Q propagation to speed up finite differences

Robert Clapp

# Standard single shot migration



## Outline

Motivation

Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples

# Variable single shot migration

## Outline

Motivation

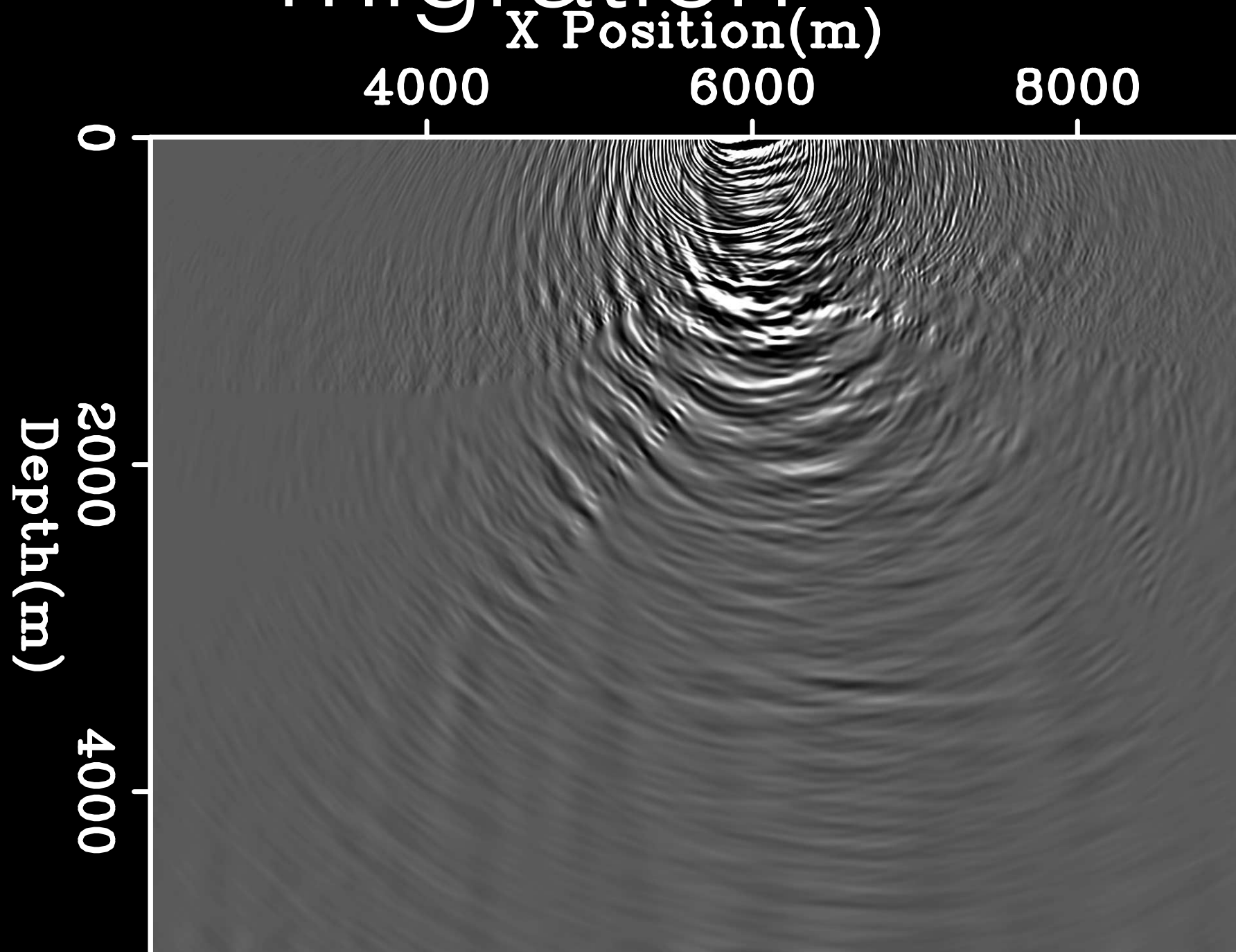
Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples



# Finite-difference constraints

## Outline

Motivation

Accelerating  
migration

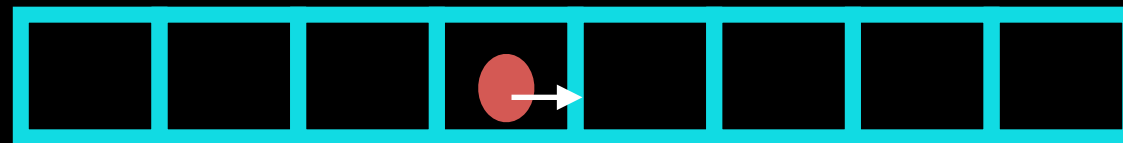
Approximating  
Q

Speedup

Implementation

Migration  
examples

Stability:  $v_{max} \frac{dt}{d_{min}} < .5$



Grid  
dispersion:

# Finite-difference constraints

## Outline

Motivation

Accelerating  
migration

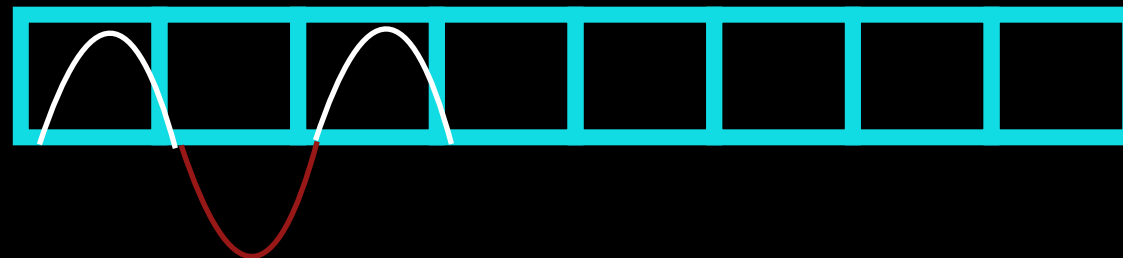
Approximating  
Q

Speedup

Implementation

Migration  
examples

Stability:  $v_{max} \frac{dt}{d_{min}} < .5$



Grid  
dispersion:  $\frac{v_{min}}{f_{max} d_{max}} > 3.2$

# Finite-difference constraints

## Outline

Motivation

Accelerating  
migration

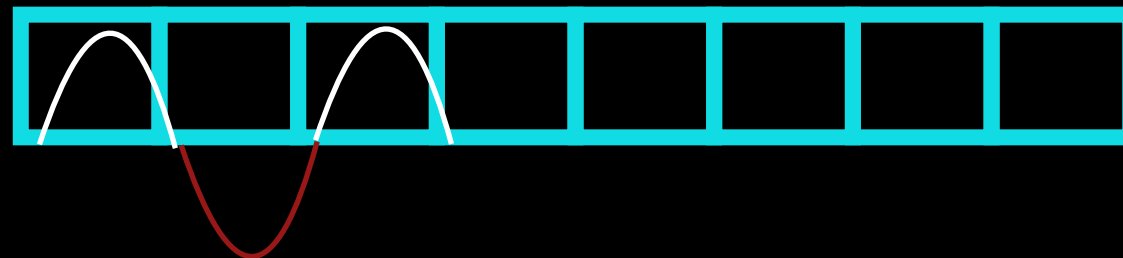
Approximating  
Q

Speedup

Implementation

Migration  
examples

Stability:  $v_{max} \frac{dt}{d_{min}} < .5$



Grid  
dispersion:  $\frac{v_{min}}{f_{max} d_{max}} > 3.2$

Higher order  
stencil less  
grid dispersion  
error

# Go through data using a pencil pattern

## Outline

Motivation

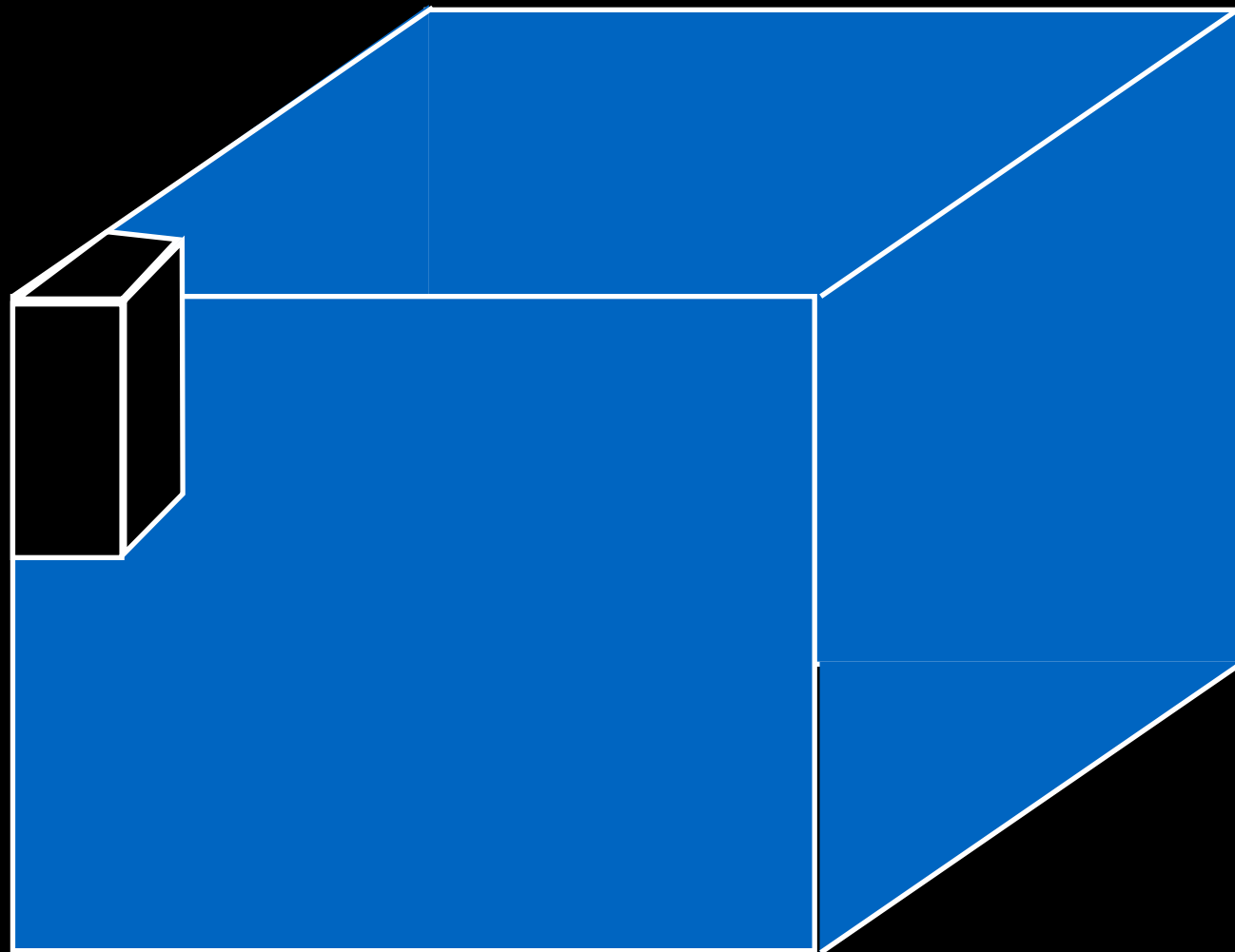
Accelerating  
migration

Approximating  
 $Q$

Speedup

Implementation

Migration  
examples



# Vectorize your code

## Outline

Motivation

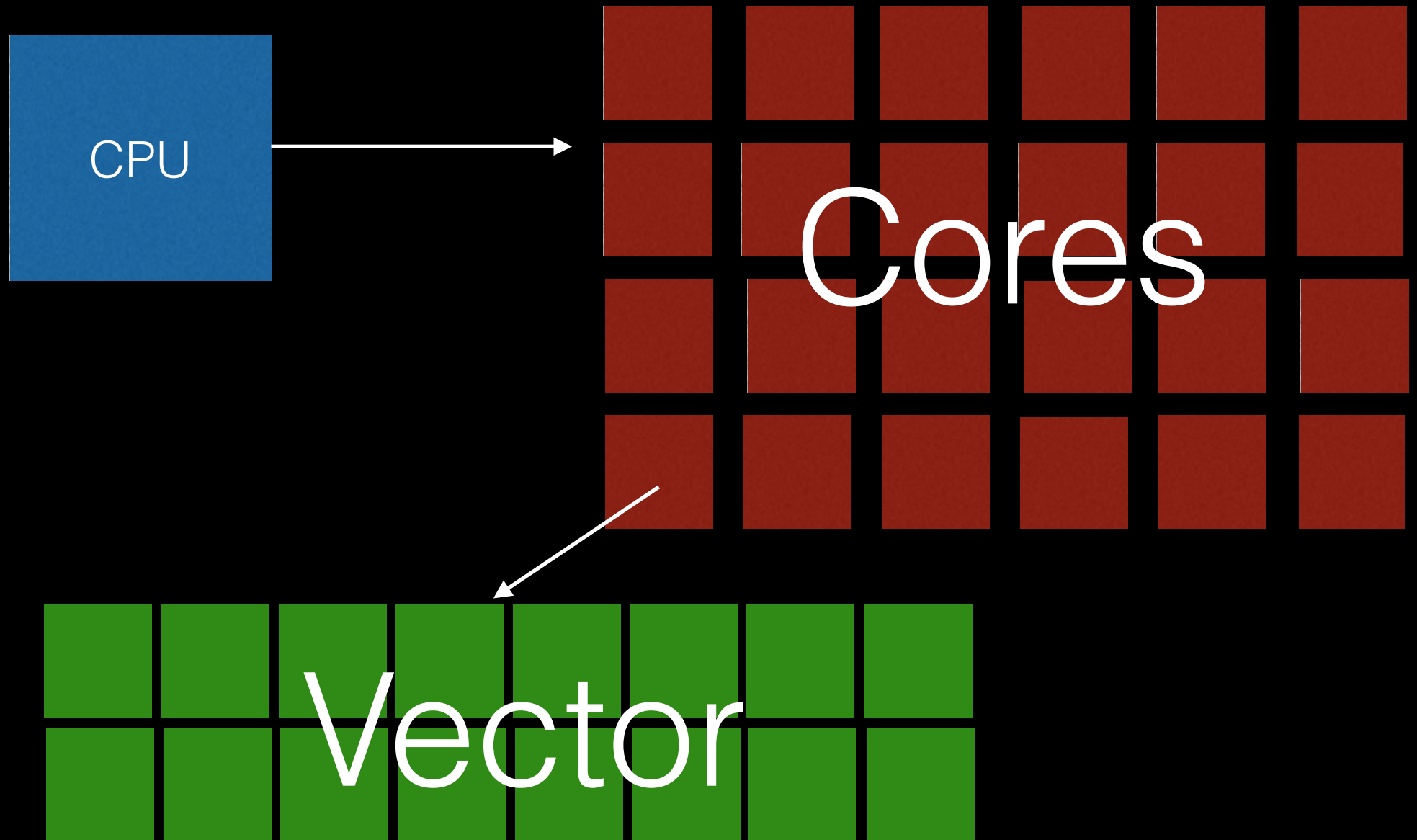
Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples





# Pencils less bandwidth contention on multi-core

## Outline

Motivation

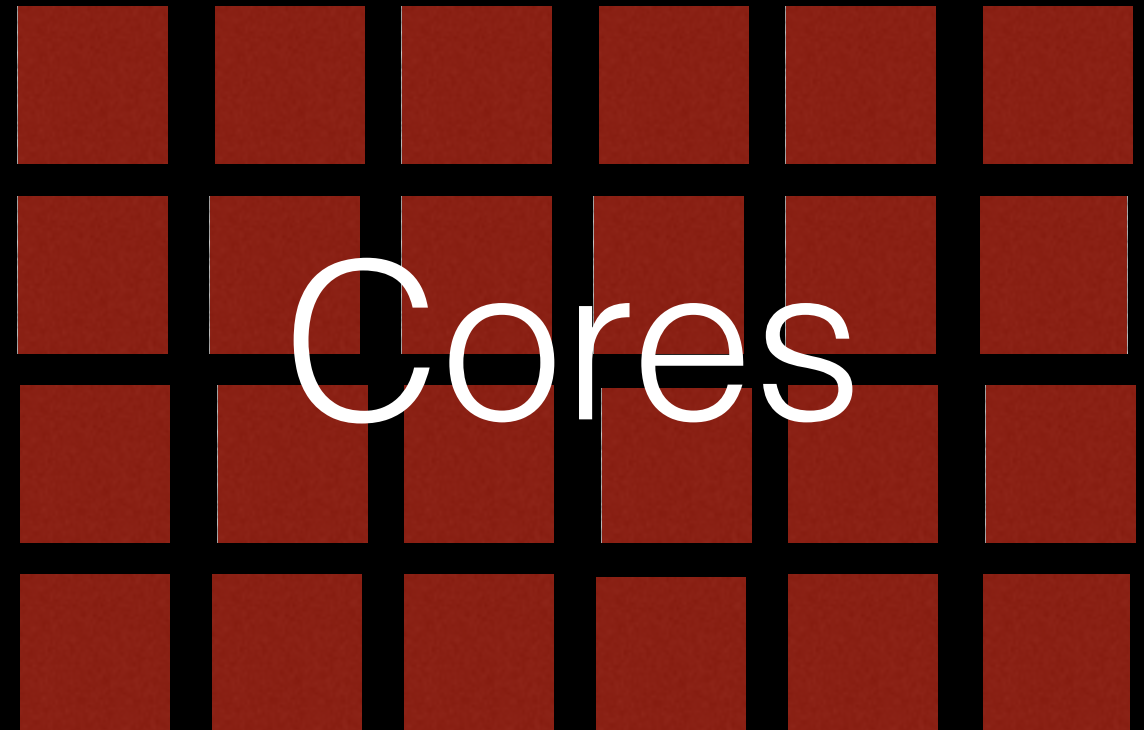
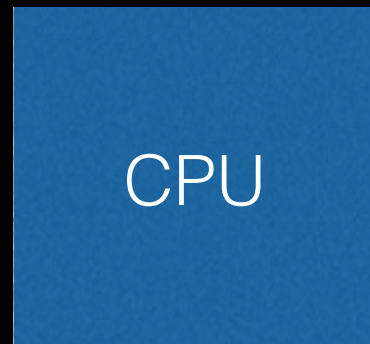
Accelerating  
migration

Approximating  
 $Q$

Speedup

Implementation

Migration  
examples



# Follow the wave-field

## Outline

Motivation

Accelerating  
migration

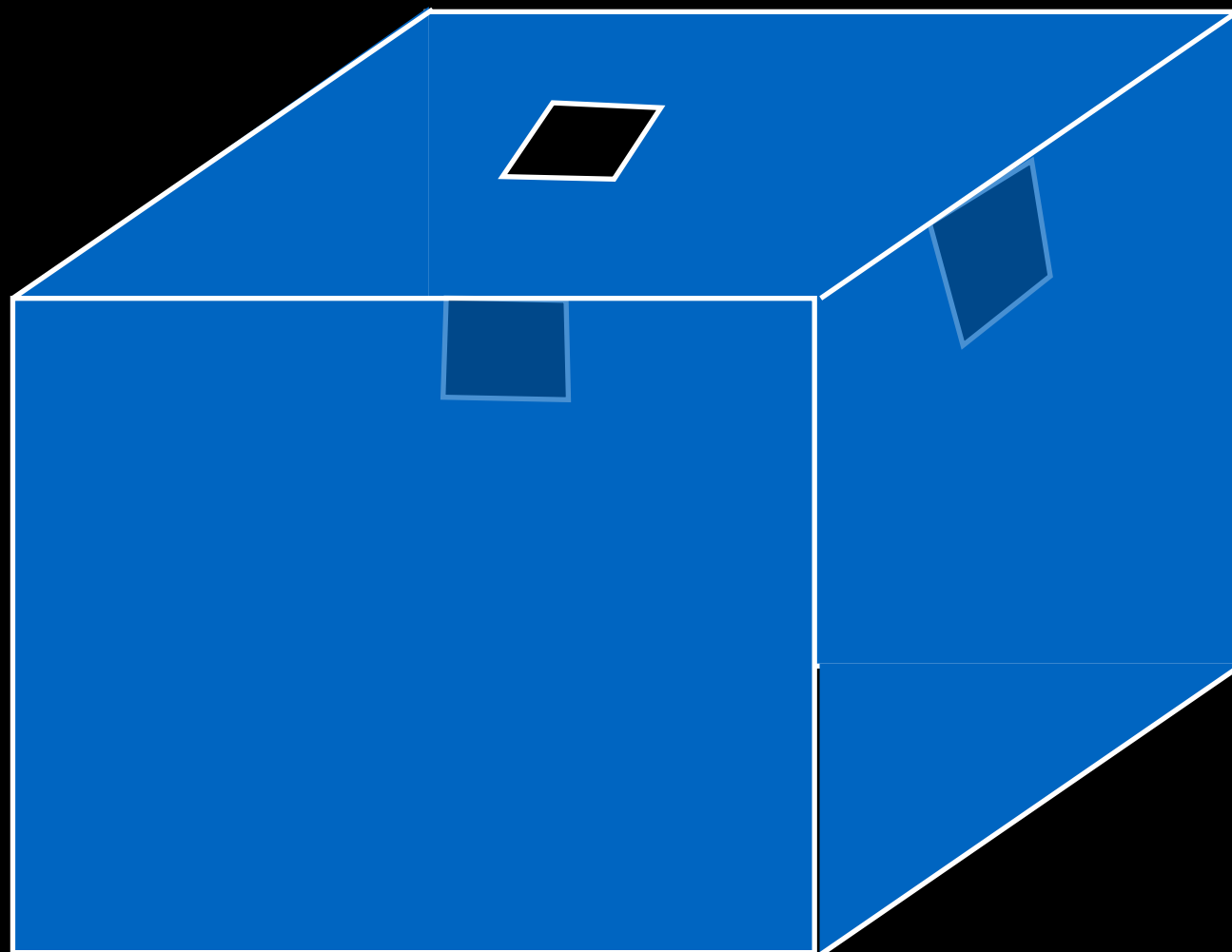
Approximating  
Q

Speedup

Implementation

Migration  
examples

$t < .1$



# Follow the wave-field

## Outline

Motivation

Accelerating  
migration

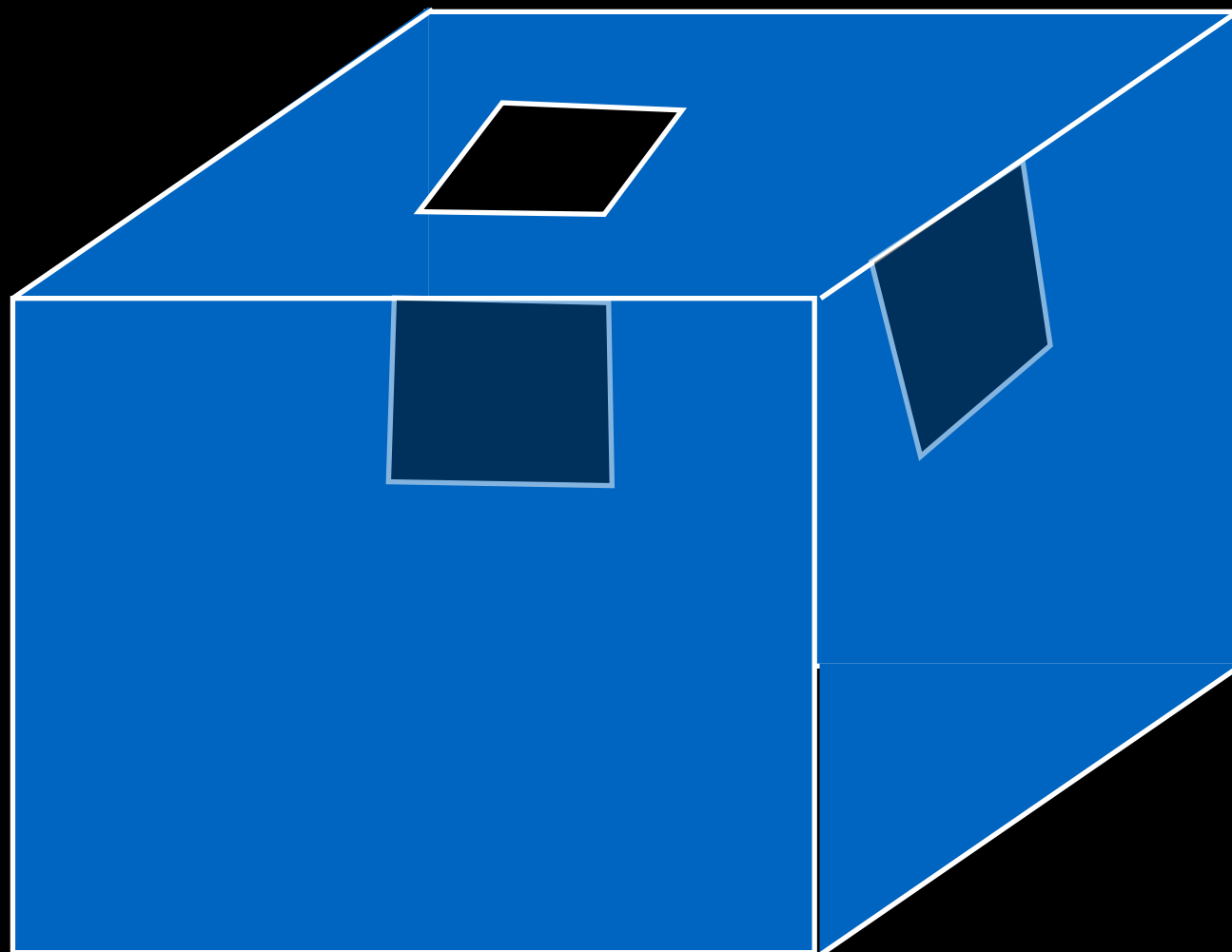
Approximating  
Q

Speedup

Implementation

Migration  
examples

$t < 1.5$



# Follow the wave-field

## Outline

Motivation

Accelerating  
migration

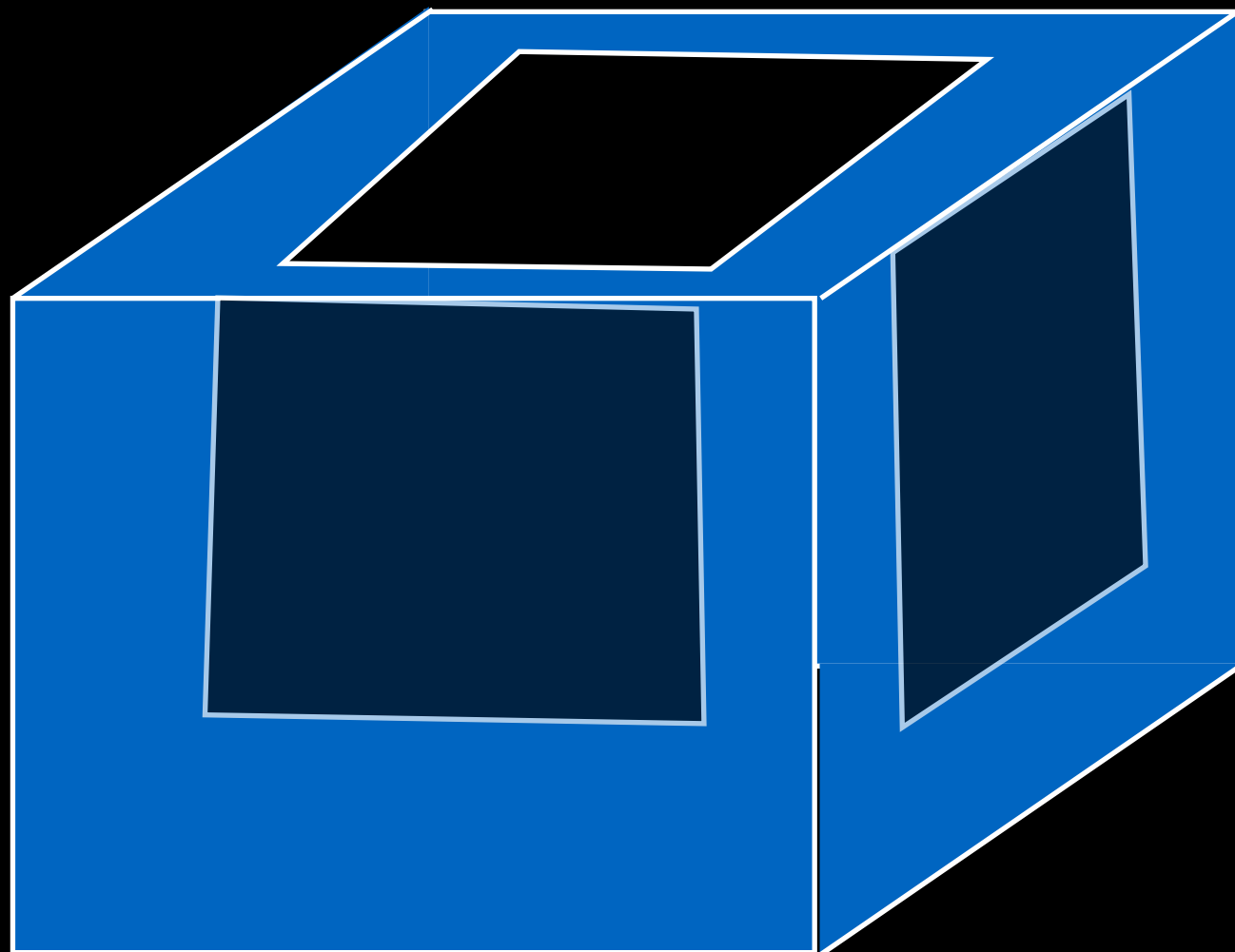
Approximating  
 $Q$

Speedup

Implementation

Migration  
examples

$t < 3.$



# Speeding up RTM

## Outline

Motivation

Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples

- High-order stencils
  - Space domain 12-30th order
  - 4th order in time
- Go through grid in pencil shapes
- Vectorize the code
- Follow the wave-field

# Speeding up Downward continuation

## Outline

Motivation

Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples

- FFD and/or smart choice of reference velocities
- Follow the wave-field
- Use different grid sampling at different frequencies (large cells at larger frequencies)
- Stop downward continuing higher frequencies at certain depths

# Speeding up Downward continuation

## Outline

Motivation

Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples

- FFD and/or smart choice of reference velocities
- Follow the wave-field
- Use different grid sampling at different frequencies (large cells at larger frequencies)
- Stop downward continuing higher frequencies at certain depths

# Models of Q

## Outline

Motivation

Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples

- Q varies as a function of frequency and material (spatially)
- Q is constant as a function of frequency and varies spatially
- As we increase in depth frequencies have decayed to the point of being unimportant
- As we increase in time useful frequencies decrease



# Fractional Laplacian

## Outline

Motivation

Accelerating  
migration

Approximating  
 $Q$

Speedup

Implementation

Migration  
examples

$$\left[ \eta \mathbf{L} + \tau \mathbf{H} \frac{d}{dt} - v^{-2} \frac{\partial^2}{\partial t^2} \right] P(t) = f(t)$$

$$\mathbf{L} = (-\nabla^2)^{\gamma+1}$$

$$\mathbf{H} = (-\nabla^2)^{\gamma+\frac{1}{2}}$$

$$\eta = -v^{2\gamma} w_0^{-2\gamma} \cos \pi \gamma \quad \tau = -v^{2\gamma-1} w_0^{-2\gamma} \sin \pi \gamma$$

$$\gamma = \frac{1}{\tan^{-1} \frac{1}{Q}}$$

Zhu(2014)

# Fractional Laplacian

## Outline

Motivation

Accelerating  
migration

Approximating  
 $Q$

Speedup

Implementation

Migration  
examples

$$\left[ \eta \mathbf{L} + \tau \mathbf{H} \frac{d}{dt} - v^{-2} \frac{\partial^2}{\partial t^2} \right] P(t) = f(t)$$

Dispersion  
component from  
attenuation

$$\mathbf{H} = (-\nabla^2)^{\gamma + \frac{1}{2}}$$

$$\cos \pi \gamma \quad \tau = -v^{2\gamma-1} w_0^{-2\gamma} \sin \pi \gamma$$

$$\gamma = \frac{1}{\tan^{-1} \frac{1}{Q}}$$

Zhu(2014)

# Fractional Laplacian

## Outline

Motivation

Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples

$$\left[ \cancel{\eta \mathbf{L}} + \tau \mathbf{H} \frac{d}{dt} - v^{-2} \frac{\partial^2}{\partial t^2} \right] P(t) = f(t)$$

$$\mathbf{L} = (-\nabla^2)$$

Amplitude decay  
term

$$\mathbf{H} = (-\nabla^2)^{\gamma + \frac{1}{2}}$$

$$\eta = -v^{2\gamma} w_0^{-2\gamma} \cos \pi \gamma$$

$$\tau = -v^{2\gamma-1} w_0^{-2\gamma} \sin \pi \gamma$$

$$\gamma = \frac{1}{\tan^{-1} \frac{1}{Q}}$$

Zhu(2014)

# Fractional Laplacian

## Outline

Motivation

Accelerating  
migration

Approximating  
 $Q$

Speedup

Implementation

Migration  
examples

$$\left( \nabla^2 - \tau \nabla^2 \frac{d}{dt} - v^{-2} \frac{\partial^2}{\partial t^2} \right) P(t) = f(t)$$

$$\tau = -v^{2\gamma-1} w_0^{-2\gamma} \sin \pi \gamma$$

$$\gamma = \frac{1}{\tan^{-1} \frac{1}{Q}}$$

# RTM imaging condition

## Outline

Motivation

Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples

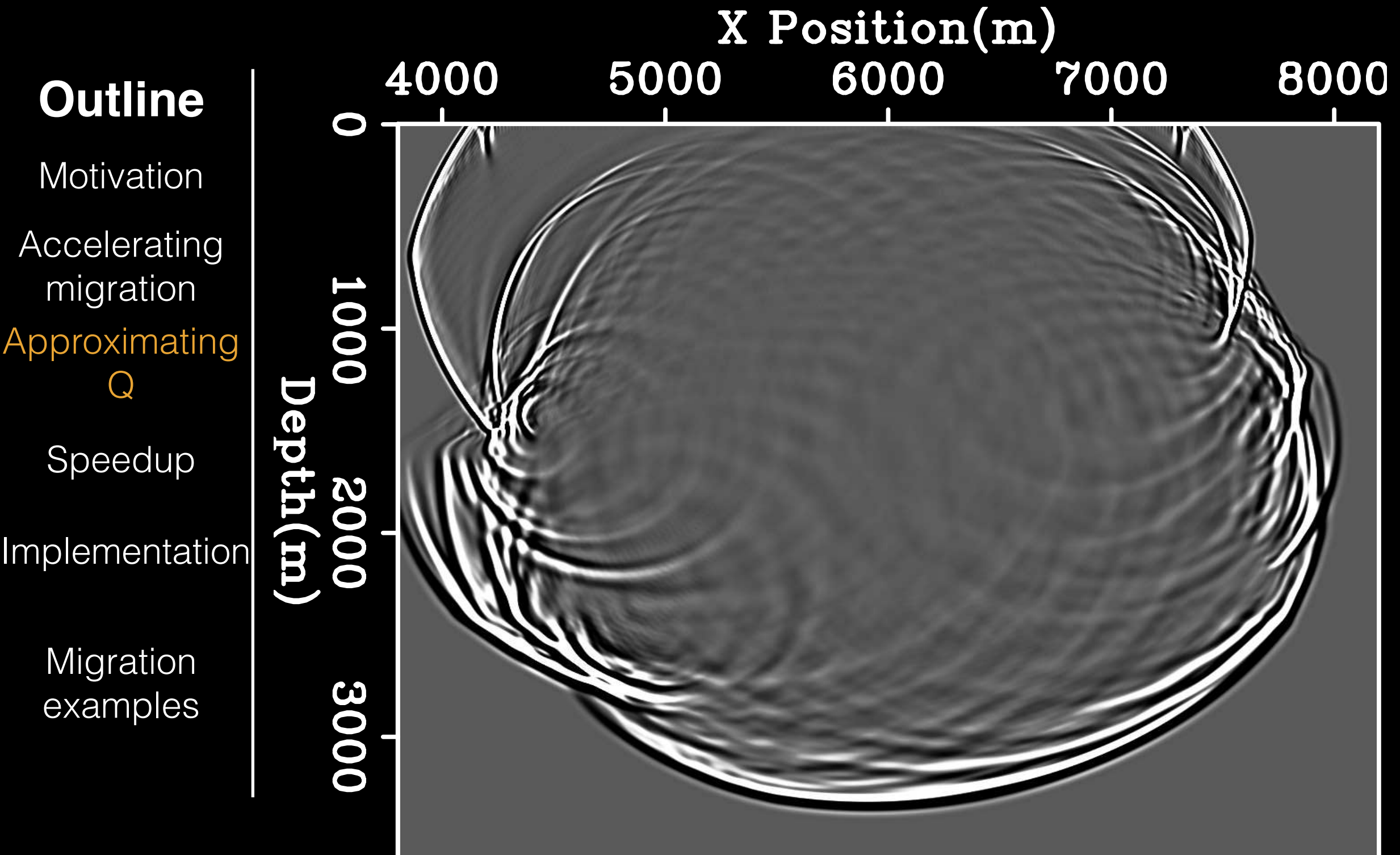
$$(P_s) * (P_d)$$

**s,d** - Source and data

**P** - Propagation

- Standard approach
  - Higher frequencies don't exist because absent from data at larger times
- Approximate Q approach
  - Higher frequencies don't exist, absent from source and data

# Standard propagation



# Q with changing grid

## Outline

Motivation

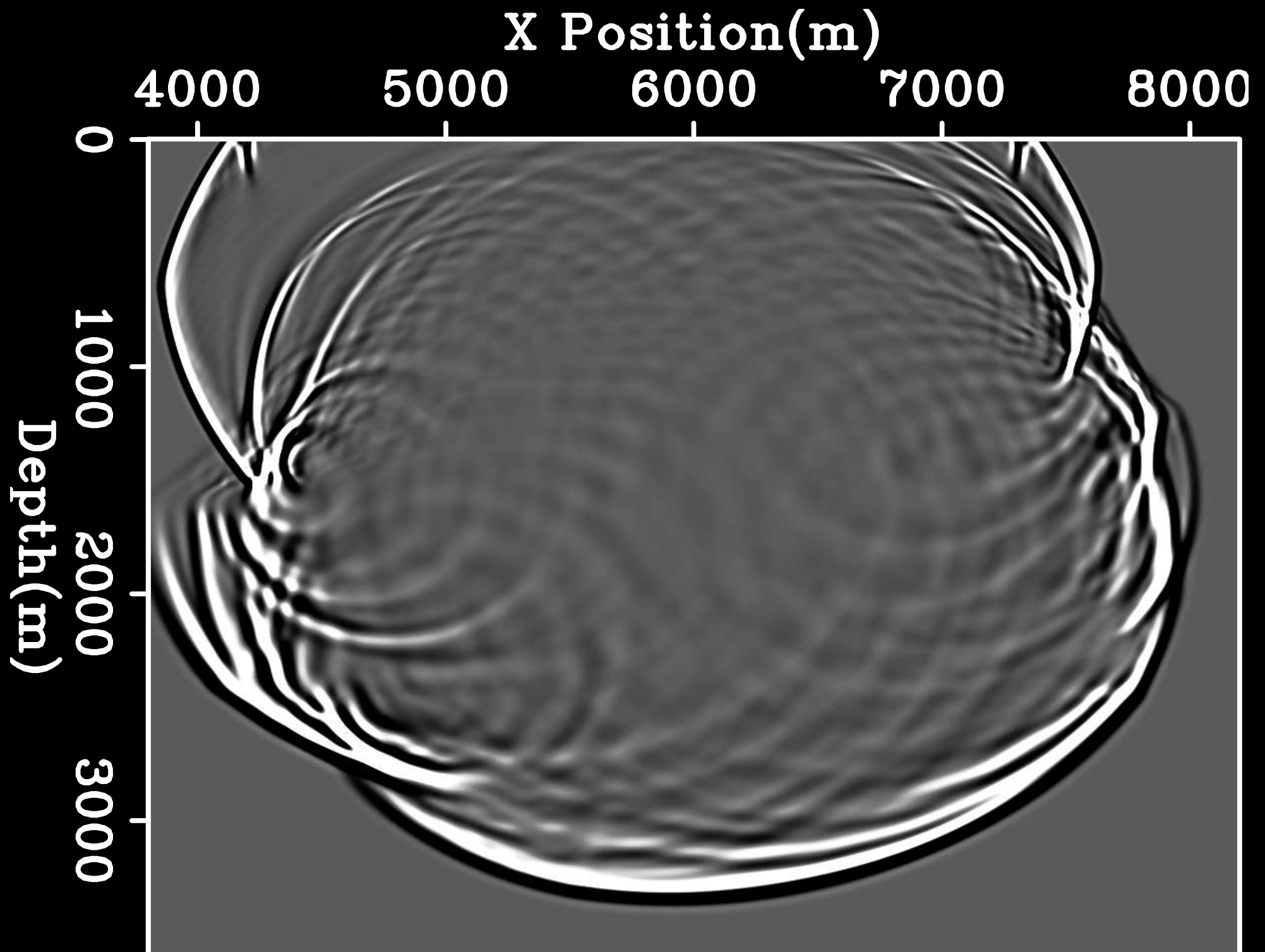
Accelerating  
migration

Approximating  
Q

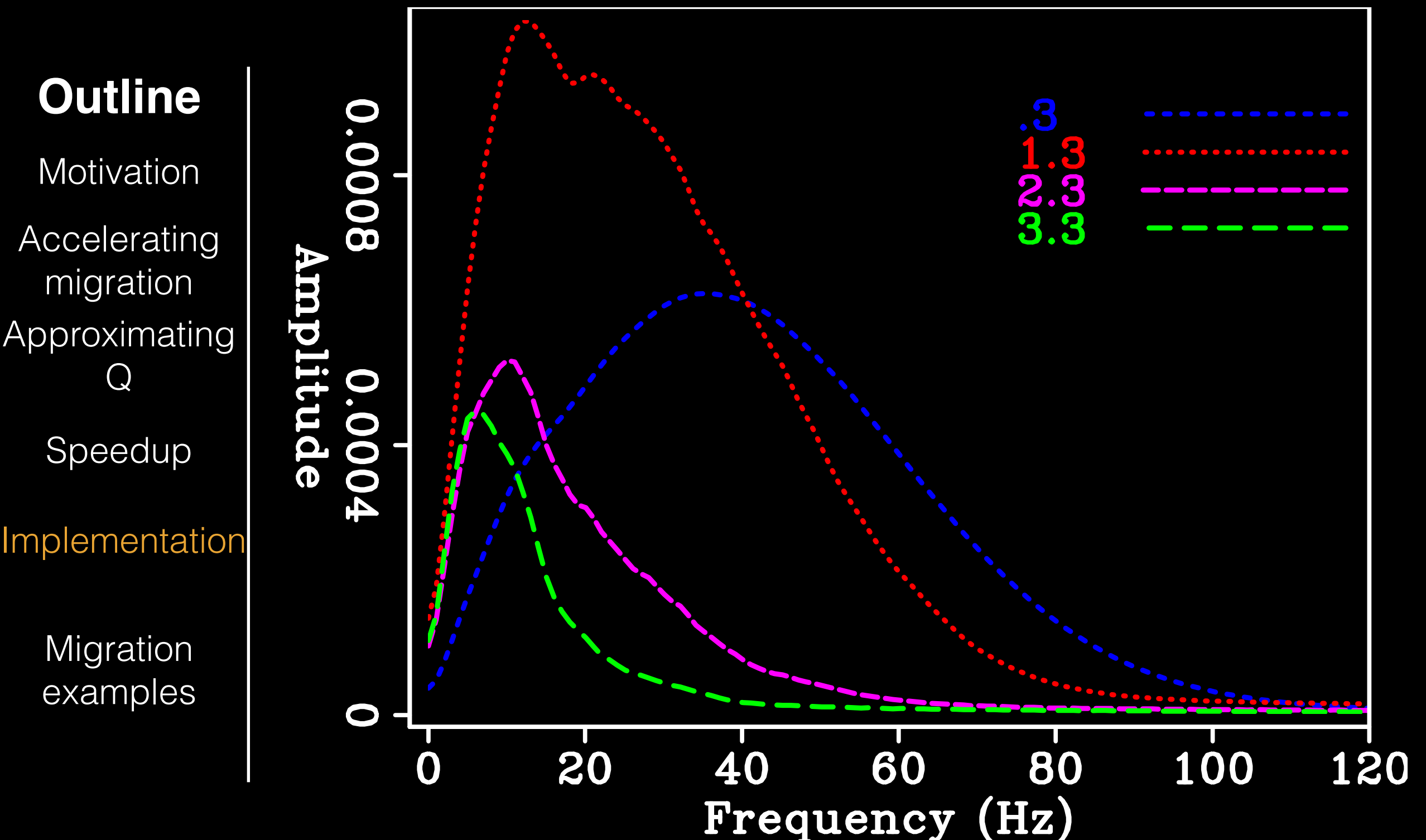
Speedup

Implementation

Migration  
examples



# Frequency change over time





# Speedup vs Q

## Outline

Motivation

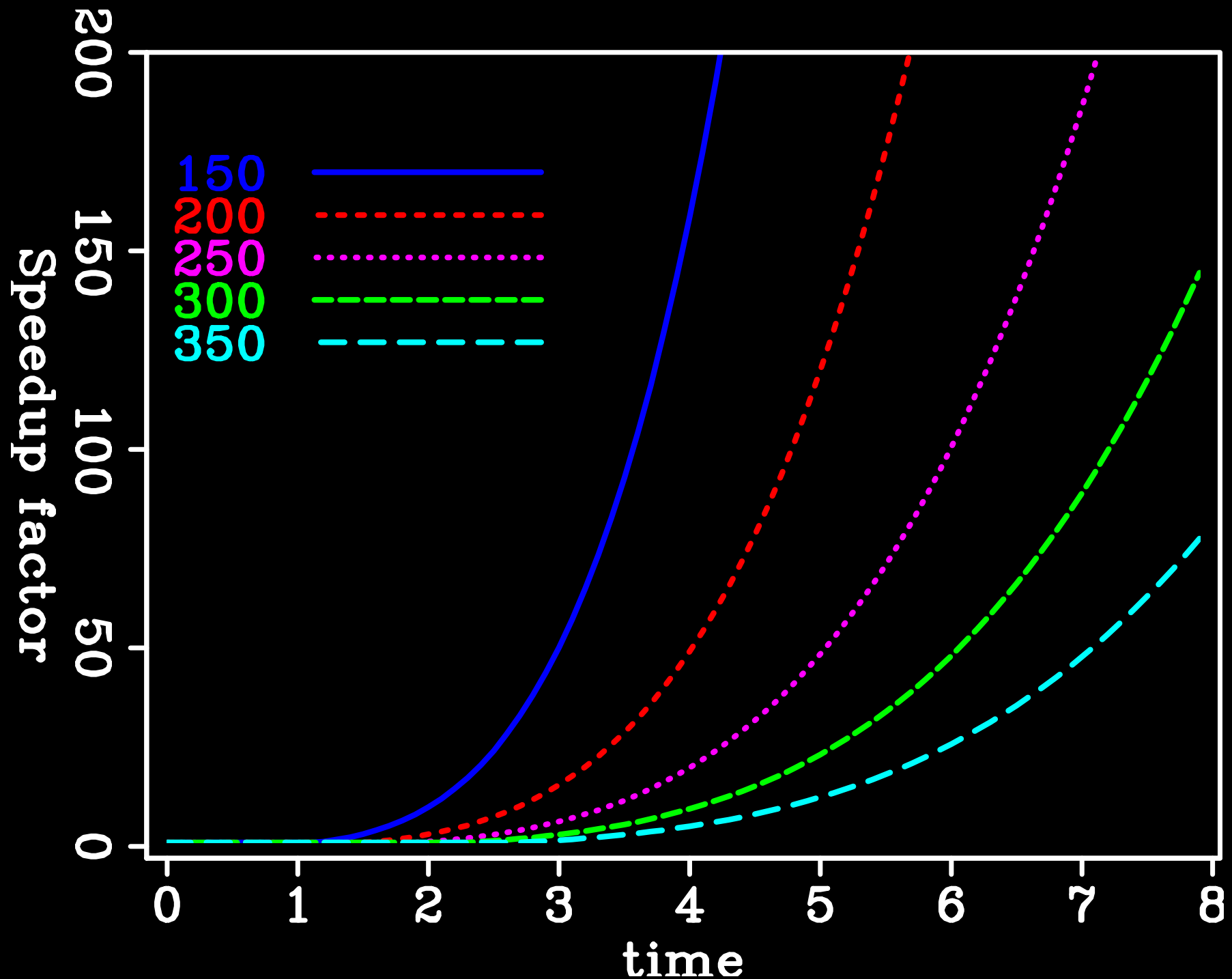
Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples



# Following wave-field

## Outline

Motivation

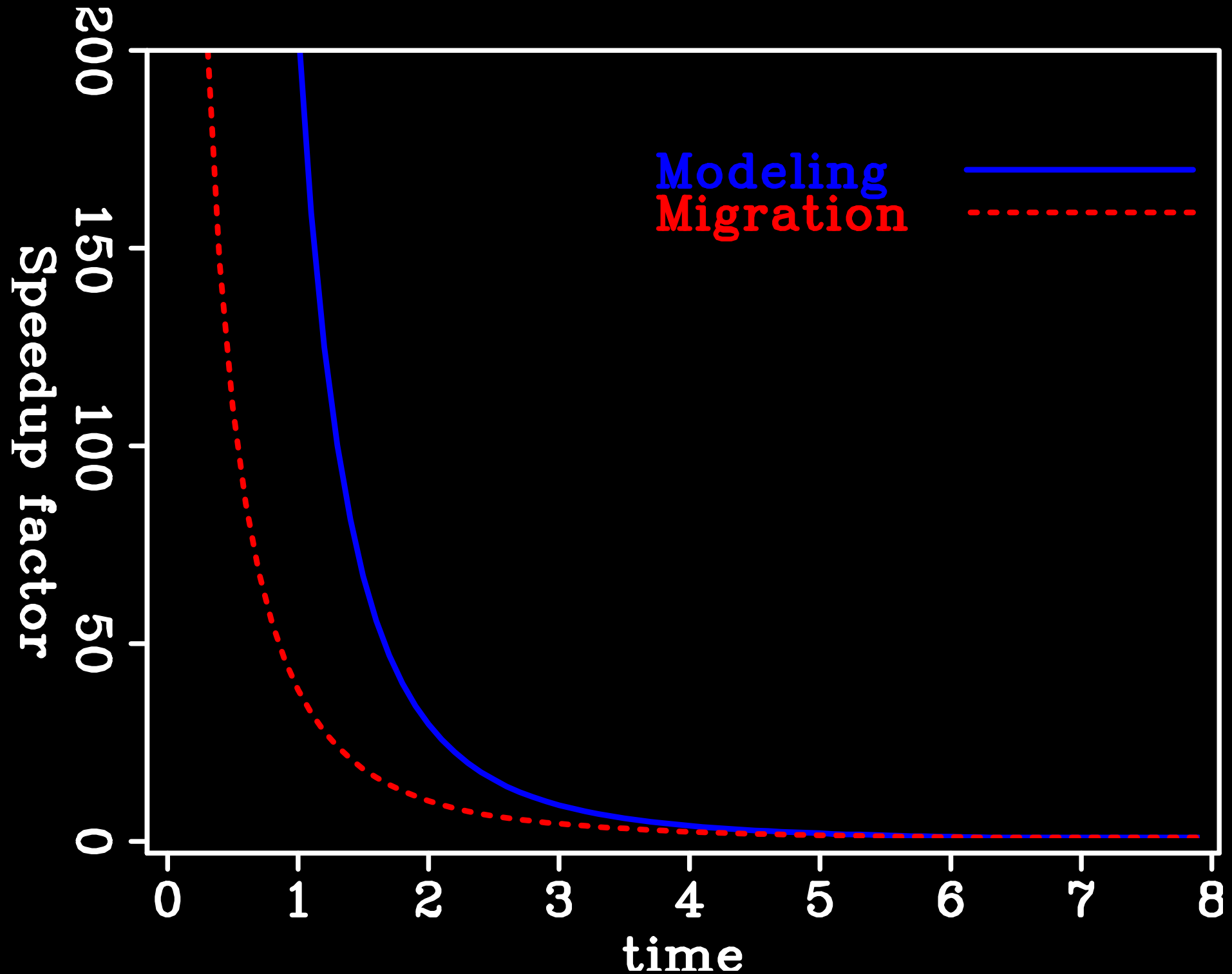
Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples



# Speedup over time

## Outline

Motivation

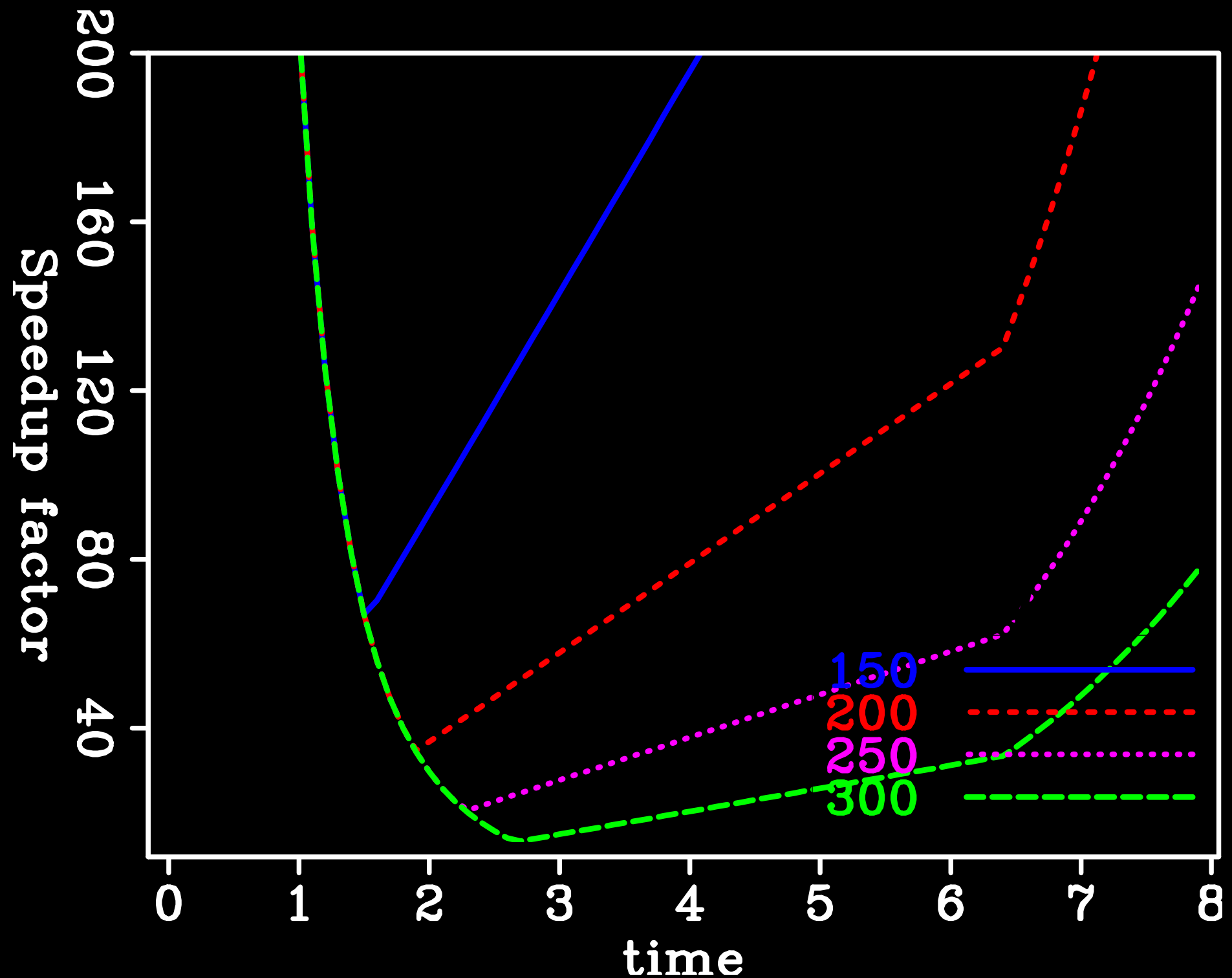
Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples



# Total speedup

## Outline

Motivation

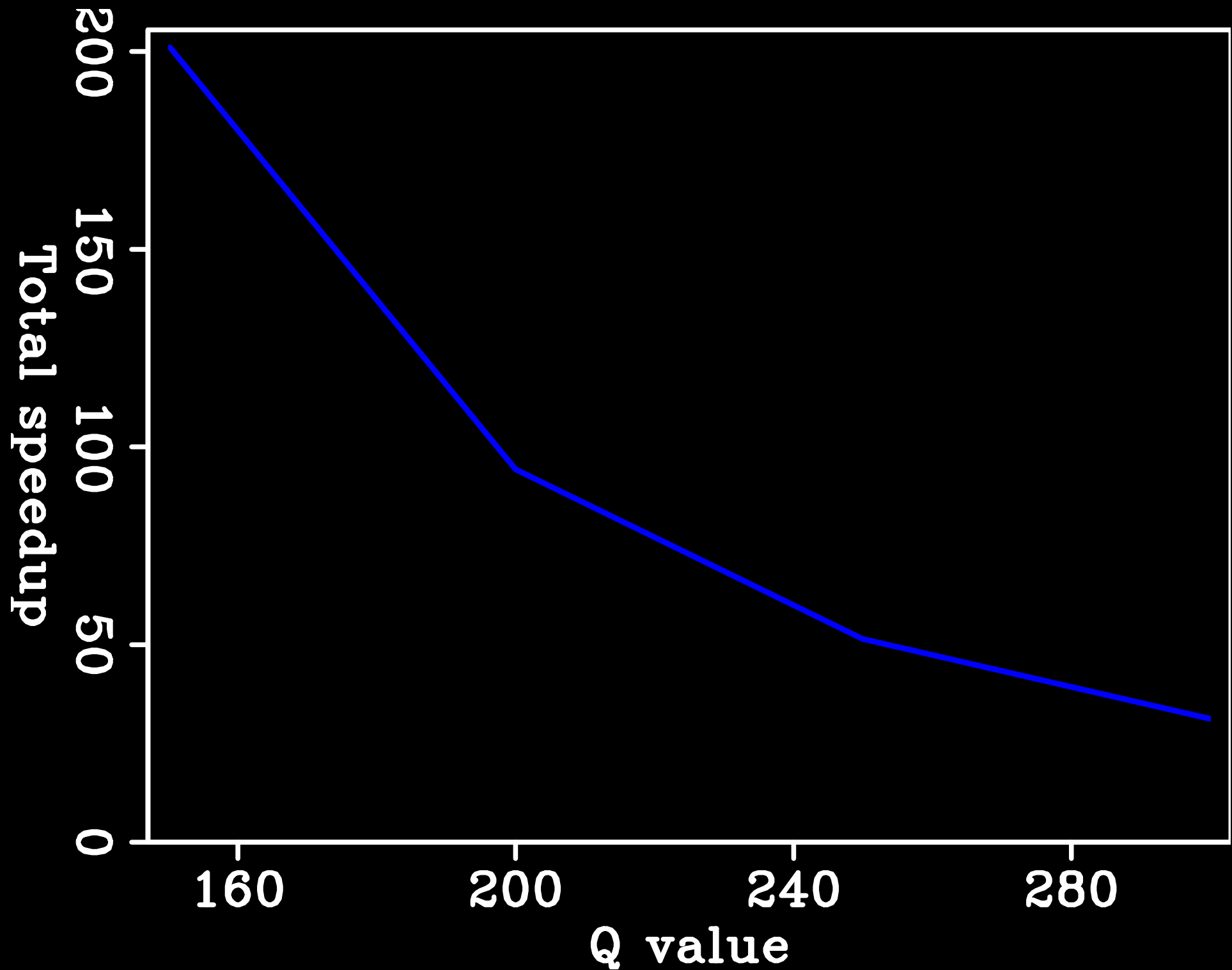
Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples



# Modified RTM algorithm

## Outline

Motivation

Accelerating  
migration

Approximating  
Q

Speedup

## Implementation

Migration  
examples

Pre-calculation

For each source:

Source specific initialization

Forward propagate source

Back propagate source

# RTM: Pre-calculation

## Outline

Motivation

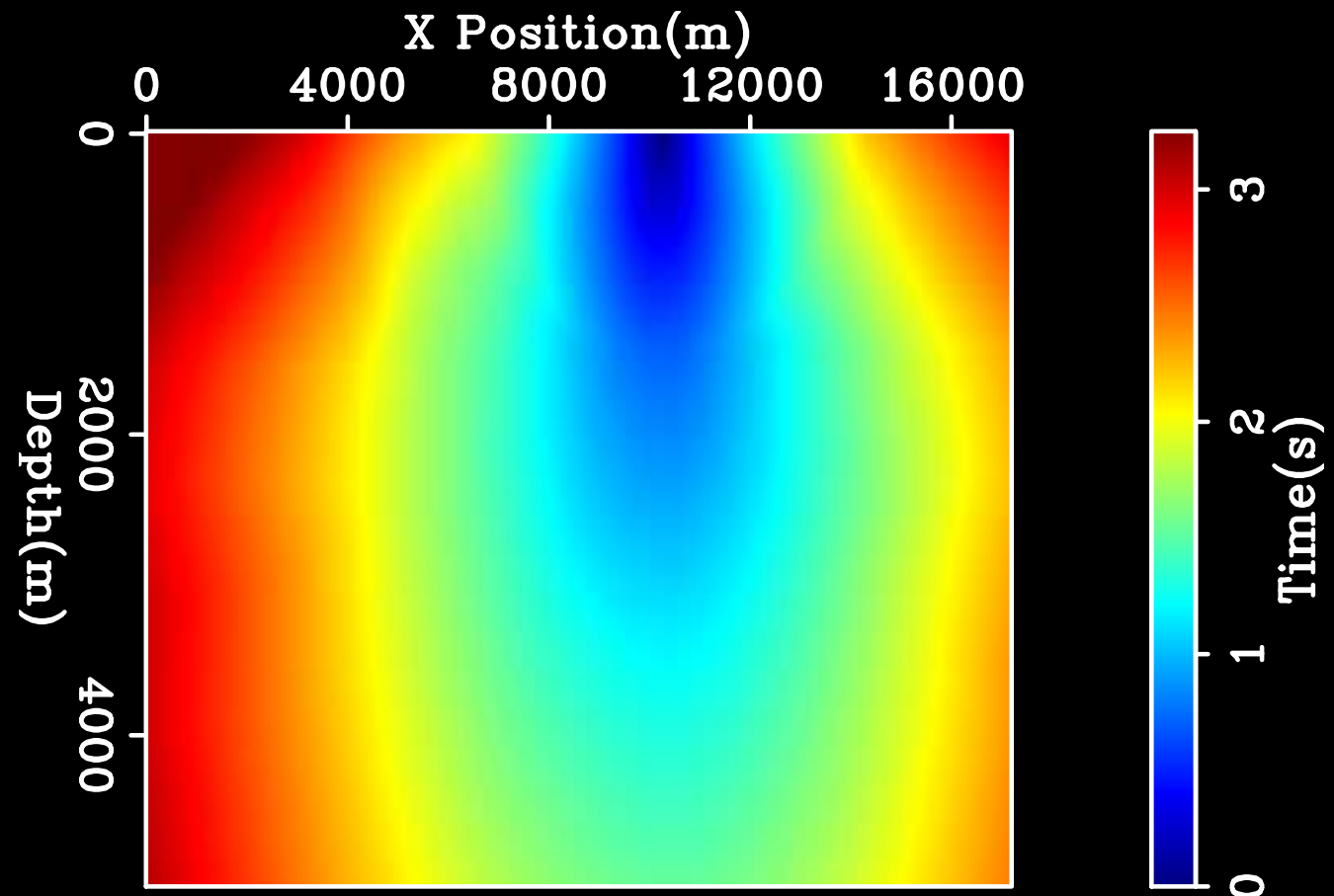
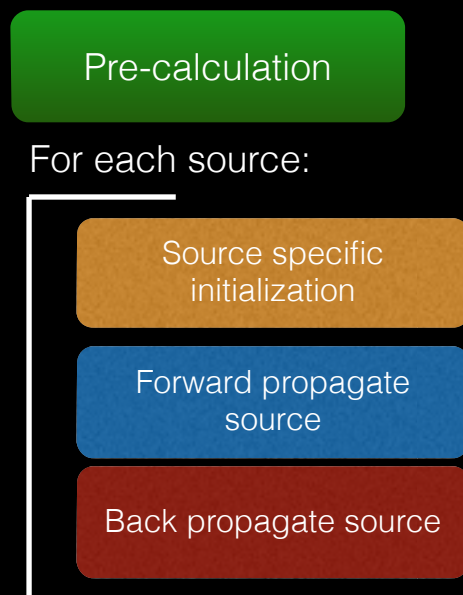
Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples



Calculate first arrival travel time  
maps from every source/  
receiver position to every  
model point

# RTM: Source initialization

## Outline

Motivation

Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples

Pre-calculation

For each source:

Source specific  
initialization

Forward propagate  
source

Back propagate source

- Break time axis into block
- For each time block
  - Find max propagation location for end of time block
  - Find max frequency which hasn't decayed below noise level
  - Calculate dt, damp based on max freq, min/max vel

# RTM: Forward propagate source

## Outline

Motivation

Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples

Pre-calculation

For each source:

Source specific  
initialization

Forward propagate  
source

Back propagate source

- For each time block
  - Resample wave-fields, velocity to current sampling
- Loop over time in block
  - Forward wave-fields using attenuated wave equation
  - Store source wave-field at imaging times



# RTM: Backward propagate receiver

## Outline

Motivation

Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples

- For each time block
  - Resample wave-fields, velocity, image to current sampling
- Loop over time in block
  - Backward propagate receiver using standard wave equation
  - Apply imaging condition using stored source wave-field and current receiver wave-field

Pre-calculation

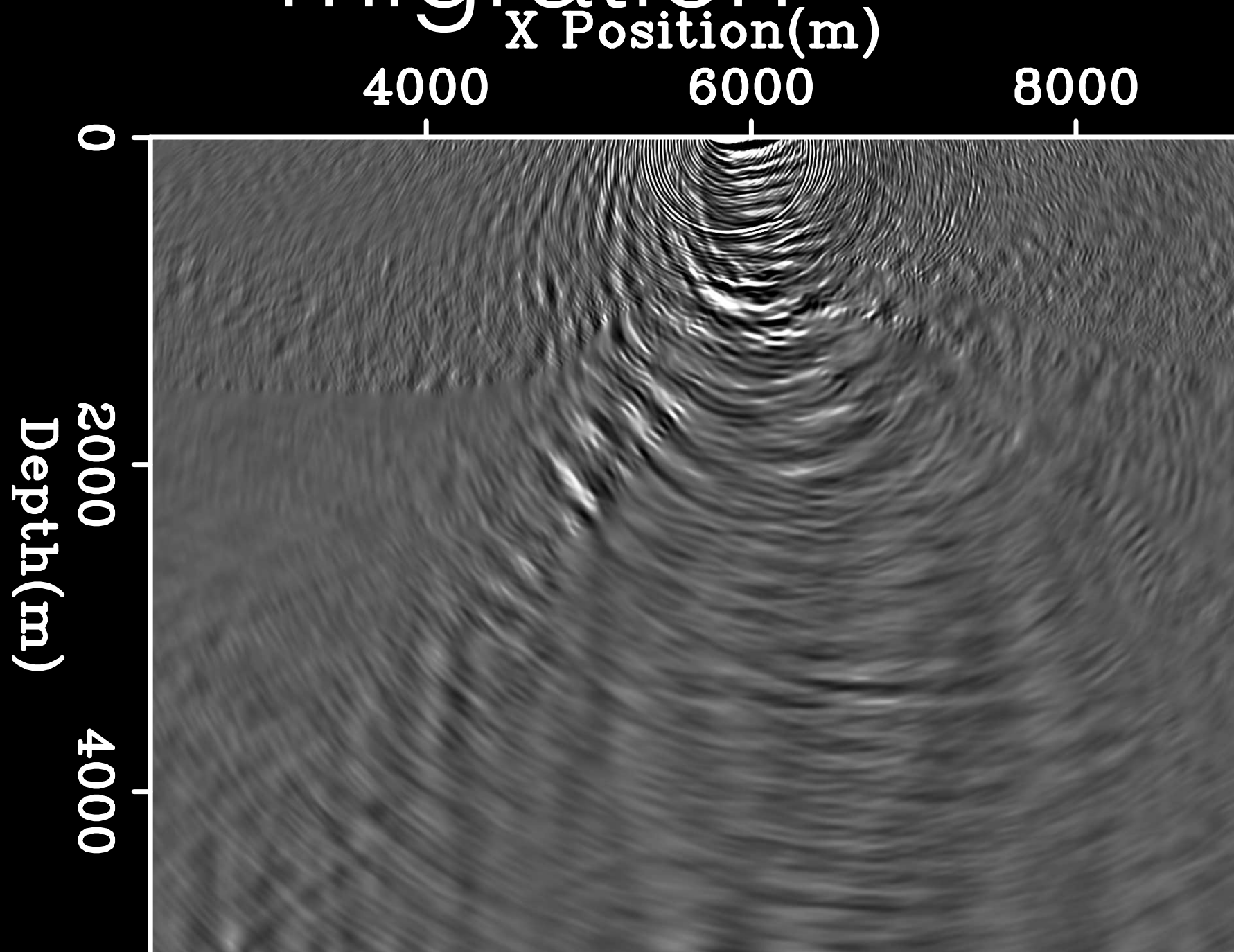
For each source:

Source specific  
initialization

Forward propagate  
source

Back propagate source

# Standard single shot migration



## Outline

Motivation

Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples

# Variable single shot migration

## Outline

Motivation

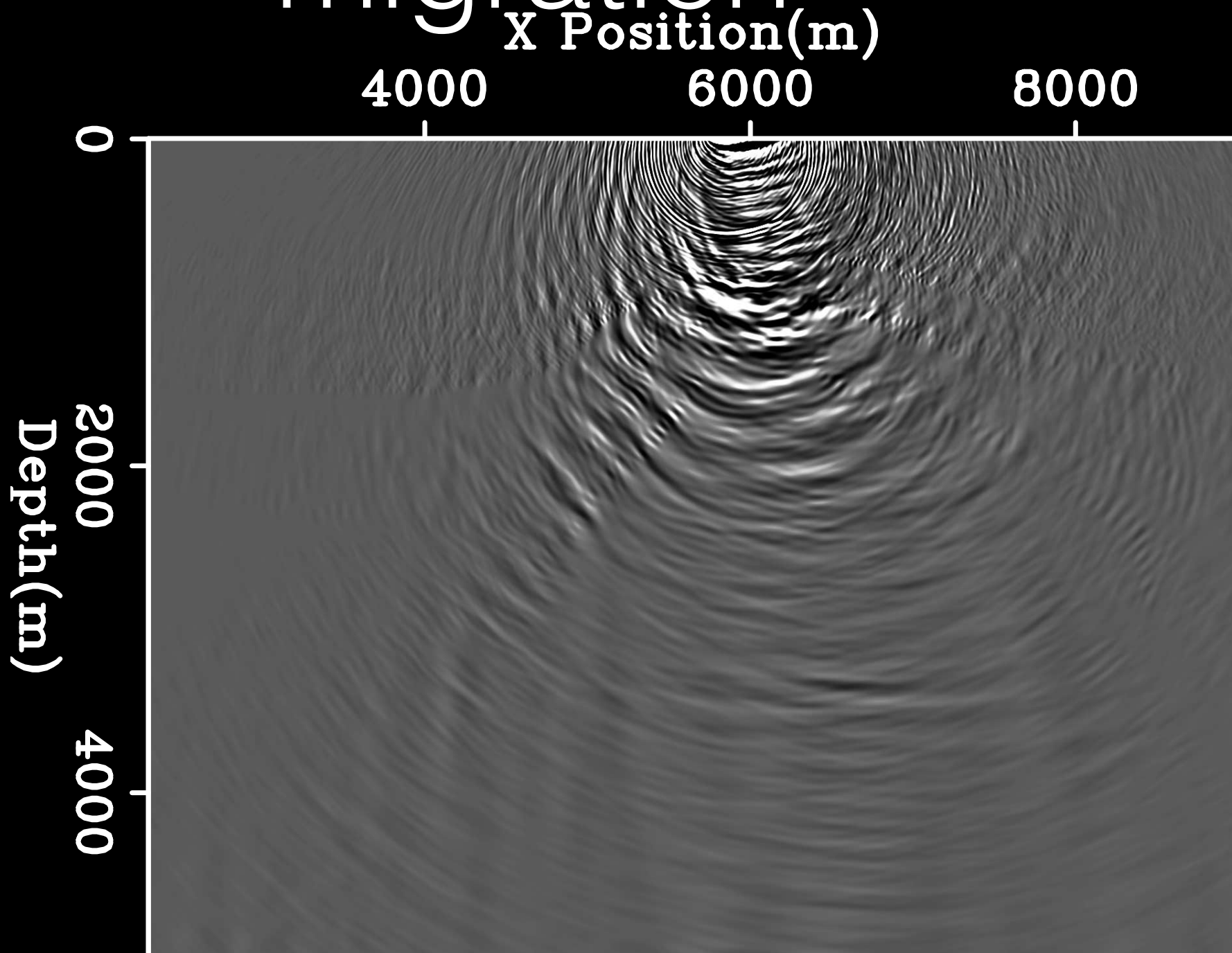
Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples





## Outline

Motivation

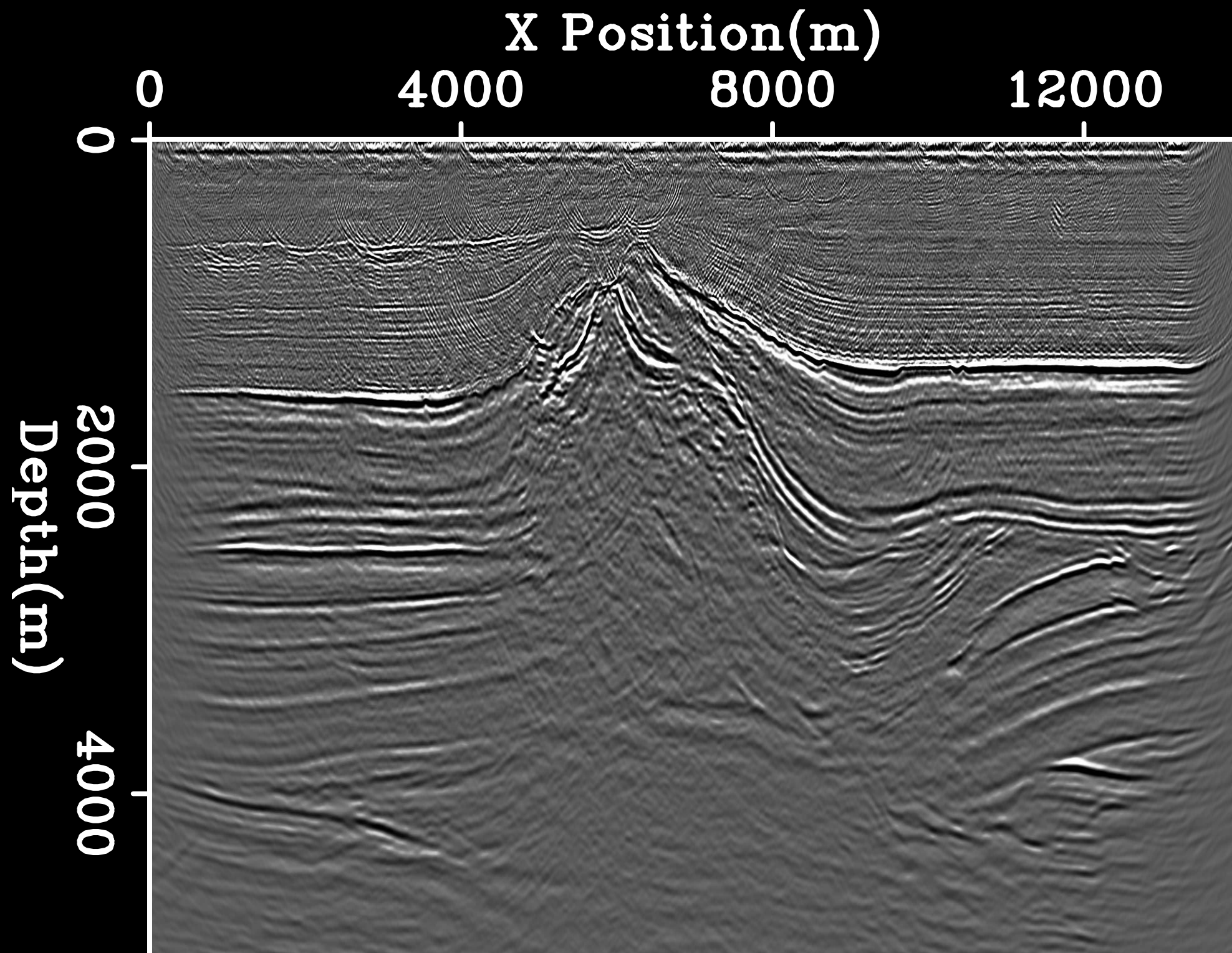
Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples



## Outline

Motivation

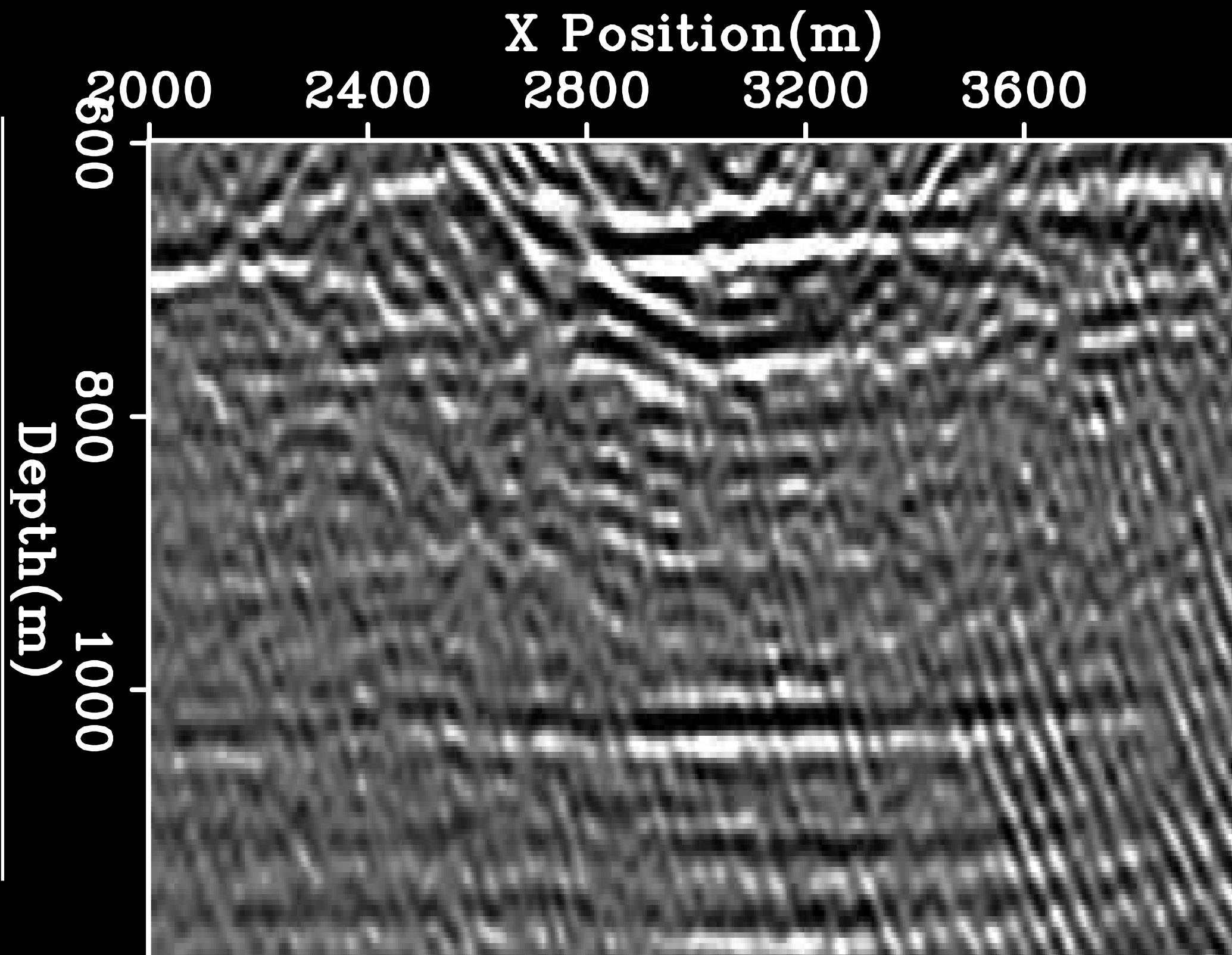
Accelerating  
migration

Approximating  
 $Q$

Speedup

Implementation

Migration  
examples



# Conclusions

## Outline

Motivation

Accelerating  
migration

Approximating  
Q

Speedup

Implementation

Migration  
examples

- Constant Q can be cheaply approximated into a standard time/space domain propagation
- Approach allows for larger grid cells at later times
- Combined with following the wave-field it can lead to significant speedups
- ***Messing with the wavelet***