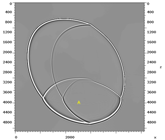
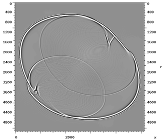
Pseudo-acoustic modeling for tilted anisotropy with pseudo-source injection

Computationally efficient modeling for anisotropic media

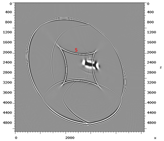


Musa Maharramov and Stewart Levin

Stanford Exploration Project



May 19, 2015



Existing methods and motivation

- Computationally efficient pseudo-acoustic (no shear waves) modeling is of significant value for applications (Fowler et al, 2010)
- FD methods are fast but suffer from *pseudo-shear* artifacts (Alkhalifah, 2000; Zhang and Zhang, 2008; Fletcher et al., 2009)
- Artifact-free spectral methods require multiple FFTs per time step (Etgen and Brandsberg-Dahl, 2009)
- Innovation: a fast FD method with a significant reduction of pseudo-shear artifacts

Convert the directional pressure velocity curve $V(\theta)$ (Tsvankin 2001) into a pseudo-differential equation

$$\frac{V^2(\theta)}{V_P^2} = 1 + \epsilon \sin^2\theta - \frac{1}{2} \pm \frac{1}{2}\sqrt{\left(1 + 2\epsilon\sin^2\theta\right)^2 - 2(\epsilon - \delta)\sin^2 2\theta},$$
$$\sin\theta = \frac{V(\theta)\left[\frac{\partial}{\partial x}\right]}{\left[\frac{\partial}{\partial t}\right]}, \quad \cos\theta = \frac{V(\theta)\left[\frac{\partial}{\partial z}\right]}{\left[\frac{\partial}{\partial t}\right]} \qquad \text{(VEL)}$$

and solve it using a spectral method plus interpolation.

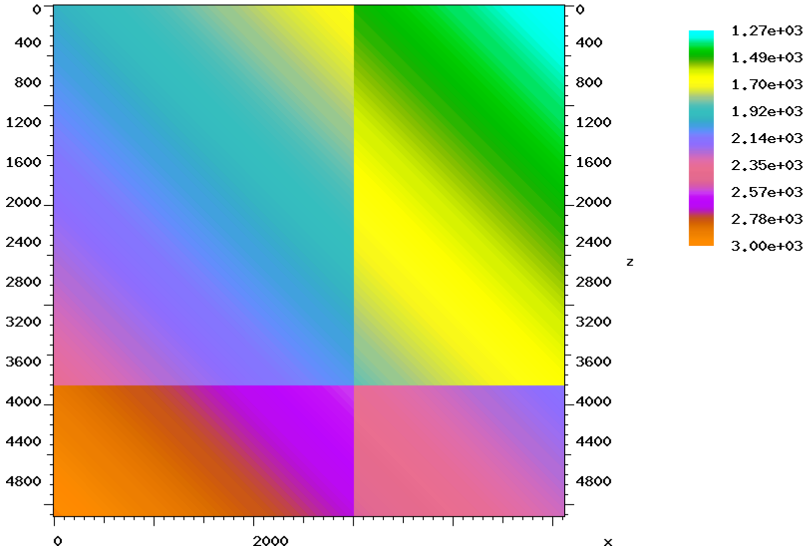
Evolutionary pseudo-differential operator

Convert the directional pressure velocity curve $V(\theta)$ (Tsvankin 2001) into a pseudo-differential equation

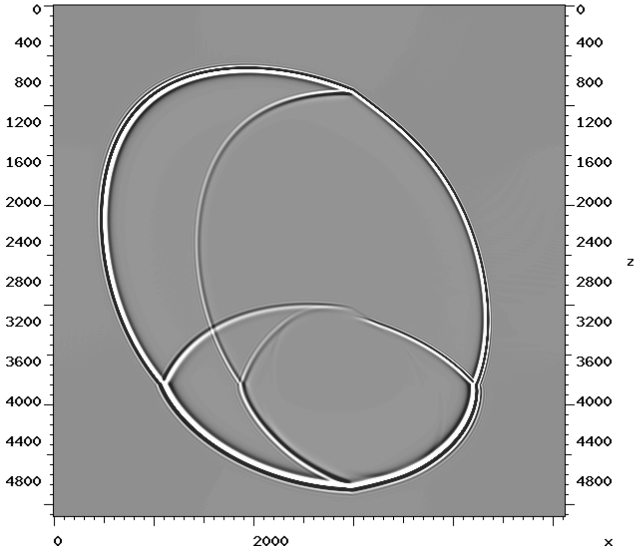
$$\begin{aligned} \frac{V^2(\theta)}{V_P^2} &= 1 + \epsilon \sin^2\theta - \frac{1}{2} \pm \frac{1}{2}\sqrt{\left(1 + 2\epsilon \sin^2\theta\right)^2 - 2(\epsilon - \delta)\sin^2 2\theta}, \\ \sin\theta &= \frac{V(\theta)\left[\frac{\partial}{\partial x}\right]}{\left[\frac{\partial}{\partial t}\right]}, \quad \cos\theta = \frac{V(\theta)\left[\frac{\partial}{\partial z}\right]}{\left[\frac{\partial}{\partial t}\right]} \end{aligned} \tag{VEL}$$

and solve it using a spectral method plus interpolation.

Model 1 (45° homogeneous tilt)



Pseudo-differential operator result (the "**gold standard**")



A finite-difference method

Convert (VEL) into a DO by squaring the square root (Alkhalifah, 2000):

$$\begin{aligned} \frac{\partial^2 q}{\partial t^2} &= V_{P\mathrm{Hor}}^2 \frac{\partial^2 q}{\partial x^2} + V_P^2 \frac{\partial^2 q}{\partial z^2} + V_P^2 \left(V_{P\mathrm{Hor}}^2 - V_{P\mathrm{NMO}}^2 \right) \frac{\partial^4 r}{\partial x^2 \partial z^2}, \\ \frac{\partial^2 r}{\partial t^2} &= q, \end{aligned} \tag{FD}$$

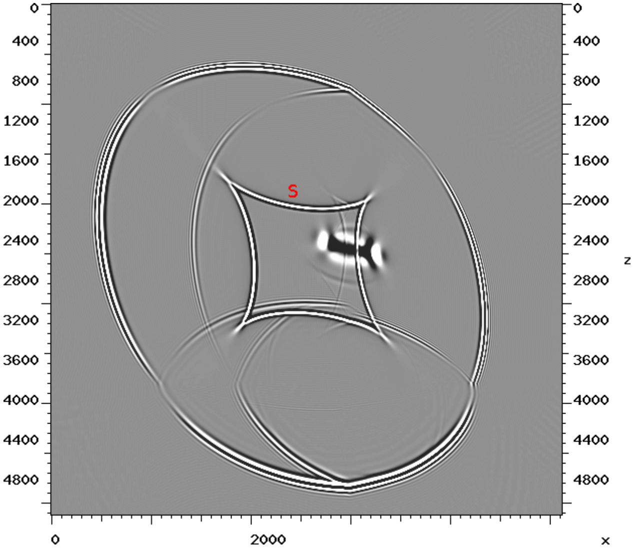
where

$$V_{P\mathrm{Hor}}^2 = V_P^2 \left(1 + 2\epsilon\right), \; V_{P\mathrm{NMO}}^2 = V_P^2 \left(1 + 2\delta\right).$$

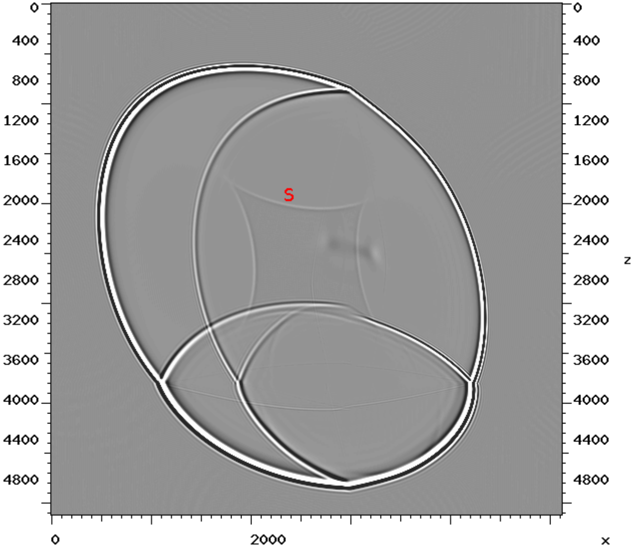
- r corresponds to solution of PDO (VEL)

- In *conventional implementations* source is injected in q

Pseudo-shear artifacts (source injected into r)



Pseudo-shear artifacts (source injected into q)

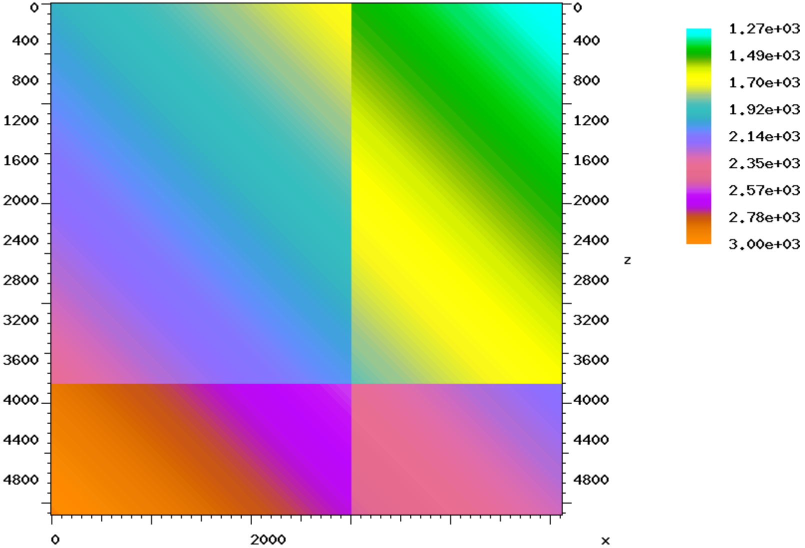


At each time step inject the active source ϕ into r and the *pseudo-source* into q:

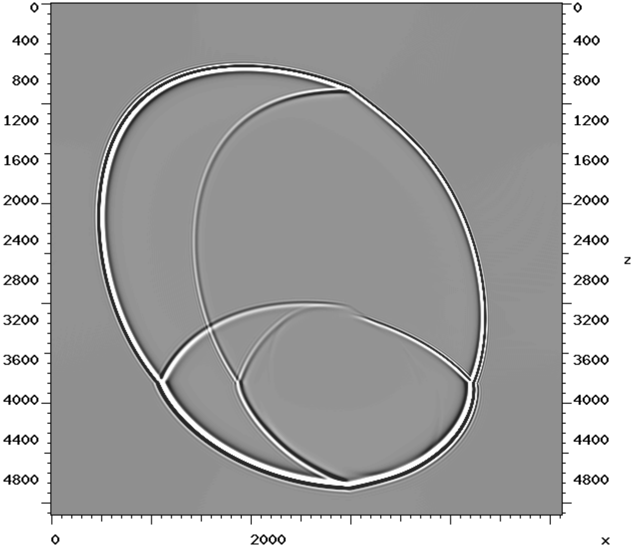
$$
\begin{aligned}
r(z,x,t_n) &\leftarrow r(z,x,t_n) + \phi(z,x,t_n), \\
q(z,x,t_n) &\leftarrow q(z,x,t_n) + V\left(z,x\right)_P^2 \left\{ \frac{\Delta}{2} + \epsilon\left(z,x\right)\frac{\partial^2}{\partial x^2} + \right. \\
&\left. \frac{\Delta}{2}\sqrt{\left[1 + 2\epsilon\left(z,z\right)\frac{\partial^2}{\partial x^2}\frac{1}{\Delta}\right]^2 - 8\left(\epsilon\left(z,x\right) - \delta\left(z,x\right)\right)\frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial z^2}\frac{1}{\Delta^2}} \right\} \phi,
\end{aligned}
\tag{INJECT}
$$

Apply the finite-difference time step of (FD).

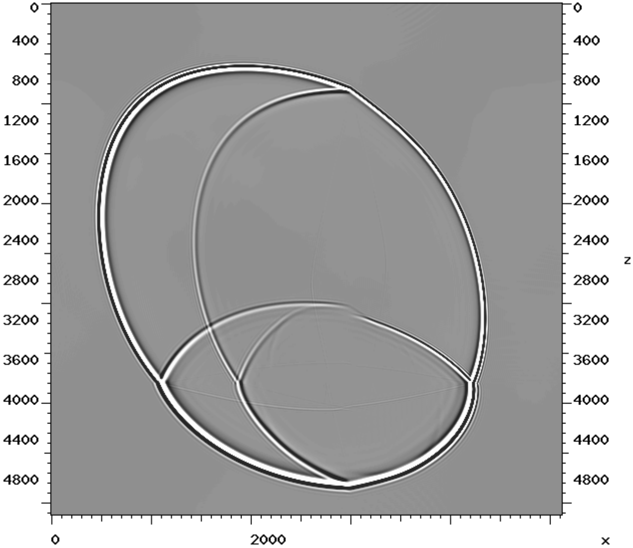
Model 1 (45° homogeneous tilt)



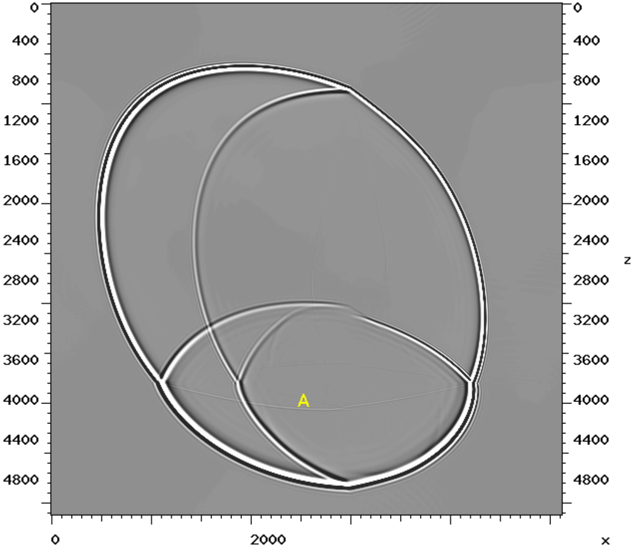
Pseudo-differential operator



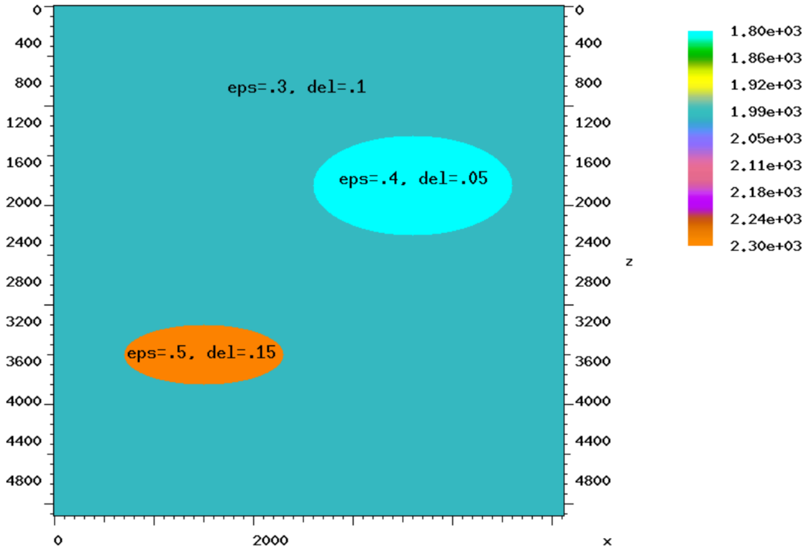
Finite-difference with pseudo-sources



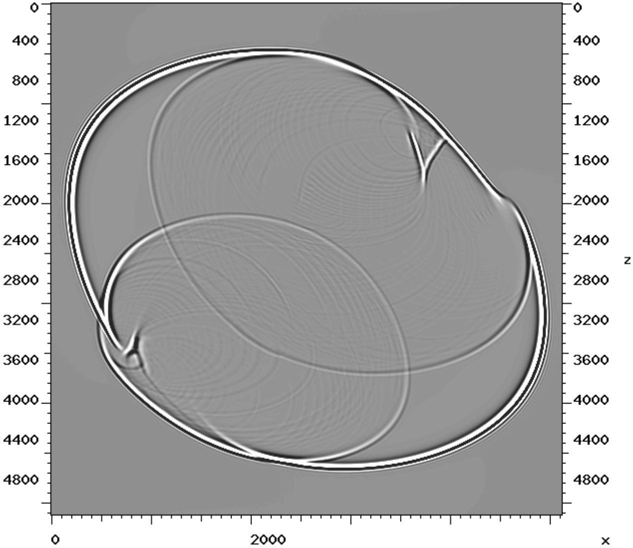
Finite-difference with pseudo-sources



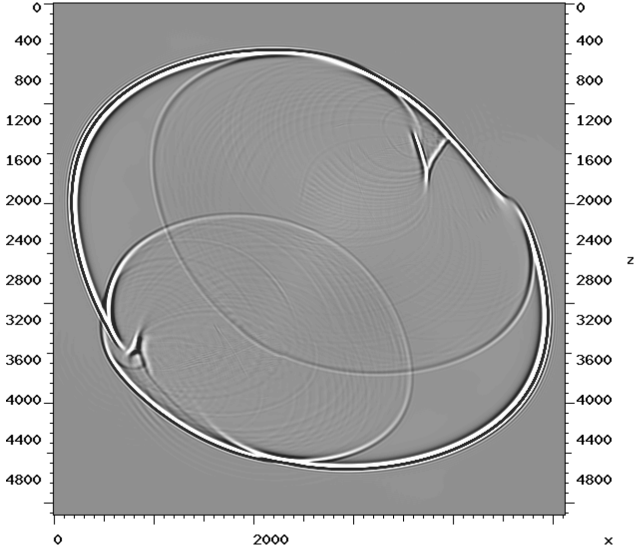
Model 2 ($35^\circ, 25^\circ$ tilt in the ovals, and 30° in the background)



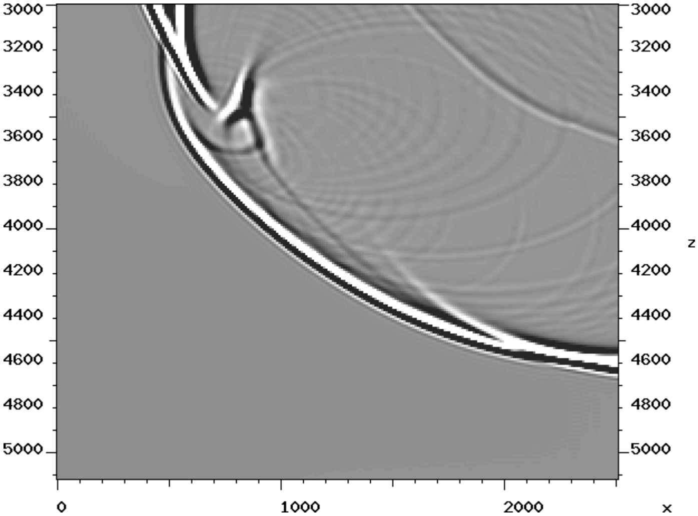
Pseudo-differential operator



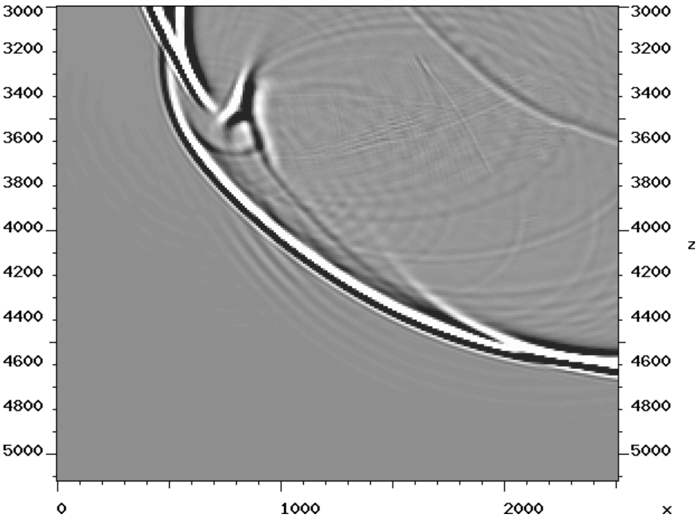
Finite-difference with pseudo-sources



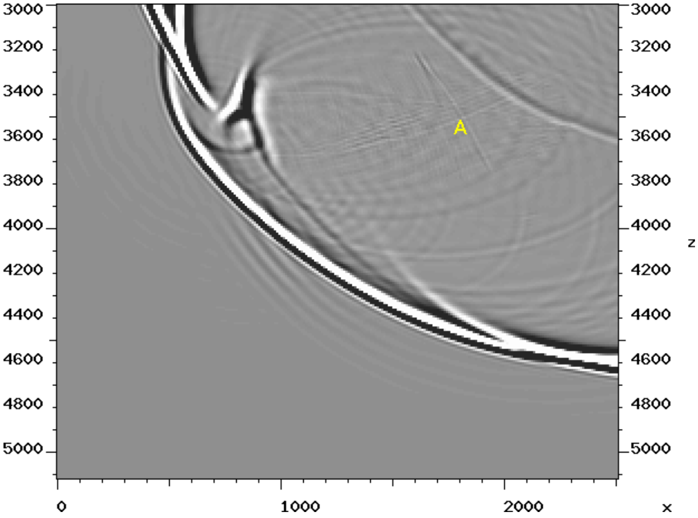
Pseudo-differential operator – θ, ϵ, δ contrasts



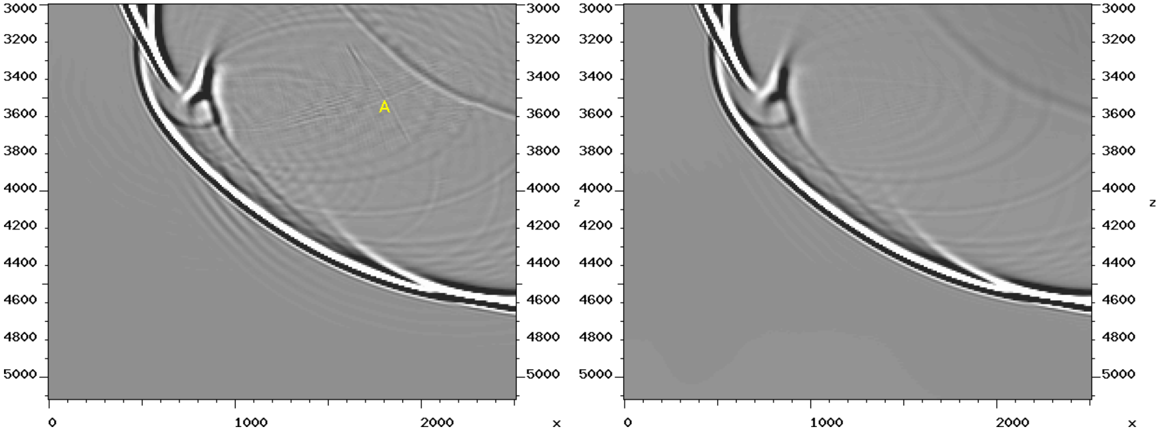
Finite-difference – θ, ϵ, δ contrasts



Finite-difference – θ, ϵ, δ contrasts



Non-smoothed vs smoothed



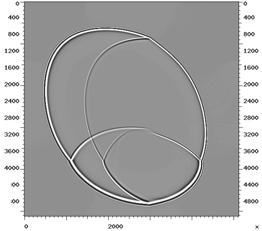
Weak high-frequency artifacts appear where FD approximation has largest errors.

Conclusions and perspectives

- Computationally cheap FD technique with artifacts suppressed by injecting a "pseudo-source"
- Savings come from spatial boundedness of pseudo-source support
- Achieves a significant reduction of shear artifacts
- Can be used with a wide range of PA systems (Fowler et al., 2010)
- Next steps: apply to energy-conserving tilted anisotropy systems (Zhang and Zhang, 2009; Macesanu, 2011)
- Combine with Lax-Wendroff to reduce dispersion

Acknowledgements

The authors thank **Biondo Biondi** for a number of useful discussions, and the **affiliate members** of Stanford Exploration Project for their support.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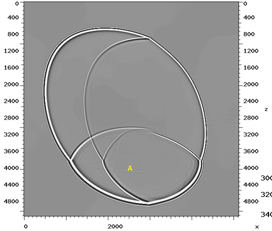
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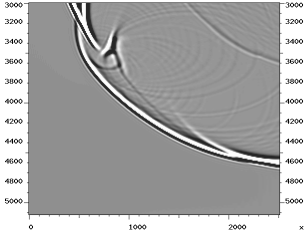
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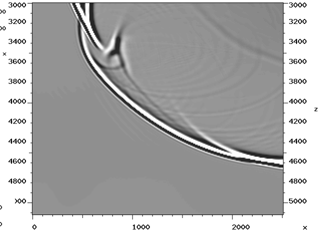
Q&A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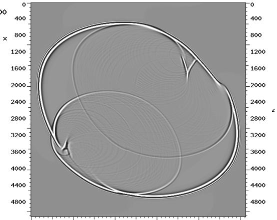
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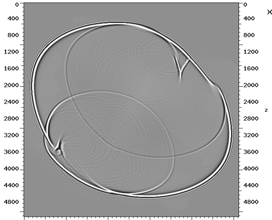
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Appendices – discussion slides

Evolutionary pseudo-differential operator

The full pseudo-differential operator equation (VEL) can be approximated with a trigonometric polynomial:

$$V^2(\theta) \approx V_P^2 \sum_{n=0}^{N} a_n \sin^{2n}(\theta), \tag{1}$$

leading to separable pseudo-differential operators used in practice:

$$\frac{\partial^2}{\partial t^2} = V_P^2 \sum_{n=0}^{N} a_n \frac{\partial^{2n}}{\partial x^{2n}} \Delta^{1-n}, \tag{2}$$

Examples of (1), the *weak anisotropy* approximation:

$$V^2(\theta) \approx V_P^2 \left(1 + \delta \sin^2\theta + \frac{\epsilon - \delta}{1 + 2\delta} \sin^4\theta\right), \tag{3}$$

and Harlan and Lazear relation (Etgen and Brandsberg-Dahl, 2009):

$$V^2(\theta) = V_P^2 \cos^2\theta + \left(V_{P\mathsf{NMO}}^2 - V_{P\mathsf{Hor}}^2\right) \cos^2\theta \sin^2\theta + V_{P\mathsf{Hor}}^2 \sin^2\theta, \tag{4}$$