Anisotropic Full Waveform Inversion

SEP163, p. 155-162

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Objective

- Full waveform inversion (FWI) can retrieve very accurate velocity models.
- **Kinematics** vs amplitude.
- Accurate kinematics requires anisotropy.

Wavefronts in isotropic and anisotropic media

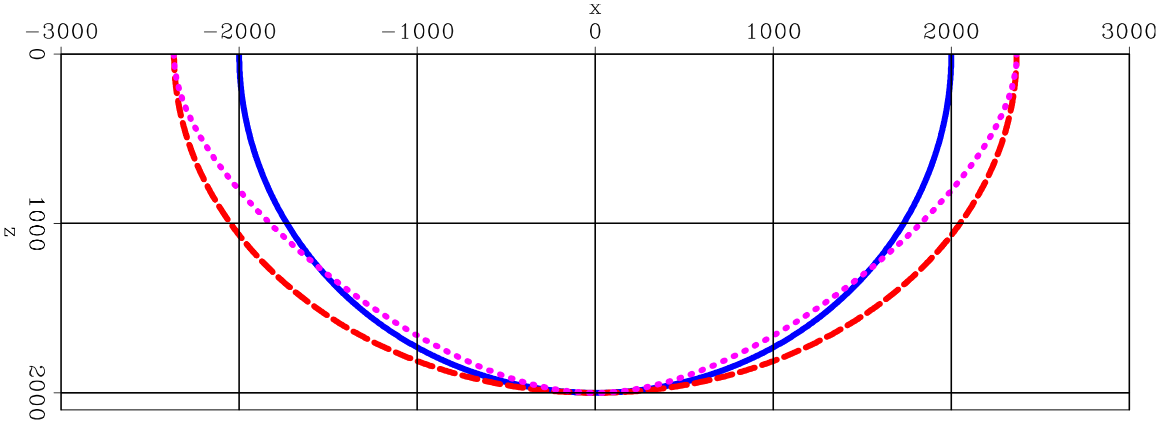

Figure 1: Wavefronts in isotropic (blue) and anisotropic (magenta and red) media (Courtesy of Elita Li).

Problems in anisotropic FWI

- Multiple parameters.
- Multiple parameterizations.
- Sensitivity.
- Crosstalk.

Problems in anisotropic FWI

- Multiple parameters.
- Multiple parameterizations: **stiffnesses** versus **velocity and Thomsen's parameters**.
- **Sensitivity**.
- **Crosstalk**.

VTI pseudo-acoustic wave equations

$$\begin{cases} \partial_t^2 p = c_{11}\partial_x^2 p + c_{13}\partial_z^2 q + f_x, \\ \partial_t^2 q = c_{13}\partial_x^2 p + c_{33}\partial_z^2 q + f_z. \end{cases}$$

- p and q are the normal stresses in the x-direction and z-direction.
- f_i are the sources.
- c_{ij} are the stiffness coefficients.

VTI pseudo-acoustic wave equations

$$
\begin{aligned}
c_{11} &= v_{pz}^2(1+2\epsilon) = v_{px}^2, \\
c_{13} &= v_{pz}^2\sqrt{1+2\delta}, \\
c_{33} &= v_{pz}^2.
\end{aligned}
$$

- v_{pz} is vertical P-velocity.
- v_{px} is horizontal P-velocity.
- ϵ and δ are Thomsen's parameters.

Wavefronts in isotropic and anisotropic media

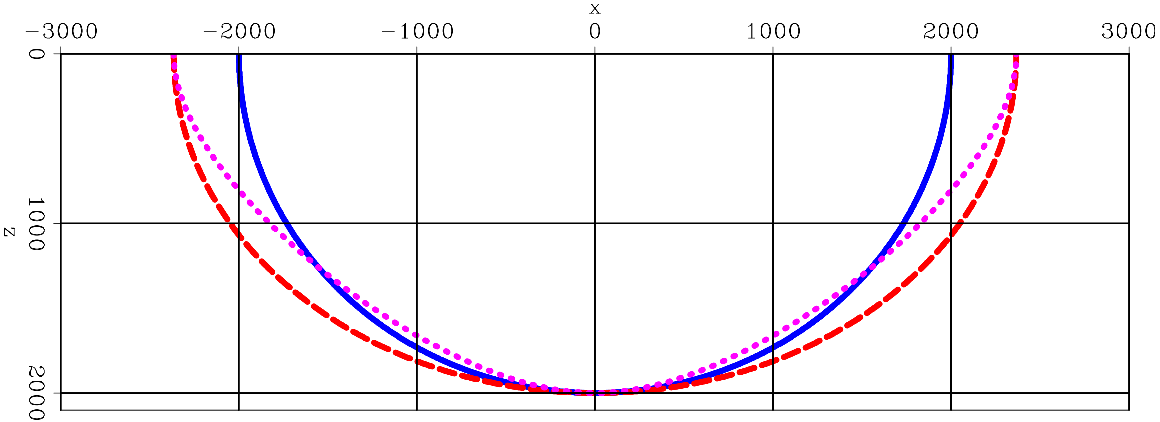

Figure 2: Wavefronts in isotropic (blue) and anisotropic (magenta and red) media (Courtesy of Elita Li).

Objective function

Modeled data:

$$d = \frac{1}{2}(p + q)\delta(x - x_r).$$

Observed data d_0.

Objective function:

$$\chi = \frac{1}{2}\|d - d_0\|_2^2.$$

Example 1

- Stiffness parameterization c_{ij}.
- Shots and receivers every where on surface.
- 40 shots, 200 m spacing; 800 receivers, 10 m spacing.

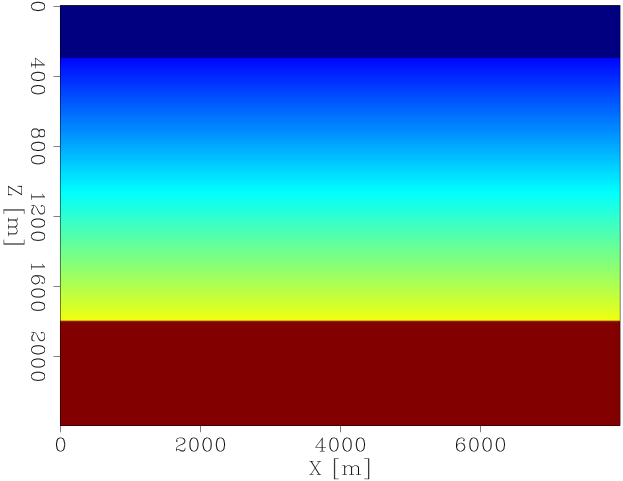

Figure 3: Initial models.

Example 1- True perturbations

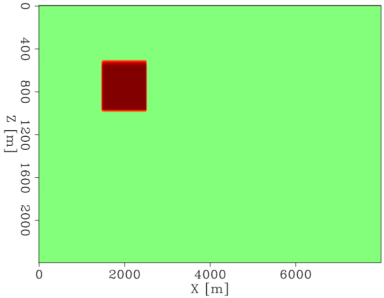

Figure 4: c_{11}.

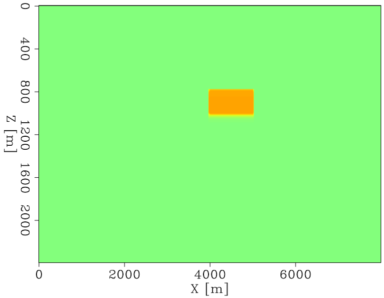

Figure 5: c_{13}.

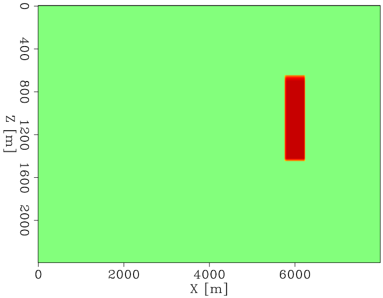

Figure 6: c_{33}.

Example 1 Inverted perturbations

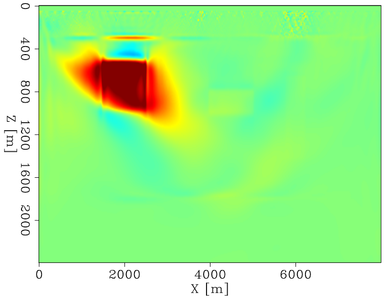

Figure 7: c_{11}.

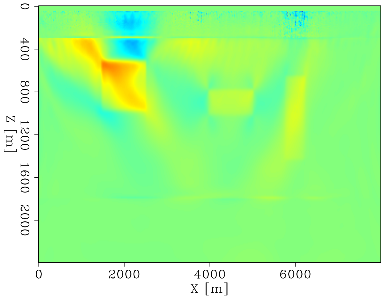

Figure 8: c_{13}.

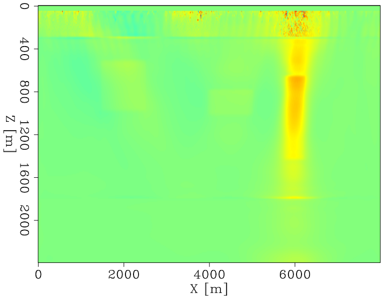

Figure 9: c_{33}.

Example 1- Radiation patterns

- Scattered energy as a function of angle.
- Partial derivatives of wavefields with respect to model parameters.

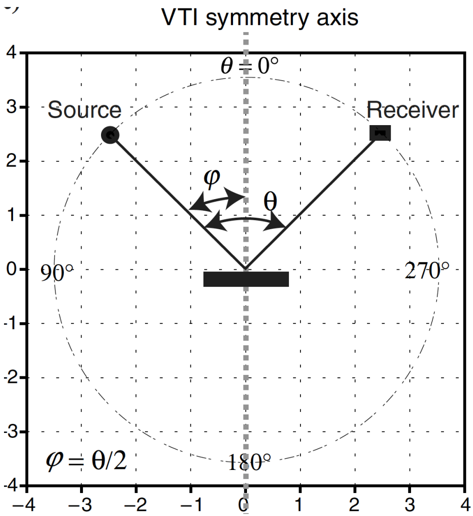

Figure 10: Radiation patterns (Gholami et al., 2013, Geophysics, 78, No. 2, R81-R105).

Example 1- Radiation patterns

- c_{13} is the least resolved.
- Strong crosstalk from c_{11} and c_{33} to c_{13}.
- Very weak crosstalk between c_{11} and c_{33}:

$$c_{11} = v_{px}^2,$$
$$c_{33} = v_{pz}^2.$$

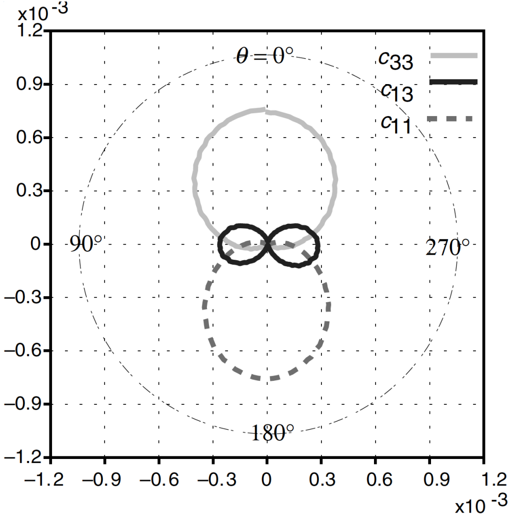

Figure 11: Radiation patterns (Gholami et al., 2013, Geophysics, 78, No. 2, R81-R105).

Example 2

- v_{pz}, ϵ, and δ parameterization.
- Simultaneous inversion inverts for all parameters.
- Fixed-δ inversion inverts for v_{pz} and ϵ.

Example 2- True models

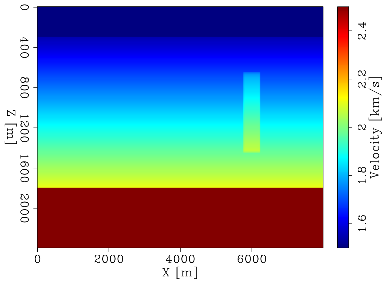

Figure 12: v_{pz}

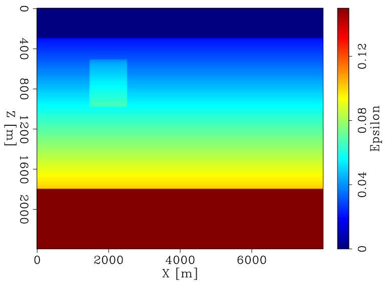

Figure 13: ϵ

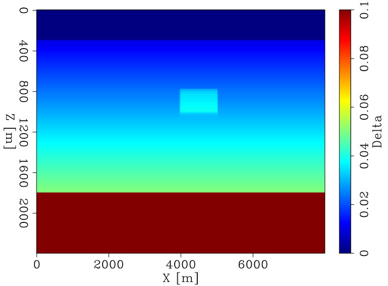

Figure 14: δ

Example 2- Simultaneous inversion

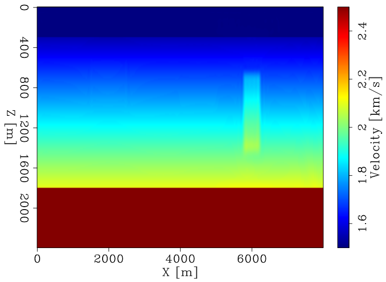

Figure 15: v_{pz}

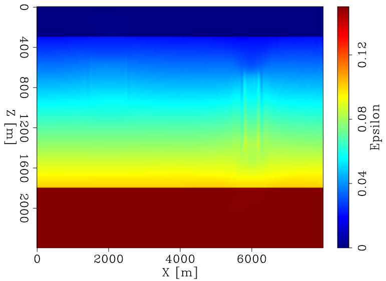

Figure 16: ϵ

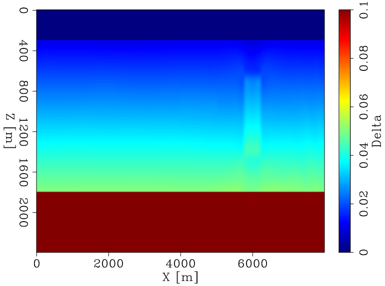

Figure 17: δ

Example 2- Fix-δ inversion

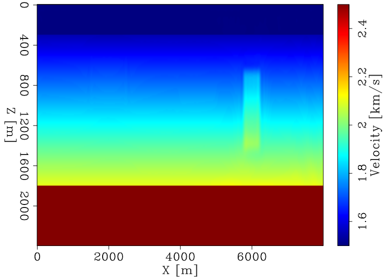

Figure 18: v_{pz}

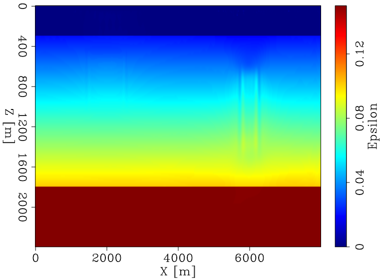

Figure 19: ϵ

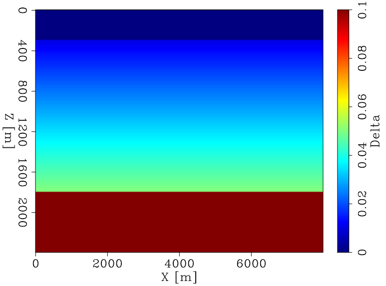

Figure 20: δ

Example 2- Radiation patterns

- v_{pz} is the best resolved and δ the least.
- Strong crosstalk from v_{pz} to ϵ and δ.

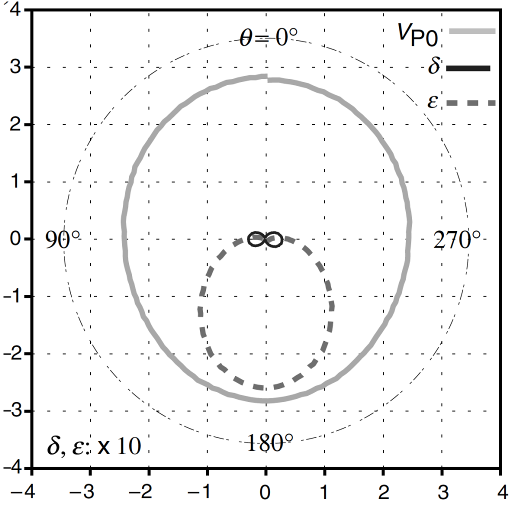

Figure 21: Radiation patterns (Gholami et al., 2013, Geophysics, 78, No. 2, R81-R105).

Question

Can the Hessian reduce crosstalk?

Newton search direction

$$\mathbf{p} = -\mathbf{H}^{-1}\mathbf{g},$$

- $\mathbf{p}$: Newton search direction.
- $\mathbf{H}$: Hessian.
- $\mathbf{g}$: gradient.

Example 3- True perturbations

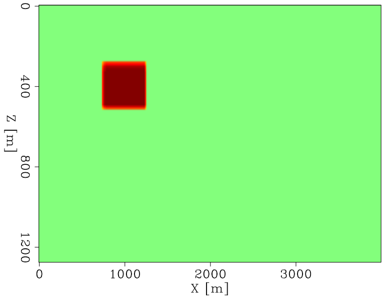

Figure 22: c_{11}

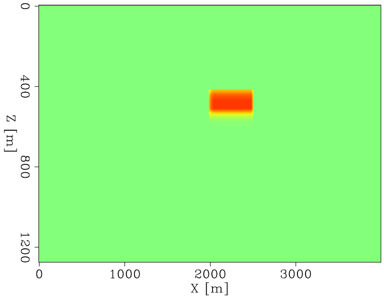

Figure 23: c_{13}

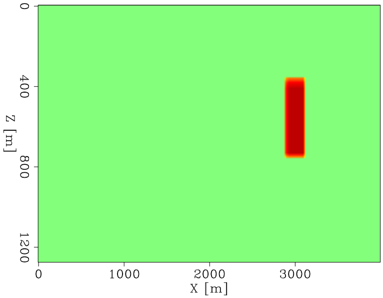

Figure 24: c_{33}

Example 3- Steepest decent direction

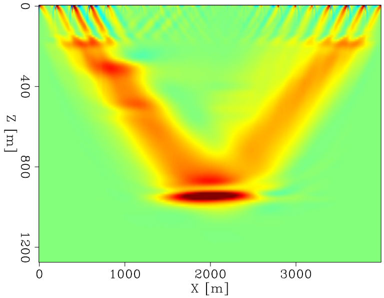

Figure 25: c_{11}

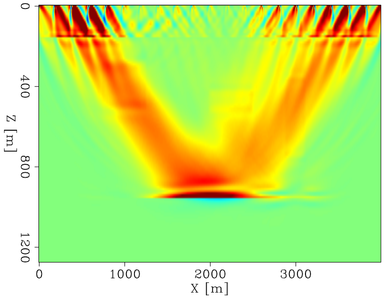

Figure 26: c_{13}

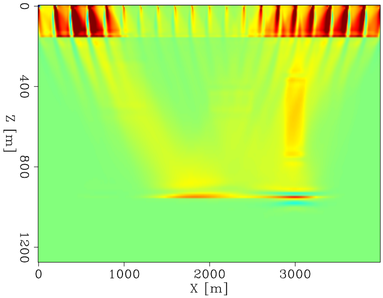

Figure 27: c_{33}

Example 3- Gauss-Newton search direction

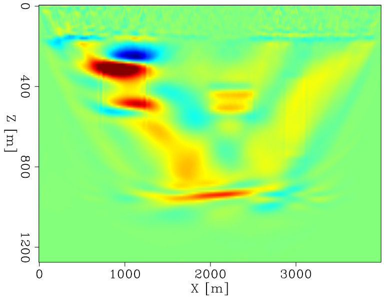

Figure 28: c_{11}

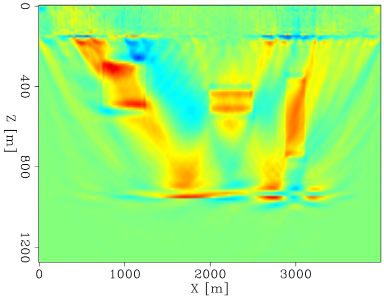

Figure 29: c_{13}

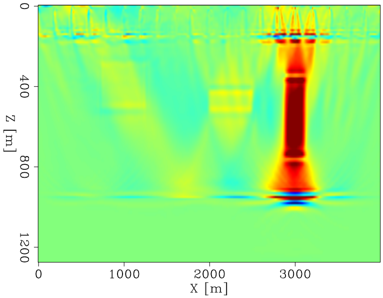

Figure 30: c_{33}

Example 3- Gauss-Newton search direction

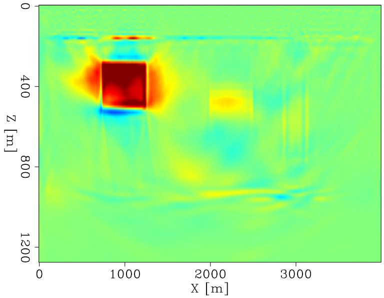

Figure 31: c_{11}

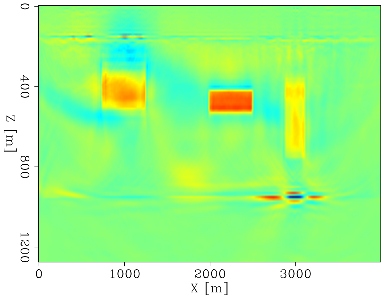

Figure 32: c_{13}

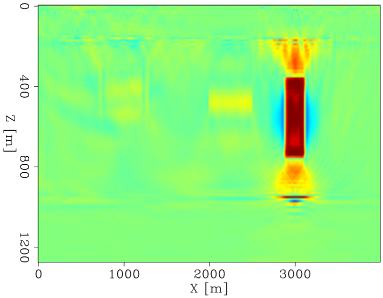

Figure 33: c_{33}

Example 3- Newton search direction

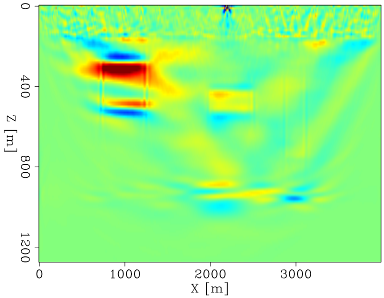

Figure 34: c_{11}

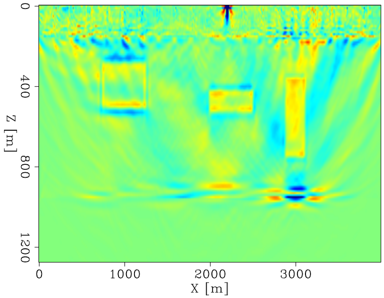

Figure 35: c_{13}

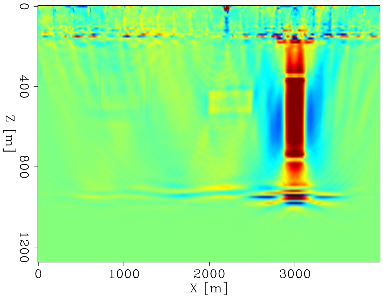

Figure 36: c_{33}

Conclusions

- Different parameterizations, different sensitivity, different crosstalk.
- Less sensitive parameters: can regularization with additional information/constraints help?
- Hessian can precondition and reduce crosstalk.
- Gauss-Newton Hessian better than full Hessian.
- Hessian to study sensitivity by eigenvalue decomposition.

Thank you!