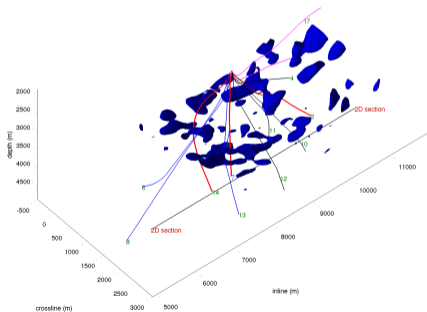# Time-lapse Inverse Theory
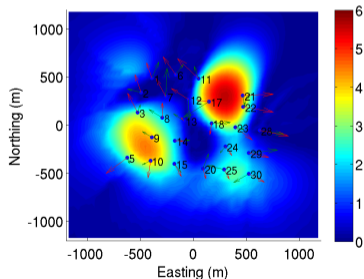
with applications in seismic imaging and geomechanics

Musa Maharramov

Stanford Exploration Project
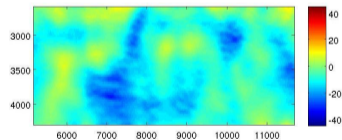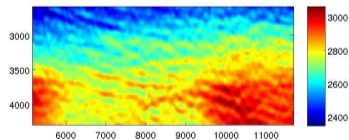
February 09, 2016
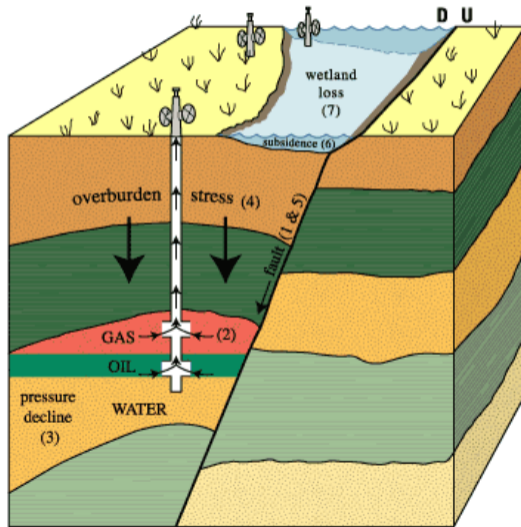

Injection-induced Δ P (MPa) 03-Jan-2008

Time-lapse seismic analysis

- Current methods: conversion of time shifts $\Rightarrow$ impedance changes

- More automated tomographic methods exist

- Goal: design a robust 4D FWI less sensitive to repeatability issues

Full-waveform inversion

Find a model $\mathbf{m}$ that minimizes misfit between the true $\mathbf{d}$ and predicted $\mathbf{u}(\mathbf{m})$ data (Lailly, 1983; Tarantola, 1984)

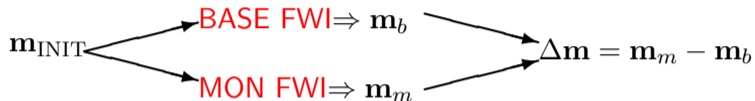$$\text{Misfit} = \|\mathbf{W}_d\left[\mathbf{d} - \mathbf{u}(\mathbf{m})\right]\|^2 \rightarrow \min, \tag{1}$$

with optional model regularization

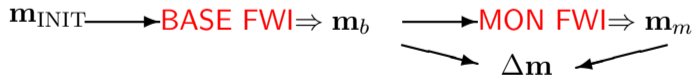$$\beta\|\mathbf{R}\mathbf{W}_m\left[\mathbf{m} - \mathbf{m}^{\text{prior}}\right]\|^2, \tag{2}$$

where $\mathbf{m}$ is a subsurface velocity model, $\mathbf{W}_d$ and $\mathbf{W}_m$ are data residual and model weighting operators, $\mathbf{R}$ is a model regularization operator, and $\mathbf{m}^{\text{prior}}$ is a model prior.

Existing model-space 4D FWI methods

► *Parallel Difference* FWI

$$\mathbf{m}_{\text{INIT}} \nearrow \text{BASE FWI} \Rightarrow \mathbf{m}_b \searrow$$
$$\searrow \text{MON FWI} \Rightarrow \mathbf{m}_m \nearrow \Delta\mathbf{m} = \mathbf{m}_m - \mathbf{m}_b$$

► *Sequential Difference* FWI

$$\mathbf{m}_{\text{INIT}} \longrightarrow \text{BASE FWI} \Rightarrow \mathbf{m}_b \longrightarrow \text{MON FWI} \Rightarrow \mathbf{m}_m$$
$$\searrow \Delta\mathbf{m} \swarrow$$

- NEW: *Simultaneous FWI* of baseline and monitor with *difference regularization*:

$$\text{Baseline Misfit} + \text{Monitor Misfit} + \qquad (3)$$

$$\alpha \| \mathbf{R} \mathbf{W}_m \left[\mathbf{m}_m - \mathbf{m}_b \right] \|_2^2. \qquad (4)$$

- NEW: *Cross-updating* Approximation to the Simultaneous FWI:

$$\mathbf{m}_{\text{INIT}} \longrightarrow \text{BASE FWI} \Rightarrow \mathbf{m}_b \longrightarrow \text{MON FWI} \Rightarrow \mathbf{m}_m$$

$$\tilde{\mathbf{m}}_m \Leftarrow \text{MON FWI} \longleftarrow \tilde{\mathbf{m}}_b \Leftarrow \text{BASE FWI}$$

$$\Delta \mathbf{m}$$
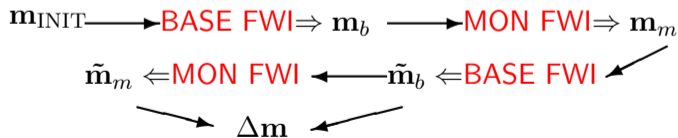
- NEW: *Simultaneous FWI* of baseline and monitor with the *total-variation difference regularization*:

$$\text{Baseline Misfit } + \text{ Monitor Misfit } + \tag{5}$$

$$\alpha ||| \nabla \mathbf{W}_m \left[\mathbf{m}_m - \mathbf{m}_b \right] |||_1. \tag{6}$$

- The Total-variation (TV) seminorm (6) provides **edge-preserving** regularization that promotes model **"blockiness"** and helps to reduce spurious oscillations ("**ROF**" model of Rudin, Osher, and Fatemi, 1992).

7 dB = 2.2 SNR synthetic (reflections + refractions)

inline (m)
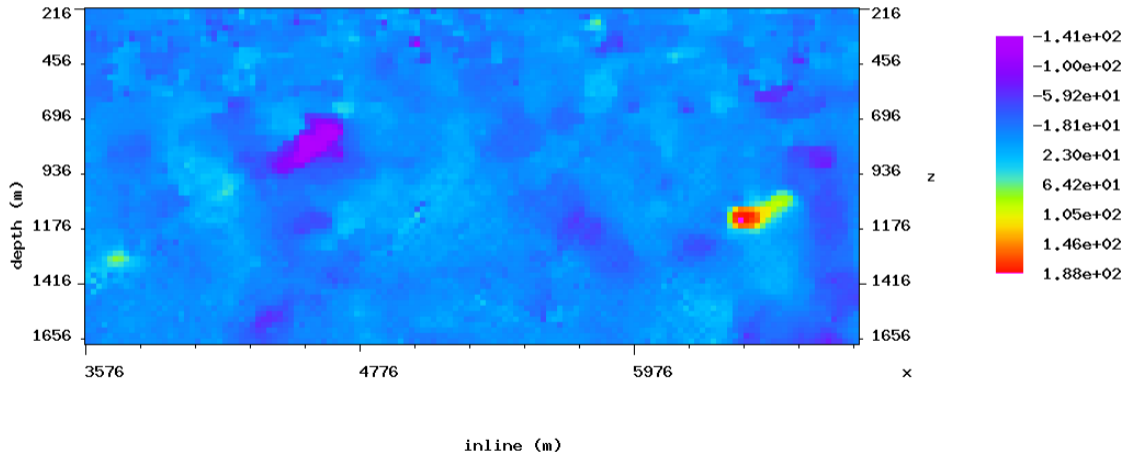
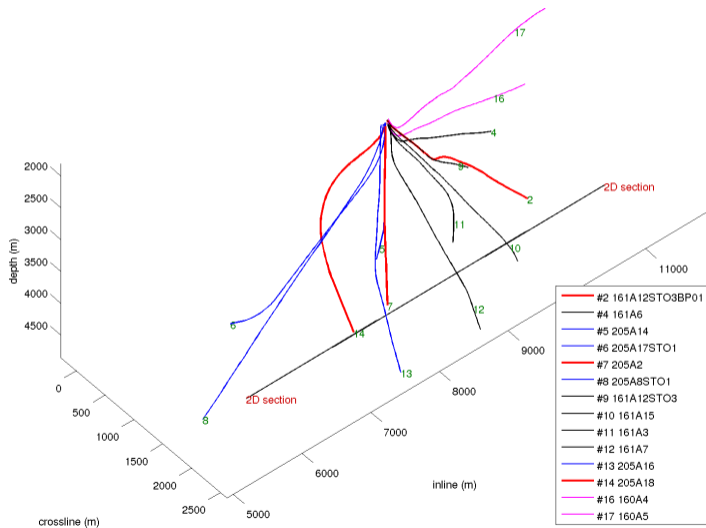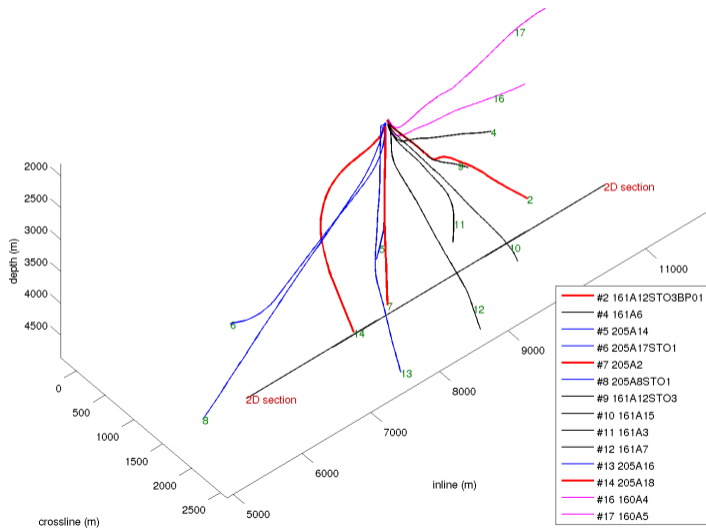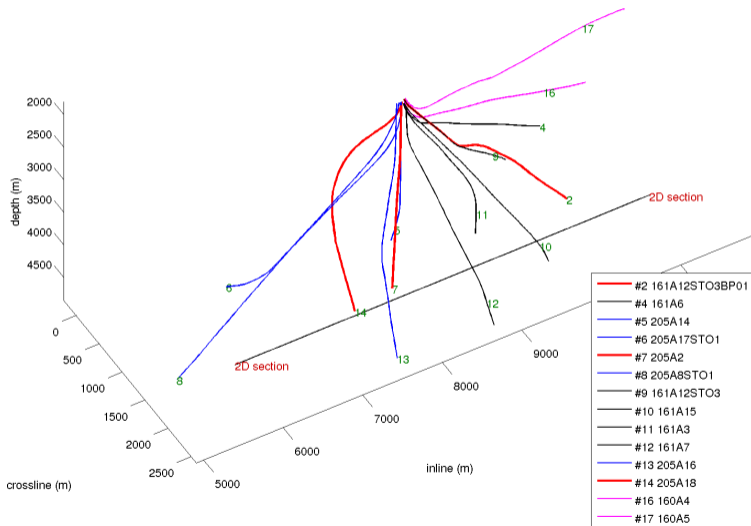# GoM Genesis field data: estimating production-induced overburden dilation

Genesis 2D FWI
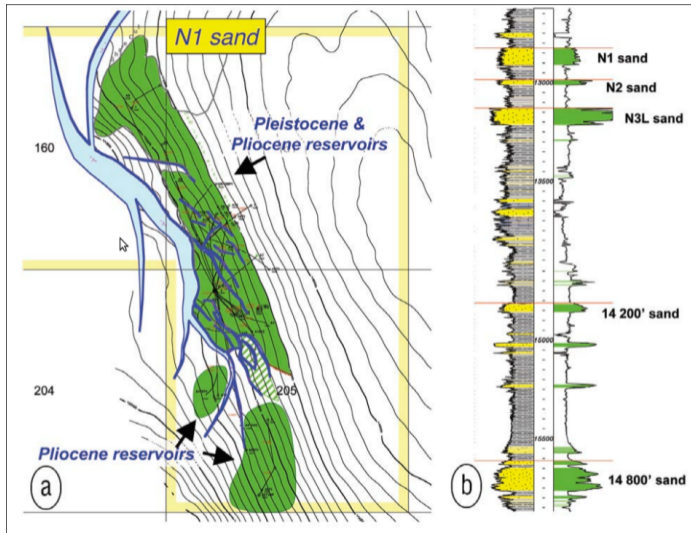
- Frequency-domain inversion, 3-30Hz

- Shot gathers interpolated from the provided sparse CDP gathers

- Surveys: 1264 shots with up to 175 receivers each (a platform gap exists)

- Phase-only inversion to recover blocky velocity anomalies

- Maximum offset 5km, maximum target depth 4km

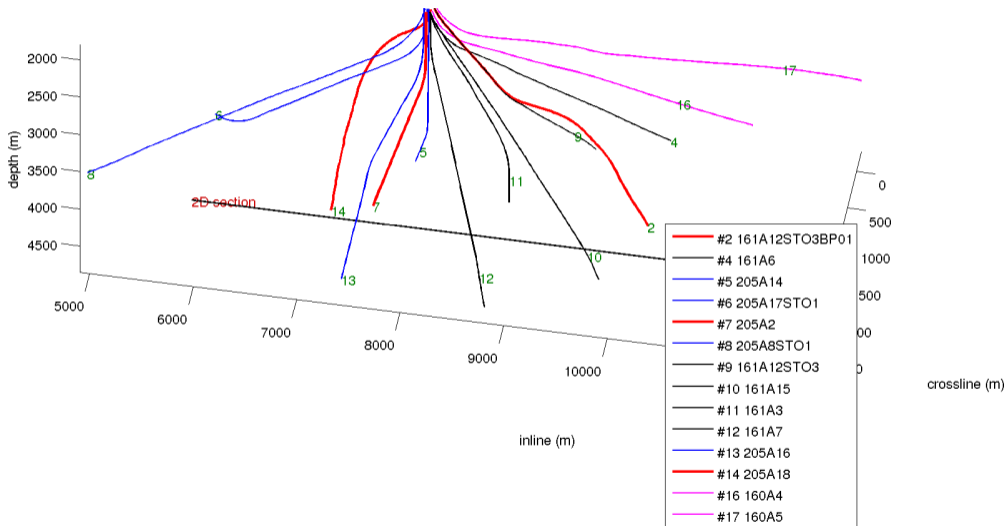- What is 4D FWI sensitivity to 1-2% overburden velocity change with such survey parameters?

- Frequency-domain inversion, 3-30Hz

- Shot gathers interpolated from the provided sparse CDP gathers

- Surveys: 1264 shots with up to 175 receivers each (a platform gap exists)

- Phase-only inversion to recover blocky velocity anomalies

- Maximum offset 5km, maximum target depth 4km

- **What is 4D FWI sensitivity to 1-2% overburden velocity change with such survey parameters?**

Sensitivity analysis for phase-only 4D FWI of reflection data
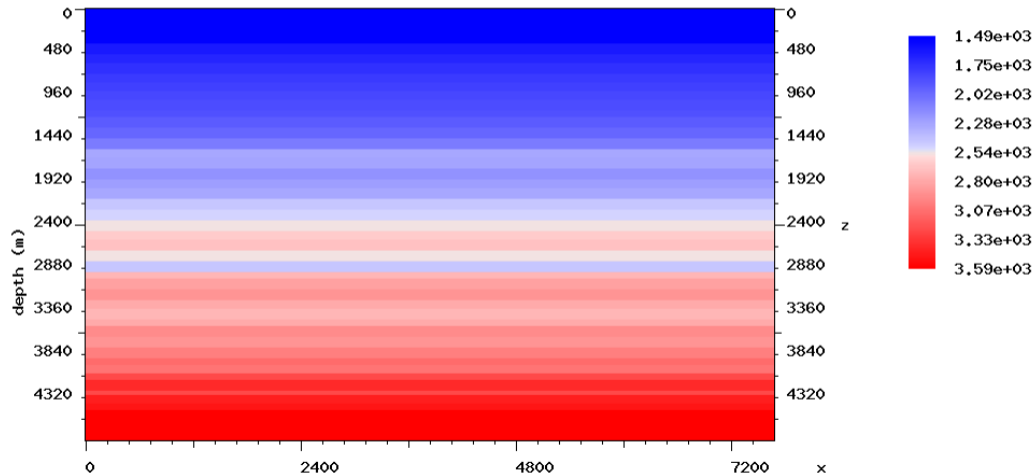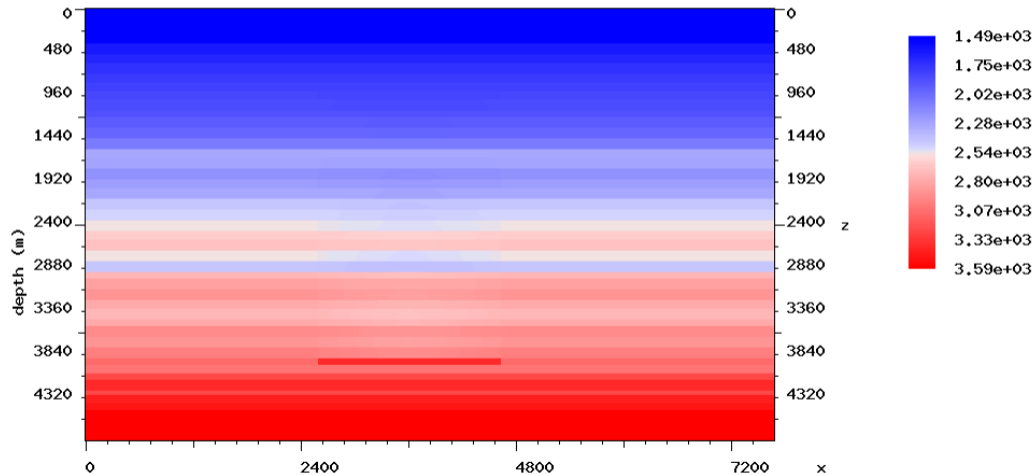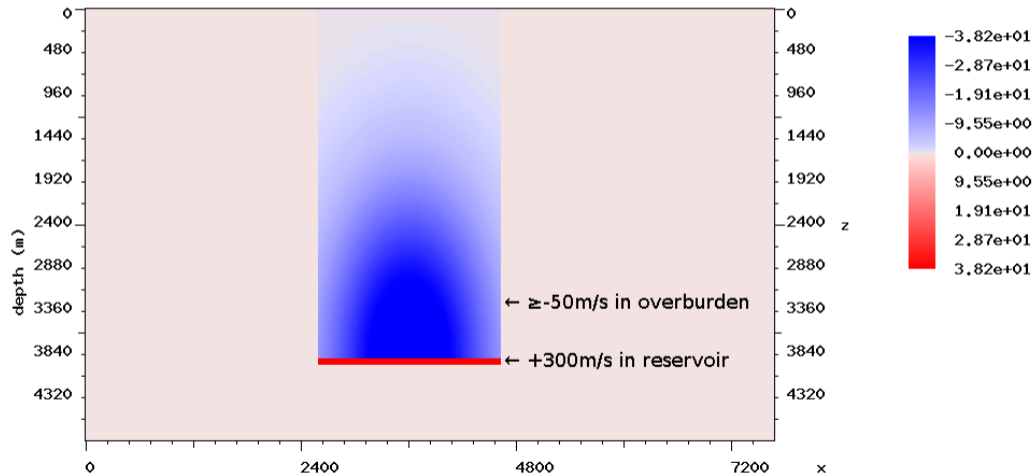
Baseline

Monitor

← ≳-50m/s

← +300m/s
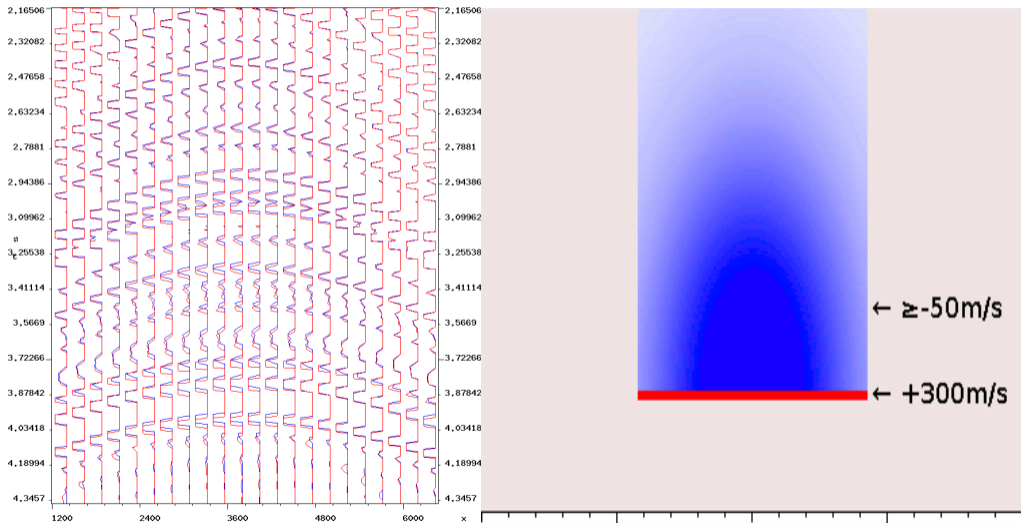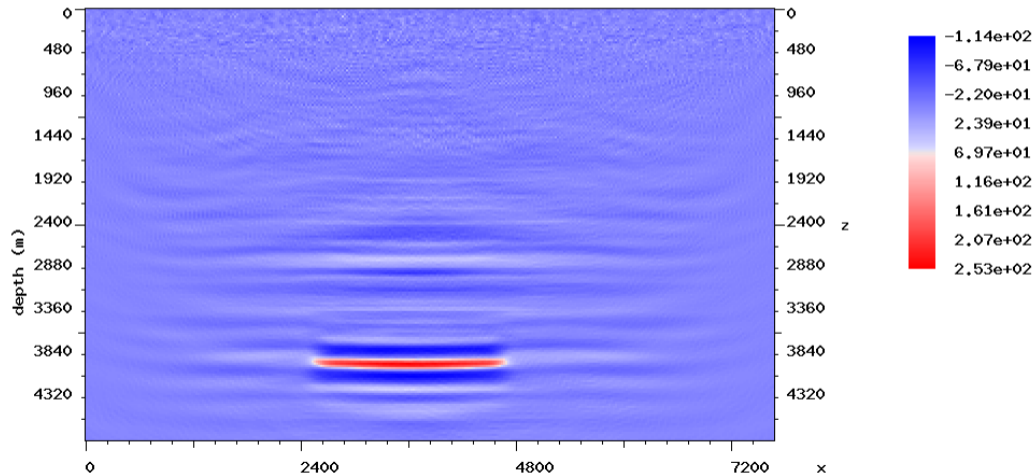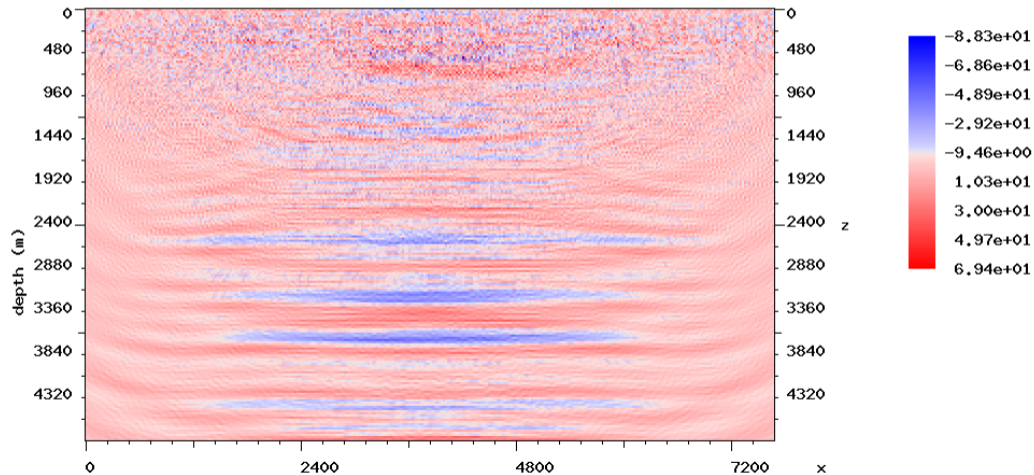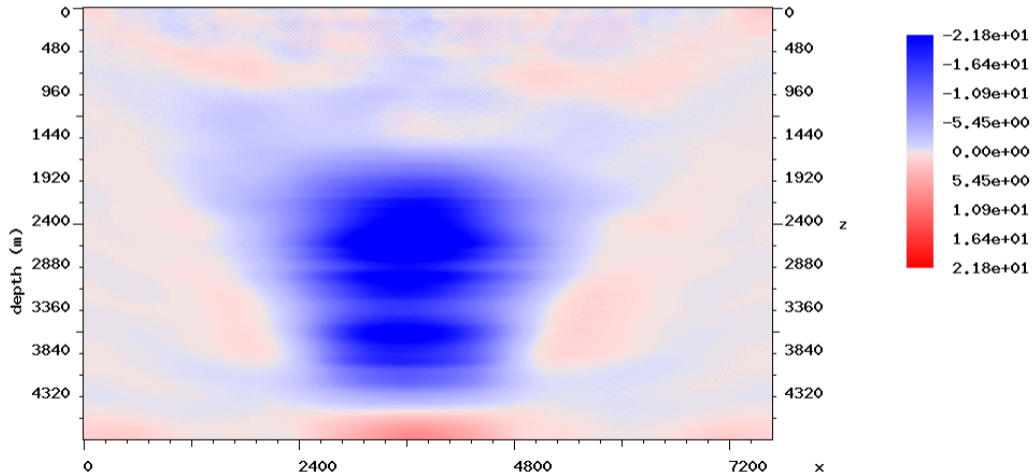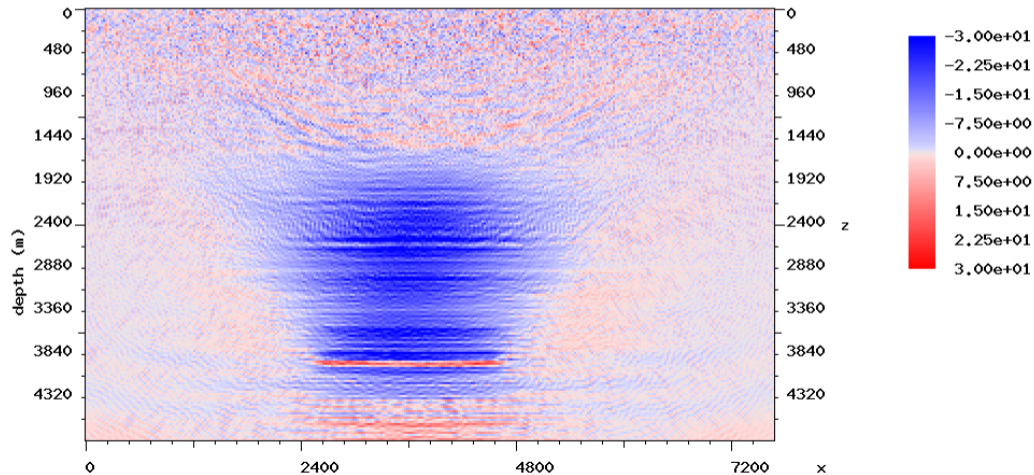
Migration with the updated velocity for image differencing
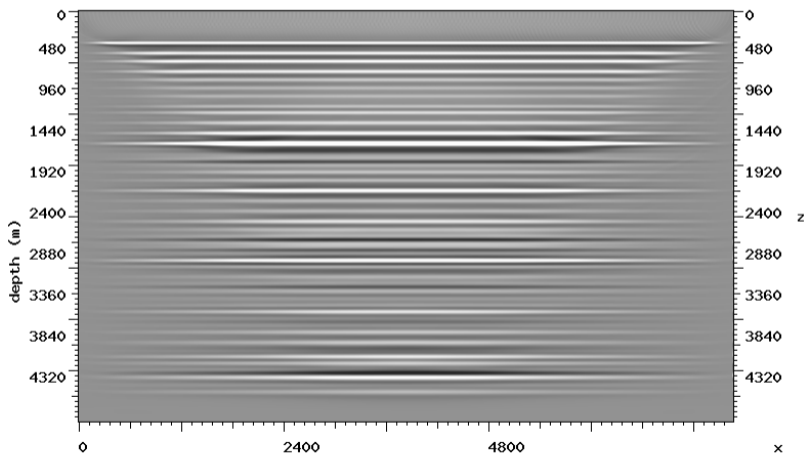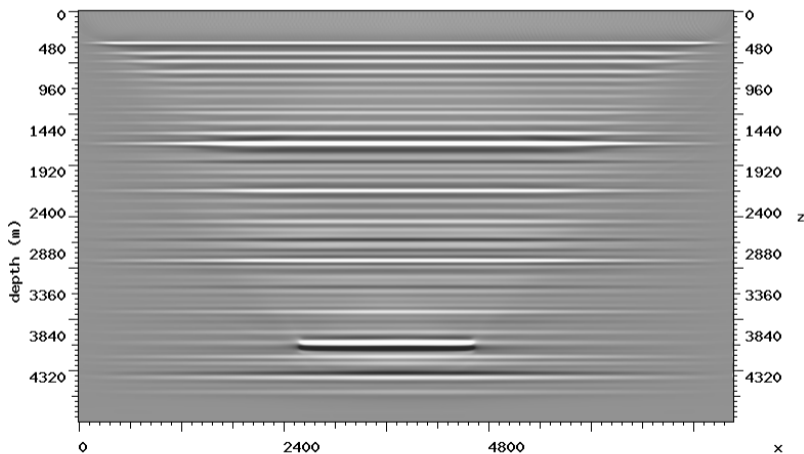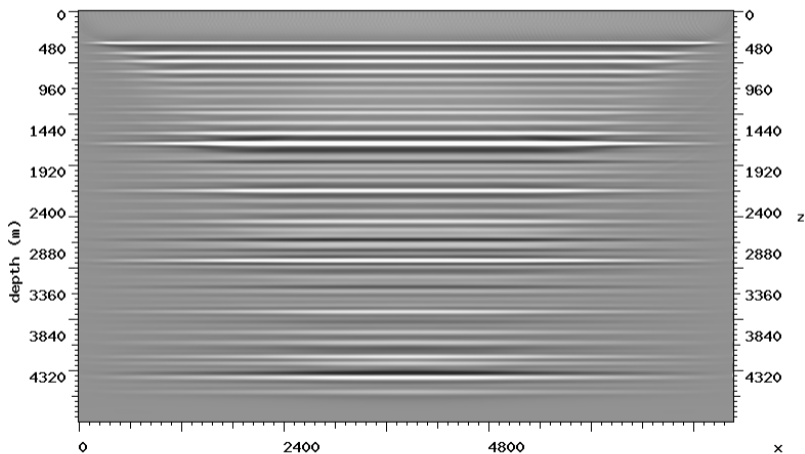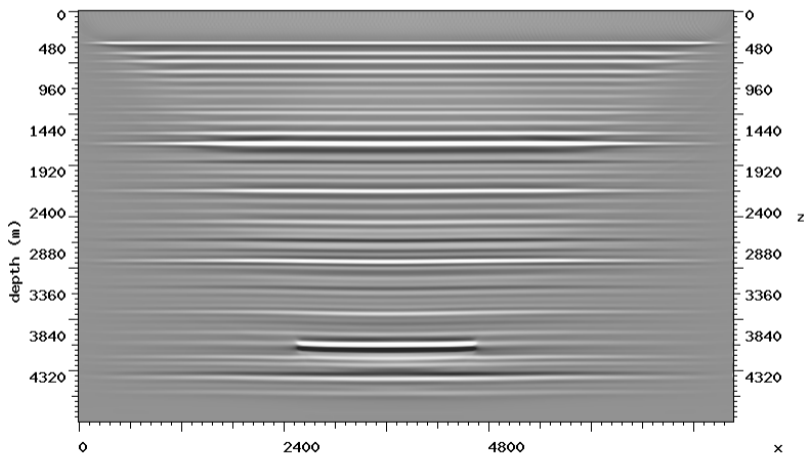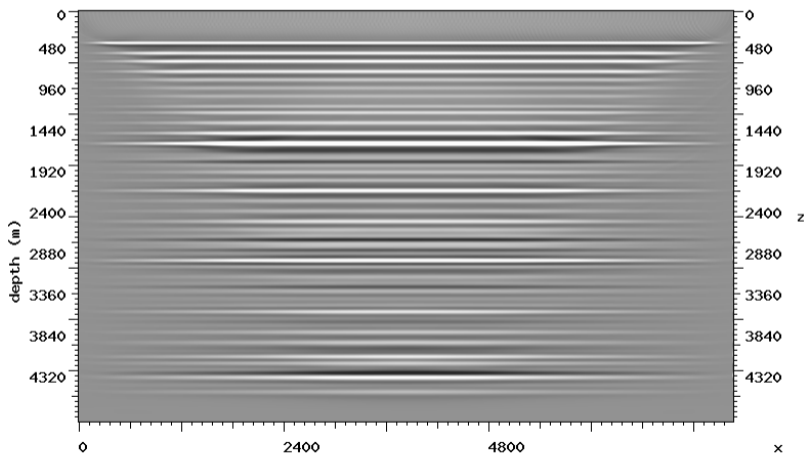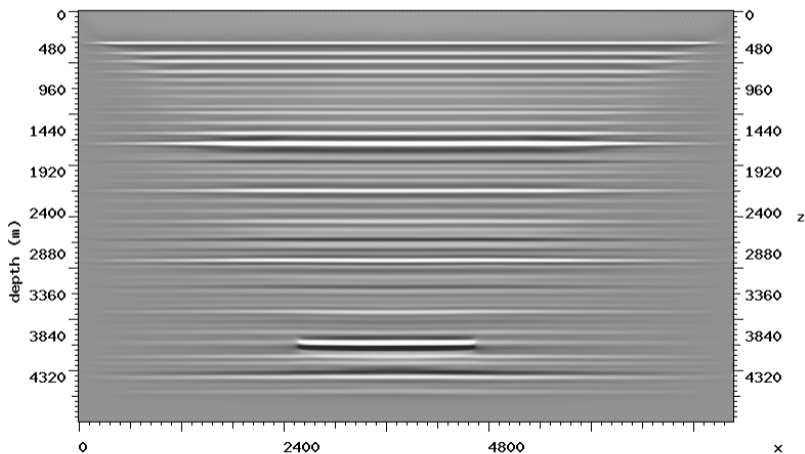
Sensitivity analysis of time-lapse overburden velocity inversion to wrong starting models
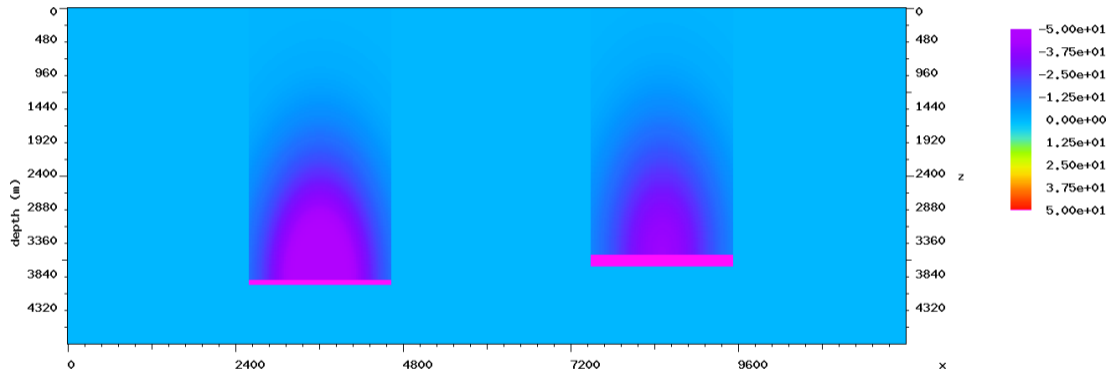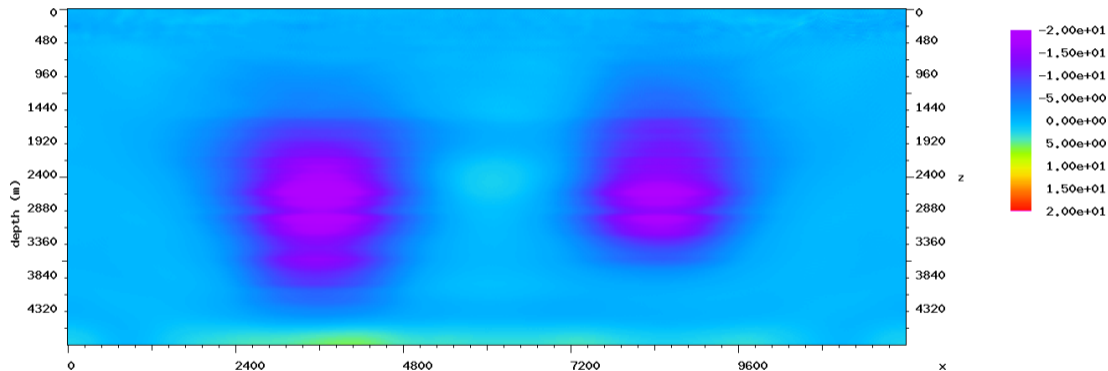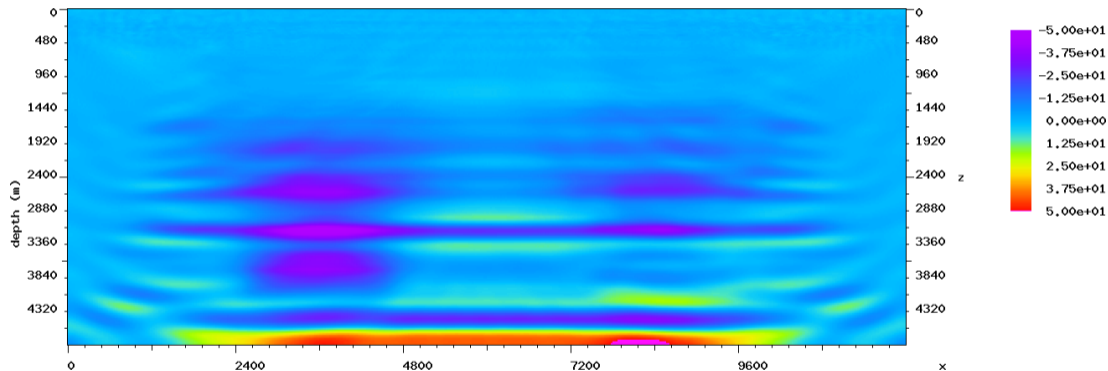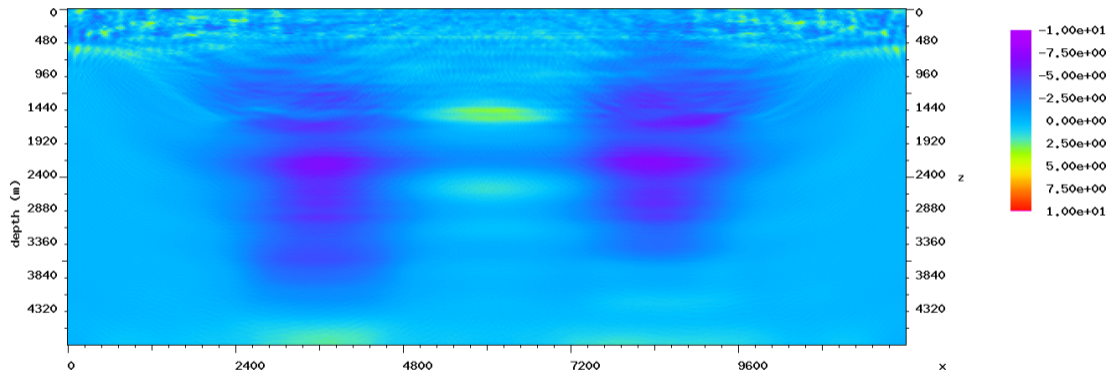
Overburden dilation: reflection field data

Baseline
Monitor

- Frequency-domain inversion, 3-30Hz
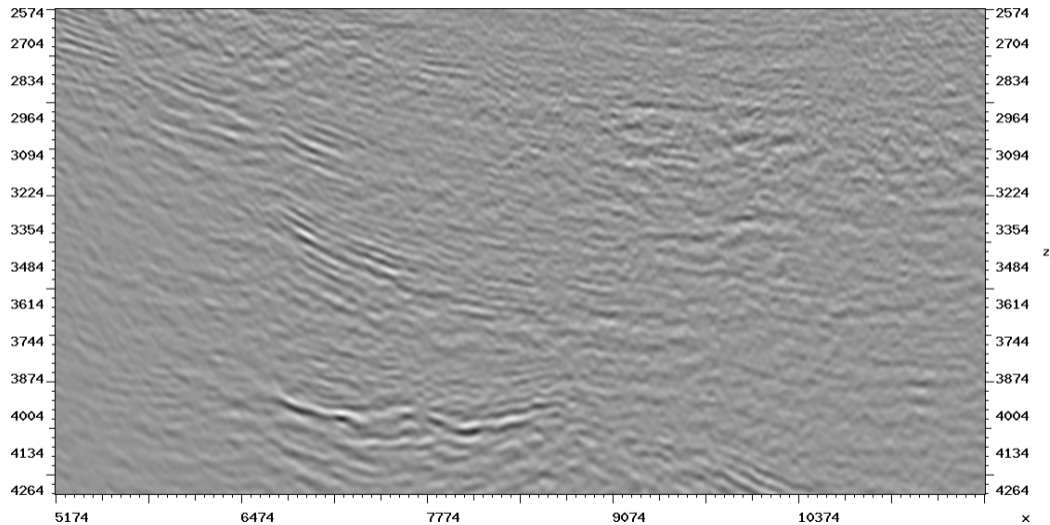
- Shot gathers interpolated from the provided sparse CDP gathers

- Total of 60 shots with up to 175 receivers each

- 3D inversion for 5208m inline $\times$ 3600m crossline $\times$ 4500m depth

- Phase-only inversion to recover blocky velocity anomalies

- Maximum offset 5km, maximum target depth 4km
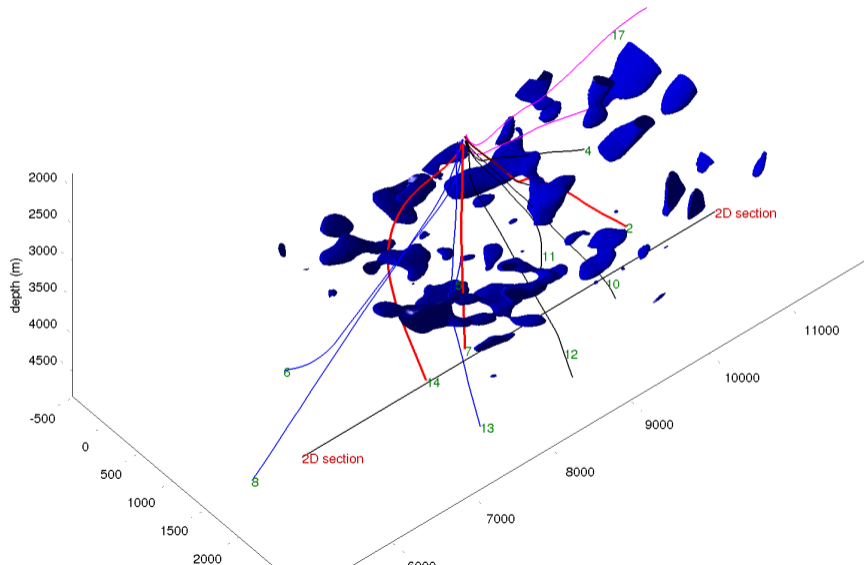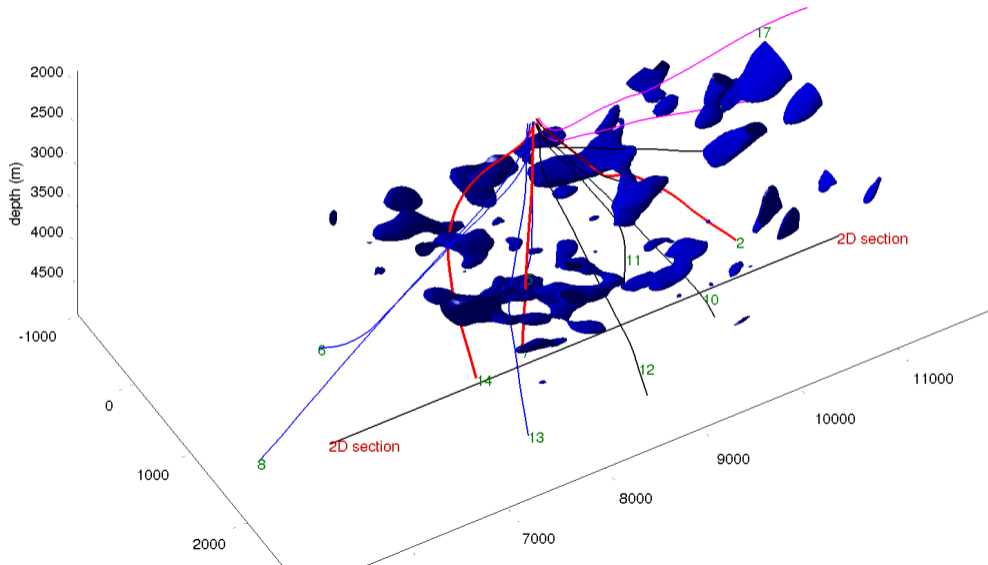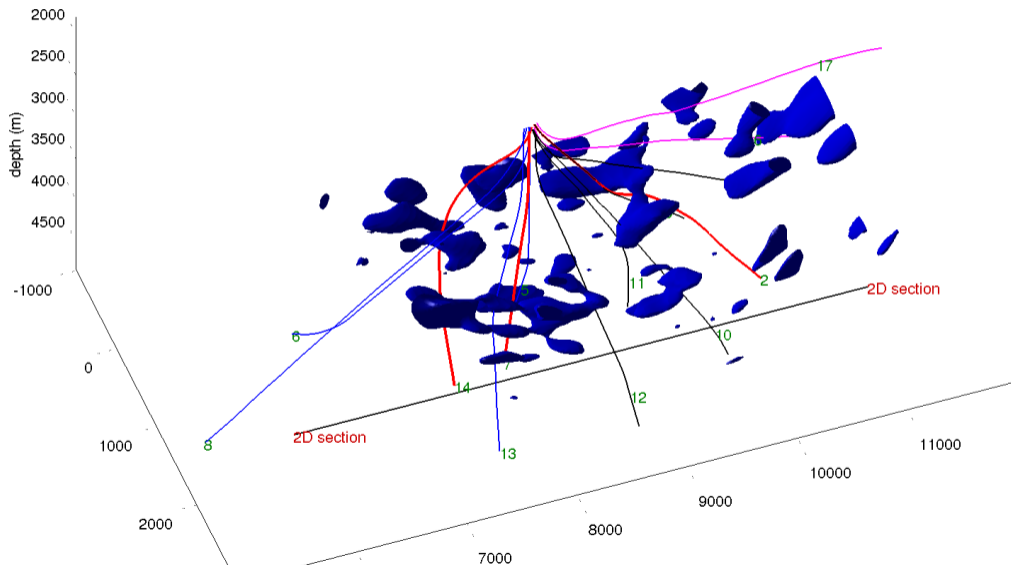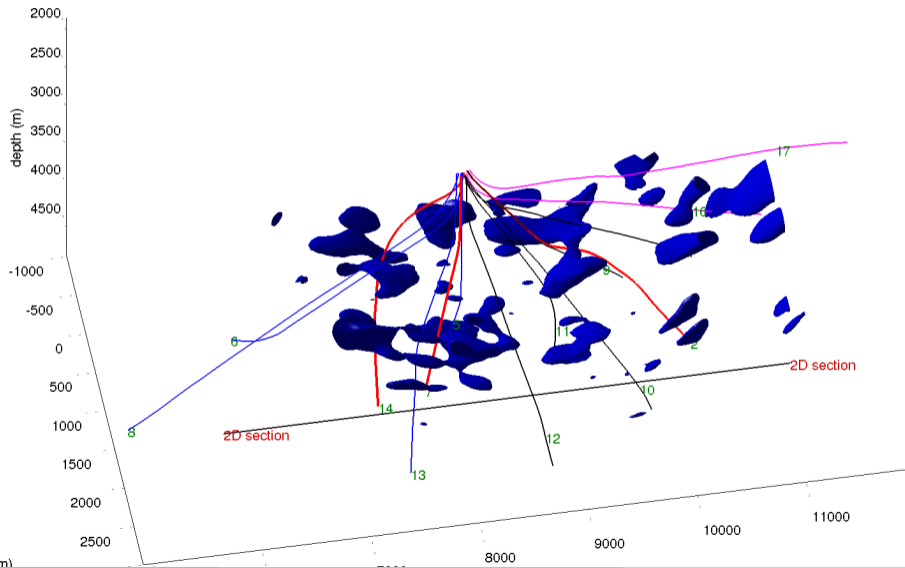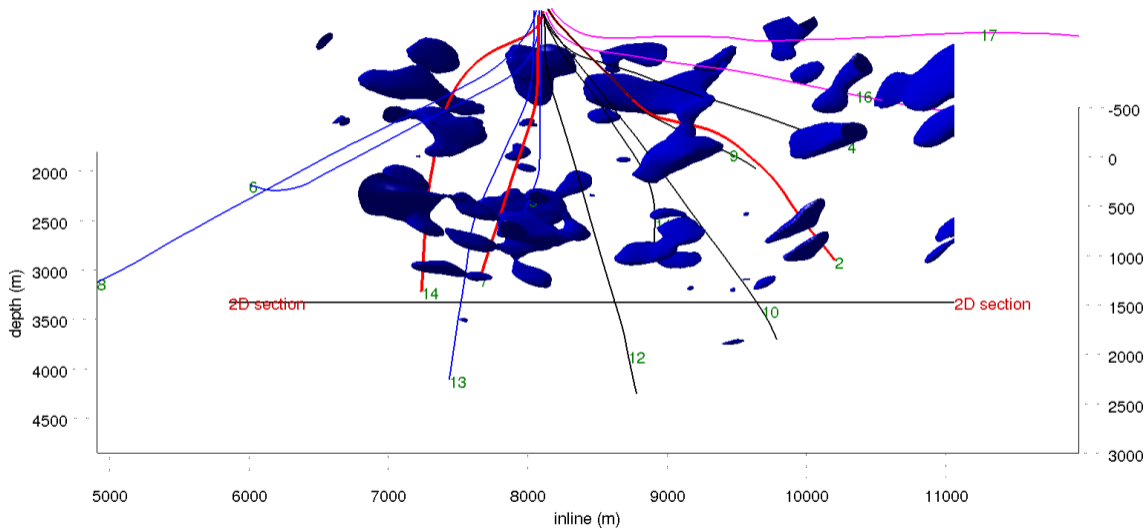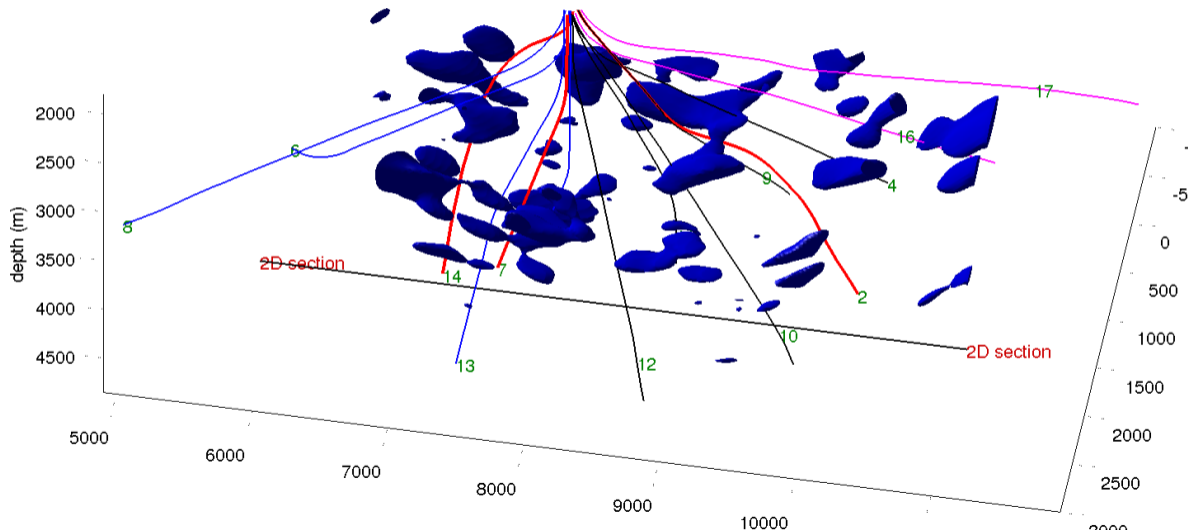
- TV-regularized simultaneous time-lapse FWI can resolve velocity changes induced by overburden dilation

- Resolving fine features of compacting reservoirs requires multi-scale or hierarchical inversion

- Computationally intensive, requires multiple inversion experiments

- Reflection 4D FWI capability $\Rightarrow$ 4D land seismic perspectives

Time-lapse geomechanical analysis
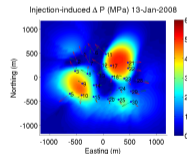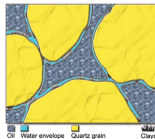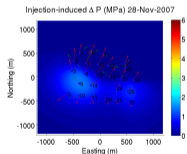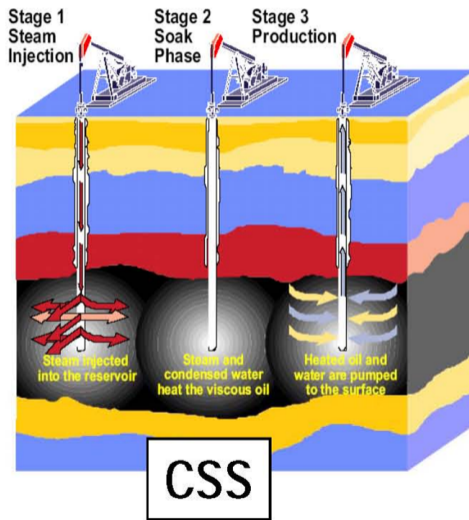
- Extraction of heavy oil from deep formations relies on Cyclic Steam Stimulation (CSS) or Steam-Assisted Gravity Drainage (SAGD).

- Effective steam injection requires understanding pressure front propagation and reservoir heterogeneity.

- Goal: design a robust computational framework for estimating pressure fronts from measurable surface deformation.

Injection Wells

Injection Wells

Injection Wells

Injection Wells

Linear Poroealstic Medium Deformation

Relate deformation and pore pressure change using Biot's theory (Rice and Cleary, 1976; Segall, 1985; Segall, 2010):

$$\mu\nabla^2 u_i + \frac{\mu}{1-2\nu}\frac{\partial^2 u_j}{\partial x_i \partial x_j} = \boxed{\alpha\frac{\partial p}{\partial x_i}} - f_i = 0, \ i = 1, 2, 3, \qquad \text{(ELAST)}$$

and

$$S_\alpha \frac{\partial p}{\partial t} - \frac{\kappa}{\eta}\nabla^2 p = -\alpha\frac{\partial}{\partial t}\left(\nabla \cdot \mathbf{u}\right), \qquad \text{(FLOW)}$$

where $\mathbf{u}$ is displacement, p is the pore pressure change, f_i is a differential body-force distribution, μ, ν, α, κ, and η are the shear modulus, Poisson's ratio, Biot coefficient, permeability, and fluid viscosity;

$$S_\alpha = \frac{3\alpha(1-2\nu)(1-\alpha B)}{2\mu B(1+\nu)}, \qquad \text{(STORAGE)}$$

where B is Skempton's coefficient.

Relate deformation and pore pressure change using Biot's theory (Rice and Cleary, 1976; Segall, 1985; Segall, 2010):

$$\mu \nabla^2 u_i + \frac{\mu}{1-2\nu} \frac{\partial^2 u_j}{\partial x_i \partial x_j} = \boxed{\alpha \frac{\partial p}{\partial x_i}} - f_i = 0, \ i = 1, 2, 3, \qquad \text{(ELAST)}$$

and

$$S_\alpha \frac{\partial p}{\partial t} - \frac{\kappa}{\eta} \nabla^2 p = -\alpha \frac{\partial}{\partial t} \left(\nabla \cdot \mathbf{u} \right), \qquad \text{(FLOW)}$$

where $\mathbf{u}$ is displacement, p is the pore pressure change, f_i is a differential body-force distribution, μ, ν, α, κ, and η are the shear modulus, Poisson's ratio, Biot coefficient, permeability, and fluid viscosity;

$$S_\alpha = \frac{3\alpha(1-2\nu)(1-\alpha B)}{2\mu B(1+\nu)}, \qquad \text{(STORAGE)}$$

where B is Skempton's coefficient.

- Regard (ELAST) as a deformation modeling operator $\mathbf{A}p = \mathbf{u}$ and **decouple** it from the flow equation (FLOW) (Vasco et al., 2000; Du and Olson, 2001; Hodgson, 2007).

- Solve the regularized constrained optimization problem

$$\|\mathbf{A}p - \mathbf{u}\|_{L_2}^2 + \epsilon\|\Delta p\|_{L_2}^2 \ \rightarrow \ \min, \ \ 0 \ \leq \ p \ \leq \ p_{\max}. \ \text{(BOUNDREG)}$$
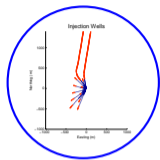
Injection-induced Δ P (MPa) 16-Nov-2007
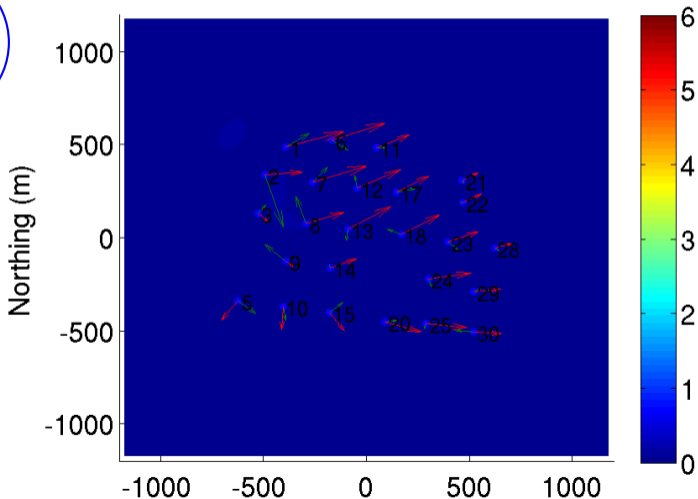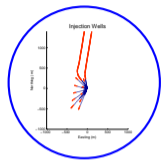
Injection-induced Δ P (MPa) 17-Nov-2007
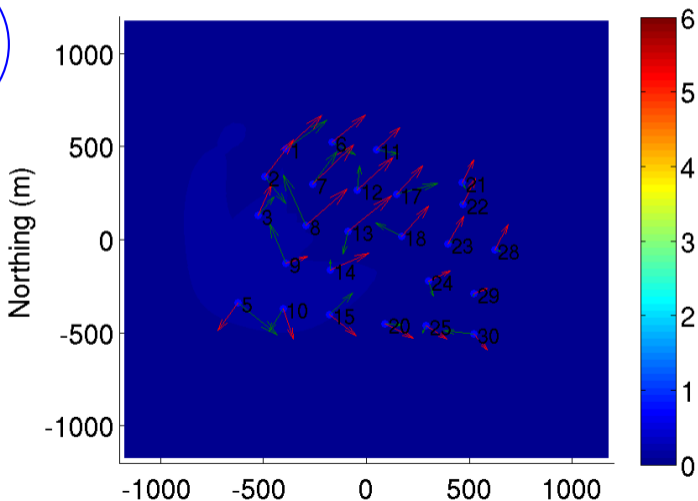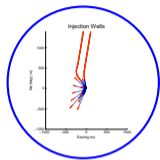
Injection-induced Δ P (MPa) 18-Nov-2007
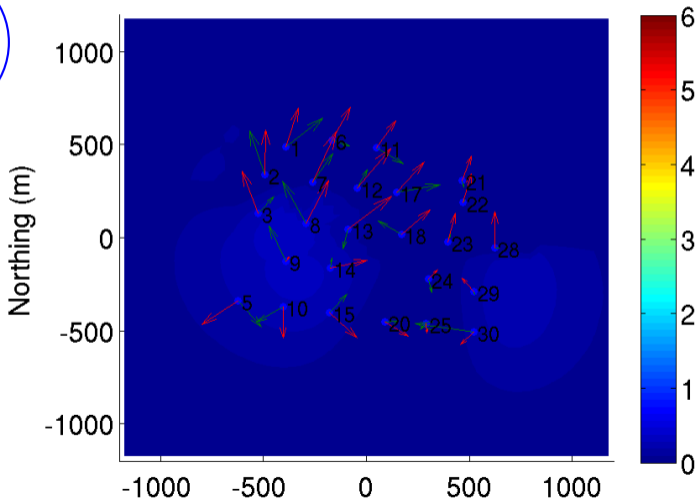
Injection-induced Δ P (MPa) 19-Nov-2007
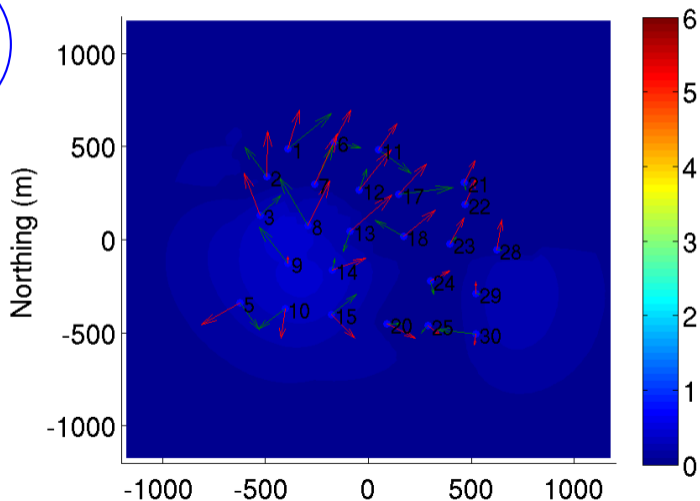
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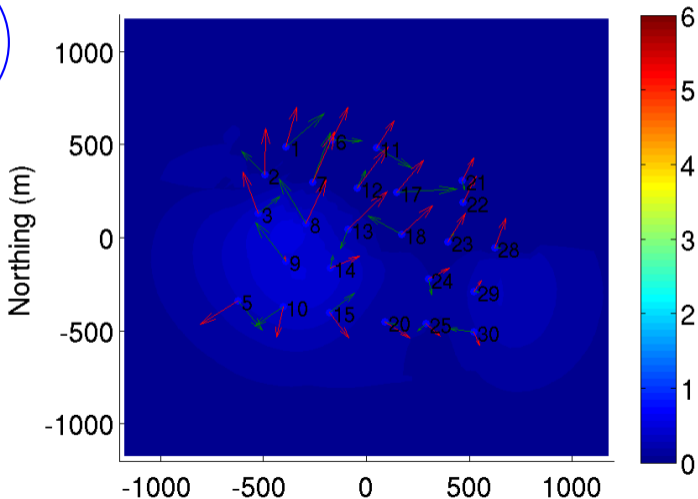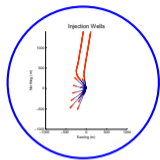
Injection-induced Δ P (MPa) 21-Nov-2007
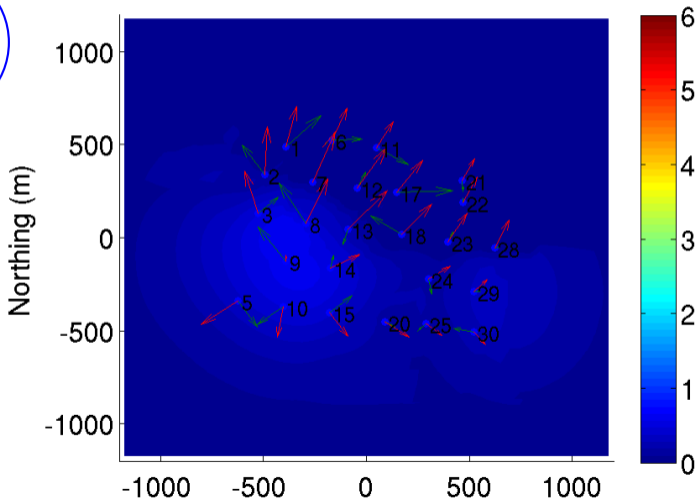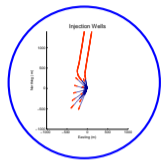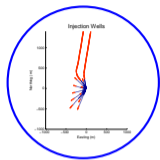
Injection-induced Δ P (MPa) 22-Nov-2007
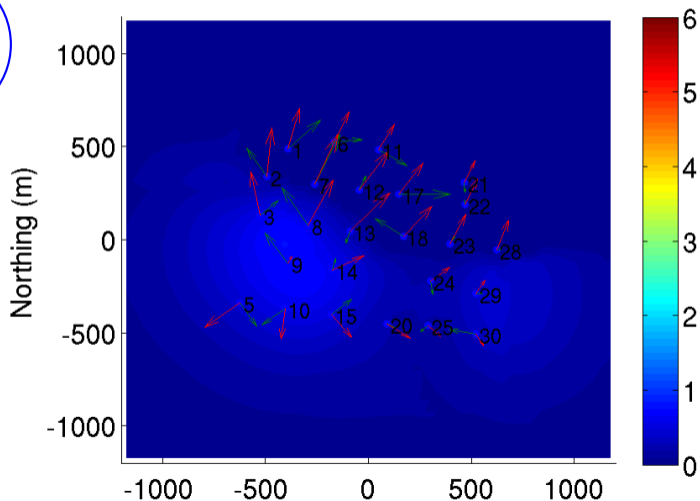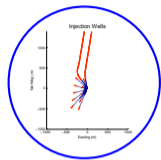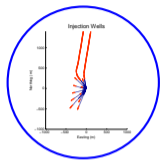
Injection-induced Δ P (MPa) 23-Nov-2007
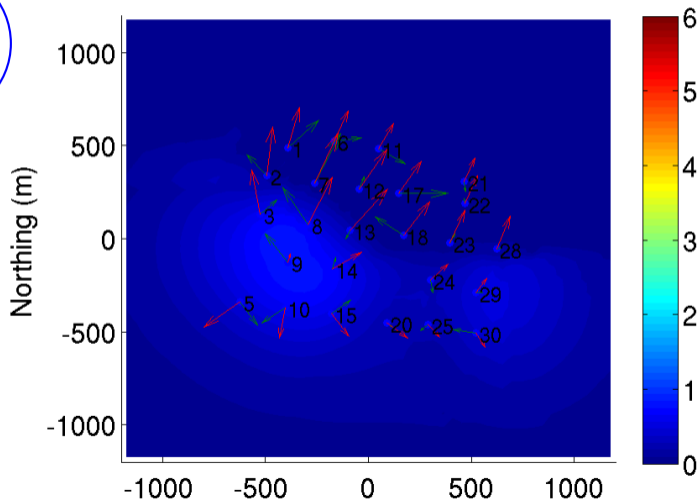
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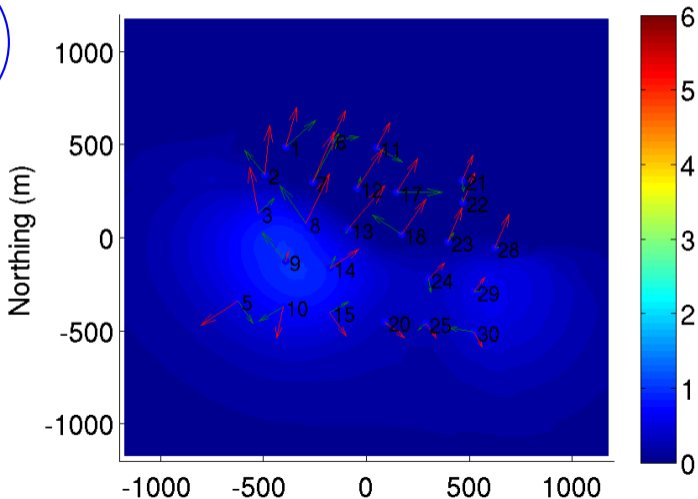
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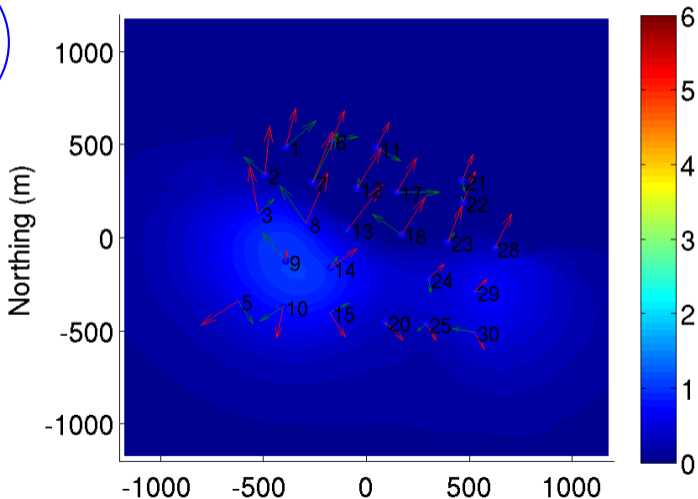
Injection-induced Δ P (MPa) 26-Nov-2007
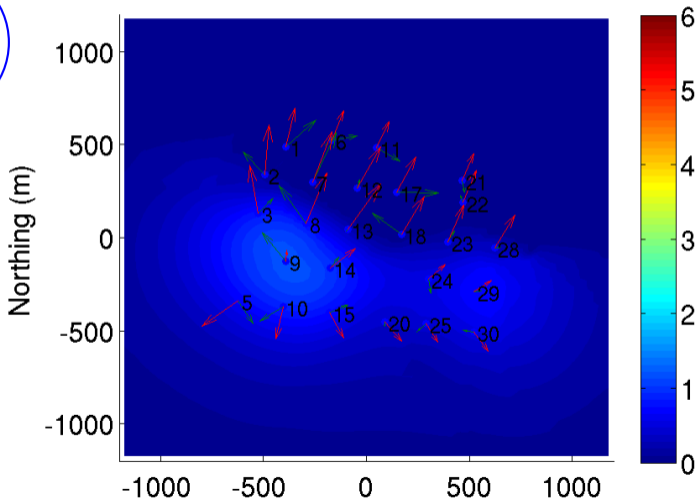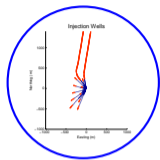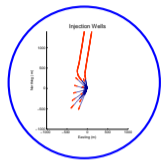
Injection-induced ΔP (MPa) 27-Nov-2007
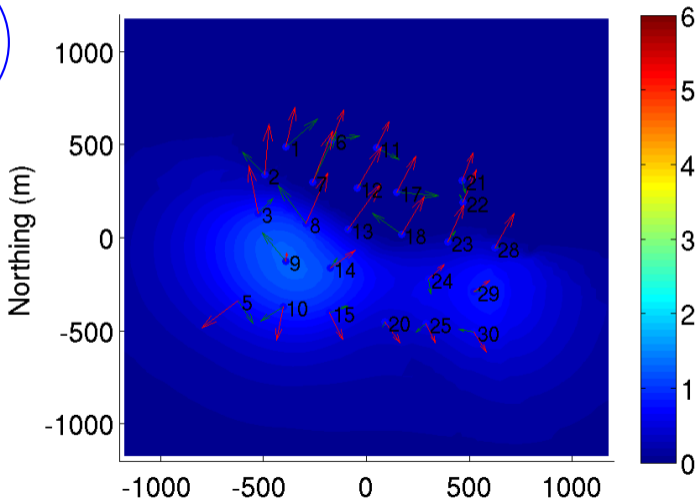
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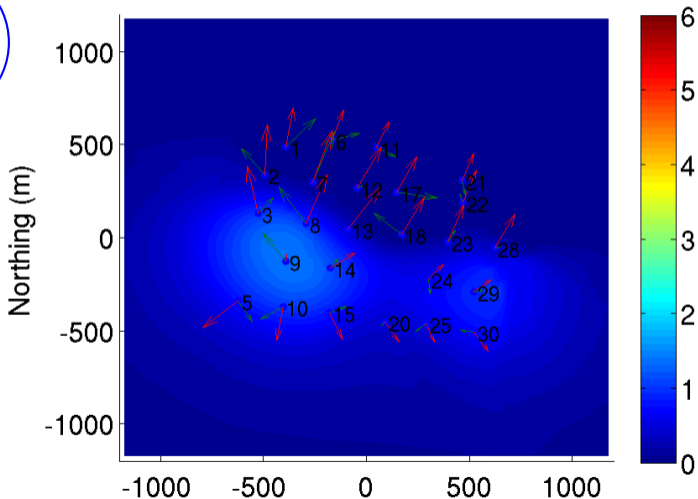
Injection-induced ΔP (MPa) 29-Nov-2007
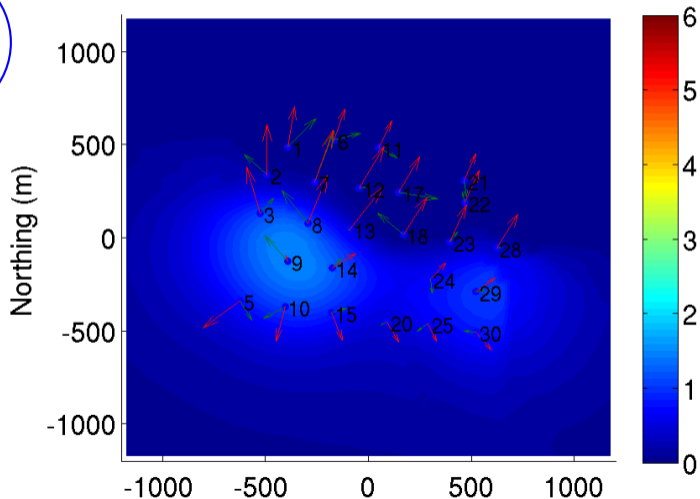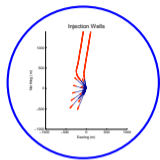
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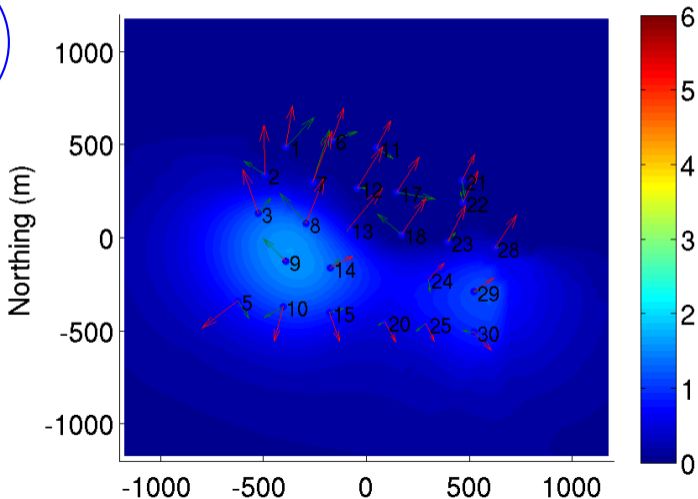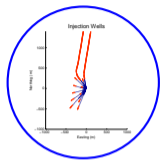
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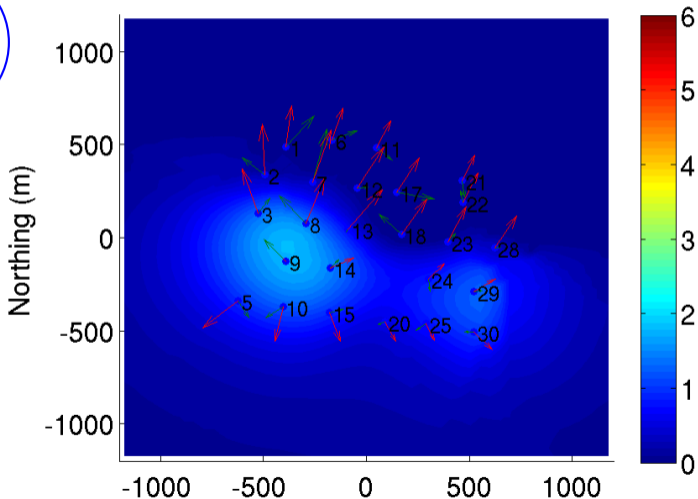
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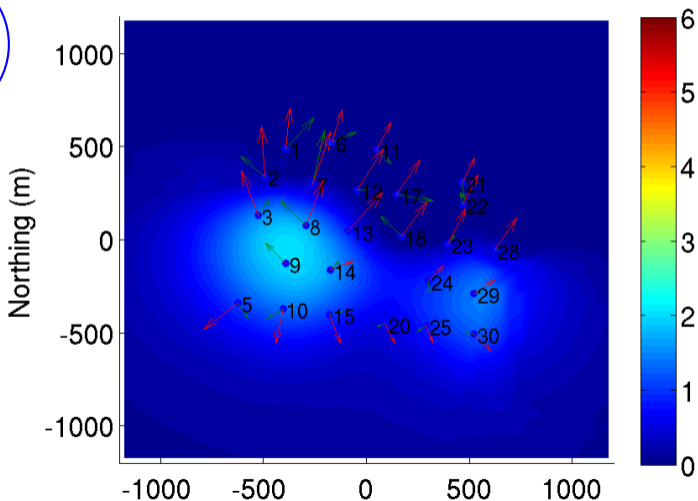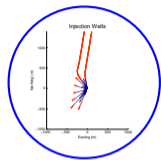
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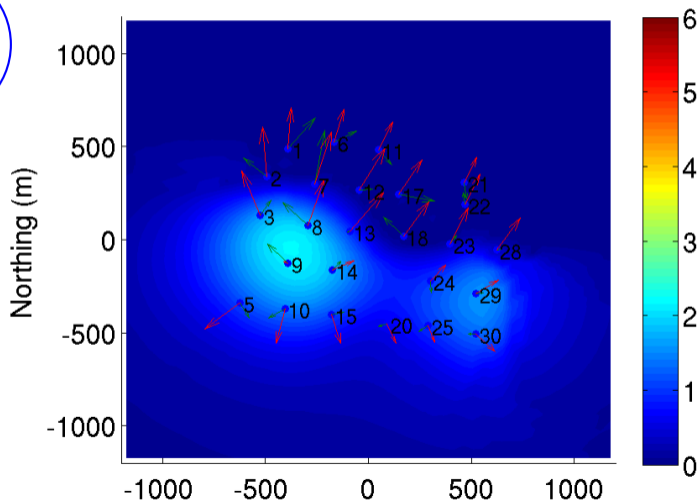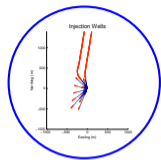
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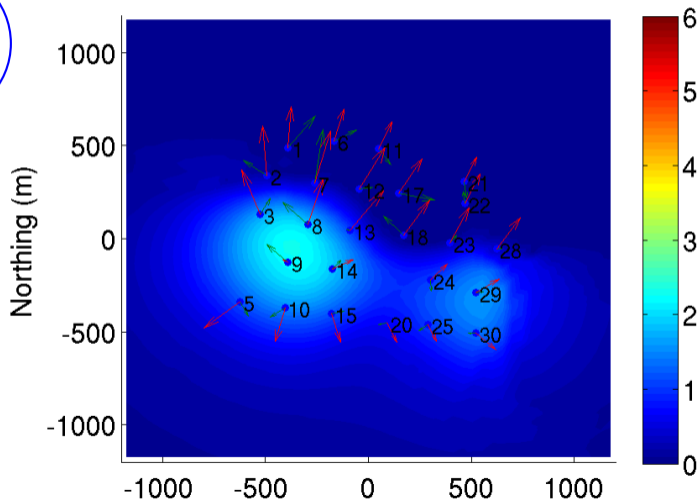
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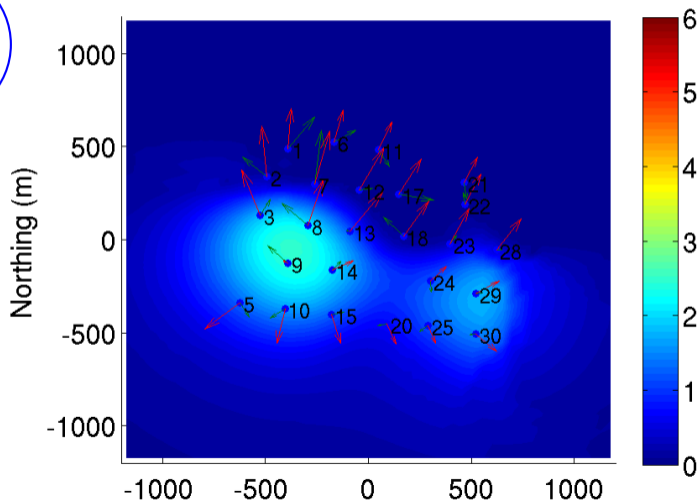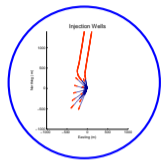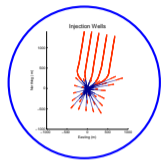
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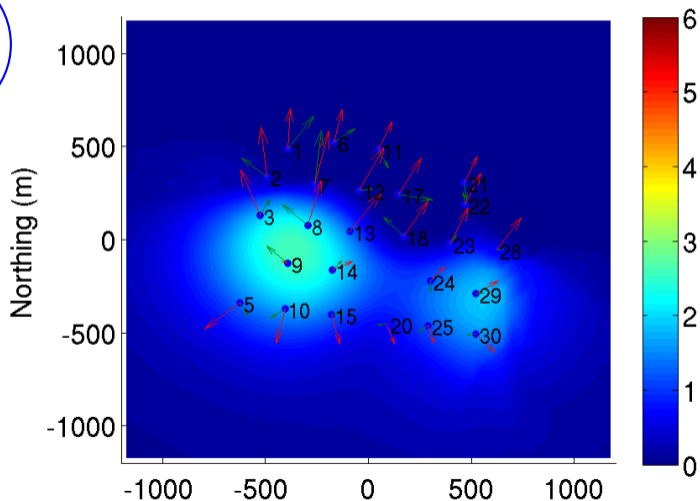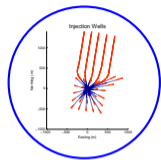
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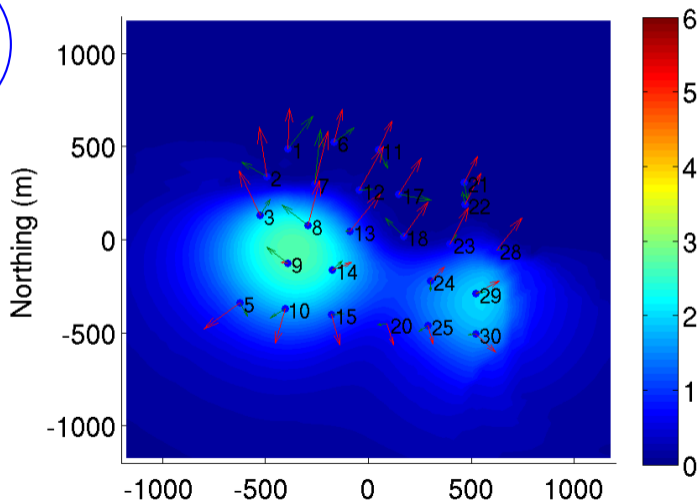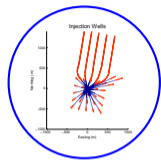
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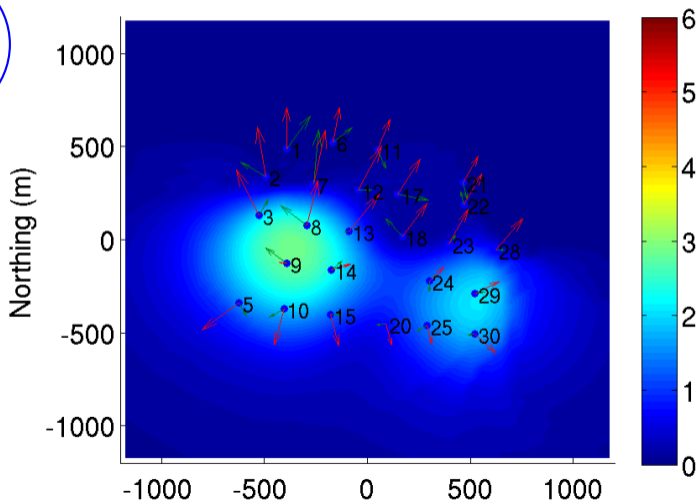
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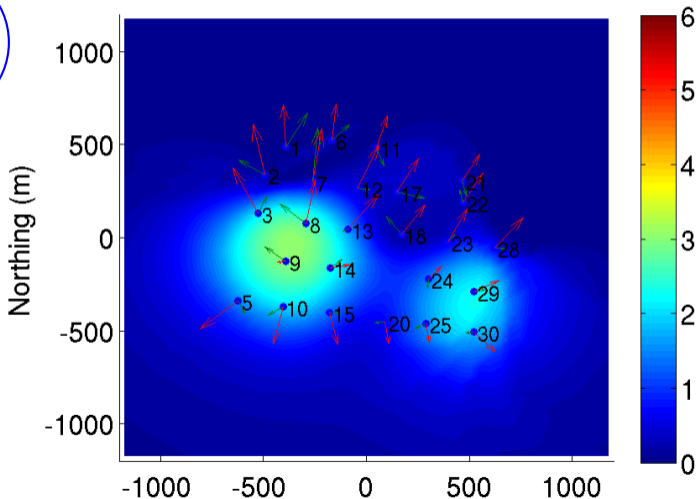
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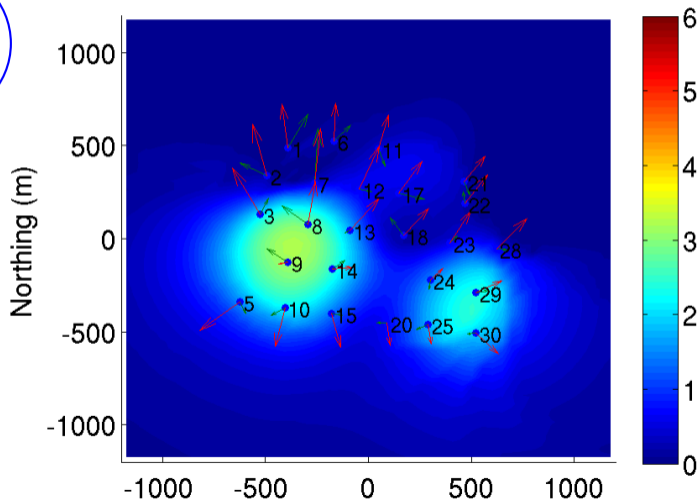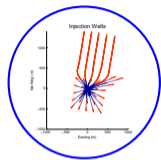
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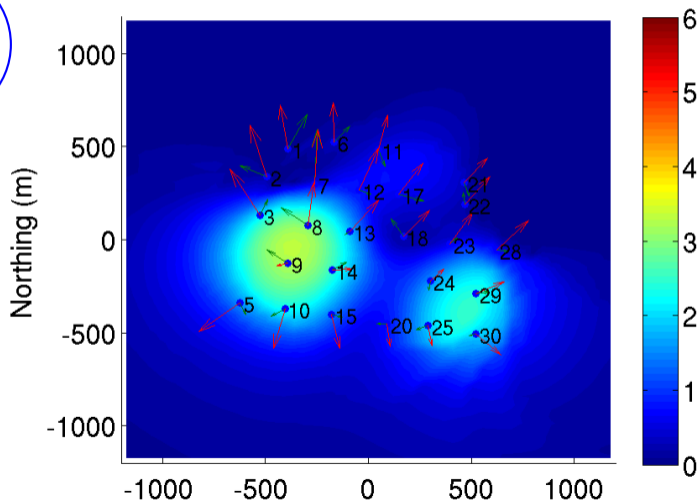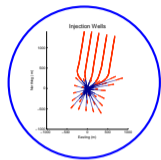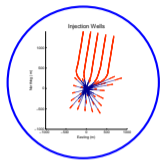
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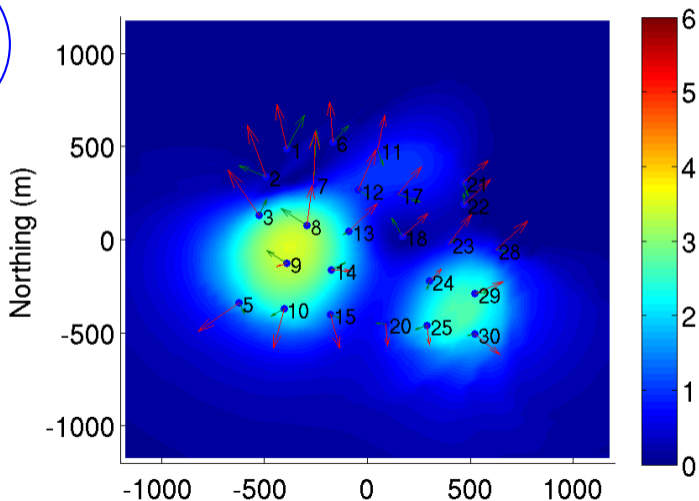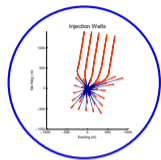
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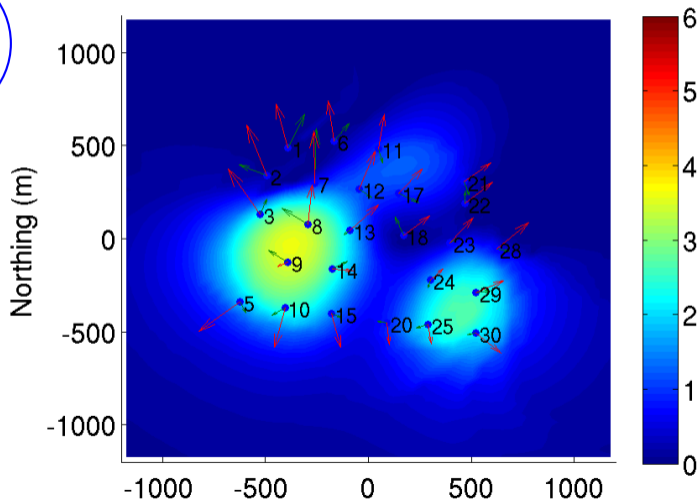
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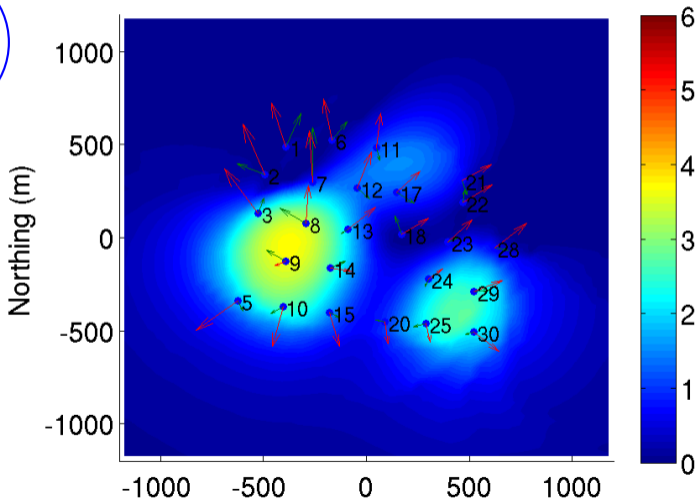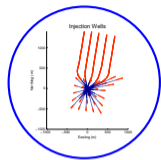
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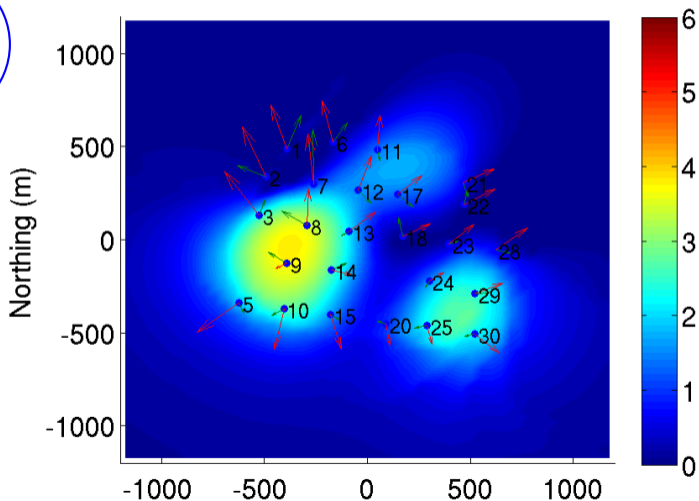
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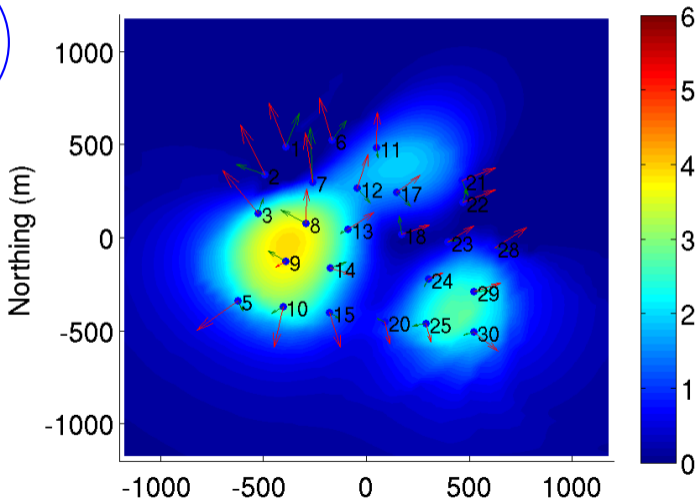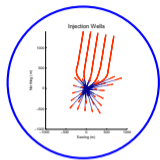
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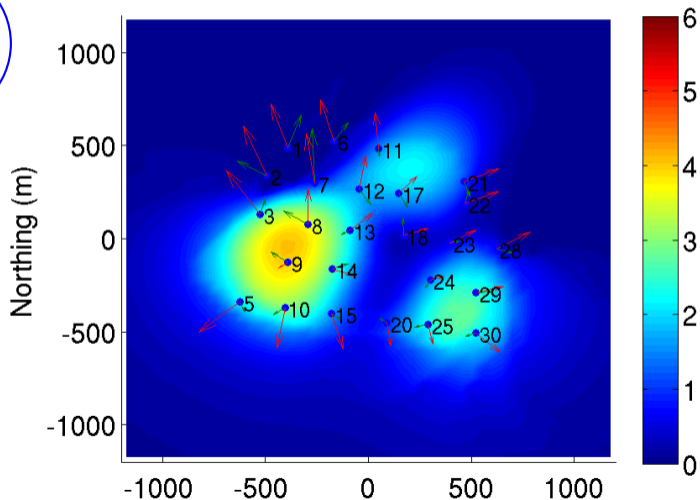
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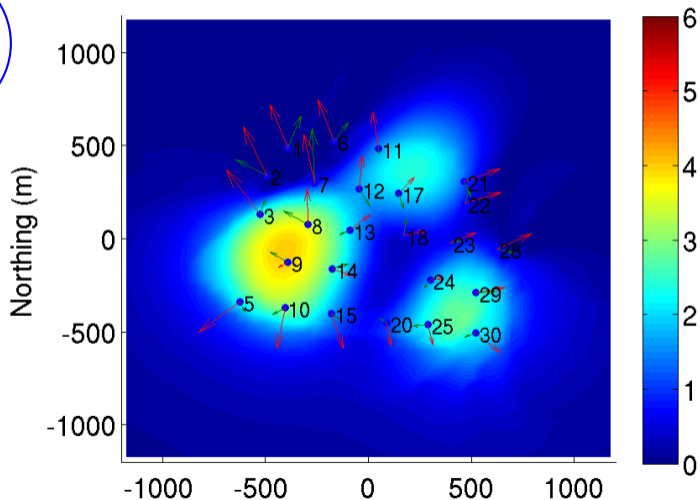
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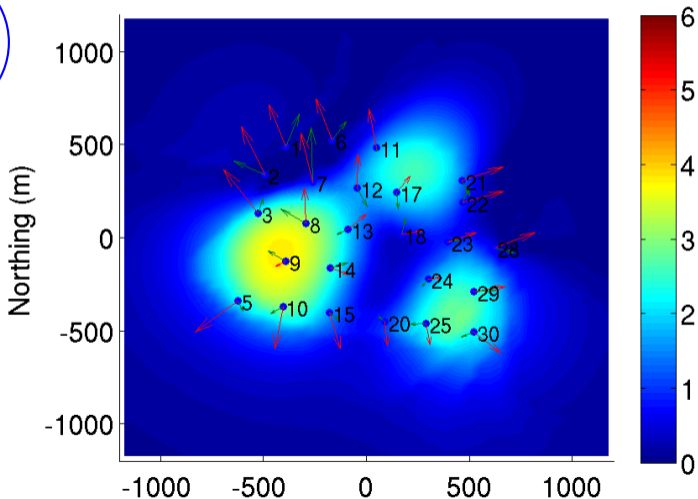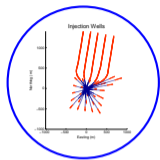
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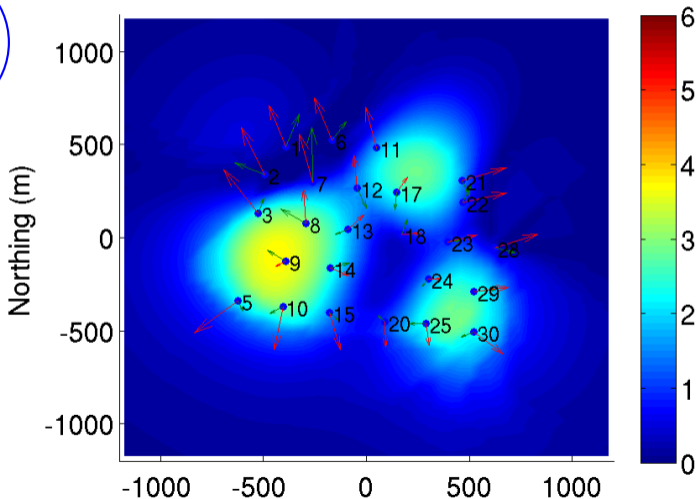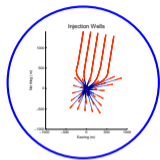
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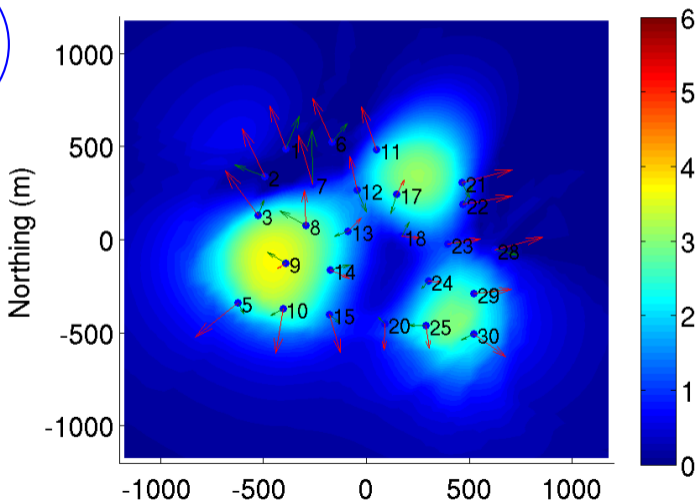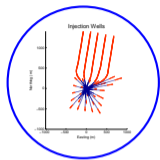
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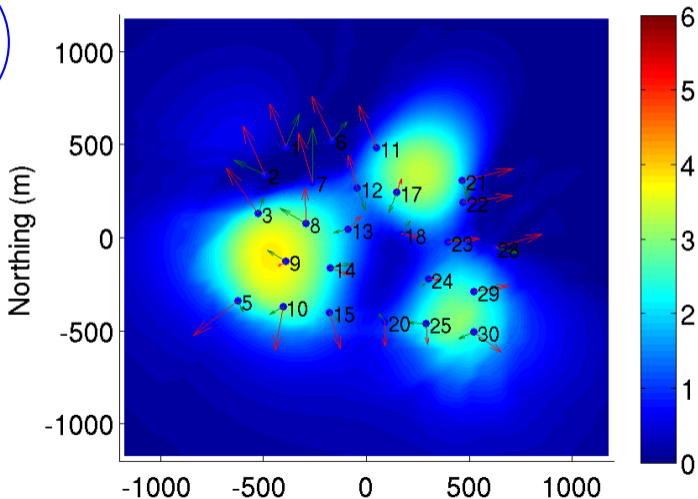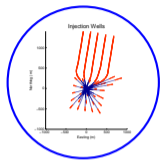
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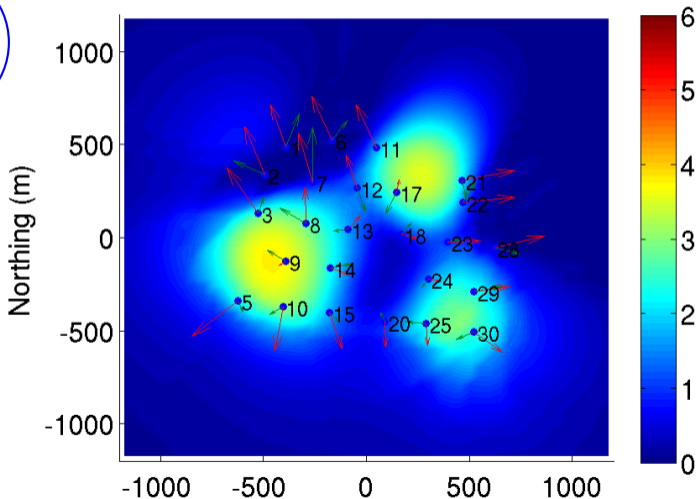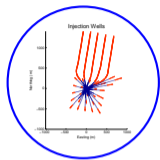
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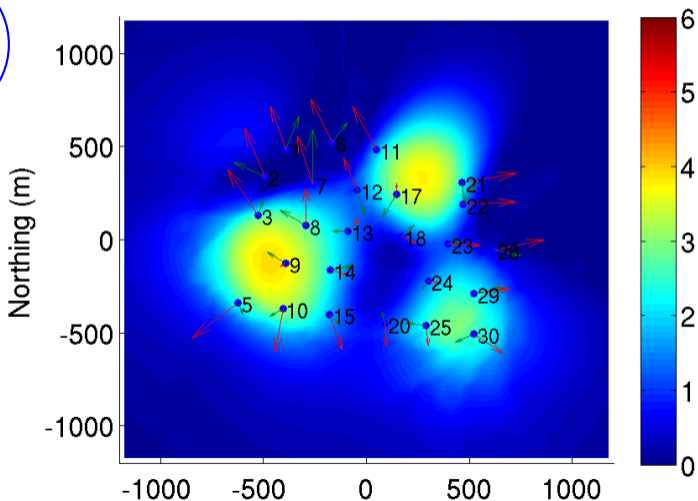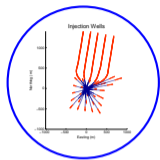
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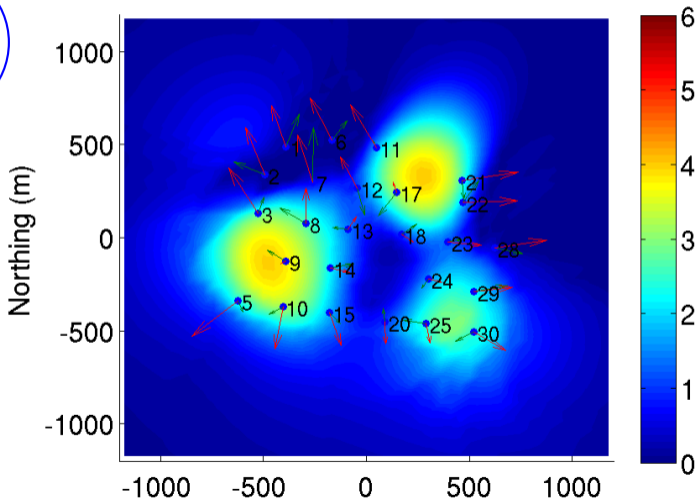
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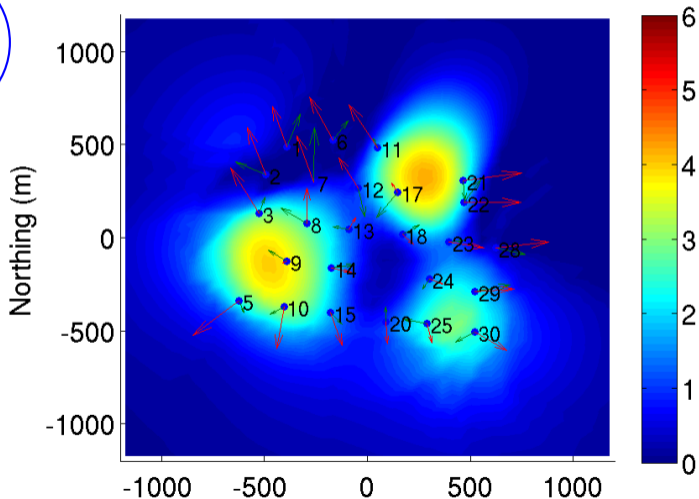
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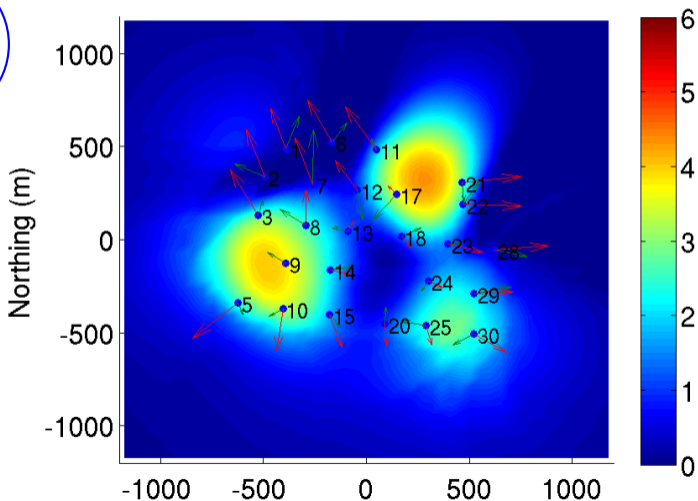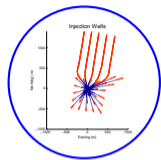
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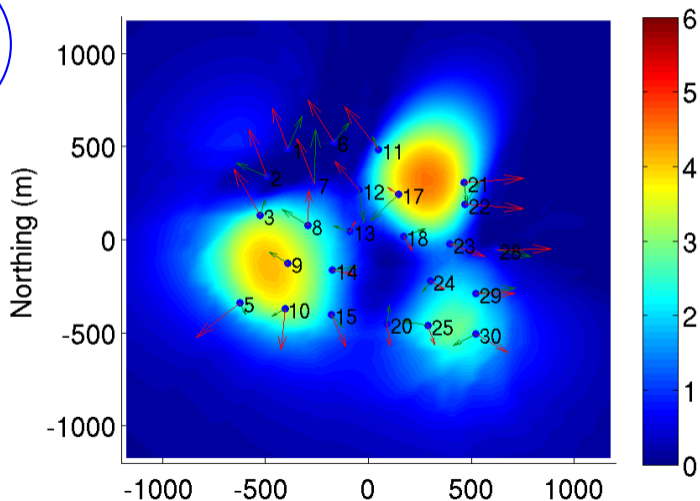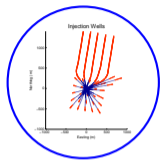
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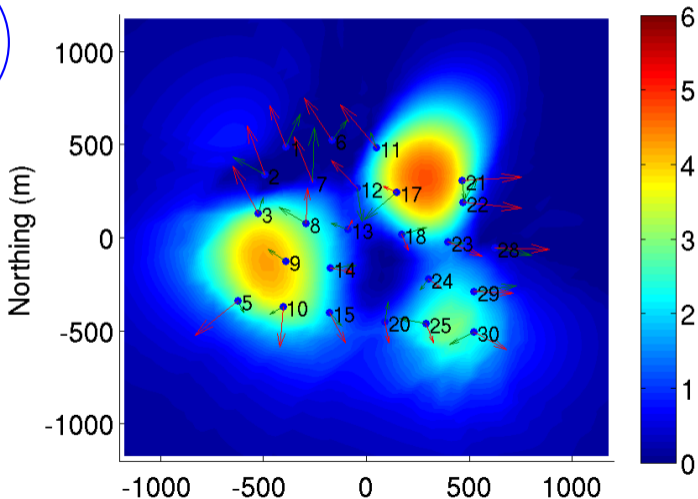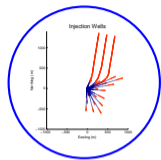
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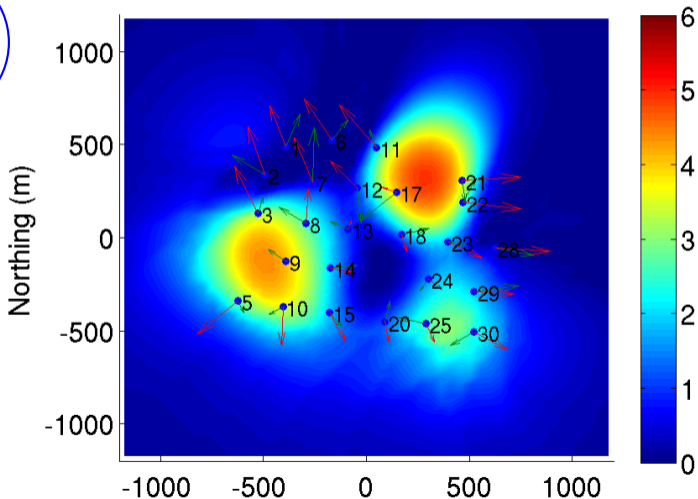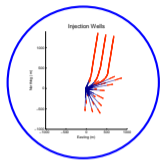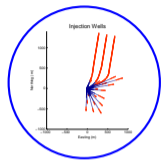
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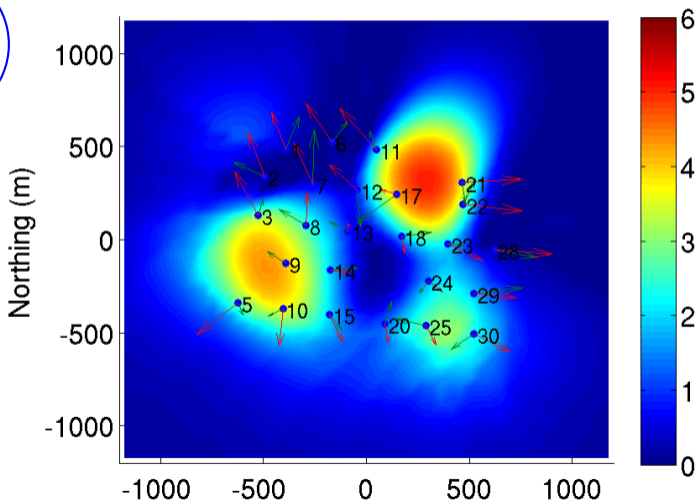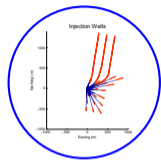
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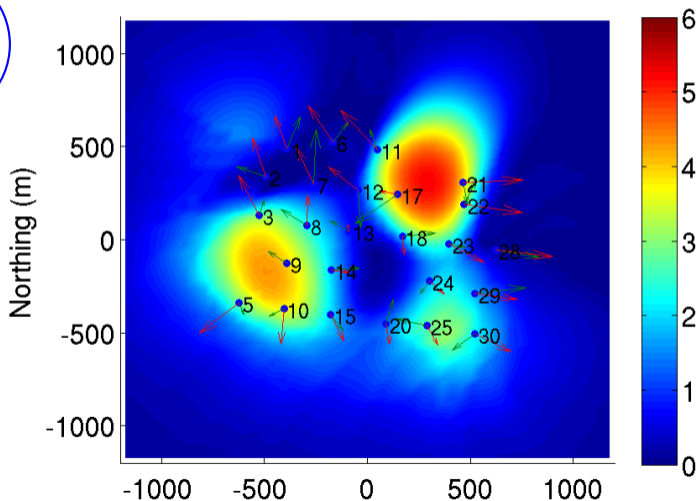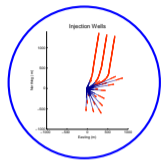
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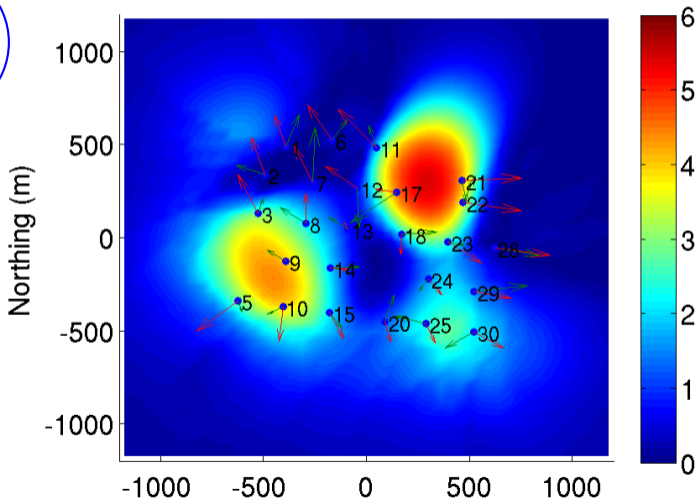
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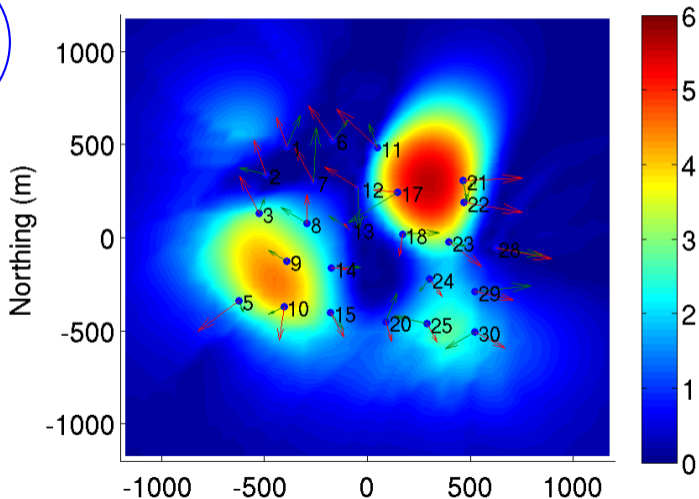
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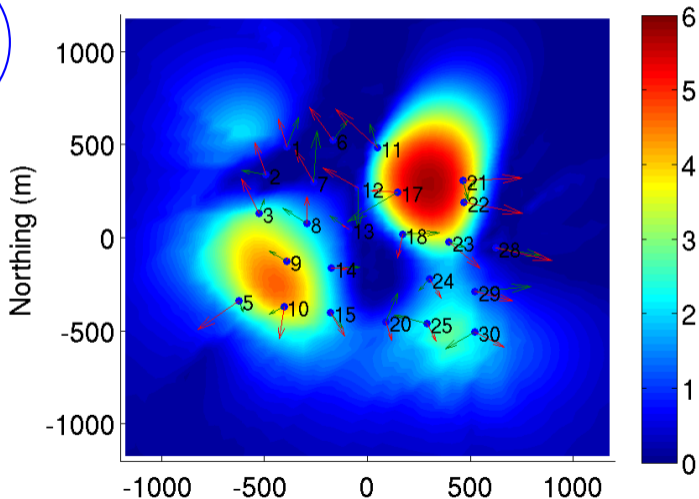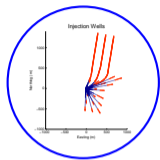
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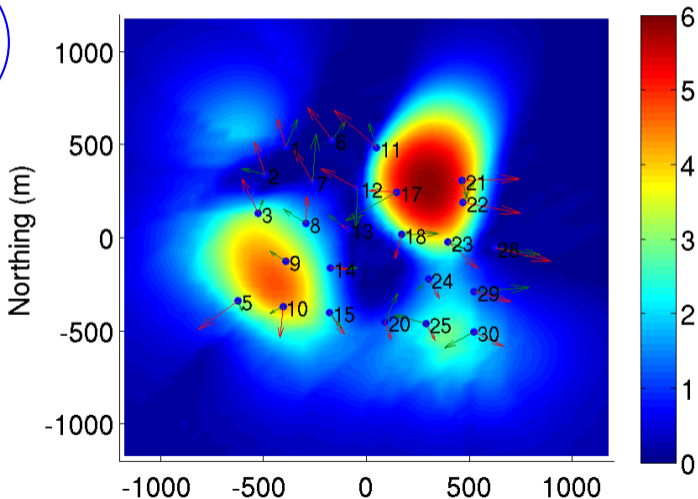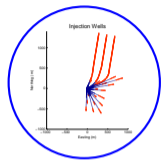
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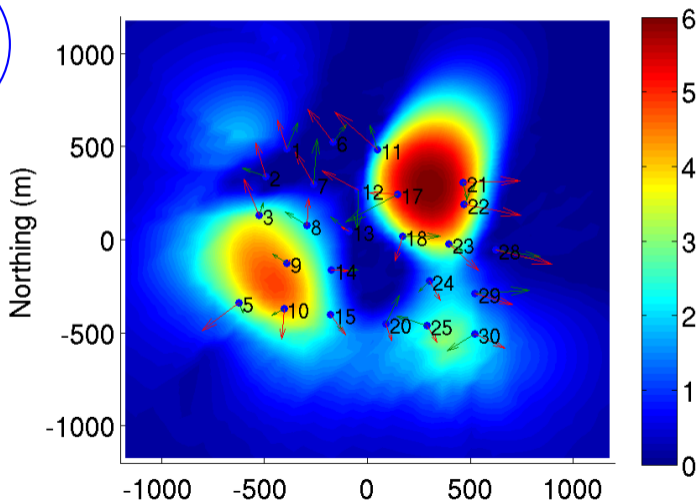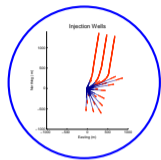
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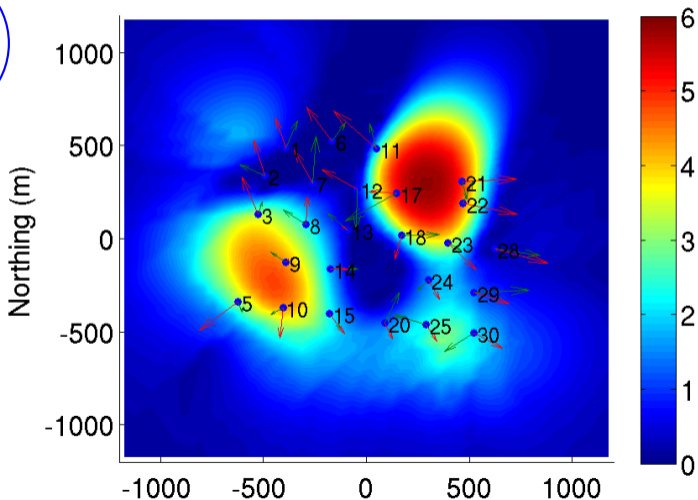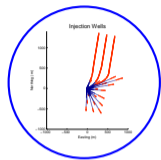
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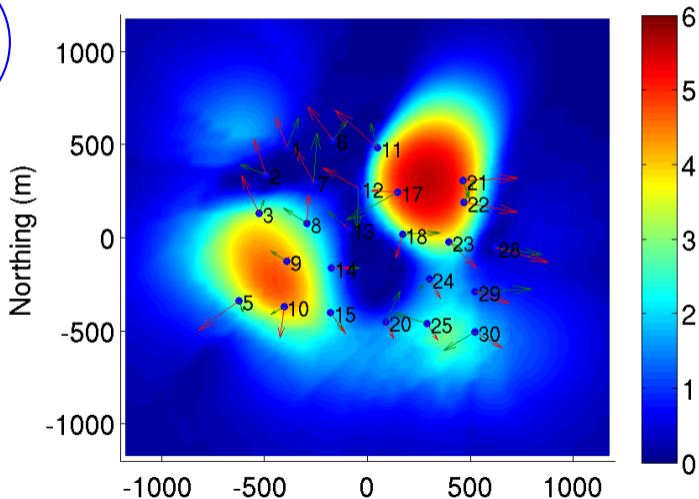
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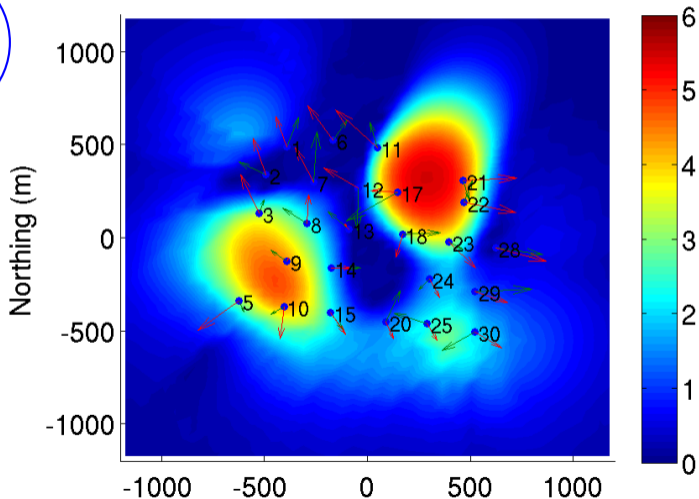
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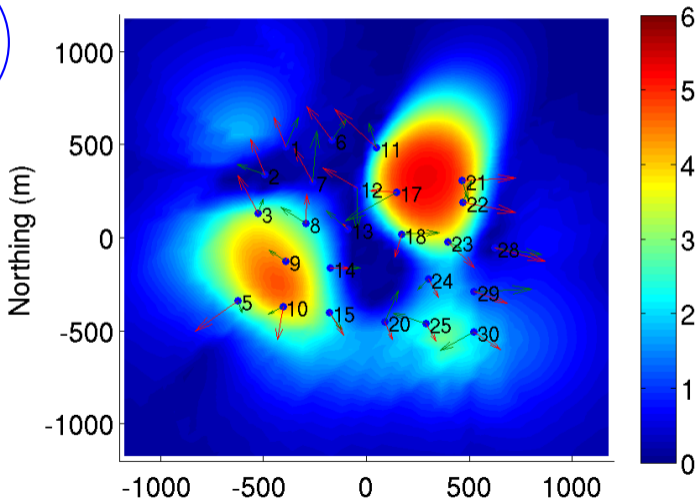
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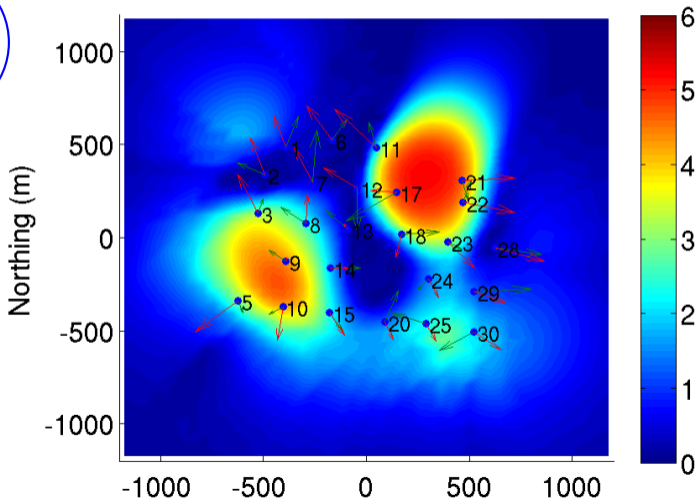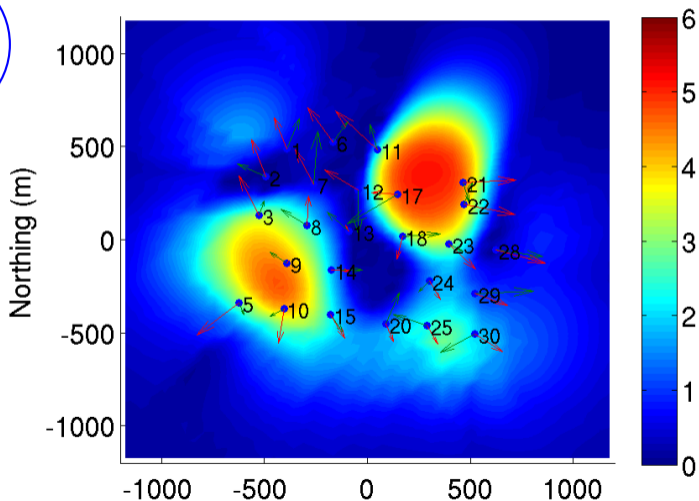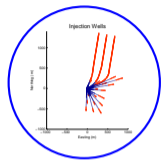
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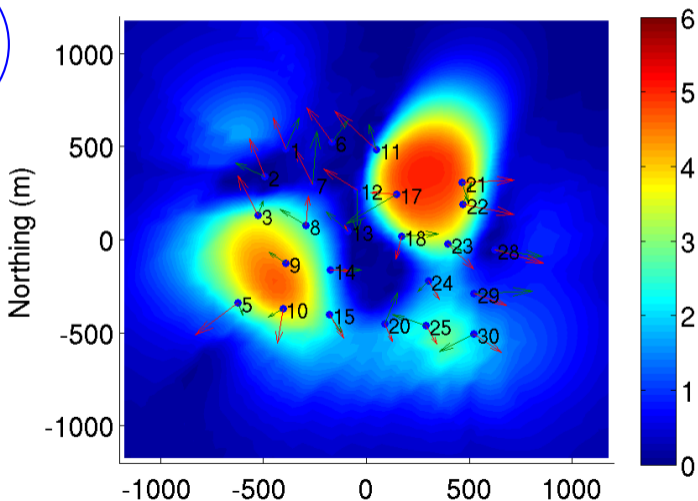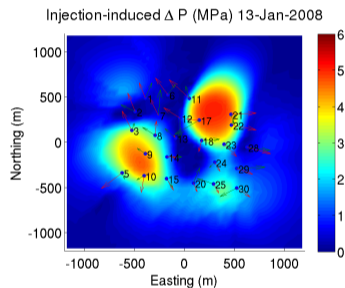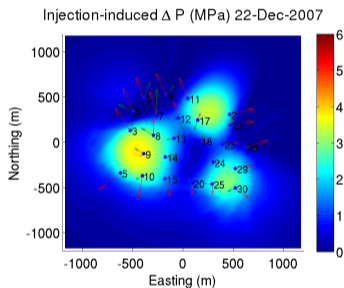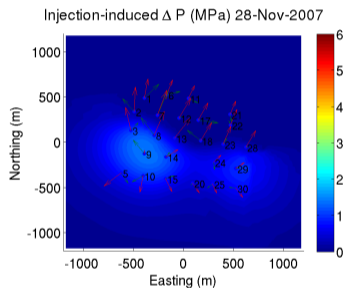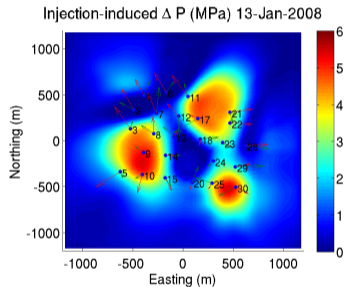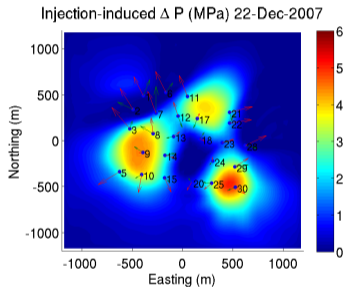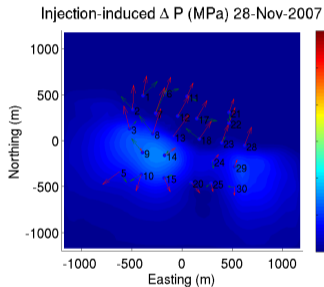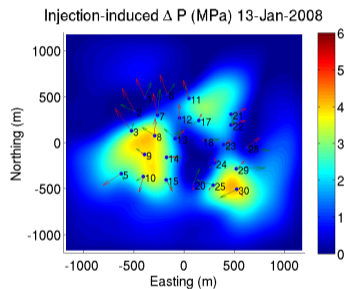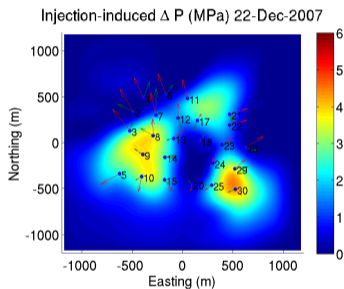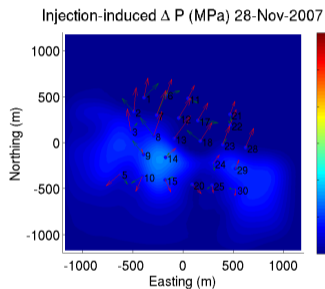
Resolving sharp pressure contrasts
(methodology and a synthetic example)

Achieve better resolution of subsurface heterogeneities (e.g, **permeability barriers**) by using edge-preserving total-variation regularization:

$$\|\mathbf{A}p - \mathbf{u}\|_{L_2}^2 + \boxed{\epsilon \|\|\nabla \mathbf{p}\|\|_{L_1}} \rightarrow \min,$$
$$p_1 \leq p \leq p_2. \qquad \text{(BOUNDTVREG)}$$

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Inversion results

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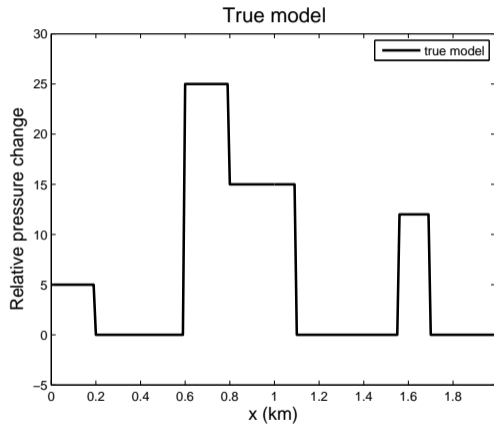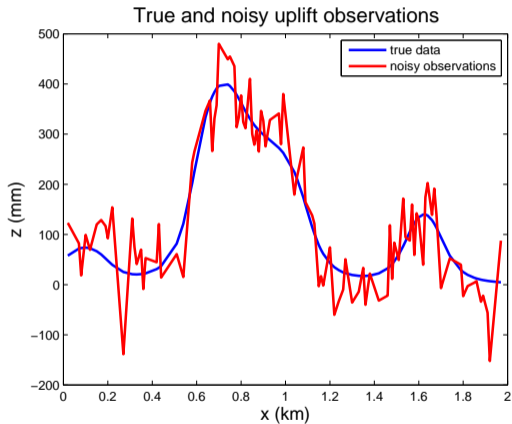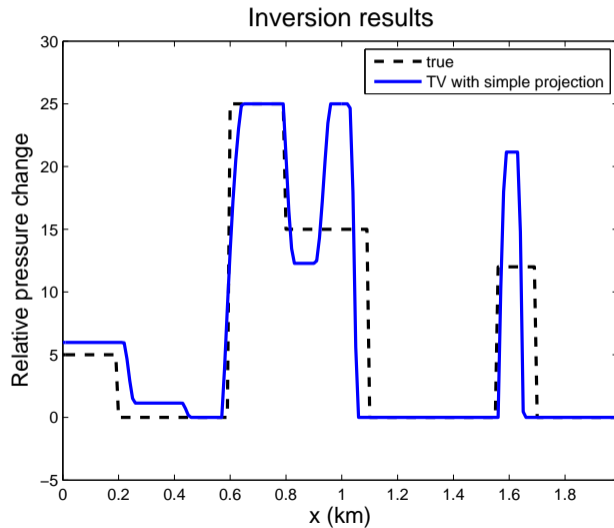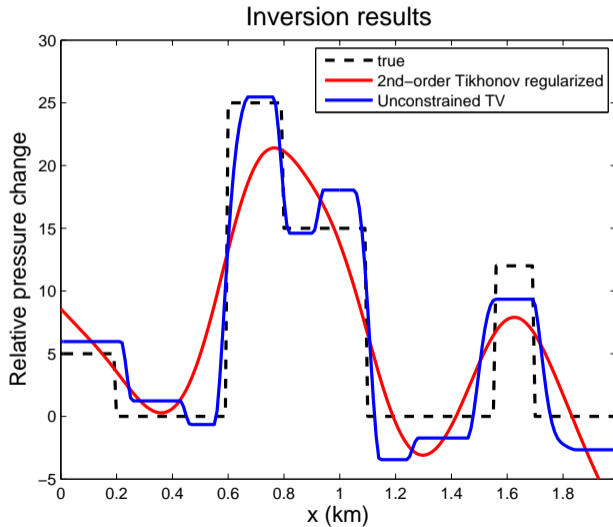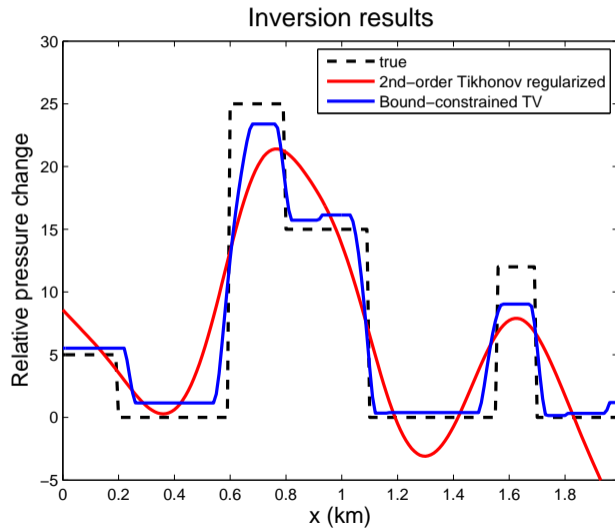
- Fluid injection is a key tool in EOR, waste water disposal, carbon sequestration, and hydraulic fracturing.

- Effective injection strategies requires understanding pressure front propagation and reservoir heterogeneity.

- Goal: design a robust computational framework for estimating **permeability and flow contrasts** from measurable surface deformation.

Formation permeability (md)

True Δ P (MPa), Day 2

Linear Poroealstic Medium Deformation

Relate deformation and pore pressure change using Biot's theory (Rice and Cleary, 1976; Segall, 1985; Segall, 2010):

$$\mu\nabla^2 u_i + \frac{\mu}{1-2\nu}\frac{\partial^2 u_j}{\partial x_i \partial x_j} = \boxed{\alpha\frac{\partial p}{\partial x_i}} - f_i = 0, \ i = 1, 2, 3, \qquad \text{(ELAST)}$$

and

$$S_\alpha\frac{\partial p}{\partial t} - \frac{\kappa}{\eta}\nabla^2 p = -\alpha\frac{\partial}{\partial t}\left(\nabla \cdot \mathbf{u}\right), \qquad \text{(FLOW)}$$

where $\mathbf{u}$ is displacement, p is the pore pressure change, f_i is a differential body-force distribution, μ, ν, α, κ, and η are the shear modulus, Poisson's ratio, Biot coefficient, permeability, and fluid viscosity;

$$S_\alpha = \frac{3\alpha(1-2\nu)(1-\alpha B)}{2\mu B(1+\nu)}, \qquad \text{(STORAGE)}$$

where B is Skempton's coefficient.

Linear Poroealstic Medium Deformation

Relate deformation and pore pressure change using Biot's theory (Rice and Cleary, 1976; Segall, 1985; Segall, 2010):

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where $\mathbf{u}$ is displacement, p is the pore pressure change, f_i is a differential body-force distribution, μ, ν, α, κ, and η are the shear modulus, Poisson's ratio, Biot coefficient, permeability, and fluid viscosity;

$$S_\alpha = \frac{3\alpha(1-2\nu)(1-\alpha B)}{2\mu B(1+\nu)}, \tag{STORAGE}$$

where B is Skempton's coefficient.

- Regard (ELAST) as a deformation modeling operator $\mathbf{A}p = \mathbf{u}$ and **decouple** it from the flow equation (FLOW) (Vasco et al., 2000; Du and Olson, 2001; Hodgson, 2007).

- Solve the regularized constrained optimization problem

$$\|\mathbf{A}p - \mathbf{u}\|_{L_2}^2 \; + \; \epsilon\|\Delta p\|_{L_2}^2 \; \to \; \min,$$
$$0 \; \leq \; p \; \leq \; p_{\max}. \tag{LS}$$

Achieve better resolution of subsurface heterogeneities (e.g, **permeability barriers**) by using edge-preserving total-variation regularization:

$$\|\mathbf{A}p - \mathbf{u}\|_{L_2}^2 \; + \; \boxed{\epsilon \||\nabla \mathbf{p}|\|_{L_1}} \; \rightarrow \; \min, \tag{TV}$$
$$0 \; \leq \; p \; \leq \; p_{\max}.$$

Achieve better resolution of subsurface heterogeneities (e.g, **permeability barriers**) by using edge-preserving total-variation regularization:

$$\|\mathbf{A}p - \mathbf{u}\|_{L_2}^2 \;+\; \boxed{\epsilon\|\|\nabla\mathbf{p}\|\|_{L_1}} \;\rightarrow\; \min,$$
$$0 \;\leq\; p \;\leq\; p_{\max}. \tag{TV}$$

True Δ P (MPa), Day 5

Uplift (m), Day 5

True Δ P (MPa), Day 10

Uplift (m), Day 10

True ΔP (MPa), Day 11

Uplift (m), Day 11

True Δ P (MPa), Day 17

Uplift (m), Day 17

True Δ P (MPa), Day 19

Uplift (m), Day 19

True Δ P (MPa), Day 21

Uplift (m), Day 21

True Δ P (MPa), Day 22

Uplift (m), Day 22

True ΔP (MPa), Day 23

Uplift (m), Day 23

True ΔP (MPa), Day 25

Uplift (m), Day 25

True ∆ P (MPa), Day 26

Uplift (m), Day 26

True Δ P (MPa), Day 27

Uplift (m), Day 27

True ΔP (MPa), Day 29

Uplift (m), Day 29

True ΔP (MPa), Day 31

Uplift (m), Day 31

True Δ P (MPa), Day 32

Uplift (m), Day 32

True Δ P (MPa), Day 34

Uplift (m), Day 34

True Δ P (MPa), Day 35

Uplift (m), Day 35

True Δ P (MPa), Day 36

Uplift (m), Day 36

True Δ P (MPa), Day 37

Uplift (m), Day 37

True Δ P (MPa), Day 42

Uplift (m), Day 42

True Δ P (MPa), Day 45

Uplift (m), Day 45

True Δ P (MPa), Day 46

Uplift (m), Day 46

True Δ P (MPa), Day 47

Uplift (m), Day 47

True Δ P (MPa), Day 49

Uplift (m), Day 49

True ΔP (MPa), Day 50

Uplift (m), Day 50

True Δ P (MPa), Day 51

Uplift (m), Day 51

True Δ P (MPa), Day 52

Uplift (m), Day 52

True Δ P (MPa), Day 54

Uplift (m), Day 54

True Δ P (MPa), Day 55

Uplift (m), Day 55

True Δ P (MPa), Day 56

Uplift (m), Day 56

True Δ P (MPa), Day 60

Uplift (m), Day 60

True ΔP (MPa), Day 25

ΔP (Mpa), day 25

ΔP (Mpa), day 25

True Δ P (MPa), Day 34   Δ P (Mpa), day 34   Δ P (Mpa), day 34

1. TV-regularization with bound constraints $\Rightarrow$ location of **sharp** pressure contrasts.

2. Combine linear poroelastostatic deformation model with a "fluid-filled chamber" model to simulate **fluid-conducting faults**.

3. Not limited to directly measurable deformation; e.g., can estimate induced deformation from time-lapse (Hodgson, 2007).

4. Application to **InSAR** field data.

- **My family**

- Principal advisor, Professor **Biondo Biondi**

- Second project advisor, Professor **Mark Zoback**

- Professor **Jon Claerbout**

Acknowledgements—Part I, seismic imaging

- **Stewart Levin**, **Mark Meadows**, and **Paul Segall**

- **Shuki Ronen**, **Robert Clapp**, **Clement Kostov**, **Joseph Stefani**, **Dimitri Bevc**, **John Etgen**, **Matthew Karazincir**, **Phuong Vu**, **Min Zhou**, **Jean Paul Gestel**, **Joseph Dellinger**, **Jean Virieux**, **Jack Dvorkin**, **Gary Mavko**, **James Mika**, **Olav Barkved**, and **David Nichols**

- My SEP colleagues for our dynamic and stimulting work environment

- Special thanks to **Chevron**, **ExxonMobil**, and **BHP Billiton** for providing Genesis field seismic data and permission to publish my work

- Affiliate members of **Stanford Exploration Project** for their support

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Acknowledgements—Part II, geomechanics

- **Stewart Levin**, **Mark Meadows**, and **Paul Segall**

- **Ran Bachrach**, **Joseph Stefani**, **Jean Paul Gestel**, **Jack Dvorkin**, **Gary Mavko**, **Amos Nur**, **Olav Barkved**, **David Nichols**, and **Randi Walters**

- Affiliate members of **Stanford Exploration Project** and **Stanford Rock Physics and Borehole Geophysics** for their support

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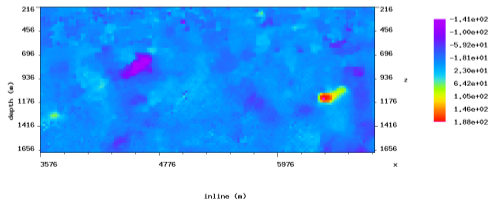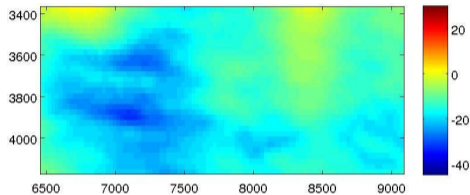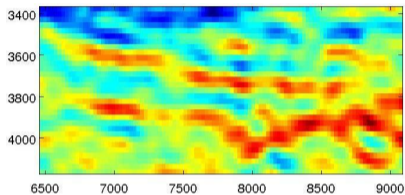
Q&A

Appendices – discussion slides

**Model decomposition into "blocky" and "wiggly" components
= cartoon+texture decomposition (Meyer, 2001):**

$$\min_{\mathbf{m},\mathbf{m}_b} \|\mathbf{F}(\mathbf{m}) - \mathbf{d}\|_2^2 + \alpha\|\|\nabla\mathbf{m}_b\|\|_1 + \beta\|\mathbf{R}\mathbf{m}_w\|_2^2, \mathbf{m} = \mathbf{m}_b + \mathbf{m}_w, \quad (7)$$

where

$$\mathbf{R} = \mathbf{I} \text{ or } \boldsymbol{\Delta}.$$
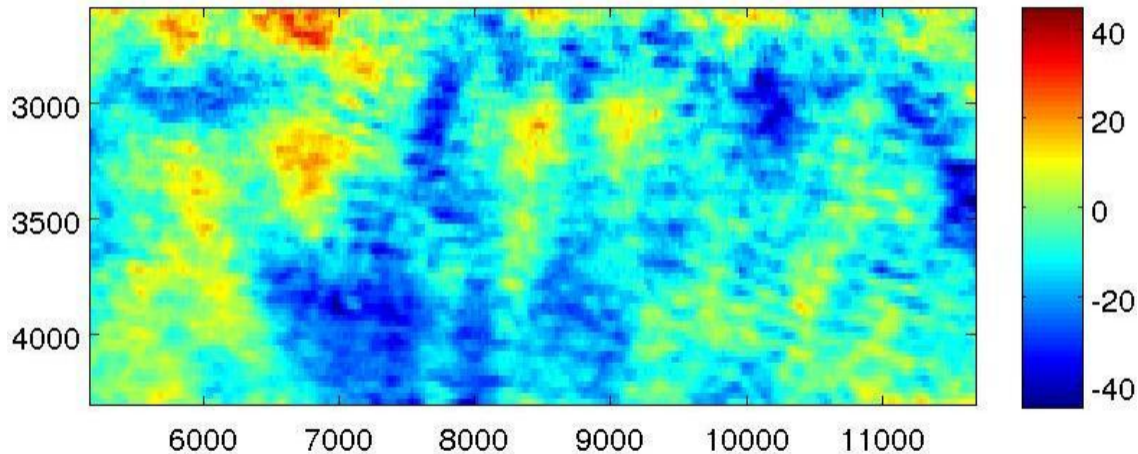
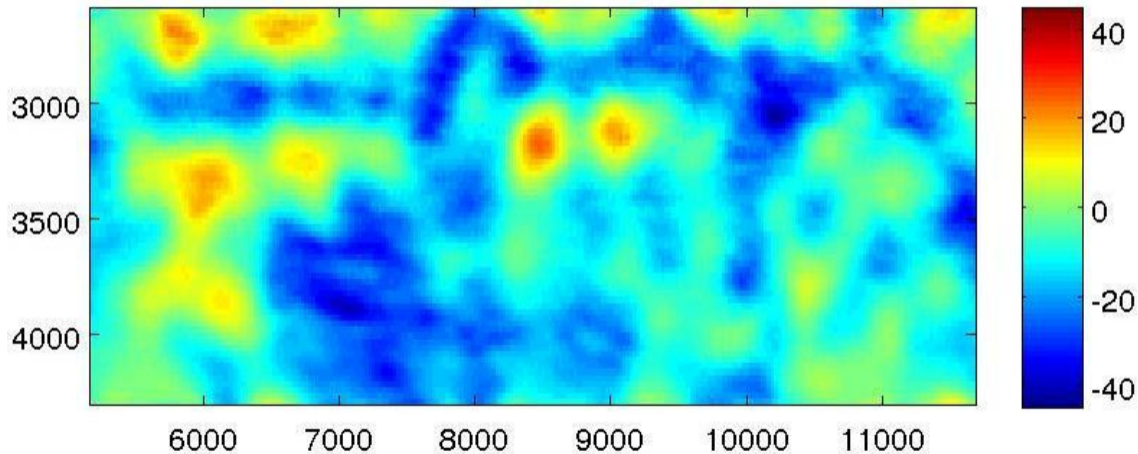$$\frac{d\tau}{dt} \approx \frac{\Delta t}{t} = \frac{\Delta z}{z} - \frac{\Delta v}{v}, \tag{8}$$
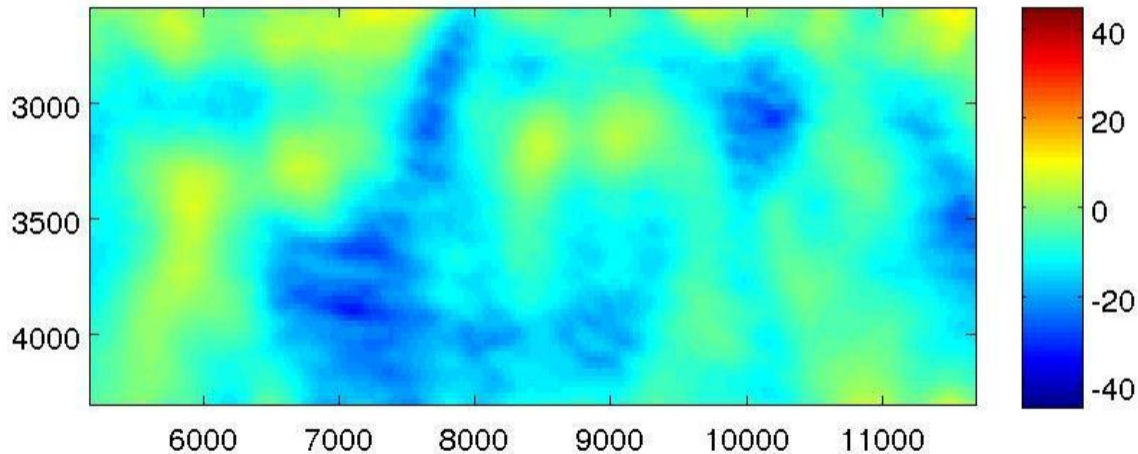
$$\frac{\Delta v}{v} = -R\frac{\Delta z}{z}, \tag{9}$$

$$\frac{\Delta v}{v} = -\frac{R}{R+1}\frac{\Delta t}{t} \approx -\frac{\Delta t}{t} \approx -\frac{d\tau}{dt}. \tag{10}$$

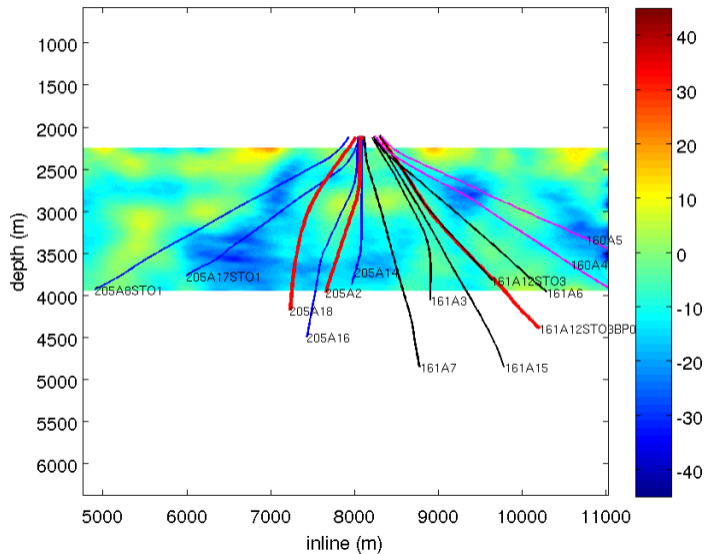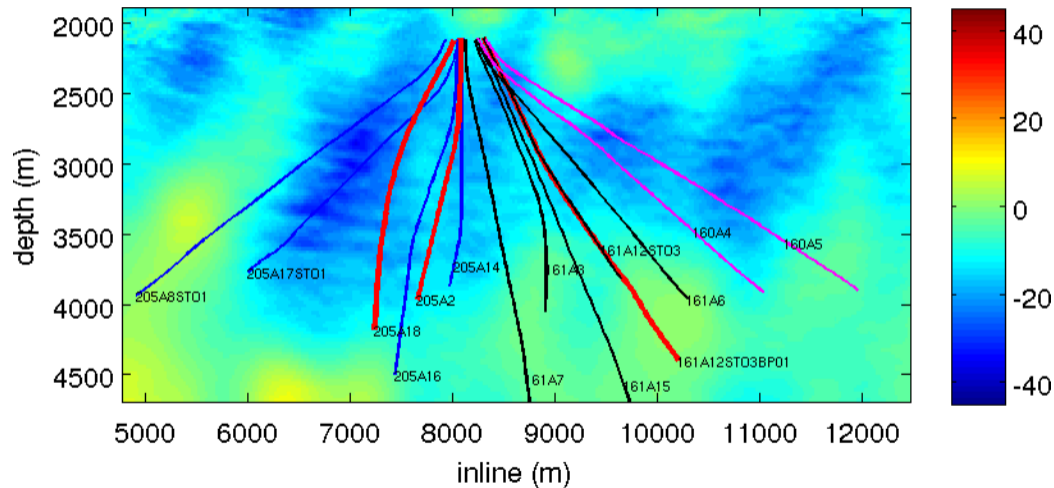$$\Delta v \approx -.02 \times 2,800 \text{ m/s} = -56 \text{ m/s}. \tag{11}$$
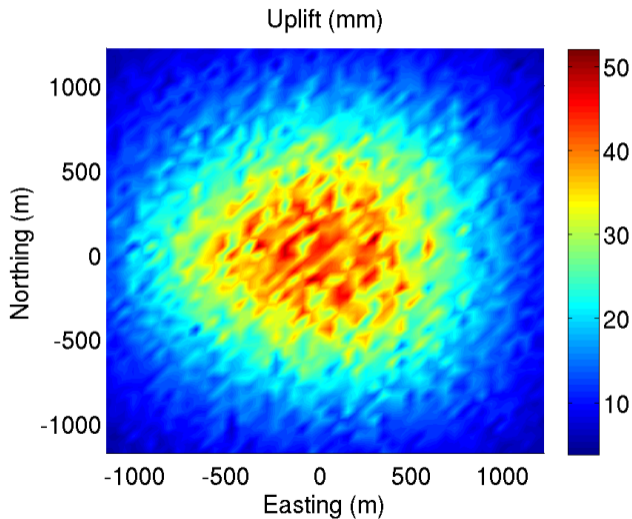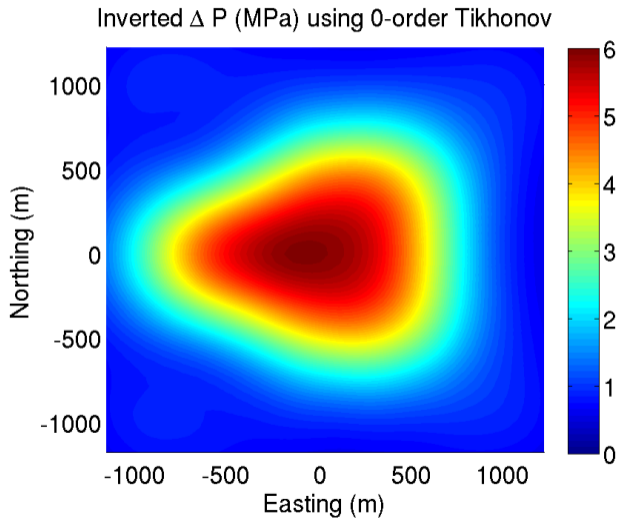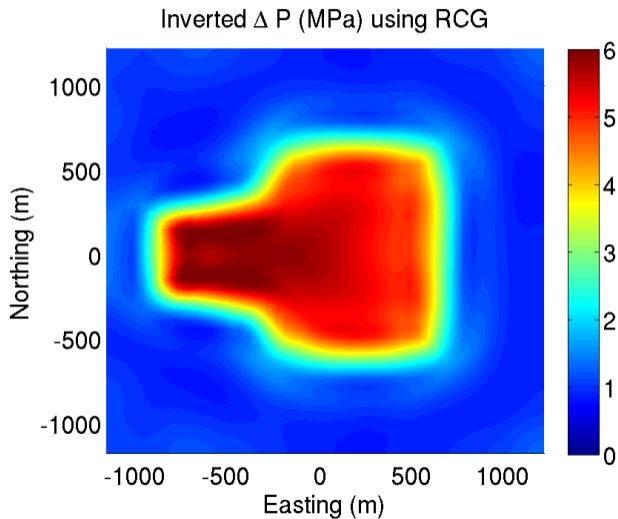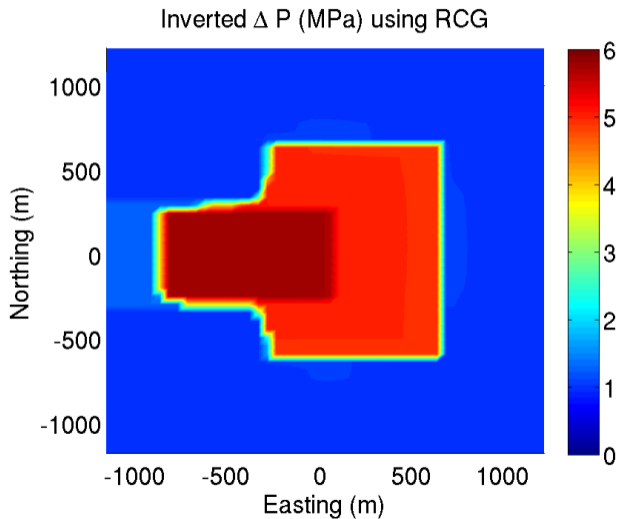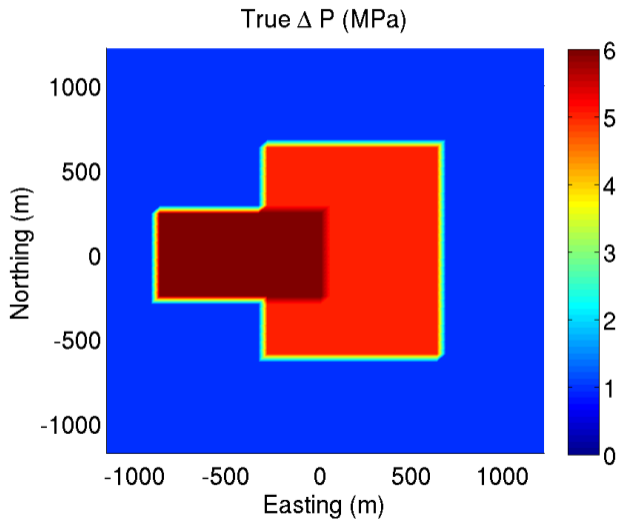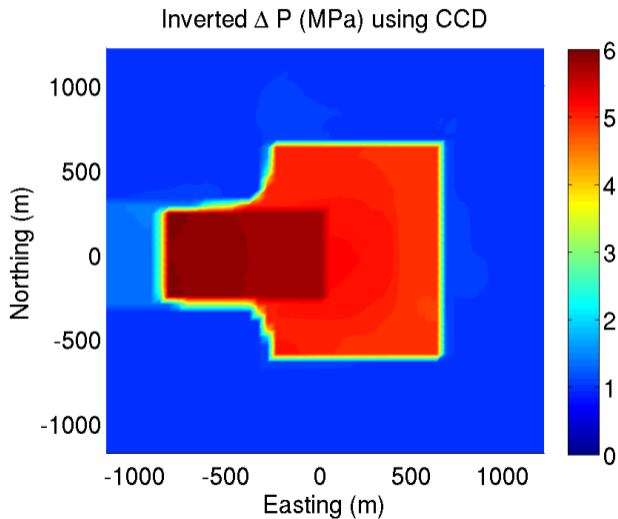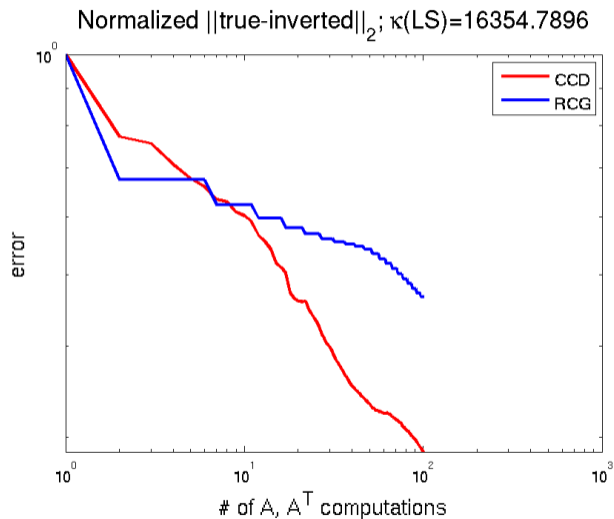
Efficient solution of L_1-TV regularized problems

True Δ P (MPa)

Inverted Δ P (MPa) using 0-order Tikhonov

Inverted Δ P (MPa) using RCG

Inverted Δ P (MPa) using RCG

True Δ P (MPa)

Inverted Δ P (MPa) using CCD

Normalized $||\text{true-inverted}||_2$; $\kappa(\text{LS})=16354.7896$

Appendix: shortcuts