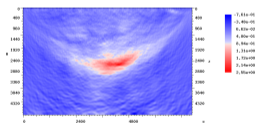


Subsurface model inversion

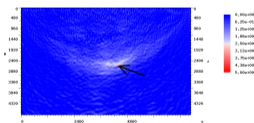
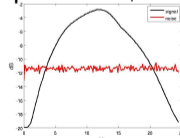
Pushing the limits of subsurface resolution

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- Fluid movement \Rightarrow subsurface velocity changes.
- Stress changes \Rightarrow subsurface velocity changes.
- **Simultaneous regularized 4D FWI** can resolve significant model changes at a reservoir/overburden scale (Maharramov et al, 2016).
- **Question:** Can FWI **resolve subtle model changes** associated with continuous subsurface stressing?
- **Question:** Can FWI **resolve relative magnitudes of changes** to understand the evolution of subsurface stress?



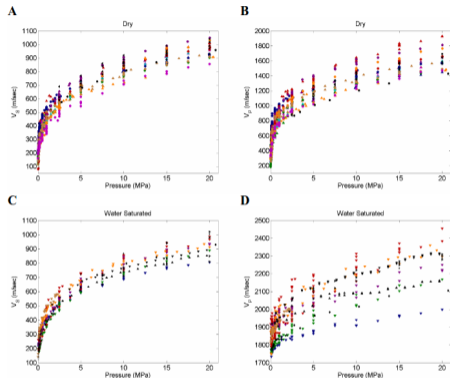
- Dependence of acoustic velocity V on isotropic effective stress P can be described empirically:

$$V = V_{\infty} \left(1 - A \exp -\frac{P}{P_0} \right), \quad (1)$$

where V_{∞} , A , P_0 are positive fitting constants for various types of rocks (Domenico, 1977; Zimmer, 2003; Lee, 2003; Johnston, 2013).

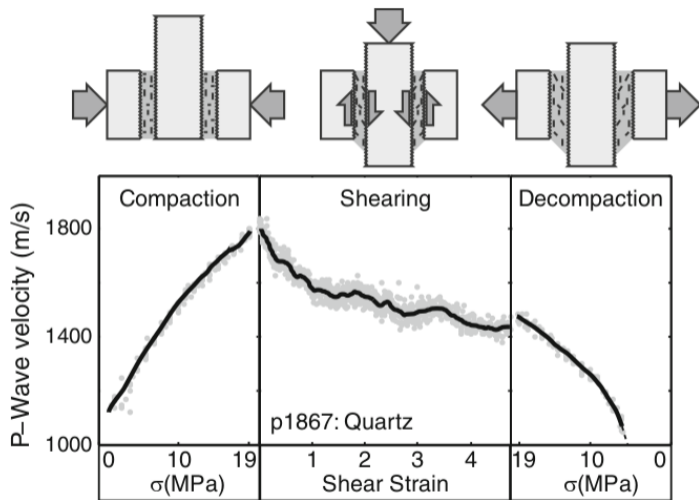
- Note the **flattening of velocity for large effective stress**.
- High-stress anomalies may be localized and **“spiky”**, e.g. near the tips of slipping faults.

Response to effective stress change (from Zimmer (2003))



- ▲ Galveston - Dry
- ▼ Galveston - Water Sat.
- ▲ Gulf of Mexico - Dry
- ▼ Gulf of Mexico - Water Sat.
- ▲ Merritt - Dry
- ▼ Merritt - Water Sat.
- ▲ Pomponio - Dry
- ▼ Pomponio - Water Sat.
- ▲ Sa Dry 1
- ▼ Sa Dry 2
- ▲ Sa Wet 3
- ▼ Sa Wet 4
- ▲ Sa Big
- ▼ Sa 35% Small
- GB Big
- GB 35% Small
- GB Tiny
- GB 35% Tiny 1
- GB 35% Tiny 2
- GB Broad
- Domenico (Ottawa) - Dry
- Domenico (Ottawa) - Water Sat.
- Yin (Ottawa) - Dry
- Yin (Ottawa) - Water Sat.
- ▼ Prasad P2 - Water Sat.
- ▲ Prasad P6 - Water Sat.
- Estes - Dry
- Estes - Water Sat.
- Robertson Dense (Ottawa) - Water Sat.

Response of loose quartz layers to shear (from Knuth et al. (2013))





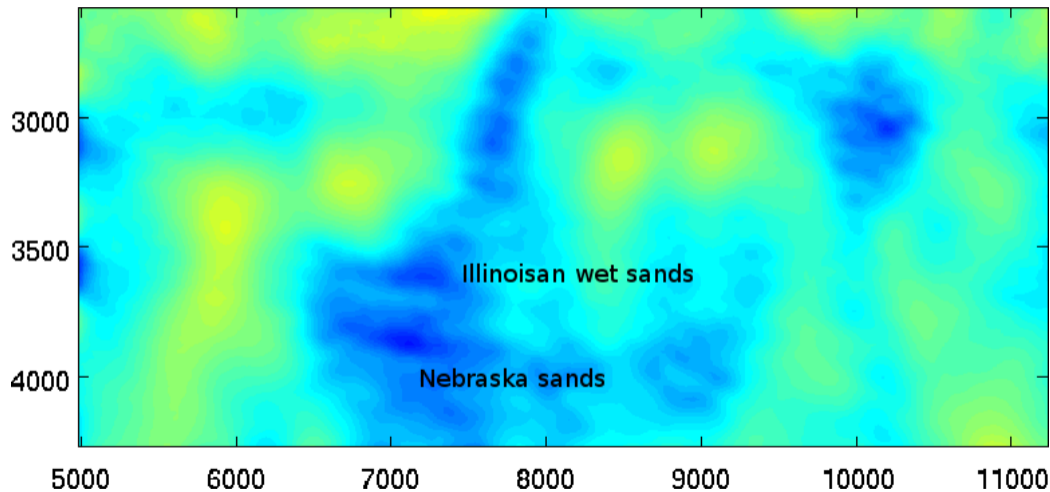
- ▶ **STAGE 1:** *Simultaneous FWI* of baseline and monitor with the *total-variation* (TV) difference regularization (Maharramov et al, 2016):

$$\text{Baseline Misfit} + \text{Monitor Misfit} + \quad (2)$$

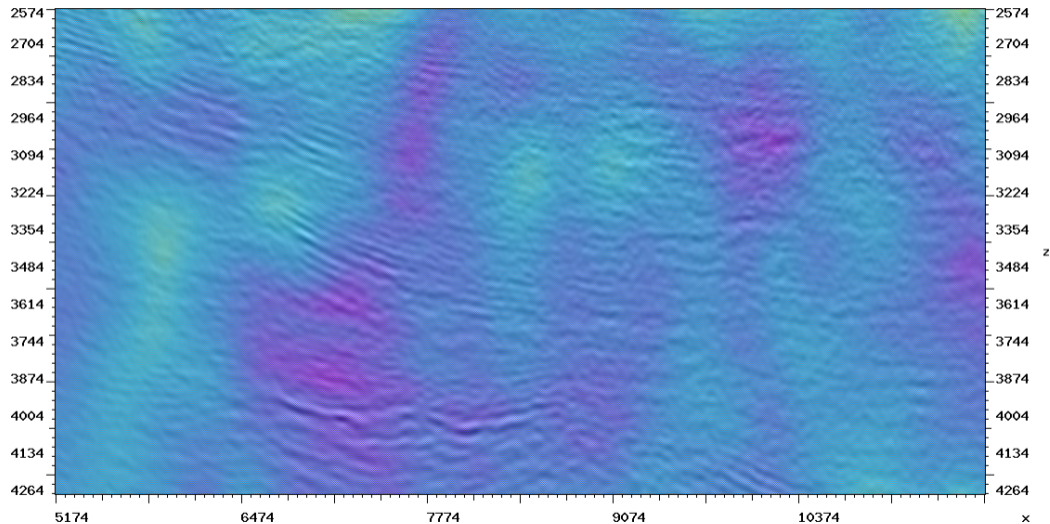
$$\alpha \|\|\nabla \mathbf{W}_m [\mathbf{m}_m - \mathbf{m}_b]\|\|_1. \quad (3)$$

- ▶ The total-variation (TV) seminorm (3) promotes model **blockiness** while reducing spurious oscillations.

Field data example: “blocky” Δv due to overburden dilation

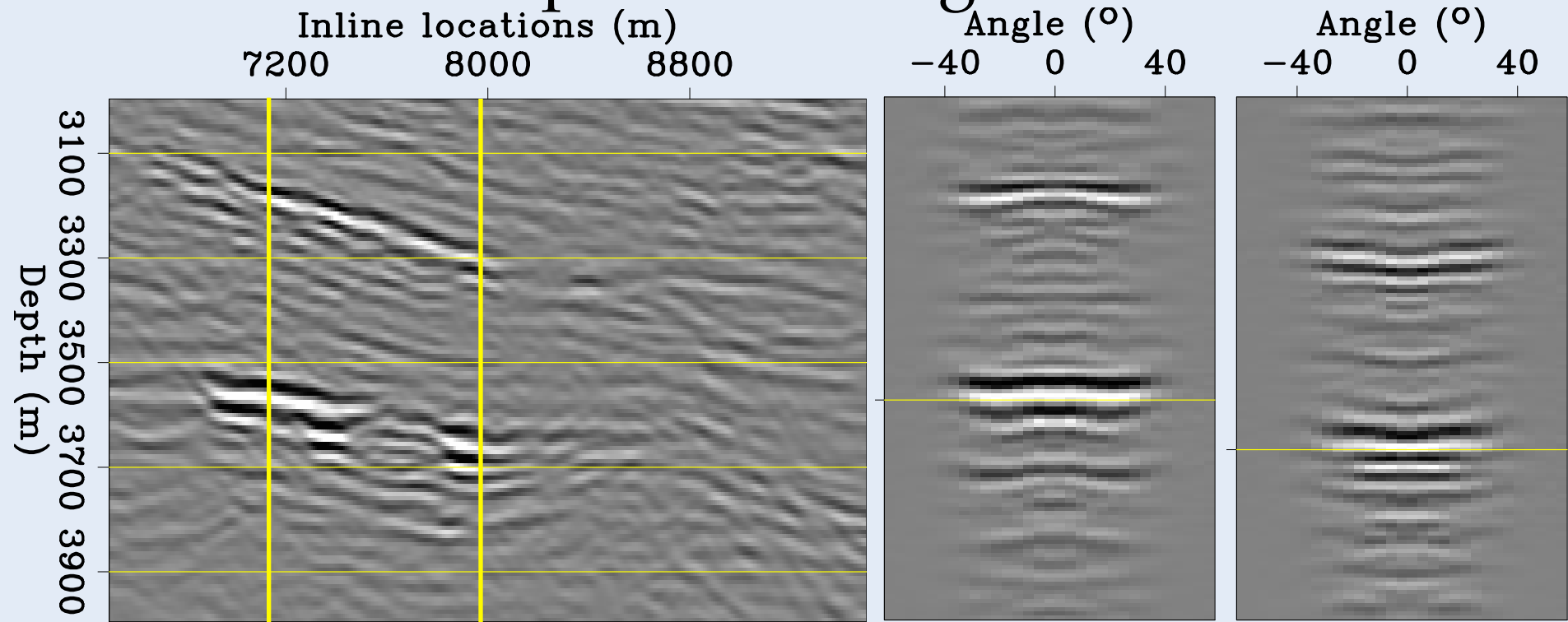


Field data: Δs vs image difference (Maharramov et al, 2016)



Analysis of depth-shifts in migrated images

prestack images

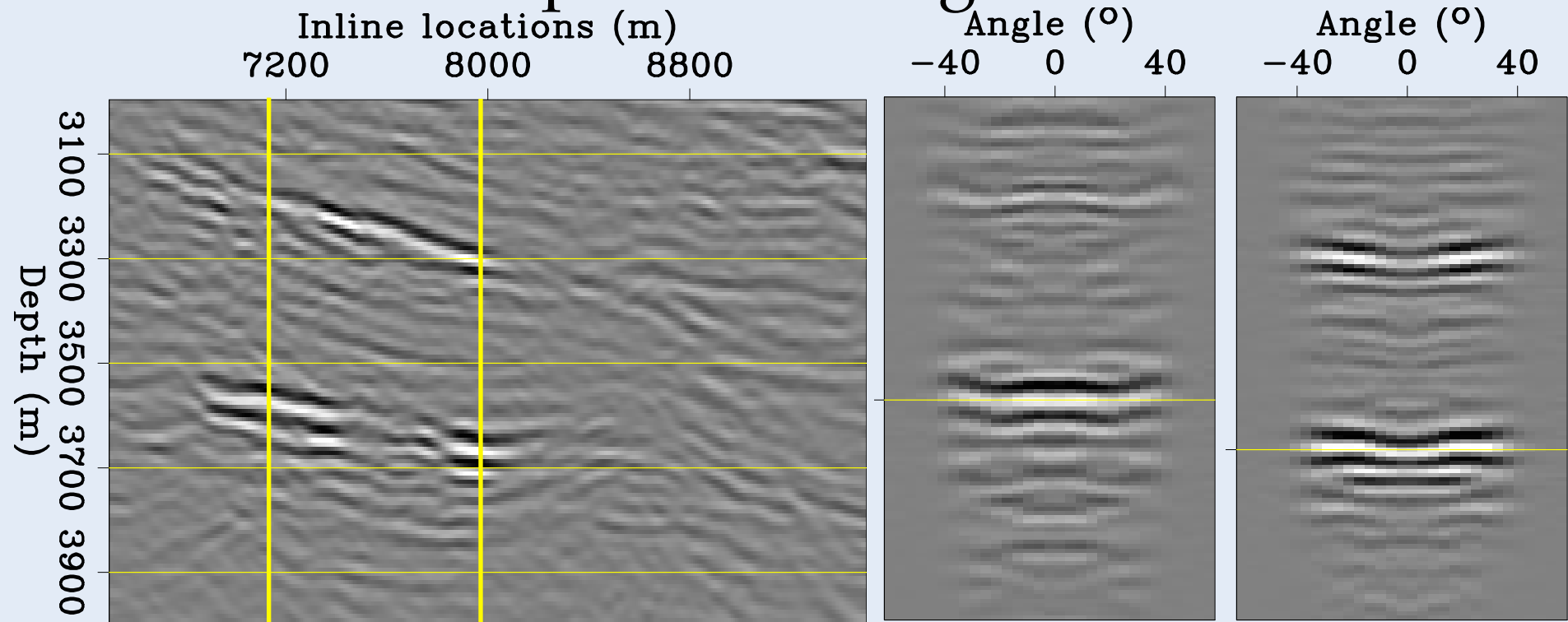


Anisotropic migration of **baseline**

Angle gathers of **baseline**

Analysis of depth-shifts in migrated images

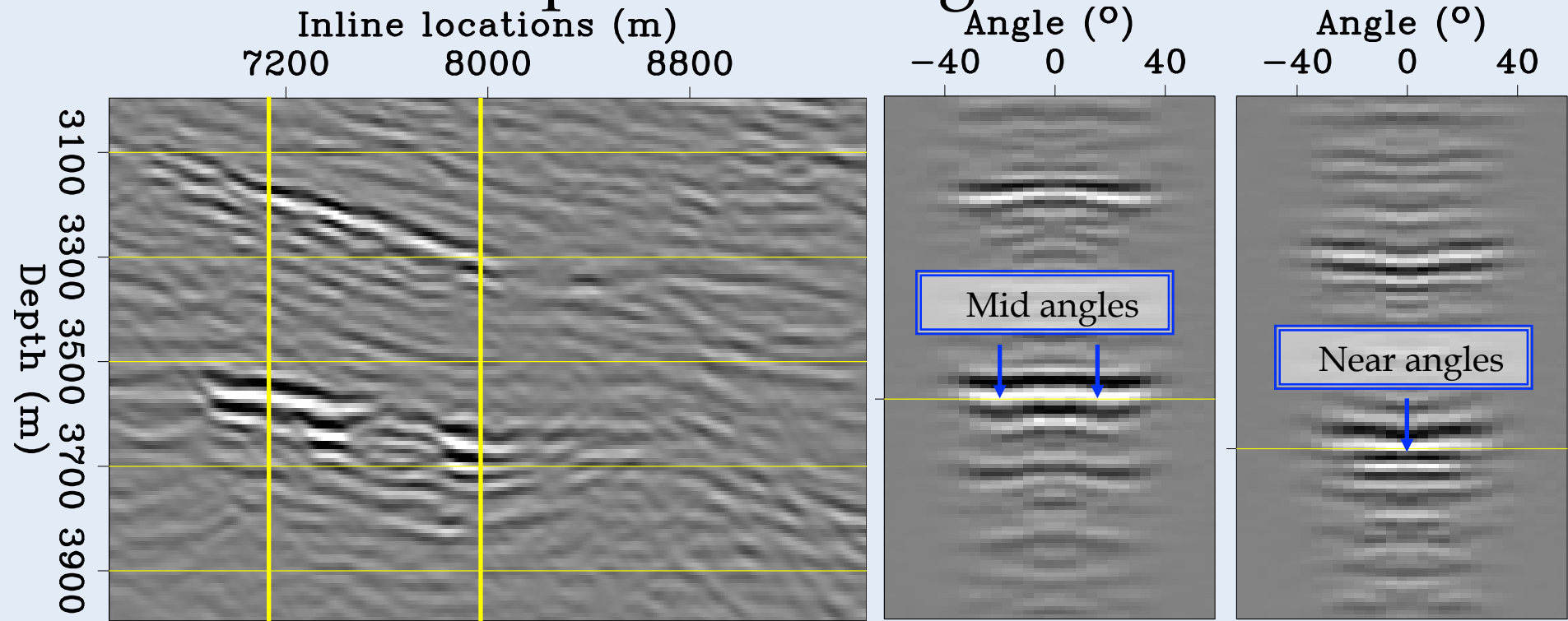
prestack images



Anisotropic migration of **monitor**

Angle gathers of **monitor**

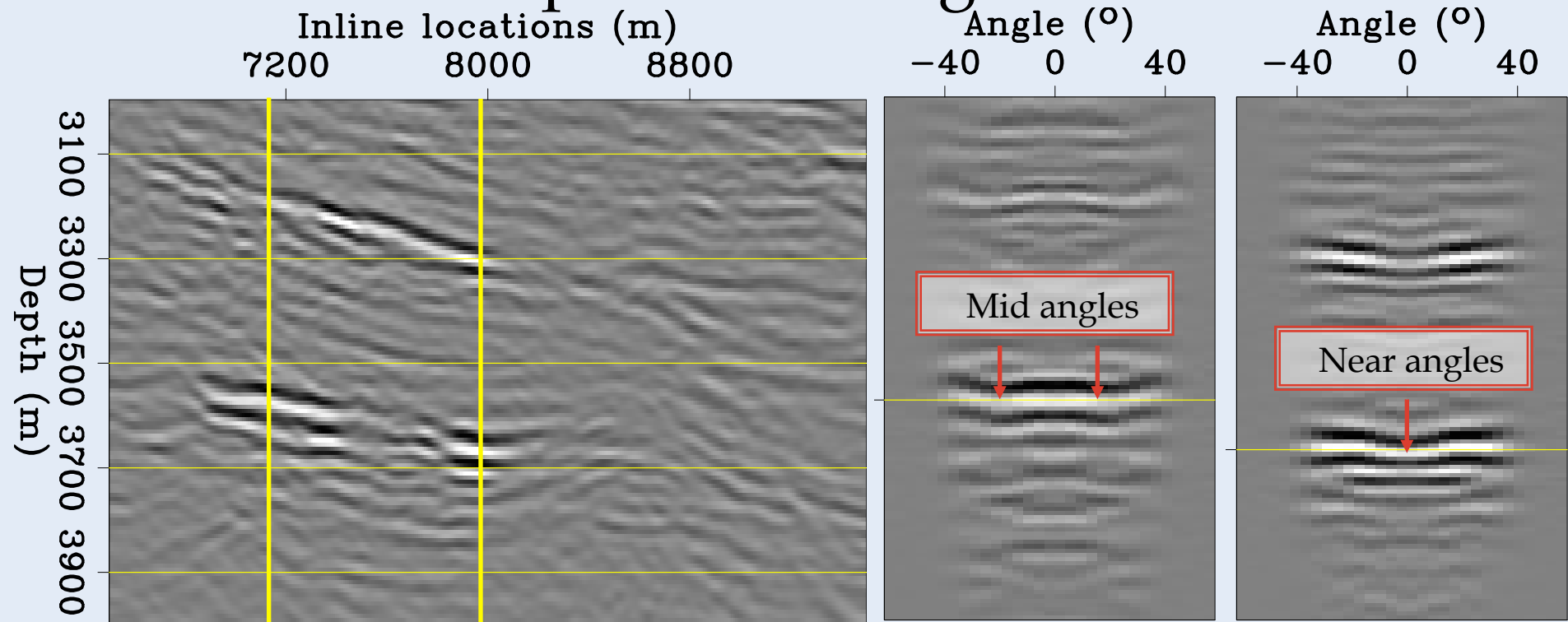
Analysis of depth-shifts in migrated images prestack images



Anisotropic migration of **baseline**

Angle gathers of **baseline**

Analysis of depth-shifts in migrated images prestack images



Anisotropic migration of **monitor**

Angle gathers of **monitor**



- ▶ **STAGE 2:** *Simultaneous FWI* of baseline and monitor with the L_1 difference regularization:

$$\text{Baseline Misfit} + \text{Monitor Misfit} + \quad (4)$$

$$\beta \|\mathbf{W}_m [\mathbf{m}_m - \mathbf{m}_b]\|_1. \quad (5)$$

- ▶ The L_1 norm (5) provides a **sparsifying** regularization that promotes model sparsity while still reducing spurious oscillations.



- ▶ 4D phase-only FWI is **to a first order \approx tomographic 4D inversion** (Maharramov and Biondi, 2016):

$$\begin{aligned}\|\mathbf{A}\delta\mathbf{s} - \delta\boldsymbol{\tau}\|_2 &< \sigma, \\ \|\delta\mathbf{s}\|_0 &= k,\end{aligned}\tag{6}$$

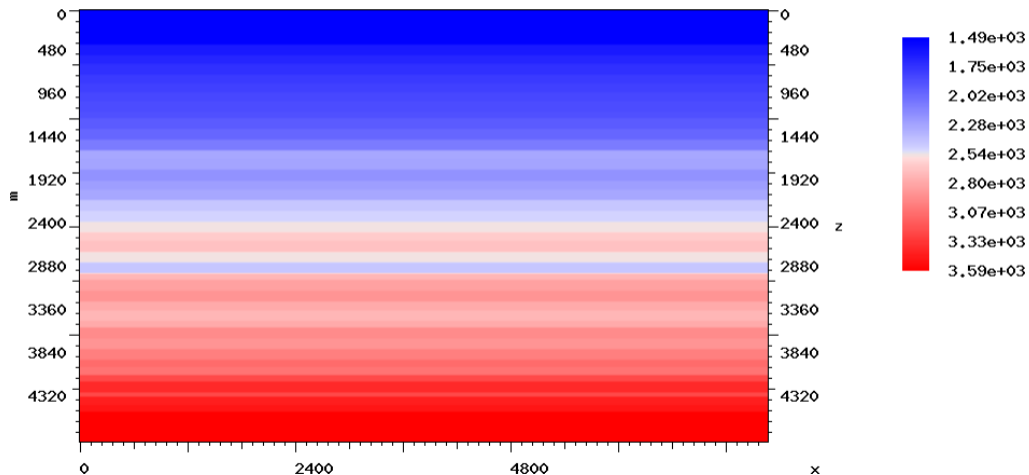
where $\delta\boldsymbol{\tau}$ = observed time shifts, $\delta\mathbf{s}$ is the unknown slowness change, \mathbf{A} is the travel-time modeling operator, σ is the 2-norm of measurement errors.

- ▶ Any minimizer $\delta\mathbf{s}$ of (6) satisfies the estimate:

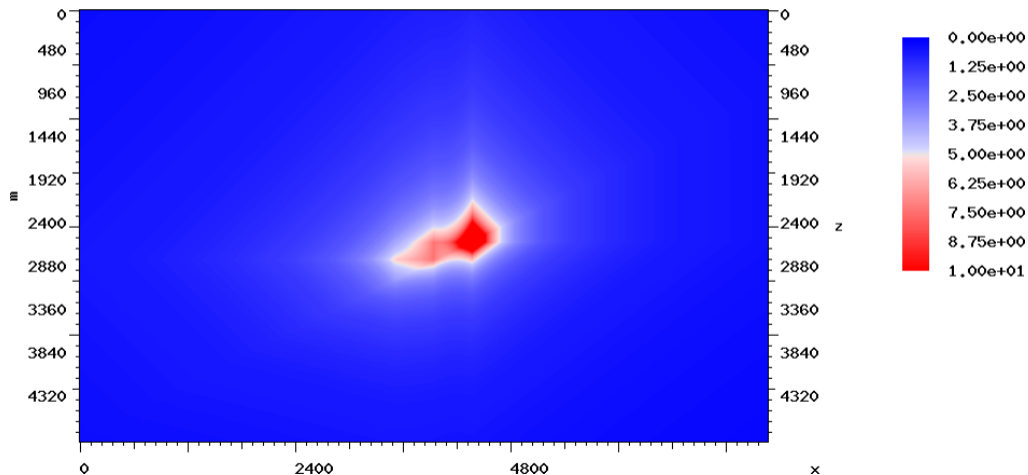
$$\|\delta\mathbf{s} - \delta\mathbf{s}_0\|_2 \leq \frac{2}{c_{2k}}\sigma,\tag{7}$$

where the lower restricted isometry constant $c_{2k} = \min_{J:|J|=2k} \lambda_{\min}(\mathbf{A}_J)$.

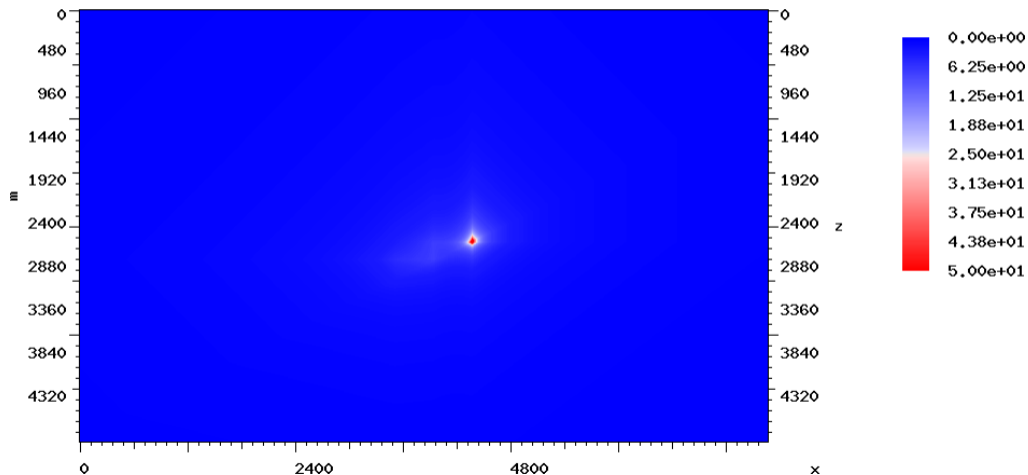
- ▶ L_1 -regularized inversion (4,5) approximates L_0 regularized inversion (Elad, 2010).
- ▶ **Continuous observations boost qualitative resolution** (the “peaks” of $\delta\mathbf{s}$ will stand out).



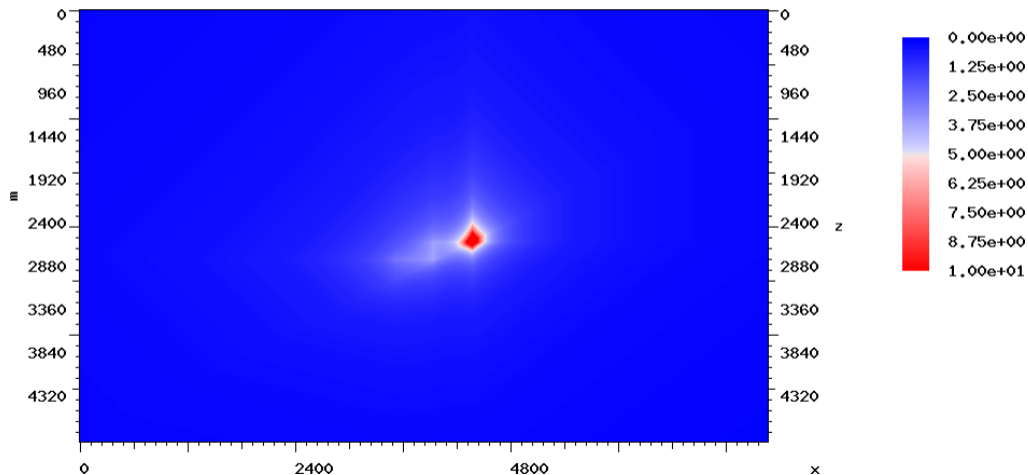
True monitor-baseline model difference at 10 m/s scale



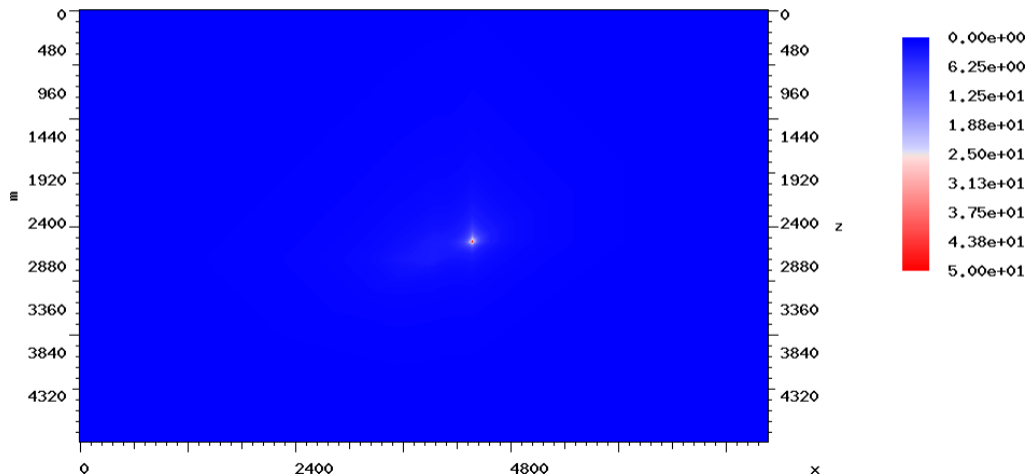
True monitor-baseline difference at 50 m/s scale

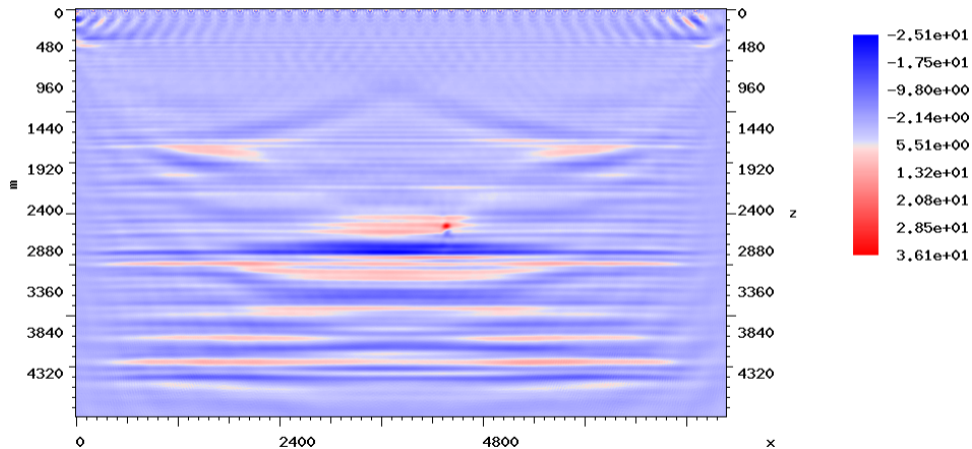


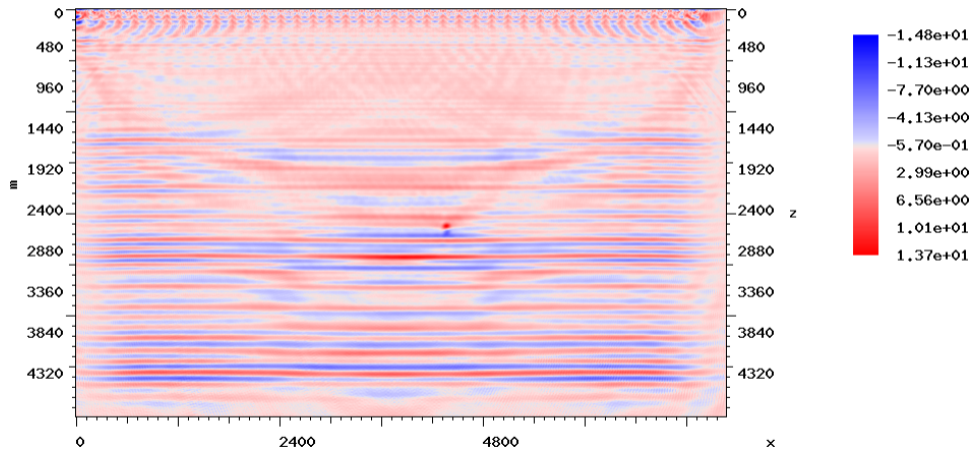
True second-first monitor model difference at 10 m/s scale

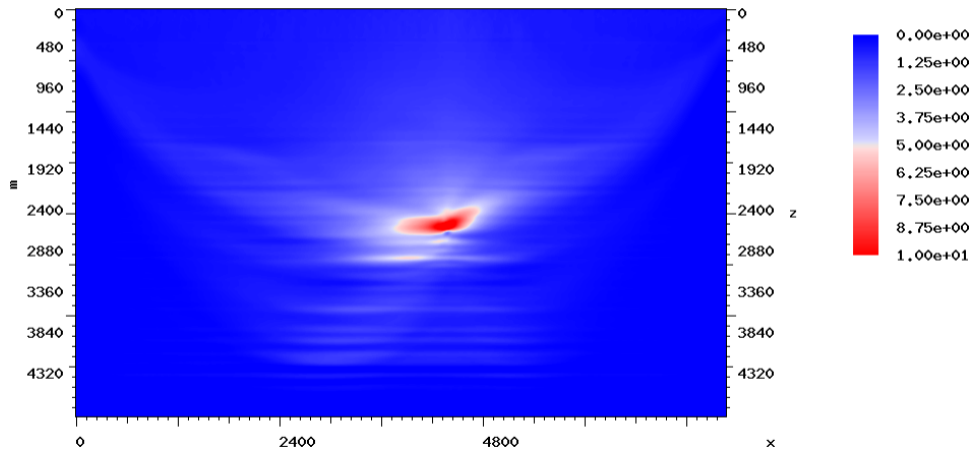


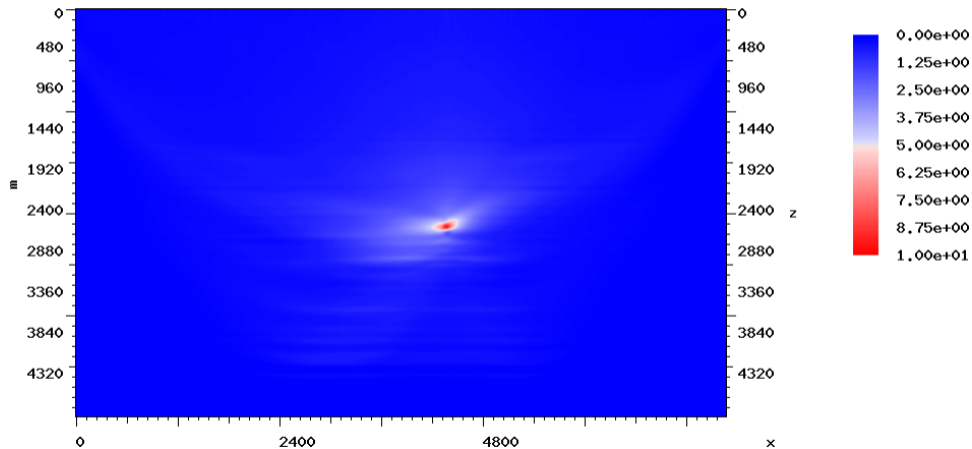
True second-first monitor model difference at 50 m/s scale



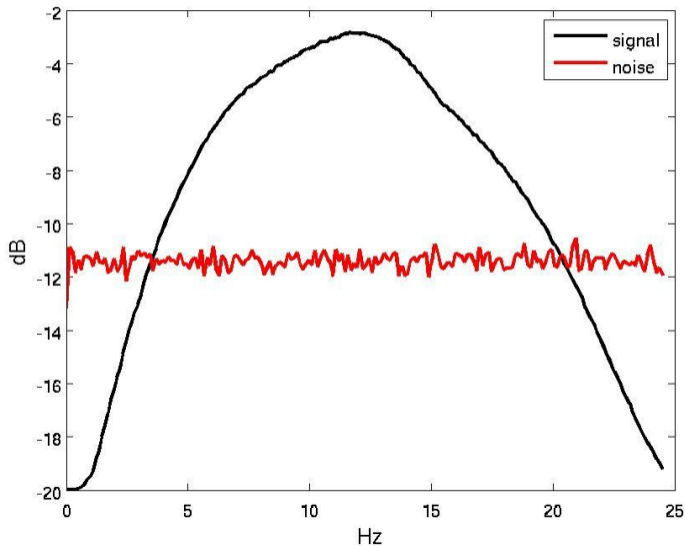




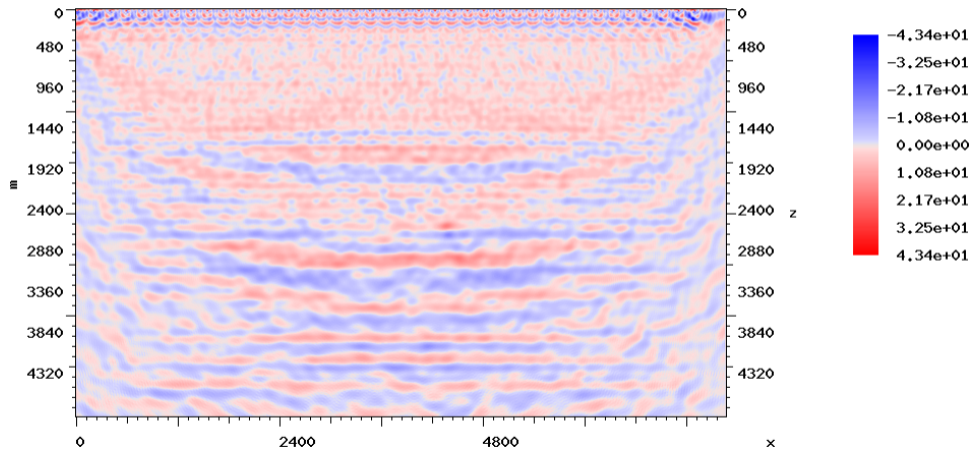


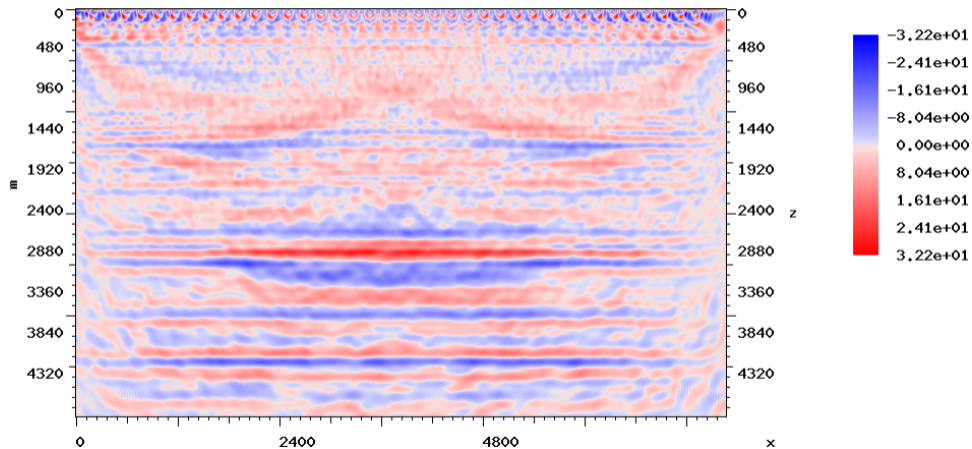


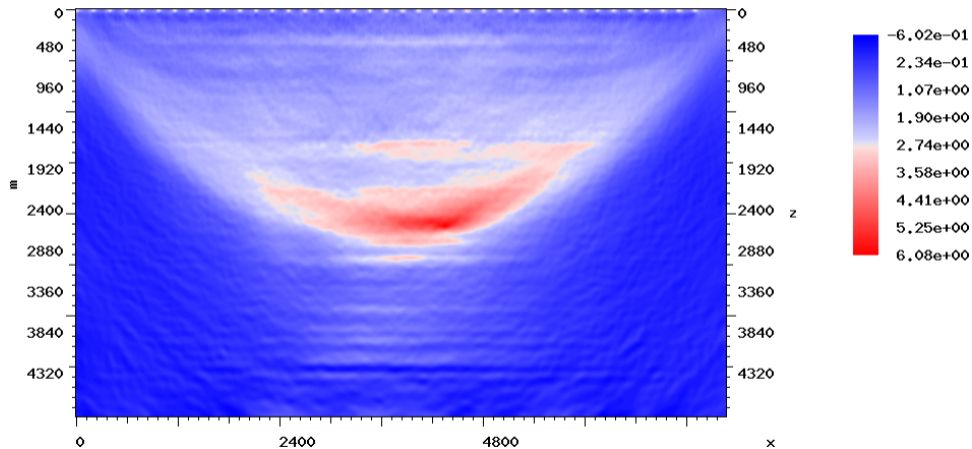
Adding white Gaussian noise



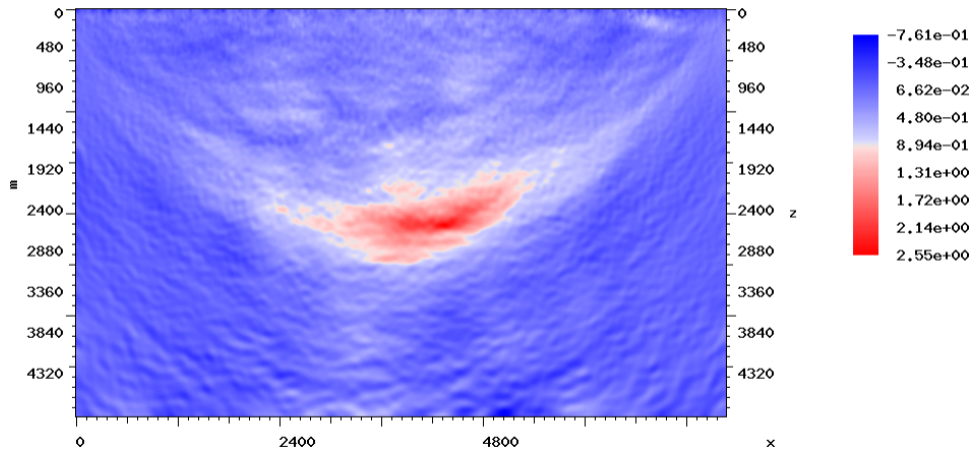
Inverted monitor-baseline difference; noisy synthetics, PD FWI

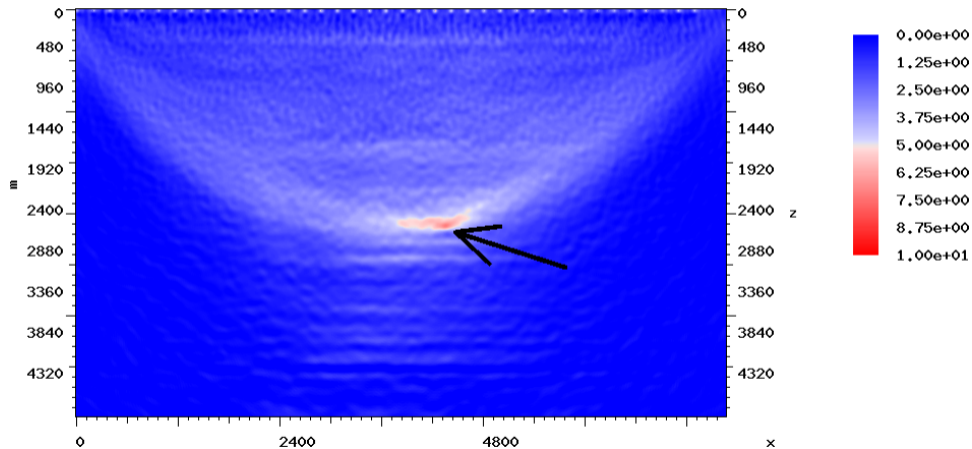


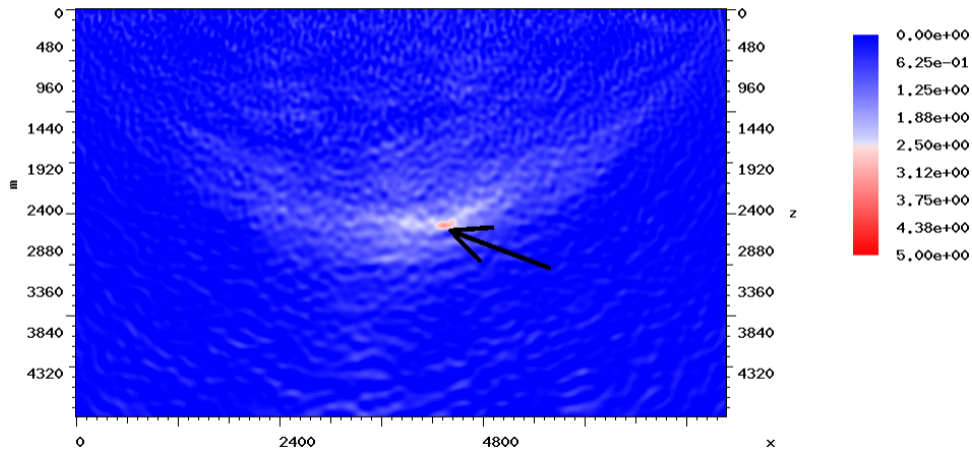




Second-first monitor; noisy synthetics, TV FWI

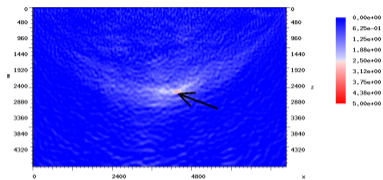
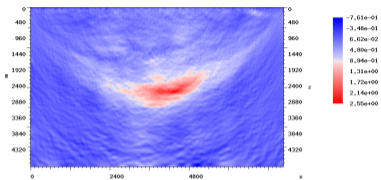








- Cascaded simultaneous regularized FWI **can resolve subtle subsurface velocity changes** associated with subsurface stressing in the presence of strong noise.
- Inversion can **detect relative magnitudes of velocity changes**.
- Potential applications in reservoir and earthquake monitoring, engineering geophysics.
- **Future work**: feasibility for continuous weak and passive sources.



Q&A

