Optimized Implicit Finite-difference migration for TTI media

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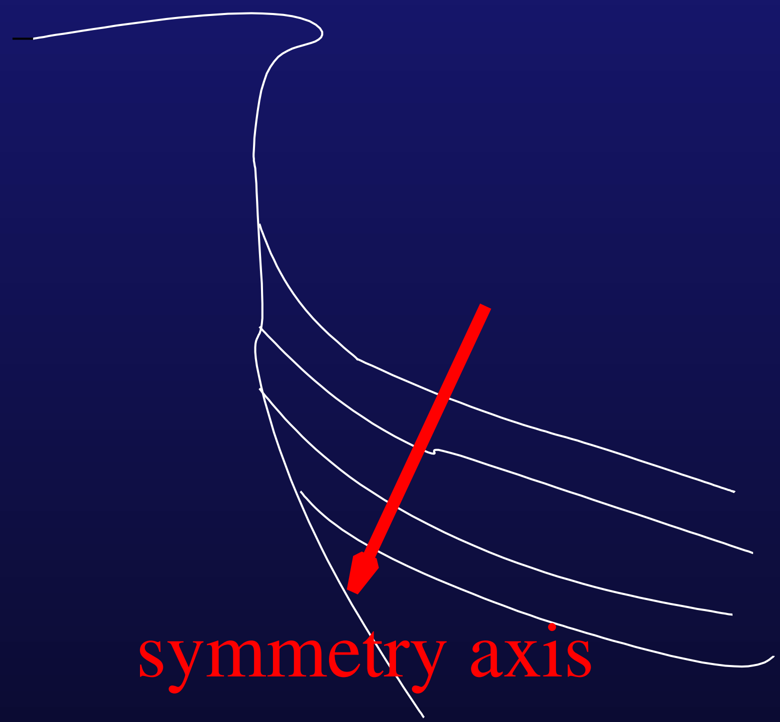
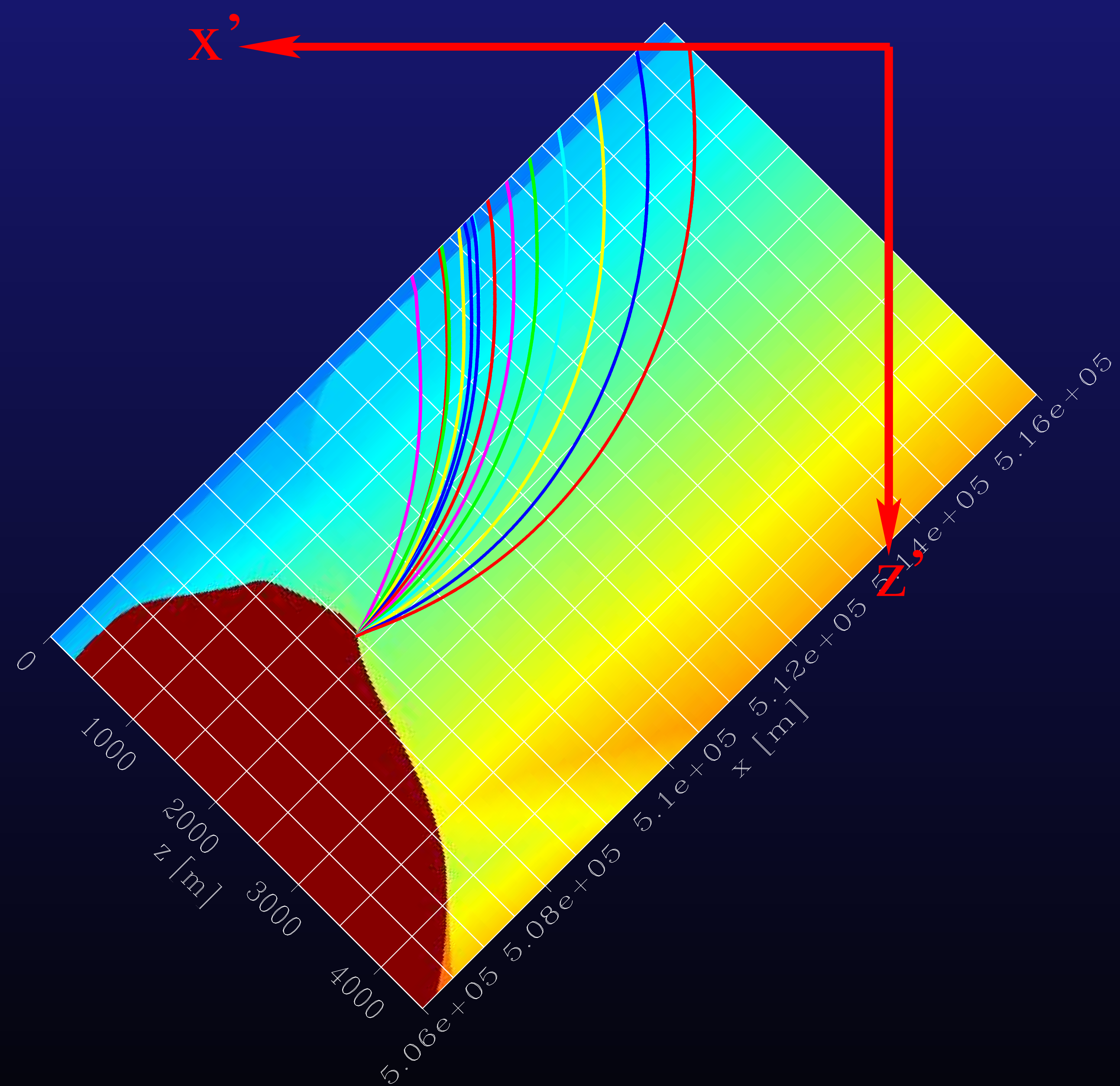
SEP125: pages 123-130,
SEP129: pages 55-63

Why Tilted TI?

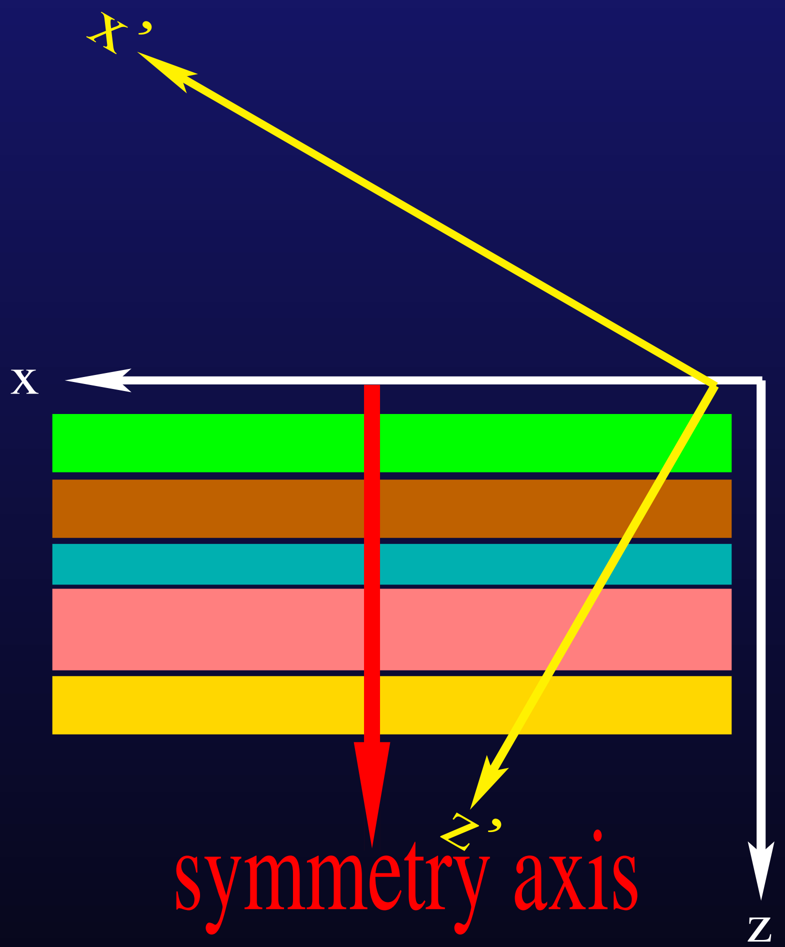


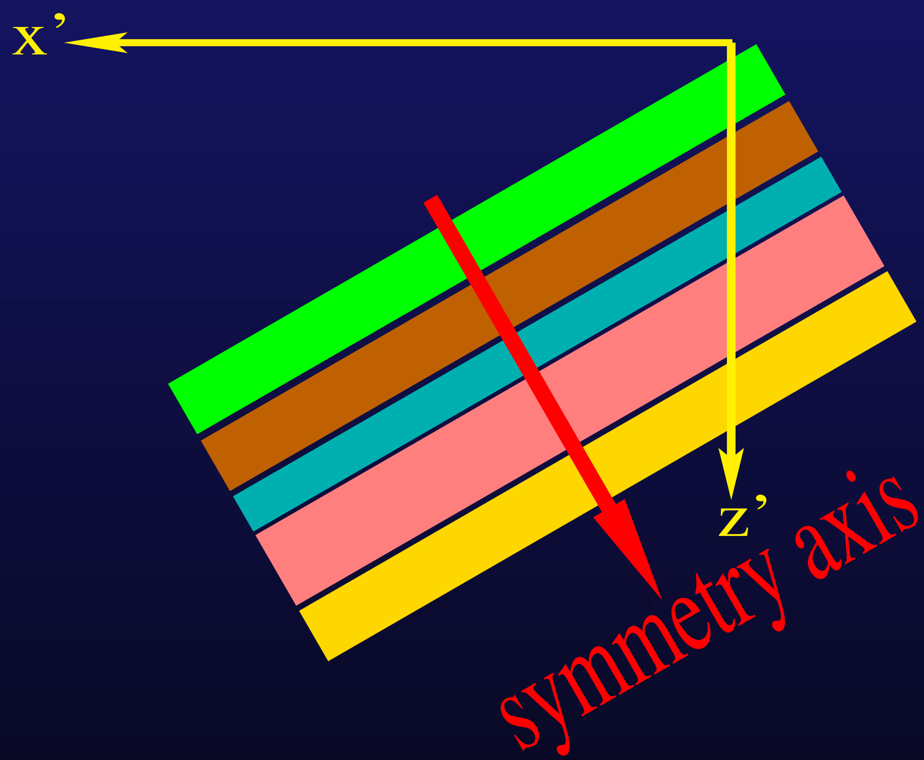
Steeply dipping sediments are often tilted TI media.

Tilted coordinates



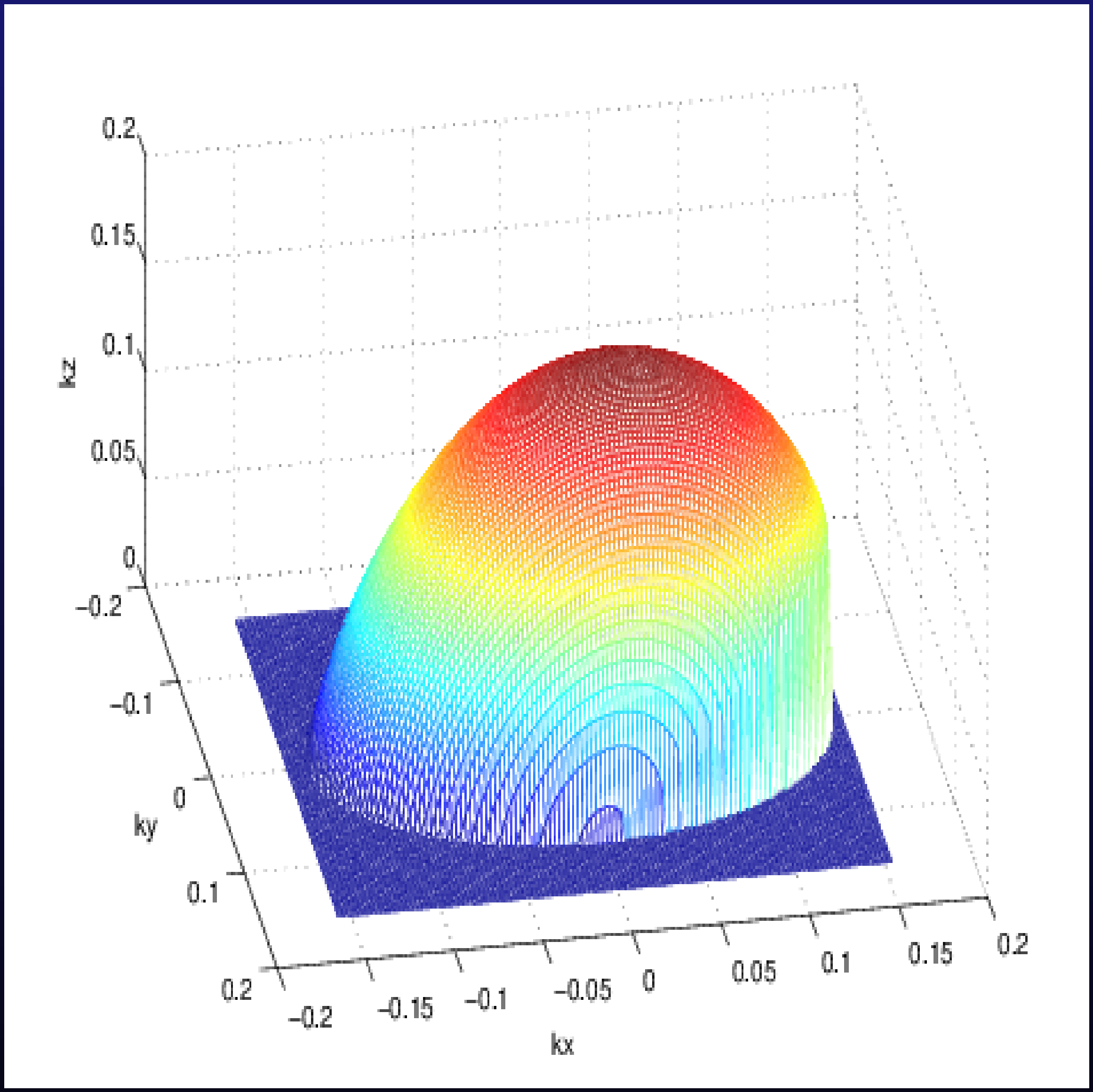
VTI media in tilted coordinates



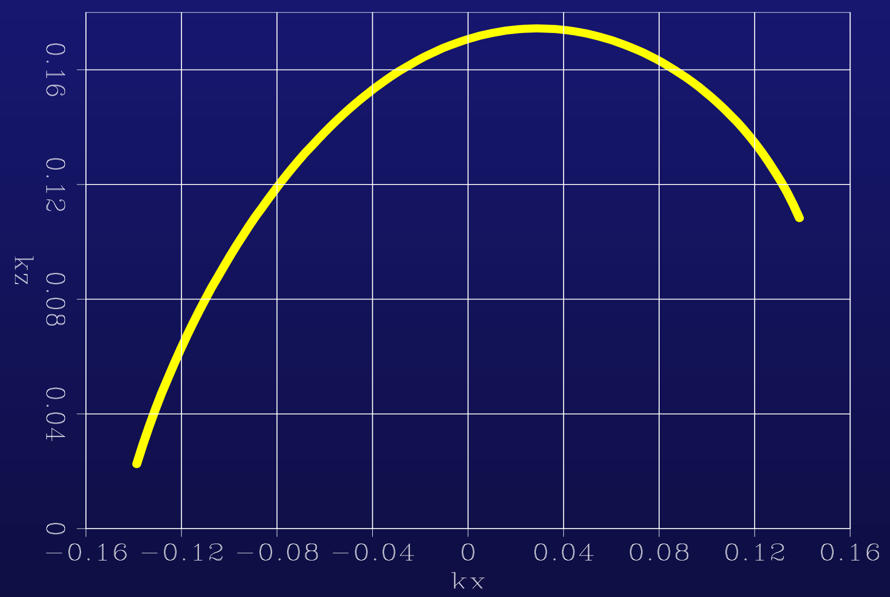


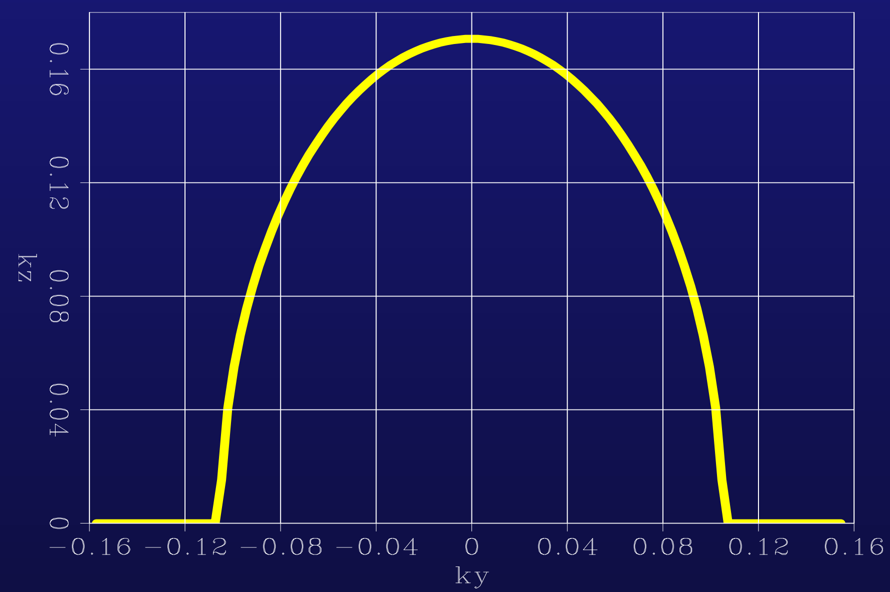
VTI media become tilted TI media in tilted coordinates.

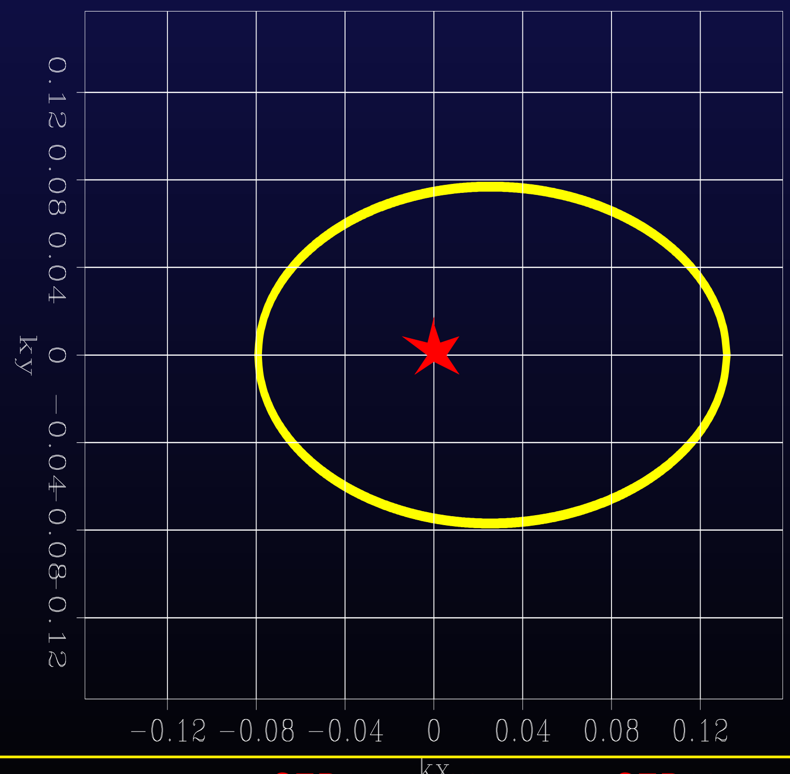
3D dispersion relation for tilted TI media



3D dispersion relation for tilted TI media

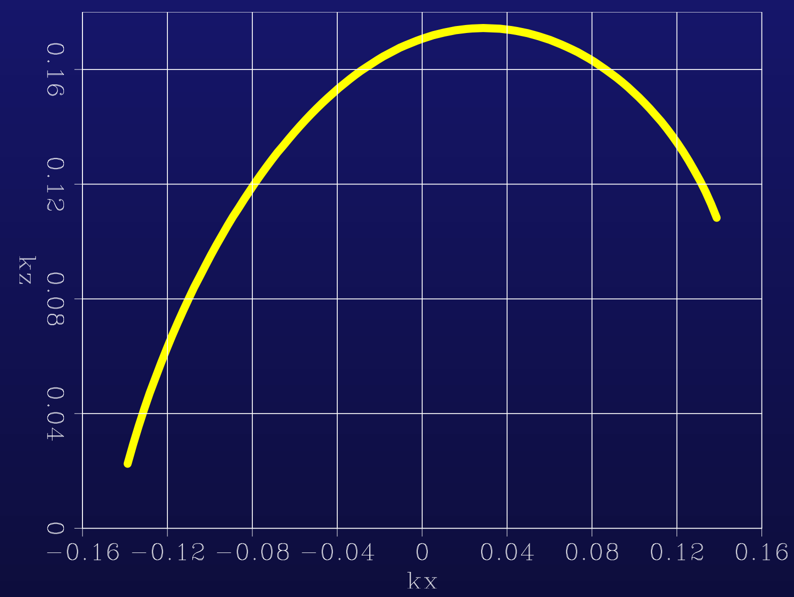






Dispersion relation in tilted TI media

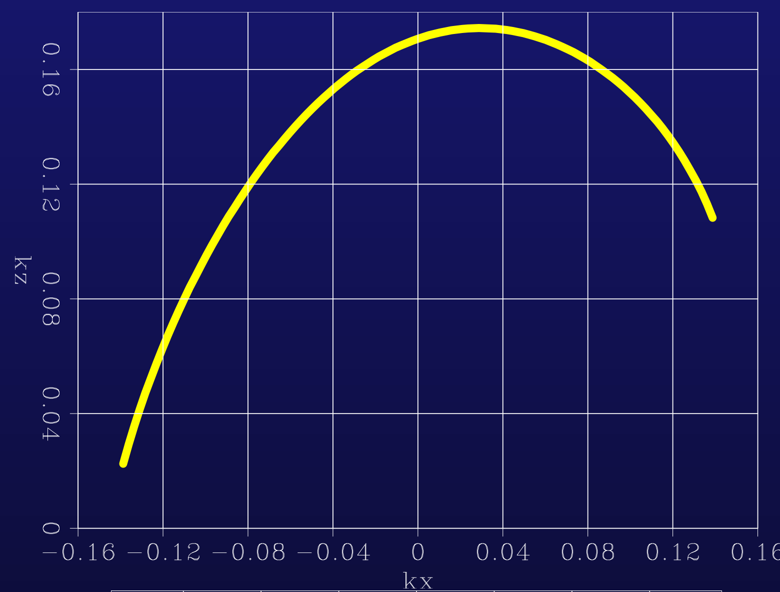
- No symmetry in x and y direction.

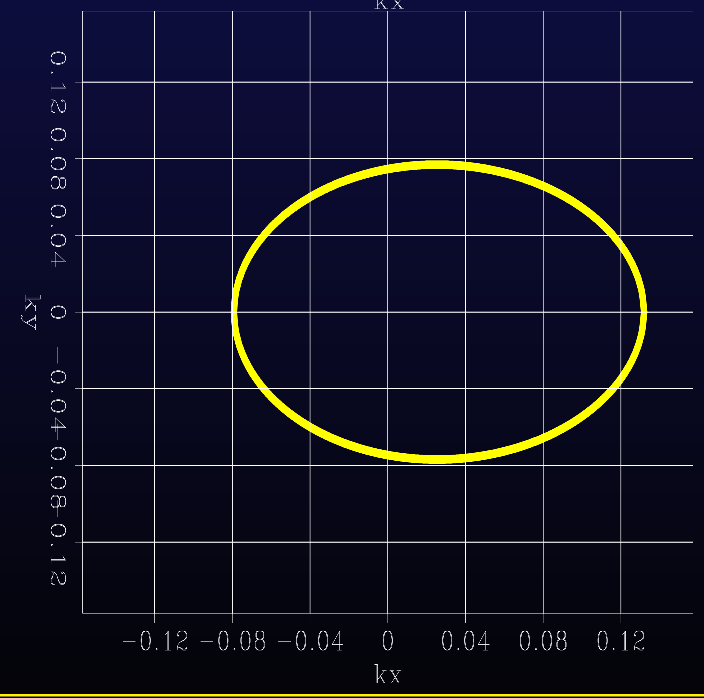


Dispersion relation in tilted TI media

- No symmetry in x and y direction.

- No circular symmetry.





Why implicit finite-difference

- PSPI: PSPI requires extrapolating large number reference wavefields and 3D interpolation.
  Five parameters: V_{P0}, ε,δ,tilting angle φ, azimuth angle α

- Explicit method: No circular symmetry. A 2D convolution operator is required for each grid.

Dispersion relation for isotropic media

$$k_z^2 + k_x^2 = \frac{\omega^2}{v^2}$$

k_z can be solved analytically as follows:

$$\frac{k_z}{\omega/v} = \sqrt{1 - \left(\frac{k_x}{\omega/v}\right)^2}$$

Approximation by Taylor series analysis

$$\sqrt{1-S^2} = 1 - \frac{1}{2}S^2 - \frac{1}{8}S^4 + \cdots$$

15° equation:

$$\frac{k_z}{\omega/v} \approx 1 - \frac{1}{2}\left(\frac{k_x}{\omega/v}\right)^2$$

45° equation:

$$\frac{k_z}{\omega/v} \approx 1 - \frac{1}{2}\left(\frac{k_x}{\omega/v}\right)^2 - \frac{1}{8}\left(\frac{k_x}{\omega/v}\right)^4 \approx 1 + \frac{-\frac{1}{2}\frac{k_x^2}{(\omega/v)^2}}{1 - \frac{1}{4}\frac{k_x^2}{(\omega/v)^2}}$$

Padé approximation: function fitting

If the function $S_z(S_r) \in C^{n+m}$, then $S_z(S_r)$ can be approximated by a rational function

$$S_z(S_r) = \frac{\sum_{i=1}^{n} a_i S_r^i}{1 + \sum_{i=1}^{n} b_i S_r^i}$$

a_i and b_i can be solved by the Least-squares optimization:

$$\min \int_0^{\sin(\theta)} \left(\sum_{i=1}^{n} a_i (S_r)^i - S_z(S_r)(1 + \sum_{i=1}^{n} b_i (S_r)^i) \right)^2 dS_r.$$

Approximation for square-root operator

Lee and Suh (1985) suggests:

The first order:

$$\sqrt{1-S_r^2} \approx 1 - \frac{0.4782S_r^2}{1-0.3764S_r^2}$$

The second order:

$$\sqrt{1-S_r^2} \approx 1 - \frac{0.4976S_r^2 - 0.4086S_r^4}{1-1.0967S_r^2 + 0.1946S_r^4}$$

Advantage of LS optimization

- More accurate scheme with the same computation cost.
- An analytical expression between k_z and k_x is not required.

3D dispersion relation for tilted TI media

$$a_4 k_z^4 + a_3 k_z^3 + a_2 k_z^2 + a_1 k_z + a_0 = 0$$

$$\begin{aligned}
a_4 &= (f-1) + 2\varepsilon(f-1)\sin^2\varphi - \tfrac{f}{2}(\varepsilon-\delta)\sin^2 2\varphi, \\
a_3 &= 2(f-1)\varepsilon\sin 2\varphi - f(\varepsilon-\delta)\sin 4\varphi, \\
a_2 &= [2(f-1)(1+\varepsilon) - f(\varepsilon-\delta)(2\cos^2 2\varphi - \sin^2 2\varphi)]k_x^2 \\
&+ 2[(f-1)(1+\varepsilon) + (f-1)\varepsilon\sin^2\varphi - f(\varepsilon-\delta)\cos^2\varphi]k_y^2 \\
&+ \left(\frac{\omega}{v_{p0}}\right)^2 (2\varepsilon\sin^2\varphi + 2 - f), \\
a_1 &= [2(f-1)\varepsilon\sin 2\varphi + f(\varepsilon-\delta)\sin 4\varphi]k_x^3 \\
&+ 2\sin 2\varphi[(f-1)\varepsilon + f(\varepsilon-\delta)]k_x k_y^2 \\
&+ 2\varepsilon\left(\frac{\omega}{v_{p0}}\right)^2 \sin 2\varphi k_x, \\
a_0 &= [(f-1)(1+2\varepsilon\cos^2\varphi) - \tfrac{f}{2}(\epsilon-\delta)\sin^2 2\varphi]k_x^4 + (f-1)(1+2\varepsilon)k_y^4 \\
&+ 2[(f-1)(1+\varepsilon+\varepsilon\cos^2\varphi) - f(\epsilon-\delta)\sin^2\varphi]k_x^2 k_y^2 \\
&+ \left(\frac{\omega}{v_{p0}}\right)^2 (2 - f + 2\varepsilon\cos^2\varphi)k_x^2 + 2\left(\frac{\omega}{v_{p0}}\right)^2 (2+\varepsilon-f)k_y^2.
\end{aligned}$$

Padé approximation: function fitting

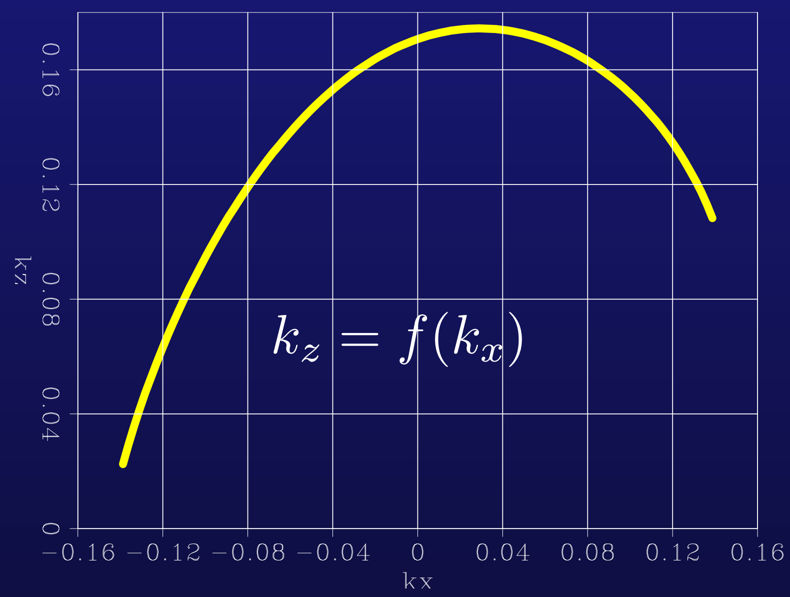
If the function $S_z(S_r) \in C^{n+m}$, then $S_z(S_r)$ can be approximated by a rational function

$$S_z(S_r) = S_z(0) + \frac{\sum_{i=1}^{n} a_i S_r^i}{1 + \sum_{i=1}^{n} b_i S_r^i}$$

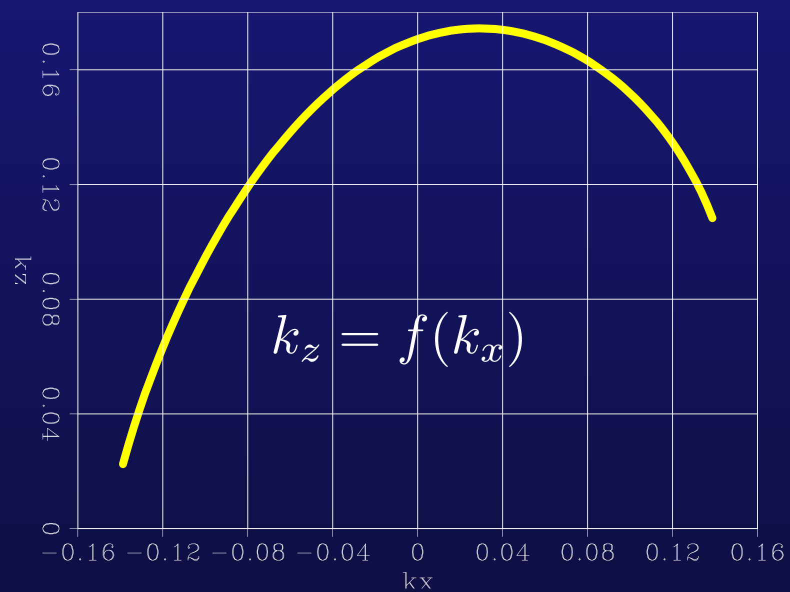
a_i and b_i can be solved by the Least-squares optimization:

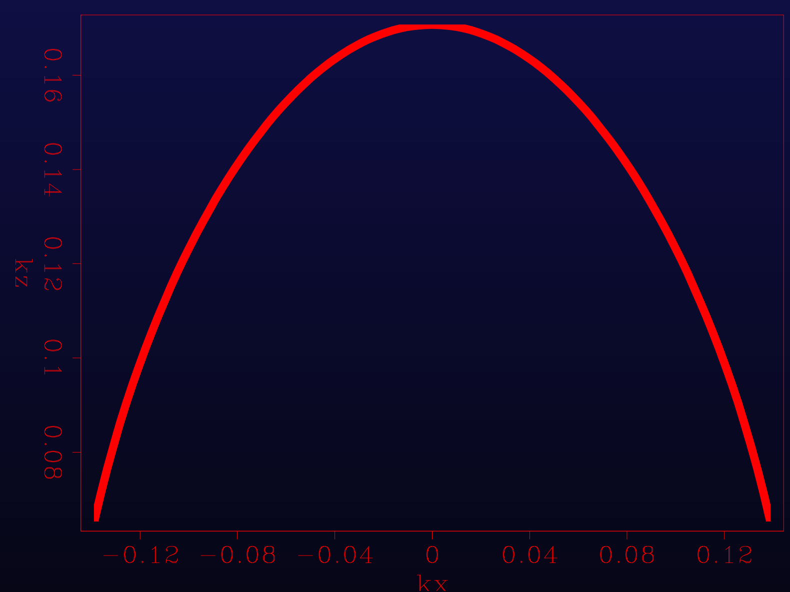
$$\min \int_0^{\sin(\theta)} \left(\sum_{i=1}^{n} a_i (S_r)^i - S_z(S_r)(1 + \sum_{i=1}^{n} b_i (S_r)^i) \right)^2 dS_r.$$

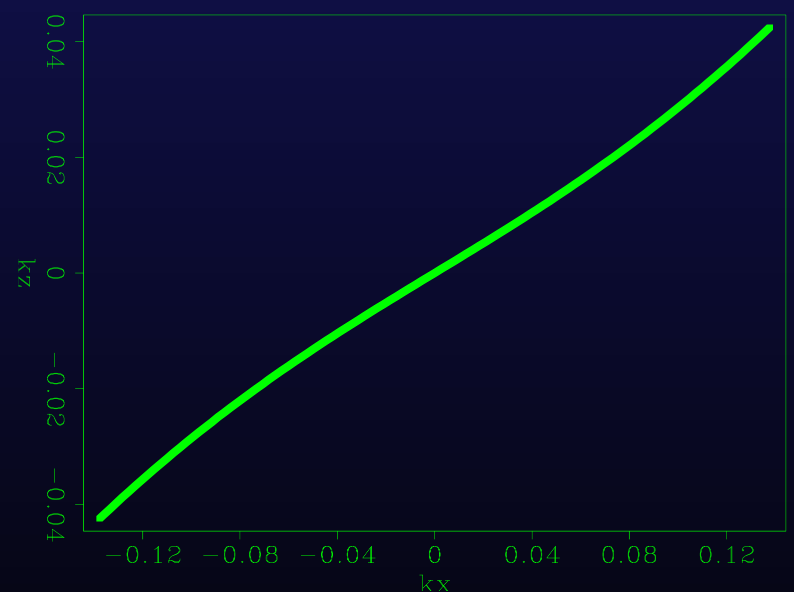
Symmetric decomposition of a function



Symmetric decomposition of a function







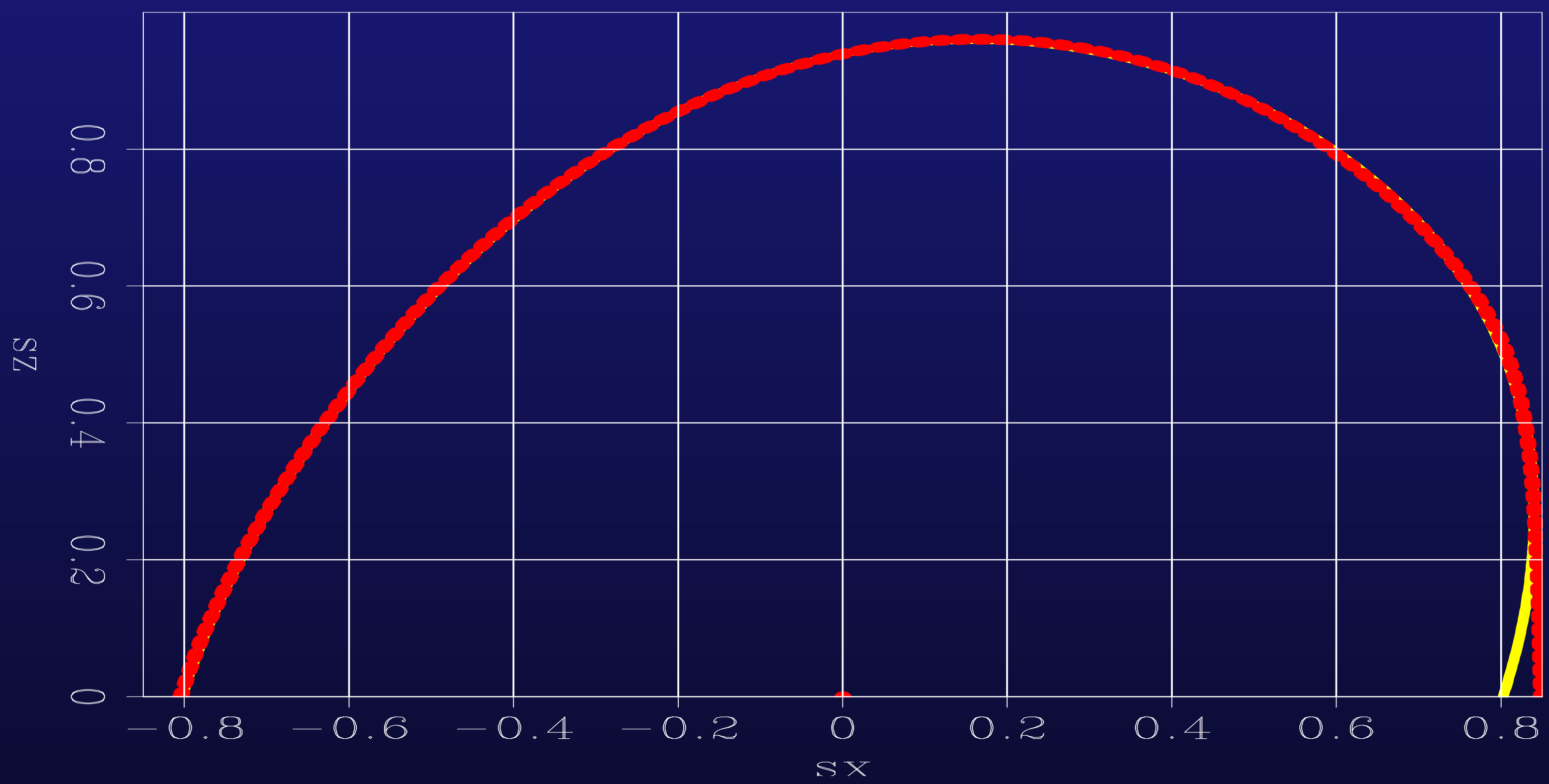
$$f^e = \frac{1}{2}[f(k_x) + f(-k_x)] \qquad f^o = \frac{1}{2}[f(k_x) - f(-k_x)]$$

Approximate dispersion relation

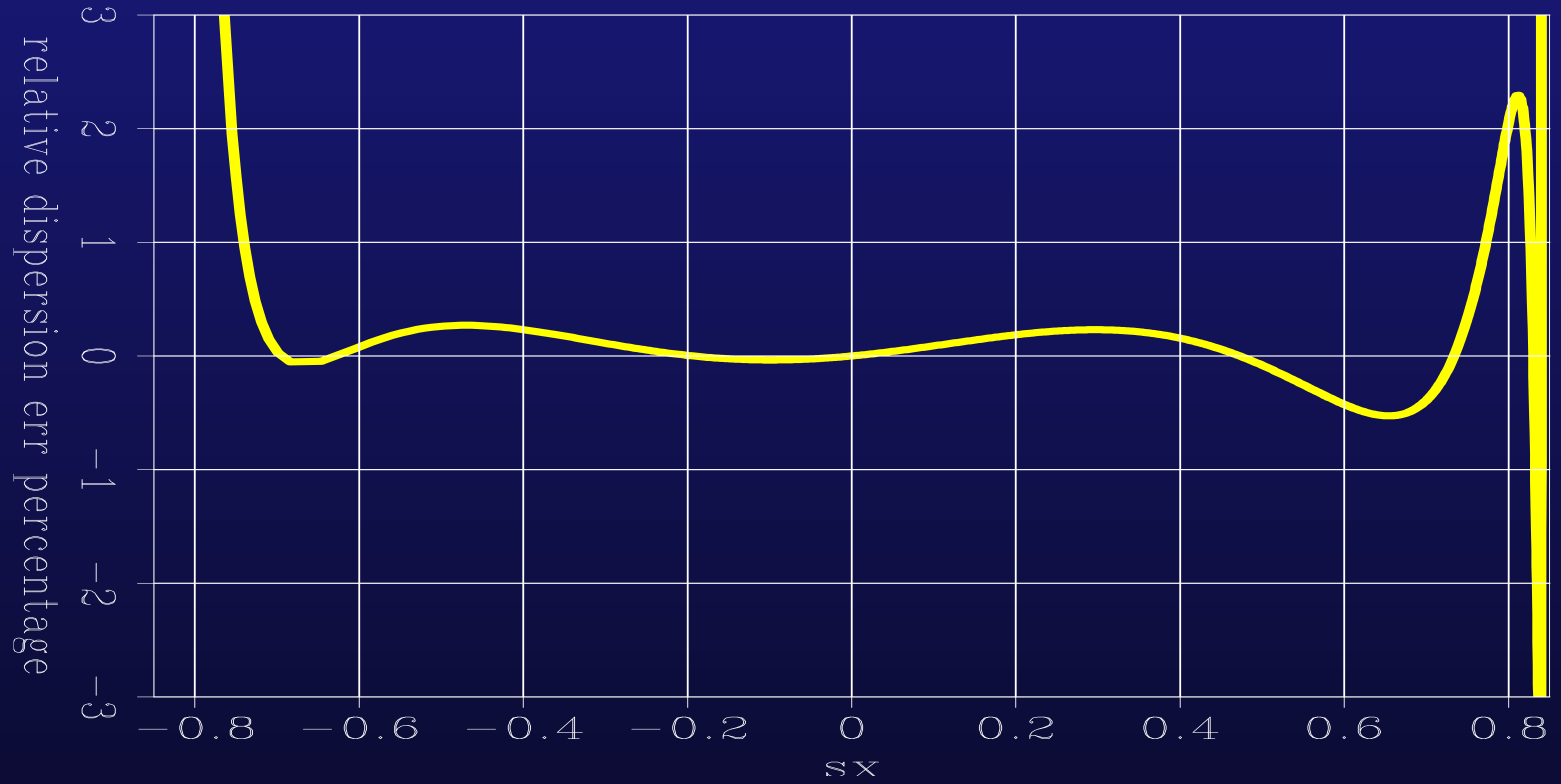
$$S_z(S_x) \approx S_z(0) + \frac{a_1 S_x^2 + a_2 S_x}{1 + b_1 S_x^2} + \frac{c_1 S_x^2 + c_2 S_x}{1 + b_2 S_x^2}$$

- a_1,a_2,b_1,b_2,c_1, and c_2 are functions of ε, δ and tilting angle θ.
- If ε, δ and θ vary laterally, a_i and b_i vary laterally.

Dispersion relation comparison



Relative dispersion error



Finite-difference scheme

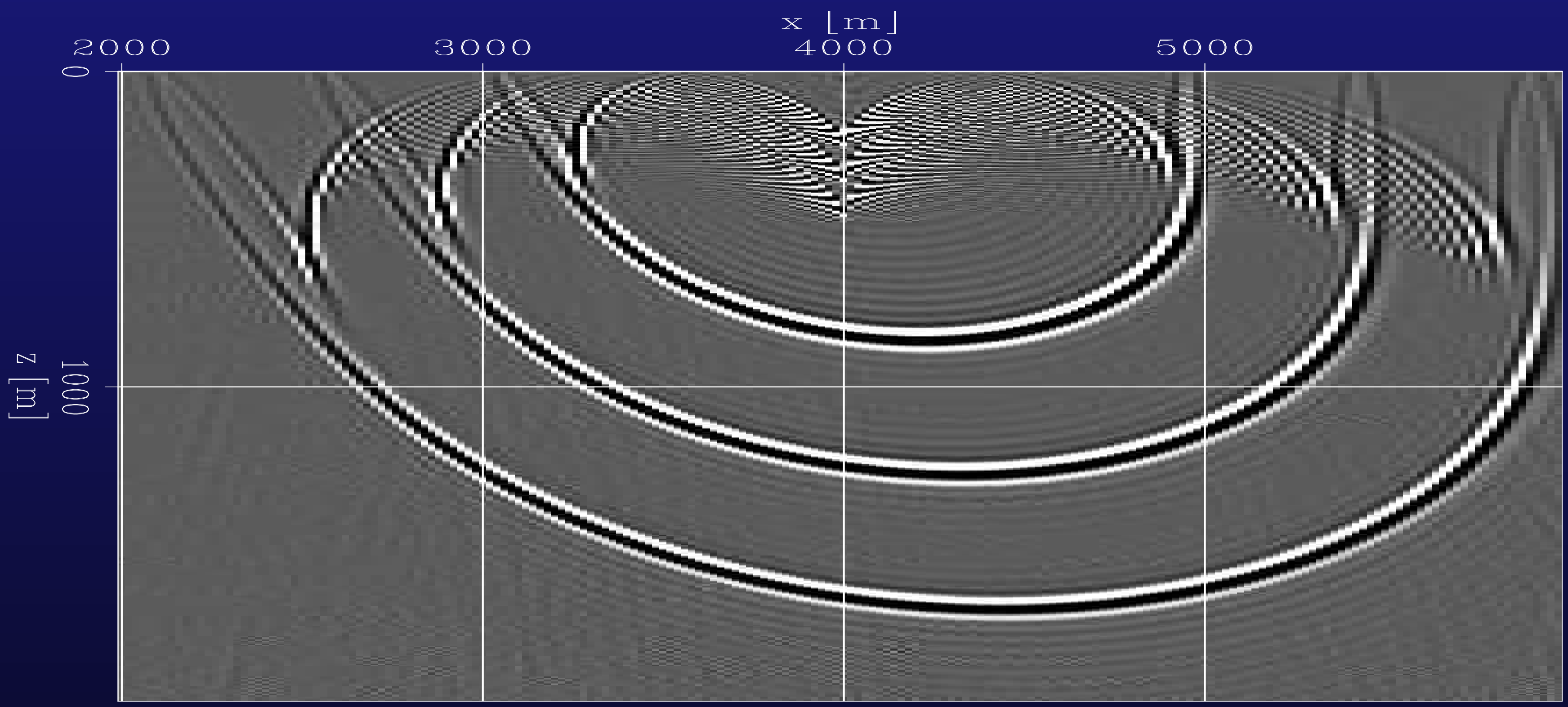
$$\frac{\partial P}{\partial z} = i\frac{\omega}{v(x)} \cdot \frac{\alpha_1(x)\left(\frac{v(x)}{\omega}\right)^2 \frac{\partial^2}{\partial x^2} + ic_1(x)\left(\frac{v(x)}{\omega}\right)\frac{\partial}{\partial x}}{1 + \beta_1(x)\left(\frac{v(x)}{\omega}\right)^2 \frac{\partial^2}{\partial x^2}} P$$

$$\frac{\partial^2}{\partial x^2} P = \frac{P_{x+\Delta x} - 2P_x + P_{x-\Delta x}}{(\Delta x)^2}, \qquad \frac{\partial}{\partial x} P = \frac{P_{x+\Delta x} - P_{x-\Delta x}}{2\Delta x}$$

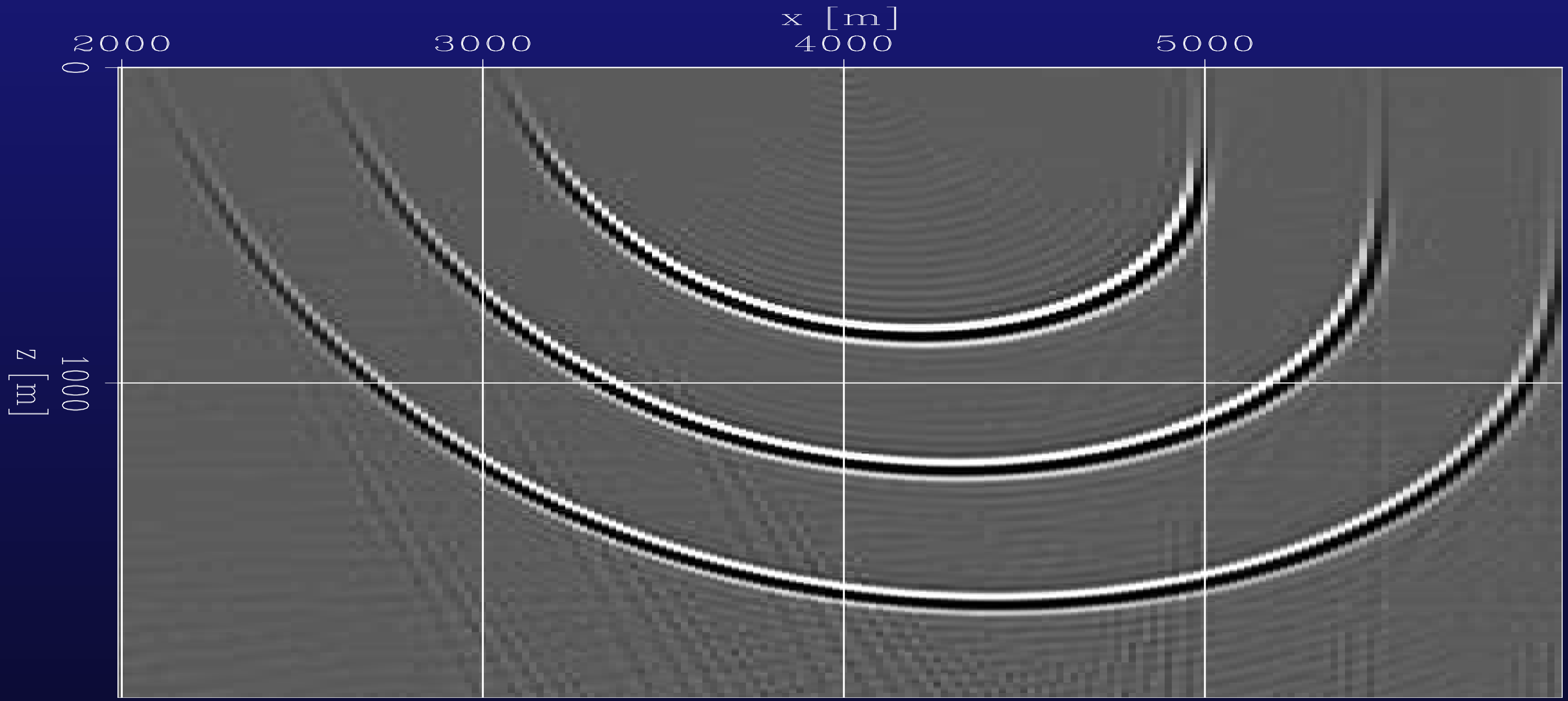
3D

- Two-way splitting
- Phase correction in the frequency-wavenumber domain

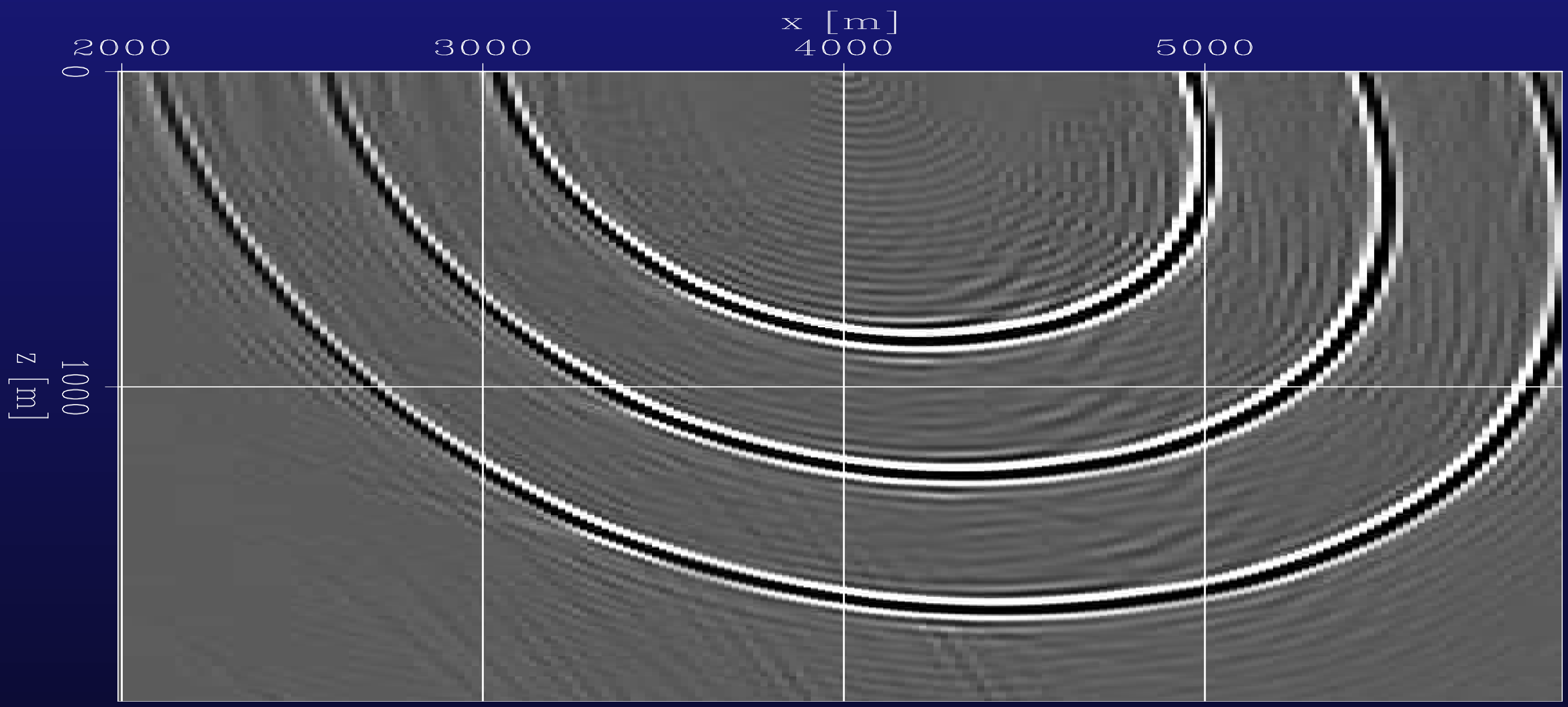
Impulse responses



Impulse responses



Impulse responses



Laterally varying TTI media

$$\frac{\partial P}{\partial z} = i\frac{\omega}{v(x)} \cdot \frac{\alpha_1(x)\left(\frac{v(x)}{\omega}\right)^2 \frac{\partial^2}{\partial x^2} + c_1(x)\left(\frac{v(x)}{\omega}\right)\frac{\partial}{\partial x}}{1 + \beta_1(x)\left(\frac{v(x)}{\omega}\right)^2 \frac{\partial^2}{\partial x^2}} P$$

α_1, β_1 and c_1 vary laterally.

Table driven implicit finite-difference

	X1	X2		Xnx-1	Xnx
Zi	ε_1 δ_1	ε_2 δ_2	• • • • • •	ε_{nx-1} δ_{nx-1}	ε_{nx} δ_{nx}

Table driven implicit finite-difference

	$X1$	$X2$		$Xnx\text{-}1$	Xnx
Zi	ε_1 δ_1	ε_2 δ_2	• • • • • •	ε_{nx-1} δ_{nx-1}	ε_{nx} δ_{nx}

	ε_{min}	$\varepsilon_{min}+d\varepsilon$			ε_{max}
δ_{min}					
$\delta_{min}+d\delta$			α_1 β_1		
δ_{max}					

Table driven implicit finite-difference

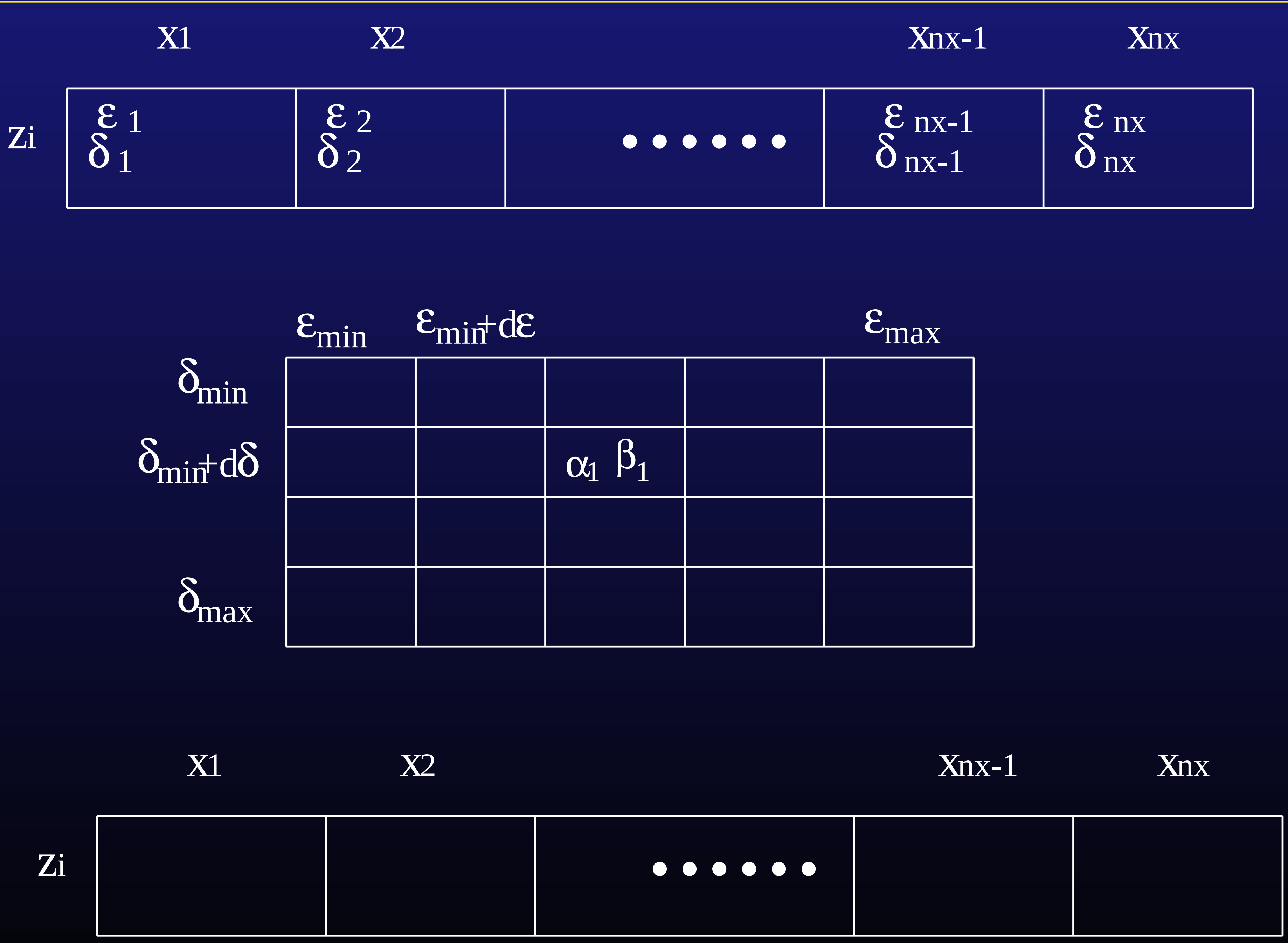


Table driven implicit finite-difference

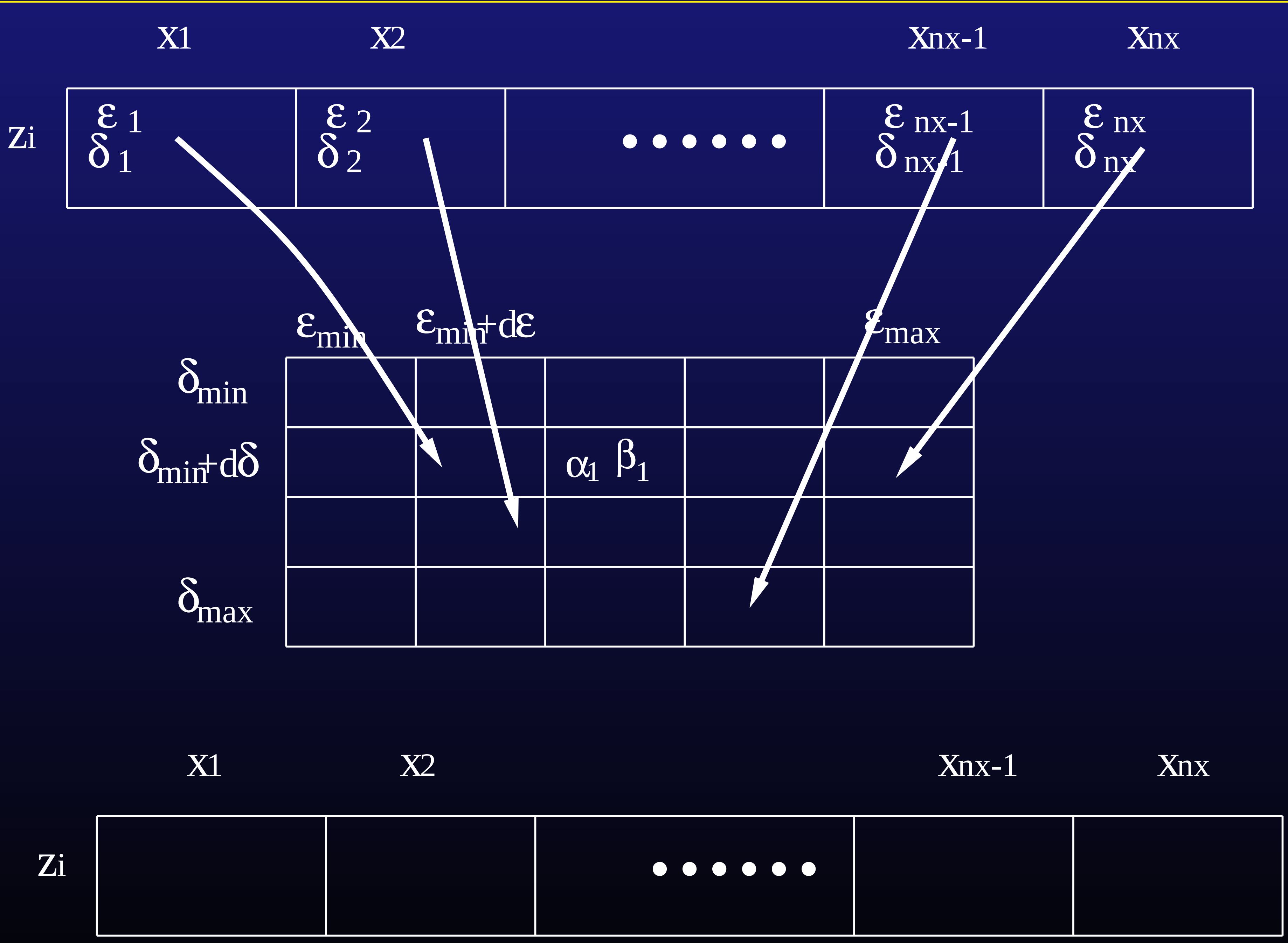
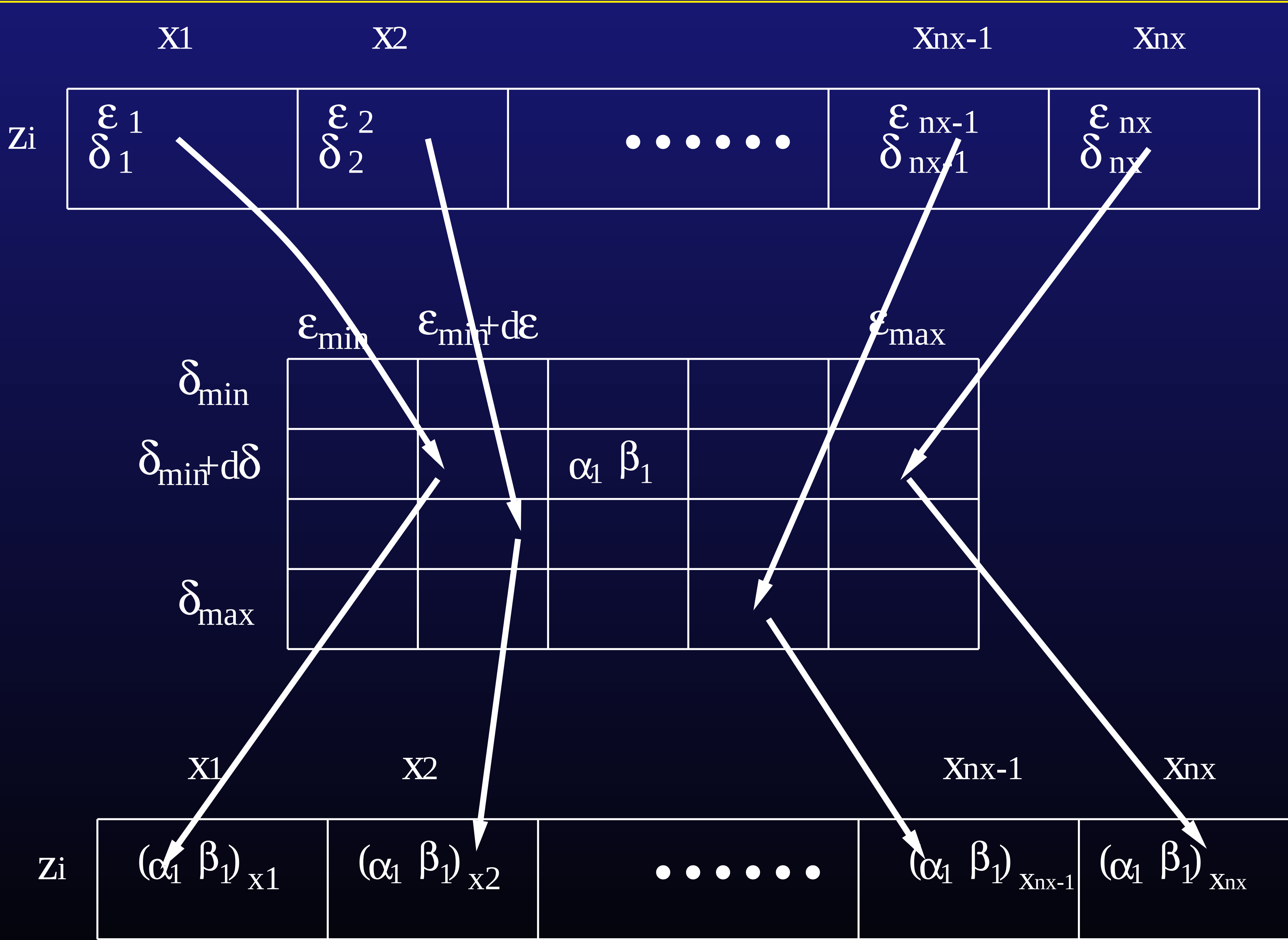


Table driven implicit finite-difference



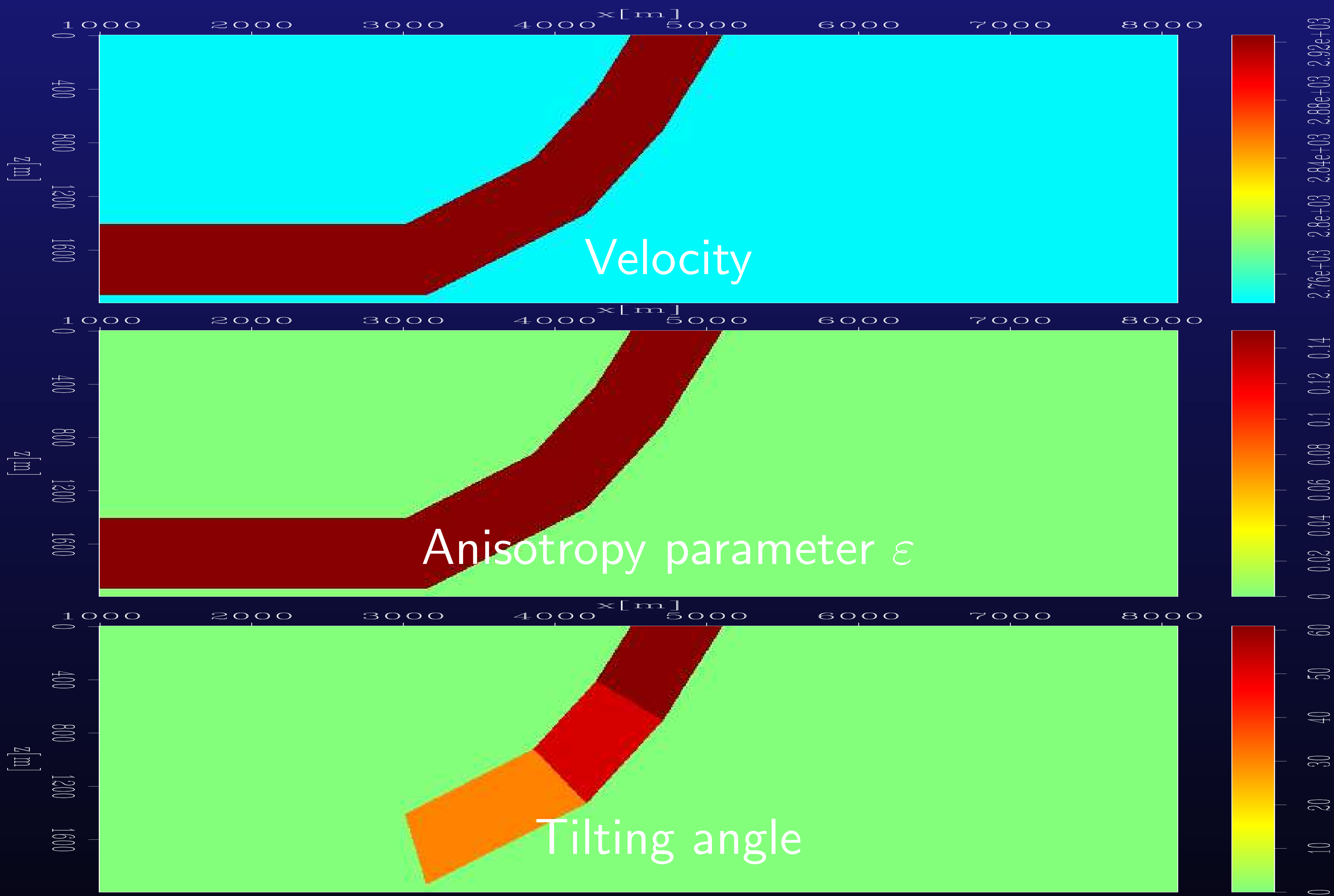
Computation cost

Cost of Implicit finite-difference
for TTI media

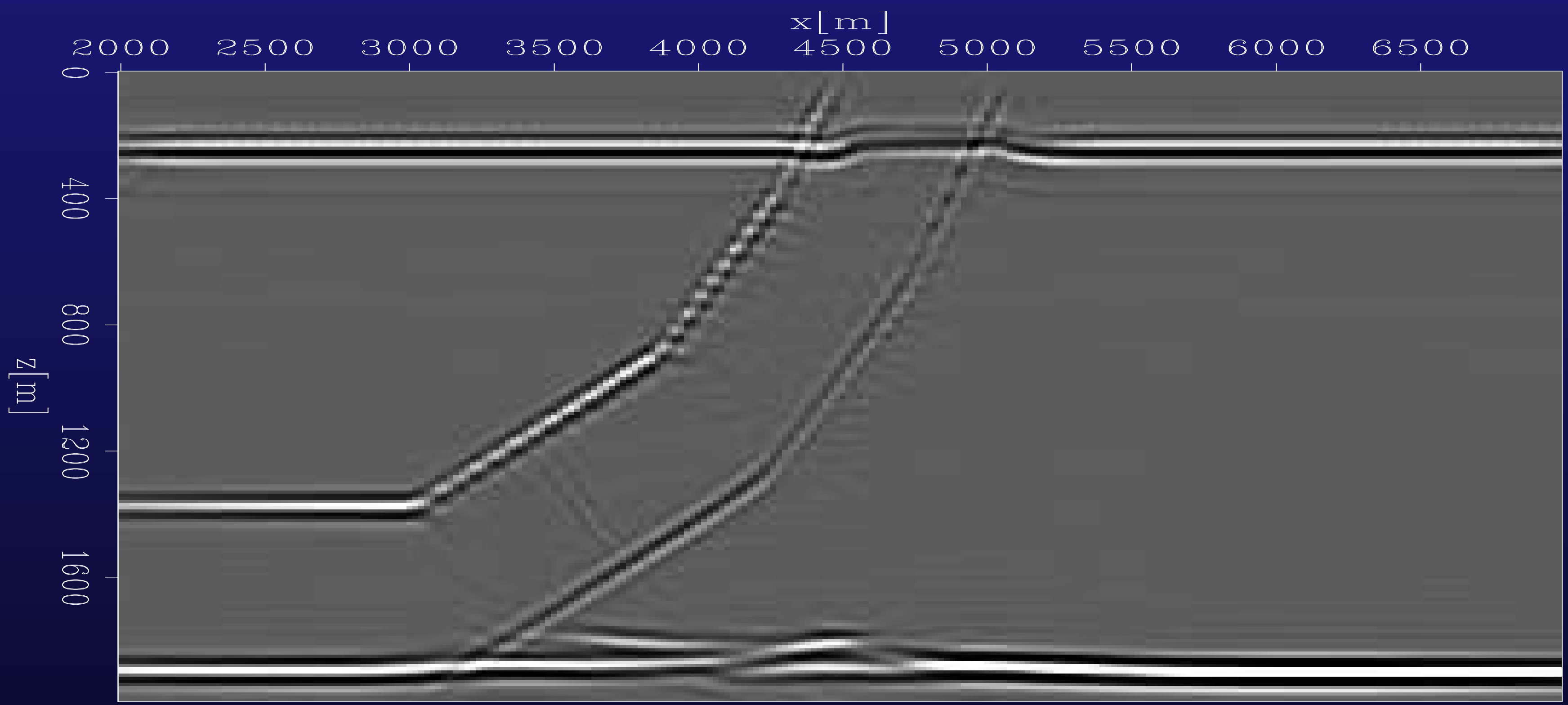
=

Cost of Implicit finite-difference for isotropic media $+$ table searching

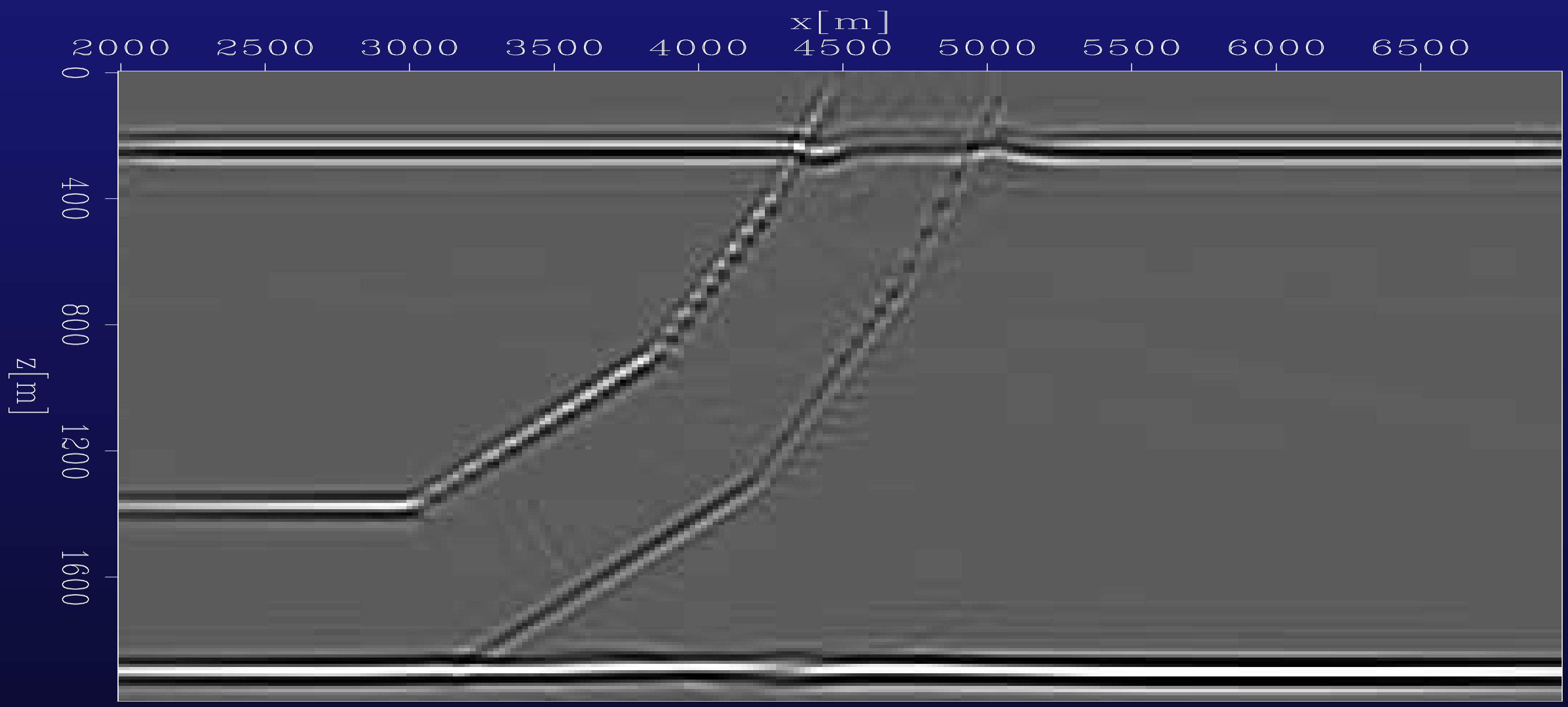
TTI model



Isotropic Migration



Migration for TTI media



Conclusion

- Optimized implicit-finite difference can handle lateral variation.
- It is stable.
- Computation cost is similar to isotropic media.

Acknowledgements

I thank BP for making the data available. I also thank Sam Gray for sending me the dataset.