Least-squares migration of incomplete data sets with regularization in the subsurface-offset domain

Yaxun Tang

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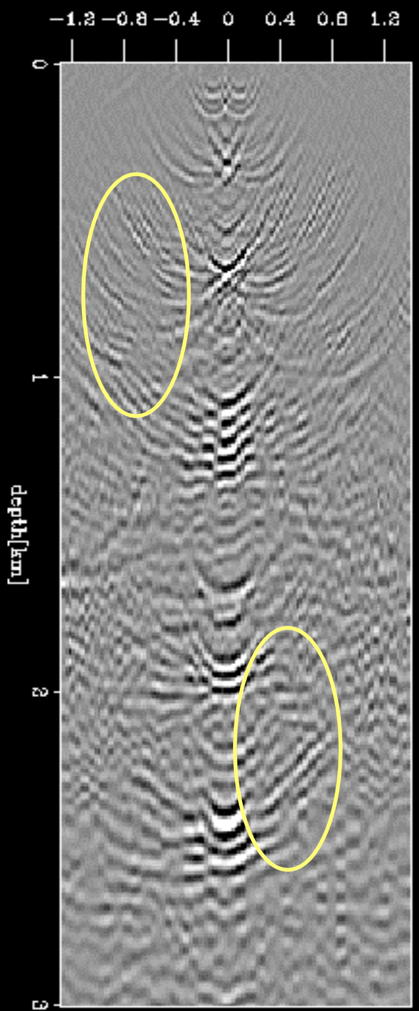
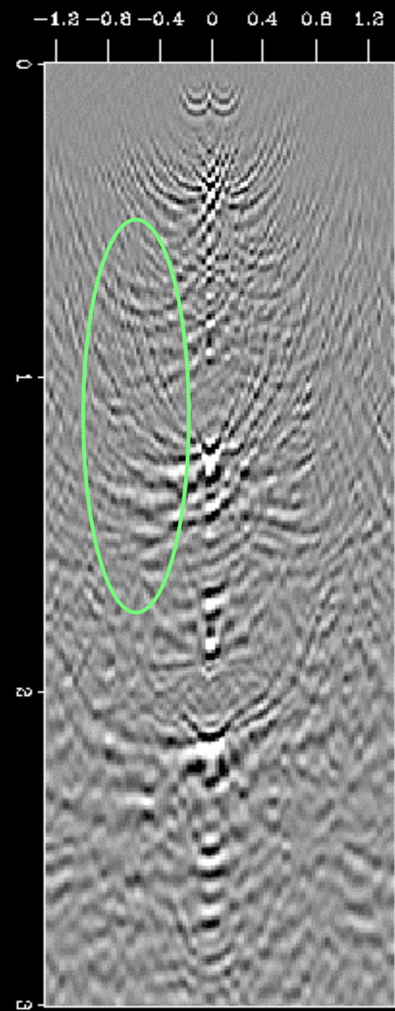
SEP-125, p159

Comparison of SODCIGs

Subsurface half offset [m]



Migration



Migration

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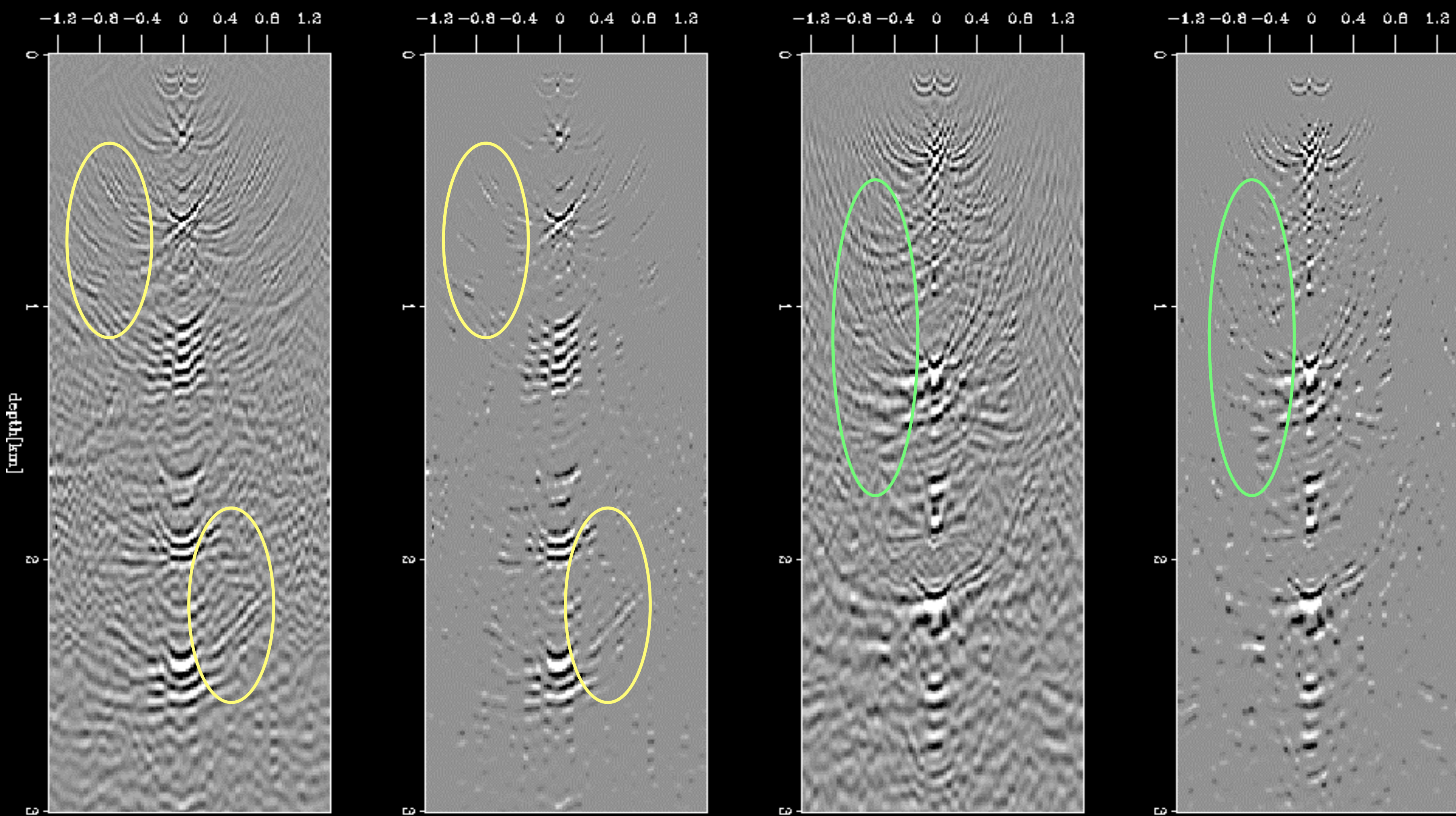
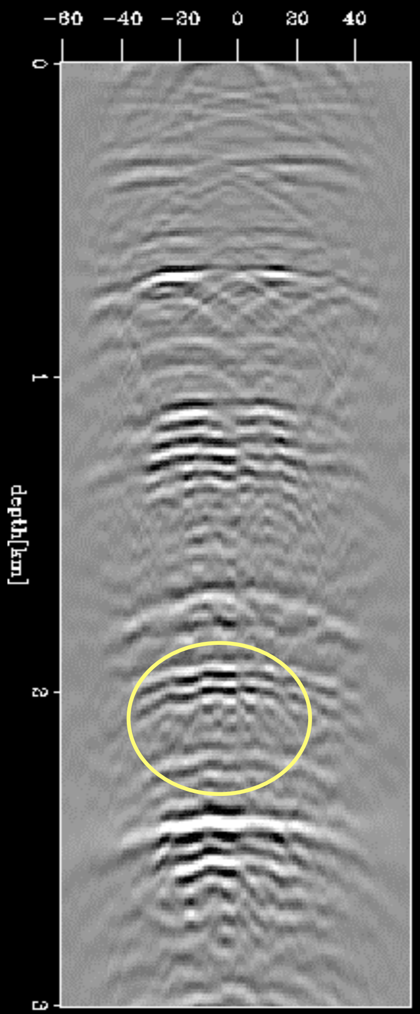


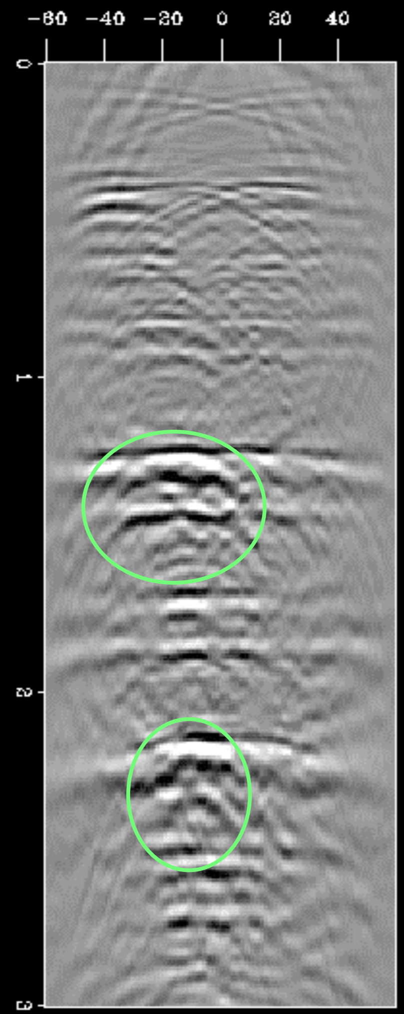
Migration | Inversion | Migration | Inversion

Comparison of ADCIGs

Angle [deg.]

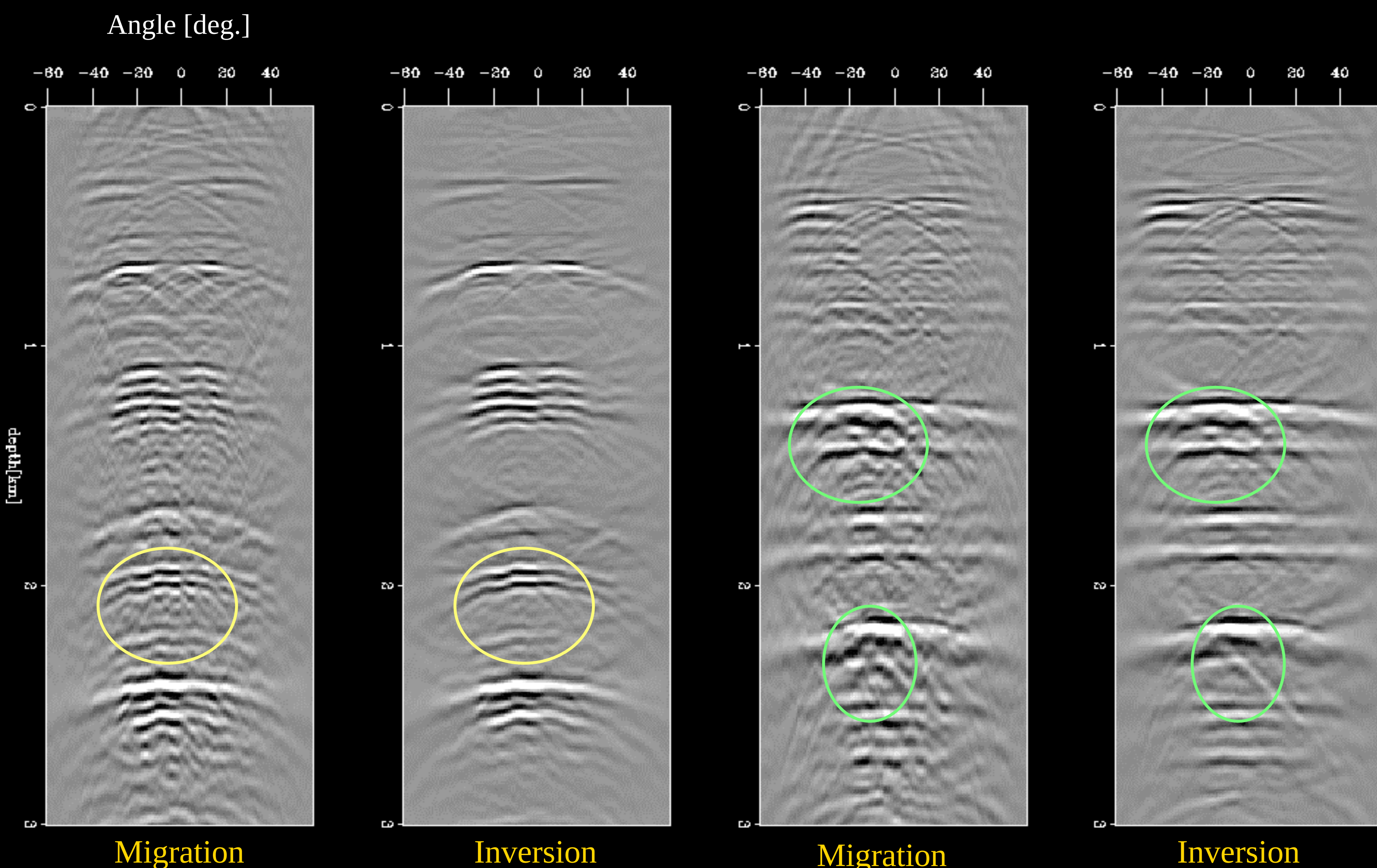


Migration



Migration

Comparison of ADCIGs



Outline

- Regularized inversion theory
- Approximate inversion
- Two-layer synthetic example
- Marmousi example
- Conclusions

Objective functions

Linear operator: $$\mathbf{d} = \mathbf{L}\mathbf{m}$$

Least-squares objective function: $$J(\mathbf{m}) = \left\| \mathbf{W_d}(\mathbf{L}\mathbf{m} - \mathbf{d}) \right\|_2$$

Regularization: $$J(\mathbf{m}) = \left\| \mathbf{W_d}(\mathbf{L}\mathbf{m} - \mathbf{d}) \right\|_2 + R(\mathbf{m})$$

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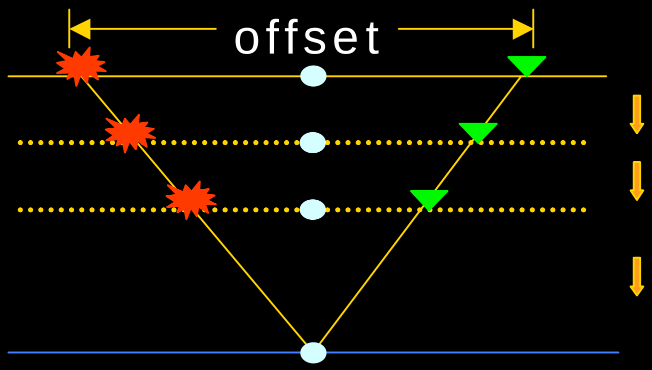
How to choose regularization terms?

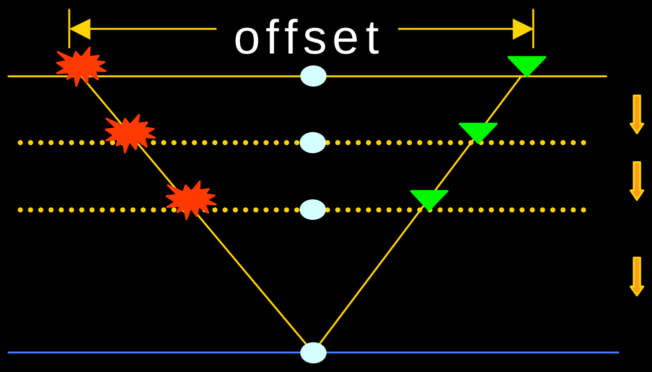


Downward continuation
shrinks offsets

Correct velocity + infinite recording geometry

- Energy in the SODCIGs will be focused at zero-offset
- The model space (SODCIGs) should be sparse

How to choose regularization terms?



Downward continuation shrinks offsets

Correct velocity + infinite recording geometry

- Energy in the SODCIGs will be focused at zero-offset
- The model space (SODCIGs) should be sparse

Require regularization terms to:

- Penalize energy at far offset
- Enforce sparseness

in SODCIGs

Penalizing energy at far offsets

- Differential semblance operator, DSO (Shen,2003;Valenciano,2006)

- Multiply h in the subsurface offset domain

$$D(\mathbf{m}) = diag(|\mathbf{h}|)\mathbf{m}$$

$$diag(|\mathbf{h}|) = diag(|h_1|,|h_2|,...,|h_m|)$$

- DSO in SODCIGs has the same effect as smoothing along ray-parameter in ADCIGs

Forcing SODCIGs to be sparse

- Sparseness constraint (Cauchy norm or L1 norm)
- Sparseness in stacked offset-ray-parameter domain (Wang and Sacchi, 2007)
- Instead, I propose to add sparseness weights in SODCIGs directly

$$W_{si} = \frac{1}{\sqrt{1+\left(D(m_i)/\sigma\right)^2}}$$

Cauchy norm

Proposed inversion scheme

Fitting goals:

$$\mathbf{0} \approx \mathbf{W_d}(\mathbf{Lm} - \mathbf{d})$$

$$\mathbf{0} \approx \varepsilon \mathbf{W_s} D(\mathbf{m})$$

$$D(\mathbf{m}) = diag(|\mathbf{h}|)\mathbf{m}$$

$$diag(|\mathbf{h}|) = diag(|h_1|, |h_2|, \ldots, |h_m|)$$

Differential semblance operator (DSO)

$$W_{si} = \frac{1}{\sqrt{1 + \left(D(m_i)/\sigma\right)^2}}$$

Sparseness constraints

Re-formulating the problem

- The cost for one iteration equals to two migrations
- Re-formulate the problem as follows

$$
\begin{aligned}
\mathbf{0} &\approx \mathbf{W_d}(\mathbf{Lm} - \mathbf{d}) \\
\mathbf{0} &\approx \varepsilon \mathbf{W}_s D(\mathbf{m})
\end{aligned}
\quad \Longrightarrow \quad
\begin{aligned}
\mathbf{0} &\approx \mathbf{Hm} - \mathbf{m_{mig}} \\
\mathbf{0} &\approx \varepsilon \mathbf{W_s} D(\mathbf{m})
\end{aligned}
$$

Approximating the Hessian

- Target-oriented Hessian via computing the Green's functions (Valenciano and Biondi, 2004)

- Approximating Hessian via reference images (Rickett, 2003)

$$\mathbf{H} \approx \mathbf{W_H} = \frac{diag\left(\mathbf{L'W_d'W_dLm_{ref}}\right)}{diag\left(\mathbf{m_{ref}}\right)}$$

Approximated inversion scheme

$$\mathbf{0} \approx \mathbf{W}_{\mathbf{H}}\mathbf{m} - \mathbf{m}_{\mathbf{mig}}$$

$$\mathbf{0} \approx \varepsilon \mathbf{W}_{\mathbf{s}} D(\mathbf{m})$$

W_H : approximated Hessian

D : differential semblance

W_s : sparseness weights

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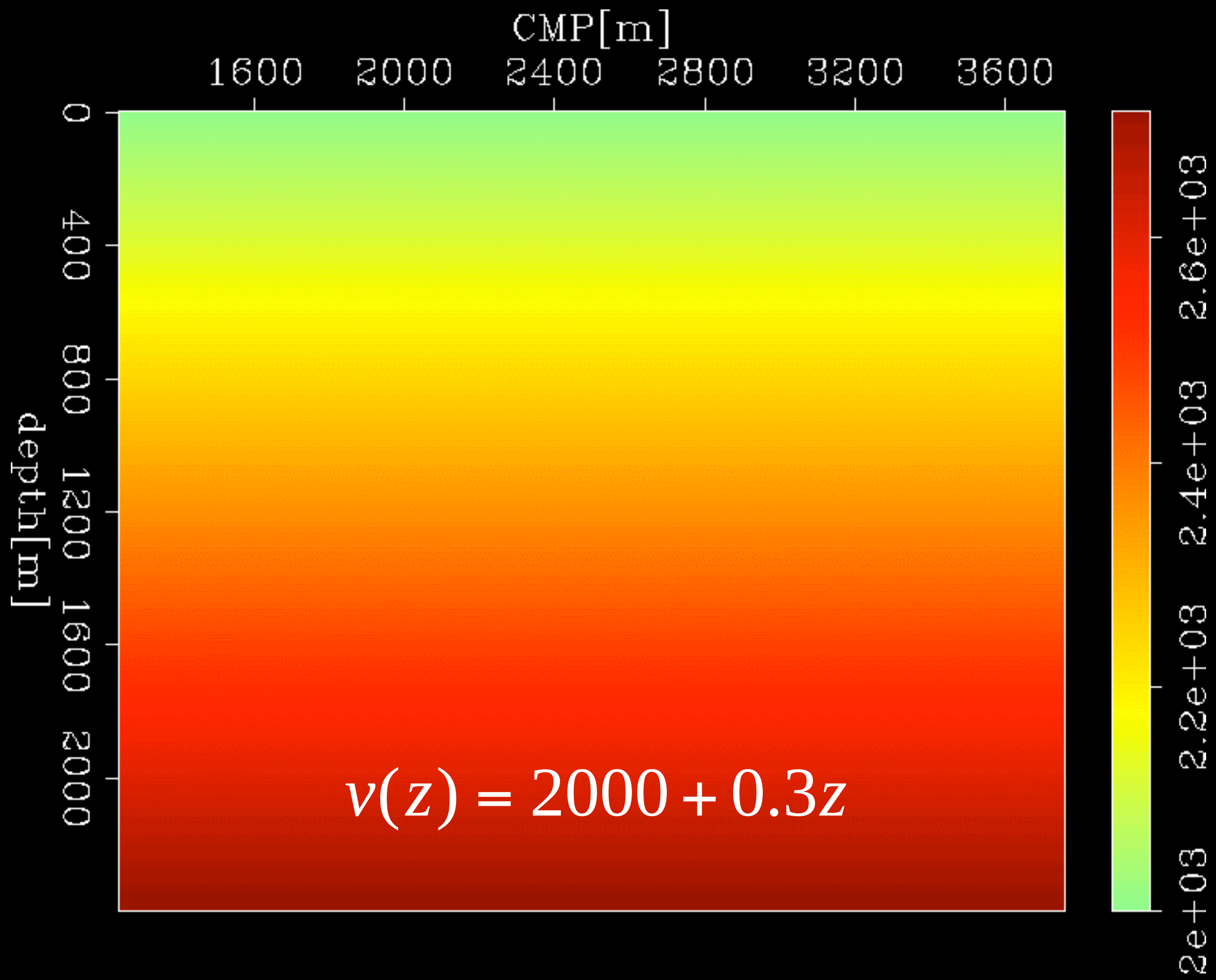
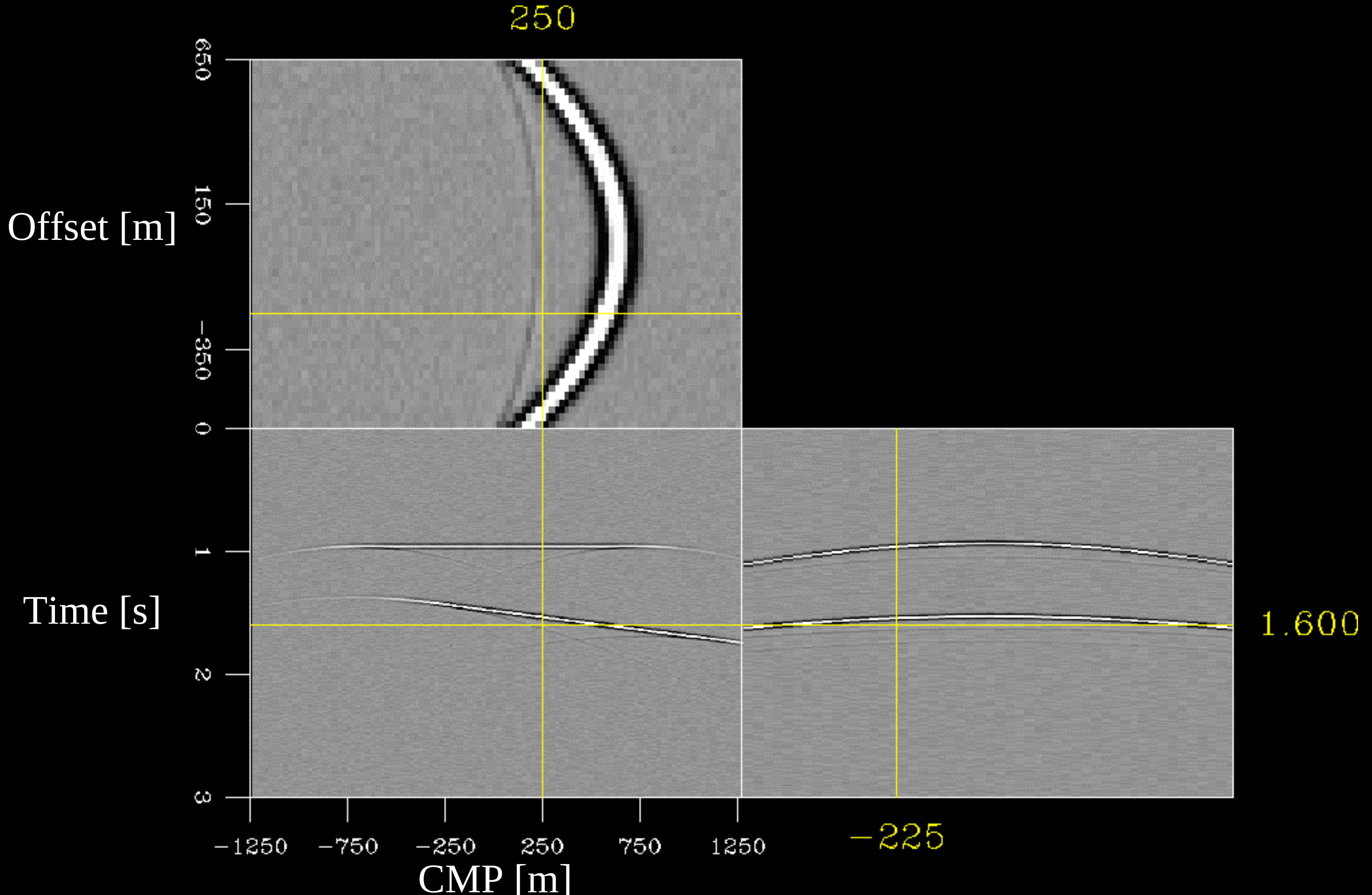
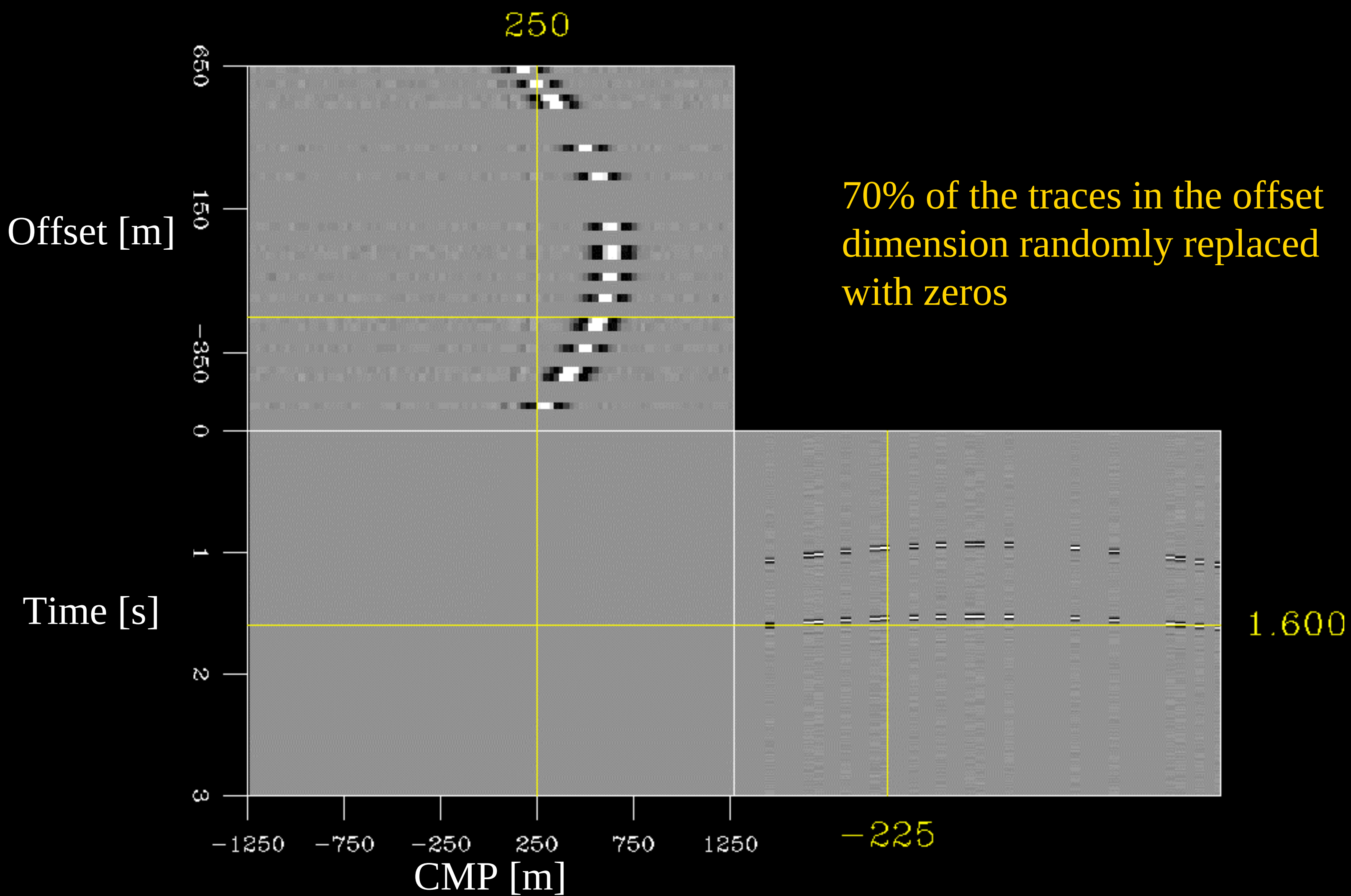
V(z) velocity model



Synthetic data

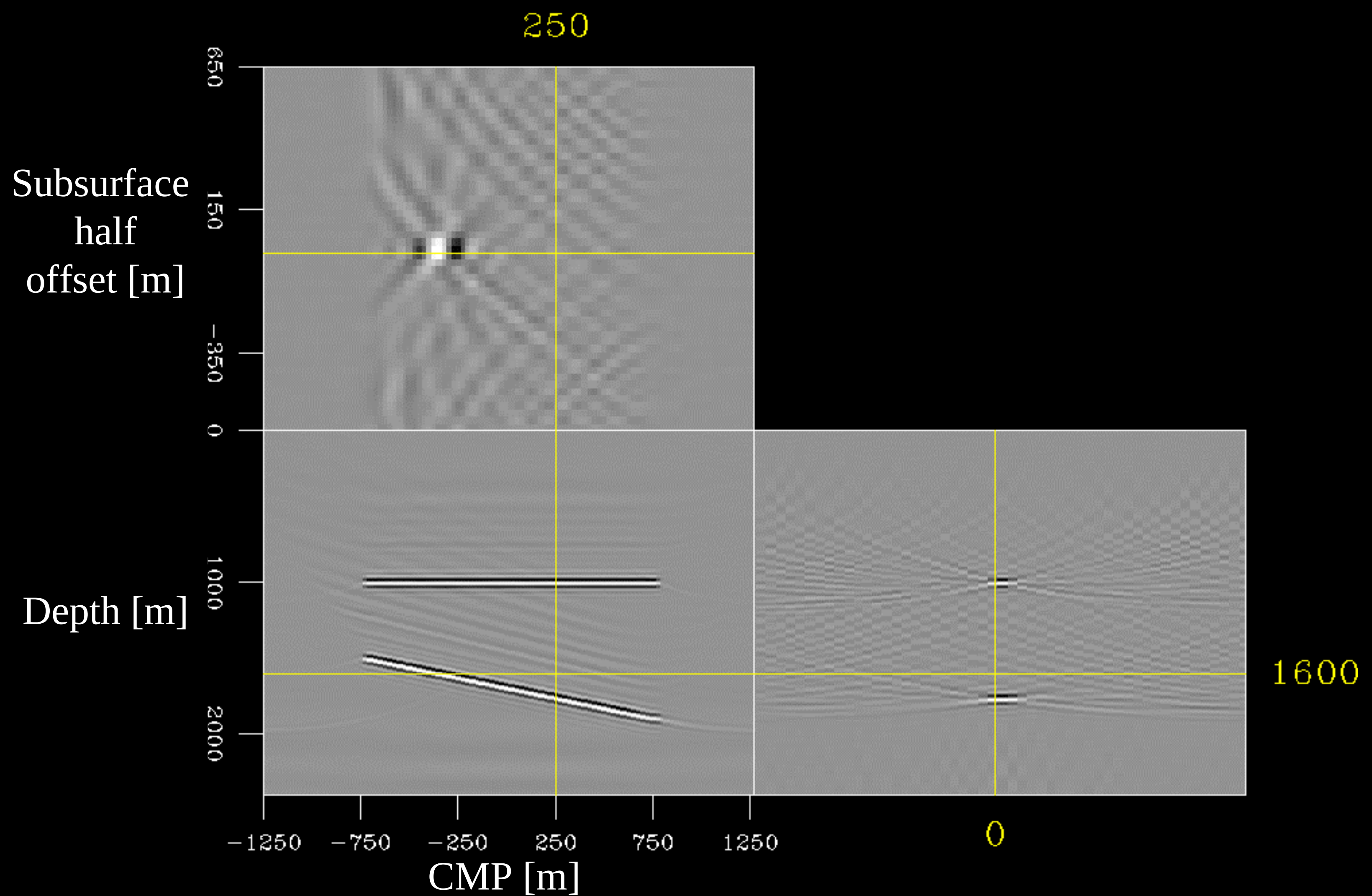


Decimated data

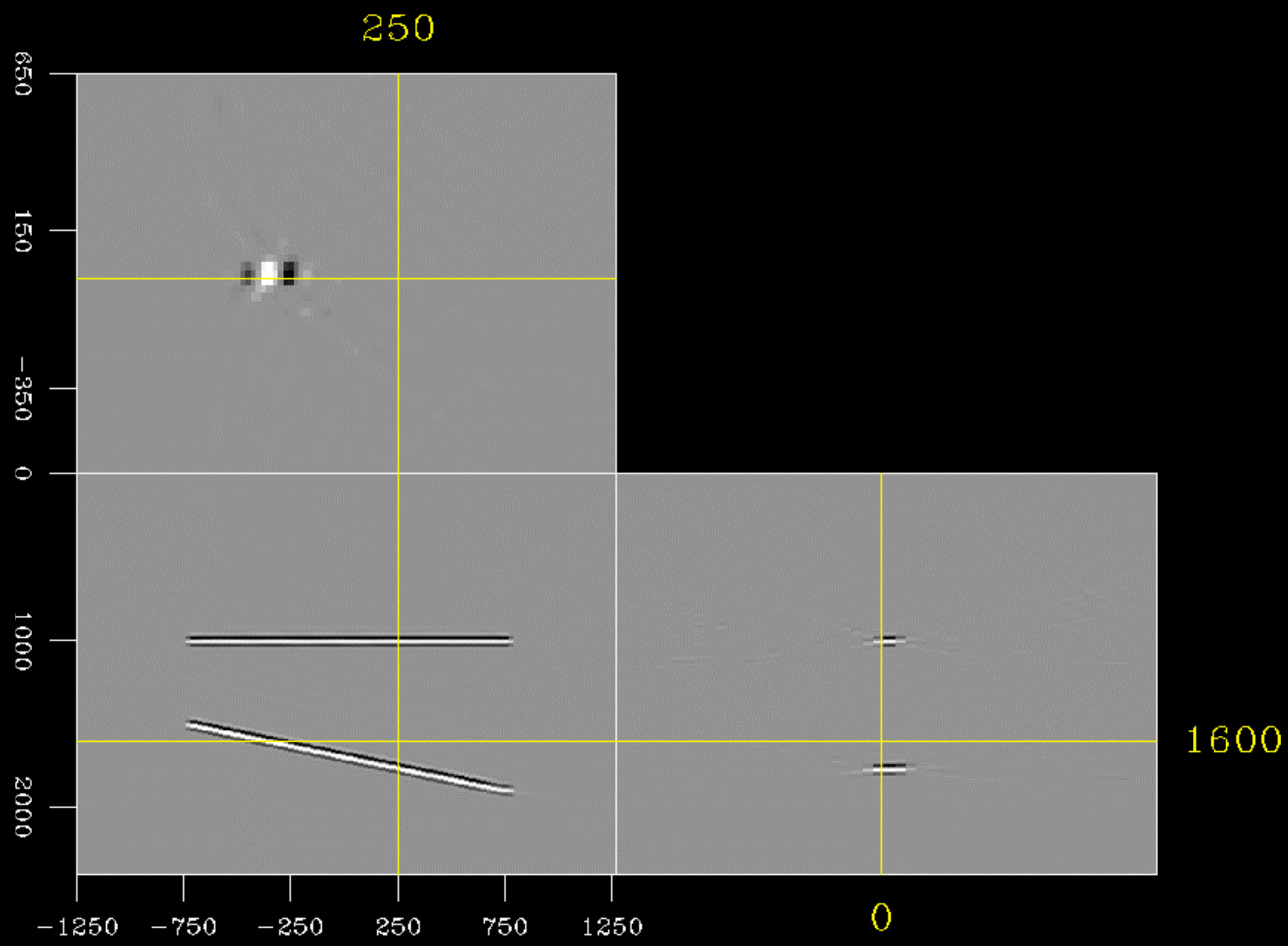


70% of the traces in the offset dimension randomly replaced with zeros

Migrated SODCIGs



Inverted SODCIGs

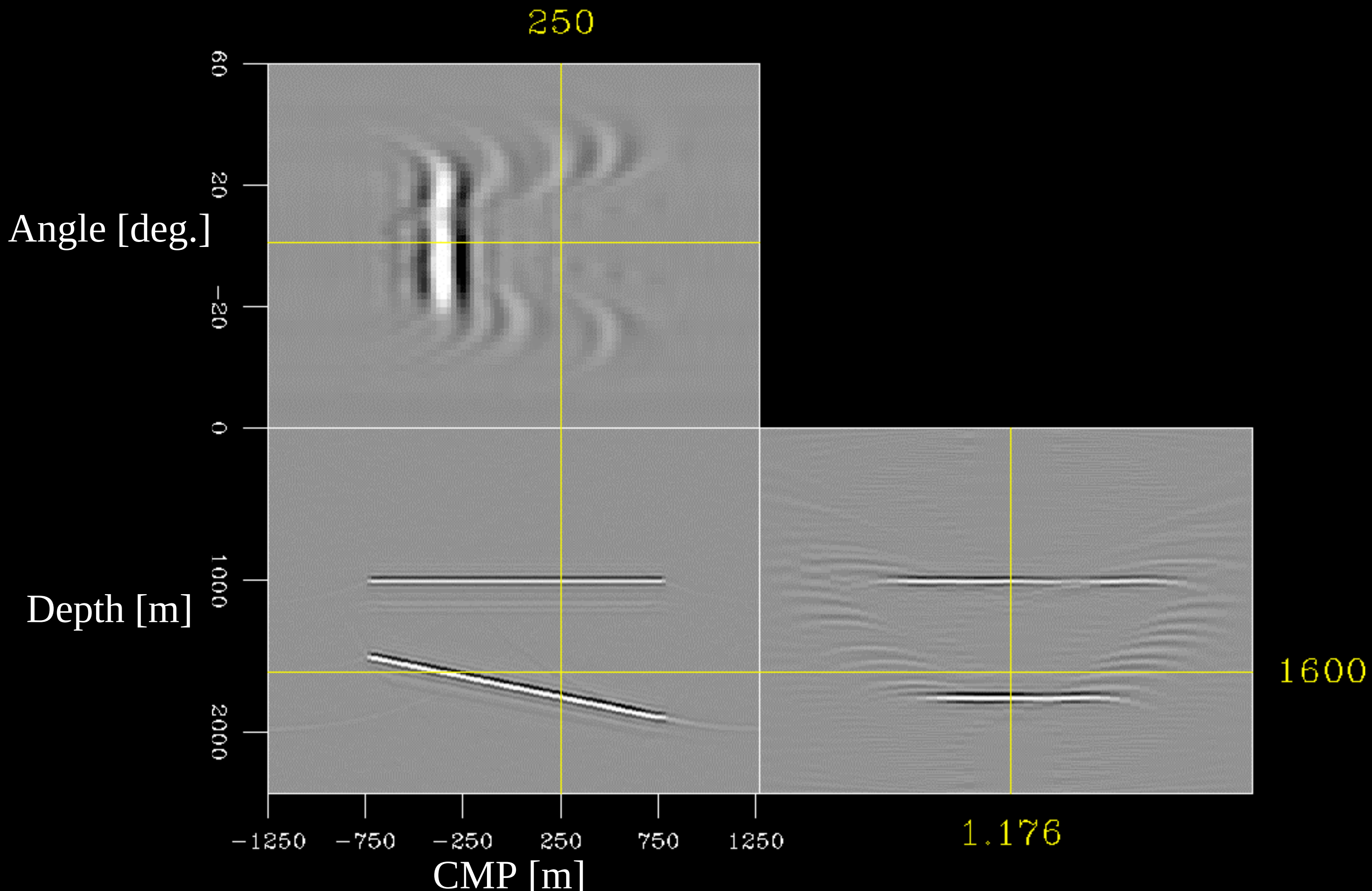


Subsurface half offset [m]

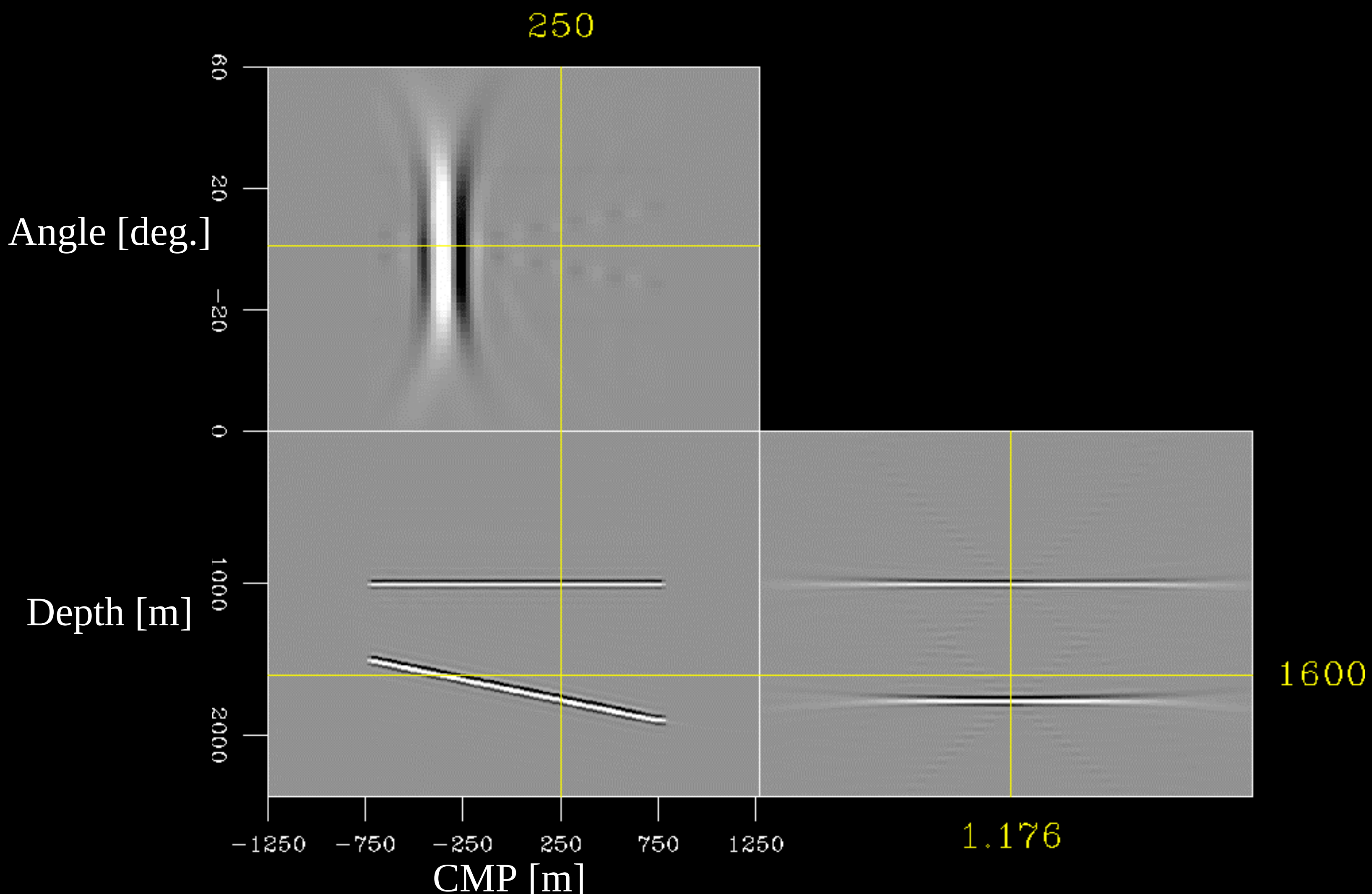
Depth [m]

CMP [m]

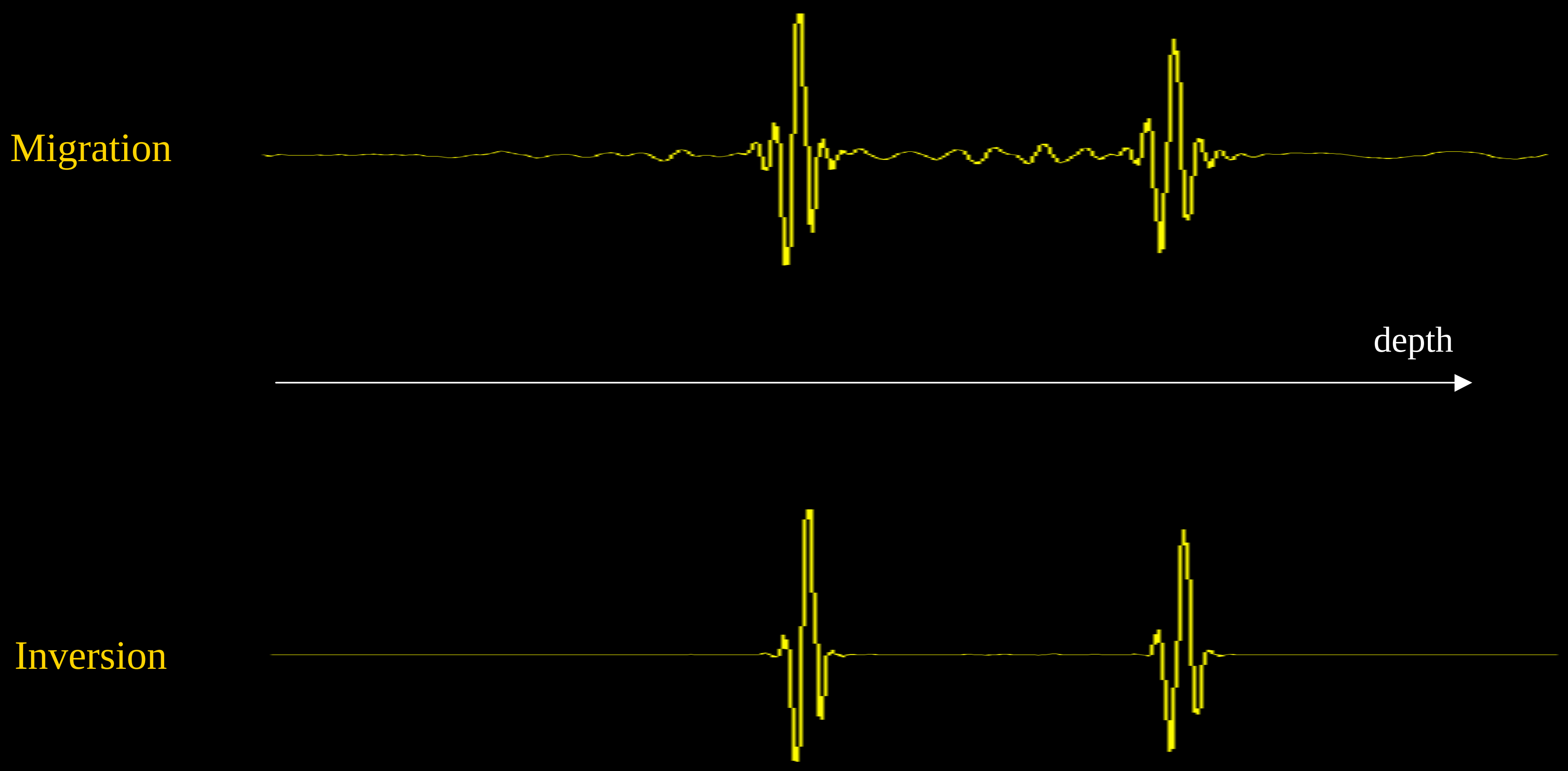
Migrated ADCIGs



Inverted ADCIGs



Traces at CMP=0 & offset=0



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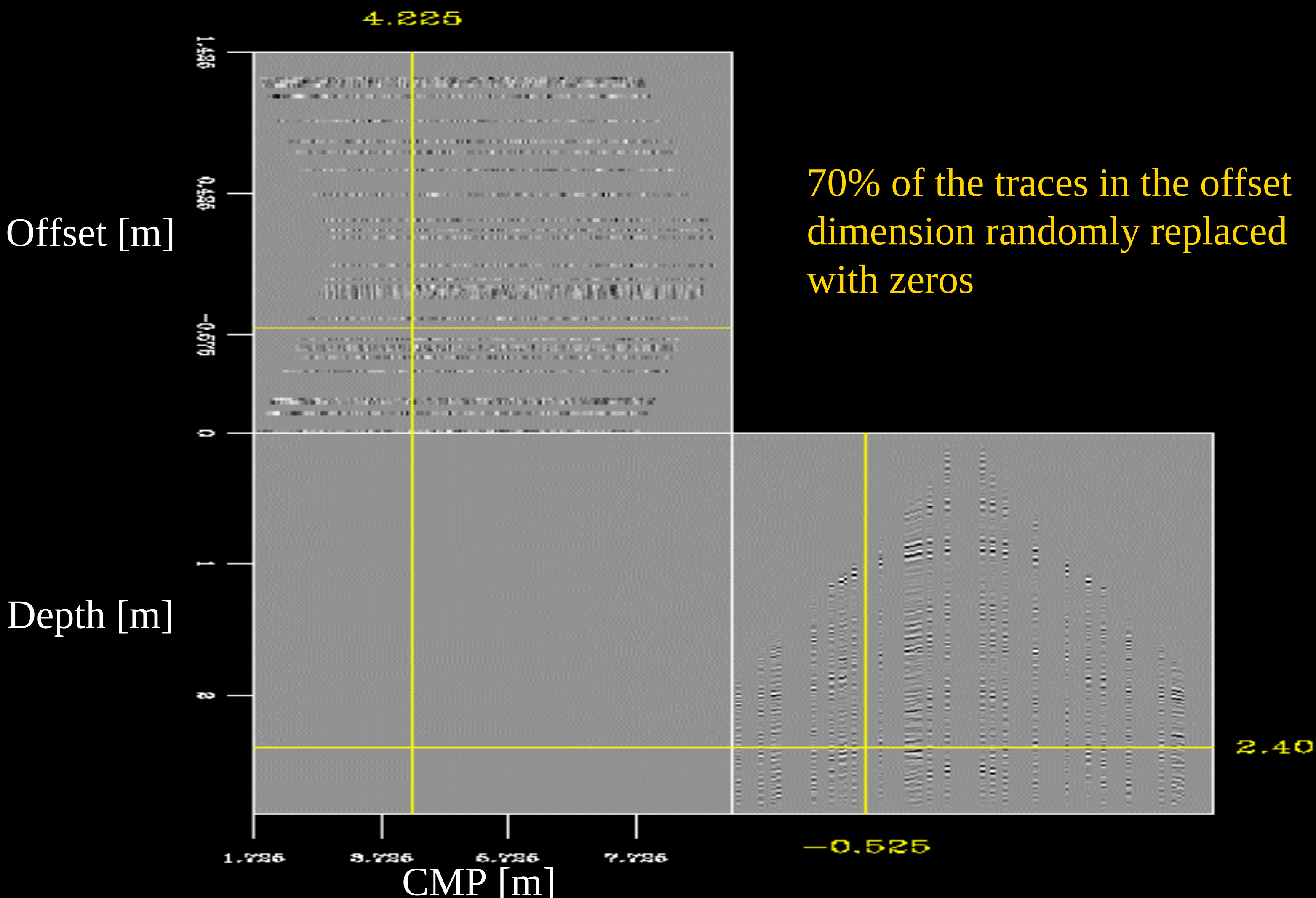

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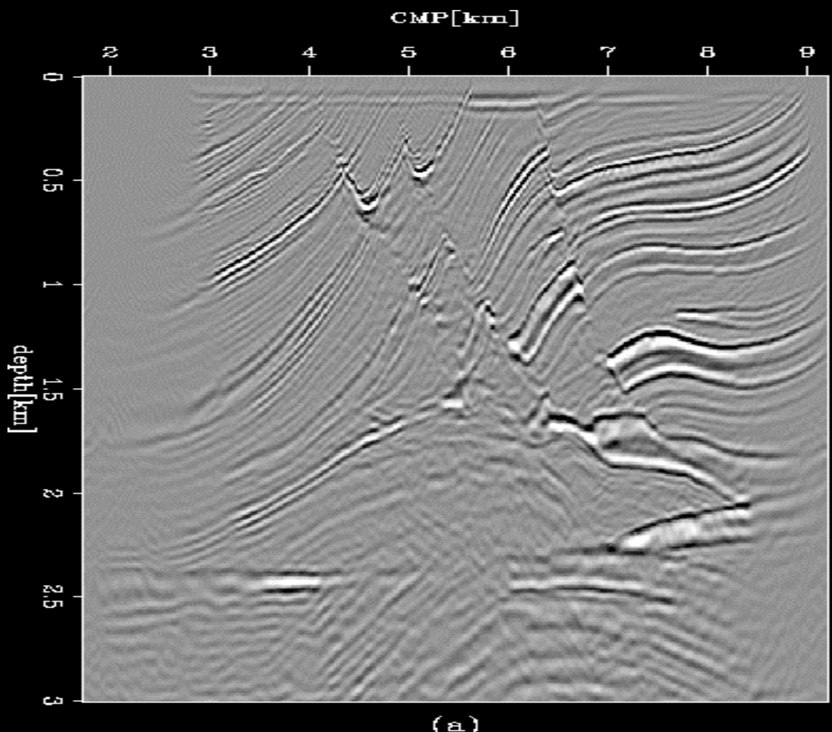
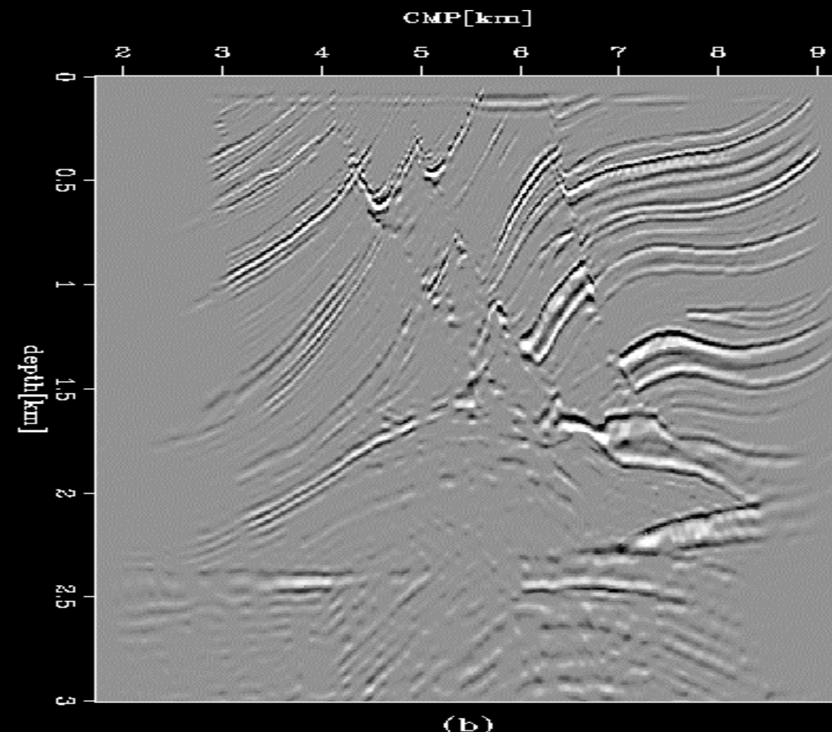
Comparison of images

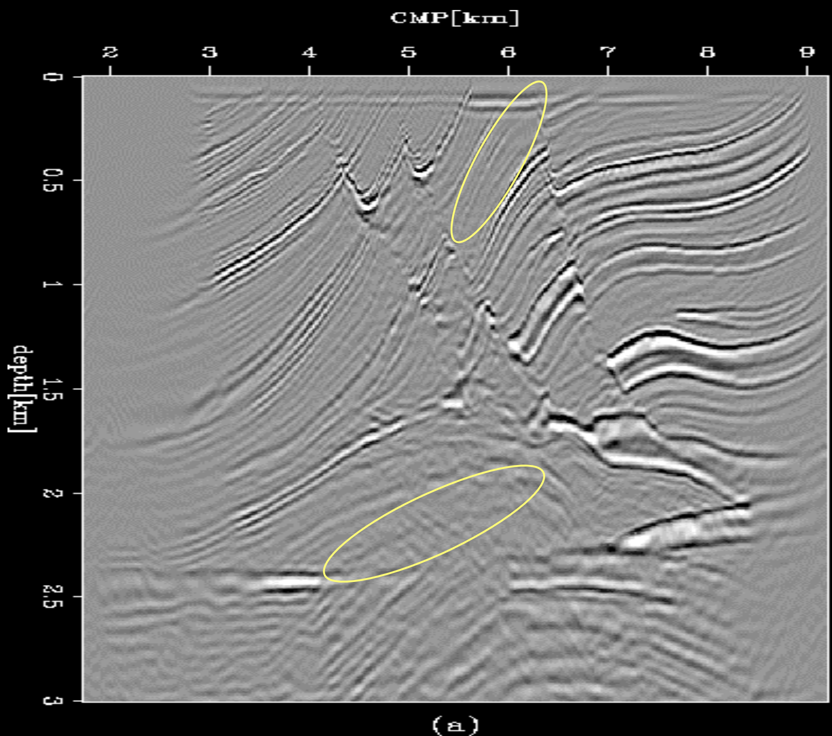


Migration

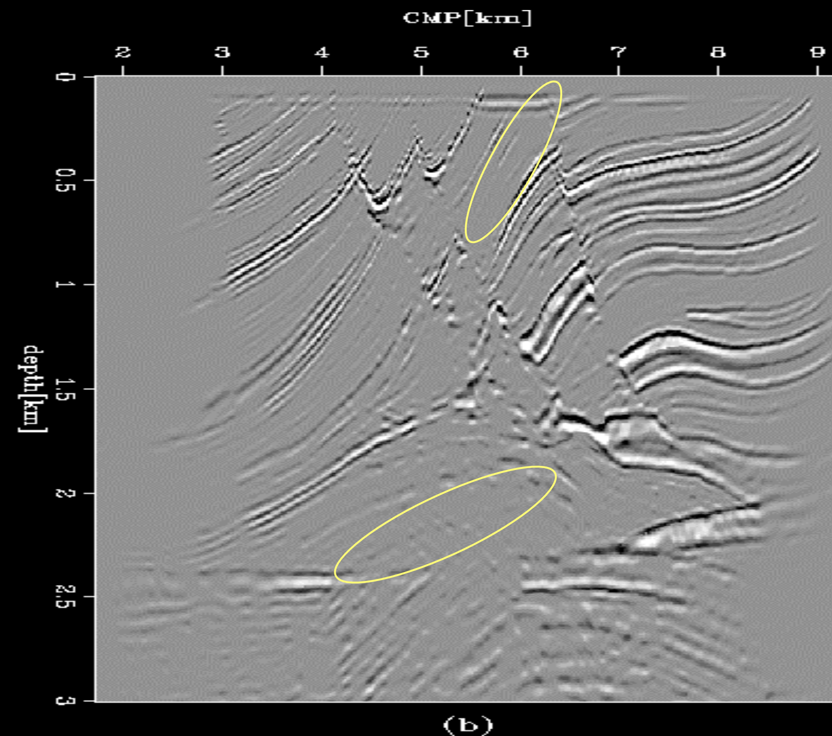


Inversion

Comparison of images



Migration



Inversion

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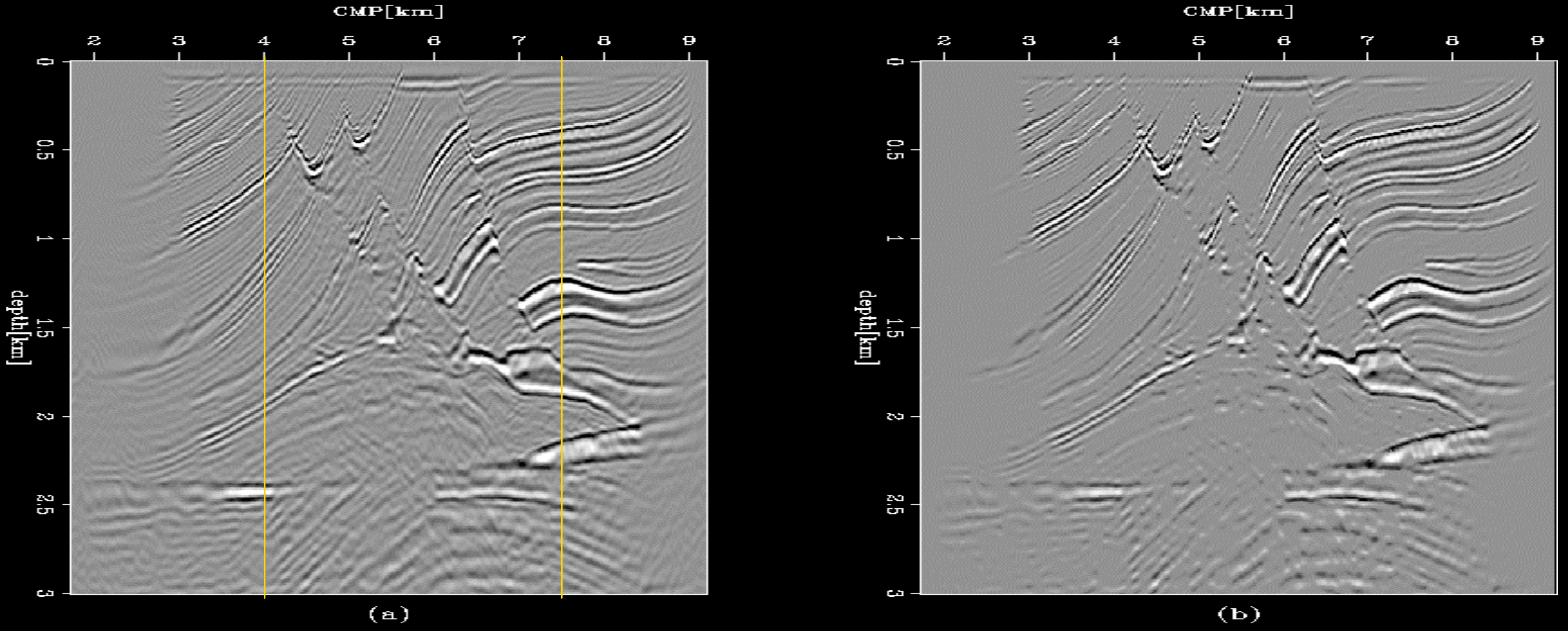
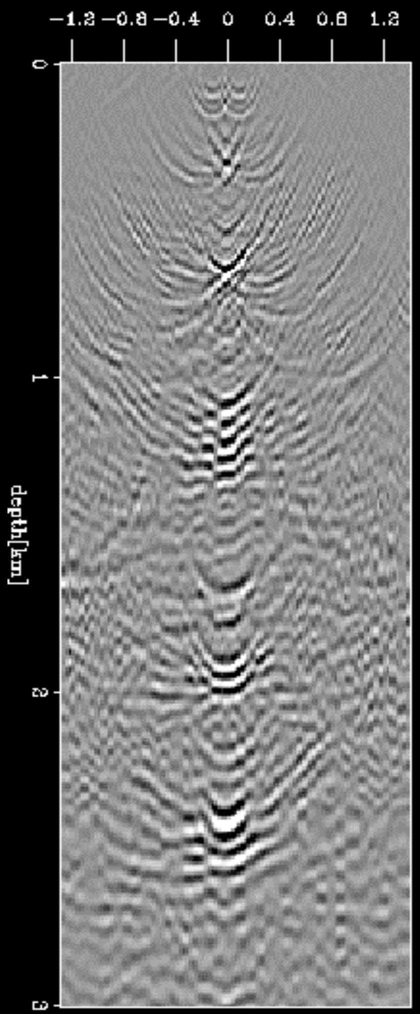


Migration

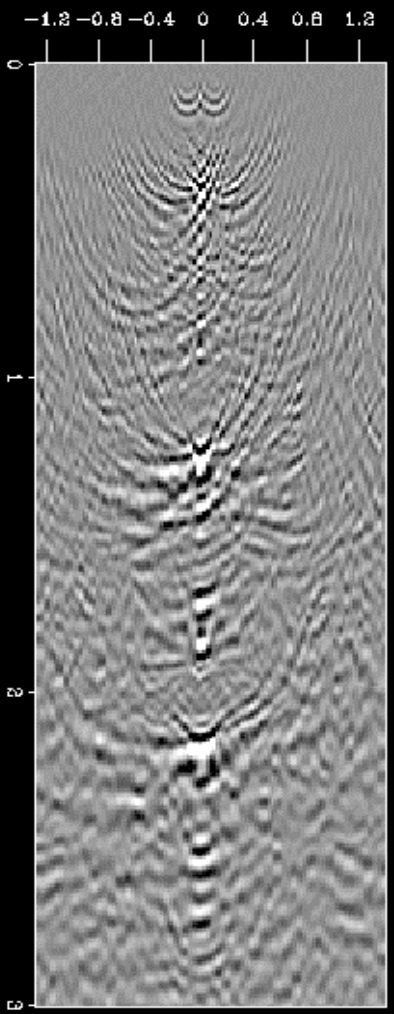
Inversion

Comparison of SODCIGs

Subsurface half offset [m]



Migration



Migration

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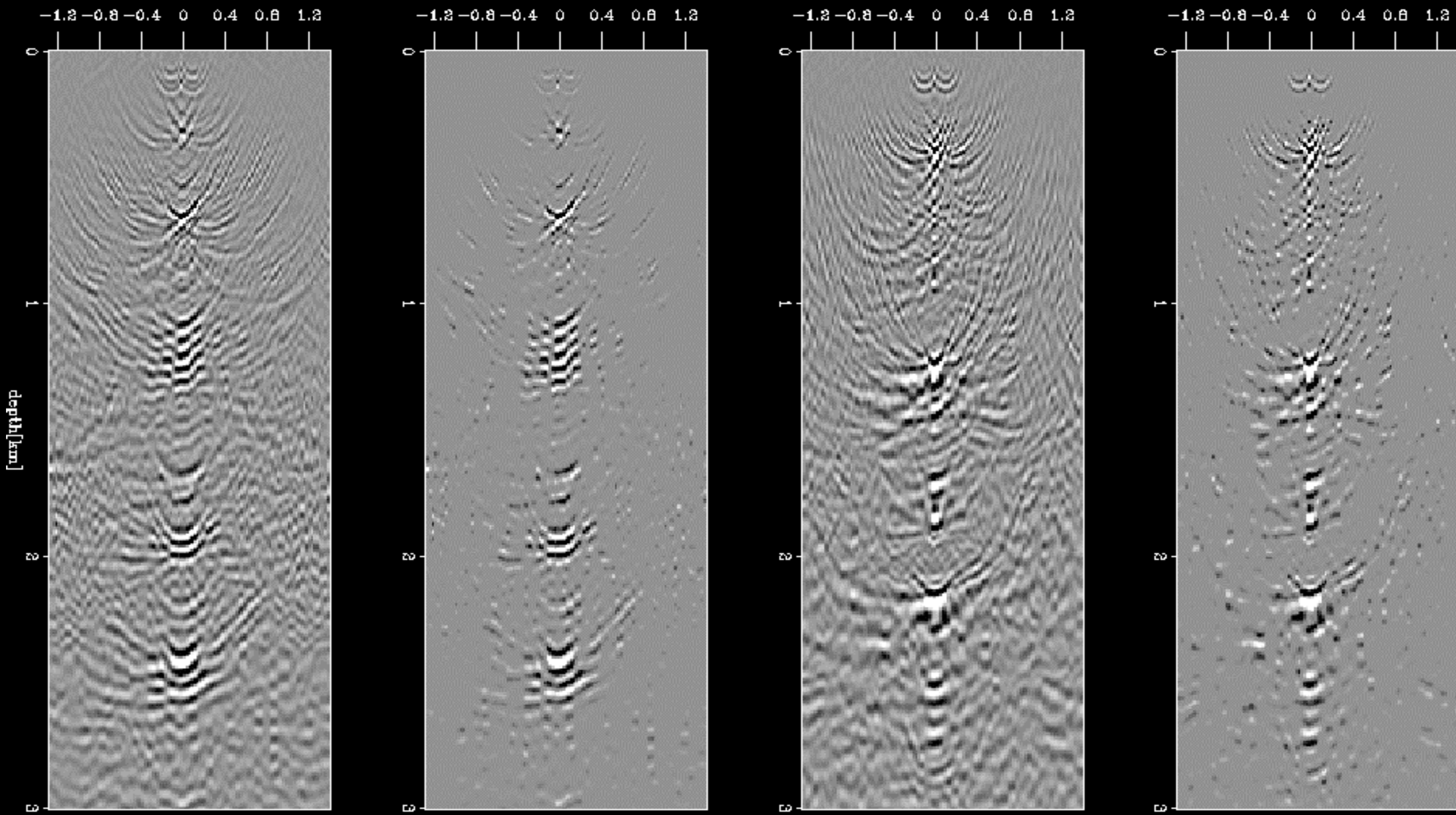
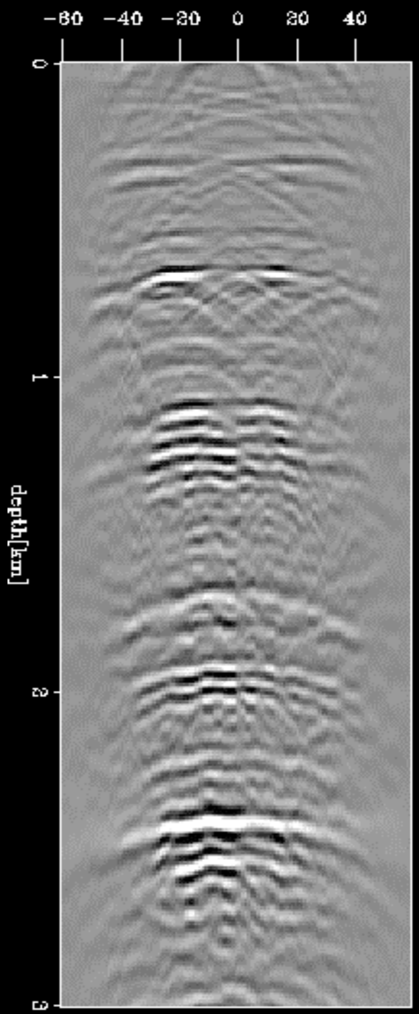


Migration | Inversion | Migration | Inversion

Comparison of ADCIGs

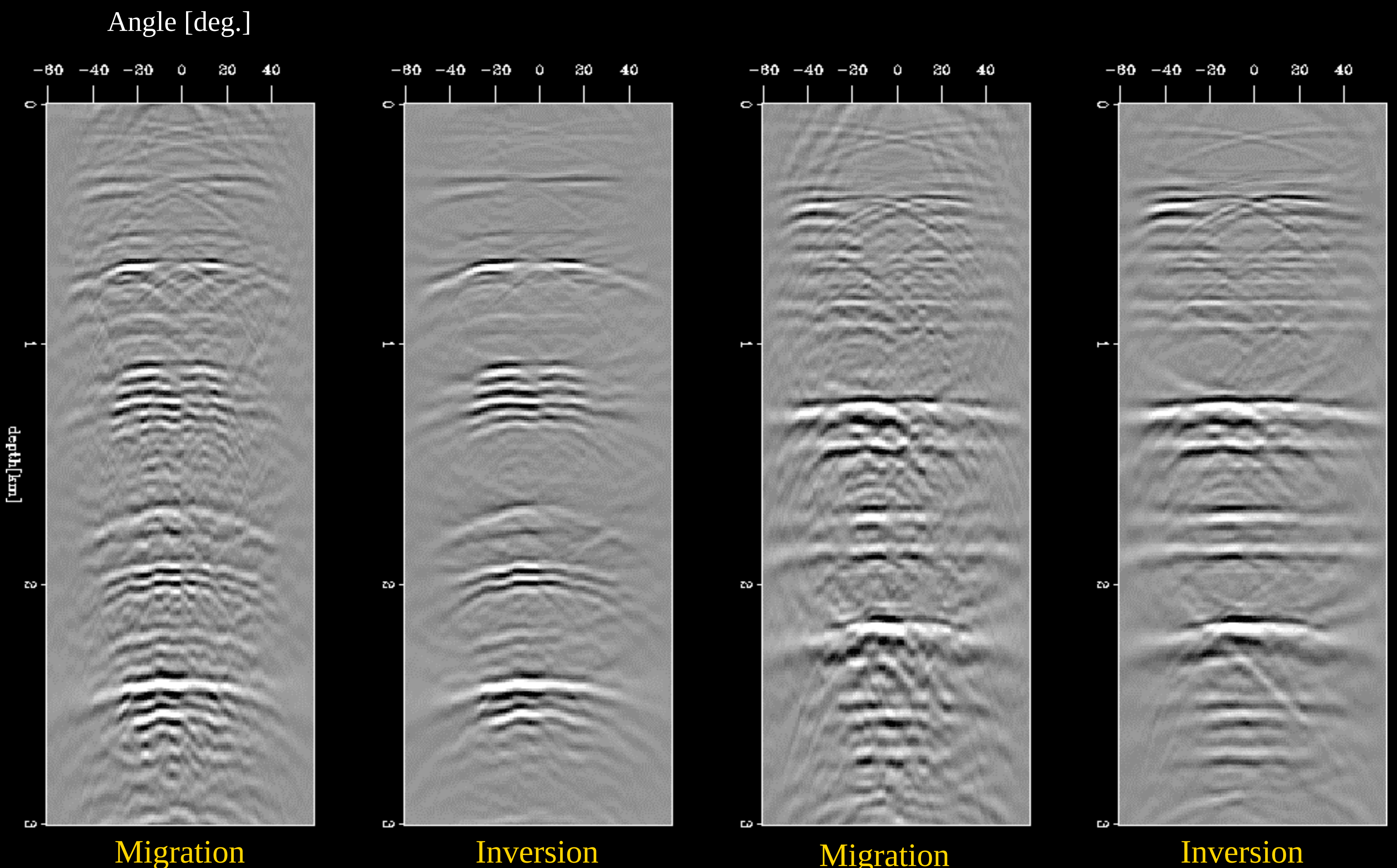
Angle [deg.]



Migration

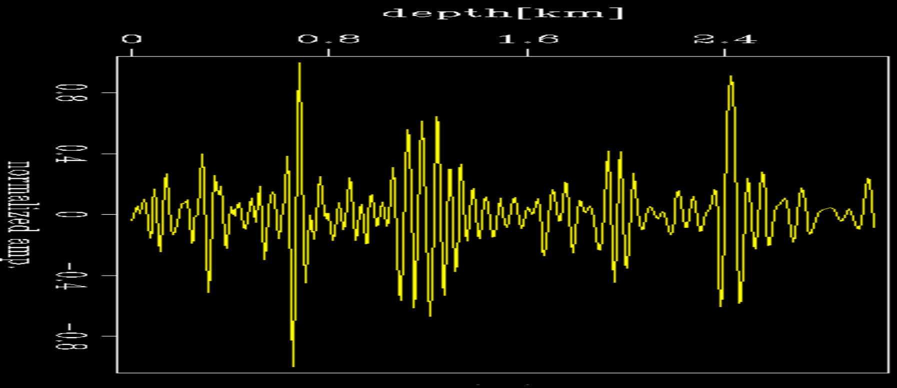
Migration

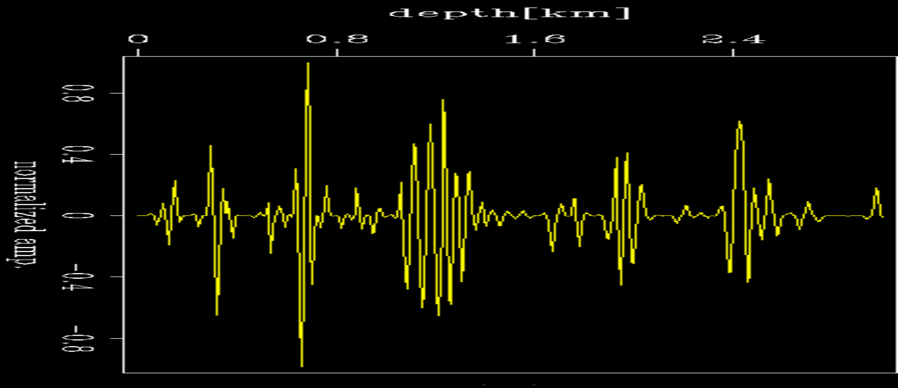
Comparison of ADCIGs



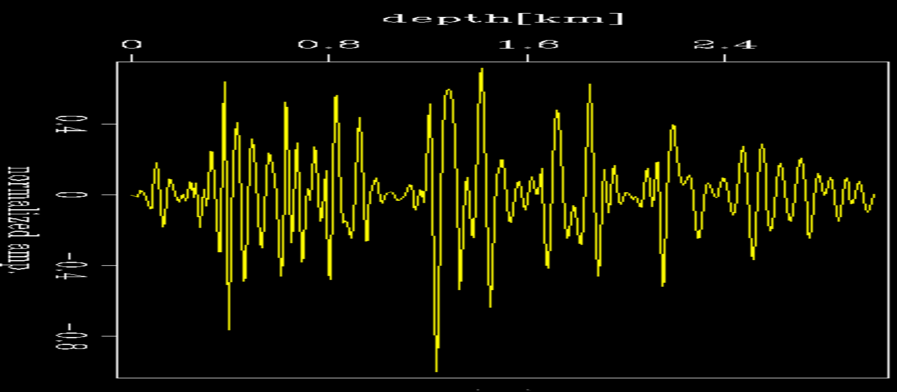
Traces from images

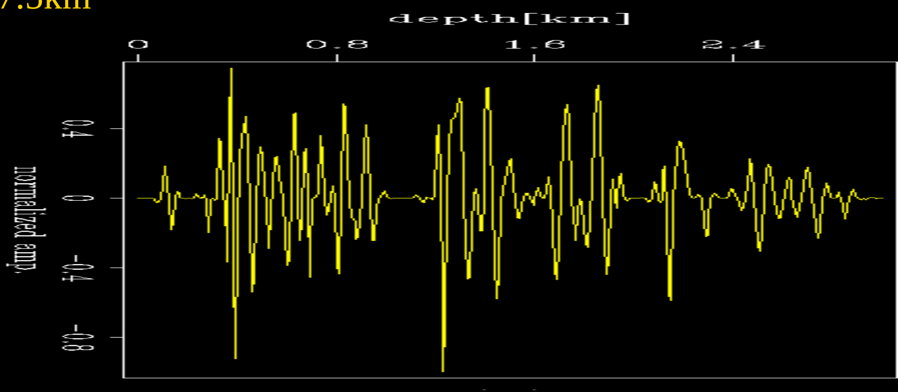
CMP=4km





CMP=7.5km





Migration

Inversion

Conclusions

- Poorly sampled data cause artifacts in the migrated gathers
- The approximate inversion scheme can attenuate those artifacts effectively
- The cost is low compared to the exact inversion scheme
- Hyper-parameters ε and σ should be chosen with care to avoid over regularization

Thanks