NMO stretch beaten! by you! in this SEP report!

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ABSTRACT
You could be the winner! Solve a longstanding problem everyone knows about. And do it in time for this coming SEP report! Here’s a guide.

INTRODUCTION
Several mechanisms remove high frequencies from wide offset traces thus leading to NMO stretch. Even without identifying the model, we have tools to rebuild missing high frequencies. It would be nice if someone could stitch together a few model fragments; but even if no-one can, we can produce examples that make it look like we did!

WHAT IS NMO STRETCH?
Presuming a horizontally layered earth, on a gather we have hyperbolic traveltimes $t^2 = \tau^2 + s^2 x^2$. Event slopes $dt/dx = s^2 x/t$ are decreasing with $t$. The nonparallel slopes mean the events are getting closer together with increasing $x$. NMO correction should be bringing them to uniform separation. NMO correction does this by stretching. Stretching makes frequencies lower. It is often that case that the wider offset traces are not higher frequency to begin with as hyperbola analysis tells us to expect which means that some other process is at work. Figure 1 is an example. It is marine data.

Figure 1: NMO stretch entirely due to shot waveform, not $Q$, because this is marine data. [from BEI]
data moved out at water velocity. So the water bottom has been flattened, but directly beneath it is another event with a significantly higher velocity. These velocity measurements measure velocity integrated from the surface to the event. When two nearby events measure significantly different velocities as we see here, the velocity in the thin layer between them must be astounding. In fact, other processes are at work that have suppressed the theoretically higher frequencies on the wider offsets. Three processes that might account for it are shot waveform, differential absorption, and sender/receiver arrays. One of the three is easy to understand and offers us a convenient tool suitable to restore higher frequencies to the wide offset traces. That tool is $Q$ compensation. It offers time variable frequency restoration which because slopes are time variable, may be a useful tool in all situations.

THE TOOLS

Figure 2 shows the Futterman operator for positive traveltime depth $\tau$. The other half (negative $\tau$) lies above and left of the origin. It contains the anti-causal inverse Futterman operators.

Use your imagination for a few moments. After NMO stretches your far offset traces they will have lower frequency content. Maybe you could use some of those filter responses from the upper left quadrant? That is where you find a high-frequency boost that diminishes with traveltime depth (the horizontal axis in Figure 2). Hooray! Not sure if they are boosting and diminishing at the rate we all want though. To find that rate, maybe we need to play around, better yet find the rate that various physical phenomena call for. Maybe we’ll happen onto a model that fits the data. Perhaps we’ll discover that a byproduct of eliminating NMO stretch happens to give an estimate of $Q$! Or perhaps it is something else.

Figure 2: This Futterman matrix operator $\mathbf{F}$ converts a model space (function of traveltime depth) to a data space (function of time) $\mathbf{d} = \mathbf{Fm}$. Mathematically, each column is the spectral factorization of $e^{-|\omega|\tau/Q}$. Imagine this plot extended to negative $\tau$. The operators up there would remove absorptive travel path. In so doing, they would oscillate instead of smoothing.
SOME THINGS YOU CAN BUILD

Figure 3 shows a spike model displayed through various Futterman windows. What is magic here? You see the same signal viewed through various frequency bands. Is it really magic? We might see something similar with a bank of sinc functions. But sinc functions are not time variable. They’re not the right tool.

Figure 3: An illustration stolen from the Futterman paper shows a signal through several time-variable spectra. Real models have many more spikes. No problem.

MODELS EXPLAINING NMO STRETCH

The most obvious explanation for NMO stretch seems to be $Q$. Waves going vertically from some layer have a shorter less-absorptive path than waves at any other angle. So, we may want to undo the $Q$ effect of that extra propagation distance. Still, we do not know if that is what the data calls for. $Q$ is not the only explanation.

Shot waveform

For years I imagined that NMO stretch could be blamed entirely on the shot waveform. That’s surely true of the marine bubble. The first bubble often follows the first arrival by about 120 ms, and then there may be more bubbles. But the ghost typically comes in much closer. This Ricker wavelet like thing is pretty short. People try inverting it, but I have not heard of it as the solution to NMO stretch. Perhaps shot waveform must be considered jointly with $Q$. 
Receiver array filtering

Shots and receivers occur as horizontal arrays. As such, they are natural high-cut filters on the space axis. Do these arrays have any filtering effect on time? Yes, spatial and temporal frequencies become linked by the angle of wave propagation. This effect is again to diminish higher frequencies at wider offsets, hence again, something like an inverse Futterman filter might be what we need to pull back the higher frequencies.
RELATED PROJECTS YOU COULD COAUTHOR

1. Assume NMO stretch is caused by shot waveform. Say how to suppress it.

2. Assume NMO stretch is caused by Futterman. Say how to suppress it.

3. How to better estimate shot waveforms by understanding the effect of $Q$?

4. How do $Q$ and shot waveform affect various migration and velocity algorithms?

5. Wanting to routinely use the unitary operator (Futterman phase correction) on field data, how do we default the parameter $Q$?

6. What is the derivative of a seismogram with respect to $Q$?

7. Try estimating $Q(\tau)$ on field data.

8. Reorganize code here to be easily accessible with SEPLIB main program and linkable operators. Include zero padding. Verify the dot product test.

9. My code does filtering by matrix multiplication. But the matrix is really very sparse. How do we recode to take advantage of sparsity?

10. What parameters and defaults should SEP’s production anti-Futterman code have for nonzero offset on land data? on marine data?

11. Put some sparse and semi-sparse models into Futterman and test inversion, and test prediction error, and test missing data estimation.

12. Our present basis functions for seismograms are impulses at all lags. But, we are oversampled at late times. Were we to switch to the model space of Futterman filters we would not need so many late time basis functions. Suppose a sparseness optimization program were able to turn off unneeded basis functions at late time, would the remaining ones correlate with nearby seismograms illuminating event slopes? or would we see only noise?

13. The shot waveform operator $S$ is a time-invariant convolution matrix clustered fairly near the main diagonal (because the shot energy is within a few dozen millisec.) The Futterman operator $F$ is also a filter matrix but it changes slowly down the main diagonal. These matrices approximately commute, but thinking of their non-commutivity (perhaps pedantically) one should come before the other. Describe migration or velocity or tomography environments in which you know which filter comes first?