Stanford Exploration Project, Report 100, April 20, 1999, pages 51–58

Short Note

Why tau tomography is better than depth tomography

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keywords: *tau, tomography, velocity*

INTRODUCTION

Seismic tomography is a non-linear problem. A standard technique is to itteratively assume a linear relation between the change in slowness and the change in travel times (Biondi, 1990; Etgen, 1990) and then re-linearize around the new model. In ray-based methods, this amounts to assuming stationary ray paths and reflection locations to construct a back projection operator (Stork and Clayton, 1991). The change in this back projection operator from non-linear iteration to non-linear iteration can be thought of as an important second order effect.

By formulating our back projection operator in terms of vertical travel-time (τ) rather than depth our reflector locations become more stable (Biondi et al., 1997; Clapp and Biondi, 1998). We show that the corresponding back projection operator is less sensitive to our initial velocity estimate. Therefore, our back projection operator changes less from non-linear to non-linear iteration, making the estimation less likely to get stuck in local minima.

THEORY

Velocity estimation is fundamentally an inverse problem. The correct solution is to do full wave-form inversion (Tarantola, 1986; Mora, 1987), but is generally impractical. Instead we start from the idea that there is a non-linear operator that relates slowness ($\mathbf{s}$) and travel time ($\mathbf{t}$),

$$\mathbf{t} \approx \mathbf{T_n s}. \tag{1}$$

We then attempt to approximate $\mathbf{T_n}$ by doing a two term Taylor expansion around our initial guess at the slowness field (a version of Newton's minimization method):

$$\mathbf{t} \approx \mathbf{T_n s_0} + \mathbf{T_0'} \Delta \mathbf{s}$$

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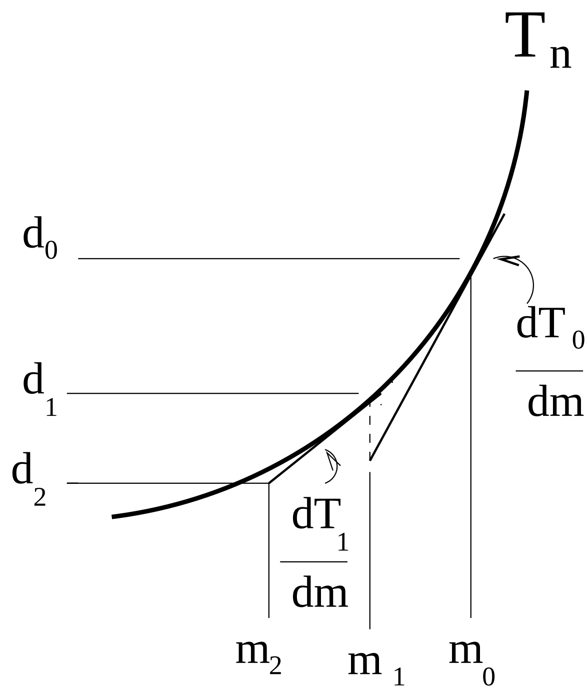

Figure 1: Newton's method applied to ray based tomography. bob2-newton [NR]

$$\begin{aligned} \mathbf{t} &\approx \mathbf{t_0} + \mathbf{T'_0}\Delta\mathbf{s} \\ \Delta\mathbf{t} &\approx \mathbf{T'_0}\Delta\mathbf{s}. \end{aligned} \tag{2}$$

where

$\mathbf{s_0}$ is our initial guess at slowness,

$\mathbf{T'_0}$ is a linear operator describing the relationship between, slowness and travel times given the initial slowness model. In ray-based methods we usually use some stationary ray paths based on the initial slowness model,

$\Delta\mathbf{s}$ is the change in slowness,

$\mathbf{t_0}$ are the modeled travel times applying $\mathbf{T'_0}$ to $\mathbf{s_0}$,

$\Delta\mathbf{t}$ are the difference between the modeled travel times, $\mathbf{t_0}$, and the measured travel times, $\mathbf{t}$.

After inverting for $\Delta\mathbf{s}$, we have a new estimate for our slowness field:

$$\mathbf{s_1} = \mathbf{s_0} + \Delta\mathbf{s}. \tag{3}$$

We can then re-linearize around this new model ($\mathbf{s_1}$), constructing a new tomography operator $\mathbf{T'_1}$. We repeat this procedure until $\Delta\mathbf{t}$ is negligible. Figure 1 is a graphical representation of the method. There are two problems with this approach. First, Newton's method is only guaranteed to converge to a local minima, we hope that by applying regularization (Clapp and Biondi, 1999) we can avoid this problem. And second, we are only using the first term

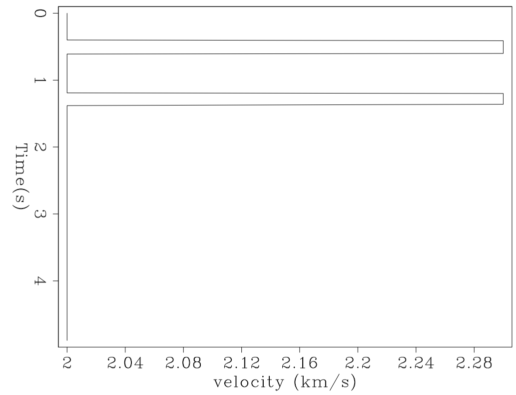

Figure 2: Synthetic 1-D velocity function in τ. bob2-cor [ER]

in our Taylor expansion, which means that when our higher order derivatives are large, are descent direction will be wrong, and we will converge at a much slower rate. When using rays, this problem occurs when the initial guess at ray paths and reflector locations are too far from their *correct* locations.

We can obtain a measure of how inaccurate our linear approximation is by looking at how much our linearized tomography operator changes from non-linear iteration to non-linear iteration (the difference erence between $\mathbf{T'_0}$ and $\mathbf{T'_1}$.) The smaller the difference, the more accurate our linearization, and the less likely our estimate well diverge. By forming our tomography in (τ, x) rather than (z, x) space, we reduce the change in $\mathbf{T'_1}$ from $\mathbf{T'_0}$. The fundamental reason is that our data is in time rather than depth. In depth, reflector positions and layer boundaries change significantly from iteration to iteration, while in tau, they hardly change at all (Biondi et al., 1997).

SIMPLE TEST

To demonstrate how the tau back projection operator is less affected by our initial slowness model, we constructed a simple 1-D synthetic. The model, Figure 2, is composed of two 2.3 km/s zones in a constant 2 km/s background. For this test we assumed that our slowness model had correctly resolved the bottom anomaly in vertical travel time. Our choice of vertical travel time is quite important, as when doing velocity estimation, we must always preserve zero-offset travel time. In this simple 1-D synthetic, that means that the vertical travel-time to the layer boundaries and to the reflector must be kept constant. Therefore, in depth, we will misplace the location of of the bottom high velocity zone but preserve the correct vertical travel times to the layer top and bottom. After constructing the model we found the ray that hit the reflector at 2 km depth, 2 km away from source in both (τ, x) and (z, x) space (Figure 3.) Following the method outlined in Clapp and Biondi (1998), we built the tomography operator for both tau ($\mathbf{T}'_{\mathbf{0},\tau}$) and depth ($\mathbf{T}'_{\mathbf{0},\mathbf{z}}$), Figure 4. For comparison, we ray traced through the 'correct' velocity model in both spaces (Figure 5) and calculated the corresponding operators. By comparing the correct and initial operator for tau and depth, or by looking at the difference between the two operators (Figure 7), we can clearly see that our initial guess for our tau operator is overall better than our initial guess for our depth

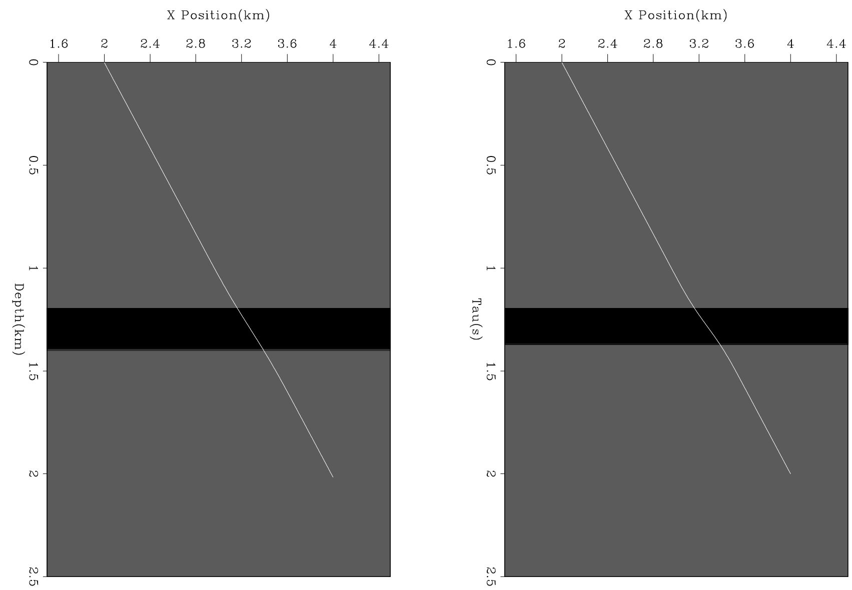

Figure 3: Initial guess at the velocity function overlaid by ray hitting reflector at 4 km with a half-offset of 2 km. Left panel is in depth, right panel is in tau. bob2-vel0 [ER]

operator. In the upper layer, we see marginally more change in the tau operator but at the lower reflector boundaries (which move in the case of depth but remain constant in tau) we see significantly more error in depth. In addition, the change in reflector position has caused a spike in the difference panel for the depth case. Finally, the change in the tau operator is smooth, while the change in the depth operator shows dramatic jumps. Our successive relinearizing have an underlying assumption that we are smoothly converging to the correct operator. In tau space, this assumption seems to be more valid. With a more complicated model our positioning of layer boundaries, will be subject to more change, making the tau compared to depth difference even more dramatic.

CONCLUSIONS

We showed that for this simple model the tau back projection operator is less affected by our initial velocity estimate than a depth back projection operator. We hypothesize that makes tau tomography to some extent *more* linear and, therefore, less likely to diverge.

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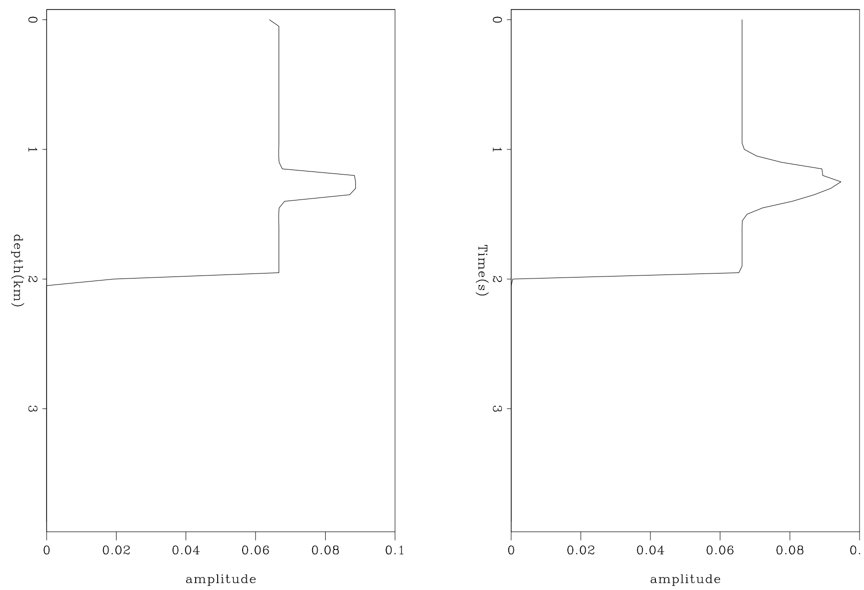

Figure 4: The operator calculated from our initial guess at velocity and the resulting ray paths in depth (left) and tau (right). bob2-operator0 [ER]

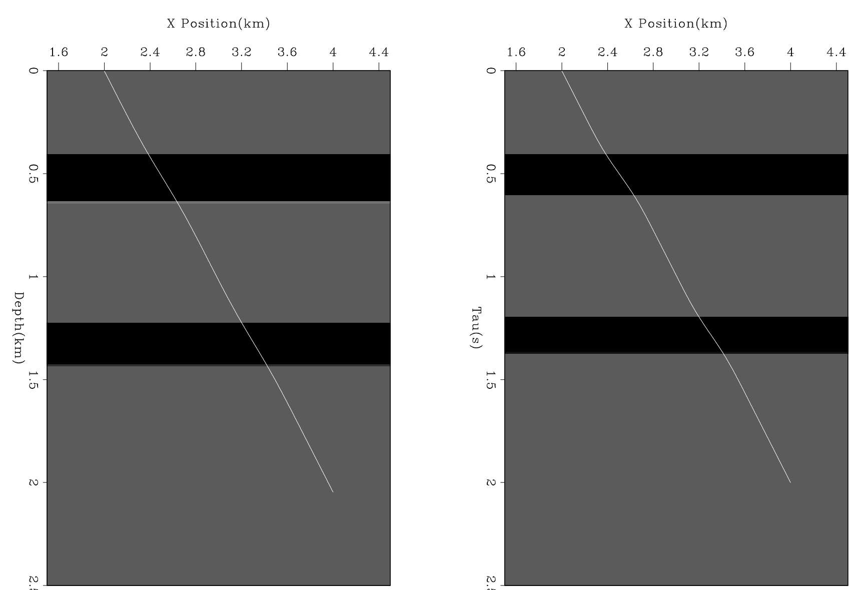

Figure 5: “Correct” velocity function overlaid by ray hitting reflector at 4 km with a half-offset of 2 km. Left Panel is in depth, right panel is in tau. bob2-vel1 [ER]

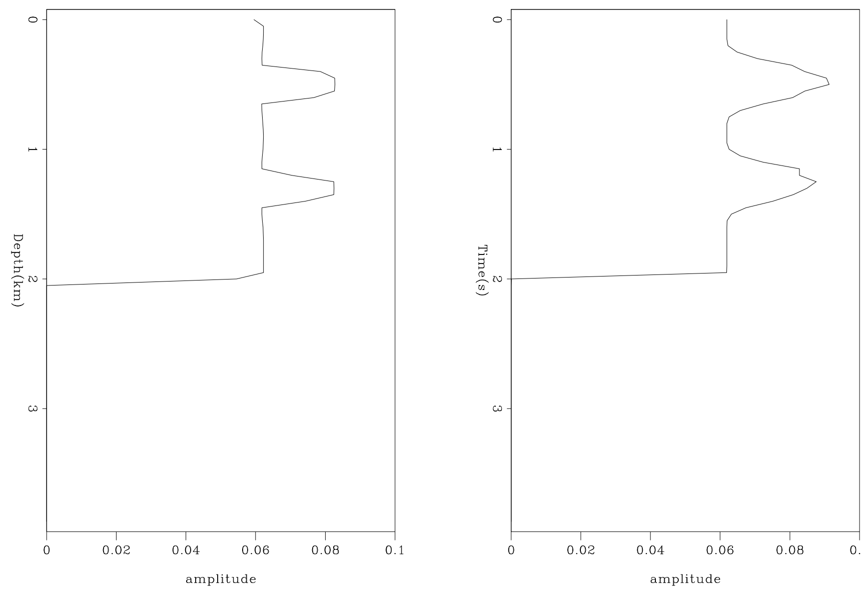

Figure 6: The operator calculated from the “correct” velocity and the resulting ray paths in depth (left) and tau (right). bob2-operator1 [ER]

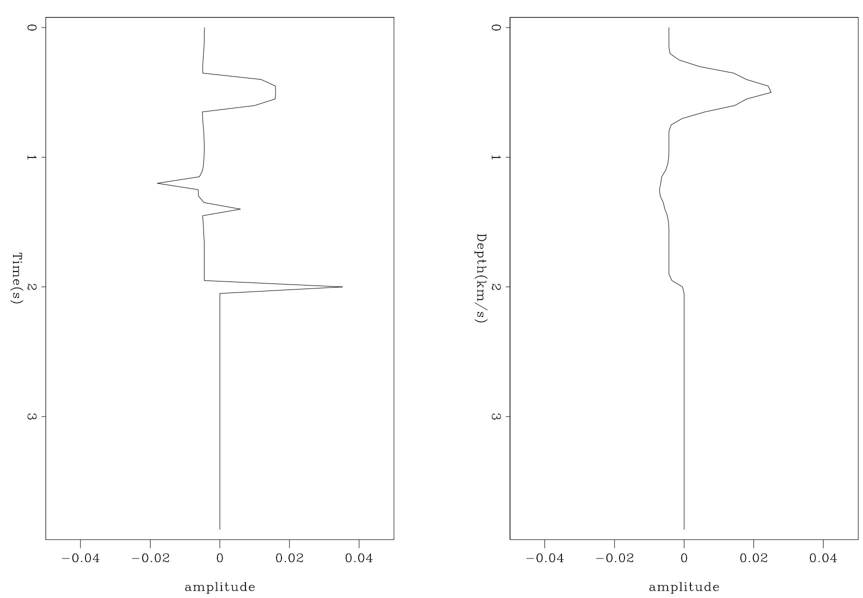

Figure 7: The difference between the operators calculated from the correct and our initial guess at velocity, for depth (left) and tau (right. Note the significant spikes at the reflector and at the lower layer boundary in the depth case. bob2-diff [ER]

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