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Short Note

Backus revisited: Just in time

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INTRODUCTION

The essence of Backus theory (Backus, 1962) is that it allows a bunch of layers to be replaced by a single layer. The new homogeneous medium has elastic properties identical in the long-wavelength limit so that mass and travel-time are conserved, but wavelet shape is not. In this paper I show that for normal incidence plane waves the three elastic layer parameters of thickness, compliance, and density can be replaced by the two, travel-time and impedance, without losing reflection and transmission information.

DEVELOPMENT

In the normal incidence case of Backus averaging, the layer properties of thickness, ΔZ_J, compliance, S_J, and mass density, R_J, are replaced by the corresponding properties of the equivalent homogeneous medium, Z_{equiv}, S_{equiv}, and R_{equiv}:

$$Z_{\text{equiv}} = \sum \Delta Z_J \tag{1}$$
$$S_{\text{equiv}} = \sum S_J \Delta Z_J / \sum \Delta Z_J \tag{2}$$
$$R_{\text{equiv}} = \sum R_J \Delta Z_J / \sum \Delta Z_J \tag{3}$$

that is, the thickness of Z_{equiv} is the sum thickness of the layers, $Z = \sum \Delta Z_J$, and the equivalent medium mechanical properties are the thickness-weighted averages of those of the layered medium. However, these layers can also be described in terms of the layer properties of one-way travel-time, ΔT_J, and impedance, I_J, with slowness, L_J acting as the means for changing the independent variable between depth and time. It is well known that:

$$I_J = \sqrt{R_J / S_J} \tag{4}$$
$$L = \sqrt{R_J S_J} \tag{5}$$

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and from these:

$$R_J = I_J L_J \tag{6}$$
$$S_J = I_J^{-1} L_J \tag{7}$$

and thus:

$$\begin{aligned} I_{\text{equiv}} &= \sqrt{R_{\text{equiv}}/S_{\text{equiv}}} \\ &= \sqrt{\sum R_J \Delta Z_J / \sum S_J \Delta Z_J} \\ &= \sqrt{\sum I_J L_J \Delta Z_J / \sum I_J^{-1} L_J \Delta Z_J} \end{aligned} \tag{8}$$

but $L_J \Delta Z_J = \Delta T_J$, so:

$$I_{\text{equiv}} = \sqrt{\sum I_J \Delta T_J / \sum I_J^{-1} \Delta T_J} \tag{9}$$

and by similar reasoning the slowness equivalent:

$$L_{\text{equiv}} = (\sum \Delta Z_J)^{-1} \sqrt{\sum I_J \Delta T_J \sum I_J^{-1} \Delta T_J} \tag{10}$$

and, since $T_{\text{equiv}} = Z_{\text{equiv}} L_{\text{equiv}}$:

$$T_{\text{equiv}} = \sqrt{\sum I_J \Delta T_J \sum I_J^{-1} \Delta T_J} \tag{11}$$

DISCUSSION

What have we done? Firstly we have shown that Backus exactly translates from depth and elastic properties to time and impedance. In other words we have transplanted Backus from the physical world to the data processor's world of traveltime and reflectivity as characterized by Goupillaud's ladder and Chapter 8 of FGDP (Claerbout, 1976). Secondly we have swapped two elastic parameters, compliance and density, for one, impedance, without any loss of information. This may represent a key to extending Backus theory sufficiently away from zero frequency to provide useful approximations to changes in wavelet shape due to scattering.

PREVIOUS PUBLICATION

This result is new to me but it is possible — even likely — that it has been previously published in this explicit form. If this is so, then the author would much appreciate getting the citation. In fact, Bill Symes, in a personal communication, has let me know that Bamberger came up with an integral representation of my summation form.

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