Stanford Exploration Project, Report 100, June 30, 1999, pages 151–159

Short Note

On Stolt prestack residual migration

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keywords: *Stolt, residual migration*

INTRODUCTION

Residual migration has proved to be a useful tool in imaging and in velocity analysis.

Rothman (1983) shows that post-stack residual migration can be successfully used to improve the focusing of the migrated sections. He also showed that migration with a given velocity v_m is equivalent to migration with a reference velocity v_0 followed by residual migration with a velocity v_r that can be expressed as a function of v_0 and v_m.

Residual migration has also been used as a tool in velocity analysis. Al-Yahya (1987) discusses a residual migration operator in the prestack domain, and shows that it can be posed as a function of a nondimensional parameter γ that is the ratio of the correct velocity and the reference velocity used for the initial migration. Etgen (1988, 1989) defines a kinematic residual migration operator as a cascade of NMO and DMO, and shows that it, again, is only a function of the nondimensional parameter γ defined by Al-Yahya. Finally, Stolt (1996) defines a prestack residual migration operator in the (f, k) domain, and shows that it depends on the reference (v_0) and the correct (v_m) migration velocities.

In this short note, I review the prestack residual Stolt migration, and show that it also can be formulated as a function of a nondimensional parameter that is the ratio of the reference (v_0) and correct (v_m) velocities. Consequently, we can use Stolt residual migration in the prestack domain to obtain a better focused image without making any assumption about the velocity. This approach has a direct application to migration velocity analysis, for instance in cases when we repeatedly do residual migration on data that have been depth-migrated with an arbitrary velocity function that cannot be approximated by a constant velocity (Biondi and Sava, 1999).

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STOLT MIGRATION

Prestack Stolt migration (PrSM) was summarized as (Claerbout, 1985)

$$p(t,y,h) \rightarrow P(\omega,k_y,k_h) \rightarrow P'(k_z,k_y,k_h) \rightarrow p'(z,y,h).$$

An important component of PrSM is the remapping from the (ω, k_y, k_h) domain to the (k_z, k_y, k_h) domain, where ω, k_z represent, respectively, the frequency and the vertical wavenumber, and k_y, k_h represent the midpoint and offset wavenumbers.

If we consider the alternative representation of the input data in shot-geophone coordinates, the mapping takes the form

$$k_z = \frac{1}{2}\left(\sqrt{\frac{\omega^2}{v^2} - k_g^2} + \sqrt{\frac{\omega^2}{v^2} - k_s^2}\right), \tag{1}$$

where k_g and k_s stand for, respectively, the geophone and the source wavenumbers. It is desirable to implement the remapping as a pull operator, to avoid numerical problems in the inverse Fourier transform. A detailed discussion on the advantages and disadvantages of the different mappings is done by Levin (1994). We can, therefore, express ω as a function of k_z from Equation (1) as:

$$\omega^2 = \frac{v^2}{16k_z^2}\left[4k_z^2 + (k_g - k_s)^2\right]\left[4k_z^2 + (k_g + k_s)^2\right] \tag{2}$$

or

$$\omega^2 = \frac{v^2}{k_z^2}\left[k_z^2 + k_h^2\right]\left[k_z^2 + k_y^2\right]. \tag{3}$$

2-D RESIDUAL STOLT MIGRATION

In general, residual migration represents a method of improving the quality of the image without having to remigrate the original data, but rather only applying a transformation to the current migration image.

In residual prestack Stolt migration (RPrSM), we attempt to correct the effects of migrating with an inaccurate reference velocity by applying a transformation to the data that have been transformed to the Fourier domain (Figure 1). Supposing that the initial migration was done with the velocity v_0, and that the correct velocity is v_m, we can then write

$$\begin{cases} k_{z_0} = \frac{1}{2}\left(\sqrt{\frac{\omega^2}{v_0^2} - k_g^2} + \sqrt{\frac{\omega^2}{v_0^2} - k_s^2}\right) \\ k_{z_m} = \frac{1}{2}\left(\sqrt{\frac{\omega^2}{v_m^2} - k_g^2} + \sqrt{\frac{\omega^2}{v_m^2} - k_s^2}\right). \end{cases} \tag{4}$$

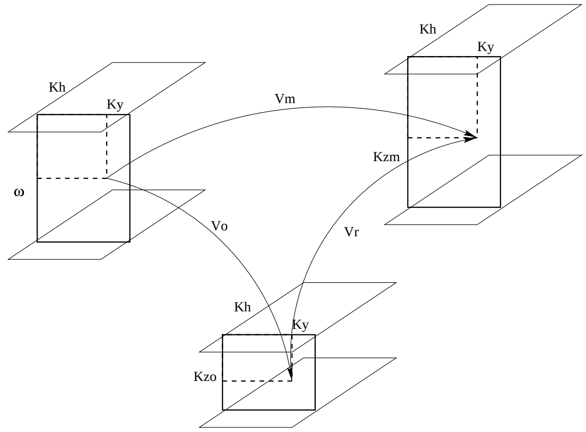

Figure 1: A sketch of Stolt residual migration paul1-stolt [NR]

The goal of RPrSM is to obtain k_{z_m} from k_{z_0}. If we use the first equation of (4) to substitute ω in the second equation of (4), we obtain

$$k_{z_m} = \frac{1}{2}\sqrt{\frac{v_0^2}{v_m^2}\frac{\left[4k_{z_0}^2+(k_g-k_s)^2\right]\left[4k_{z_0}^2+(k_g+k_s)^2\right]}{16k_{z_0}^2} - k_g^2} + \frac{1}{2}\sqrt{\frac{v_0^2}{v_m^2}\frac{\left[4k_{z_0}^2+(k_g-k_s)^2\right]\left[4k_{z_0}^2+(k_g+k_s)^2\right]}{16k_{z_0}^2} - k_s^2} \tag{5}$$

or

$$k_{z_m} = \frac{1}{2}\sqrt{\frac{v_0^2}{v_m^2}\frac{\left[k_{z_0}^2+k_h^2\right]\left[k_{z_0}^2+k_y^2\right]}{k_{z_0}^2} - (k_y+k_h)^2} + \frac{1}{2}\sqrt{\frac{v_0^2}{v_m^2}\frac{\left[k_{z_0}^2+k_h^2\right]\left[k_{z_0}^2+k_y^2\right]}{k_{z_0}^2} - (k_y-k_h)^2}. \tag{6}$$

Equation (6) represents the RPrSM equation in two dimensions. For post-stack data, the same equation takes the familiar form

$$k_{z_m} = \sqrt{\frac{v_0^2}{v_m^2}\left[k_{z_0}^2 + k_y^2\right] - k_y^2}. \tag{7}$$

3-D RESIDUAL STOLT MIGRATION

The RPrSM equations in three dimensions can be written similarly to those in two dimensions if we consider the relationship between source-geophone and midpoint-offset coordinates as follows:

$$\vec{k_g} = \vec{k_y} + \vec{k_h}$$
$$\vec{k_s} = \vec{k_y} - \vec{k_h}.$$

The equivalent prestack residual migration equation in source-geophone coordinates thus becomes

$$\begin{aligned} k_{z_m} = {} & \frac{1}{2}\sqrt{\frac{v_0^2}{v_m^2}\frac{\left[4k_{z_0}^2+(|\vec{k_g}|-|\vec{k_s}|)^2\right]\left[4k_{z_0}^2+(|\vec{k_g}|+|\vec{k_s}|)^2\right]}{16k_{z_0}^2} - |\vec{k_g}|^2} \\ & + \frac{1}{2}\sqrt{\frac{v_0^2}{v_m^2}\frac{\left[4k_{z_0}^2+(|\vec{k_g}|-|\vec{k_s}|)^2\right]\left[4k_{z_0}^2+(|\vec{k_g}|+|\vec{k_s}|)^2\right]}{16k_{z_0}^2} - |\vec{k_s}|^2}. \end{aligned} \tag{8}$$

In midpoint-offset coordinates, the same equation becomes

$$k_{z_m} = \frac{1}{2}\sqrt{\frac{v_0^2}{v_m^2}\frac{\left[4k_{z_0}^2+(|\vec{k_y}+\vec{k_h}|-|\vec{k_y}-\vec{k_h}|)^2\right]\left[4k_{z_0}^2+(|\vec{k_y}+\vec{k_h}|+|\vec{k_y}-\vec{k_h}|)^2\right]}{16k_{z_0}^2} - |\vec{k_y}+\vec{k_h}|^2} + \frac{1}{2}\sqrt{\frac{v_0^2}{v_m^2}\frac{\left[4k_{z_0}^2+(|\vec{k_y}+\vec{k_h}|-|\vec{k_y}-\vec{k_h}|)^2\right]\left[4k_{z_0}^2+(|\vec{k_y}+\vec{k_h}|+|\vec{k_y}-\vec{k_h}|)^2\right]}{16k_{z_0}^2} - |\vec{k_y}-\vec{k_h}|^2}, \tag{9}$$

which, for the post-stack case, takes the form

$$k_{z_m} = \sqrt{\frac{v_0^2}{v_m^2}\left[k_{z_0}^2 + |\vec{k_y}|^2\right] - |\vec{k_y}|^2}. \tag{10}$$

EXAMPLES

In this section, I present two 2-D post-stack synthetic examples, shown in Figures 2 and 3, to prove the applicability of the equations derived in the preceding sections.

In the first example, the input is a set of three spikes. Initially, I do forward modeling with a velocity of $v_m = 3.0$ km/s. Next, I do Stolt migration with a velocity of $v_0 = 3.6$ km/s, and residual Stolt migration with a ratio $v_0/v_m = 1.2$ (Figure 2). I then take the same input and perform Stolt migration with a velocity of $v_0 = 2.4$ km/s, followed by residual Stolt migration with a ratio $v_0/v_m = 0.8$ (Figure 3). In both cases, the data are correctly collapsed at the location of the original spikes. Residual migration can be done without knowing the absolute values of the velocity.

In the next example, shown in Figure 4, I apply the same methodology to a real dataset (Ecker, 1998). All three panels are the result of residual migration as described in the preceding theory sections. The corresponding ratios are 0.96 for the top panel, 0.98 for middle panel, and 1.00 for the bottom panel. Residual migration with ratio=1.0 is equivalent to no residual migration at all. It is apparent that different images are focused better in one region or another, although the image corresponding to the ratio 0.98 seems to have the highest overall energy. We can use such an observation to obtain an image that has the best focusing in all the regions. This can be achieved by doing residual migration for a range of velocity ratios, and then interpolating the image that is best focused everywhere. One possible application is in wave-equation migration velocity analysis, where we can improve the focusing of a depth-migrated dataset by residual migration, without making any assumption about the original velocity distribution (Biondi and Sava, 1999).

CONCLUSIONS

Equations (5) and (6) show that residual prestack Stolt migration can be done without actual knowledge of the reference and correct velocities, but only by knowing or assuming their ratio. This understanding has important practical consequences, for example, in applications

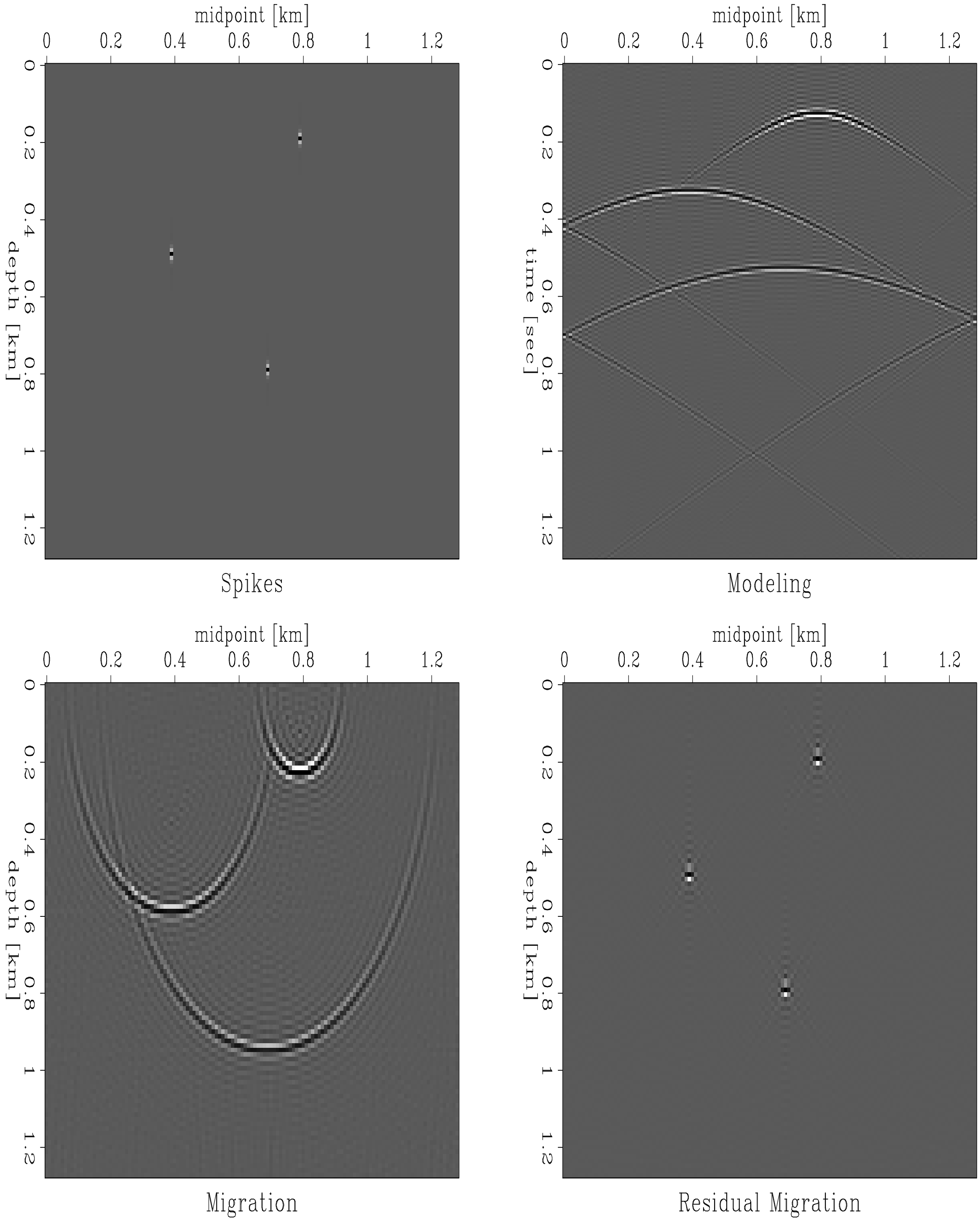

Figure 2: Top left: input data. Top right: Stolt forward modeling with the correct velocity $v_m = 3.0$ km/s. Bottom left: Stolt migration with an incorrect velocity of $v_0 = 3.6$ km/s. Bottom right: Stolt residual migration with the ratio $v_0/v_m = 1.2$. paul1-posp [ER]

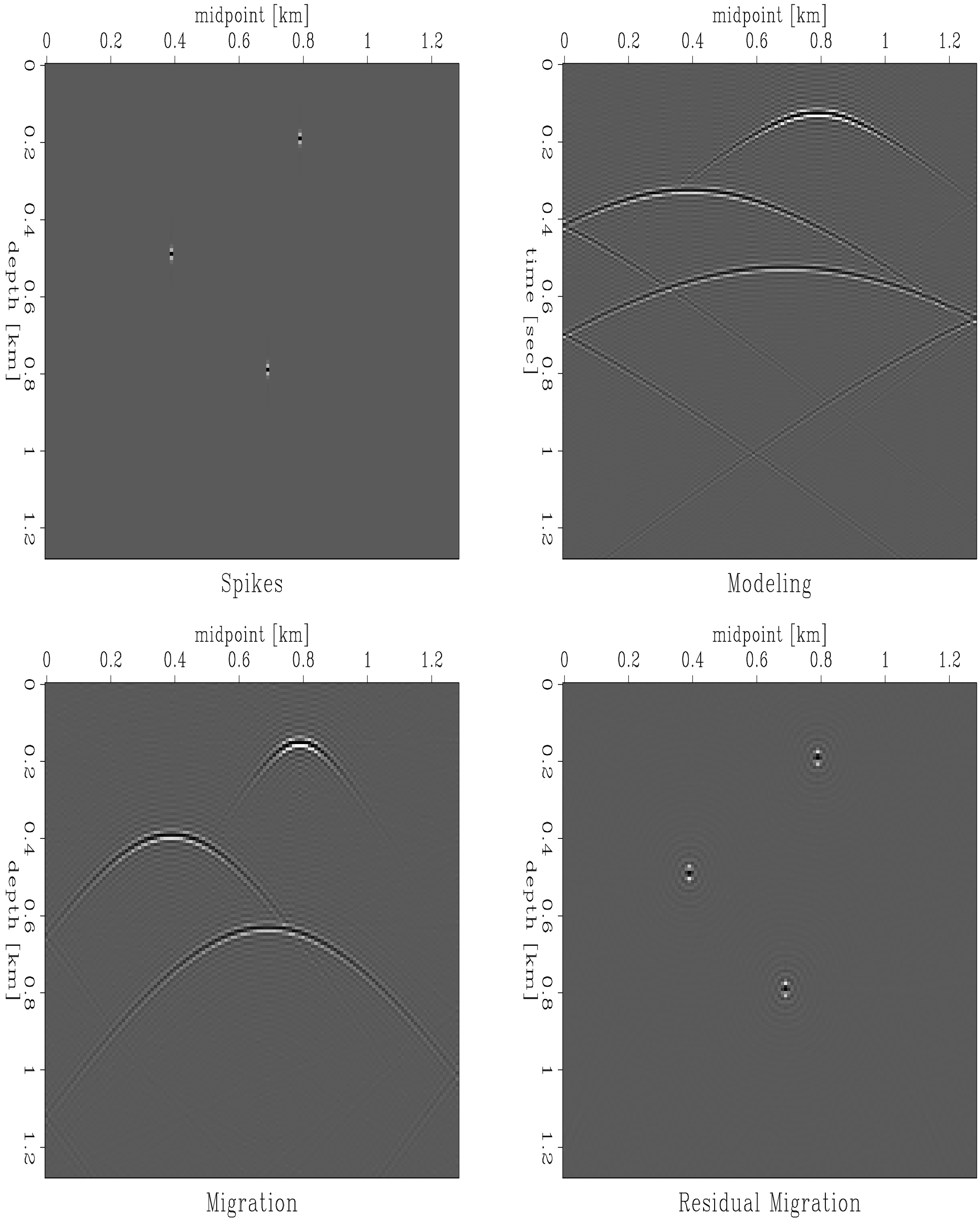

Figure 3: Top left: input data. Top right: Stolt forward modeling with the correct velocity $v_m = 3.0$ km/s. Bottom left: Stolt migration with an incorrect velocity of $v_0 = 2.4$ km/s. Bottom right: Stolt residual migration with the ratio $v_0/v_m = 0.8$. paul1-posm [ER]

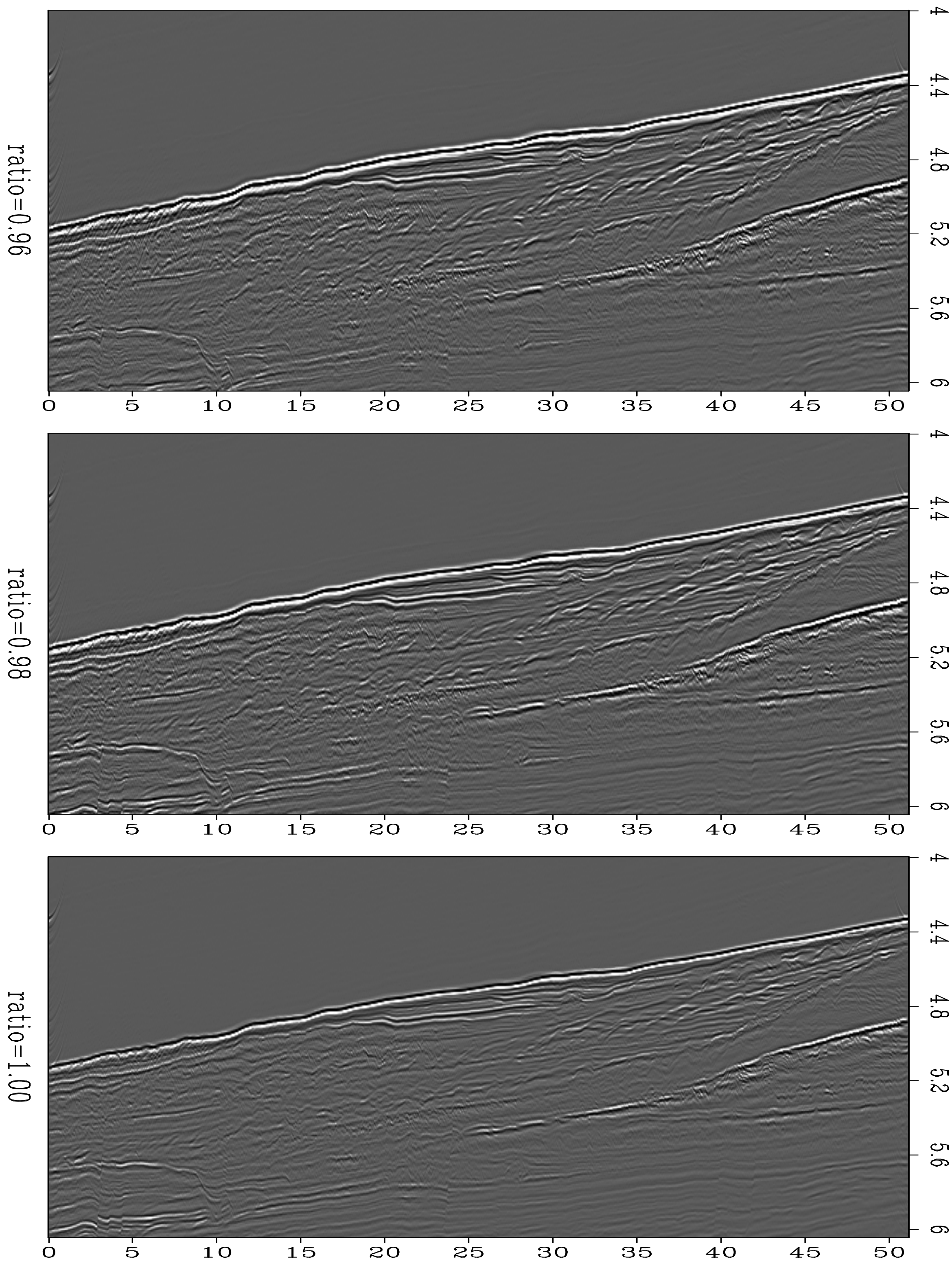

Figure 4: Images obtained after residual migration. Ratio = 1.00 means no residual migration. Different images are focused better in different regions. The image corresponding to ratio = 0.98 seems to have the best focusing. paul1-hydrates [CR]

that use RPrSM as a method of improving the focusing of an image that has been depth-migrated with an arbitrary velocity function.

ACKNOWLEDGMENTS

I would like to thank Sergey Fomel for allowing me to use his interpolation code in the Stolt programs.

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