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Anti-aliasing multiple prediction beyond two dimensions

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ABSTRACT

Theoretically, the Delft method of surface-related multiple elimination can be applied in three dimensions, as long as the source and receiver coverage is dense enough. In reality, such a dense coverage is still far from reach, using the available multi-streamer acquisition system. One way to fill the gap is to massively interpolate the missing sources and receivers in the survey, which requires a huge computational cost. In this paper, I propose a more practical approach for the multi-streamer system. Instead of using large-volume missing-streamer interpolation, my method finds the most reasonable proxy from the collected dataset for each missing trace needed in the multiple prediction. Although this approach avoids missing-streamer interpolation, another problem pops up in the multi-streamer case, the aliasing noise caused by the sparse sampling in the cross-line direction. To solve this problem, I introduce a new concept, the partially-stacked multiple contribution gather (PSMCG). Using multi-scale prediction-error filter (MSPEF) theory, this approach interpolates the PSMCG in the cross-line direction to remove the aliasing noise.

INTRODUCTION

The Delft approach to surface-related multiple elimination (Berkhout and Vershcuur, 1997; Vershcuur and Berkhout, 1997) formulated the demultiple process as a two-step inversion problem based on the Huygens principle, that is, first predicting the multiple and then subtracting it from the original dataset.

The multiple prediction step, crucial for the success of the whole algorithm, involves one important assumption about the data acquisition geometry, namely, a source/receiver pair is needed wherever a multiple reflects. The Delft approach is quite successful in solving 2-D problems (Verschuur and Prein, 1999), in which the assumption is relatively easily satisfied. However, in many 3-D surveys, there is a large gap between this assumption and the reality (Dragoset and Jeričević, 1998).

Two different directions have been taken to solve the problem. One is to interpolate the trace at the missing source and receiver positions massively to attain a dense coverage

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of the surface (van Dedem and Verschuur, 1998). However, the computational cost of this method is huge. The other approach is to predict the multiple based on the 2-D theory and then extend the subtraction step to handle incorrectly predicted multiples (Ross, 1997; Ross et al., 1997). The success of this approach is restricted to simple 3-D cases.

This paper proposes a method designed for multi-streamer geometry. Two distinctive features make it more practical. First, this approach finds the most reasonable proxy from the collected dataset for any missing trace. There is no need to interpolate missing streamers and shotlines. Second, I introduce a concept, the partially-stacked multiple contribution gather (PSMCG). Using the multi-scale prediction-error filter (MSPEF) theory (Claerbout, 1992), the proposed approach interpolates the PSMCG in the cross-line direction. This gives us a densely sampled multiple contribution from all possible Huygens secondary sources. The following summation step can remove aliasing noise better.

Two numerical examples in this paper demonstrate how the approach works.

MULTIPLE PREDICTION BEYOND TWO DIMENSIONS

One of key issues in 3-D demultiple is that there are many missing traces. Instead of interpolating the missing traces, the critical points of my approach are:

1. This approach tries to find proxies for the missing traces.
2. To be qualified as a proxy, a trace must have the same offset and either a similar CMP location or a similar azimuth angle.
3. Those proxies for the missing traces are used to predict multiples with first-order accuracy in multi-streamer geometry.

In order to better understand the approach, let's assume that we have a multi-streamer acquisition system, as shown in Figure 1, with one shotline and seven streamers. Supposing that we want to predict the multiple from source S_0 to receiver R_4, we need to consider the contributions from all the possible multiple reflection points between S_0 and R_4 by cross-convolution. For instance, we need to collect all the traces with sources located at S_i ($i = 1, ..., 7$) and a receiver located at R_4. In Figure 1, the thin solid line represents the corresponding trace collected in the survey, and the thin dashed line stands for a missing trace in the survey. The challenge is to find appropriate proxies for such missing traces.

There is one well-known geophysical concept that can help us meet the challenge, the common-midpoint (CMP), which assumes that traces with the same CMP location and the same offset contain the same information about one location in the earth. Although the common-midpoint assumption is a first-order approximation when the structure is not strictly flat, I will demonstrate that it is useful in our search for the substitute traces.

For example, for the virtual trace $\overrightarrow{S_1 R_4}$, the real trace $\overrightarrow{S_4 R_1}$ shares the same CMP location and has the same offset as well. The only difference is the azimuth angle. Therefore, trace

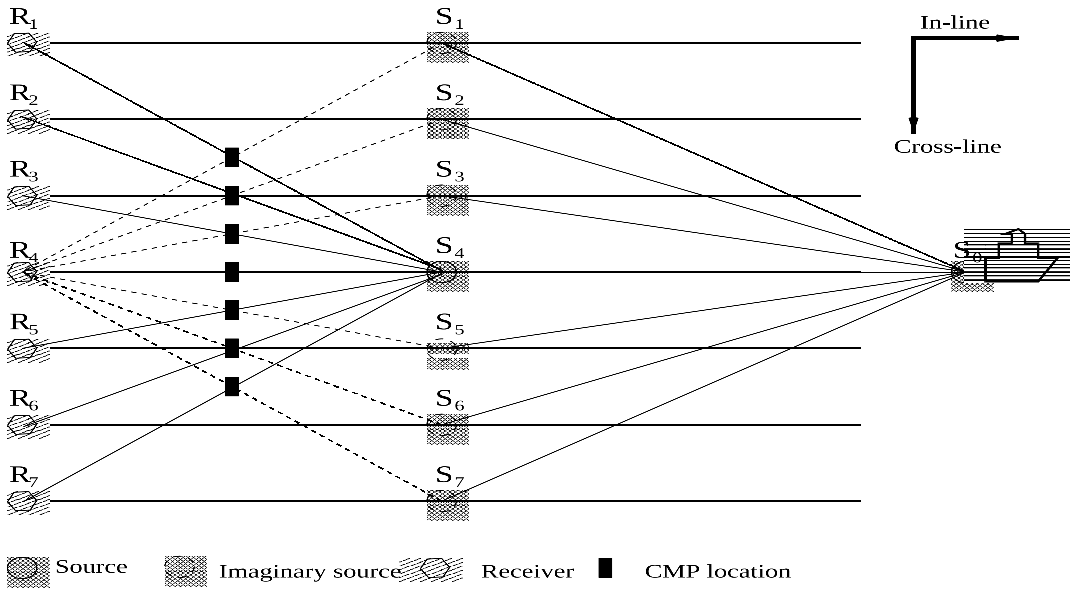

Figure 1: A multi-streamer geometry. The $\overrightarrow{S_4 R_1}$ path is a proxy for the absent out-of-plane shot $\overrightarrow{S_1 R_4}$, since the two paths share the same midpoint and have the same offset. Similarly, $\overrightarrow{S_2 R_4}$ is replaced by $\overrightarrow{S_4 R_2}$, $\overrightarrow{S_3 R_4}$ by $\overrightarrow{S_4 R_3}$, $\overrightarrow{S_5 R_4}$ by $\overrightarrow{S_4 R_5}$, $\overrightarrow{S_6 R_4}$ by $\overrightarrow{S_4 R_6}$, and $\overrightarrow{S_7 R_4}$ by $\overrightarrow{S_4 R_7}$. yalei1-multi-streamer [NR]

$\overrightarrow{S_4 R_1}$ is a proxy for trace $\overrightarrow{S_1 R_4}$ in the multiple prediction, with first-order accuracy. Similarly, we can find substitutes for other virtual traces.

The central streamer in Figure 1 is a special case, in which we can always find substitute traces for the virtual ones with the same CMP location and offset. When we try to predict other streamers' multiple reflections, though, as in Figure 2, we are not so lucky to find proxies with the same CMP location and offset. However, we can relax the definition of a substitute trace by giving up the requirement that the proxy share the same CMP location. Then we can find another group of proxies for the missing traces, as Figure 2 illustrates. Since the cross-line spreading aperture is usually smaller than the in-line aperture, this extension may be acceptable in many real applications.

There are some limitations to the method discussed in this paper. Before addressing those limitations, I would first classify the surface multiple reflections into two categories. Figure 3 depicts two types of geometries for the surface multiples, $S_0\overrightarrow{M_1}R_1$ and $S_0\overrightarrow{M_2}R_2$. The difference between these two categories is that, source S_0, surface multiple reflection position M_1, and receiver R_1 are aligned together on the surface; whereas S_0, M_2, and R_2 can not be aligned together. The embedded physical reason is that, multiple $S_0\overrightarrow{M_1}R_1$ is mainly caused by 1-D earth's structures or in-line dip reflectors (2.5-D), and multiple $S_0\overrightarrow{M_2}R_2$ by cross-line dips or scattering reflectors.

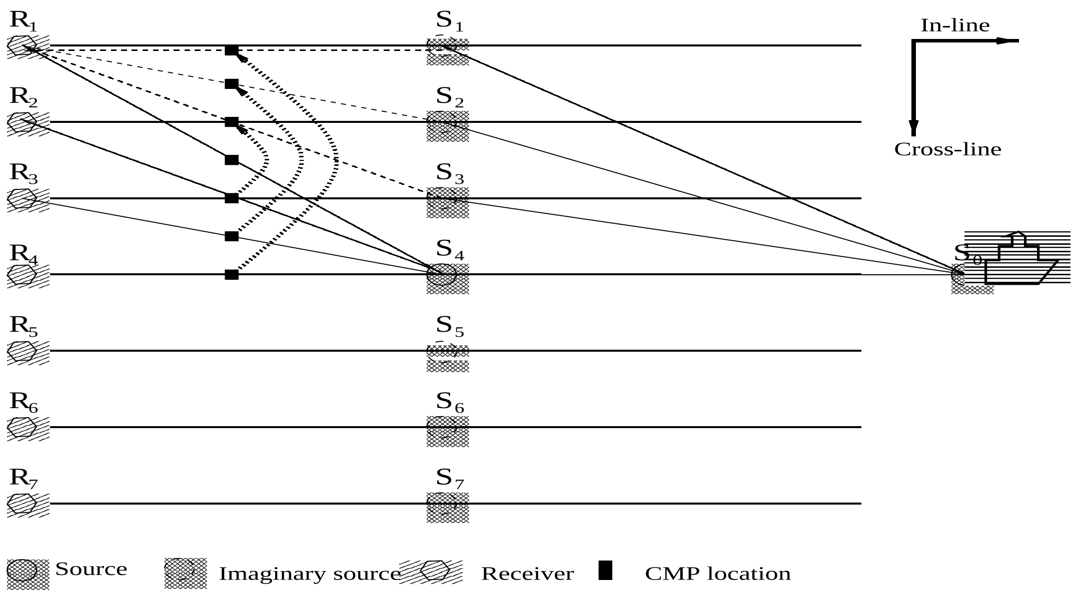

Figure 2: While working with streamers not in the middle, we have to give up the requirement that the proxy share the same CMP location, and find a group of proxies with reasonable accuracy. $\overrightarrow{S_1R_1}$ can be replaced by $\overrightarrow{S_4R_4}$, $\overrightarrow{S_2R_1}$ by $\overrightarrow{S_4R_3}$, and $\overrightarrow{S_3R_1}$ by $\overrightarrow{S_4R_2}$. Each pair shares the same offset and azimuth angle. yalei1-multi-streamer-1 [NR]

The definition of proxies in my proposal guarantees that the method in this paper is fully applicable to the multiples like $S_0\overrightarrow{M_1}R_1$ without kinematic approximations. The approximation errors occur only when we deal with the multiples like $S_0\overrightarrow{M_2}R_2$. In other words, when there are strong cross-line dips or scattering reflectors, my approach will introduce the approximation errors inevitably.

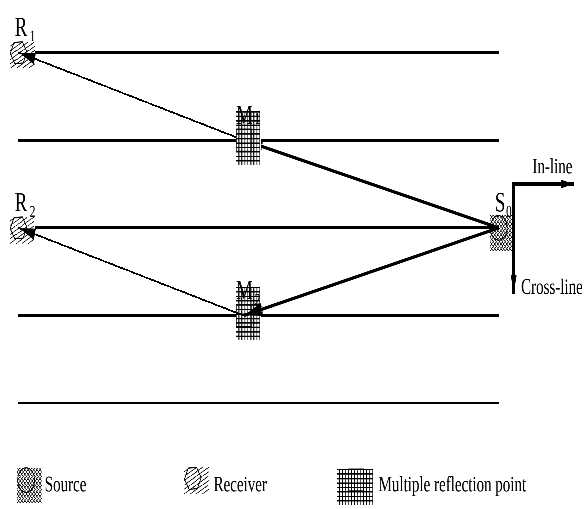

Figure 3: Two types of surface multiples. $S_0\overrightarrow{M_2}R_2$, in which the source S_0, the multiple reflection point M_2, and the receiver R_2 can be aligned, is more likely caused by cross-line dips or scattering reflectors. $S_0\overrightarrow{M_1}R_1$, in which S_0, M_1, and R_1 are aligned together, occurs when the earth's structures are approximately 1-D or there are only in-line dip reflectors. yalei1-multiple-type [NR]

A MULTIPLE CONTRIBUTION GATHER

The multiple prediction can be further divided into two sub-steps: trace cross-convolution and multiple contribution summation, which, in practice, people usually collapse into a single procedure. In order to gain more insight into the method, however, I consider them as two separate steps.

Figure 4 schematically demonstrates 2-D multiple prediction. In order to predict the multiple from source S to receiver R, we need to cross-convolute all the possible contributing traces marked by M_i in the middle, since we do not know where exactly the multiple reflection occurs on the surface. Then the summation step locates the exact multiple reflection position as long as the contributing traces are densely sampled on the surface.

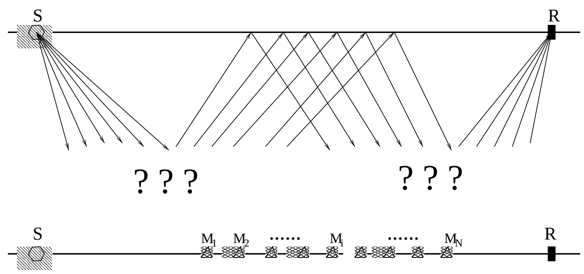

Figure 4: 2-D multiple prediction. All the contributing traces form a 2-D MCG prior to summation. The question marks in the plot indicate that the underground structures are unknown. yalei1-noah2d [NR]

Prior to summation, if we lay out the cross-convoluted traces from left to right, we get an intermediate result, a multiple contribution gather (MCG). Figure 5 shows a 2-D multiple contribution gather and the corresponding summation result. The MCG is a section in 2-D and a cube in 3-D. The restriction of the multi-streamer geometry makes the 3-D MCG cube densely sampled in the in-line direction and coarsely sampled in the cross-line direction. Therefore, we can safely apply the summation in the in-line direction first and get a partially-stacked MCG (PSMCG).

Figure 6 displays two PSMCGs with different sampling intervals and the corresponding stacked multiples. Unfortunately, a brutal summation in the sparsely sampled cross-line direction introduces a large amount of aliasing noise into the predicted multiple. The next section describes a method of avoiding such noise.

ANTI-ALIASING IN THE MULTIPLE PREDICTION

The multiple prediction proposal discussed in the preceding section suggests that we can estimate 3-D multiples without trace interpolation. However, as Figure 6 shows, the other problem—aliasing noise—has to be dealt with carefully if there is no missing-streamer interpolation. Like any other Kirchhoff-style operation, anti-aliasing is an important issue in multiple prediction. This issue deserves even more attention in three dimensions, since the cross-line sampling is more sparse than the in-line sampling.

The 3-D estimation of a multiple trace is achieved by stacking a 3-D MCG. As discussed in the preceding section, we can safely stack the 3-D MCG into a 2-D PSMCG along the

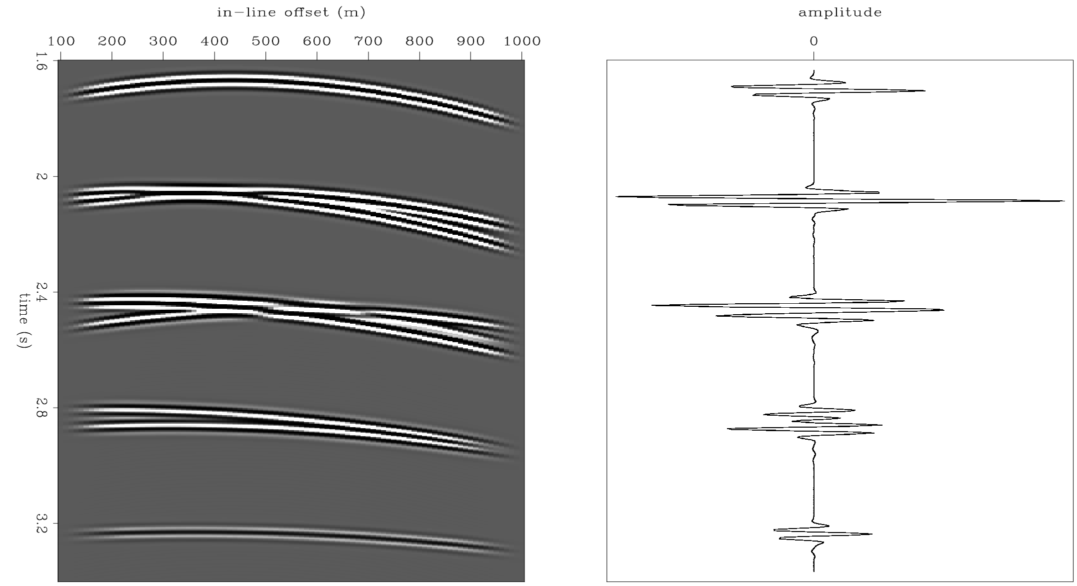

Figure 5: Left: a 2-D MCG. The amplitude has been tapered off to avoid edge effects. Crossing events imply that the multiples are split into different branches. Right: the stacking of the MCG. The locations of predicted multiples correspond to the tops of the curved events. yalei1-mcg-2d [ER]

in-line direction. In the cross-line direction, we must sample the PSMCG more densely to avoid the aliasing noise. Therefore, I propose interpolating the PSMCG directly and then stack it into a multiple trace.

We can interpolate the aliased data in either the F-X (Spitz, 1991) or the T-X domain (Claerbout, 1992). I have chosen the time-space domain multi-scale prediction-error filter (MSPEF) theory discussed in Section 8.4 of Claerbout (1992) to interpolate the PSMCG. The basic idea of the theory is that large objects often resemble small objects. Supposing that we have input data with alternate missing traces, we can estimate a PEF with the following shape:

$$\begin{array}{ccccccccc} a & \cdot & b & \cdot & c & \cdot & d & \cdot & e \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \end{array} \quad (1)$$

Then we can make the filter smaller by throwing away the zeros (represented by dots) in filter (1) to get

$$\begin{array}{ccccc} a & b & c & d & e \\ \cdot & \cdot & 1 & \cdot & \cdot \end{array} \quad (2)$$

which has the same dip characteristics as filter (1).

Figure 7 shows two PSMCGs containing crossing events, before and after interpolating the alternative missing traces and the corresponding stacking results. The aliasing noise has decreased significantly after trace interpolation.

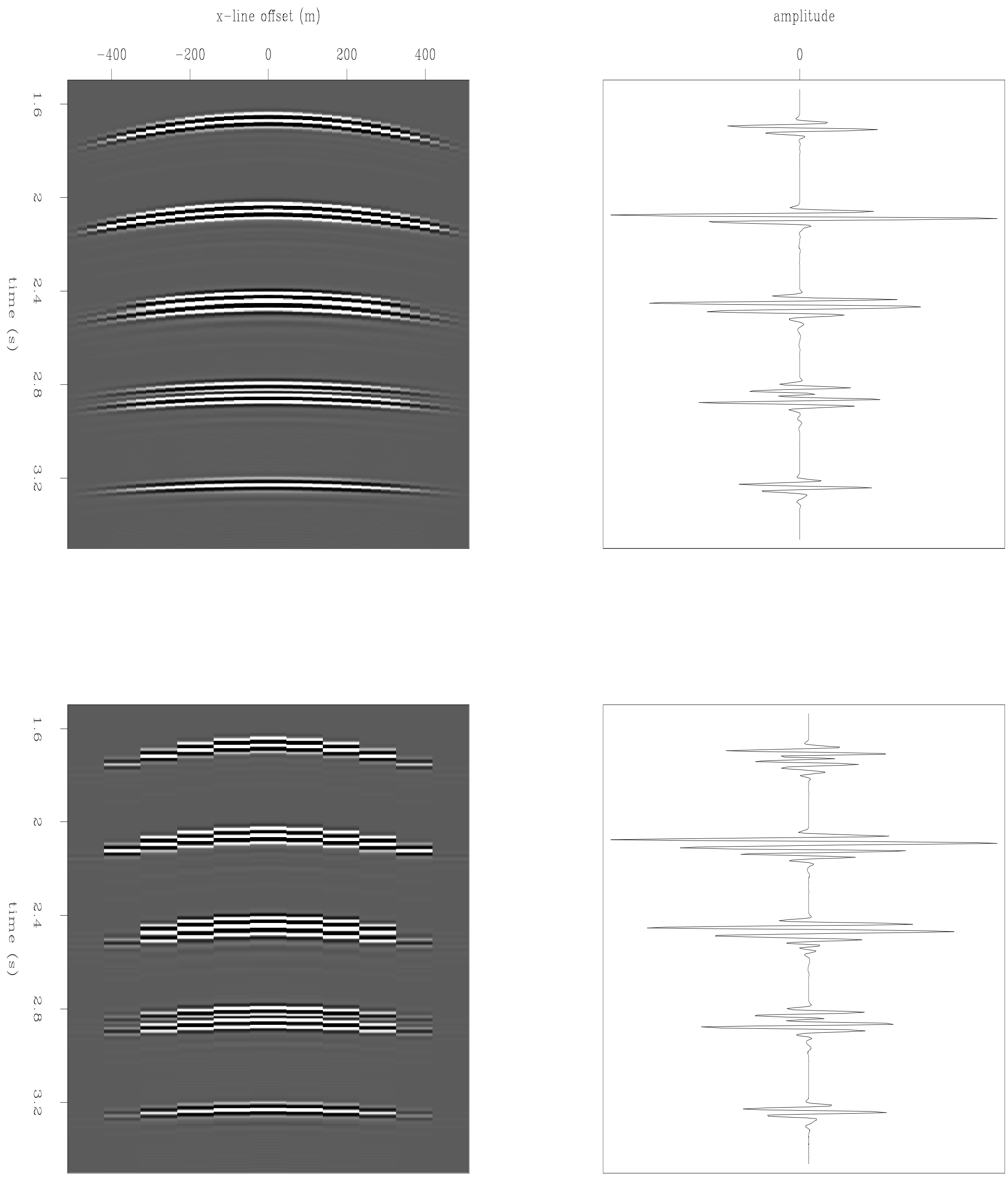

Figure 6: Top: a densely sampled (Δ_{streamer}=25m) PSMCG and its stacking result. Bottom: a sparsely sampled (Δ_{streamer}=100m) PSMCG and its stacking result. Stacking of the bottom PSMCG introduces aliasing noise to the multiple trace, especially to the top two wavelets in the plot. yalei1-mcg-ps [CR]

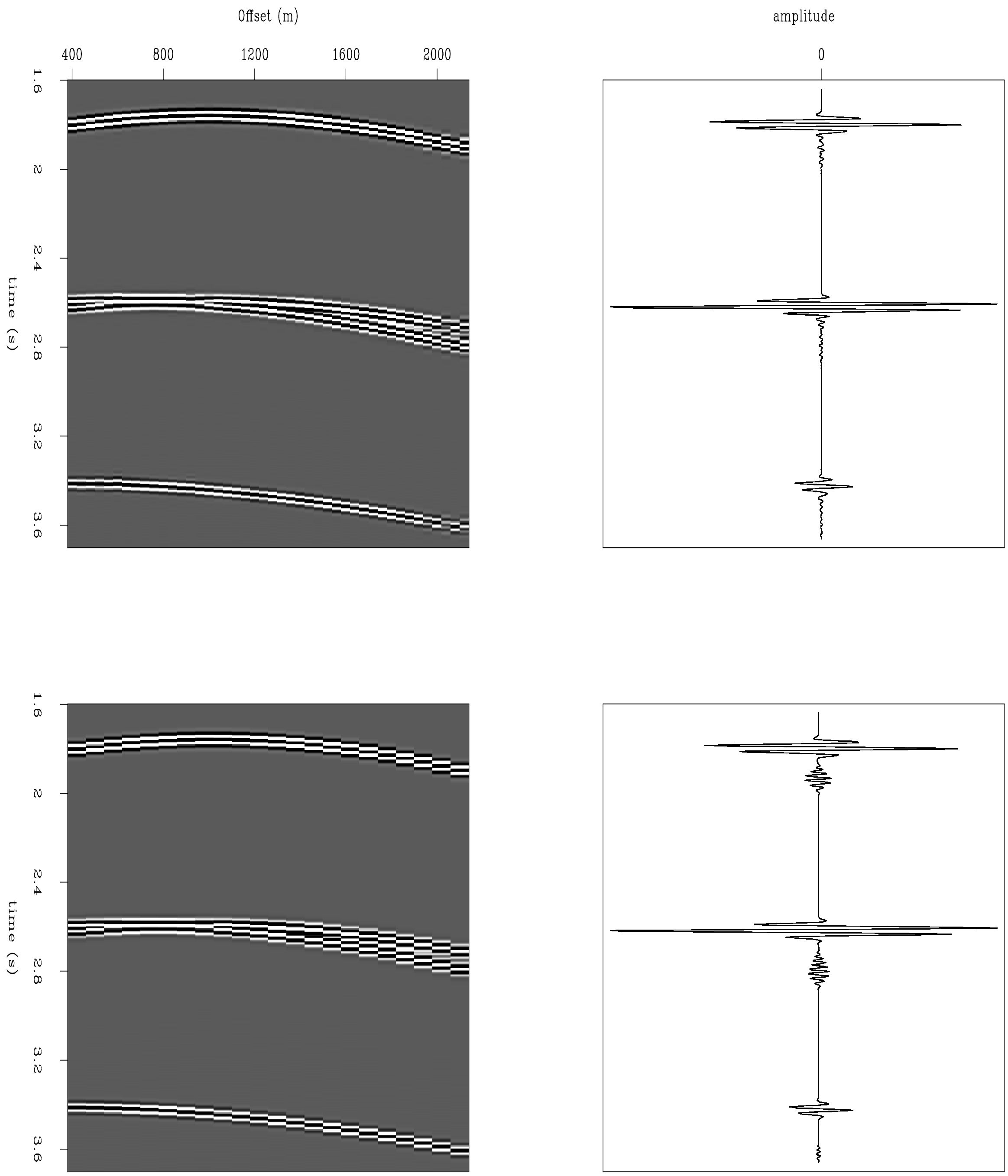

Figure 7: Top: a densely sampled ($\Delta_{streamer}$=25m) PSMCG and its stacking result. Bottom: a sparsely sampled ($\Delta_{streamer}$=50m) PSMCG and its stacking result. The aliasing noise has been greatly decreased after the trace interval is halved from 50m to 25m. yalei1-mcg-interp [ER]

NUMERICAL EXAMPLES

Combining the multiple prediction and the PSMCG interpolation, we get a new, practical multiple prediction scheme beyond two dimensions. In this section, two 3-D synthetic datasets with similar acquisition geometries are used to evaluate the new scheme. The corresponding model and acquisition parameters appear in Tables 1 and 2.

Model	In-line dip (°)	X-line dip (°)	Layer velocity (km/s)
A	(0.0,5.0,10.0)	(0.0, 0.0, 0.0)	(1.5, 2.0, 2.5)
B	(0.0,5.0,10.0)	(0.0, -5.0, 5.0)	(1.5, 2.0, 2.5)

Table 1: Model parameters of Models A and B

Model	Δ_{source} (m)	Δ_{receiver} (m)	Δ_{streamer} (m)
A	20	20	100
B	20	20	50

Table 2: Acquisition parameters of Models A and B

Model A is designed so that the shotline and streamers are deployed along the in-line dip direction. Therefore, there is no approximation error in our approach. With 11 streamers covering from $-500m$ to $+500m$ and a $100m$ streamer interval in the cross-line direction, this is a wide azimuth survey.

For a given shot location, Figure 8 shows the ideal multiple gather and the predicted one. Each 3-D MCG cube in this example is first stacked along the in-line direction into a 2-D PSMCG containing at most 11 traces sampled at intervals of $100m$. The 2-D PSMCG is then interpolated in the cross-line direction to be sampled at $25m$ intervals. The interpolated PSMCG is further stacked into a trace.

Model B is a relatively narrow azimuth survey that still contains 11 streamers covering from $-250m$ to $+250m$ at $50m$ streamer intervals in the cross-line direction. The bottom two reflectors in the model have the opposite cross-line angles, which inevitably introduce approximation error into the estimation. However, as shown in Figure 9, as long as the cross-line dips have no dominant direction, the error can possibly be compensated for in the subtraction step.

CONCLUSIONS AND FUTURE WORK

This paper has proposed a method of predicting 3-D multiples in multi-streamer geometry that does not require massive missing-streamer interpolation. Two numerical examples suggest that this approach can be used in the multi-streamer survey if the cross-line dip is mild. I am now working on the subtraction step. My future work will focus on evaluating and improving this approach with real data.

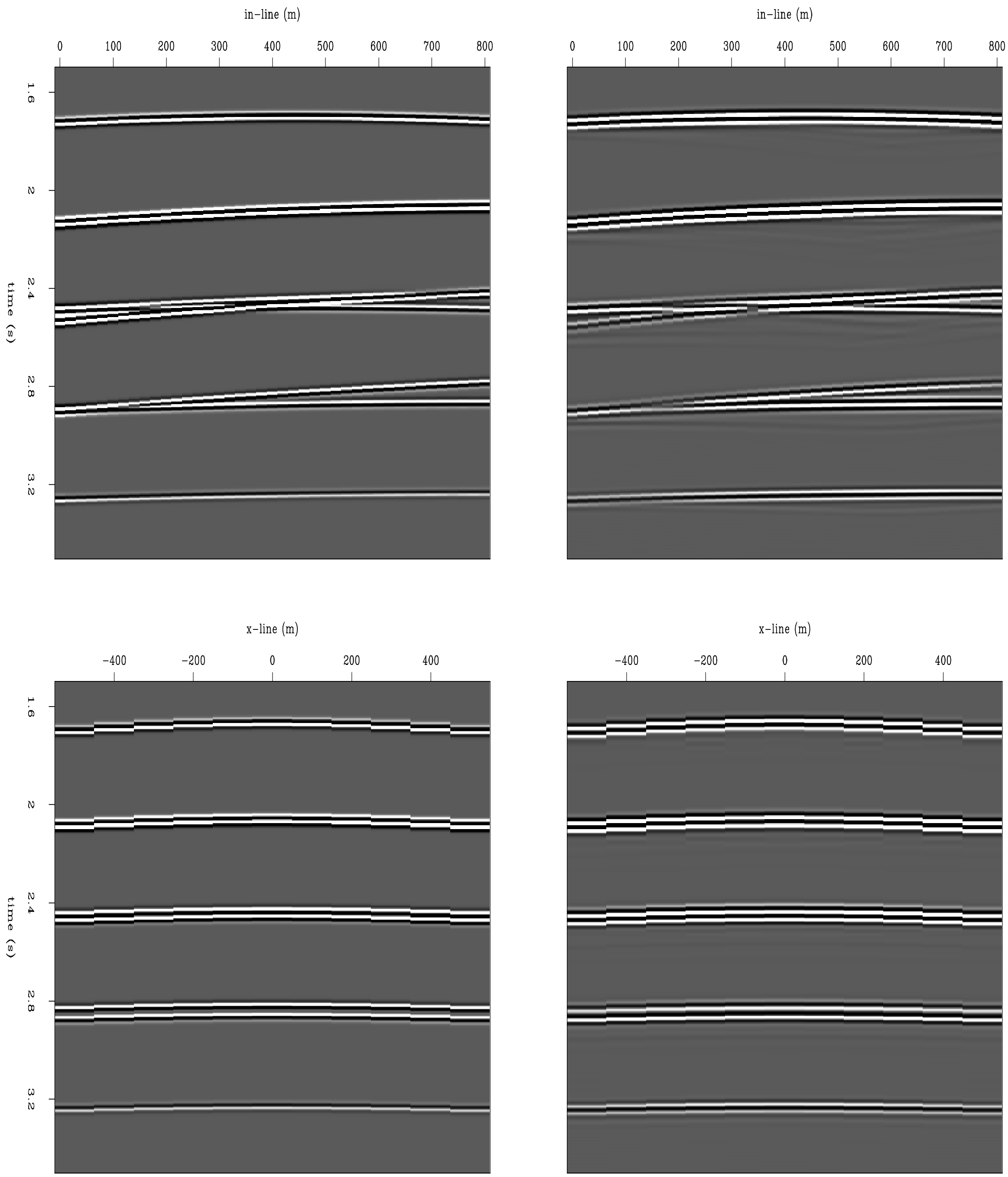

Figure 8: An in-line dip model. Left: the ideal multiple reflection. Right: the estimated multiple reflection. yalei1-wa2d-mult [CR]

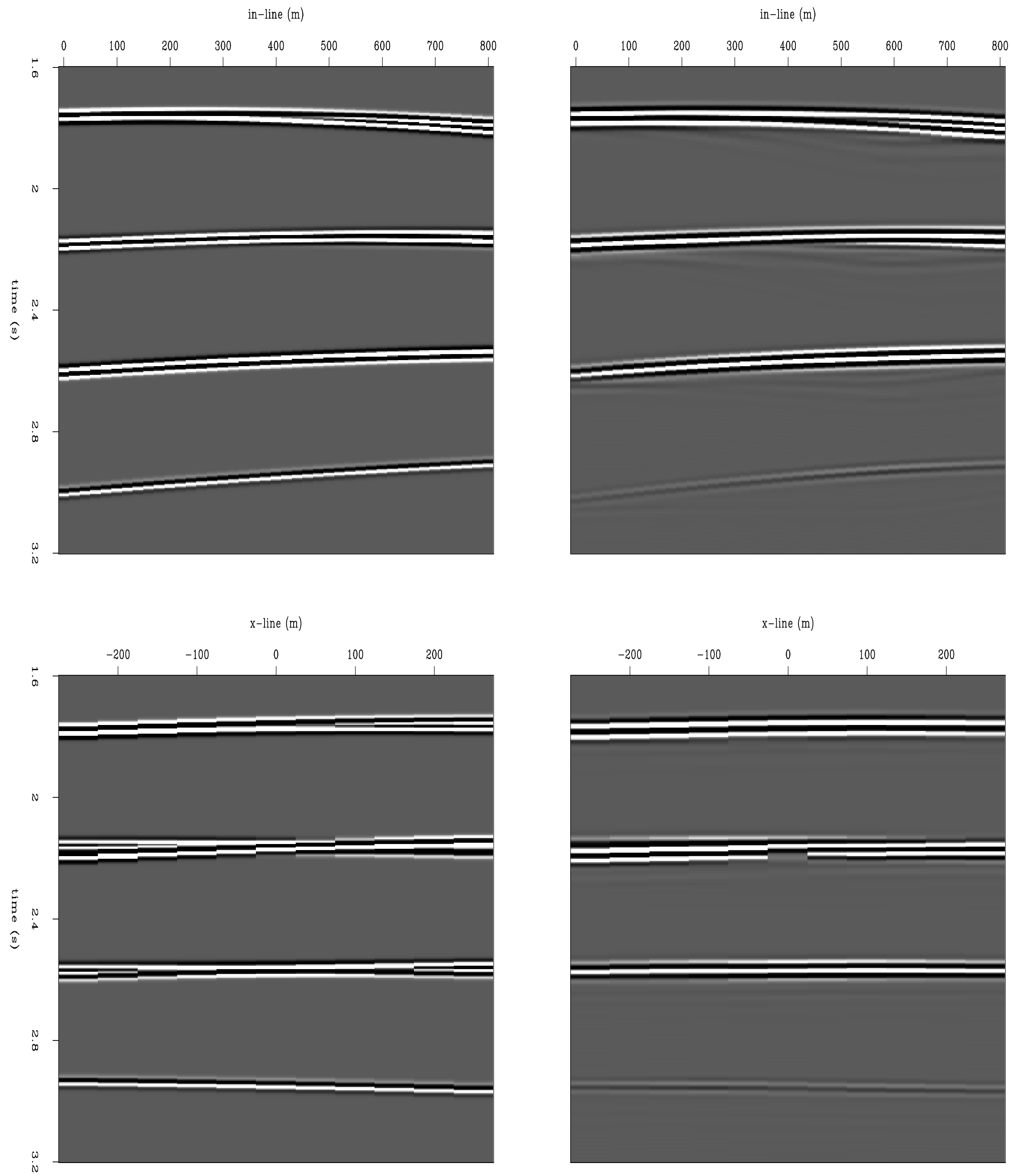

Figure 9: A mixing dip model. Left: the ideal multiple reflection. Right: the estimated multiple reflection. yalei1-na3d-mult [CR]

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