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$(t-x)$ domain, pattern-based multiple separation

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ABSTRACT

Pattern-based signal/noise separation is a common technique to suppress multiples. It can be formulated in the $t-x$ domain using non-stationary Prediction Error Filters (PEF). One can obtain a kinematically correct model of the multiples by downward continuation. The CMP gather and the corresponding multiple estimate are characterized by a space varying PEF. After applying a simple separation technique one can obtain CMP gathers where the multiple energy is significantly attenuated. The method is applied to synthetic and 2-D field CMP gathers.

INTRODUCTION

Multiple suppression is one of the largest problems facing the seismic industry. One common technique are the family of approaches generally refered to as 'model based' (Berryhill and Kim, 1986; Wiggins, 1988). These methods work by first getting an estimate of the models through downard continuation (Berryhill and Kim, 1986), computing the first term of the Neuman series (Ikelle et al., 1997), or some other method. Next, the primaries are estimated through some type of filtering operation using the estimated multiples. Recently, the problem has been formulated as a signal-noise separation problem in the frequency domain (Spitz, 1999; Bednar and Neale, 1999). These methods operate in the $f-x$ domain with the limiting assumption that the data are time-stationary.

Until recently the signal-noise method proposed by Spitz (1999) could not be formulated in the time domain because it involves dividing by a filter describing the multiple. Claerbout (1998) discovered that multi-dimensional PEFs can be mapped into 1-D, therefore making it possible to do inverse filtering in the time domain. The stationarity assumption inherent in PEF estimation can be overcome by estimating non-stationary filters (Crawley et al., 1998). As a result, Spitz's (1999) method can be formulated to work with time domain PEFs (Clapp and Brown, 1999).

In this paper we show how the time domain formulation of Spitz's approach can effectively attenuate multiples. We apply the method to a 2-D synthetic dataset and show that it is effective in both simple and complex areas. We then apply it on a 2-D real CMP gather. We show that our technique is successful in the attenuating most of the multiple energy with little loss of primary energy.

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METHODOLOGY

Signal to noise separation

Consider the recorded data $\mathbf{d}$ to be the simple superposition of "signal" $\mathbf{s}$, i.e., reflection events, and "noise" $\mathbf{n}$, i.e., multiples: $\mathbf{d} = \mathbf{s} + \mathbf{n}$. For the special case of uncorrelated signal and noise, the so-called *Wiener estimator* is a filter, which when applied to the data, yields an optimal (least-squares sense) estimate of the embedded signal (Castleman, 1996). The frequency response $\mathbf{H}$ of this filter is

$$\mathbf{H} = \frac{\mathbf{P_s}}{\mathbf{P_n} + \mathbf{P_s}}, \tag{1}$$

where $\mathbf{P_s}$ and $\mathbf{P_n}$ are the signal and noise power spectra, respectively. Abma (1995) and Claerbout (1999) solved a constrained least squares problem to separate signal from spatially uncorrelated noise:

$$\begin{array}{rcl} \mathbf{Nn} & \approx & 0 \\ \epsilon \mathbf{Ss} & \approx & 0 \\ \text{subject to} & \leftrightarrow & \mathbf{d} = \mathbf{s} + \mathbf{n} \end{array} \tag{2}$$

where the operators $\mathbf{N}$ and $\mathbf{S}$ represent $t - x$ domain convolution with non-stationary PEF which whiten the unknown noise $\mathbf{n}$ and signal $\mathbf{s}$, respectively, and ϵ is a Lagrange multiplier. Minimizing the quadratic objective function suggested by equation (2) with respect to $\mathbf{s}$ leads to the following expression for the estimated signal:

$$\hat{\mathbf{s}} = \left(\mathbf{N^T N} + \epsilon^2 \mathbf{S^T S}\right)^{-1} \mathbf{N^T N}\, \mathbf{d} \tag{3}$$

By construction, the frequency response of a PEF approximates the inverse power spectrum of the data from which it was estimated. Thus, we see that the approach of equation (2) is similar to the Wiener reconstruction process. Spitz (1999) showed that for uncorrelated signal and noise, the signal can be expressed in terms of a PEF, $\mathbf{D}$, estimated from the data $\mathbf{d}$, and a PEF, $\mathbf{N}$, estimated from the noise model:

$$\mathbf{S} = \mathbf{D}\mathbf{N}^{-1}. \tag{4}$$

Spitz' result applies to one-dimensional PEF's in the $f - x$ domain, but our use of the helix transform (Claerbout, 1998) permits stable inverse filtering with multidimensional $t - x$ domain filters. Substituting $\mathbf{S} = \mathbf{D}\mathbf{N}^{-1}$ and applying the constraint $\mathbf{d} = \mathbf{s} + \mathbf{n}$ to equation (2) gives

$$\begin{array}{rcl} \mathbf{Ns} & \approx & \mathbf{Nd} \\ \epsilon \mathbf{D}\mathbf{N}^{-1}\mathbf{s} & \approx & \mathbf{0}. \end{array} \tag{5}$$

Iterative solutions to least-squares problems converge faster if the data and the model being estimated are both uncorrelated. To precondition this problem, we again appeal to the Helix transform to make the change of variables $\mathbf{x} = \mathbf{S}\mathbf{s} = \mathbf{D}\mathbf{N}^{-1}\mathbf{s}$ or $\mathbf{s} = \mathbf{N}\mathbf{D}^{-\mathbf{1}}\mathbf{x}$ and apply it to equation (5):

$$\begin{aligned} \mathbf{N}\mathbf{N}\mathbf{D}^{-1}\mathbf{x} &\approx \mathbf{N}\mathbf{d} \\ \epsilon\mathbf{x} &\approx \mathbf{0} \end{aligned} \tag{6}$$

After solving equation (6) for the preconditioned solution $\mathbf{x}$, we obtain the estimated signal by reversing the change of variables: $\hat{\mathbf{s}} = \mathbf{N}\mathbf{D}^{-\mathbf{1}}\mathbf{x}$.

Filter estimation

A PEF (**a**) by definition is the filter that minimizes the energy when convolved with the data (**d**). To estimate a space-invariant filter, this amounts to applying the fitting goal,

$$\mathbf{0} \approx \mathbf{D}\mathbf{a}. \tag{7}$$

When estimating a space-varying PEF, the number of filter coefficients can quickly become more than the number of data points, creating an underdetermined problem. Crawley et al. (1999) proposed estimating space varying filter with radial patch. The fitting goals become

$$\begin{aligned} \mathbf{0} &\approx \mathbf{D}\mathbf{A}^{-\mathbf{1}}\mathbf{p} \\ \mathbf{0} &\approx \epsilon\mathbf{A}^{-\mathbf{1}}, \end{aligned} \tag{8}$$

where $\mathbf{A}^{-\mathbf{1}}$ is a preconditioning operator that smoothes in a radial direction (assuming that dips will be more consistent along radial lines, Figure 1.

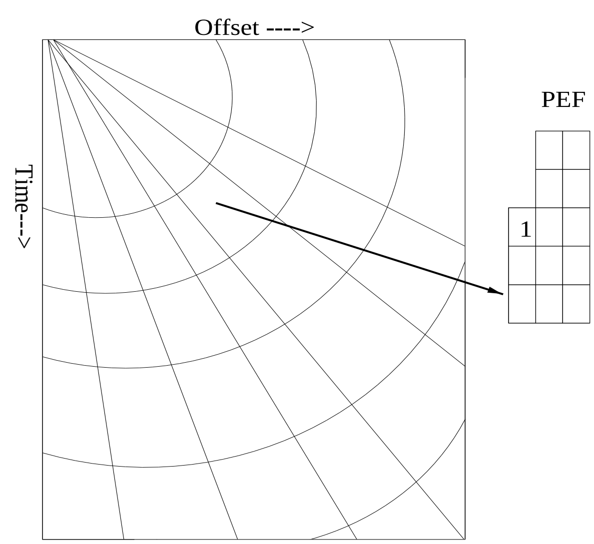

Figure 1: Space varying filter composition. A different filter is placed inside each micro-patch. The filter estimation problem is done globally, with the filter coefficient smoothed in a radial direction. bob2-pef [NR]

EXAMPLE

Synthetic example

To test the method, we chose an elastic synthetic data generated by BP. Figure 2 is the p-wave velocity model used to construct the data. Within the model, multiple behavior ranged from rather simple (away from the salt body) to much more complex (along the edge and under the salt). We then modeled the multiples by doing frequency Kirchoff upward continuation of both the sources and receivers to the sea-floor. For our first test we chose a CMP gather at 1 kft, away from the salt edge. The left panel of Figure 3 shows the original CMP gather. The center panel shows the upward continued gather at the same location. The right panel shows the predicted signal for the CMP gather. Note that we have done a good job eliminating the multiples with little loss of primary energy. Our second test was more of challenge. We chose

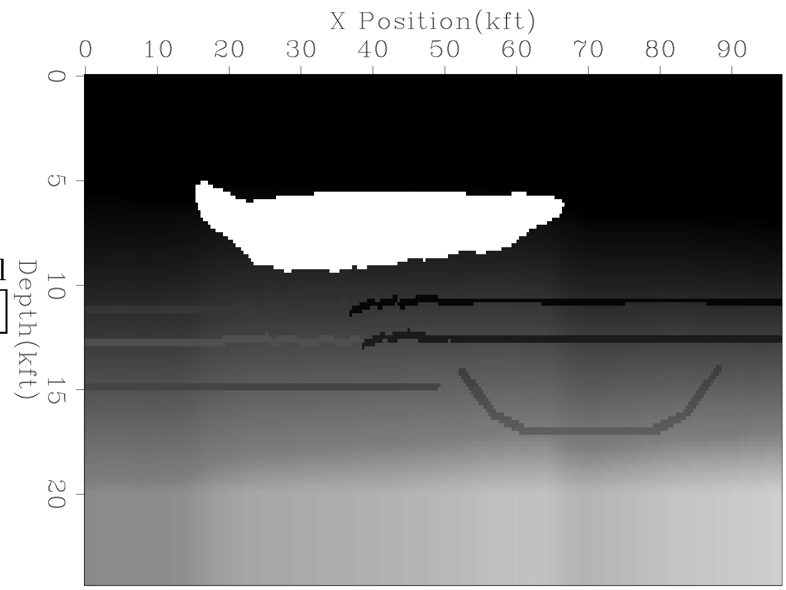

Figure 2: Synthetic velocity model for the BP dataset. bob2-bp-vel [ER]

a CMP through the salt at 44 kft (Figure 4). Note that both the primary and multiple energy is significantly more complicated than in the previous example. The right panel of Figure 4 again shows our primary estimate. In this case our primary estimate isn't quite as good as Figure 3 but we still have done an acceptable job reducing the multiple energy.

Real data

For the real data example, we chose a dataset provided by Mobil. The data previously was by Lumley et al. (1994) using a hyperbolic radon technique and by Guitton (2000) with L1 hyperbolic multiple attenuation scheme. We found the water bottom and upward continued the data. The left panel of Figure 5 is a CMP gather from the data and the middle panel is our multiple estimate. If we look at our primary estimate (the right panel of Figure 5) we can see that we have removed a little primary energy and have little multiple energy in the gather, but overall, we have done an acceptable job attenuating the multiples.

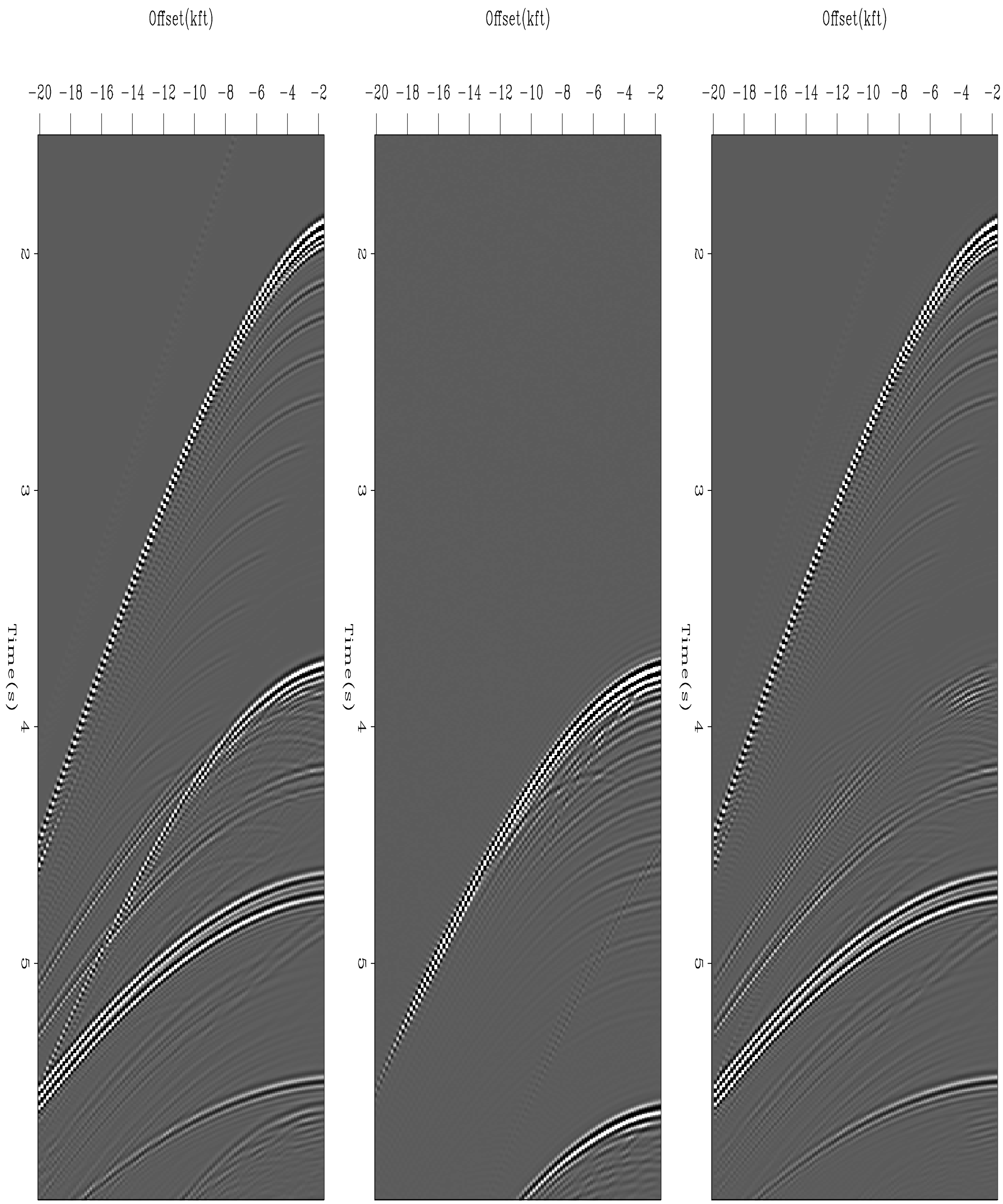

Figure 3: CMP gather at one kft. The left panel is the original CMP gather, the middle is the multiple estimate, and the right panel is our signal estimate. bob2-bp-cmp1 [ER]

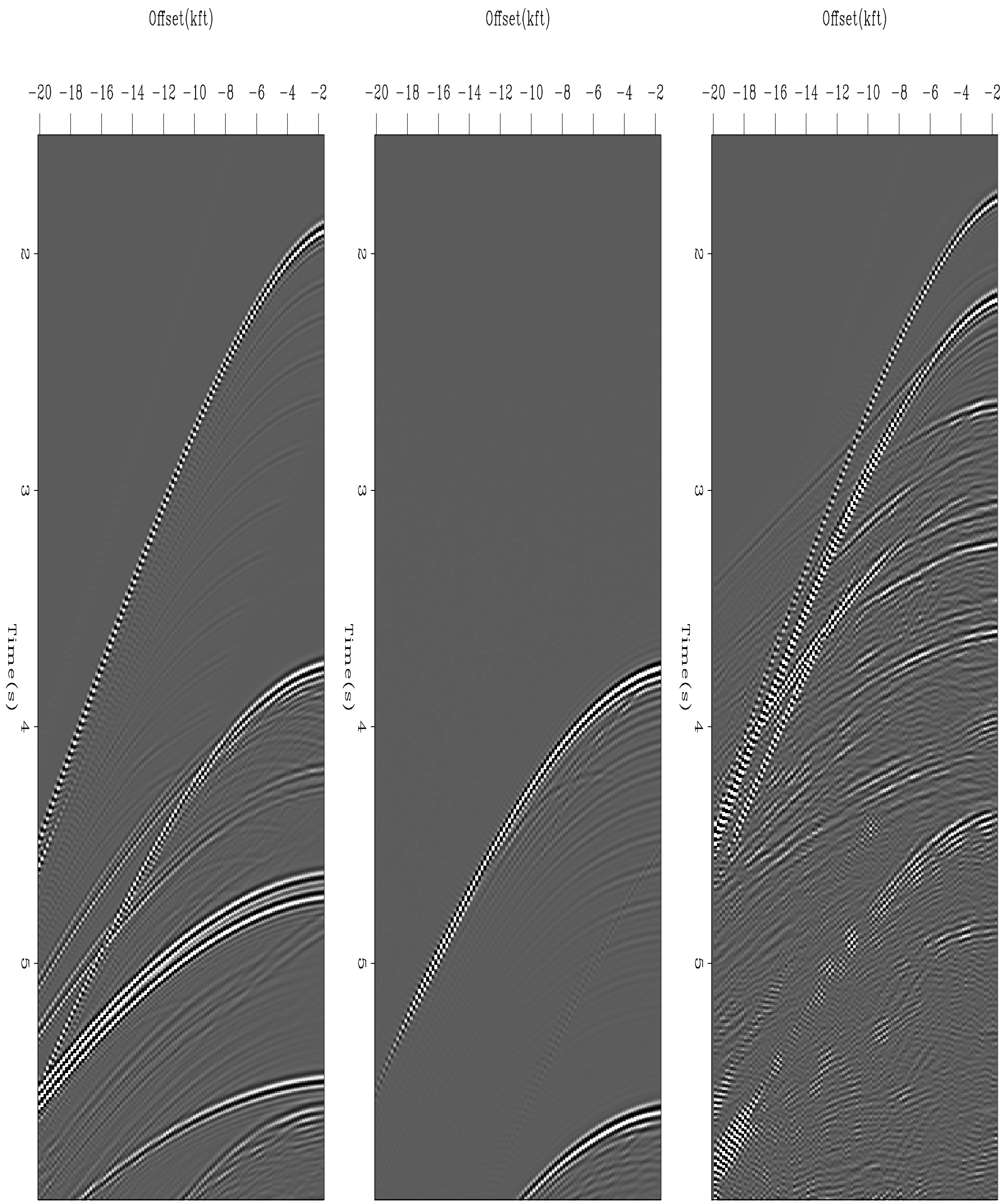

Figure 4: CMP gather at 44 kft. The left panel is the original CMP gather, the middle is the multiple estimate, and the right panel is our signal estimate. bob2-bp-cmp2 [ER]

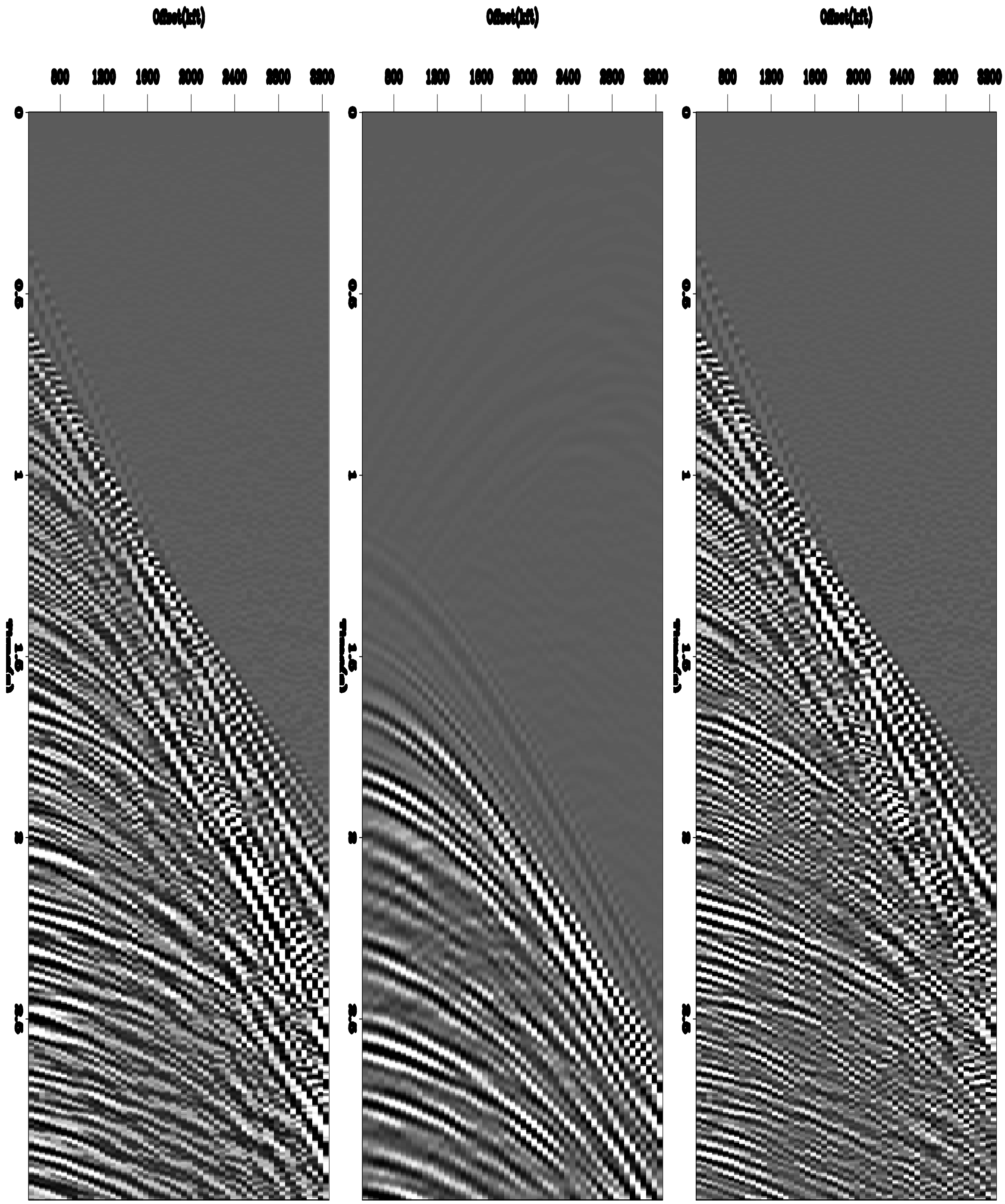

Figure 5: Multiple suppression result on real data. The left panel is the original CMP gather, the middle is the multiple estimate, and the right panel is our signal estimate. bob2-mobil-cmp2 [ER]

PROBLEMS

Currently, the major weakness of this approach is its sensitivity to parameter choice. The separation fitting goals (6) apply the inverse of a non-stationary PEF. If that PEF isn't stable, the separation of the multiples and primaries is not possible. To get a stable filter we can increase ϵ in our filter estimation (9). Unfortunately, increasing ϵ decreases the quality of our prediction. By changing the size of our micro-patches, we can usually get a stable filter while obtaining a good prediction. At this stage we haven't figured an algorithm that can automatically change micro-patch size to obtain the desired combination, a stable non-stationary PEF that can satisfactorily predict the data.

CONCLUSIONS

On early tests, the separation technique was successful in suppressing multiples. Until the method can be made more stable, it can not be used in a production environment.

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