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Regularizing tomography with non-stationary filters

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ABSTRACT

The ideal regularizer is the inverse of the model covariance matrix. Often the model covariance matrix has a complicated structure that is difficult to characterize. Non-stationary prediction error filters (PEF) have the ability to describe complicated model behavior. Non-stationary filters are effective regularizers for missing data and tomography problems.

INTRODUCTION

Most geophysical problems are either under-determined or mixed-determined, requiring some type of regularization. The ideal regularizer is the inverse model covariance (Tarantola, 1986). In previous papers I have shown that a space-varying operator composed of small plane-wave annihilation filters, or *steering filters*, can be an effective regularization operator (Clapp et al., 1997, 1998; Clapp and Biondi, 1998, 2000). Steering filters are best suited to describing models with relatively simple covariance functions. For a certain class of velocity models, such as models with discontinuities, steering filters have difficulty accurately describing model covariance.

PEFs are able to describe a much wider class of models than steering filters. To robustly estimate a PEF we must have a model with stationary statistics, something that is rarely true with seismic problems. We can often satisfy the stationarity requirement by breaking up our problem into small patches (Claerbout, 1992b). Unfortunately, we can only make our patch size so small before we can't generate sufficient statistics to estimate our PEF (Crawley et al., 1999).

An alternative approach, proven to be effective when dips change quickly, is to estimate PEFs in micro patches with a non-stationary PEF (Crawley et al., 1999; Clapp and Brown, 1999, 2000). When dealing with discontinuities, regularizing with a non-stationary PEF can be more effective in describing the model covariance than steering filters. Non-stationary filters do a better job honoring sharp boundaries and characterizing complex models.

I will begin by showing how steering filters perform poorly at discontinuities. I will then show how to build and estimate a non-stationary PEF. I will use the non-stationary PEF in the context of a missing data problem. I will conclude by using a non-stationary PEF for

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regularization to tomography.

BACKGROUND

In general, geophysical inverse problems (inverting for some model (**m**), given some (**d**), while applying some operator (**L**)) are ill-posed. A classic example of this is the missing data problem (Claerbout, 1999). The goal of the missing data problem is to interpolate intelligently between a sparse set of known points. For example, let's take a synthetic velocity model with an upper horizontal reflector, an anticline between two unconformities, and updipping layer at the bottom of the model. Suppose we have velocity measurements at several wells (Figure 1) and you would like to interpolate it onto a regular 2-D mesh.

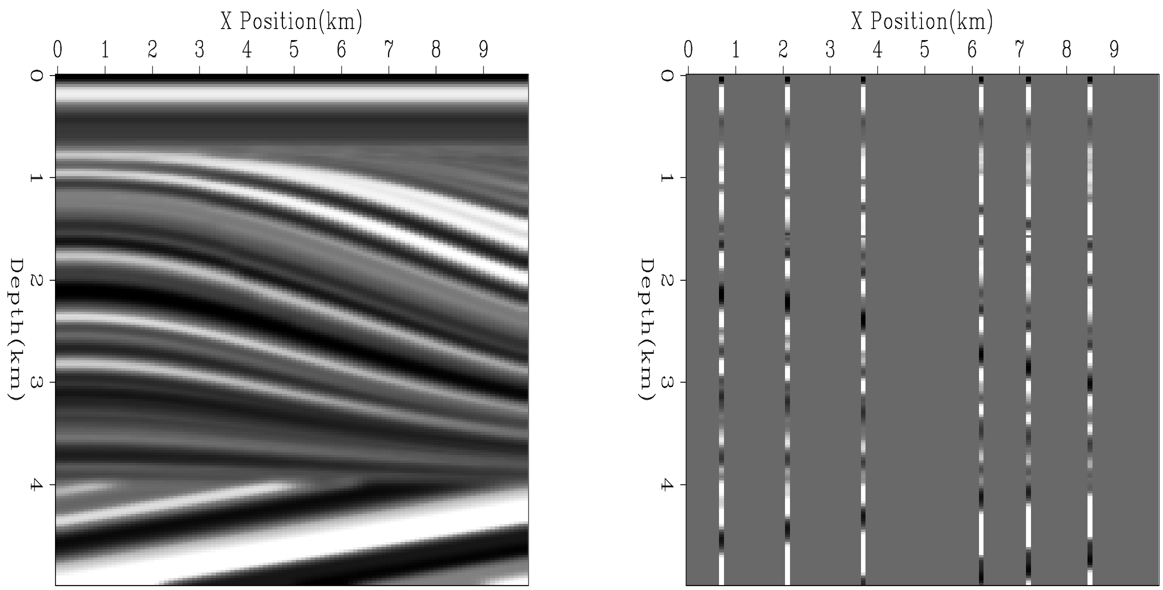


Figure 1: Left panel shows a synthetic velocity model, right panel shows a subset of that data chosen to simulate well log data. bob3-well-logs [ER]

The geophysicist might follow the approach described by Claerbout (1999), first interpolating the irregular data onto a regular mesh by applying some type of binning operator, **B**, then defining a fitting goal that requires the model to fit the data exactly at the known points (**J**),

$$\mathbf{JBd} \approx \mathbf{Jm}. \tag{1}$$

At model locations where there are no data values, we want the model to be 'smooth', therefore we will use Tikhonov regularization to minimize the output of a roughening operator applied to the model,

$$\mathbf{0} \approx \mathbf{Am}. \tag{2}$$

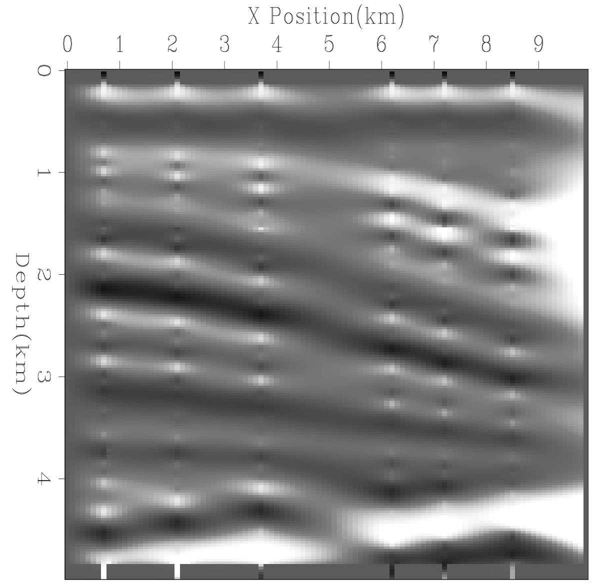


Figure 2: Interpolation result after 200 iterations using an inverse Laplacian regularization operator. Note the edges effects at the top and bottom of the model due to using a internal convolution operator. bob3-qdome-lap [ER]

If we don't have any other knowledge about our model, an isotropic operator like the Laplacian might be a logical choice for **A** since it leads to the "minimum energy" solution. If I apply the fitting goals implied by (1) and (2) for 200 iterations using the Laplacian for **A** I get Figure 2. The result is what has been euphemistically referred to as the 'ice cream cone result' (Brown, 1998). By spreading information isotropically, the model goes smoothly from our known points to some local average. We see little to no continuation of layers, which is generally a thoroughly unsatisfactory result.

Covariance

With no other information, the Laplacian might be the best regularization operator that we could use. But if we know something else about out model, can we do better? According to Tarantola (1987), we should be using the inverse model covariance for our regularization operator. The statistics of the model vary spatially, but by breaking up into four patches, one above the anticline, one below the anticline, and two within the anticline we can at least approximate stationary statistics. If we calculate the covariance within patches where the statistics are relatively stable, we get Figure 3.

Steering filters

If we examine our desired model, it is apparent that the covariance function varies within at least two of our four patches (we can also see this in the covariance function of patch 2 and 4 of Figure 3). Therefore, it follows that we should get a better image by making smaller and smaller patches. Crawley (1998) showed that this is true when solving a data interpolation problem. Traditional methods for characterizing the model like PEFs and variograms can only grow so small before we have insufficient statistics to calculate them.

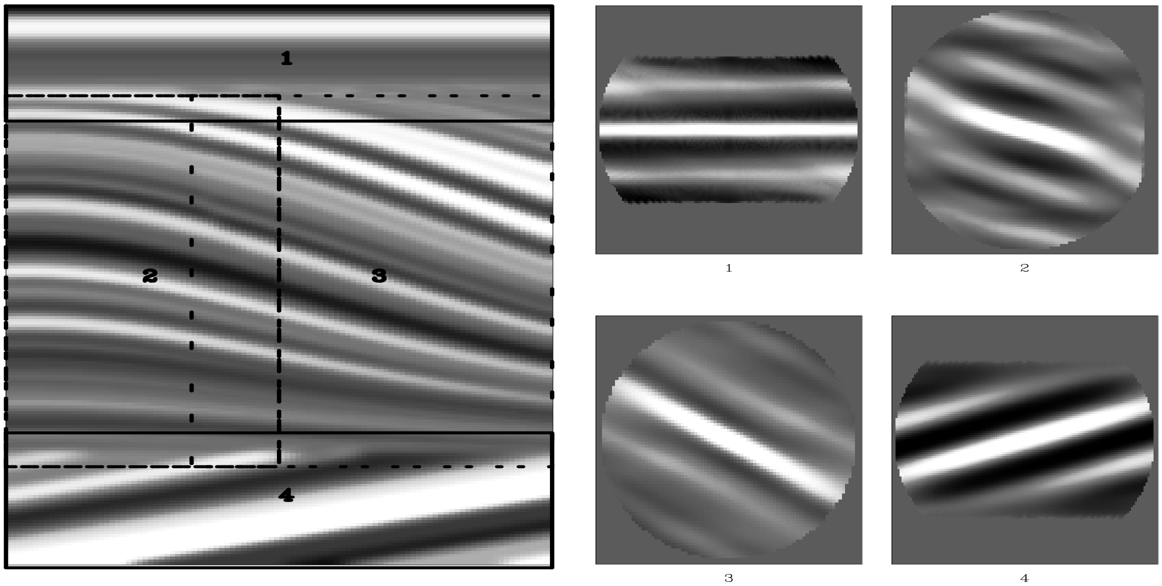
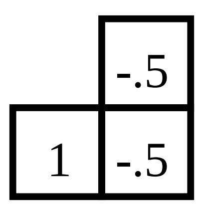


Figure 3: The covariance at four different regions of our model (left panel of Figure 1.) The top left is above the upper unconformity; top right, the upper portion of the anticline; bottom left, lower portion of the anticline; and bottom right, bellow the lower anticline. bob3-covar-change [ER]

When our stationary assumption is valid, such as in regions one and four of Figure 3 the covariance matrix is fairly simple. We have a primary trend oriented along the dip of the velocity field that slowly dies out and a ringing effect due to the sinusoidal nature of our model. We would like to come up with a way to emulate the primary trend of the covariance matrix through minimal information.

To do this it is important to remember that our regularization operator should have the inverse spectrum of the covariance matrix. Therefore if the covariance function is primarily a dipping event, our regularization operator should be destroying that dip. Claerbout (1990; 1992a) showed how to estimate the primary dip in a region and how to construct a filter that could destroy that dip. These small filters, which I refer to as *steering filters* can be as simple as a two or three point filter, Figure 4. A steering filter consists of a fixed ‘1’ and one or more coefficients in the next column. The location of the filter coefficients in the second column determines the dip that the filter will destroy. Figure 5 is the inverse impulse response of Figure 4. Note how the general orientation of the impulse response is approximately the same as the covariance function below the lower unconformity. If we assume that velocity

Figure 4: A steering filter which annihilates dips of 22.5 degrees. bob3-small-filter [NR]



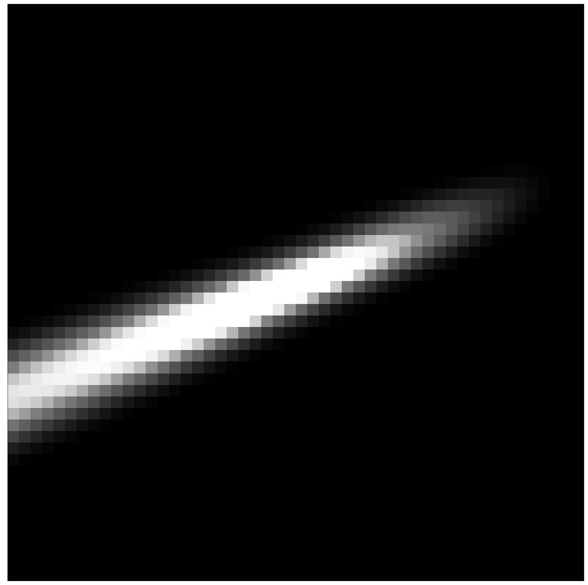


Figure 5: The impulse response of the inverse of Figure 4. bob3-small-response [ER]

follows structure and we have some guesses at reflector position, we can use this information to build our steering filters. For this problem we will assume that we have the location of four reflectors, one above the top unconformity, two between the unconformities and one below the lower unconformity (left-panel of Figure 6). If we interpolate these dips to our entire model space we have all we need to construct a space-varying operator composed of steering filters. If we use this operator as our regularizer, we get Figure 7 as our interpolation result. The steering filters did a significantly better job than the isotropic regularizer.

DISCONTINUITIES AND STEERING FILTERS

Now let's move onto a model with discontinuities. Figure 8 is similar to Figure 1, with the exception that we now have a listric fault in the middle of the anticline structure. If we follow the same interpolation path as we did in the last section, the right panel of Figure 8 is our interpolation result. Instead of reproducing the fault we have created a model that smoothly changes from horizons on the left of the fault to the corresponding horizons to the right side of the fault. In many cases a smooth change is not only acceptable, but desirable (for example, ray-based methods require a smooth model). In other cases the smooth change is unrealistic and something we want to avoid (salt boundaries and some fault boundaries). Our interpolation fails at the fault because we are not correctly describing the covariance along the edge of the fault. The problem is that the covariance function at the edge of the fault is not symmetric. Along the left edge of the fault we have good correlation with points to the left but our correlation with points to the right our correlation is shifted. This asymmetric behavior is difficult to describe with steering filters.

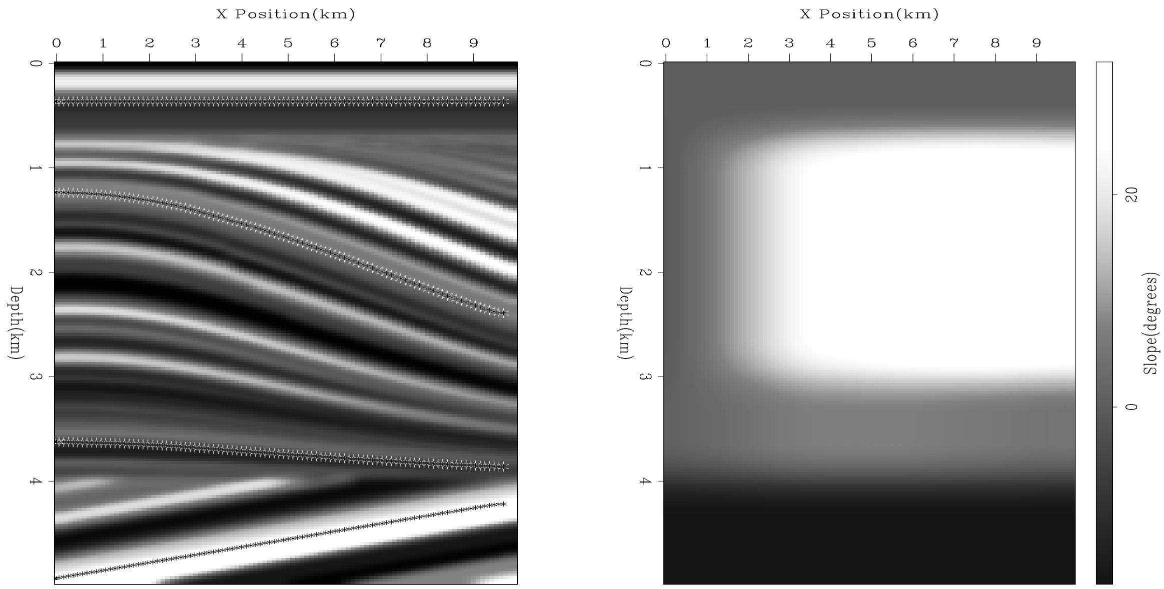


Figure 6: Left panel are four reflectors chosen to represent our *a priori* information. The right panel is interpolated slope calculated from the reflectors that will form the basis of our steering filter. bob3-qdome-refs [ER]

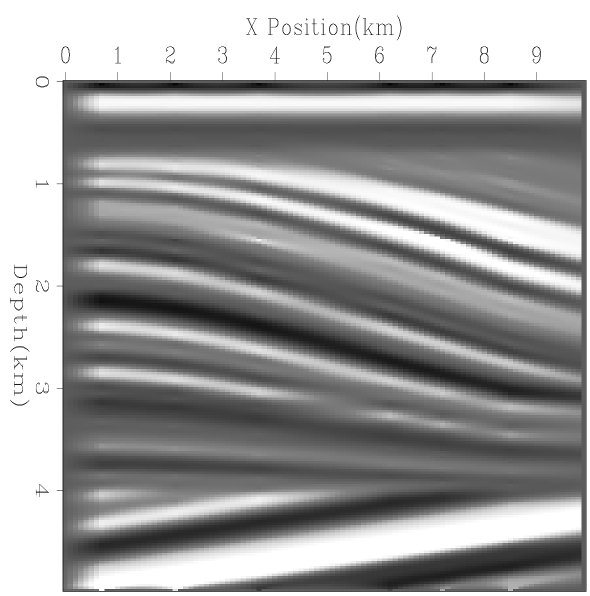


Figure 7: The result of using our steering filter operator as regularizer to the missing data problem. bob3-qdome-reg-cont [ER]

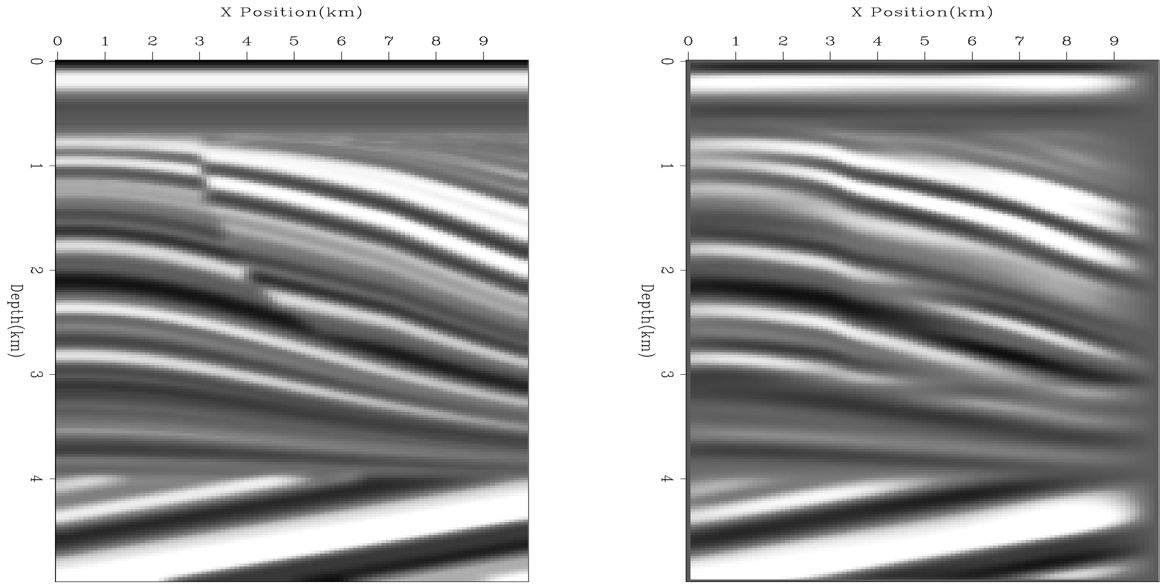


Figure 8: Left panel is a synthetic velocity model with the beds being offset by a listric fault. The right panel is the result of interpolating the model using steering filters. bob3-fault-model [ER]

ESTIMATING A NON-STATIONARY FILTER

Another option is to characterize our model in terms of PEFs rather than steering filters. Normally we estimate a PEF by solving

$$\mathbf{M}\mathbf{a} \approx \mathbf{0}, \tag{3}$$

where $\mathbf{M}$ is convolution with a field that has the same properties as the model and $\mathbf{a}$ is our PEF. The output of this convolution is white (Claerbout, 1992a). Therefore $\mathbf{a}$ must have the inverse spectrum of the model. When the model varies continuously we must add a slight twist to our PEF estimation. Instead of breaking up our model space into regions where our stationary assumption is valid we are going to modify the PEF. Our PEF ($\mathbf{a}$) is now going to be composed of several different PEFs operating in micro-patches (Figure 9). With so many filters, and therefore filter coefficients, our filter estimation problem goes from being over-determined to under-determined. We can force the system to again be overdetermined by adding a regularization equation to our original filter estimation fitting goals,

$$\begin{aligned} \mathbf{0} &\approx \mathbf{M}\mathbf{a} \\ \mathbf{0} &\approx \epsilon\mathbf{F}\mathbf{a} \end{aligned} \tag{4}$$

where the regularization operator $\mathbf{F}$ smoothes the filter coefficients (Clapp et al., 1999).

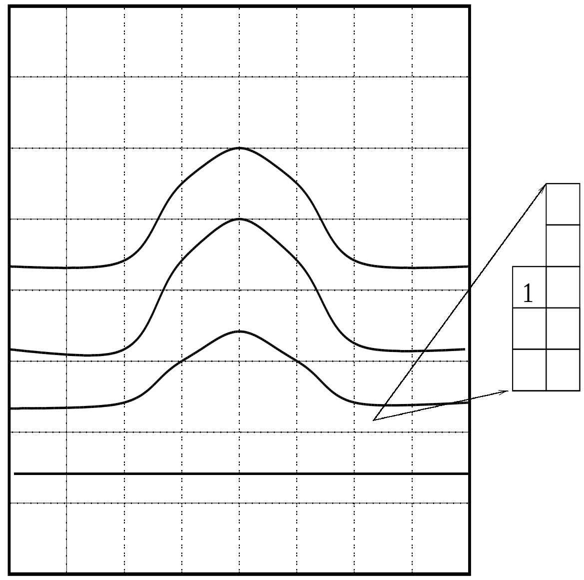


Figure 9: Non-stationary PEF construction. The model is broken up into micro-patches. Each micro-patch has its own PEF. bob3-patch [NR]

MISSING DATA

To see the power of a non-stationary filter to characterize model covariance let 's return to the fault model missing data problem (Figure 8). For our interpolation we will use the known model as the basis for our PEF, and have a micro-patch size of one sample in x and z. The resulting interpolation, Figure 10 isn't quite as high-frequency as our initial model and we have a minimal amount of continuation of the layers over the fault, but we generally do an excellent job recovering the model.

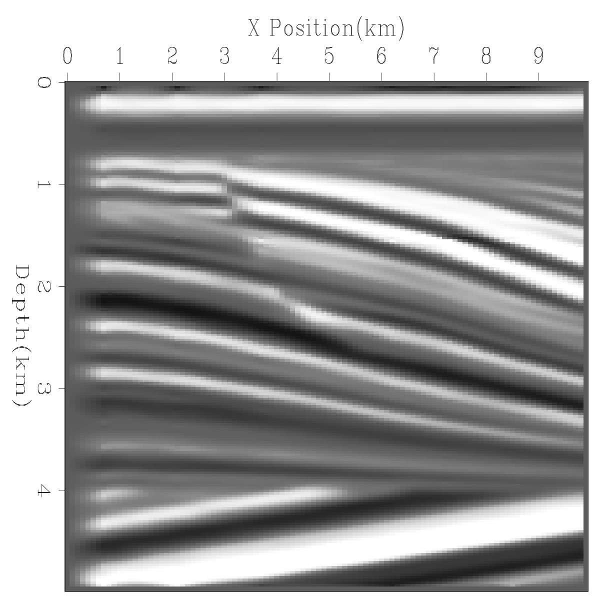


Figure 10: The result of finding non-stationary PEF using the correct model and then applying it to the missing data problem. The returned image is almost exact and does a better job with bed discontinuities than Figure 8. bob3-fault.cont [ER]

TOMOGRAPHY

Review

The next step is to see how well our non-stationary filter regularizes a tomography problem. I constructed a synthetic anticline model (Figure 11) with six reflectors,one above the anticline, four within the anticline, and one flat reflector representing basement rock. For added difficulty, there is a low velocity layer between the second and third reflector. The model was used to do acoustic wave modeling, with the resulting dataset having 32 meter CMP spacing and 80 offsets spaced 64 meters apart.If we use as our initial estimation of the slowness, an $s(z)$ function from outside the anticline, the reflectors are pulled up due to using too low a velocity within the anticline (Figure 12). Following the methodology of Clapp and Biondi (1999) we

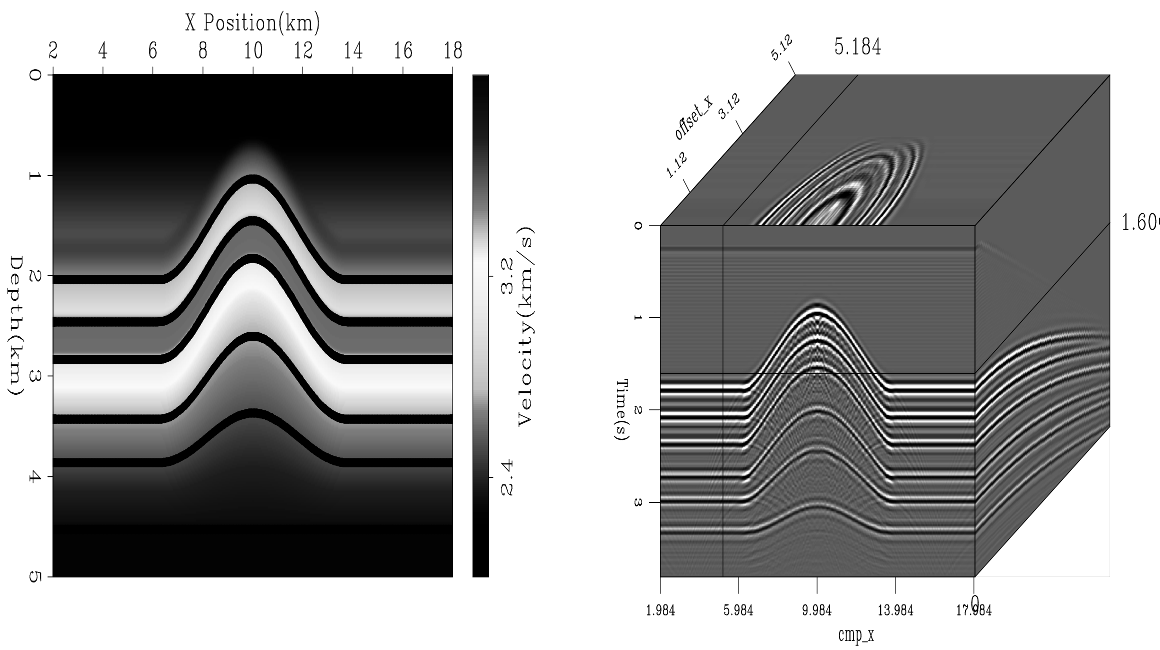


Figure 11: Left panel is the synthetic velocity model with six reflectors spanning the anticline. The right panel shows the data generated from this model. bob3-synth-model [ER]

will begin by considering a regularized tomography problem. We will linearize around an initial slowness estimate and find a linear operator in the vertical traveltime domain $\mathbf{T}$ between our change in slowness $\Delta\mathbf{s}$ and our change in traveltimes $\Delta\mathbf{t}$. We will write a set of fitting goals,

$$\begin{aligned} \Delta\mathbf{t} &\approx \mathbf{T}\Delta\mathbf{s} \\ \mathbf{0} &\approx \epsilon\mathbf{A}\Delta\mathbf{s}, \end{aligned} \tag{5}$$

where ($\mathbf{A}$) is our steering filter operator. However, these fitting goals don't accurately describe what we really want. Our steering filters are based on our desired slowness rather than change

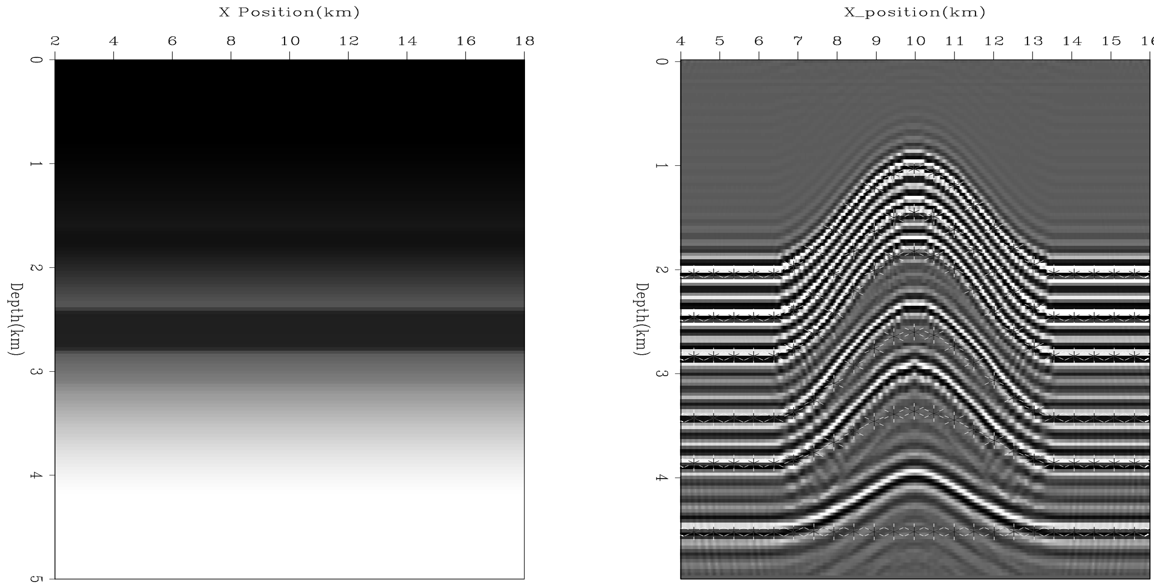


Figure 12: Left panel is our initial guess at the velocity function, the right panel shows the zero offset ray parameter reflector position using this migration velocity. The correct reflector positions are shown as '*'. Note that reflectors are significantly mispositioned. bob3-mig0 [ER]

of slowness. With this fact in mind, we can rewrite our second fitting goal as:

$$\mathbf{0} \approx \mathbf{A}(\mathbf{s_0} + \Delta\mathbf{s}) \tag{6}$$

$$-\epsilon\mathbf{A}\mathbf{s_0} \approx \epsilon\mathbf{A}\Delta\mathbf{s}. \tag{7}$$

Our second fitting goal cannot be strictly defined as regularization but we can do a preconditioning substitution:

$$\begin{aligned} \Delta\mathbf{t} &\approx \mathbf{T}\mathbf{A}^{-1}\mathbf{p} \\ -\epsilon\mathbf{A}\mathbf{s_0} &\approx \epsilon\mathbf{I}\mathbf{p}. \end{aligned} \tag{8}$$

Warping

There is one aspect of using non-stationary PEFs that I have glossed over to this point: the requirement that we have field with similar statistics to estimate the PEF from. Viewed one way this is a significant weakness to the approach, viewed another it can be seen as a useful feature. One of the largest problems in seismic imaging is how to put the geologist's conception of geology into the geophysicst's inversion problem. The non-stationary PEF can be estimated from the geologist's model. Therefore, our regularization operator directly incorporates the geologist's conception of the velocity structure into the velocity estimation.

For this simple model, we will use as our conception of geology a vertically warped version of our initial velocity estimate. We measure how much a migrated reflector positions varies from our initial flat estimate. Figure 13 shows our initial estimate.

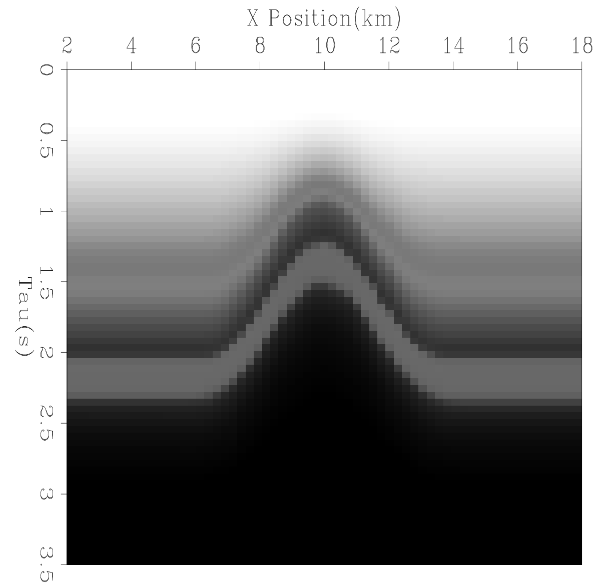


Figure 13: Warped velocity model used to estimate non-stationary filters from. bob3-warp [ER]

Results

If we do three non-linear iterations of tomography using fitting goals (8) each time, we get Figure 14 as our velocity estimate. The velocity estimate does a good job recovering the anticline shape. However, it doesn't do a good job recovering the low velocity layer. Migrating with Figure 14 we get Figure 15. Overlaid on top of the migrated image are the correct reflector positions. Overall we did a good job positioning the reflectors. In addition, we can see that we have little residual moveout in our CRP gathers (Figure 15) but they are relatively flat.

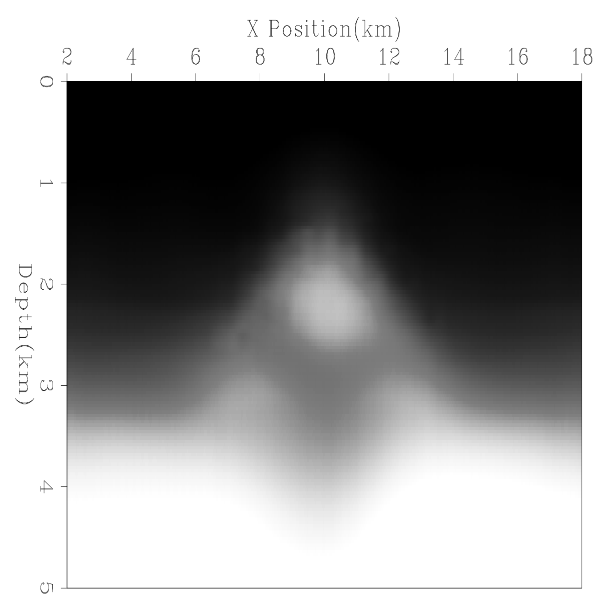


Figure 14: Velocity model after three non-linear iterations of non-stationary filter regularization. bob3-vel3 [CR]

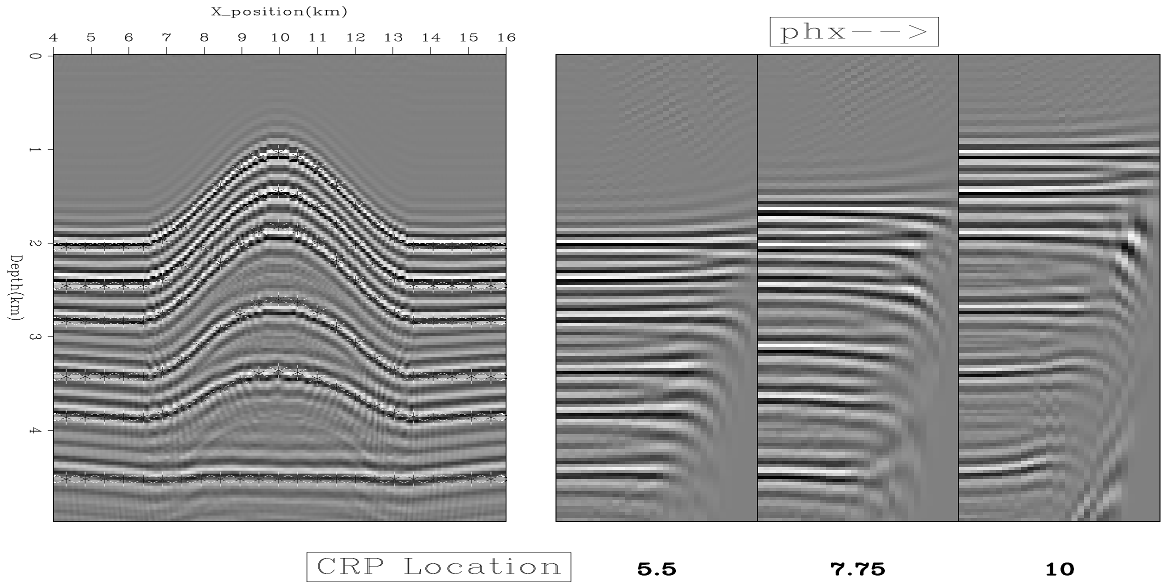


Figure 15: Migration result using Figure 14 as the velocity model. The left panel is the zero angle image overlaid by the correct reflector positions. The right three panels shows CRP gathers at six, eight, and ten kilometers. bob3-res.vel3.from0 [CR]

CONCLUSIONS

Non-stationary PEFs are an effective interpolator, especially when the model has discontinuities, for missing data problems and an effective regularizer for tomography problems. The weakness of the approach, the requirement that we have field to estimate the PEF from, can be turned into a strength by allowing a geologist's conception of the velocity to be directly encoded into the regularization. A more illuminating test of the non-stationary regularization of tomography would be on a model with sharp, rather than smooth, boundaries.

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