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Efficient 3-D wavefield extrapolation with Fourier finite-differences and helical boundary conditions

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ABSTRACT

Fourier finite-difference (FFD) migration combines the complementary advantages of the phase-shift and finite-difference migration methods. However, as with other implicit finite-difference algorithms, direct application to 3-D problems is prohibitively expensive. Rather than making the simple $x - y$ splitting approximation that leads to extensive azimuthal operator anisotropy, I demonstrate an alternative approximation that retains azimuthal isotropy without the need for additional correction terms. Helical boundary conditions allow the critical 2-D inverse-filtering step to be recast as 1-D inverse-filtering. A spectral factorization algorithm can then factor this 1-D filter into a (minimum-phase) causal component and a (maximum-phase) anti-causal component. This factorization provides an LU decomposition of the matrix, which can then be inverted directly by back-substitution. The cost of this approximate inversion remains $O(N)$ where N is the size of the matrix.

INTRODUCTION

Within the exploration industry, geophysicists are realizing the inherent limitations of Kirchhoff methods when it comes to accurately modeling the effects of finite-frequency wave propagation. This is fueling interest in "wave-equation" migration algorithms, such as those based on wavefield extrapolation, that do accurately model finite-frequency effects.

As with all migration algorithms, there is a tradeoff amongst extrapolators: cost versus accuracy. For wavefield extrapolators, however, the tradeoff goes three ways: accuracy at steep dips versus the ability to accurately handle lateral velocity variations versus cost again. Fourier finite-difference migration (Ristow and Ruhl, 1994) strikes an effective balance between the accuracy priorities, combining the steep dip accuracy of phase-shift migration in media with weak lateral velocity contrasts, and the ability to handle lateral variations with finite-difference.

Unfortunately, as with other implicit finite-difference, the cost does not scale well for three-dimensional problems without additional approximations that often expensive and may compromise accuracy. In an earlier paper, Rickett et al. (1998) solved the costly matrix inversion for implicit extrapolation with the 45° equation with an approximate LU decomposition

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based on the helical transform (Claerbout, 1998b). In this paper, the same approach allows me to extrapolate with a more accurate operator.

THEORY

Fourier finite-difference migration

Three-dimensional FFD extrapolation is based on the equation (Ristow and Ruhl, 1994),

$$\frac{\partial P}{\partial z} = i\left[\sqrt{\frac{\omega^2}{c^2}+\nabla^2_{x,y}}+\left(\frac{\omega}{v}-\frac{\omega}{c}\right)+\frac{\omega}{v}\left(1-\frac{c}{v}\right)\frac{\frac{v^2}{\omega^2}\nabla^2_{x,y}}{a+b\frac{v^2}{\omega^2}\nabla^2_{x,y}}\right]P, \tag{1}$$

where $v = v(x,y,z)$ is the medium velocity, c is a reference velocity ($c \leq v$), and a and b are coefficients subject to optimization. The first term describes a simple Gazdag phase-shift that must be applied in the $(\omega, \mathbf{k})$ domain; the second term describes the first-order split-step correction (Stoffa et al., 1990), applied in $(\omega, \mathbf{x})$; and the third term describes an additional correction that can be applied as an implicit finite-difference operator (Claerbout, 1985), also applied in $(\omega, \mathbf{x})$.

In areas with strong lateral velocity variations ($c/v \approx 0$), FFD reduces to a finite-difference migration, while in areas of weak lateral velocity variations ($c/v \approx 1$), FFD retains the steep-dip accuracy advantages of phase-shift migration. As a full-wave migration method, FFD also correctly handles finite-frequency effects.

For constant lateral velocity, the finite-difference term in equation (1) can be rewritten as the following matrix equation,

$$(\mathbf{I}+\alpha_1\mathbf{D})\,\mathbf{q}_{z+1} = (\mathbf{I}+\alpha_2\mathbf{D})\,\mathbf{q}_z \tag{2}$$

$$\mathbf{A}_1\mathbf{q}_{z+1} = \mathbf{A}_2\mathbf{q}_z \tag{3}$$

where $\mathbf{D}$ is a finite-difference representation of the x,y-plane Laplacian, $\nabla^2_{x,y}$, and $\mathbf{q}_z$ and $\mathbf{q}_{z+1}$ represent the diffraction wavefield at depths z and $z+1$ respectively . Scaling coefficients, α_1 and α_2, are complex and depend both on the ratio, ω/v, and the ratio c/v.

The right-hand-side of equation (3) is known. The challenge is to find the vector $\mathbf{q}_{z+1}$ by inverting the matrix, $\mathbf{A}_1$. For 2-D problems, only a tridiagonal matrix must be inverted; whereas, for 3-D problems the matrix becomes blocked tridiagonal. For most applications, direct inversion of such a matrix is prohibitively expensive, and so approximations are required for the algorithm to remain cost competitive with other migration methods.

A partial solution is to split the operator to act sequentially along the x and y axes. Unfortunately this leads to extensive azimuthal operator anisotropy, and necessitates expensive additional phase correction operators.

Helical factorization

The blocked-tridiagonal matrix of the 3-D extrapolation, $\mathbf{A}_1$, represents a two-dimensional convolution operator. Following Rickett et al.'s (1998) approach to factoring the 45° equation, I apply helical boundary conditions (Claerbout, 1998b) to simplify the structure of the matrix, reducing the 2-D convolution to an equivalent problem in one dimension.

For example, helical boundary conditions allow a two-dimensional 5-point Laplacian filter to be expressed as an equivalent one-dimensional filter of length $2N_x + 1$ as follows

$$d = \begin{bmatrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{bmatrix} \xrightarrow{\text{helical boundary conditions}} (1, 0, \,...\, 0, 1, -4, 1, 0, \,...\, 0, 1).$$

The operator, $\mathbf{D}$, in equation (2) could represent convolution with this filter; however, I use a more accurate, but equivalent, 9-point filter.

Unfortunately, the complex scale-factor, α_1, means $\mathbf{A}_1$ is symmetric, but not Hermitian, so the filter, a_1, is not an autocorrelation function, and standard spectral factorization algorithms will fail. Fortunately, however, the Kolmogoroff method can be extended to factor any cross-spectrum into a pair of minimum phase wavelets and a delay (Claerbout, 1998a).

With this algorithm, the 1-D convolution filter of length $2N_x + 1$ can be factored into a pair of (minimum-phase) causal and (maximum-phase) anti-causal filters, each of length $N_x + 1$. Fortunately, filter coefficients drop away rapidly from either end, and in practice, small-valued coefficients can be safely discarded.

By reconsituting the matrices representing convolution with these filters, I obtain an approximate LU decomposition of the original matrix. The lower and upper-triangular factors can then be inverted efficiently by recursive back-substitution.

While we have only described the factorization for $v(z)$ velocity models, the method can also be extended to handle lateral variations in velocity. For every value of ω/v and c/v, we precompute the factors of the 1-D helical filters, a_1 and a_2. Filter coefficients are stored in a look-up table. We then extrapolate the wavefield by non-stationary convolution, followed by non-stationary polynomial division. The convolution is with the spatially variable filter pair corresponding to a_2. The polynomial division is with the filter pair corresponding to a_1. The non-stationary polynomial division is exactly analogous to time-varying deconvolution, since the helical boundary conditions have converted the two-dimensional system to one-dimension.

EXAMPLES

Figure 1 compares depth-slices through impulse responses of FFD migration (with $c/v = 0.8$) for the splitting approximation, (a), and the helical factorization, (b). The azimuthal anisotropy is noticeably reduced with the helical factorization.

Figure 2 shows extracts from a three-dimensional FFD depth migration of a zero-offset subset from the SEG/EAGE salt dome dataset. This rugose lateral velocity model initially

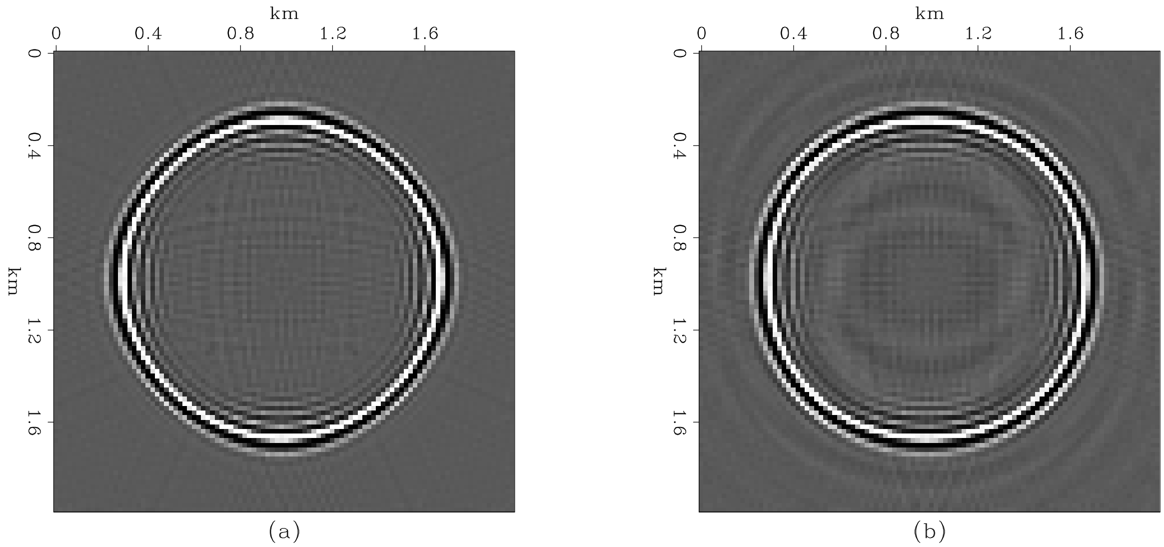

Figure 1: Depth-slices of centered impulse response corresponding to a dip of 45° for $c/v = 0.8$. Panel (a) shows the result of employing an $x - y$ splitting approximation, and panel (b) shows the result of the helical factorization. Note the azimuthally isotropic nature of panel (b). james1-timeslices [CR]

caused mild stability problems for the FFD migration, and I had to smooth the velocity model to produce the results shown in Figure 2. Biondi (2000) presents an unconditionally stable formulation of the FFD algorithm; however, that formulation does not easily fit with the approximate helical factorization discussed here.

CONCLUSIONS

Helical boundary conditions allow the critical 2-D inverse-filtering step in FFD migration to be recast as 1-D inverse-filtering. A spectral factorization algorithm can then factor this 1-D filter into a (minimum-phase) causal component and a (maximum-phase) anti-causal component. This factorization provides an LU decomposition of the matrix, which can then be inverted directly by back-substitution. The cost of this approximate inversion remains $O(N)$ where N is the size of the matrix.

I demonstrate this alternative factorization retains azimuthal isotropy without the need for additional correction terms, and apply the migration algorithm to the 3-D SEG/EAGE salt dome synthetic dataset.

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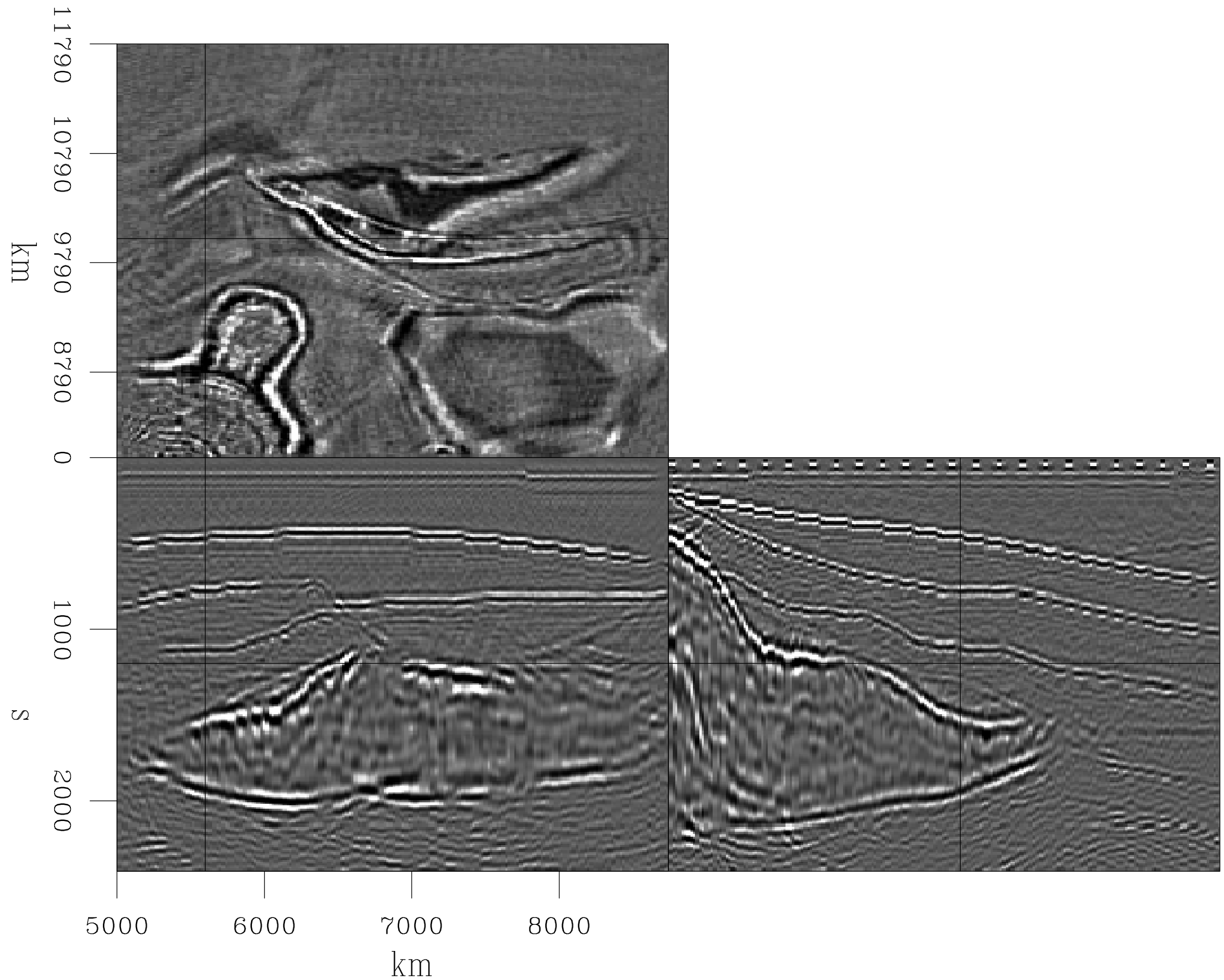

Figure 2: Migration of a three-dimensional zero-offset subset from the SEG/EAGE salt dome dataset by Fourier finite-differences with helical boundary conditions. james1-cubeplot [CR]