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Spectral factorization of 2-D reflection seismic data

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ABSTRACT

I propose spectral factorization of 2-D seismic data. Boulders strewn on the water bottom of an otherwise horizontally layered earth imply that the multidimensional minimum-phase wavelet of a zero-offset section is a common midpoint gather.

INTRODUCTION

Recently, math professor George Papanicolaou delivered a seminar to our Geophysics Department in which he presented to us an amazing proposition: Random scatterers can give us superresolution because they can enlarge the effective aperture.

I will propose it this way: Scatterers give rise to seismic coda. Maybe we can use it effectively even though we may not know the location of the scatterers.

In this paper I sketch how this could happen in the realistic case of near-surface scattering and I indicate how it could be tested and demonstrated.

Except for one essential feature, the earth model that we examine in this initial exploratory phase is a two-dimensional horizontally layered earth. The essential departure is that the top roughness that acts as point scatterers. We might think of it as fine scale surface topography. Alternately, we might think of it as a thin water layer with boulders strewn around, all acting as point scatterers of random amplitude, polarity and location. The one-dimensional earth model itself has arbitrary velocity $v(z)$, multiple reflections, shear waves, anisotropy, etc. Visualize this geometry:

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                s     g
.          r1 r2 r3 r4 r5 r6 r7 r8 r9
.
.
.
.
.
.          __________________________
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We are interested in ray paths like the one from the shot s to the reflector, to the rock r9, to the reflector, to the geophone g. Both paths to and from the rock r9 include all arrivals,

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both direct and reflected. We convolve the 1-D simple earth's s to r9 response with its r9 to g response. Whatever the response is to one rock, the response to all the rocks will be the one-rock response convolved horizontally by random numbers.

There will be several conjectures. The simplest is that from a zero-offset section, spectral factorization (Kolmogoroff, 1939) via the helix (Claerbout, 1998) manufactures a common midpoint gather from which we can do velocity analysis.

SUBTRACT ANY TWO SHOT GATHERS

We will soon be doing some data-guided theory. Carefully examine Figure 1. I have only this one shot gather (a photo image) but I'd like you to imagine two of them. Imagine the earth really is a one-dimensional layered medium with surface scatterers. Then the two shot gathers would look identical EXCEPT for the diffraction at 250m offset. This diffraction would be differently positioned on each shot gather depending on the distance of the shot from the surface scatterer. Let us subtract these two gathers. Now the layered media reflectors all go away and we have a gather containing only two copies of the surface diffraction.

To study the surface scatter events, we need to be rid of the layered media primary events. These could be gotten rid of by the simple subtraction or they could be gotten rid of by a spatial lowpass filter or a spatial PEF, or in the event of gentle dip, by various kinds of "steering filters". We can return to the practical issue of separating the simple reflections from the surface scatter after we have covered some matters of principle.

2-D AUTOCORRELATION OF SURFACE SCATTERED RETURNS

Here we show theoretically that the 2-D autocorrelation (or 2-D spectrum) of surface scattered reflections is the same as that of the primary reflections. Thus by autocorrelation we will concentrate information that could be widely distributed in time and space. Later, we'll convert the autocorrelation to something more familiar.

Let the layered earth response from shot s to geophone g be $u(s,g,t) = u(0,g-s,t) = u(g-s,t)$ or in Fourier space, $u(g-s,\omega)$ or simply $u(g-s)$. When an upcoming wave hits the earth surface at g_1 it encounters a scattering object which reflects the primary wave with a random scaling $\xi(g_1)$. The signal at g_1 then takes off for a second flight like a multiple reflection, but departing in all directions. We are going to build the theoretical 2-D spectrum of this surface scattered wave w from the theoretical 2-D spectrum of u, the layered media primary reflection.

First we express the cascade of the two bounces. The arrival w at g_2 at time t is the sum of the time of each bounce, τ and $t-\tau$. Since this is a convolution in the time domain, we express it as a product in the frequency domain. Then we form the complex conjugate of this

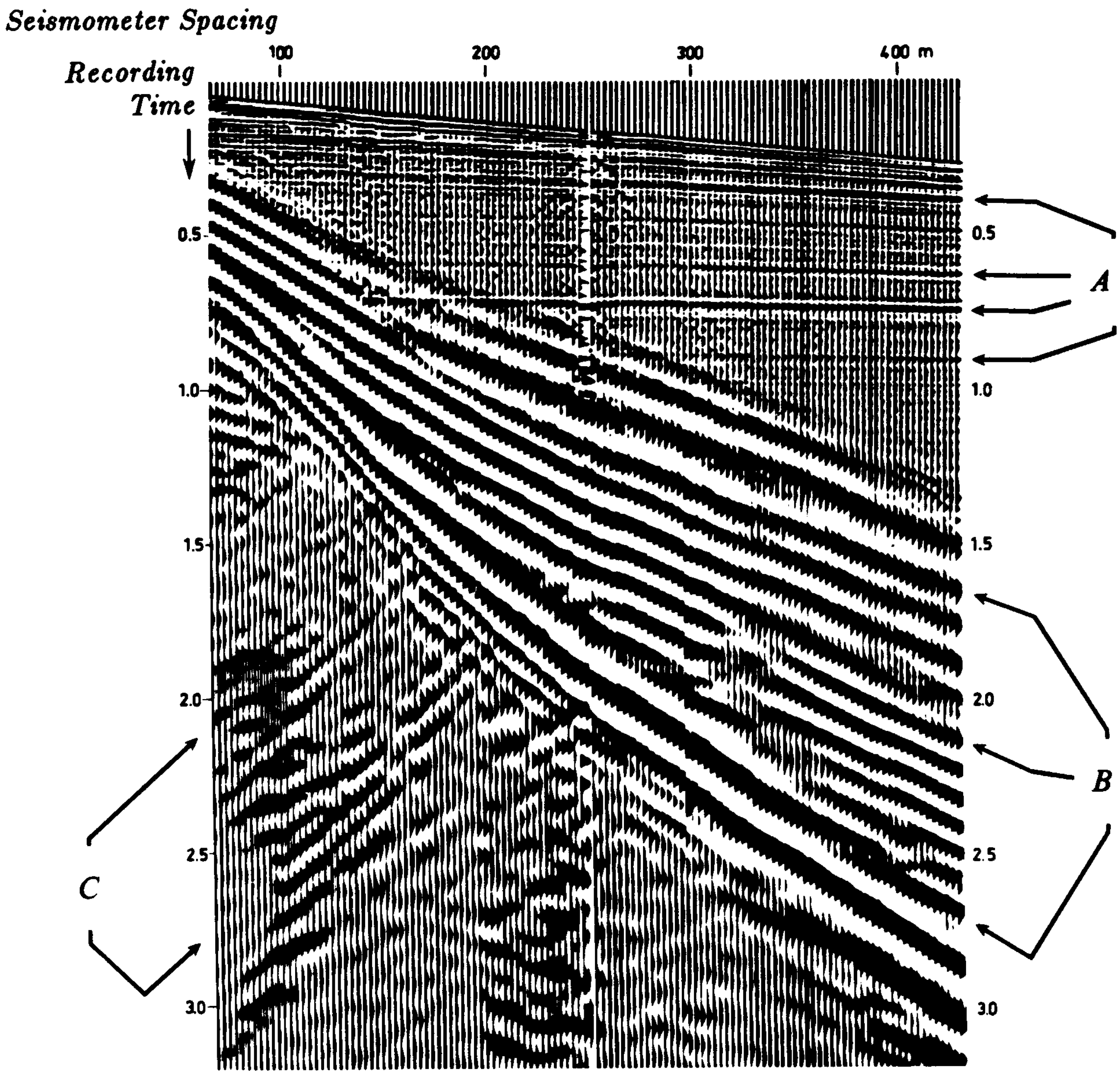

Figure 1: Shell shot gather from IEI page 172 (paper version) section 3.2. (/book/iei/Fig/ofs/shell.ps) Observe the scattered ground roll with its apex at 250 m offset. It is interesting to identify the ray paths of ALL the backscattered energy. Might you believe the diffraction at 250 m offset is a weaker copy of the entire shot gather but shifted 250 m and .17 s? If you do, you will recognize that this copy also contains (albeit weakly) the near zero offset traces that are missing from the the recording of the gather itself. jon4-shell [NR]

expression in preparation for autocorrelation on the x-axis.

$$w(s,g_2,t) \quad = \quad \sum_{g_1}\sum_{\tau} u(s,g_1,\tau)\, u(g_1,g_2,t-\tau)\, \xi(g_1) \tag{1}$$

$$w(s,g_2,\omega) \quad = \quad \sum_{g_1} u(g_1-s,\omega)\, u(g_2-g_1,\omega) \quad \xi(g_1) \tag{2}$$

$$\bar{w}(s,g_2+x,\omega) \quad = \quad \sum_{g_3} \bar{u}(g_3-s,\omega)\, \bar{u}(g_2+x-g_3,\omega)\, \bar{\xi}(g_3) \tag{3}$$

We insert the last two expressions into the expression for spatial autocorrelation.

$$A(s,x) \quad = \quad \sum_{g_2} w(s,g_2)\, \bar{w}(s,g_2+x) \tag{4}$$

We will determine $A(s,x)$ experimentally as described earlier. Here we will see its theoretical relation to the primary reflected field u.

$$\begin{aligned}
A(s,x) \quad &= \quad \sum_{g_1}\sum_{g_2}\sum_{g_3} u(g_1-s)\, u(g_2-g_1)\, \bar{u}(g_3-s)\, \bar{u}(g_2-g_3+x)\, \xi(g_1)\, \bar{\xi}(g_3) && (5)\\
&= \quad \sum_{g_1}\sum_{g_2}\sum_{g_3} u(g_1-s)\, u(g_2-g_1)\, \bar{u}(g_3-s)\, \bar{u}(g_2-g_3+x)\, \delta(g_1-g_3) && (6)\\
&= \quad \sum_{g_1}\sum_{g_2} \quad u(g_1-s)\, u(g_2-g_1)\, \bar{u}(g_1-s)\, \bar{u}(g_2-g_1+x) && (7)\\
&= \quad \sum_{g_1} u(g_1-s)\, \bar{u}(g_1-s) \quad \sum_{g_2} u(g_2-g_1)\, \bar{u}(g_2-g_1+x) && (8)\\
&= \quad \left(\sum_{h} u(h)\, \bar{u}(h)\right) \quad \left(\sum_{h} u(h)\, \bar{u}(h+x)\right) && (9)
\end{aligned}$$

We Fourier transform over x. The first factor above is not a function of space. It is merely a function of ω, say a filter $|f(\omega)|^2$. Thus our main result:

$$|w(s,\omega,k_x)|^2 \quad = \quad |f(\omega)|^2 \;\; |u(\omega,k_x)|^2 \tag{10}$$

We see that in principle, for each shot point s, we measure the spectrum of the impulse response of the layered medium.

Hazardous cross terms

Suppose that we had not gotten rid of the layered media terms. The observations would be $u+w$ and we would be autocorrelating that. Both u and w have essentially the same autocorrelation, but their cross-term is dangerous. It has a different form (as you can verify). This form multiplies the expectation of $\xi(g)$ which might be theoretically zero but in practice might often not be zero. In principle we eliminate the cross term by eliminating u from the observation $u+w$, but in practice this difficulty remains to haunt us wherever we find the earth is not sufficiently horizontally layered.

AUTOCORRELATION

Need for all autocorrelations to be the same

The surface diffracted events that we might see on a shot gather are probably all much the same. (Anyway, I am not prepared to discuss their differences. It seems to be an implication of the theory. This deserves a better explanation or more study.)

Assuming the diffractions are all the same, their autocorrelations are all the same. (Anyway, if not, we are in big trouble, because then we get a smearing together of all their different autocorrelations.) The autocorrelation combines all the diffractions despite their different locations. By combining them, it enhances them. The autocorrelation does the job of merging the energy of all the scatterers.

There is an issue of the cross correlation of one diffractor with another. We'll suppose the cross correlations cancel out because of the random superposition of many shifts and directions. In practice this could fail if there are a small number of very strong ones.

Several people suggested that I should investigate the effect of random scatterers spread throughout the earth instead of having them all at the surface. I agree that is an interesting model to study, but I feared it because we all recognize that reflectors at all different depths will produce different hyperbolas. We dare not autocorrelate such data until we have processed it so that all hyperbolas look the same.

Spectral factorization

The trouble with the autocorrelation of the CSG is that we are not accustomed to it. We don't know how to think about it. We would rather have the CSG itself. This suggests spectral factorization. In one dimension it is ancient knowledge that spectral factorization finds us an impulse response function of the system. Using the helix (Claerbout, 1998), in helioseismology (Rickett and Claerbout, 1999a) (Rickett and Claerbout, 1999b) we found that we can recover a multidimensional causal acoustic impulse response of the sun instead of "autocorrelation wavelets" that helioseismologists had been getting. Now we conjecture that we can do something similar here:

> CONJECTURE: A spectral factorization of the (autocorrelation of the) shot-geophone reflection data should give us the "multidimensional impulse response" of the earth.

There are some pitfalls with spectral factorization, but they are fairly well understood and they have often been overcome in the past.

What if the autocorrelation is bigger than the cable?

Consider the case where the cable is much shorter than the spatial autocorrelation of the diffractor. This would appear to be a troublesome case. We may notice from the Shell gather in Figure 1 that a limited range of offsets could allow us to see all offsets because of the presence of the scatterers at all ranges. If, however, we measure an autocorrelation over a small range, could spectral factorization possibly construct a minimum phase wavelet over a large range? I fear not. On the other hand, we can enlarge the aperture of a single shot by the idea of synthetic aperture. Let us see how this might be done.

Data layout

Ordinarily we think of reflection data as three dimensional, $P(t,g,s)$. That is because we redefine time to begin anew at $t = 0$ for each shot. Now let us use the more natural time, the wall clock time during data acquisition. Suppose the gun fires every 10 s for 10,000 s. Thus we have 1000 shots along a horizontal survey line. At each receiver we have this entire 10,000 s signal. We have one such a signal at each geophone. There is no shot axis. Thus the data is intrinsically two-dimensional, $P(t,g)$. Next we use the helix, as always, to wrap both t and g into one super-long signal. Apply spectral factorization, and unwrap the helix. What we should have is an estimate of the simple CSG we began with.

What is new, however, is very new and very interesting. When data is autocorrelated, it is averaged. In any average, some of the terms may be omitted if the sum is normalized properly.

I hypothesize that we'll have a very similar autocorrelation if we are missing many of the recordings. In particular, I propose to consider only the zero-offset traces. Forget about that long recording streamer! I hypothesize that the 2-D spectral factorization of the ZOS can give us a shot profile with all 1000 receivers.

> CONJECTURE: The spectral factorization of the (autocorrelation of the) zero-offset section is the common midpoint gather.

This conjecture seems plausible when we recall that the ZOS amounts to the simple CSG convolved on the horizontal axis with a line of random numbers. The autocorrelation eliminates the random numbers and the spectral factorization recovers the CSG.

The proposal is really amazing: We could throw away our marine streamer and have only one receiver and hence only one point on the offset axis, yet the rocks randomly placed on the water bottom create for us a CMP gather that we could use for for velocity analysis. We better try it!

Finally, perhaps we can produce Cheops' pyramid.

> CONJECTURE: The spectral factorization of the 2-D seismic data is Cheop's pyramid.

Spatial aliasing

An omnipresent theme in geophysical surveying is that we never have enough spatial coverage. In my book GEE, a theme is that we cope with insufficient data not via autocorrelation calculation but via prediction-error filter (PEF) estimation. Multidimensional PEFs are also a natural way to handle non stationarity. No doubt, we could return to the PEF approach for dealing with scattering too.

ACKNOWLEDGMENT

Professor Papanicolaou asserted that the superresolution phenomena is related to "sending a time reversed signal" back into the earth.

I note that if the impulse response of a system is time reversed and then fed into the same system, the output is an autocorrelation function. Many physical systems satisfy the conditions of reciprocity, that is, they have a symmetric matrix that carries source locations to receiver locations. On such a system you can interchange the sender and receiver and record the same signal.

My view here is that the magic comes from using autocorrelation. It can gather the scattered information that the main antenna is not prepared for.

I'd like also to thank James Rickett for his critical interest in this work, for helping me get my algebra correct, and for pointing out the danger with the cross term of $u+w$.

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