Stanford Exploration Project, Report 103, April 27, 2000, pages 205–333

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Short Note

Test case for PEF estimation with sparse data

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Are PEFs going to be a useful tool on very sparse data sets? GEE disappoints. It throws away regression equations that depend on missing data. With sparse data, we have none left.

We need a test case. I offer one here. A test case with some realistic aspects. We should be able to get perfect results while nobody else does. And they'll all have to admit it :-)

THE TEST CASE

We begin with a rough one-dimensional function. A random walk would be nice, the integral of random numbers (possibly coin flips). Call it $r(x)$. Actually, I'd like a random walk that crosses the zero axis a couple times. We could try several seeds until we find an "attractive" one. Maybe leaky integrate random numbers.

Next, flex a piece of paper so that along the x-axis it matches $r(x)$. Now any line parallel to the x-axis should match $r(x)$. Let us tilt this on the y-axis, so our altitude function is $h(x,y) = r(x) + y$. Notice that we have a function whose second y-derivative vanishes everywhere. So δ_{yy} is a PEF for it. Its Gaussian curvature $h_{xx}h_{yy} - h_{xy}^2$ vanishes. Now, to confuse people, we rotate it 45°. Thus, I propose the test function $h(x,y) = r(x+y) + (x-y)$. Its perfect PEF is

```
.  0  0
.  0  0
1  0  0
0 -2  0
0  0  1
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and we need to show that we can find it.

What will we give for data? I offer a track of dense data along the x axis (like a well log or seismogram). Already you can see with this and with the PEF, and one more data point off the track, the data everywhere is known. Well, almost everywhere. Actually, there is a shadow. Now we sprinkle a dozen data values around the plane. We might choose those points aliased on our rough function in such a way that everyone but us would be completely confused.

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HOW ARE WE GOING TO GET THE PEF?

How are we going to get the PEF? I propose we invoke stationarity and scale invariance. Notice that we have an interesting kind of scale invariance here. We can expand the correct PEF and it is still a correct PEF. (It is interesting to notice that an apparent scale or resonant spectrum visible along the observation track does not prevent the 2-D function from being scale invariant).

We have a non-linear optimization problem to solve. You know it, $0 \approx A(Z)P(Z)$ where $a_0 = 1$ and some of the p_i are known. It is non-linear. This problem is linearized in (Claerbout, 1992) and (Claerbout, 2000) but I don't trust an ignorant descent. I've had some disappointments doing that (but that was before I learned about scale invariance). How would you try?

REFERENCES

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