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Ground roll and the Radial Trace Transform – revisited

Morgan Brown and Jon Claerbout[1]

ABSTRACT

The Radial Trace Transform (RTT) is an attractive tool for wavefield separation because it lowers the apparent temporal frequency of radial events like ground roll, making it possible to remove them from the data by simple bandpass filtering in the Radial Trace (RT) domain. We discuss two implementations of the RTT. In the first, and better known, the RT domain is well-sampled, and thus suitable for post-filtering, but is prone to interpolation errors. We present an alternate implementation, which is pseudo-unitary in the limit of an infinitely densely sampled RT space, with the side effect that the RT domain has missing data. Using a simple 2-D filter for regularization, we estimate the missing data in the RT domain by least squares optimization, without affecting the invertibility of the RTT. Our implementation suppresses radial noise while preserving signal, including static shifts. Although it runs into trouble when noise is spatially aliased, we show that application of a linear moveout correction prior to processing increases our scheme's effectiveness.

INTRODUCTION

The Radial Trace Transform (RTT), is a simple coordinate transform of normal (t,x) domain seismic gathers; a horizontal deformation, accomplished by the following linear mapping

$$\begin{aligned} t &\rightarrow t \\ x &\rightarrow v = \frac{x}{t} \end{aligned} \tag{1}$$

The radial coordinate is termed "v" because the RTT sorts the data by apparent velocity. Neglecting dispersion effects, ground roll maps to zero temporal frequency in the RT domain. The RTT is not a new concept. Nearly twenty years ago, this coordinate transform was an active subject of research for use in multiple suppression (Taner, 1980), migration (Ottolini, 1982), and even for the subject of this paper, ground roll removal (Claerbout, 1983). Henley's (1999) recent paper reminded the world of the RTT's usefulness in attacking ground roll. Sava and Fomel (2000) use the RTT to compute angle gathers, exploiting the fact that slant-stack in the (t,x) domain is equivalent to computing the RTT in the Fourier domain.

Figure 1 shows 30 radial traces (thick lines) overlain on a rectangular (t,x) mesh. Points along radial traces rarely fall exactly on (t,x) data locations, making the RTT an exercise in

[1] **email:** morgan@sep.stanford.edu,claerbout@stanford.edu

interpolation. In matrix notation, let us denote the RTT of data **d** as follows:

$$\mathbf{R}\mathbf{d} = \mathbf{r} \tag{2}$$

R is generally non-square, and even if square, is usually noninvertible. Often, however, such interpolation operators are nearly unitary: $\mathbf{R}^T\mathbf{R} \approx \mathbf{I}$. We define the interpolation error as follows

$$\mathbf{e} = \left[\mathbf{I} - \mathbf{R}^{\mathbf{T}}\mathbf{R}\right]\mathbf{d}. \tag{3}$$

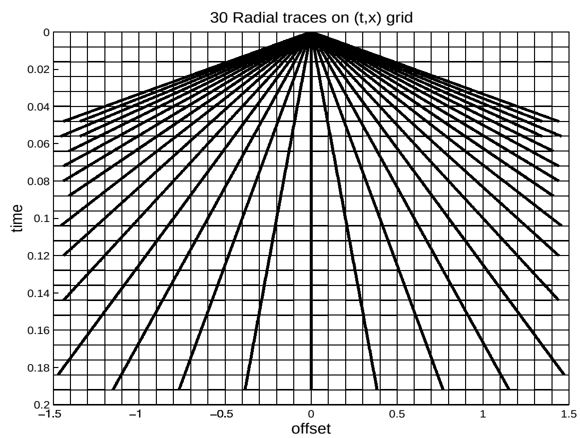

Figure 1: 30 radial traces overlaying a (t,x) grid. morgan1-figure1 [NR]

METHODOLOGY

In this paper, two implementations of the RTT are investigated. Each is illustrated schematically in Figure 2.

1. v**-interpolation method:** For fixed (t,x) bins, linearly interpolate between radial traces.
2. x**-interpolation method:** For fixed (t,v) bins, linearly interpolate between offsets.

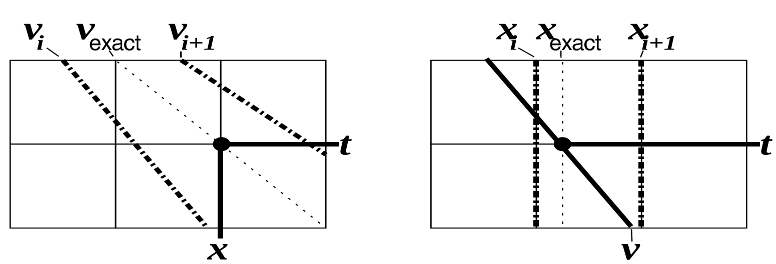

Figure 2: Left: "v-interpolation" implementation. Right: "x-interpolation implementation. morgan1-schem [NR]

In the v-interpolation approach, **R** "pushes" energy — weighted by linear interpolation coefficients — from fixed (t,x) bins into the two radial trace (t,v) bins that bracket them. This implementation will cause interpolation error in regions where many (t,x) bins lie between adjacent radial traces (see Fig. 1). However, the RT domain is nonphysical in the sense that as many radial traces can be used as computer memory permits, so the interpolation error can essentially be driven to zero by sampling densely enough in RT space. Unfortunately, by sampling finely, in many regions of RT space there are pairs of radial traces which bracket no (t, x) bin, so "holes" are introduced into the RT space which inhibit later filtering operations.

In the x-interpolation approach, **R** "pulls" energy into fixed (t, v) bins from the two offset (t, x) bins that bracket them. The interpolation error of this implementation depends only on the trace spacing of the data. The net effect of applying the operator is to smooth laterally, making this implementation dangerous if the data has even small static shifts. At typical trace spacing, **R** is not pseudo-unitary, but since the RT space is guaranteed to have no "holes", this implementation is appropriate for post-filtering operations. Henley (1999) used the x-interpolation approach, as did the author of the SEPlib program `Radial`.

The two implementations of the RTT discussed above illustrate a fundamental, and oft-ignored duality in the analysis of interpolation operators. Intuition supports the x-interpolation method — any given unknown model point is the weighted average of the two known data points which bracket it. In applications like NMO, where averaging is done along the well-sampled time axis, this intuition is sensible, but it breaks down when the averaging is across offsets. The alternate approach (v-interpolation) is less intuitive, as the interpolation is done across radial traces, in the "virtual" space of the model. Since the model space is not constrained by the parameters of data acquisition, it can be sampled as densely as needed to minimize interpolation error. In fact, in the limit of infinitely dense sampling in model space, simple nearest neighbor binning drives interpolation error to zero.

The idea of this paper is to use the v-interpolation RTT to take advantage of its minimal interpolation error, and then handle the problem of "holes" in RT space by missing data estimation. Ideally, the seismic gather contains nearly-horizontal primary reflections and radial events, so the RT space is composed of nearly-vertical ($h(v)$) and nearly-horizontal ($l(t)$) events which a cascade of derivative operators extinguishes:

$$\frac{\partial}{\partial v}\frac{\partial}{\partial t}\,[h(v)+l(t)] = 0 \tag{4}$$

A finite difference stencil approximating $\partial^2/\partial v \partial t$ is

$$\mathbf{a} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \tag{5}$$

Filter **a** is well-suited for the helical least-squares missing data estimation methodology of Claerbout (1999). The missing data problem is driven by two fitting goals: 1) Honor the known data points exactly, and 2) impose any prior knowledge on the unknown model parameters via regularization. Define a known data mask **K**, simply a diagonal operator of the same dimension as **m**; 1 where data is known, 0 otherwise. The prior assumptions about the model are contained in filter **a**. Define **A** as the convolution matrix which applies the filter **a** and combine the two fitting goals into a single regularized optimization problem.

$$\mathbf{Km} = \mathbf{KRd} \tag{6}$$

$$\epsilon \mathbf{Am} \approx 0. \tag{7}$$

Equation (6) forces the model to match the data where the latter is known. Equation (7) minimizes the power of the convolution of **a** with **m**, i.e., optimality is achieved when the unknown model is filled with horizontal and vertical events. ϵ is a user-chosen scale factor.

In this paper, we seek to use the RTT to do noise suppression. Specifically, we map a 2-D seismic gather to RT space, then apply a conservative (6.5 Hz cutoff) highpass filter – call it **B** – to remove the noise, and finally transform back to (t,x) space. In symbols, we can write the estimated signal, $\hat{\mathbf{s}}$, as follows

$$\hat{\mathbf{s}} = \mathbf{R}^T \mathbf{B} \mathbf{R} \mathbf{d}. \tag{8}$$

The corresponding estimate of the noise, $\hat{\mathbf{n}}$, is obtained simply by subtracting the estimated signal from the data:

$$\hat{\mathbf{n}} = \mathbf{d} - \hat{\mathbf{s}} = \left(I - \mathbf{R}^T \mathbf{B} \mathbf{R} \right) d. \tag{9}$$

RESULTS

Figure 3 shows a 2-D shot gather from a multicomponent survey in Venezuela, which exhibits dispersive ground roll with an apparent velocity range of 200-500 m/s. A relatively fine trace spacing of 17 m partially mitigates spatial aliasing. Some weak backscattered energy may be present. Additionally, static shifts of $\pm$ one sample were introduced to the data randomly, in order to emphasize the effect of the RTT on data whose lateral coherency may be degraded. Figure 4 shows the RTT of the data in Figure 3 for both implementations. Each panel contains 300 radial traces. As mentioned by Henley (1999), the origin of the RTT can be placed anywhere. Often in land surveys, the near-zero offset traces are not recorded, meaning that the apparent origin of radial events will lie off the section. The following results were obtained with the the origin of the RTT placed at 0.25 seconds. As expected, the raw v-interpolation panel has many "holes", while the x-interpolation panel does not. The missing data estimation algorithm described above has plausibly filled the missing data, insofar as visual similarity to the x-interpolation panel is a valid measure.

Figure 5 compares the error arising from both implementations of the RTT (equation (3). In the RTT panels shown in Figure 4, each containing 300 radial traces, the error in both of the v-interpolation panels is negligible. This figure gives visual proof of the fact that the missing data infill process does not harm the original data, i.e., that the missing data points in RT space are in the nullspace of $\mathbf{R}^T$. As expected, the x-interpolation error is nonzero, particularly for high-wavenumber events like ground roll and likely backscatter. Additionally, and less obviously, the primary events between 1.75 and 2.5 seconds suffer energy loss and a noticeable lateral smoothing as a result of the transform.

Figure 6 is the estimated signal (equation (8)). Notice that missing data infill has markedly improved the quality of noise suppression obtained by the v-interpolation technique. Still, by visual inspection, we must conclude that the x-interpolation result is the best of the three for noise suppression.

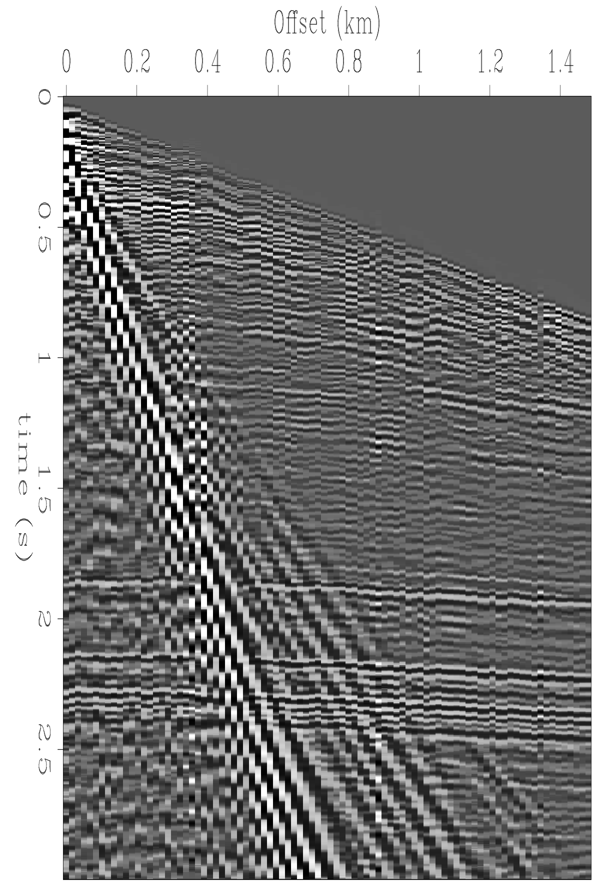

Figure 3: 2-D shot gather. morgan1-hector-dat [ER]

Figure 7 is the estimated noise (equation (9)). First notice that x-interpolation result contains more than just the radial ground roll that the simple physical model accounts for. From Figure 5, we know that the presence of the primary energy between 1.75 and 2.5 seconds and backscattered noise in the x-interpolation panel is due to interpolation errors, and not to the highpass filtering. The non-infilled v-interpolation result contains some energy (either artifacts or primary energy) around 1 second, while the infilled result does not. From the standpoint of signal preservation, the v-interpolation result with missing data infill is the best of the three. Philosophically, by using a pseudo-unitary RTT operator and thus ensuring that the only thing modifying the original data is the bandpass filter, the v-interpolation implementation honors the physics which drives the problem in the first place.

Aliased Data

Ground roll is nearly always spatially aliased, so the relatively unaliased example of Figure 3 is a somewhat unrealistic exception to the practical rule. To inject some realism, we decimated the original 2-D shot gather (Figure 3) by a factor of two in offset, as shown in Figure 8, so that the ground roll is quite aliased. Figure 9 compares the RTT of the decimated data. The results are disappointing. Looking at the v-interpolation without infill panel (top), the human eye can easily interpolate vertically to reconstruct the radial events in RT space. Unfortunately, the

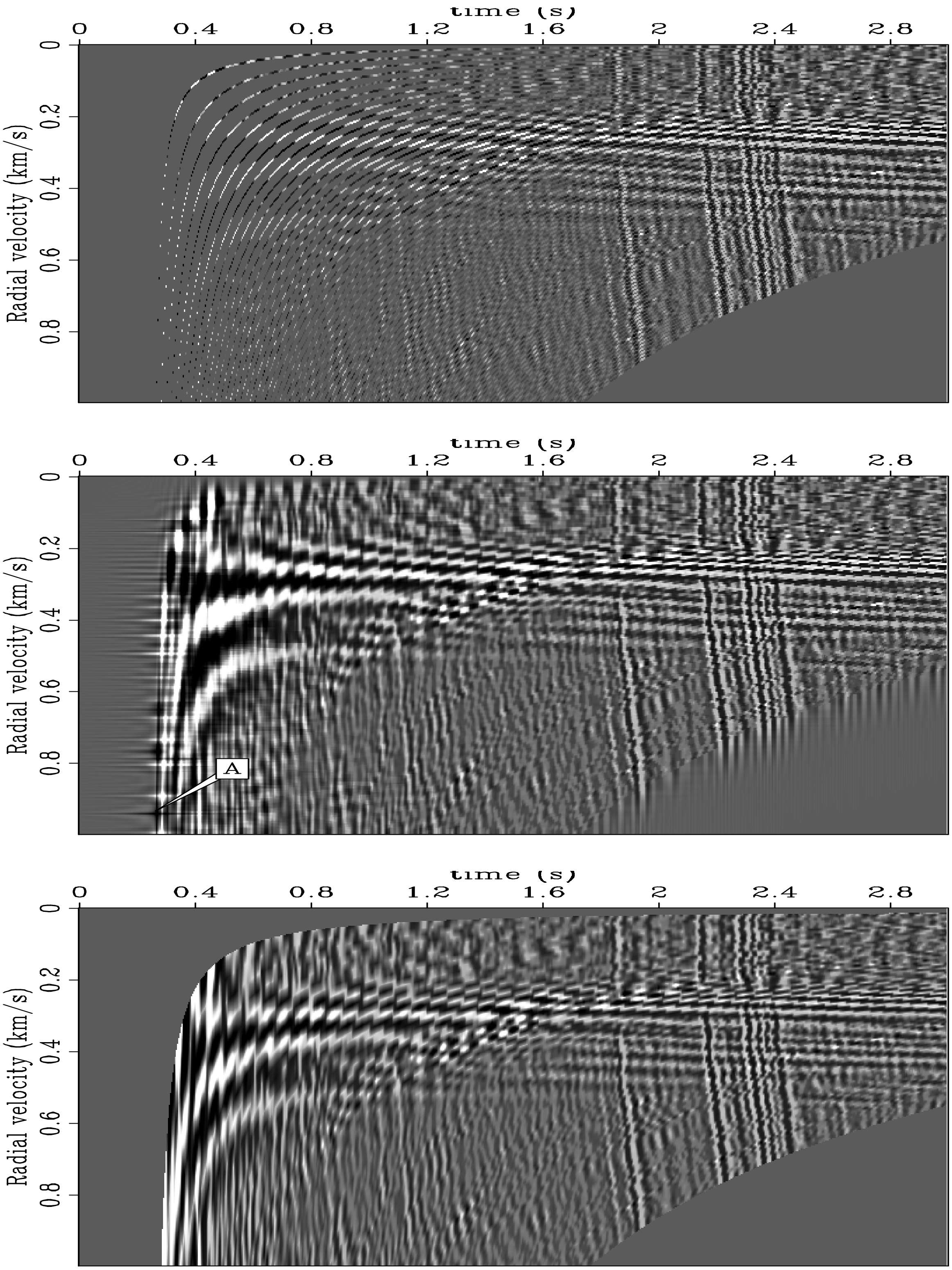

Figure 4: Radial Trace Transform. Top: v-interpolation without infill. Middle: v-interpolation with infill. "A" points to an example of the "+"-shaped impulse response of the missing data filter of equation (7). Bottom: x-interpolation. morgan1-hector-radial-comp [ER,M]

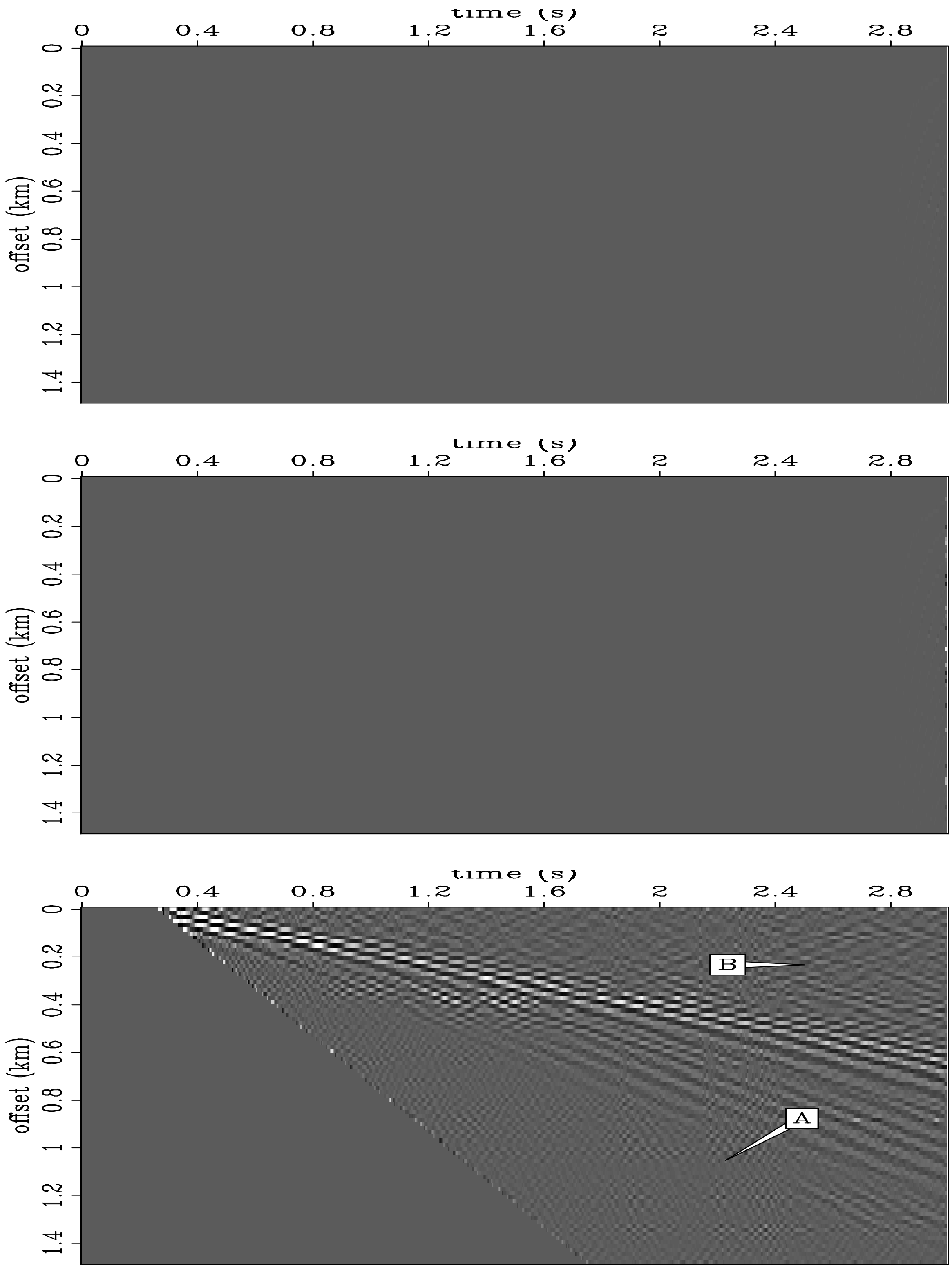

Figure 5: Interpolation error. Top: v-interpolation error without infill. Middle: v-interpolation error with infill. Bottom: x-interpolation error. “A” points to lost energy from the primary events around 2 seconds. “B” points to removed backscattered noise. morgan1-hector-raderr-comp [ER,M]

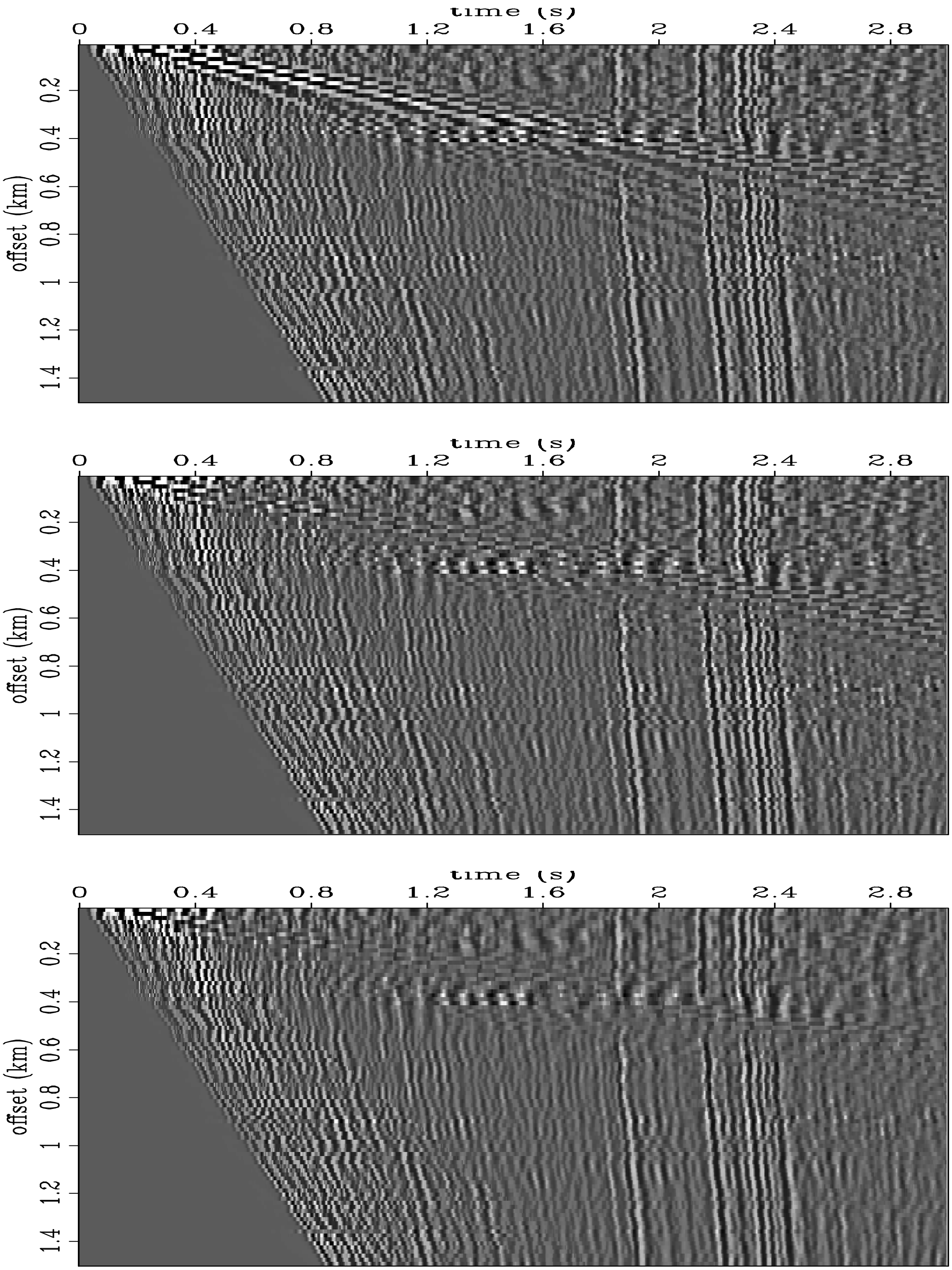

Figure 6: Estimated signal. Top: v-interpolation without infill. Middle: v-interpolation with infill. Bottom: x-interpolation. morgan1-hector-estsig [ER,M]

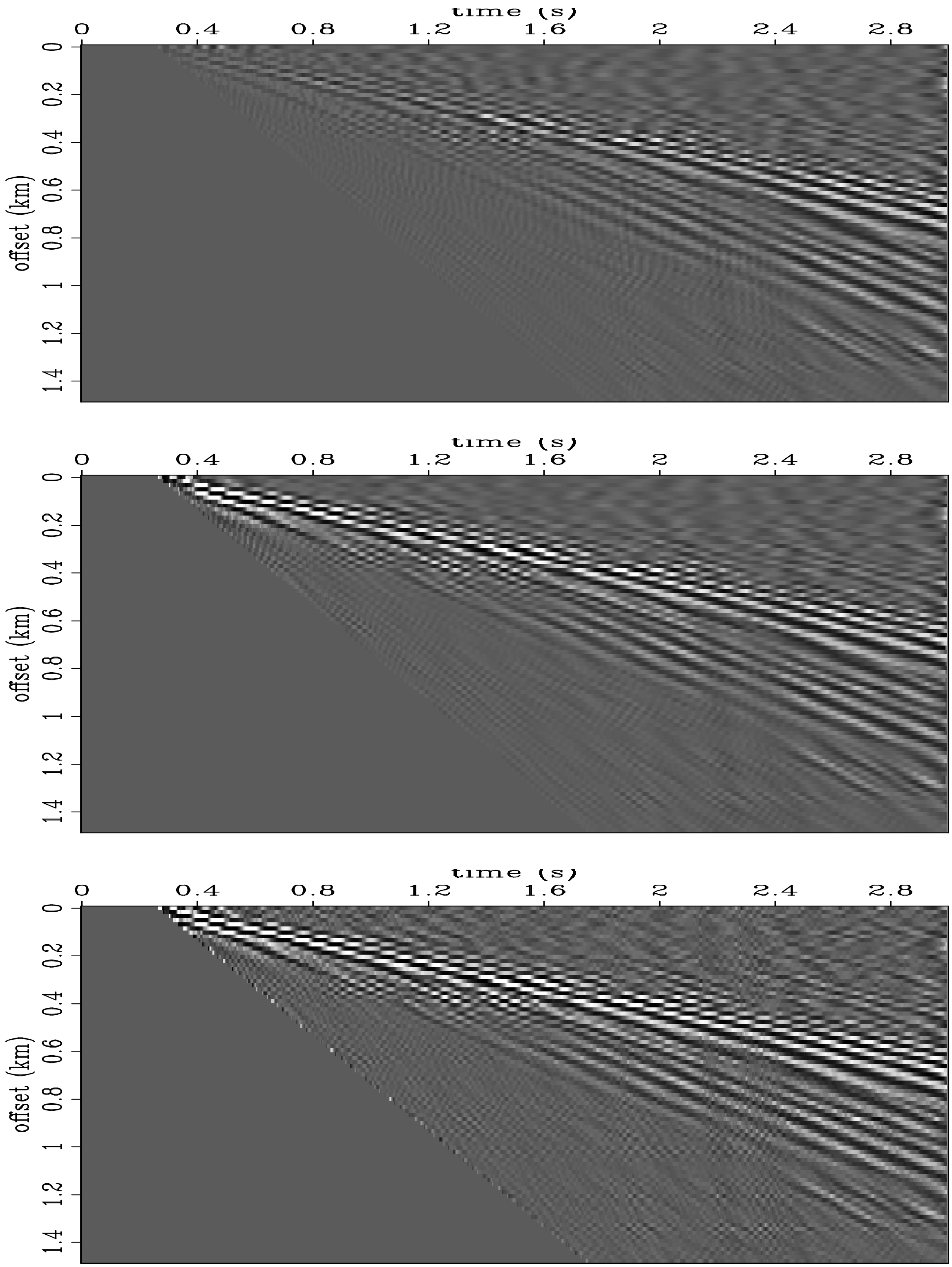

Figure 7: Estimated noise. Panels defined as in Figure 6. morgan1-hector-estnoiz [ER,M]

v-interpolation panel with infill does not have the desired vertical coherence. In fact, it would seem that the central premise motivating this paper — that the RTT maps ground roll to zero temporal frequency — is violated. Figures 10 and 11 are analogous to Figures 6 and 7 — they are the estimates of signal and noise, respectively. All implementations (v-interpolation with and without infill, and x-interpolation) do an relatively poor job of noise suppression.

A simple way to dealias linear ground roll is to apply a linear moveout (LMO) correction. Figure 12 shows the result of applying a 1.5 km/sec LMO correction to the decimated data of Figure 8. The ground roll is no longer spatially aliased, but the primaries are also no longer "flat", as they were originally. As a result, interpolation errors for the x-interpolation RTT will increase. Figure 13 compares the RTT panels for the decimated/LMO'ed data. The ground roll now occupies a higher effective velocity band, and more importantly, is much closer to zero temporal frequency than in Figure 9. The noise suppression achieved (Figure 14) is better than the case in which LMO was not used (Figure 10). As expected, and mentioned above, the x-interpolation RTT leads to severe losses of signal energy, quite a bit more severe than either of the two v-interpolation implementations, as can be seen in Figure 15. Unfortunately, both v-interpolation implementations seem to suffer some small signal losses, which suggests that LMO may actually be "aliasing" the primaries by mapping them to low temporal frequency in the RT domain.

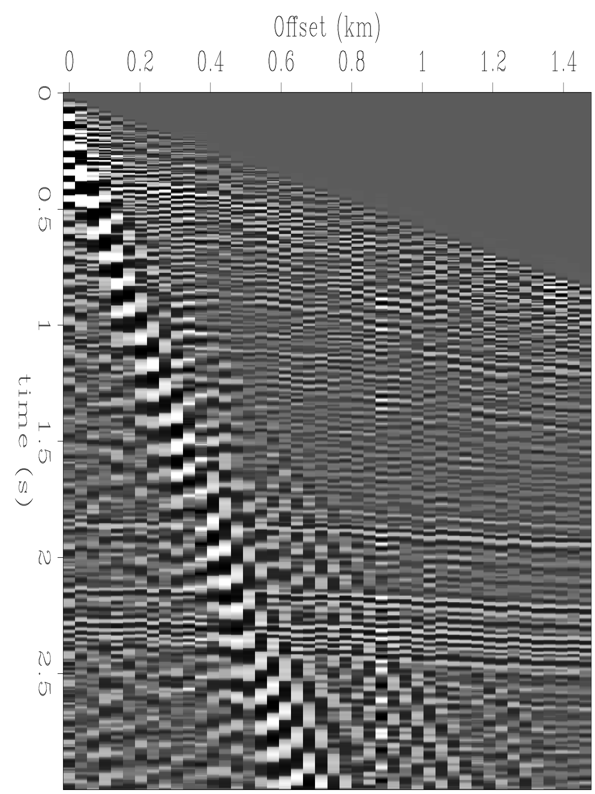

Figure 8: Same 2-D shot gather as Figure 3, only decimated by a factor of two in offset. morgan1-hectoralias-dat [ER]

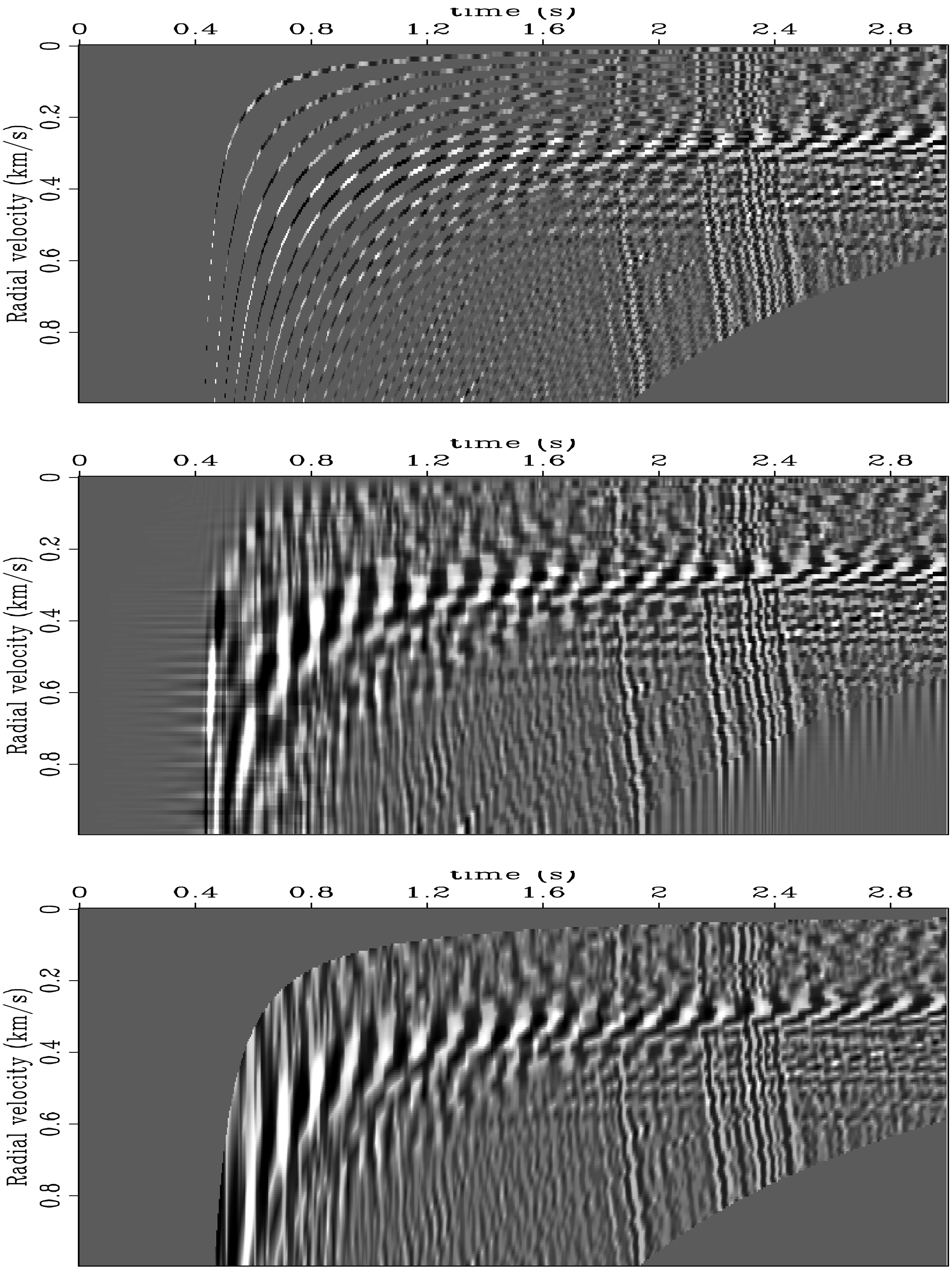

Figure 9: Top: v-interpolation without infill. Middle: v-interpolation with infill. Bottom: x-interpolation. morgan1-hectoralias-radial-comp [ER,M]

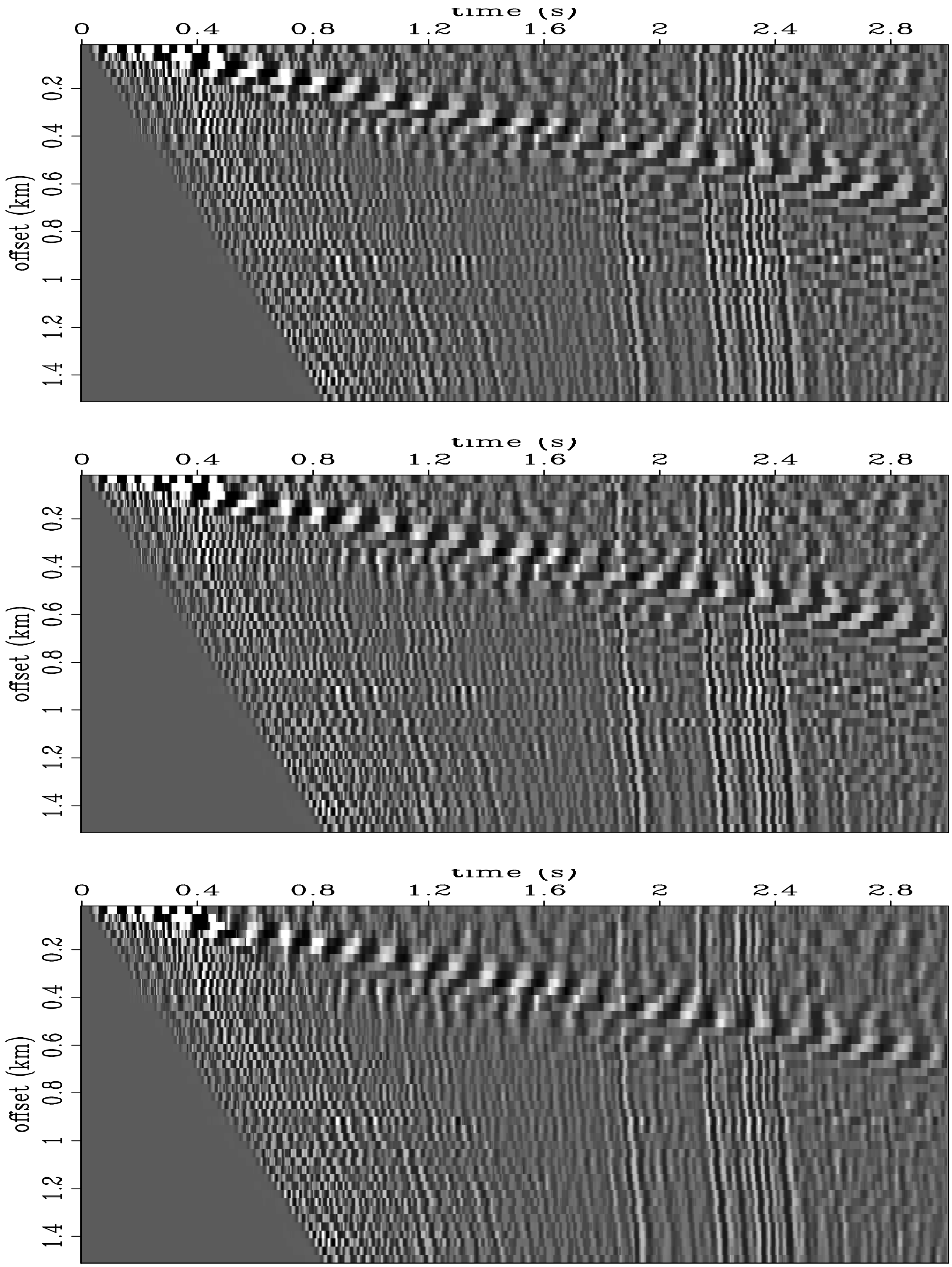

Figure 10: Estimated signal. Top: v-interpolation without infill. Middle: v-interpolation with infill. Bottom: x-interpolation. morgan1-hectoralias-estsig [ER,M]

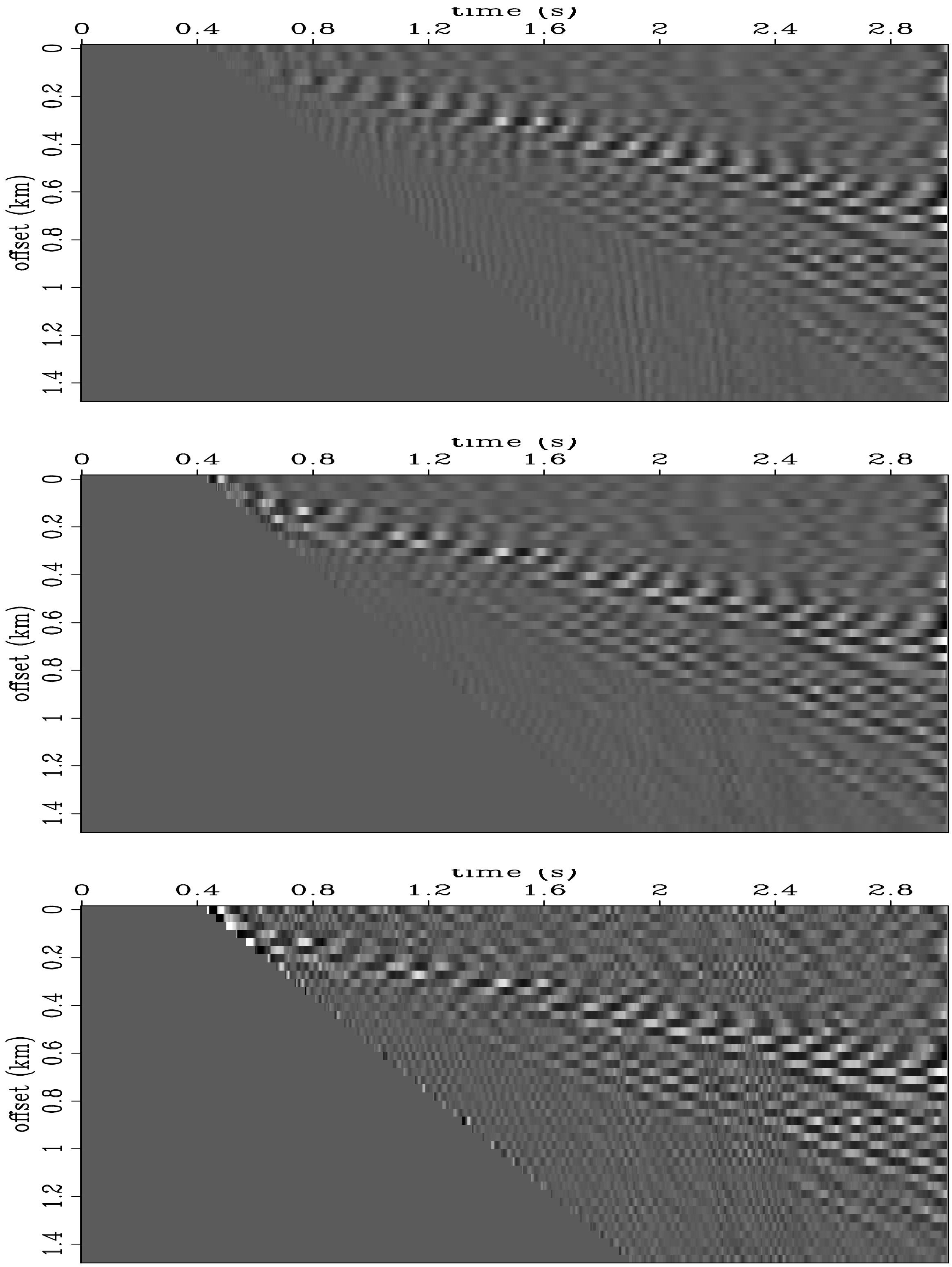

Figure 11: Estimated noise. Panels defined as in Figure 10. morgan1-hectoralias-estnoiz [ER,M]

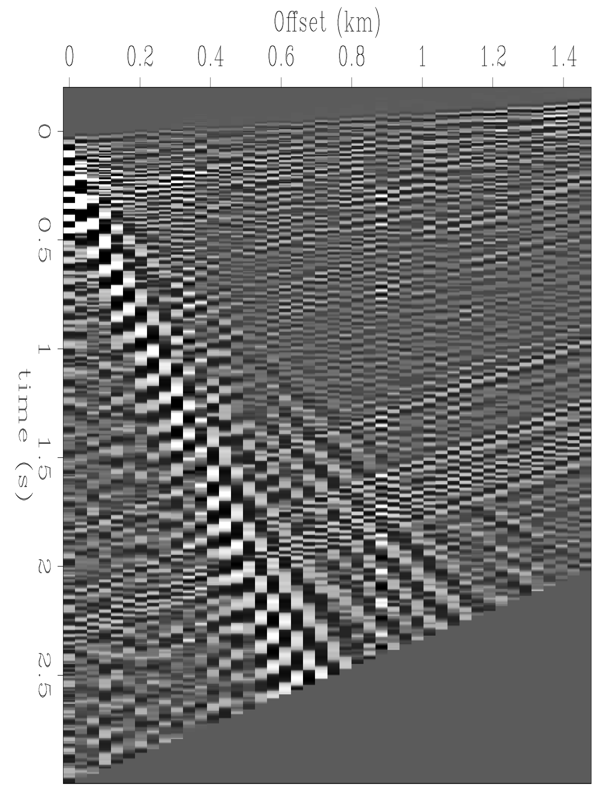

Figure 12: Decimated 2-D shot gather (Figure 8), after 1.0 km/sec linear moveout correction. morgan1-hectorlmo-dat [ER]

CONCLUSIONS

Our implementation of the RTT effectively suppresses unaliased radial noise while preserving signal, including static shifts. In its current form, our missing data interpolation technique did a relatively poor job of coherently interpolating spatially aliased radial noise events to vertical events in the RT domain, although we have high hopes that success is only a small conceptual leap away. To combat aliasing, we applied an LMO correction to the data to dealias the noise events with an LMO correction, leading to improved noise suppression, at the cost of some lost signal.

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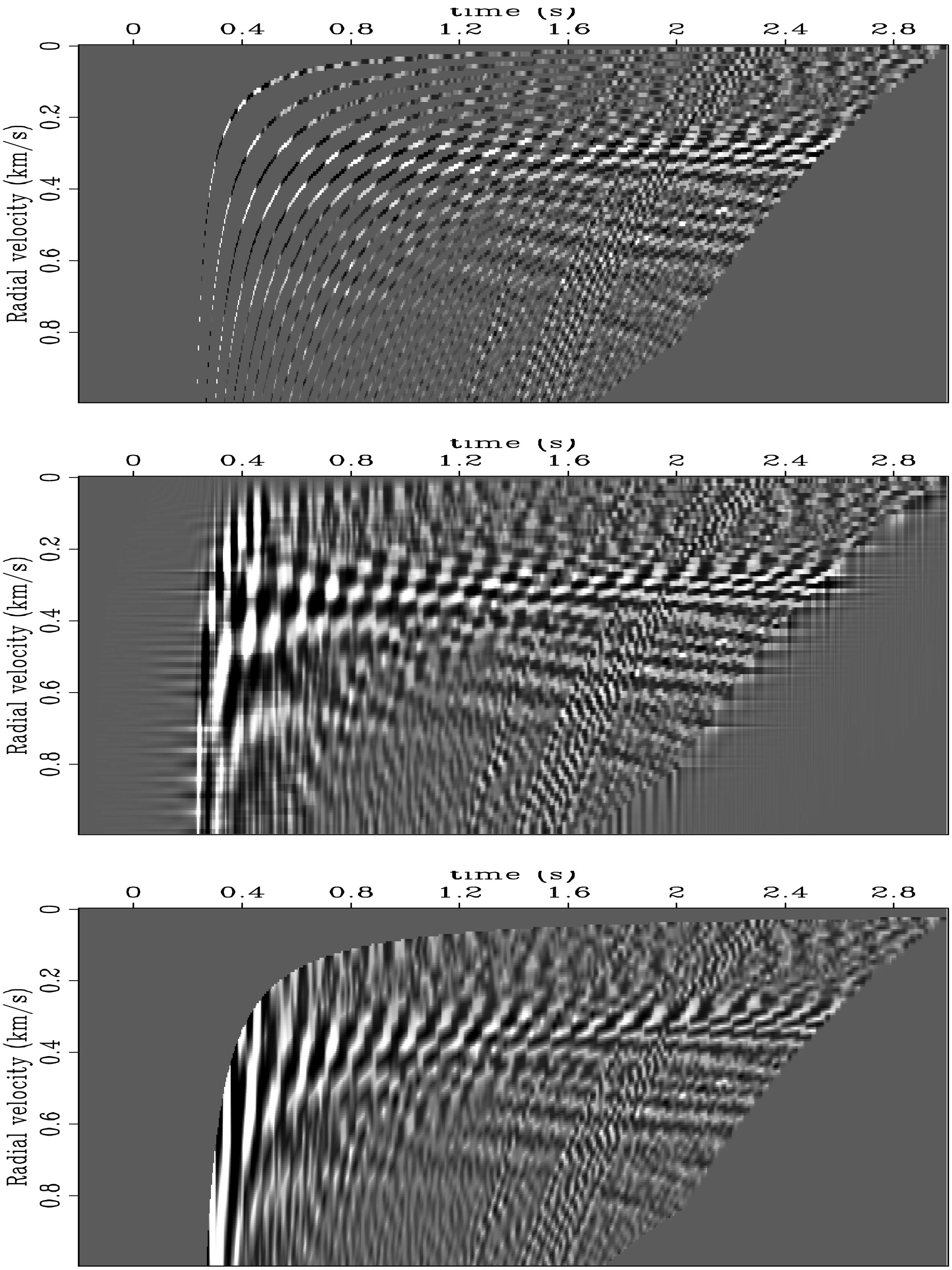

Figure 13: Top: v-interpolation without infill. Middle: v-interpolation with infill. Bottom: x-interpolation. morgan1-hectorlmo-radial-comp [ER,M]

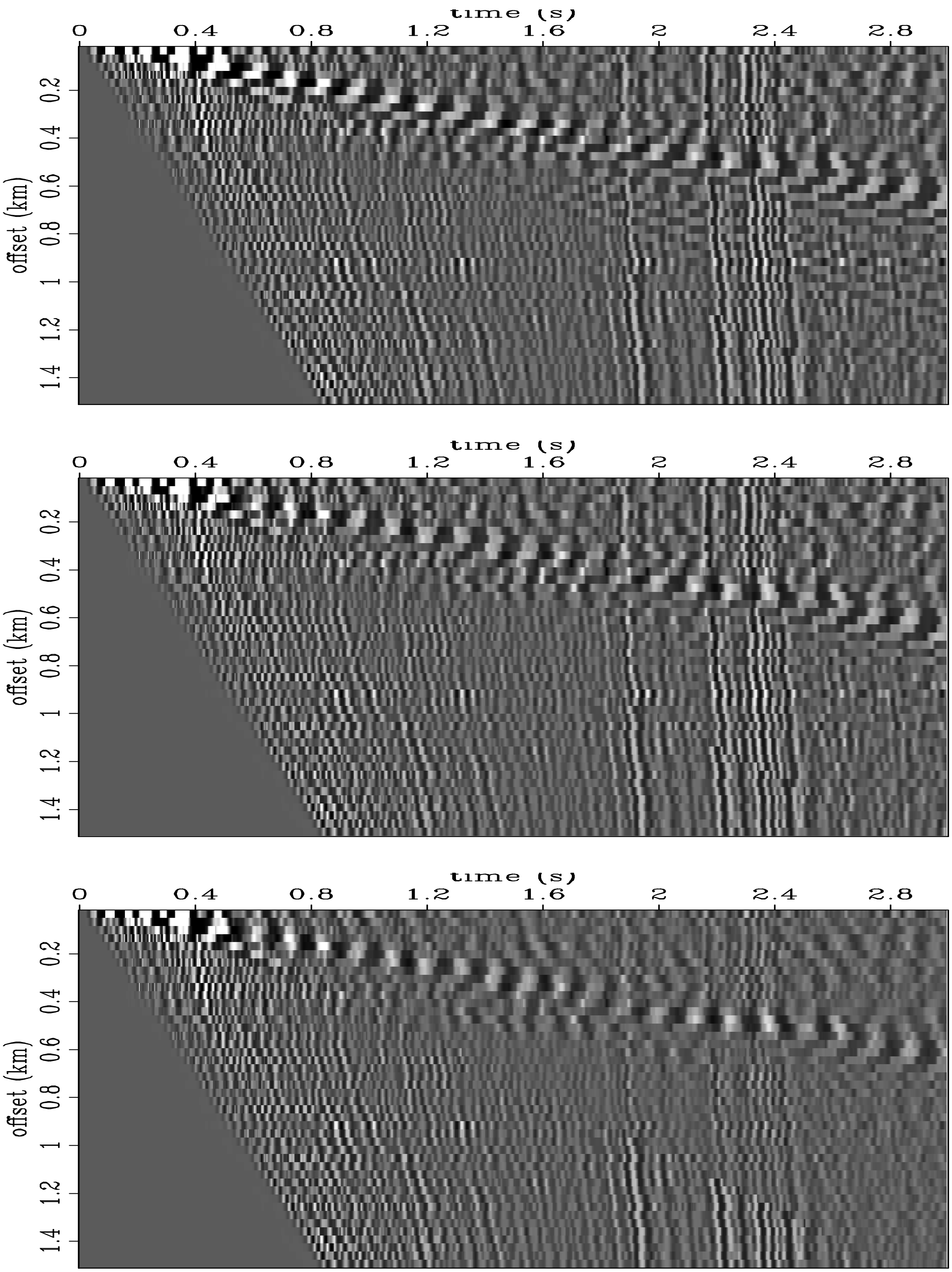

Figure 14: Estimated signal. Top: v-interpolation without infill. Middle: v-interpolation with infill. Bottom: x-interpolation. morgan1-hectorlmo-lmo-estsig [ER,M]

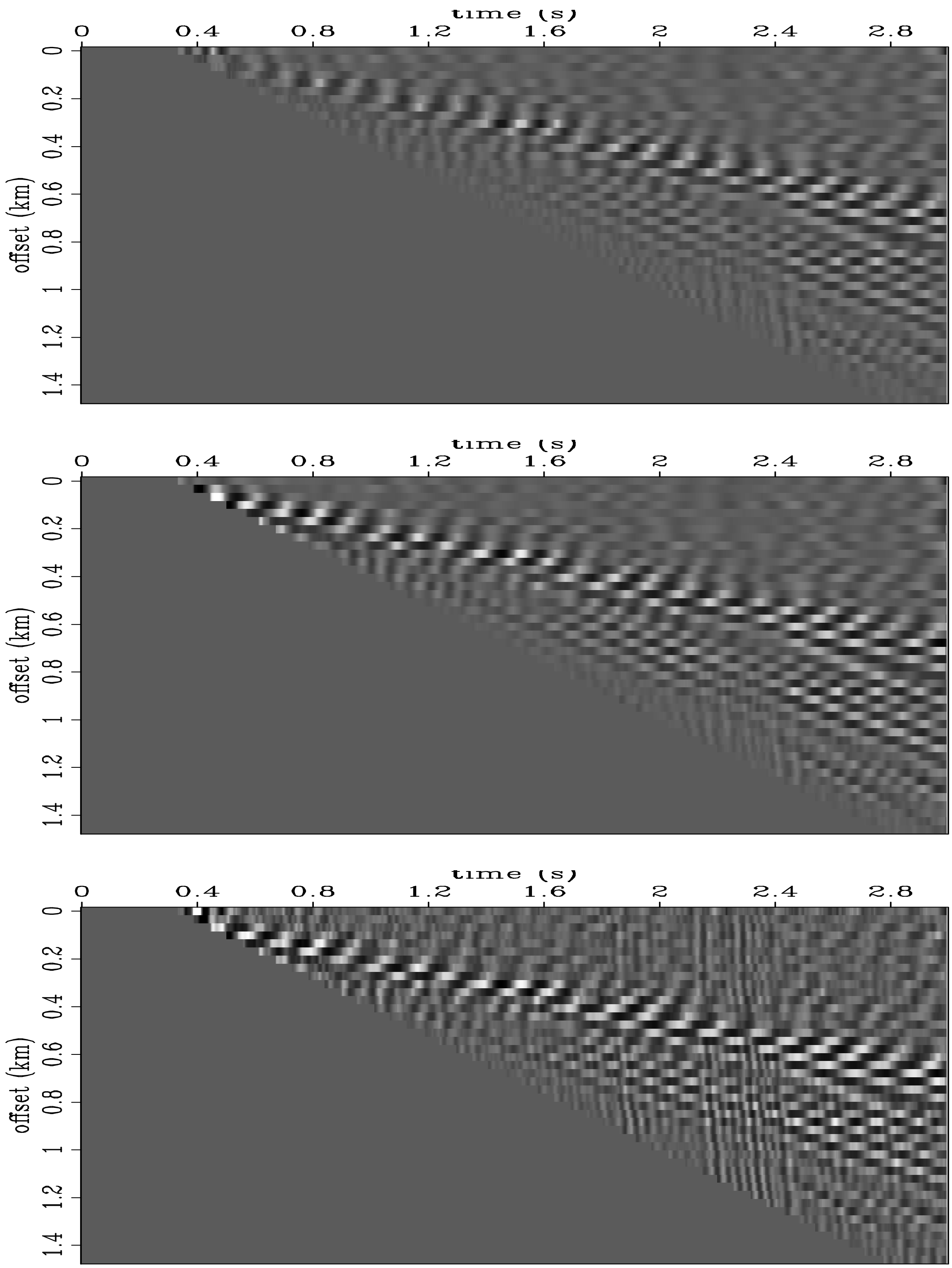

Figure 15: Estimated noise. Panels defined as in Figure 14. morgan1-hectorlmo-lmo-estnoiz [ER,M]

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