Stanford Exploration Project, Report 103, April 27, 2000, pages 205–249

Stanford Exploration Project, Report 103, April 27, 2000, pages 205–249

Seismic data interpolation with the offset continuation equation

Sergey Fomel[1]

ABSTRACT

I propose a finite-difference offset continuation filter for interpolating seismic reflection data. The filter is constructed from the offset continuation differential equation and is applied on frequency slices in the log-stretch frequency domain. Synthetic data tests produce encouraging results: nearly perfect interpolation of a constant-velocity dataset with a complex reflector model and reasonably good interpolation of the Marmousi dataset.

INTRODUCTION

As early as 20 years ago, researchers at SEP considered data interpolation as one of the most important problems of seismic data processing (Claerbout, 1980, 1981; Thorson, 1981). In 2-D exploration, the interpolation problem arises because of missing near and far offsets, spatial aliasing and occasional bad traces. In 3-D exploration, the importance of this problem increases dramatically because 3-D acquisition almost never provides a complete regular coverage in both midpoint and offset coordinates (Biondi, 1999). Data regularization in 3-D can solve the problem of Kirchoff migration artifacts (Gardner and Canning, 1994), prepare the data for common-azimuth imaging (Biondi and Palacharla, 1996), or provide the spatial coverage required for 3-D multiple elimination (van Dedem and Verschuur, 1998).

Claerbout (1992, 1999) formulates the following general principle of missing data interpolation:

> A method for restoring missing data is to ensure that the restored data, after specified filtering, has minimum energy.

How can one specify an appropriate filtering for a given interpolation problem? Smooth surfaces are conveniently interpolated with Laplacian filtering (Briggs, 1974; Fomel, 2000). Steering filters help us interpolate data with predefined dip fields (Clapp et al., 1997; Fomel et al., 1997; Fomel, 1999). Prediction-error filters in time-space or frequency-space domain successfully interpolate data composed of distinctive plane waves (Spitz, 1991; Claerbout, 1999). Because prestack seismic data is not stationary in the offset direction, non-stationary prediction-error filters need to be estimated, which leads to an accurate but relatively expensive method with many adjustable parameters (Crawley, 1999; Clapp et al., 1999).

[1]**email:** sergey@sep.stanford.edu

A simple model for reflection seismic data is a set of hyperbolic events on a common midpoint gather. The simplest filter for this model is the first derivative in the offset direction applied after the normal moveout correction.[2] Going one step beyond this simple approximation requires taking the dip moveout (DMO) effect into account (Deregowski, 1986). The DMO effect is fully incorporated in the offset continuation differential equation (Fomel, 1994, 1995a).[3]

Offset continuation is a process of seismic data transformation between different offsets (Deregowski and Rocca, 1981; Bolondi et al., 1982; Salvador and Savelli, 1982). Different types of DMO operators (Hale, 1995) can be regarded as continuation to zero offset and derived as solutions of an initial-value problem with the revised offset continuation equation (Fomel, 1995b). Within a constant-velocity assumption, this equation not only provides correct traveltimes on the continued sections, but also correctly transforms the corresponding wave amplitudes (Fomel and Bleistein, 1996). Integral offset continuation operators have been derived independently by Stovas and Fomel (1996), Bagaini and Spagnolini (1996), and Chemingui and Biondi (1994). The 3-D analog is known as azimuth moveout (AMO) (Biondi et al., 1998). In the shot-record domain, integral offset continuation transforms to shot continuation (Schwab, 1993; Bagaini and Spagnolini, 1993; Spagnolini and Opreni, 1996). Integral continuation operators can be applied directly for missing data interpolation and regularization (Bagaini et al., 1994; Mazzucchelli and Rocca, 1999). However, they don't behave well for continuation at small distances in the offset space because of limited integration apertures and, therefore, are not well suited for interpolating neighboring records. Additionally, as all integral (Kirchoff-type) operators they suffer from irregularities in the input geometry. The latter problem is addressed by accurate but expensive inversion to common offset (Chemingui, 1999).

In this paper, I propose an application of offset continuation in the form of a finite-difference filter for Claerbout's method of missing data interpolation. The filter is designed in the log-stretch frequency domain, where each frequency slice can be interpolated independently. Small filter size and easy parallelization among different frequencies assure a high efficiency of the proposed approach. Although the offset continuation filter lacks the predictive power of non-stationary prediction-error filters, it is much simpler to handle and serves as a good *a priori* guess of an interpolative filter for seismic reflection data. I test the proposed method by interpolating randomly missing traces in a constant-velocity synthetic and by restoring near offsets and intermediate shot gathers in the Marmousi synthetic dataset. These early tests produce encouraging results. In the final section of the paper, I discuss possible strategies for improving the method.

---

[2]A similar filter appears in velocity estimation with the differential semblance method (Symes and Carazzone, 1991; Symes, 1999).

[3]To the author's knowledge, the first derivation of the revised offset continuation equation was done by Joseph Higginbotham of Texaco in 1989. Unfortunately, Higginbotham's derivation never appeared in open literature.

PROBLEM FORMULATION

If $\mathbf{D}$ is a regularization operator, and $\mathbf{m}$ is the estimated model, then Claerbout's interpolation method amounts to minimizing the power of $\mathbf{Dm}$ ($\mathbf{m}^T\mathbf{D}^T\mathbf{Dm}$) under the constraint

$$\mathbf{Km} = \mathbf{m_k} \; , \tag{1}$$

where $\mathbf{m_k}$ stands for the known data values, and $\mathbf{K}$ is a diagonal matrix with 1s at the known data locations and zeros elsewhere. It is easy to implement a constraint of the form (1) in an iterative conjugate-gradient scheme by simply disallowing the iterative process to update model parameters at the known data locations (Claerbout, 1999).

The operator $\mathbf{D}$ can be considered as a differential equation that we assume the model to satisfy. If $\mathbf{D}$ is able to remove all correlated components from the model and produce white Gaussian noise in the output, then $\mathbf{D}^T\mathbf{D}$ is essentially equivalent to the inverse covariance matrix of the model, which appears in the statistical formulation of least-squares estimation (Tarantola, 1987).

In this paper, I propose to use the offset continuation equation (Fomel, 1995a) for the operator $\mathbf{D}$. Under certain assumptions, this equation is indeed the one that prestack seismic reflection data can be presumed to satisfy. The equation has the following form:

$$h\left(\frac{\partial^2 P}{\partial y^2} - \frac{\partial^2 P}{\partial h^2}\right) = t_n \, \frac{\partial^2 P}{\partial t_n \, \partial h} \; , \tag{2}$$

where $P(t_n, h, x)$ is the prestack seismic data after the normal moveout correction (NMO), t_n stands for the time coordinate after NMO, h is the half-offset, and y is the midpoint. Offset continuation has the following properties:

- Equation (2) describes an artificial process of prestack data transformation in the offset direction. It belongs to the class of linear hyperbolic equations. Therefore, the described process is a wave-type process. Half-offset h serves as a continuation variable (analogous to time in the wave equation).
- Under a constant-velocity assumption, equation (2) provides correct reflection traveltimes and amplitudes at the continued sections. The amplitudes are correct in the sense that the geometrical spreading effects are properly transformed independently from the shape of the reflector. This fact has been confirmed both by the ray method approach (Fomel, 1995a) and by the Kirchhoff modeling approach (Fomel and Bleistein, 1996; Fomel et al., 1996).
- Dip moveout (DMO) (Hale, 1995) can be regarded as a particular case of offset continuation to zero offset (Deregowski and Rocca, 1981). As shown in my earlier paper (Fomel, 1995b), different known forms of DMO operators can be obtained as solutions of a special initial-value problem on equation (2).
- To describe offset continuation for 3-D data, we need a pair of equations such as (2), acting in two orthogonal projections. This fact follows from the analysis of the azimuth moveout operator (Fomel and Biondi, 1995; Biondi et al., 1998).

- A particularly efficient implementation of offset continuation results from a log-stretch transform of the time coordinate (Bolondi et al., 1982), followed by a Fourier transform of the stretched time axis. After these transforms, equation (2) takes the form

$$h\left(\frac{\partial^2 \tilde{P}}{\partial y^2} - \frac{\partial^2 \tilde{P}}{\partial h^2}\right) - i\,\Omega\,\frac{\partial \tilde{P}}{\partial h} = 0\,, \tag{3}$$

  where Ω is the corresponding frequency, and $\tilde{P}(\Omega, h, x)$ is the transformed data (Fomel, 1995b). As in other F-X methods, equation (3) can be applied independently and in parallel on different frequency slices.

I propose to adopt a finite-difference form of the differential operator (3) for the regularization operator $\mathbf{D}$. A simple analysis of equation (3) shows that at small frequencies, the operator is dominated by the first term. The form $\frac{\partial^2 P}{\partial y^2} - \frac{\partial^2 P}{\partial h^2}$ is equivalent to the second mixed derivative in the source and receiver coordinates. Therefore, at low frequencies, the offset waves propagate in the source and receiver directions. At high frequencies, the second term in (3) becomes dominating, and the entire method becomes equivalent to the trivial linear interpolation in offset. The interpolation pattern is more complicated at intermediate frequencies.

TESTS

I started numerical testing of the proposed technique first on the constant velocity synthetic, where all the assumptions behind the offset continuation equation are valid. Encouraged by the results, I proceeded to tests on the Marmousi synthetic dataset, which is associated with a highly inhomogeneous velocity model.

Constant-velocity synthetic

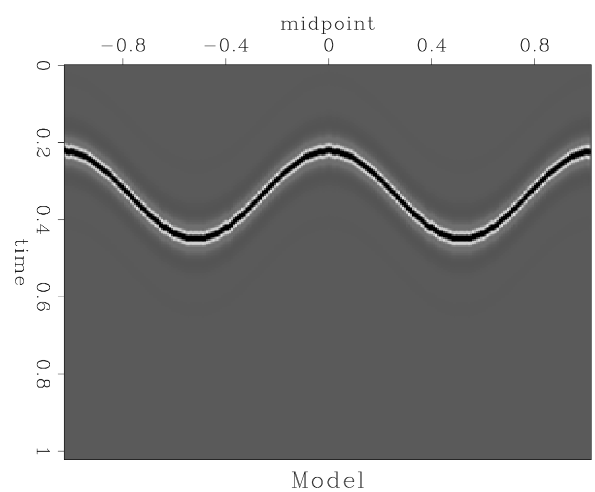

Figure 1: Reflector model for the constant-velocity test sergey2-cup [ER]

A sinusoidal reflector shown in Figure 1 creates complicated reflection data, shown in Figure 2 before and after the normal-moveout correction. The syncline parts of the reflector lead to traveltime triplications at sufficiently large offset. A mixture of different dips from the triplications would make it extremely difficult to interpolate the data in individual common-offset gathers, such as those shown in Figure 2. The plots of time slices after NMO (Figure 3) clearly show that the data are also non-stationary in the offset direction. Therefore, a simple offset interpolation scheme would also fail.

To set up an interpolation experiment, I randomly removed half of the traces in the original data and attempted to reconstruct them. Figure 4 shows the reconstruction process on individual frequency slices. Despite the complex and non-stationary character of the reflection events in the frequency domain, the offset continuation equation is able to reconstruct them quite accurately from the decimated data.

Figure 5 shows the input and the result of interpolation after transforming it back to the time domain. A comparison of the interpolation result with the ideal output (Figure 2) shows that the reflection data are nearly perfectly interpolated even in the complex triplication zones.

The constant-velocity test results allow us to conclude that, when all the assumptions of the offset continuation theory are met, we can easily accomplish an accurate interpolation. In the next subsection, I deal with the more complicated case of Marmousi.

Marmousi synthetic

The famous Marmousi synthetic was modeled over a very complicated velocity and reflector structure (Versteeg, 1994). The dataset has been used in numerous studies of various seismic processing and imaging techniques. Figure 6 shows the near and far common-offset gathers from the Marmousi dataset. The structure of the reflection events is extremely complex and contains multiple triplications and diffractions.

To test the proposed interpolation method, I set the goal of interpolating the missing near offsets in the Marmousi dataset. Additionally, I attempted to interpolate intermediate shot gathers so that all common-midpoint gathers receive the same offset fold. In the original dataset, both receiver and shot spacing are equal to 25 meters, which creates a checkerboard pattern in the offset-midpoint plane. This acquisition pattern is typical for 2-D seismic surveys.

Interpolation of near offsets can reduce imaging artifacts in different migration methods. Ji (1995) used near-offset interpolation for accurate wavefront-synthesis migration of the Marmousi dataset. He developed an interpolation technique based on the hyperbolic Radon transform inversion. Ji's method produces fairly good results, but is significantly more expensive that the offset continuation approach explored in this paper.

Figure 7 shows the input and interpolated Marmousi data in the log-stretch frequency domain. We can see that the data in the frequency slices also have a very complicated structure. Nevertheless, the offset continuation method is able to reconstruct the missing portions of the data in a visually pleasing way. The data are not extrapolated off the sides of the common-offset gathers. This behavior is physically reasonable, because such an extrapolation would involve assumptions about unilluminated portions of the subsurface.

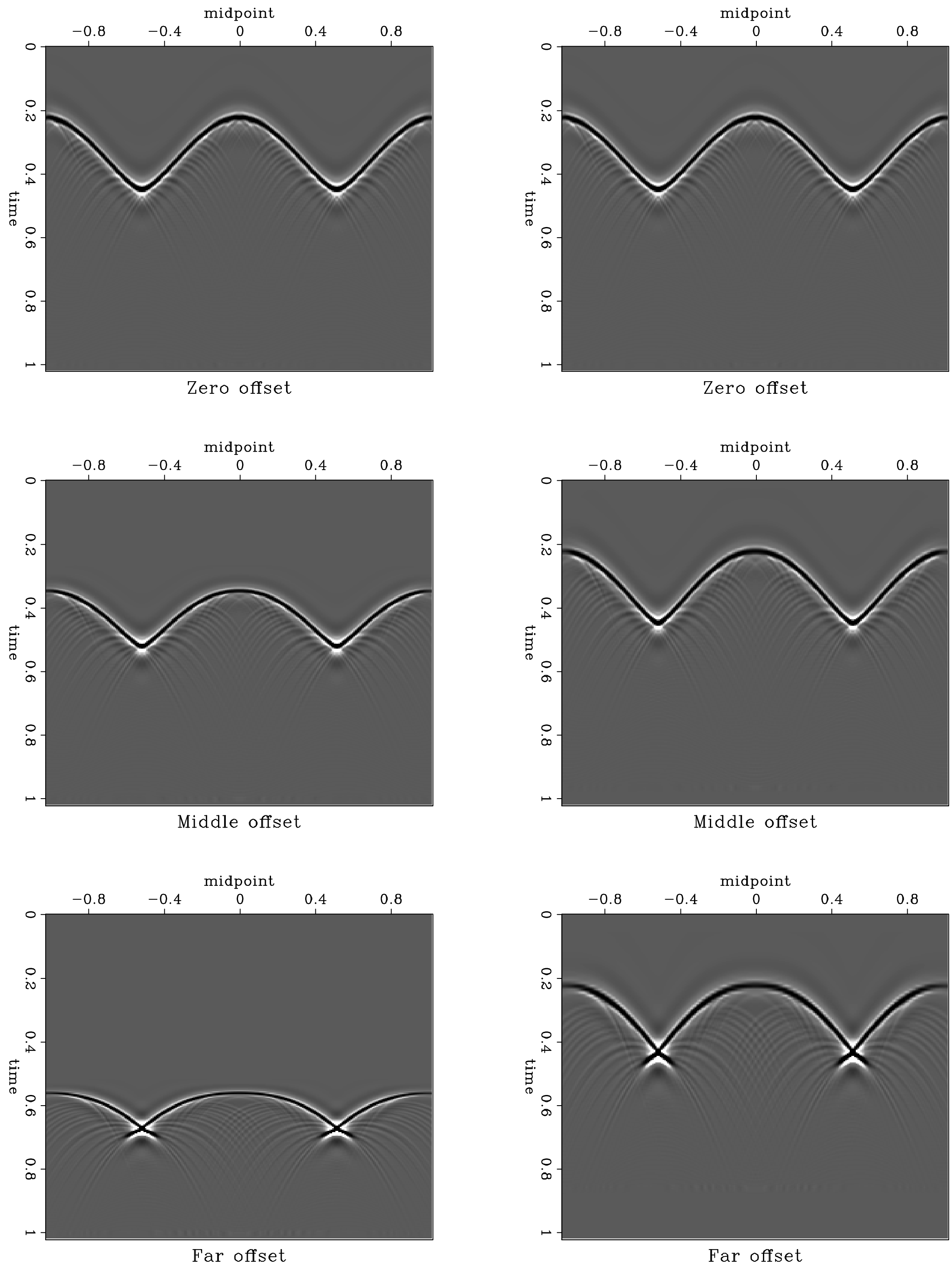

Figure 2: Prestack common-offset gathers for the constant-velocity test. Left: before NMO. Right: after NMO. Top, center, and bottom plots correspond to different offsets. sergey2-data [ER,M]

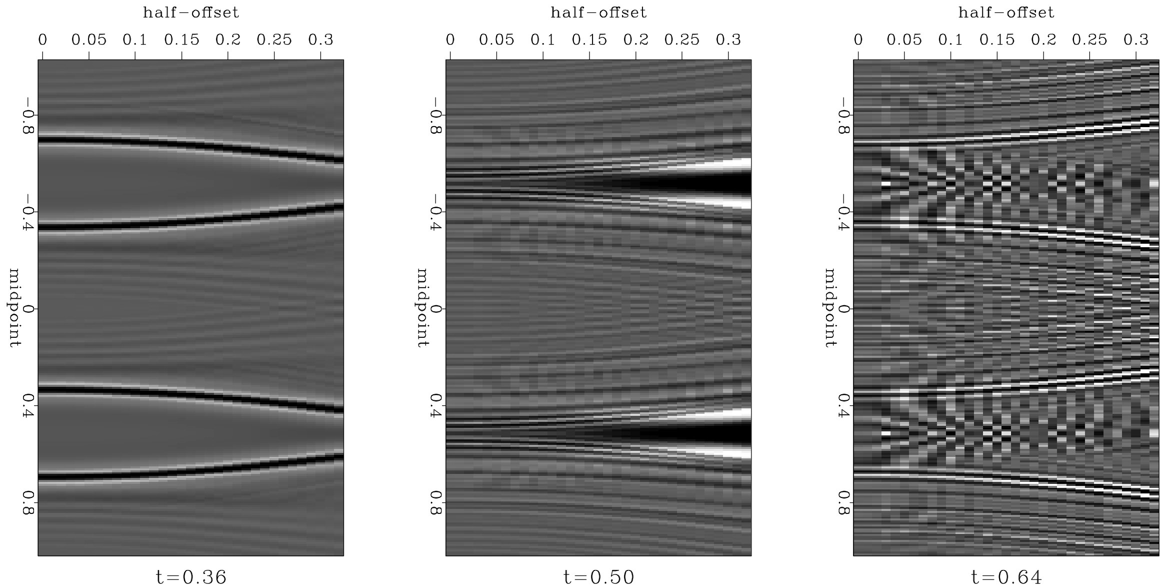

Figure 3: Time slices of the prestack data at different times (after NMO). sergey2-tslice [ER]

Figure 8 shows one of the shot gathers obtained after transforming the data back into time domain and resorting them into shot gathers. The positive offset part of the shot gather was reconstructed from a common receiver gather by using reciprocity. Comparing the top and bottom plots, we can see that many different events in the original shot gather are nicely extended into near offsets by the interpolation procedure.

In addition to interpolating near offsets, I have reconstructed the intermediate shot gathers in order to equalize the CMP fold. Figure 9 shows an example of an artificial shot gather created by such a reconstruction. An sample CMP gather before and after interpolation is shown in Figure 10. Examining the bottom part of the section, we can see that that the interpolation process tends to put more continuity in the near offsets than could be expected from the data. In other places, the interpolation succeeds in producing a visually pleasant result.

DISCUSSION

Early tests with synthetic models demonstrate that the offset continuation equation is a useful and efficient regularization operator for interpolating seismic reflection data. I plan to perform more tests in order to evaluate the performance of this method on real data. An extension to 3-D data is simple in theory, but it will require several modifications in the computational framework.

In the range of possible interpolation methods (Mazzucchelli et al., 1998), the offset continuation approach clearly stands on the more efficient side. The efficiency is achieved both by the small size of the finite-difference filter and by the method's ability to decompose and parallelize the method across different frequencies. Part of the efficiency gain could probably

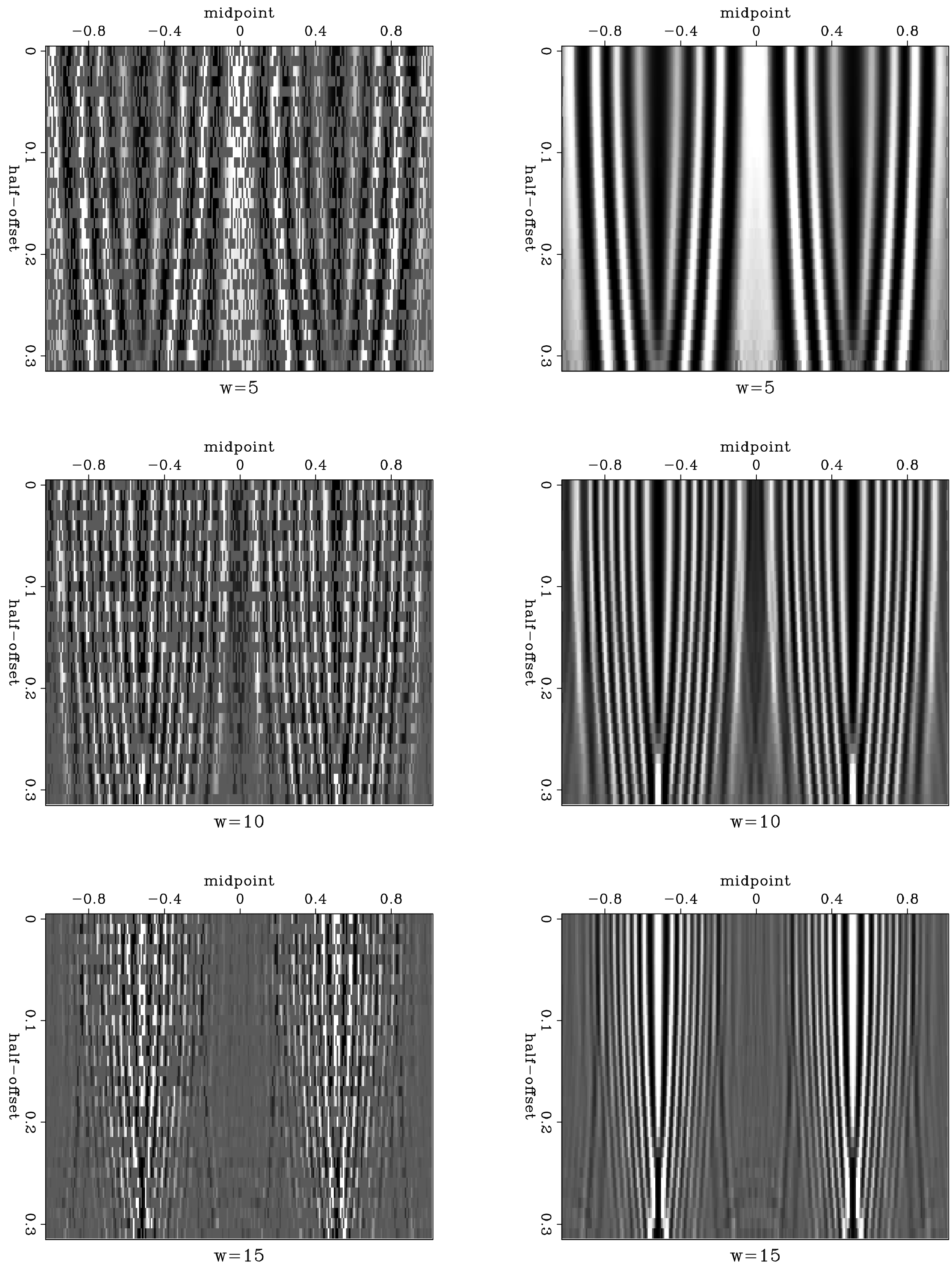

Figure 4: Interpolation in frequency slices. Left: input data 50% of the traces are randomly removed). Right: interpolation output. Top, bottom, and middle plots correspond to different frequencies. Real parts of the complex-valued data are shown. sergey2-fslice [ER,M]

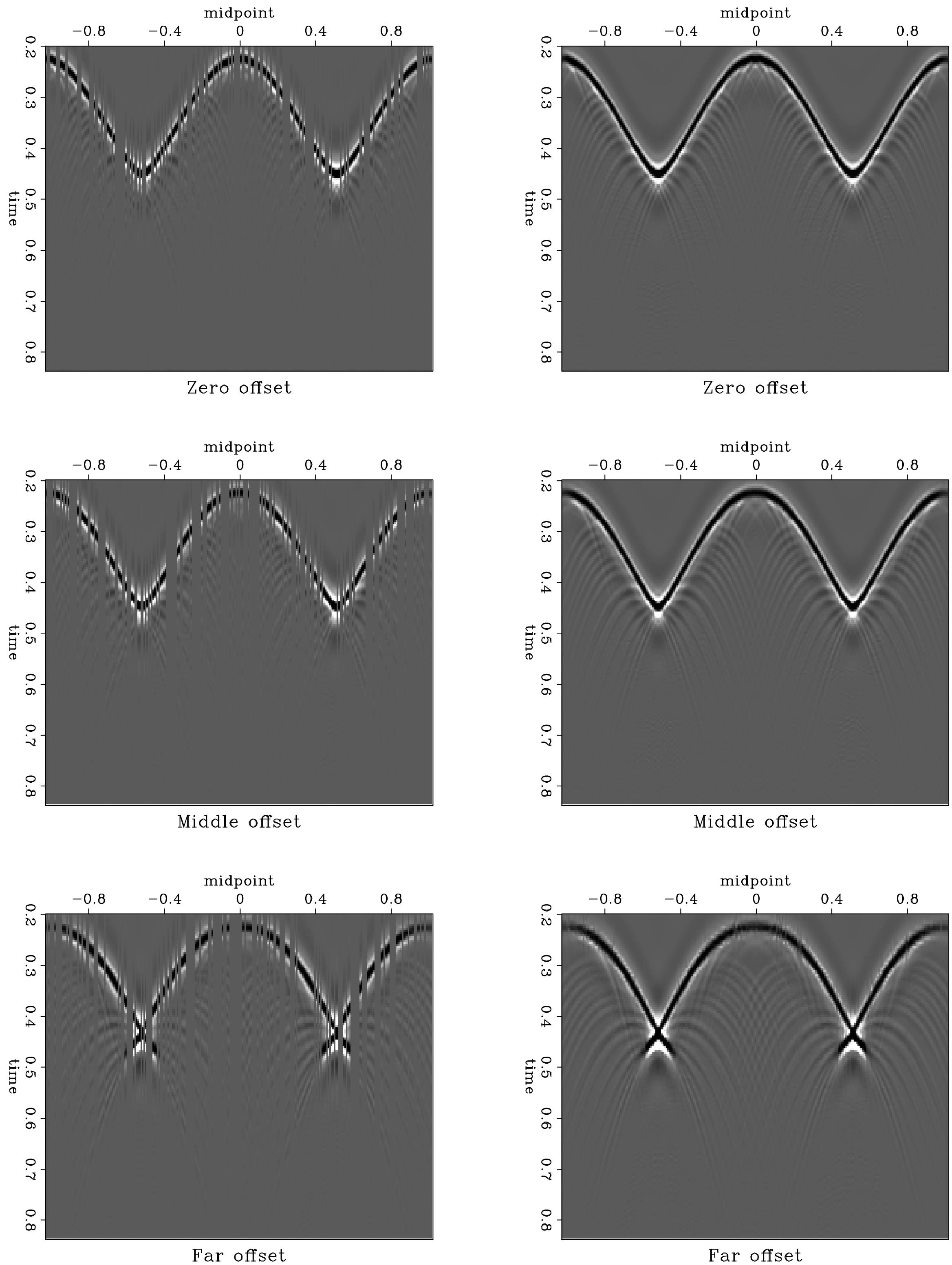

Figure 5: Interpolation in common-offset gathers. Left: input data (50% of the traces are randomly removed). Right: interpolation output. Top, center, and bottom plots correspond to different common-offset gathers. sergey2-all [ER,M]

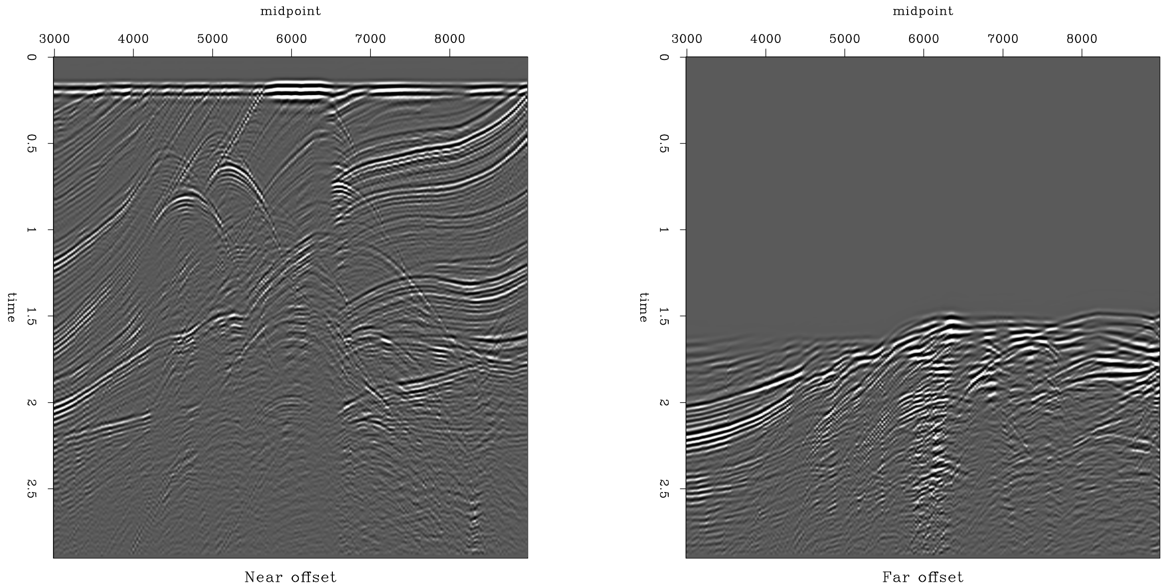

Figure 6: Common-offset gathers of the Marmousi dataset. Left: near offset. Right: far offset. sergey2-marm [ER]

be sacrificed for achieving more accurate results. Here are some interesting ideas one could try:

- Instead of fixing the offset continuation filter in a data-independent way, one could estimate some of its coefficients from the data. In particular, the second term in equation (3) can be varied to better account for the effects of variable velocity and amplitude variation with offset. Theoretical extensions of offset continuation to the variable velocity case were studied by Hong et al. (1997) and Luo et al. (1999).
- Formulating the problem in the pre-NMO domain would allow us to consider several velocities by convolving several continuation filters. This could be an appropriate approach for interpolating both primary and multiple reflections.
- Missing data interpolation problems can be greatly accelerated by preconditioning (Fomel, 1997; Fomel et al., 1997). Finding an appropriate preconditioning for the offset continuation method is an open problem. The non-stationary nature of the continuation filter make this problem particularly challenging.

I plan to devote my remaining time at the Stanford Exploration Project to investigating these fascinating ideas.

REFERENCES

Bagaini, C., and Spagnolini, U., 1993, Common shot velocity analysis by shot continuation operator: 63rd Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 673–676.

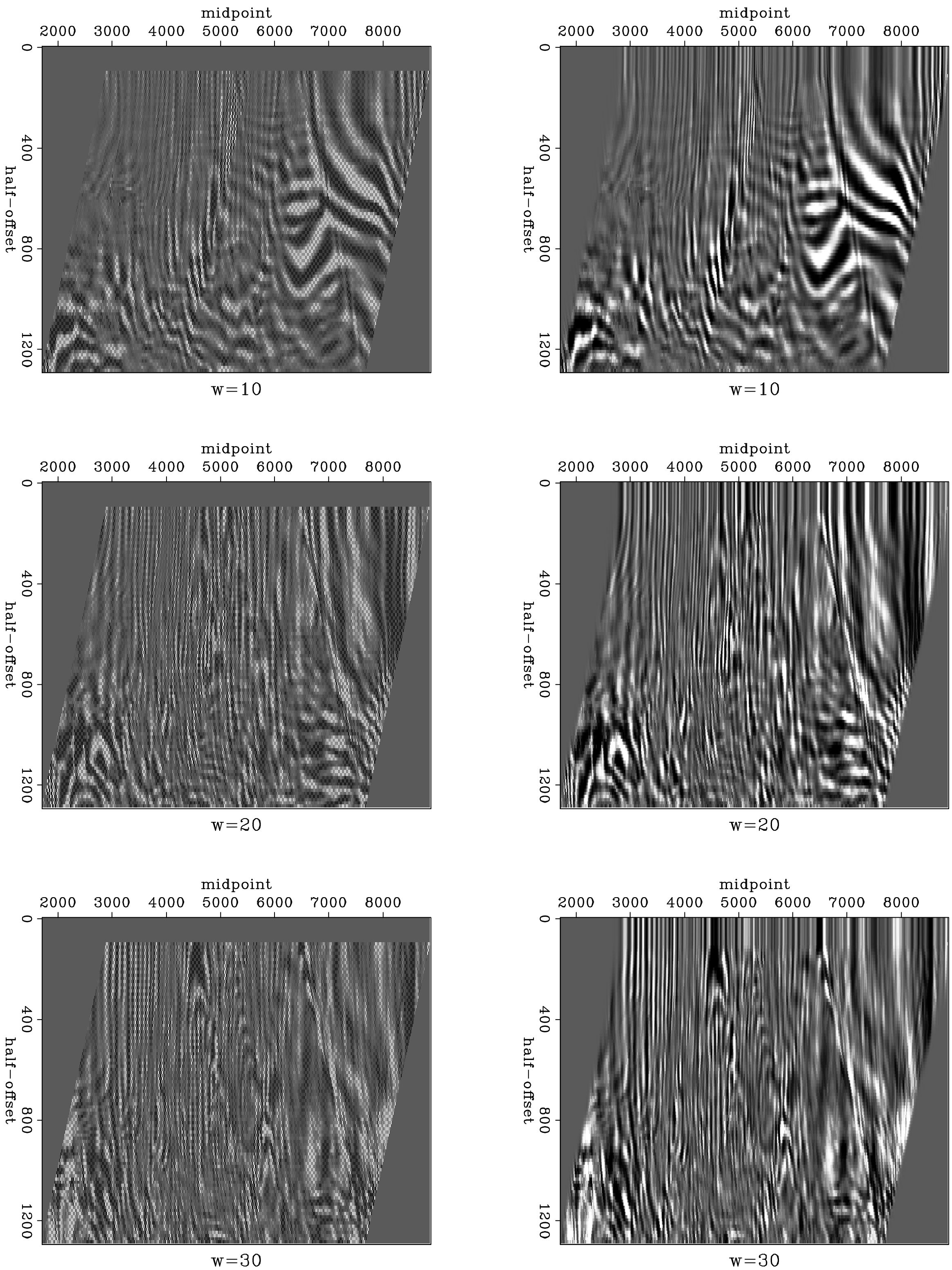

Figure 7: Interpolation of the Marmousi dataset in frequency slices. Left: input data. Right: interpolation output. Top, center, and bottom plots correspond to different frequencies. Real parts of the complex-valued data are shown. sergey2-mslice [CR,M]

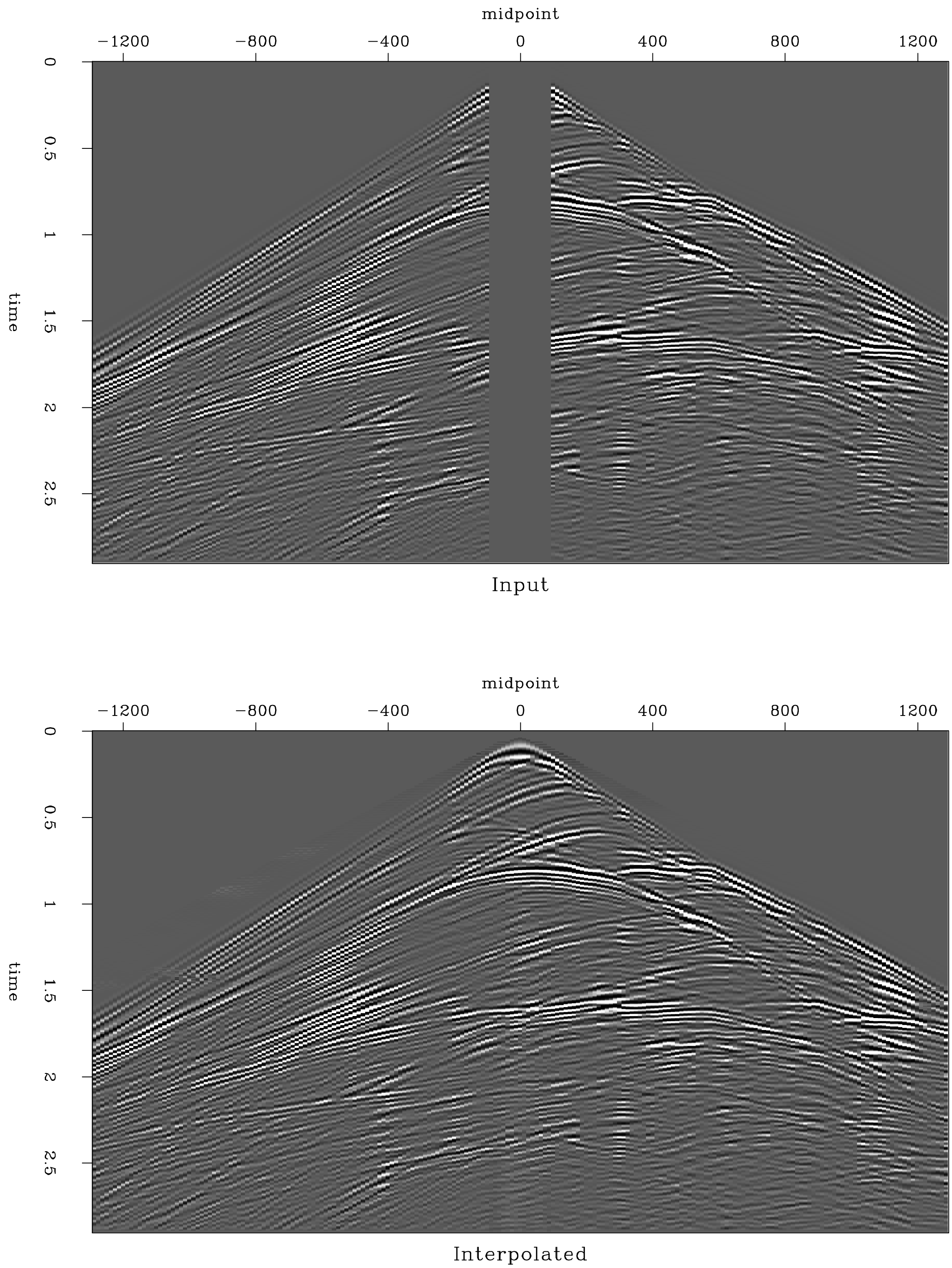

Figure 8: Interpolation of near offsets in a Marmousi shot gather. The shot position is 4500 m. Top: input data. Bottom: interpolation output. sergey2-mshot [CR,M]

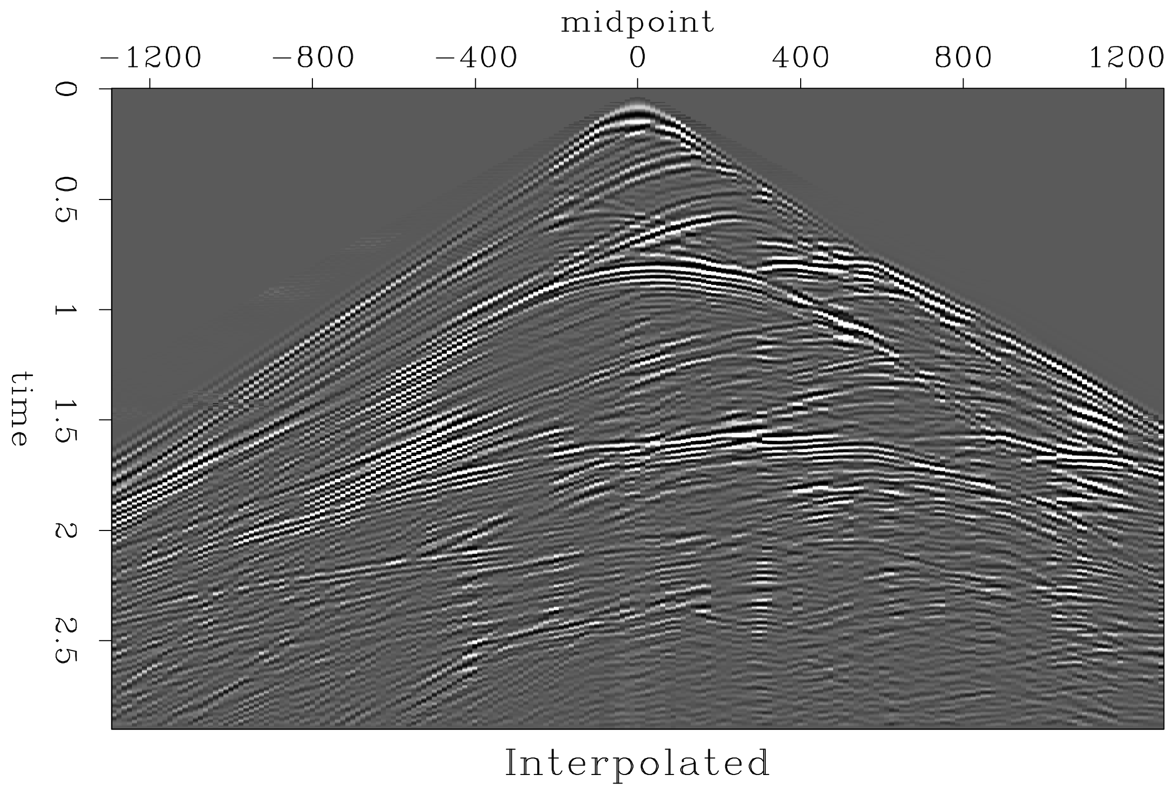

Figure 9: This shot gather at 4525 m is a result of data interpolation. sergey2-mishot [CR]

Bagaini, C., and Spagnolini, U., 1996, 2-D continuation operators and their applications: Geophysics, **61**, no. 06, 1846–1858.

Bagaini, C., Spagnolini, U., and Pazienza, V. P., 1994, Velocity analysis and missing offset restoration by prestack continuation operators: 64th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1549–1552.

Biondi, B., and Palacharla, G., 1996, 3-D prestack migration of common-azimuth data: Geophysics, **61**, no. 06, 1822–1832.

Biondi, B., Fomel, S., and Chemingui, N., 1998, Azimuth moveout for 3-D prestack imaging: Geophysics, **63**, no. 2, 574–588.

Biondi, B. L., 1999, 3-D Seismic Imaging: Stanford Exploration Project.

Bolondi, G., Loinger, E., and Rocca, F., 1982, Offset continuation of seismic sections: Geophys. Prosp., **30**, no. 6, 813–828.

Briggs, I. C., 1974, Machine contouring using minimum curvature: Geophysics, **39**, no. 1, 39–48.

Chemingui, N., and Biondi, B., 1994, Coherent partial stacking by offset continuation of 2-D prestack data: SEP–**82**, 117–126.

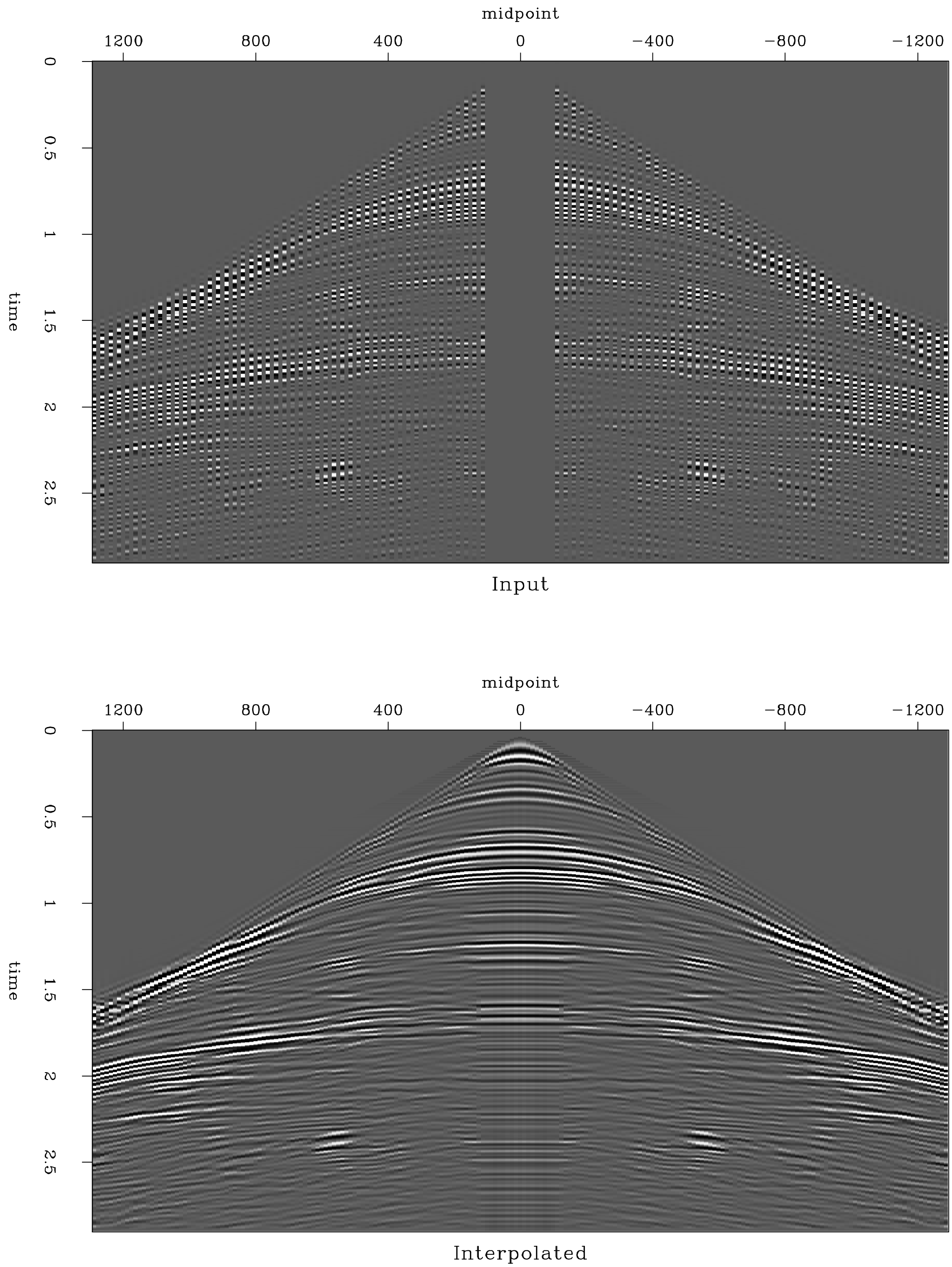

Figure 10: Interpolation of near and intermediate offsets in a Marmousi CMP gather. The midpoint position is 4500 m. Top: input data. Bottom: interpolation output. sergey2-mcmp [CR,M]

Chemingui, N., 1999, Imaging irregularly sampled 3D prestacked data: Ph.D. thesis, Stanford University.

Claerbout, J. F., 1980, Restoration of missing data by least squares optimization: SEP–**25**, 1–16.

Claerbout, J. F., 1981, Missing data: Wanted, good convergence in a few steps: SEP–**26**, 23–38.

Claerbout, J. F., 1992, Earth Soundings Analysis: Processing Versus Inversion: Blackwell Scientific Publications.

Claerbout, J., 1999, Geophysical estimation by example: Environmental soundings image enhancement: Stanford Exploration Project, http://sepwww.stanford.edu/sep/prof/.

Clapp, R. G., Fomel, S., and Claerbout, J., 1997, Solution steering with space-variant filters: SEP–**95**, 27–42.

Clapp, R. G., Fomel, S., Crawley, S., and Claerbout, J. F., 1999, Directional smoothing of non-stationary filters: SEP–**100**, 197–209.

Crawley, S., 1999, Interpolation with smoothly nonstationary prediction-error filters: SEP–**100**, 181–196.

Deregowski, S. M., and Rocca, F., 1981, Geometrical optics and wave theory of constant offset sections in layered media: Geophys. Prosp., **29**, no. 3, 374–406.

Deregowski, S. M., 1986, What is DMO: First Break, **4**, no. 7, 7–24.

Fomel, S., and Biondi, B., 1995, The time and space formulation of azimuth moveout: SEP–**84**, 25–38.

Fomel, S., and Bleistein, N., 1996, Amplitude preservation for offset continuation: Confirmation for Kirchhoff data: SEP–**92**, 219–227.

Fomel, S., Bleistein, N., Jaramillo, H., and Cohen, J. K., 1996, True amplitude DMO, offset continuation and AVA/AVO for curved reflectors: 66th Ann. Internat. Meeting, Soc. Expl. Geophys., 1731–1734.

Fomel, S., Clapp, R., and Claerbout, J., 1997, Missing data interpolation by recursive filter preconditioning: SEP–**95**, 15–25.

Fomel, S. B., 1994, Kinematically equivalent differential operator for offset continuation of seismic sections: Russian Geology and Geophysics, **35**, no. 9, 122–134.

Fomel, S., 1995a, Amplitude preserving offset continuation in theory Part 1: The offset continuation equation: SEP–**84**, 179–198.

Fomel, S., 1995b, Amplitude preserving offset continuation in theory part 2: Solving the equation: SEP–**89**, 109–132.

Fomel, S., 1997, On model-space and data-space regularization: A tutorial: SEP–**94**, 141–164.

Fomel, S., 1999, Plane wave prediction in 3-D: SEP–**102**, 101–110.

Fomel, S., 2000, Helical preconditioning and splines in tension: SEP–**103**, 289–301.

Gardner, G. H. F., and Canning, A., 1994, Effects of irregular sampling on 3-D prestack migration: 64th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1553–1556.

Hale, D., Ed. **DMO processing**. Society Of Exploration Geophysicists, 1995.

Hong, L., Xingyao, Y., Youming, L., Changchun, Y., and McMechan, G. A., 1997, The offset continuation in complex medium: 67th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1901–1904.

Ji, J., 1995, Sequential seismic inversion using plane-wave synthesis: Ph.D. thesis, Stanford University.

Luo, M., Li, Y., and Liu, H., 1999, Amplitude preserving offset continuation in lateral varied media: 69th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1933–1936.

Mazzucchelli, P., and Rocca, F., 1999, Regularizing land acquisitions using shot continuation operators: effects on amplitudes: 69th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1995–1998.

Mazzucchelli, P., Rocca, F., Spagnolini, U., and Spitz, S., 1998, Wavefield interpolation - Continuation or prediction filter techniques?: 61st Mtg. Eur. Assoc. Expl Geophys., Extended Abstracts, Session:02–51.

Salvador, L., and Savelli, S., 1982, Offset continuation for seismic stacking: Geophys. Prosp., **30**, no. 6, 829–849.

Schwab, M., 1993, Shot gather continuation: SEP–**77**, 117–130.

Spagnolini, U., and Opreni, S., 1996, 3-D shot continuation operator: 66th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 439–442.

Spitz, S., 1991, Seismic trace interpolation in the f-x domain: Geophysics, **56**, no. 6, 785–794.

Stovas, A. M., and Fomel, S. B., 1996, Kinematically equivalent integral DMO operators: Russian Geology and Geophysics, **37**, no. 2, 102–113.

Symes, W. W., and Carazzone, J. J., 1991, Velocity inversion by differential semblance optimization: Geophysics, **56**, no. 5, 654–663.

Symes, W. W., 1999, All stationary points of differential semblance are asymptotic global minimizers: Layered acoustics: SEP–**100**, 71–92.

Tarantola, A., 1987, Inverse problem theory: Elsevier.

Thorson, J. R., 1981, Restoration of missing data by parsimony in the frequency domain: SEP–**26**, 1–22.

van Dedem, E. J., and Verschuur, D. J., 1998, 3-D surface-related multiple elimination and interpolation: 68th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1321–1324.

Versteeg, R., 1994, The marmousi experience: Velocity model determination on a synthetic complex data set: The Leading Edge, **13**, no. 09, 927–936.