



## Short Note

# Amplitudes and inversion in the reflection angle domain

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### INTRODUCTION

Reflection angle domain (RAD) imaging is gaining interest as an alternative to shot and offset domain methods (Xu et al., 1998). The advantage of RAD imaging is that it reduces the number and strength of artifacts seen in complex areas, particularly artifacts caused by multipathing. RAD imaging reduces artifacts but does not overcome the inherent limitations in surface seismic recording geometries. As a result, areas where the RAD is most useful can also benefit from replacing migration with inversion. Although inversion is more expensive from a computational standpoint, it can effectively address amplitude problems (Duquet and Marfurt, 1999) and, with intelligent preconditioning, null space concerns caused by limited survey geometry. In this paper, we use a preconditioned inversion approach described in Prucha et al. (1999b). By using preconditioning, we are attempting to fill in model components that have no corresponding data. This raises the concern of whether or not the preconditioning is creating a reasonable and realistic model, especially in terms of amplitude.

Amplitude analysis is difficult in complex areas, even when we have an accurate velocity model (Castagna and Backus, 1993). Limited recording geometry and shadow zones caused by overburden can create artificial, erroneous amplitude variation with angle (AVA) results. Since our preconditioned inversion helps fill this model null space, we would like our created amplitudes to be as reasonable as possible. In this paper, we will show that our chosen preconditioning operator fills the model null space intelligently with amplitudes that are more accurate than those obtained by migration alone.

To show the benefits of preconditioning in the RAD, we will first expand on why it is necessary to use an inversion process. We will then explain how we carried out our preconditioned inversion. Finally, we will apply our method to a synthetic model and show that the amplitude response after RAD preconditioning is more accurate than that after simple RAD migration.

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## THEORY

### Motivation

Creating common image gathers (CIGs) in the RAD can be done by Kirchhoff methods (Xu et al., 1998) or wave-equation methods (Prucha et al., 1999a). However, there are issues in both that must be taken into account to properly image the subsurface and handle the amplitudes intelligently.

The RAD has some inherent limitations caused by our finite survey geometries. One of these is a depth dependency. As depth increases, the maximum reflection angle that can be imaged decreases (Figure 1). This creates a null space that increases with depth. It is also easy to see that the maximum reflection angle that can be imaged for a particular point in the subsurface is dependent on the dip of the reflector being imaged (Figure 2), so the model null space is less predictable than a simple function of depth.

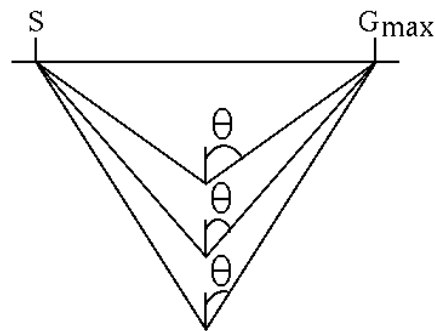


Figure 1: Depth limitation of maximum reflection angle imaged. `marie1-depth` [NR]

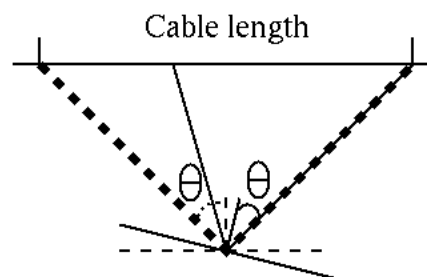


Figure 2: Dip limitation of maximum reflection angle imaged. The dashed lines relate to the zero dip reflector, the solid lines to the dipping reflector. The maximum reflection angle possible for the dipping reflector is much smaller than that for the zero dip reflector. `marie1-dip` [NR]

It is clear that these limitations will result in missing data where the energy from a reflector within the survey area arrives at the surface at a point outside of the survey geometry. This is especially troublesome in complex areas with rapidly varying velocity, where the behavior can vary quickly spatially. This "missing" data will create a model null space. We want to fill this null space intelligently, so we turn to inversion.

Another reason inversion is superior to migration in complex areas is that it is easier to get accurate amplitudes. In areas where the subsurface is simple, obtaining reasonable amplitudes

is easy to do with almost any migration method. As the problem becomes more complex, correctly weighting arrivals becomes essential (Albertin et al., 1999; Bloor et al., 1999). For the complex environments that the reflection angle domain is most useful in, the weighting function that would have to be applied to the migration process is very difficult to find, let alone code effectively. When we use inversion, our problem converges to give us accurate amplitudes without needing the additional weighting function, so the problem is less complex than the migration problem would be to get an equivalent answer.

### Inversion in the RAD

We want to fill the model null space with information that is as reasonable as possible. We do not want to allow artificial amplitude variations that could lead to disastrous AVA analysis. To put information in the model's null space that is based on the known information, we can use regularization in the inversion process.

Regularization is a familiar process that can be represented by these fitting goals:

$$\mathbf{d} \approx \mathbf{Lm} \quad (1)$$

$$0 \approx \epsilon \mathbf{Am} \quad (2)$$

where  $\mathbf{d}$  is the data,  $\mathbf{m}$  is the model,  $\mathbf{L}$  is a linear operator,  $\mathbf{A}$  is the regularization operator, and  $\epsilon$  determines the strength of the regularization. To reduce the number of iterations needed, we can change this regularized problem into a preconditioned one with this substitution:  $\mathbf{m} = \mathbf{Sp}$ , where  $\mathbf{S} = \mathbf{A}^{-1}$  so our fitting goals become:

$$\mathbf{d} \approx \mathbf{LSp} \quad (3)$$

$$0 \approx \epsilon \mathbf{p}. \quad (4)$$

In this paper,  $\mathbf{L}$  is the modeling operator that is the adjoint to the wave-equation method of creating RAD CIGs explained in Prucha et al. (1999b). The preconditioning operator  $\mathbf{S}$  was chosen based on two known facts. First, assuming that we have the correct velocity function when we carry out the migration, the resultant model will have horizontal events along the reflection angle axis. Second, the AVA response is expected to be smoothly varying for a particular point in the subsurface. Therefore, we chose an operator that would smooth horizontally along the reflection angle axis. In this paper, we use a steering filter (Clapp et al., 1997) that will smooth in a narrow path along the reflection angle axis.

This process has already been introduced by Prucha et al. (2000) but we are now interested in the amplitudes that result from it. To do so, we experimented with a simple synthetic model.

## APPLICATION

We have two goals in performing inversion rather than migration. The first is to obtain accurate amplitudes that migration would require a complicated weighting function to get. The second

goal is to compensate for the limitations in our recording geometry and the effects of complex overburden that can create null spaces. By using preconditioning with our inversion problem, we hope to fill the null spaces with reasonable amplitudes.

To test the effect of preconditioning on amplitude, we created a simple, flat reflector model with a constant amplitude for all reflection angles. We then generated the data for this model by using the adjoint of the migration process explained in Prucha et al. (1999b), essentially upward continuing the wavefield through the synthetic Elf velocity model (Figure 3) with a very long half offset of 6400 meters. We chose this velocity model and reflector location because there is a shadow zone in this area (Prucha et al., 1998) and should therefore create amplitude problems with simple migration.

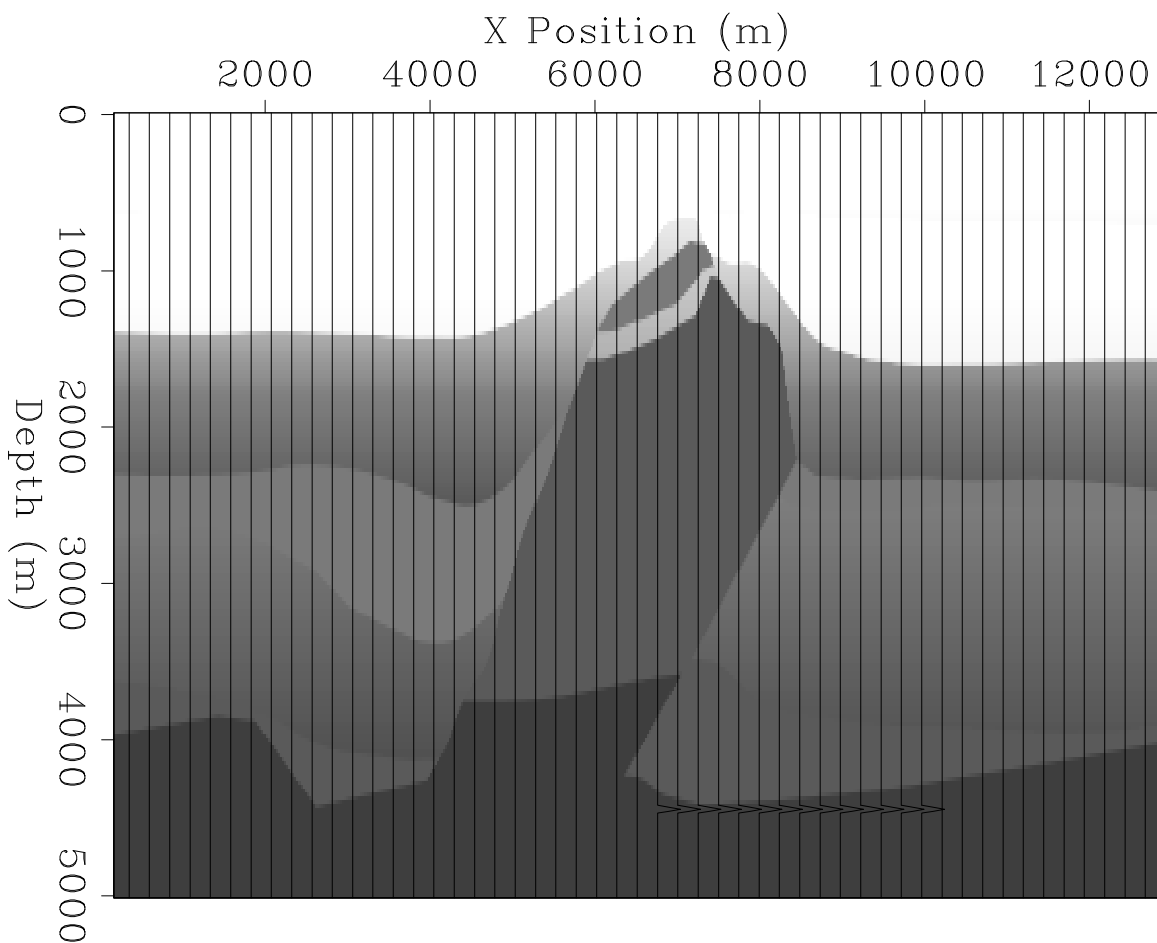


Figure 3: Synthetic model of a single, flat reflector in the Elf velocity model. marie1-model  
[ER]

To create a "correct" answer to the amplitude question, we migrated the long offset data using wave-equation RAD migration. The extremely long offset allowed us to minimize the model null space. The resultant amplitude response along reflection angle, which is ideally constant for continuous, infinite survey geometry, we considered to be our ideal solution (panel A in Figure 4). The response is almost constant for all reflection angles but the wavelet broad-

ens slightly with increasing angle, which is expected because of effects similar to normal moveout (NMO) stretch.

We then cut our survey geometry to one quarter of its original size by making the maximum half offset 1600 meters, which is the size of the offset in the real dataset that the synthetic model is based on. Prucha et al. (1998) has shown that with this survey geometry for this synthetic model, migration in the offset domain results in amplitudes that are very low at short and long offsets and fairly constant at mid-offsets. The AVA response obtained from RAD migration of this reduced dataset (panel C in Figure 4) is quite different from our "correct" answer. It has very low amplitudes at small angles and no response at large angles, which is not surprising given the response seen in the offset domain. There is a sharp increase in amplitude in the mid-range of the reflection angles which is clearly not correct. There even appears to be some curvature to the event, despite the fact that we know we used the correct velocity model.

Finally, we applied our preconditioned inversion to the limited offset data (panel B in Figure 4). Although the response is not perfect, it is closer to our "correct" response after only one iteration. It is a nice, smooth response that does not vary as much with angle as the migration result. It is still weak at small and large angles, but it extends farther into the large angles than does the short-offset migration. It is also flat, like the ideal response.

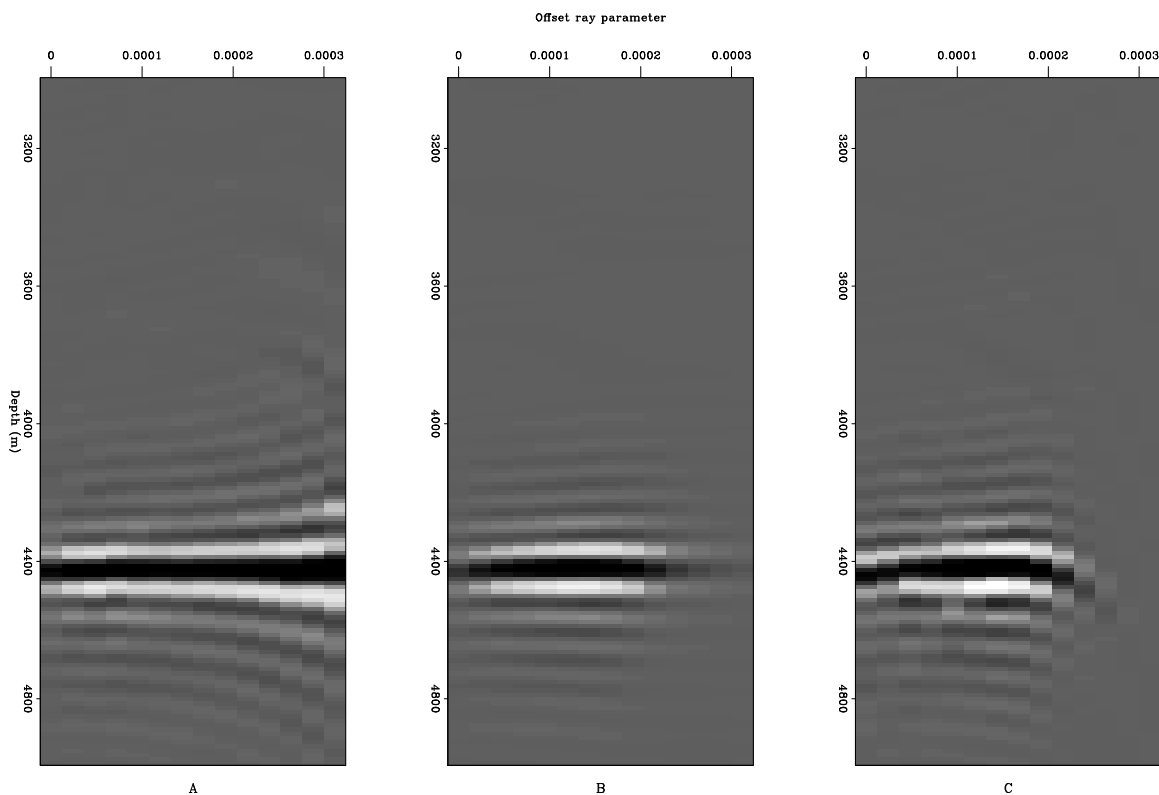


Figure 4: Comparison of the amplitude variation with reflection angle of (A) the long offset data (the "correct" answer), (B) the preconditioned inversion for short offset, and (C) the migration for short offset, extracted at x position 8025. marie1-compamp.315 [CR]

## CONCLUSIONS

Although they are not perfect, the amplitudes we obtained through our preconditioned inversion are more accurate than the amplitudes obtained by migration alone.

## ACKNOWLEDGMENTS

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