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Offset and angle domain common-image gathers for shot-profile migration

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ABSTRACT

In order to estimate elastic parameters of the subsurface, geophysicists need reliable information about angle-dependent reflectivity. In this paper, we describe how to image non-zero offsets during shot-profile migration so that they can be mapped to the angle domain with Sava and Fomel's (2000) transformation. CIGs also contain information about how well focused events are at depth, and so provide a natural domain for migration-focusing velocity analysis.

INTRODUCTION

For simple velocity models (e.g. constant v) a one-to-one mapping exists between recording offset ($|\vec{s}-\vec{g}|$) on the surface and incidence angle. Early amplitude versus offset (AVO) studies took advantage of this fact: authors looked at amplitude variation of events in common-midpoint gathers after normal moveout (NMO). It soon became clear, however, that NMO does not adequately correct for the focusing aspects of wave propagation, and that amplitude studies should be conducted after migration (Resnick et al., 1987).

Kirchhoff migration algorithms can easily migrate data recorded at different offsets separately, and produce common image point gathers (CIGs) with migrated amplitude versus offset. Alternatively, interpreters often compare amplitude on near-offset migration images with far-offset images to look for anomalies linked to the presence of hydrocarbons. However, offset on the surface is a *data-space* parameter, whereas incidence angle is a *model-space* parameter; even for many relatively simple $v(z)$ velocity models, no simple relationship exists between surface offset and incidence angle at the reflector.

For Kirchhoff amplitude versus angle (AVA) studies, all offsets need to be migrated, and the incidence angle determined at the imaging step by ray-tracing. Two disadvantages of this process are that it is sensitive to the stability of the ray-paths, and complicates the algorithm's I/O requirements.

de Bruin et al. (1991) first described how to calculate angle-dependent reflectivity with a wave-equation migration process. Their algorithm is appropriate for shot-profile algorithms, and involves a local slant-stack as part of the image-forming step. Similarly, Prucha et al.

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(1999) describe how to extract angle domain common-image-point gathers during shot-geophone migration in the offset-midpoint domain. Since these two migration methods are based on finite-frequency solutions to the wave-equation, there is no sensitivity to ray-traced angles. There is also no significantly increased I/O burden over zero-offset imaging with the same algorithm other than the need to output another model-space axis.

After wave equation migration, offset no longer refers to the recording offset at the surface, but rather refers to the subsurface offset between upgoing and downgoing wavefields. Offset after migration is therefore a model-space rather than a data-space parameter. Figure 1 graphically illustrates the meaning of offset before, during, and after migration.

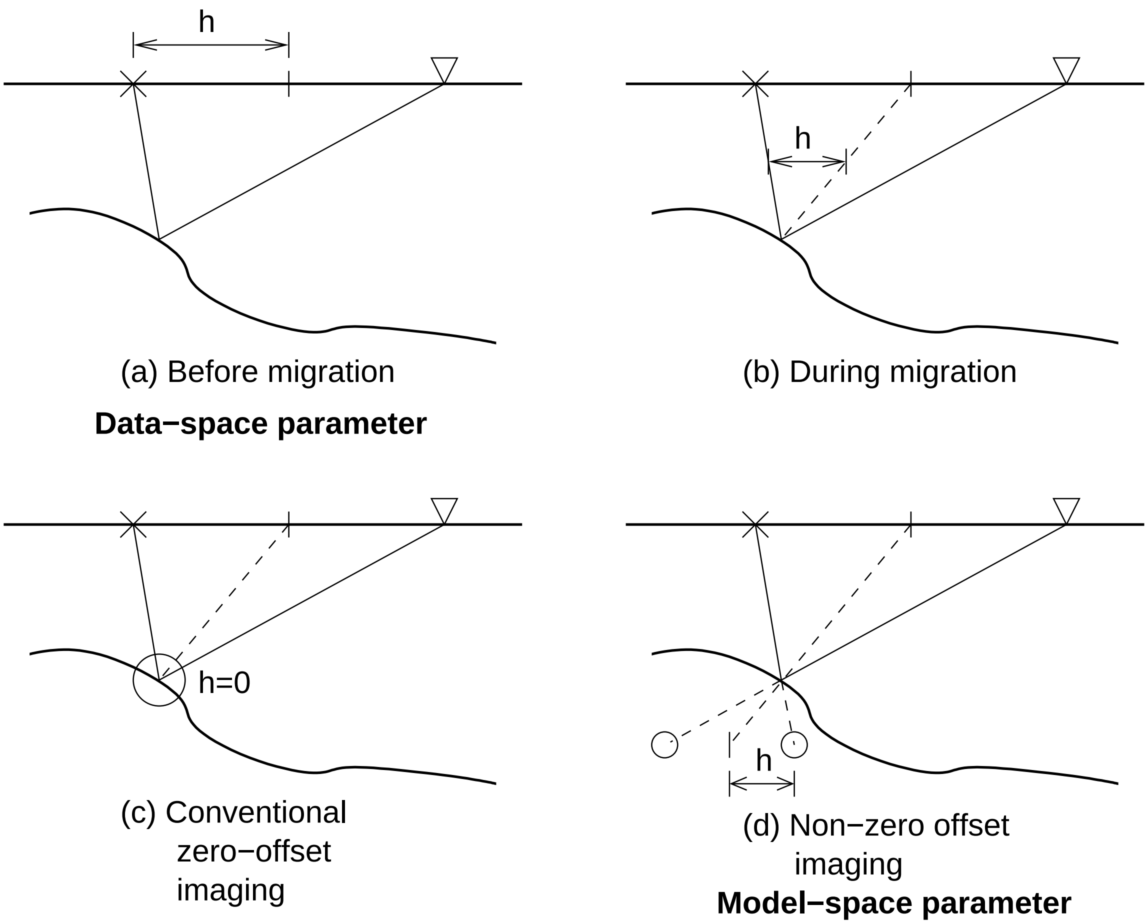

Figure 1: Illustration of offset before (a), during (b), and after (c,d) migration. james2-offsetexp [NR]

For shot-geophone migration in the offset-midpoint domain, Sava and Fomel (2000) provide an algorithm that converts CIGs in the offset domain to angle gathers. The authors offer this as an alternative to Prucha et al.'s approach which requires extracting offset ray-parameter gathers as part of the migration imaging condition.

MULTI-OFFSET IMAGING

We produce angle gathers for shot-profile migrations by combining de Bruin et al.'s (1991) approach with that of Sava and Fomel (2000). Rather than extracting a single zero-offset/zero-time reflectivity image, we extract multiple zero-time images with a range of offsets. In 2-D this can be performed with the following sum over frequency:

$$I(x,h,z) = \sum_{\omega} q_-(x-h,z,\omega)\, q_+(x+h,z,\omega)^*. \tag{1}$$

Gathers produced this way contain off-diagonal elements of Berkhout's reflectivity matrix (1985), and are equivalent to those produced by imaging multiple non-zero offsets in an offset-midpoint shot-geophone migration. Consequently, the offset axes can be mapped to angle with Sava and Fomel's (2000) transformation, which is based on the relationship

$$\tan\gamma = - \left.\frac{\partial z}{\partial h}\right|_{t,x} = \frac{|\vec{k_h}|}{k_z}, \tag{2}$$

where γ is the half-opening (incidence) angle. The imaging condition ($t=0$) and their common-midpoint nature (constant x) allow image gathers to be related to the partial derivative at constant t and x.

Shot to midpoint transformations

Implicit in the multi-offset imaging scheme outlined above is a transformation from shot-receiver to midpoint-offset space. Although this transformation is buried within the migration process, the subtleties associated with the conversion remain.

If the wavefields are sampled with spacing Δx, then equation (1) will image with half-offset spacing $\Delta h = \Delta x$, as shown in Figure 2 (a). Sampling in offset can be refined further by considering Figure 2 (b); however, to do so requires imaging onto midpoints which do not lie on the propagation grid.

This problem is experienced whenever data are transformed from shot-geophone to midpoint-offset space, and no perfect solution exists. A typical workaround is to refine the midpoint grid, and fill empty bins with zeros; however, this doubles the data-volume and hence also doubles the cost of migration. Another alternative is to process even and odd offset separately; the disadvantage of this approach is that each half of the dataset may be undersampled.

By working in the shot-geophone domain, these problems are avoided until after the migration is complete. Migration decreases the data-volume, increases the signal-to-noise ratio, and resolves locally conflicting dips. Therefore, it is easier to resample the data on whatever grid suits the interpreter after migration.

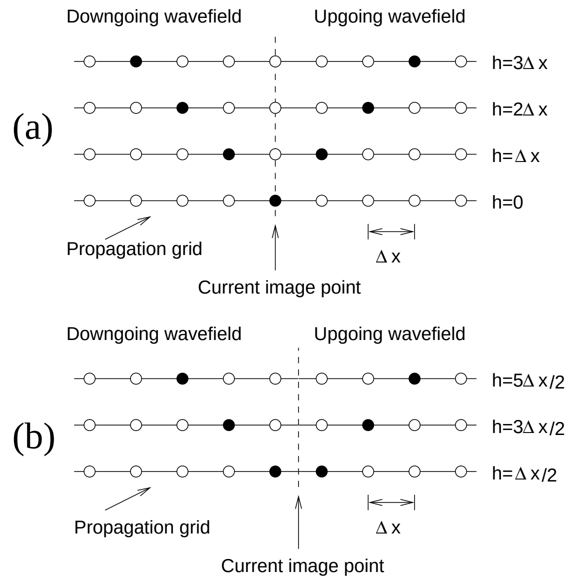

Figure 2: Imaging offsets (a) with $\Delta h = \Delta x$ based on equation (1) alone, and (b) with $\Delta h = \Delta x/2$, where the midpoint lies between propagation grid nodes. james2-cmpboth [NR]

Computational considerations

The cost of downward continuing a 3-D wavefield one depth step in the Fourier domain is proportional to $N_{xy} \log(N_{xy})$. On the other hand, the cost of imaging with a 3-D version of equation (1) is proportional to $N_{xy} N_{h_{xy}}$. Clearly, if the number of offsets is large, then the cost of the imaging step may be comparable (or even greater) than the cost of the downward continuation.

This increase in computational cost is not a problem with shot-geophone migration. In that instance, the cost of the imaging step (proportional to $N_{xy} N_{h_{xy}}$), will always be less than the cost of the downward continuation [proportional to $N_{xy} N_{h_{xy}} \log(N_{xy} N_{h_{xy}})$].

INTERPRETATION

As with AVO studies, the rock-physics interpretation of angle-domain CIGs after shot-profile migration is fraught with pitfalls: two of the most important being caused by velocity problems and sampling problems.

The first major pitfall is common to all angle-dependent reflectivity studies: if events are not correctly imaged, they will not be flat in the angle domain, and artifacts will appear in AVA interpretations. For these methods to be successful, velocities must be very well understood.

While the second major pitfall may also cause problems for other migration algorithms, geometries suitable for shot-profile migration are particularly susceptible to sampling problems on the shot axis.

Example common-image gathers

Figure 3 illustrates the effect velocity plays on offset-domain common-image gathers. The three panels show CIGs produced by migrating the Marmousi synthetic dataset with three different velocity models: the correct velocity [panel (a)], a velocity that is too low [panel (b)], and a velocity model that is too high [panel (c)]. Interpreting patterns in the offset-domain CIGs is difficult; however, after transformation to the angle domain (Figure 4, standard residual-moveout patterns indicates the sign of the velocity error: events curving up meaning too low, and events curving down meaning too high.

To illustrate the problems associated with sampling for shot-profile migrations, Figure 5 shows the same image gathers as Figure 3, but after migrating only every twentieth shot. Even if the velocity is correct, energy does not cancel at non-zero offsets, and so events are not flat in the angle-domain (Figure 6). When the velocities are incorrect, the angle gathers remain chaotic: shot aliasing has effectively rendered the angle-gathers uninterpretable in terms of velocities.

Although both de Bruin's (1991) original methodology and the approach described here provide means of obtaining common-image gathers from shot-profile migration, the problem of shot-aliasing remains important for the geometries that are best suited to shot-profile migration.

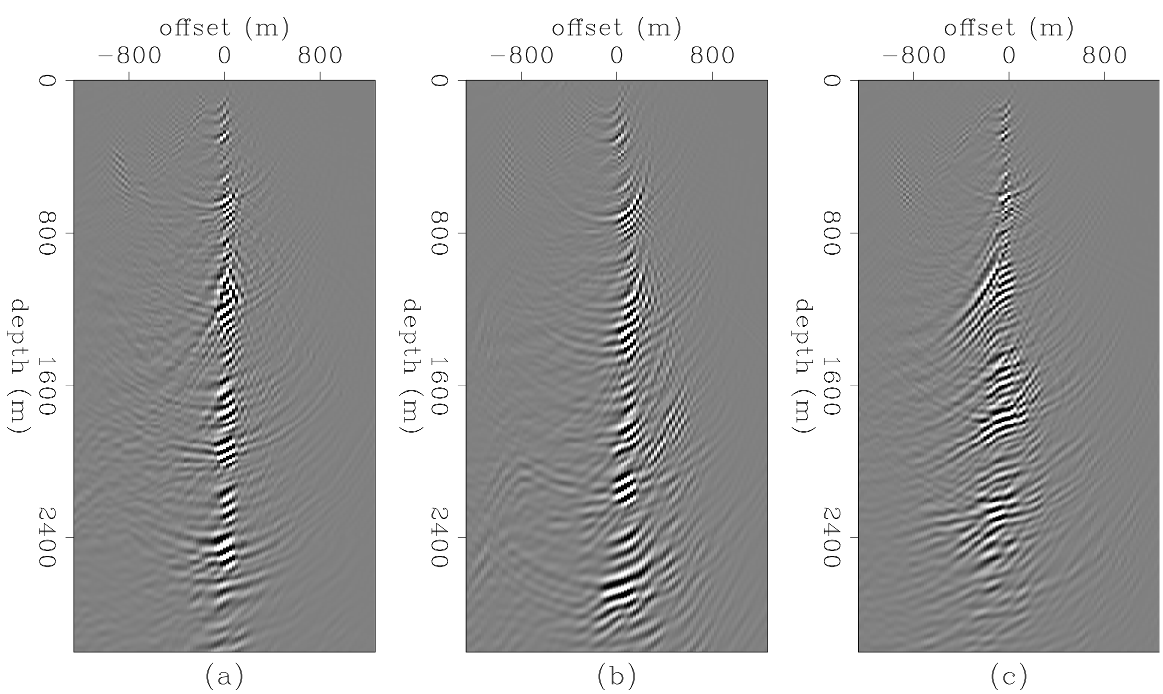

Figure 3: Offset-domain common-image gathers. Panels (a), (b), and (c) were migrated with velocity models that were correct, 6% too low, and 6% too high, respectively. james2-offvel4000 [CR,M]

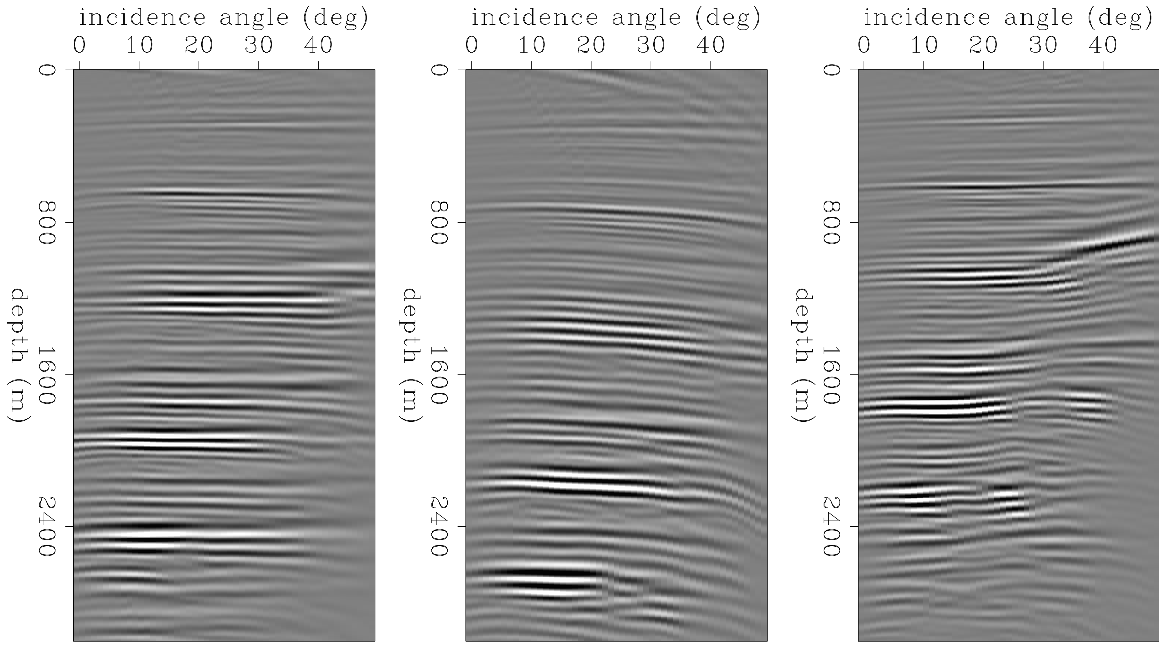

Figure 4: Angle-domain common-image gathers. Panels (a), (b) and (c) were migrated with velocity models that were correct, too low, and too high, respectively. james2-angvel4000 [CR,M]

CONCLUSIONS

We construct offset-domain common-image gathers after shot-profile migration. The offset-domain gathers then can be transformed to the angle domain by Sava and Fomel's (2000) transformation where residual moveout can be used for velocity analysis or image enhancement. The angle domain also potentially contains information about reflectivity as a function of angle that can be used to infer rock properties across the reflecting interface.

For sparse-shot geometries, suitable for shot-profile migration, however, the problem of shot-aliasing remains. Shot aliasing can cause corruption of the angle domain gather, rendering simple moveout-based velocity analysis very difficult.

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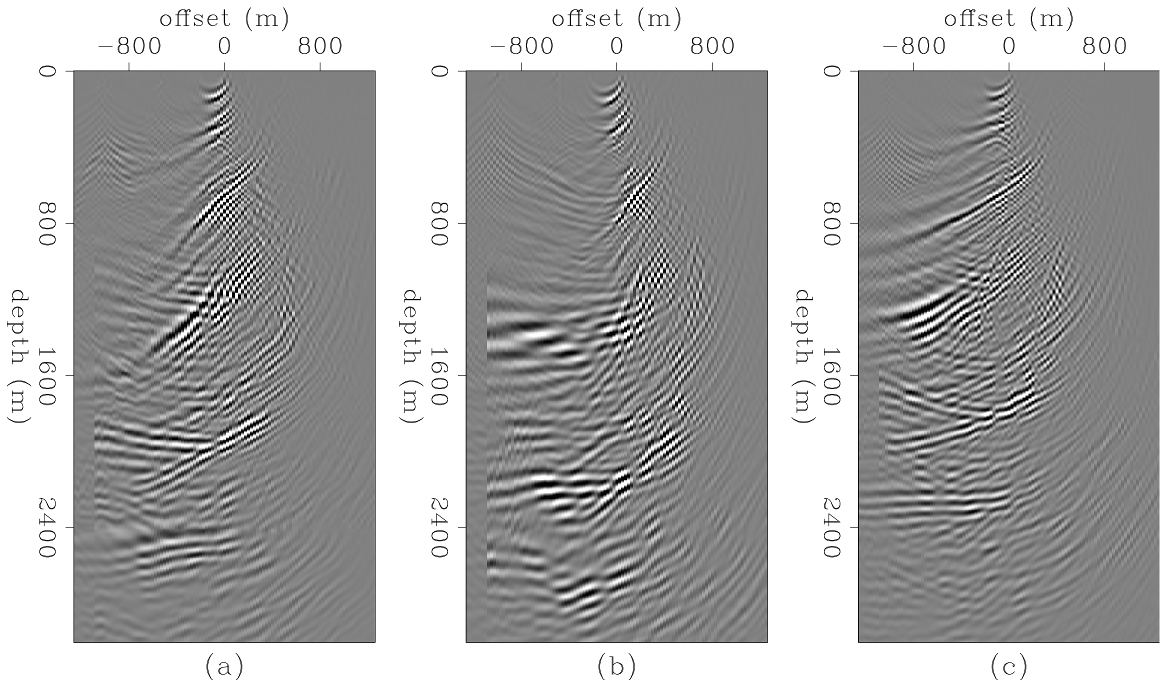

Figure 5: Poorly illuminated offset-domain common-image gathers. Panels (a), (b), and (c) were migrated with velocity models that were correct, 6% too low, and 6% too high, respectively. Shot spacing was 500 m, instead of 25 m for Figure 3. james2-offcov4000 [CR,M]

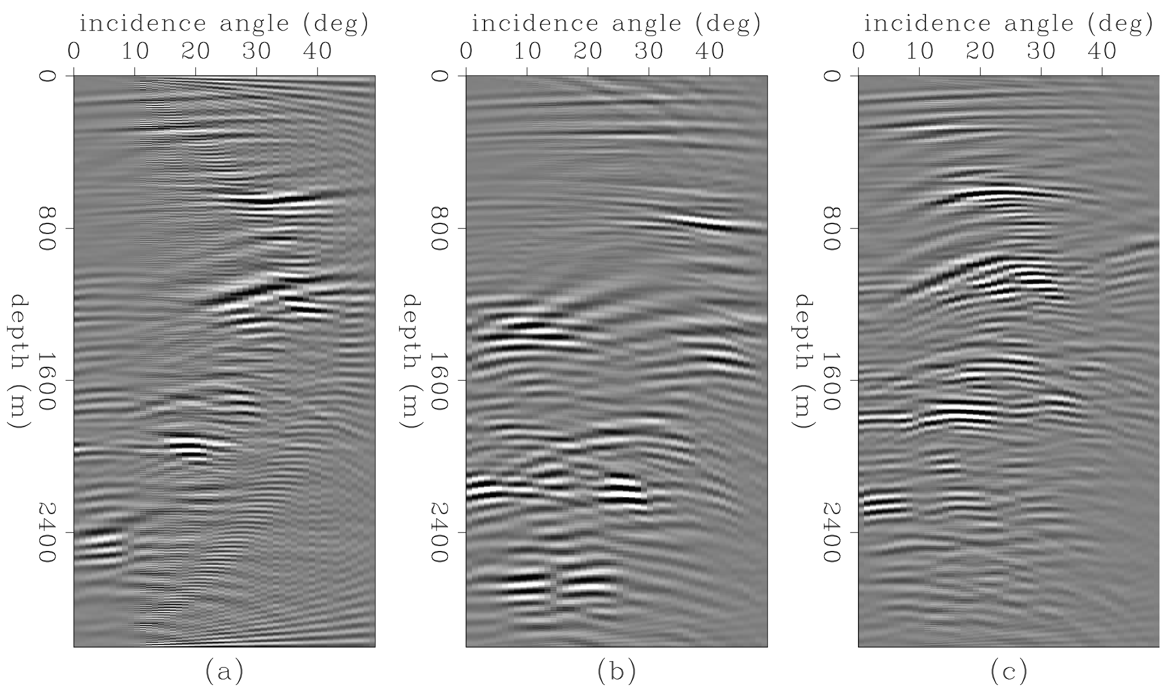

Figure 6: Poorly illuminated angle-domain common-image gathers. Panels (a), (b) and (c) were migrated with velocity models that were correct, 6% too low, and 6% too high, respectively. Shot spacing was 500 m, instead of 25 m for Figure 4. james2-angcov4000 [CR,M]

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