Acoustic daylight imaging:
Introduction to the underlying concept:
A prospect for the instrumented oil field

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ABSTRACT

Why and how it is that the autocorrelation of natural noise gives us a reflection seismogram.

There I was, stuck in a dreary lecture, staring up at the fire sprinklers near the ceiling. I would much rather be conjuring up prospects for the instrumented oil field. What if that water nozzle near the ceiling was a geophone, and I was down here in the reservoir? If that geophone was recording all the noise in this room, all the people chattering and banging about, from all that sound, would I be able to figure out the shape of this room and the location of all the furniture in it? It reminds me of that old question, "Can you hear the shape of a drum?"

We are taught that sound waves and light waves are both waves, so why do we need to control the illumination in one case, but not the other? The idea of acoustic daylight imaging is that we should be able to make seismic images from the natural ambient seismic noises just as our eyes do with the natural ambient light.

One geophone is not enough. A line of geophones is not enough either. Imagine your eye had a retina with just one light sensing cell, or even a linear array of rods and cones. That still would not be enough. You need a retina with an areal array of sensors.

So, if the ceiling was covered with geophones, from those sounds, I could tell you the shape of this room and everything in it. I could even tell you if somebody moved any pieces of furniture. This is exactly what James Rickett and I showed the readers of The Leading Edge in August 1999 when we told them about the seismology of our sun, Sol.

THE LESSONS OF SOLAR SEISMOLOGY

To make a long story short, solar physicists figured out how to build an instrument like a video camera which when pointed at the sun uses the concept of Doppler shift to give us a virtual array of 1000×1000 seismometers on the sun's surface! Further, they can position and space their million seismometers just as easily as you can point and zoom a camera!

We had a lot of fun with those solar seismograms. We couldn't let off any shots up there

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on the sun, but we did have an experiment much like that of geophones on the ceiling of the lecture hall or the hypothetical instrumented oil field. We got a magnificent common-midpoint gather. Disappointingly, the sun is a fluid and all its CMP gathers are almost exactly the same. No reflections. No shear waves either. But there are plunging waves that come back up and reflect off the sun surface, and go back down again for multiple bounces, about 5-6 clear multiple reflections. Their velocity was 10 km/sec at the sun's surface growing rapidly with depth.

We call imaging based on the ambient noise, "acoustic daylight imaging". Our excursion into solar seismology left us with the embarrassing situation that we had demonstrated this concept on the sun before we were able to demonstrate it on earth.
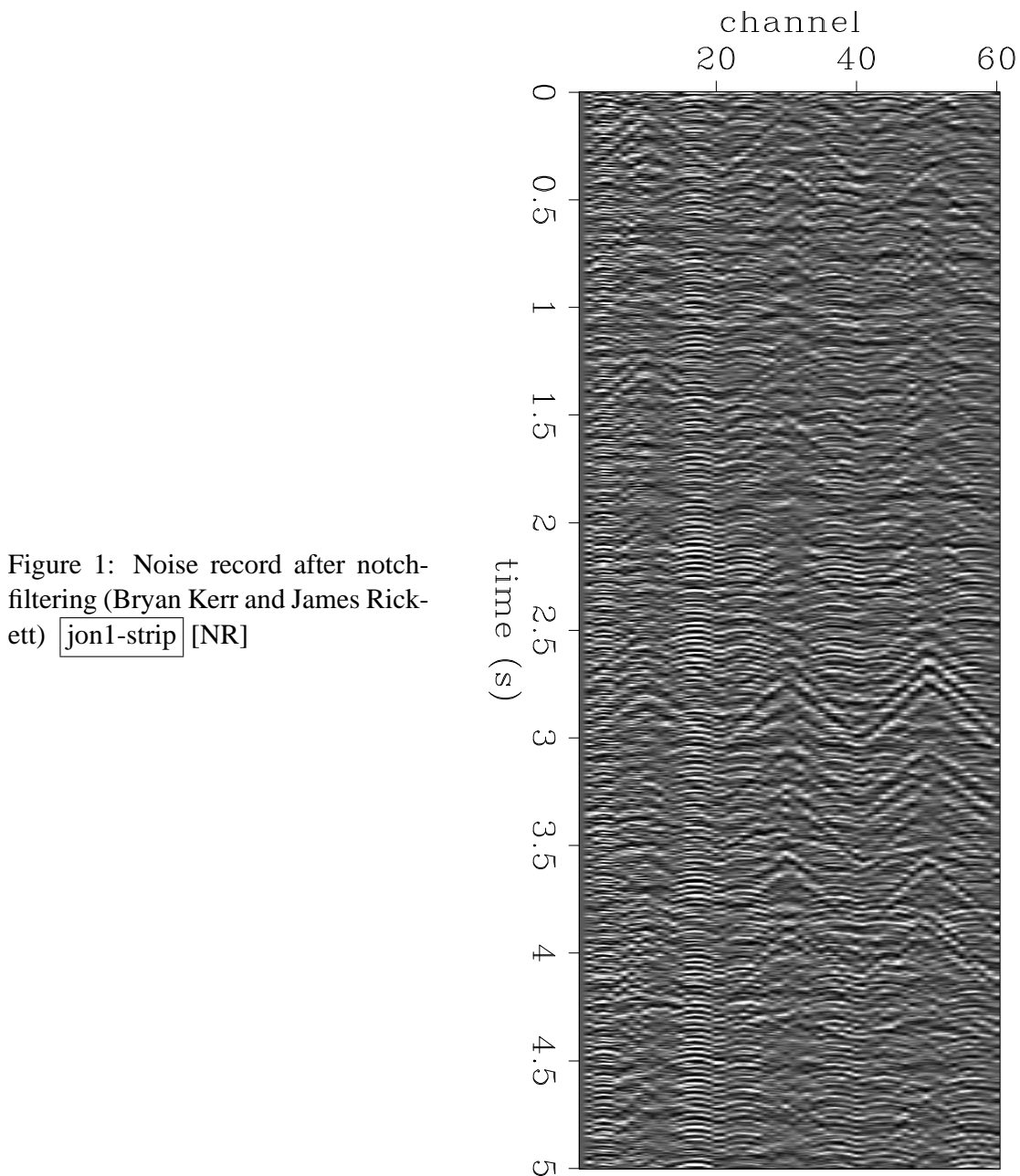

Figure 1: Noise record after notch-filtering (Bryan Kerr and James Rickett) jon1-strip [NR]

Since then we have had better luck on earth. Some students[2] set up a local geophone array. On their first try, without using any sources, they got the ground roll loud and clear, and some hints at reflections. Naturally, we feel optimistic about the instrumented oil field. We only hope they don't botch it when they build it. We live in a 3-D world, and we don't think 2-D data will work.
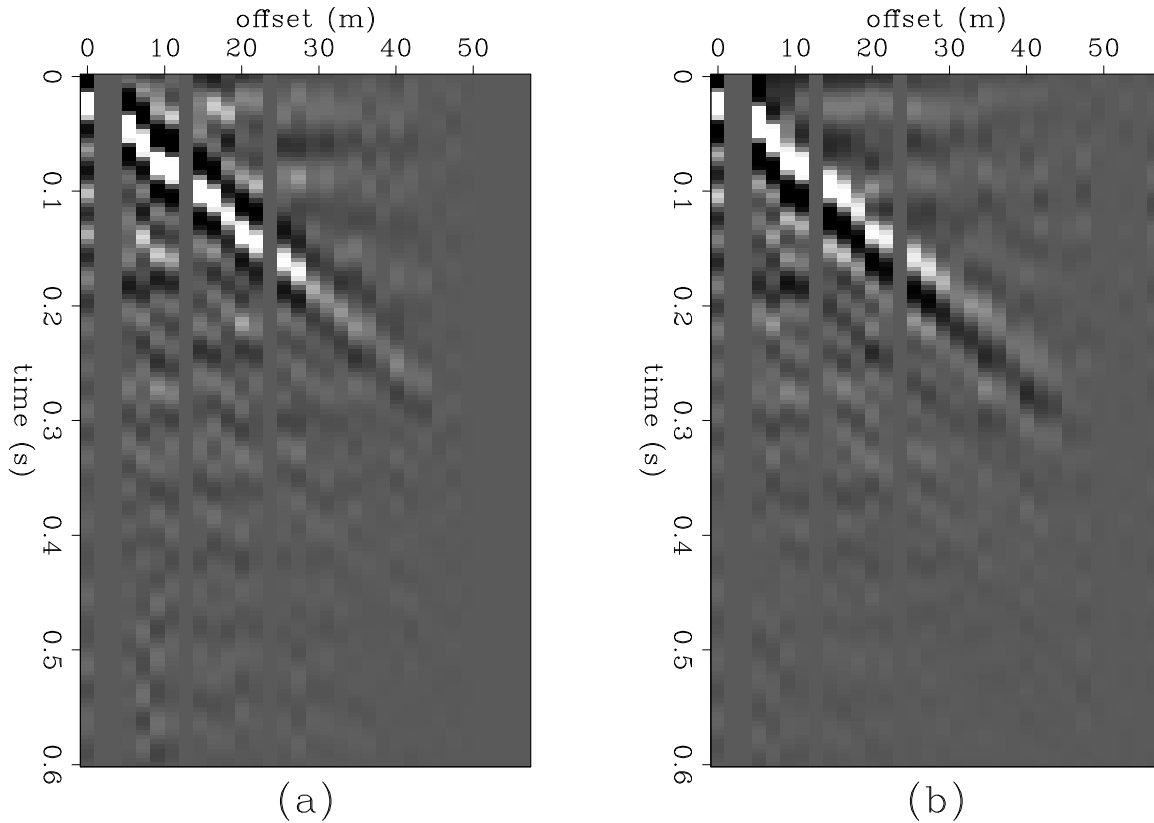

Figure 2: 3-D impulse response summed over azimuth calculated by two methods, (a) 3-D autocorrelation, and (b) spectral factorization. (Bryan Kerr and James Rickett) jon1-longimp [NR,M]

HOW IT WORKS

Since I first started (with some dense algebra in Geophysics), I figured out a way to explain the basic idea with almost no math. Almost none, but a little bit will help. Since we are going ahead with almost no math, you'll need to be satisfied with getting only the vertically incident seismogram at zero-offset, but you will get all the multiple reflections of all orders with correct amplitudes too.

Let us start off with a triangle function, say the sampled function $(3, 2, 1)$ which looks like

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[2]Preliminary results from a small-scale 3-D passive seismic study in Long Beach, CA, by Bryan Kerr and James Rickett (**?**), in an upcoming report of the Stanford Exploration Project

a triangle when you plot it. Suppose we have a downgoing wave which is a triangle function. We like to write it as $D = 3 + 2Z + Z^2$. You can think of Z as the the "unit delay operator" meaning that the "2" comes a moment after the "3" and the "1" comes two moments later corresponding to Z^2. Some of you will recognize Z as the Z-transform variable but that isn't necessary here.

One of the very most interesting things you can do with a signal is take its autocorrelation or compute its energy as a function of frequency. We do this for the triangle with this expression $\overline{D}D = (3 + 2/Z + 1/Z^2)(3 + 2Z + Z^2)$. If you multiply it out and look at the term that has no Z you see it is $3^2 + 2^2 + 1$ which is the total energy. Coefficients of other powers of Z are the autocorrelation at other lags. If you remember Fourier series, you will recall that when $Z = e^{i\omega\Delta t}$ we have the Fourier transform and spectrum. The expression $\overline{D}D = (3 + 2/Z + 1/Z^2)(3 + 2Z + Z^2)$ represents both the autocorrelation and the spectrum of the downgoing triangle wave. It represents the autocorrelation when you look at the coefficients of the product, and it represents the spectrum when you consider numerical values for Z.

From all of this Z business, the main thing we need to recognize is that $D = 3 + 2Z + Z^2$ is a causal function meaning that the "3" occurs at time $t = 0$ and everything else happens later. Likewise $\overline{D} = 3 + 2/Z + 1/Z^2$ is an anticausal function meaning that the action is before $t = 0$.

OK, now with all that theoretical background out of the way, let us put it to some use. Examine Figure 1. It depicts a pancake earth with two different experiments. On the left is the
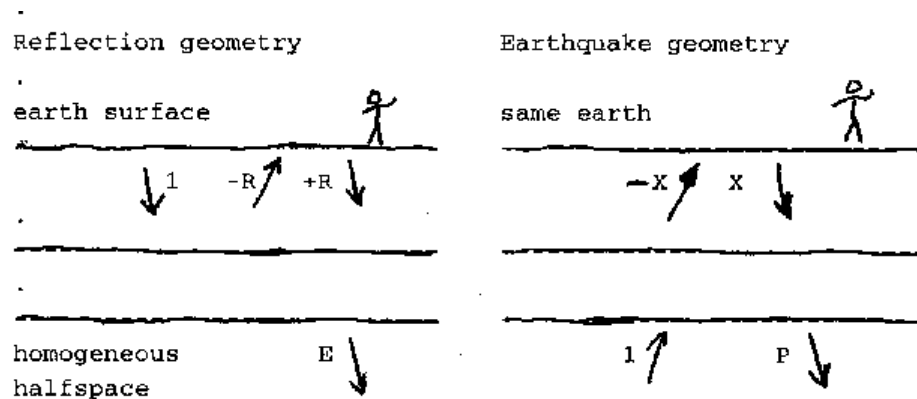
Reflection geometry   Earthquake geometry

earth surface   same earth

      ↓1   −R↗ +R↓   −x↗ x↓

homogeneous   E↓   1↗   P↓
halfspace

Figure 3: Pancake earth with prospector's boundary conditions (left) and earthquake boundary conditions (right) jon1-eqcor [NR]

one we are familiar with, reflection seismology. Theoretically, the geophysicist on the surface generates a downgoing impulse, the "1". The earth surface is called a free surface which means that it is perfectly reflecting. Any upcoming wave $U = u_1 Z + u_2 Z^2 + u_3 Z^3 + \cdots$ at the earth surface will create a downgoing wave D of opposite polarity $D = -U$. Of course the downgoing wave hits subsurface reflectors and makes another upcoming wave ad infinitum, and it all gets very complicated with lots and lots of multiple reflections. When all is said and done, however, at the earth's surface, the upcoming wave is opposite the downgoing wave

$U = -D$ at all times except the very first. At that first moment (actually the zeroth moment) there is a downgoing wave with no upcoming wave.

The story is different for the earthquake picture. Here the impulse "1" comes from the opposite side of the earth. The up- and down-going waves have exactly the opposite polarity at the earth surface at all times, so we write $U = -X$ and $D = X$.

Now we have finished with all our definitions and we are ready to see the most amazing correspondence between the prospectors' world and the earthquake world, and this correspondence is what gives us the potential to make big bucks in the instrumented oil field by knowing what is going on at all times.

We need the concept of conservation of energy flux. For this we need to make another simplifying assumption. You will recognize that this one isn't quite true, but it turns out to be a good engineering approximation. We assume that the earth does not absorb any acoustic energy.

We define the net downgoing energy flux as the downgoing energy minus the upgoing energy. If you are willing to think of numerical values for Z (Fourier transform) the net energy flux is actually a function of frequency. Let $D(Z)$ represent the downgoing wave and $U(Z)$ represent the upcoming wave. The net downgoing energy flux is $\overline{D}D - \overline{U}U$. By our assumption that no energy is absorbed, and since energy isn't piling up at any layer boundary, the downward energy flux in any one layer must equal to that in the next layer. Applying this idea to all the layers, we find that the net downgoing energy flux in the top layer matches that in the bottom halfspace. In other words, what goes in must come out, so the downgoing net energy flux anywhere is simply that at the bottom $\overline{E}E$. Thus

$$\overline{D}D - \overline{U}U = \overline{E}E$$

Next we apply this energy flux concept.

In the prospectors' world, at the earth's surface the downgoing wave is $1 + R$ and the upcoming wave is $-R$. We insert these values into the net energy flux and then simplify the algebra a little.

$$\overline{(1+R)}(1+R) - \overline{R}R = \overline{E}E$$
$$\overline{R} + 1 + R = \overline{E}E$$

OK, the hard part is done. We are done with the math and now we only need to understand what it means. The righthand side is a spectrum or an autocorrelation. Something like $\overline{E}E = (3 + 2/Z + 1/Z^2)(3 + 2Z + Z^2)$. The lefthand side has two parts, a causal waveform like $R = r_0 + r_1 Z + r_2 Z^2 + r_3 Z^3 + \cdots$ and an anticausal waveform like $\overline{R} = r_0 + r_1/Z + r_2/Z^2 + r_3/Z^3 + \cdots$. This is just about it, the Big Apple. Our energy flux statement says if we compute the autocorrelation $\overline{E}E$ of the escaping wave E we can look at one side of the autocorrelation to see the reflected wave R. That's amazing! We have finished with the central part of the story which is its hardest part. We are almost ready to pull it all together.

One more little step is required to get us all the way to acoustic daylight imaging. There is no more math, but there is a conceptual step, and it is a big one. (You didn't expect this to be

child's play, did you?) You may have heard of the Principle of Reciprocity. Reciprocity says a common midpoint gather is symmetric about zero offset. It says if you interchange a shot with a geophone you get the same seismogram. We all know a common midpoint gather is not *exactly* symmetric or people would never record split spread. Nobody does record marine data with split spread. Land data is different. Only with vibrators is it practical to come close to fulfilling the requirements of reciprocity since for reciprocity to apply, both the sender and receiver should be at the same depth. Let's stop all this quibbling about practicalities of reciprocity and accept it so that we can move forward to the acoustic daylight imaging concept.

Both the earthquake person listening to natural noises and the prospector are on the same earth. One experiment says we have an impulse of excitation at the surface, and a wave E goes off to the interior of the earth. The other experiment says we have an impulse incident from the deep interior of the earth and we see the wave X on the surface. Reciprocity says that $E = X$.

Putting $E = X$ into the energy flux equation gives us the main conclusion,

$$\overline{R} + 1 + R = \overline{X}X$$

which is the mathematics that says,

> "One side of the autocorrelation of the earthquake seismogram is the reflection seismogram."

LIMITING FACTORS

You might agree that the earth's natural noise comes from below, but you might not be willing to agree that it is an impulse function. Of course you are correct. What saves the day is that we depend only upon the *autocorrelation* of the natural noises. In other words, the theorem is equally true if the autocorrelation of the natural noises incident from below is an impulse function.

The potential nonimpulsiveness of the autocorrelation of the random noise excitation identifies a practical problem which leads to processing design issues.

Expressed back in the framework of our boring lecture room, if all the noise makers are tuned into the key of C, our final images are certain to resonate at that frequency. Luckily, natural noises do seem to have a fairly rich spectrum of frequencies and directions of arrival. Luckily also in imaging theory (and practice), when something is missing, it doesn't ruin the image; it merely degrades it. Imaging theory and practice are not so fragile as inversion. Thus missing frequencies and directions of illumination should not wholly frustrate our activity, but naturally they would limit us accordingly.

Another side of the same problem is more serious. When too much light is coming from one direction we need to shield ourselves from the glare. There is an analog in seismology. We have the practical problem that a great deal of the natural noises come in the form of

ground roll. We cannot escape this problem without "shielding our eyes" with adequate spatial filtering. This is why simple 2-D arrays seem to us to be inadequate. With Steve Cole's PhD dissertation, we did set out a lot of geophones (4056), and they did suppress a lot of surface noises, enough so that we could see the natural noises arriving from great distances.

I have always felt the great thing about seismology is that it really works. Hundreds and thousands of wiggles and eventually it has often come to make good sense. Our experiments are truly repeatable. In time we really do learn more. Years ago with 2-D seismology we had "noises" but now with 3-D seismology, we realize those noises were the geology, now beautifully imaged. Our field has not always treated everyone kindly, but in our field, the gap between theory and practice has always been a good place to put your eyes to see the future.