Introduction Subsurface images provided by the seismic reflection method are the single most important tool used in oil and gas exploration. Almost exclusively, our conceptual model of the seismic reflection method, and consequently our seismic data processing algorithms, treat primary reflections, those waves that are scattered back towards the surface only once, as the "signal". The traveltimes of the primary reflections are used to map the structure of lithology contrasts while their amplitudes provide information about the magnitude of the lithology contrasts as well as other information such as presence or absence of fluids in the pore spaces of the rock. All other waves such as multiples, waves that are scattered back toward the surface more than once, are considered "noise".

CLASSIFICATION OF MULTIPLES

There are many types of multiples, some of which are illustrated in Figure 1. For the purpose of this thesis, however, multiples will be classified in two main categories: specular multiples and diffracted multiples. Specular multiples are those that reflect as light rays, following Snell's law at the reflection points. Diffracted multiples, in contrast, are scattered in all directions at the diffractor location. I will further classify the diffracted first-order multiples into receiver-side and source-side multiples depending on which side the diffractor lies as shown in panels (a) and (b) of Figure 2 respectively. The dashed lines represent the arbitrary trajectories that the multiple may take from the diffractor to the receiver (for a receiver-side diffracted multiple) or from the source to the diffractor (for a source-side diffracted multiple). The travelpath from the diffractor to the receiver is independent of the source location for a receiver-side multiple and likewise the travelpath from the source to the diffractor is independent of the receiver location for a source-side multiple. This behavior makes the kinematics of specular and diffracted multiples very different in Common-midpoint (CMP) gathers. Specular multiples have a moveout curve that is symmetric around their apex at zero offset, since reciprocity requires the same traveltime for rays from the source location to the receiver location and from the receiver location to the source location. Diffracted multiples, on the other hand, do not have their apex at zero offset (**?**) and are therefore not symmetric around zero offset as shown in Figure 3. The travelpath of the receiver side multiple, for example, is not the same if the source and receiver locations are interchanged. Reciprocity is not violated, however. Receiver-side multiples just become source-side multiples and vice-versa as illustrated in Figure 2. A similar splitting of the source- and receiver-side multiple happen with peg-leg multiples from a dipping reflector (**?**).

DATA SPACE AND IMAGE SPACE

In this thesis, I refer to data space as the un-migrated space. This means data as a function of time. I consider two main sets of data: source gathers and CMP gathers. The first are function of the source coordinates, offsets and time while the second are

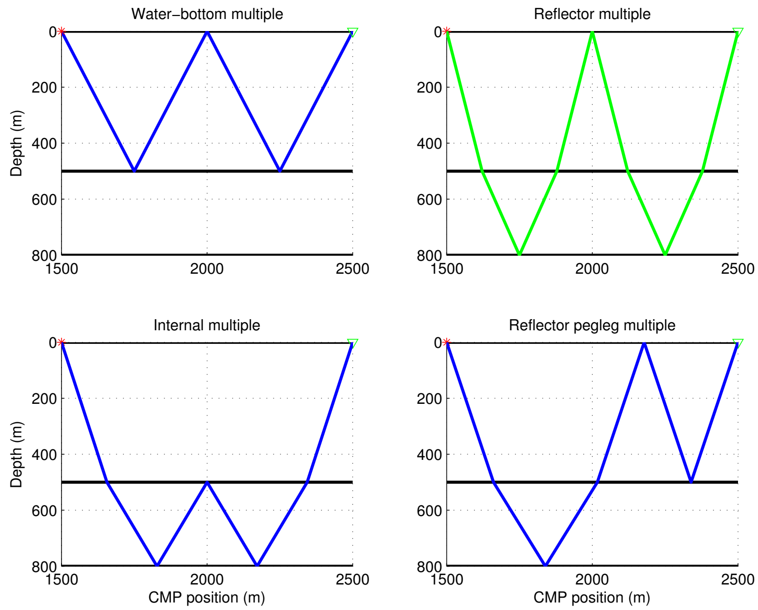

Figure 1: Examples of 2D specular multiples.

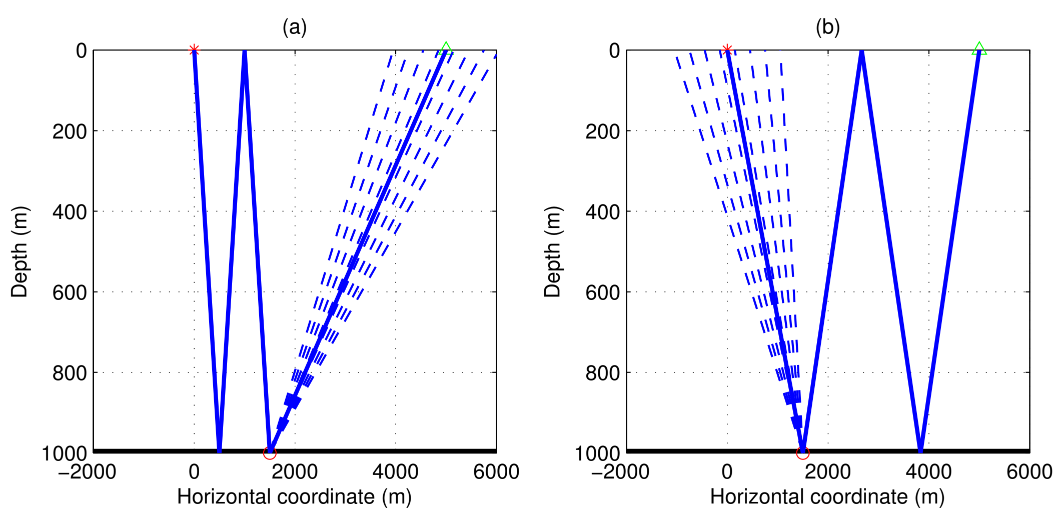

Figure 2: Schematics of receiver-side (a) and source-side (b) diffracted multiples. The asterisk represents the source and the triangle represents a receiver. The dashed lines indicate possible trajectories of the diffracted rays from the diffractor to the receiver or from the source to the diffractor. The diffractor itself is indicated by the empty circle at about 1600 m.

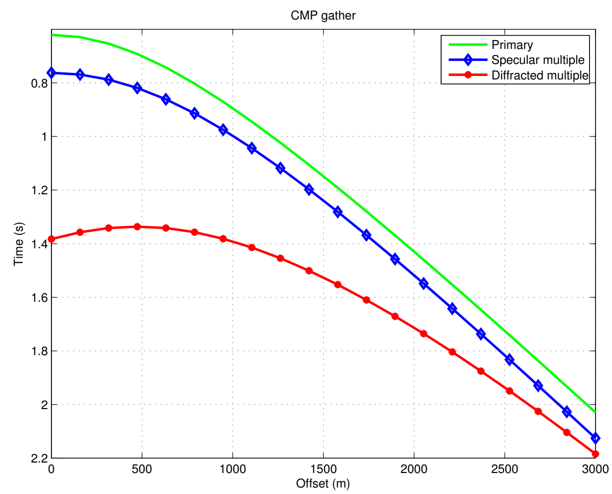

Figure 3: Comparison of the moveout curves of a primary, a specular multiple and a diffracted multiple in a CMP gather. Notice that the apex of the diffracted multiple is not at zero offset.

function of the CMP coordinates, *half*-offsets and time.

I will refer to image space as the domain of migrated data. In particular, data migrated in depth. In image space I consider two main datasets: Subsurface Offset Common Image Gathers (SODCIGs) and Angle Domain Common Image Gathers (ADCIGs). In this thesis I will use exclusively wave-equation migration algorithms.

ATTENUATION OF MULTIPLES IN DATA SPACE

The standard approach in seismic data processing is to attenuate the multiples before imaging, that is, in data space. Most algorithms for the attenuation of multiples in data space are based on three main characteristics of the multiples: (1) their periodicity in arrival time (predictive deconvolution), (2) their difference in moveout with respect to the primaries in CMPs (f-k and τ-p filtering) and (3) their predictability as the auto-convolution of the primaries (Surface Related Multiple Elimination (SRME)). Each of these approaches have distinctive advantages and disadvantages.

Predictive deconvolution

The attenuation of short-period multiples (most notably reverberations from relatively flat, shallow water-bottom) can be achieved with predictive deconvolution. The periodicity of the multiples is exploited to design an operator that identifies and removes the predictable part of the wavelet (multiples), leaving only its non-predictable part (signal). The key assumption is that genuine reflections come from an earth reflectivity series that can be considered random and therefore not predictable (Yilmaz, 1987). In general, for other than short-period multiples, only moderate success can be achieved with this simple, one-dimensional procedure.

In principle, deterministic deconvolution can be used to remove water-bottom reverberations when the exact depth and speed of sound of the water layer are known. Since these conditions are rarely met, deterministic deconvolution is not widely used, despite the elegance of its closed, exact mathematical formulation (**?**).

Moveout-based filtering

Primaries and multiples exhibit hyperbolic moveout in CMPs but their curvature is different. After Normal Moveout (NMO) correction with the NMO velocity of the primaries, ideally the primaries exhibit flat moveout whereas the residual moveout of the multiples can be approximated by parabolas or hyperbolas (**??**). This difference in moveout can be exploited to separate the primaries from the multiples in either the f-k domain or the τ-p (Radon) domain.

The performance of an f-k filter in suppressing multiples strongly depends on primary and multiple reflections being mapped to separate regions of the f-k plane. This is in general the case on far-offset traces, for which the difference in moveout can be large, but not on short-offset traces for which the difference in moveout is small. The performance of f-k filtering, therefore, is poor at small offsets even if the subsurface geology is not very complex. This usually makes f-k filtering an undesirable option for multiple elimination.

Radon demultiple in data space (**??**) has proven successful in attenuating specular multiples if the subsurface is not very complex. In complex subsurface areas, such as under salt, the hyperbolic or in fact any NMO approximation breaks down. The NMO velocities are inaccurate and therefore, after NMO, the primaries are unlikely to be flat. Furthermore, the residual moveout of the multiples is unlikely to be well approximated by parabolas or hyperbolas. The quality of the separation between primaries and multiples in the Radon domain, and their focusing, therefore, deteriorates. As a result, multiples are imperfectly attenuated and, worse, the attenuation is offset dependent. In such complex areas, Radon demultiple in data space is not a good option.

SRME

Surface-related multiple elimination (SRME) uses the recorded seismic data to predict and iteratively subtract the multiple series (**?**). The key advantage of SRME is that it needs no subsurface information whatsoever. The multiples are completely predicted from the data. 2D SRME can deal with all kinds of surface-related 2D multiples, provided all relevant data are recorded within the aperture and offset limitations of the survey line. Predicting 3D multiples with 2D SRME is hazardous because the accuracy of the prediction depends on the amount of crossline dip. Figure 4 shows an example with 3D real data. Panel (a) is an inline common-offset section with an obvious water-bottom multiple. Panel (b) is the inline offset gather taken at horizontal position 12000 m. Panel (c) is the multiple prediction with 2D SRME and panel (d) is the offset gather for the predicted data (figure courtesy of Bill Curry). The prediction is very good except at the canyon where the water-bottom has non-negligible crossline dip. Notice that the predicted multiple arrives later than the multiple in the data inside the canyon but arrives at the correct times to either side of the canyon.

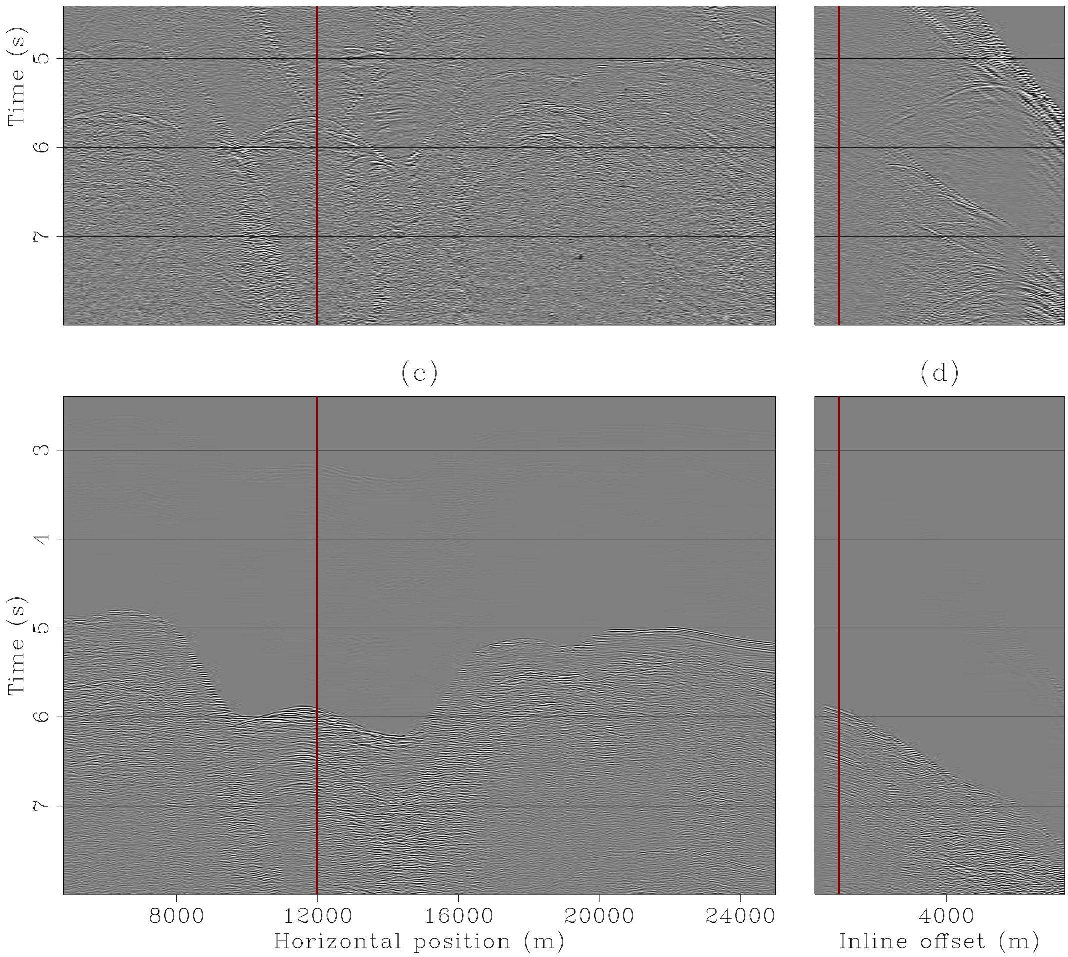

Figure 4: Prediction of 3D multiple with 2D SRME. Panel (a) is an inline common-offset section from a 3D survey. Panel (b) is an offset gather. Panel (c) is the inline common-offset section of the predicted multiple with 2D SRME and panel (d) is the offset gather of the predicted multiple. Notice that the prediction is good away from the canyon (the arrival times of the predicted multiple match those of the data) but no in the canyon where the predicted arrival times of the multiple are larger due to the crossline dip of the canyon.

Prediction of multiples from reflectors with crossline dip, and diffracted multiples, especially those from scatterers with a cross-line offset component, cannot be accurately achieved with 2D SRME. Predicting these multiples requires the much more expensive 3D SRME. If the acquisition of the 3D survey is regular and dense enough, the survey apertures large enough in both in-line and cross-line directions, and there is no feathering, 3D SRME performs well. With standard marine streamer acquisition, however, the sampling in the cross-line direction is too coarse, the cross-line aperture is too narrow, short offsets are not recorded and feathering and acquisition obstacles make the acquisition geometry irregular. Any multiple whose surface bounce is not recorded can not be predicted by 3D SRME. Again, diffracted multiples, and multiples from a reflector with significant crossline dip pose the most serious problem because their surface bounce is likely to lie way outside the relatively narrow recording patch. Figure 5 shows a schematic map view of this situation. The empty circle represents the surface bounce of the multiple and, since there is not detector at that location, 3D SRME cannot predict that multiple. We do not need a particularly convoluted subsurface for this situation to arise in practice. All it takes is crossline dip of the water-bottom or the multiple-generating surface, or the presence of diffractors. These multiples, therefore, need to be removed by other methods (**?**) or the data need to be interpolated and extrapolated to a dense, large aperture grid (**?????**). Interpolation is complicated by the coarse sampling that may introduce aliasing in the steep flanks of the multiple multiple moveout curves. Also, internal multiples cannot be predicted by SRME, unless the data is successively datumed to every multiple-generating interface. This is obviously a time-consuming process that is seldom, if ever, carried out. In most situations this is not a serious drawback, however, since internal multiples are usually weak. The exception is internal multiples from very strong reflectors such as salt boundaries.

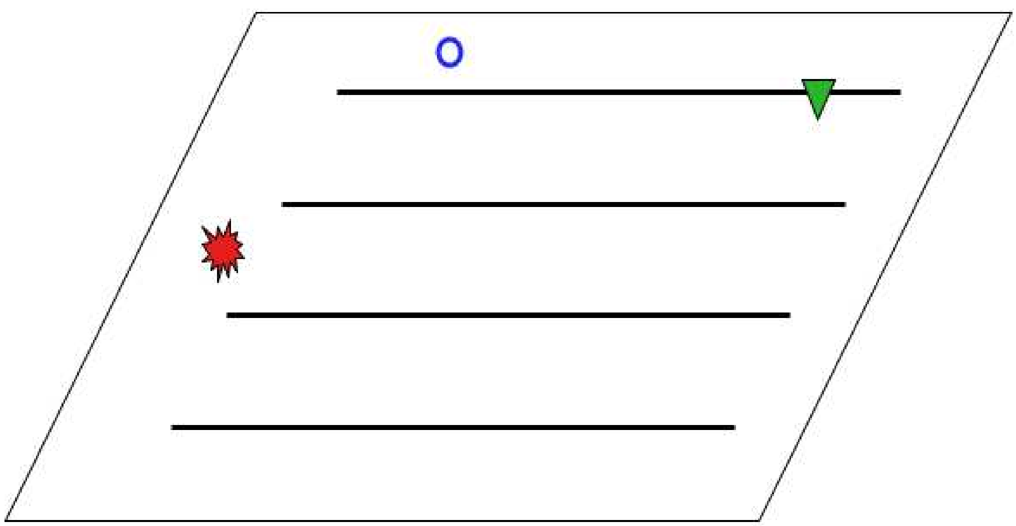

Figure 5: Schematic map view of a simple situation in which 3D SRME can not predict a multiple. The subsurface has significant crossline dip such that the surface bounce of the multiple (indicated by the empty circle), lies outside of the recording patch and therefore the multiple can not be predicted from the data.

PROBLEM DESCRIPTION

The previous section briefly described the weaknesses of the standard multiple attenuation approaches in particular when applied to sparse 3D data over complex subsurface. Data space methods cannot handle the wave distortions associated to complex wave travelpaths and 3D SRME requires data that is not usually acquired.

The issue I address in this thesis is the development of a relatively simple, practical algorithm, that can attenuate both specular and diffracted multiples for 2D and 3D data acquired with standard marine, narrow-azimuth towed-streamer geometry. The method uses only the recorded data and does not need costly and often inaccurate massive data interpolation and extrapolation. It does, however, require a reasonably accurate migration velocity field. The algorithm works in the image space, meaning it is applied after the data have been migrated. Since wave equation migration accurately handles complex wave propagation, the method works well for data acquired over complex subsurface regions such as under salt, again, provided the migration velocity field is reasonably accurate.

This thesis also makes theoretical contributions that explain the process by which prestack wave equation migration maps multiples from data space (CMP gathers) to image space (ADCIGs). In particular, I develop the equations that explain the residual moveout of canonical multiples in ADCIGs for both specular and diffracted multiples. I demonstrate that the specular multiples are focused similar to primaries whereas the diffracted multiples are not. For 3D data I demonstrate that the reflection azimuth dependency as a function of the dip angle is different for primaries and multiples. Likewise for the aperture angle dependency as a function of reflection azimuth. I develop a Radon transform that separates the primaries from the multiples as a function of both aperture and reflection azimuth angles.

I also develop in this thesis a new approach to the matching and adaptive subtraction of the multiple model from the data. Unlike SRME, the image space Radon transform allows the estimation of a primary model along with the estimation of the multiple model. I exploit this capability to design a nonlinear inversion approach that simultaneously matches and adaptively subtracts from the data both the estimate of the multiples and the estimate of the primaries. The effect is to reduce the well-known crosstalk problem, *i.e.*, that residual multiple energy that contaminates the estimate of the primaries.

THESIS OVERVIEW

The next chapters develop the subject of the thesis for both 2D and 3D data. In both cases I illustrate the results with synthetic and real data.

Chapter 2: Attenuation of 2D multiples

In this chapter I present the equations that map specular and diffracted 2D multiples from data space to image space. I implement an apex-shifted Radon transform that separates primaries from specular and diffracted multiples in the Radon domain as a function of depth, residual moveout curvature, and apex shift. I then apply the method to a real 2D line from the Gulf of Mexico plagued with strong subsalt specular and diffracted multiples. I show that most of the multiples can be attenuated without significantly affecting the amplitudes of the primaries.

Chapter 3: Simultaneous matching and adaptive subtraction of primaries and multiples

In this chapter I address the issue of subtracting an estimated multiple model from data containing primaries and multiples. This is a key step in multiple attenuation methods such as SRME in which the multiple model is expected to have wavelet differences with respect to the data. Instead of just matching and adaptively subtracting the multiple model, I simultaneously match the estimates of both the primaries and the multiples to the data. This has the key advantage of reducing the crosstalk from the multiples in the final estimate of the primaries. I illustrate the method with both synthetic and real 2D data in CMP and angle gathers. Furthermore, I show that the method can be used beyond the attenuation of multiples by applying it to the matching and adaptive subtraction of ground-roll. The adaptive subtraction presented in this chapter is used to compute the results of Chapter **??**.

Chapter 4: Mapping of 3D multiples to image space: Theory and a synthetic data example

The main problem in the attenuation of multiples is with narrow-azimuth towed-streamer 3D data. In this chapter I extend the basic equations for the residual moveout of specular multiples in ADCIGs for 3D data. I show that the mapping of the multiples is similar to the 2D case, except that the crossline dip generates an azimuth dependency that is different for primaries and multiples. I use a simple 3D synthetic prestack dataset with two primary reflections and two specular multiples to illustrate the mapping of the multiples to 3D ADCIGs. Since the 3D ADCIGs are function of both aperture angle and reflection azimuth, their interpretation is not trivial. I describe in some detail the information contained in the 3D ADCIGs and show how the multiples and the primaries have very different residual moveout. This chapter lays the foundation for the application of the method to the real 3D dataset of Chapters **??** and **??**.

Chapter 5: Imaging and mapping 3D multiples to image gathers: Example with a Gulf of Mexico dataset.

In this chapter I illustrate the mapping of 3D multiples to image space with a real 3D dataset from the Gulf of Mexico. The data has all the usual shortcomings associated with marine streamer acquisition: sparse sampling in the crossline direction, small crossline aperture, strong feathering, irregularity in the sail lines and uneven midpoint fold. I use shot-profile migration to compute SODCIGs and from them compute 3D ADCIGs. I show that in this case we can discriminate between primaries and multiples in both inline and the crossline subsurface offsets in SODCIGs and aperture and reflection azimuth in ADCIGs, despite the relatively narrow range of aperture angles that illuminate the subsalt reflectors.

Chapter 6: Attenuation of 3D subsalt multiples with Gulf of Mexico dataset

In this chapter I tie up the different components presented in the previous chapters. I attenuate subsalt multiples from the real dataset migrated in the previous chapter. I show that some multiple attenuation can be achieved by muting the multiple energy away from zero subsurface offsets in SODCIGs and stacking the results. I obtain much better attenuation of the multiples by applying Radon filtering on azimuth-stacked ADCIGs.

Chapter 7: Conclusions

This last chapter summarizes the conclusions of the thesis.