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Separation of converted modes in marine data

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ABSTRACT

Two methods are tested to separate the converted modes from the dominant P wavefield. The first one uses the critical slowness for P waves as the discriminant factor and is ineffective for deep reflectors. A better result is obtained with another approach, in which the upcoming wavefield is divided into zones sharing a common range of ray parameter; a filtering process using the horizontal slowness as the discriminant factor is then applied to each zone. Although more efficient, this method is more sensitive to contamination by multiples. For this reason the suppression of multiples and peg-legs prior to the separation process becomes of vital importance.

INTRODUCTION

Wave propagation in a marine environment can be separated into two parts: the propagation through the water, in which the acoustic wave equation applies, and the propagation in the subsurface strata, in which the elastic wave equation applies. Propagation in the water is described by the scalar pressure wavefield recorded by the hydrophones; in solid layers the displacement vector, or elastic field separation should be used. The coupling of the two potentials can be achieved by the application of the continuity of the normal components of stress and displacement at the liquid-solid interface.

The conversion of the scalar recorded field into the elastic wavefield at the ocean floor and the subsequent separation of the converted wavefield can be achieved by the application of successive steps.

First the recorded pressure field is decomposed into the upcoming and downgoing wavefields (Claerbout, 1971), and since the downgoing wave at the cable is just the upcoming wave reflected at the water surface (or ghost), it is possible to express the upcoming wavefield as a function of the recorded wavefield. Then, the principle of survey sinking (Schultz and Sherwood, 1980; Claerbout, 1985) is used to extrapolate the upcoming pressure field down to the ocean floor. The next step is the conversion

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of the scalar pressure field into the displacement amplitude field of the P waves in the water; this conversion is accomplished through a simple relation in the $w - k_x$ domain. Finally, a slowness filter is used to obtain the displacement amplitudes of P and SV waves below the water bottom interface. This filtering process shows to be more effective if a prior separation of the data into *P wave Snell rays domains* is performed, so that a different filter can be used for each domain.

UPCOMING PRESSURE FIELD AT SEA FLOOR

As shown in Figure **??**, the recorded pressure field can be decomposed into the downgoing and upcoming wavefields at the cable depth

$$\phi(x,z,t) = \acute{\phi}(x,z,t) + \grave{\phi}(x,z,t), \tag{1}$$

and, assuming a perfect reflection at the water's surface,

$$\phi(x, z = z_0, t) = \acute{\phi}(x, z = z_0, t) - \acute{\phi}(x, z = -z_0, t).$$

Let $\mathcal{L}(z)$ be the extrapolation operator for upcoming waves

$$\acute{\phi}(x, z = -z_0, t) = \mathcal{L}(-2z_0)\acute{\phi}(x, z = z_0, t),$$

then

$$\acute{\phi}(x, z = z_0, t) = [1 - \mathcal{L}(-2z_0)]^{-1}\phi(x, z = z_0, t).$$

and since $\qquad \acute{\phi}(x,z,t) = \mathcal{L}(z - z_0)\acute{\phi}(x, z = z_0, t) \qquad$ we get

$$\acute{\phi}(x,z,t) = \mathcal{H}(z, z0)\ \phi(x, z = z_0, t), \tag{2}$$

where

$$\mathcal{H}(z, z_0) = \mathcal{L}(z - z_0)[1 - \mathcal{L}(-2z_0)]^{-1}. \tag{3}$$

The operator defined in equation (3) performs two distinct processes: the conversion of the recorded pressure field into the upcoming pressure field, and its downward continuation to the sea bottom.

In the ω-k_x domain, the downward extrapolation operator for upcoming waves can be expressed by

$$\mathcal{L}(z) = e^{ik_z z} = e^{-i\sqrt{\frac{\omega^2}{v^2} - k_x^2}\, z},$$

which can be substituted into the Fourier transform of equation (2) to obtain

$$\acute{\Phi}(k_x, z, \omega) = \mathcal{H}\ \Phi(k_x, z = z_0, \omega), \tag{4}$$

where

$$\mathcal{H}(z, z_0, k_x, \omega) = \frac{e^{ik_z z}}{2\ i\ \sin k_z z_0}. \tag{5}$$

It is evident from equations (3) and (5) that the operator $\mathcal{H}$ has poles at $k_z z_0 = n\pi$, corresponding to vertical wavelengths of $\frac{2\,z_0}{n}$.

For waves with these wavelengths, the downgoing and upcoming fields will cancel each other at the cable depth (assuming perfect reflection at the water's surface), and the result is a null recorded wavefield. Of course no information can be obtained for the upcoming field in this case, but since it represents only a couple of lines [2] in the ω-k_x plane we can neglect its contribution for the final, downward continued wavefield.

SEPARATION OF P AND S WAVEFIELDS

To apply scattering theory we need to transform the pressure field into a displacement field. The pressure wavefield obeys the scalar wave equation

$$\nabla^2\phi(x, z, \omega) = -\frac{\omega^2}{v^2}\phi(x, z, \omega).$$

The P wave displacement $\vec{p}$ is related to the pressure by the following expression:

$\phi = -K\,\nabla\cdot\vec{p},$ where K is the bulk modulus of water.

Recalling that the medium (water) is homogeneous and that the displacement field is irrotational, we can use the relation $\nabla^2 = \nabla\cdot\nabla$ and combine the two previous equations to get

$$\vec{p} = \frac{v^2}{\omega^2 K}\nabla\phi.$$

[2] For $\triangle t$ = 4ms and z_0 = 10m, only n=0 and n=1.

This equation relates the displacement vector field to the scalar pressure field and can be rewritten in the following form:

$$\begin{pmatrix} p_x \\ p_z \end{pmatrix} = \frac{v^2}{\omega^2 K} \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial z} \end{pmatrix} \phi(x, z, \omega).$$

And after a double spatial Fourier transform, it will be expressed by

$$\begin{pmatrix} P_x(k_x, k_z, \omega) \\ P_z(k_x, k_z, \omega) \end{pmatrix} = i \frac{v^2}{\omega^2 K} \begin{pmatrix} k_x \\ k_z \end{pmatrix} \Phi(k_x, k_z, \omega).$$

Finally, the displacement amplitude $\acute{P}$ of the upcoming P wave on the water can be expressed as a function of the upcoming pressure field:

$$\acute{P} = i \frac{v^2}{\omega^2 K} \sqrt{k_x^2 + k_z^2}\, \acute{\Phi}$$

or

$$\acute{P} = \frac{v}{-i\omega K} \acute{\Phi}.$$

and, as one should expect, the pressure field differs from the velocity field (time derivative of the displacement) by just a scale factor $\frac{v}{K}$.

Figure **??** shows how the upcoming P and S waves are scattered at the ocean bottom interface. The displacement amplitude of the upcoming pressure field just above the sea floor can be expressed as functions of the displacement amplitudes of all the incident waves:

$$\acute{P}_1 = [\acute{P}\acute{P}]_{2-1} \acute{P}_2 + [\acute{S}\acute{P}]_{2-1} \acute{S}_2 + [\grave{P}\acute{P}]_{1-1} \grave{P}_1, \tag{6}$$

where $[\acute{P}\acute{P}]_{2-1}$ is the transmission coefficient for upcoming P into P waves, $[\acute{S}\acute{P}]_{2-1}$ is the transmission coefficient for upcoming S into P waves , and $[\grave{P}\acute{P}]_{1-1}$ is the reflection coefficient for downgoing P into P waves (Aki and Richards, 1980). The last term in the previous equation corresponds to the ocean floor primary reflection and all its associated multiples and peg-legs. The conditions most suitable for the generation of converted waves in marine surveys are the same ones that are responsible for the occurrence of strong multiple contamination. While the ocean floor primary will not interfere with any converted wave, the multiples and peg-legs must be removed in order to isolate the PSSP wavefield. The importance of this step and the method used for its accomplishment are the subjects of Appendix A.

The two remaining terms in equation (6) (which correspond to the P and S wavefields) can be reasonably separated by the use of the critical angle for P waves at the first layer as a discriminant factor (Cunha, 1989), as illustrated on Figure **??**. Waves of the type PPPP, PPSP and PSPP will be found only with horizontal slownesses smaller than the slowness corresponding to the ocean floor critical angle for P waves (30 degrees on the figure), while only the PSSP mode will be found at higher horizontal slownesses. As a first approach we consider that only the dominant mode (PPPP) is present for pre-critical horizontal slownesses.

Under these assumptions, a "P-sensible receiver" coupled at the sea bottom would record a displacement

$$\acute{P}_2 = \frac{1}{[\acute{P}\acute{P}]_{2-1}} \acute{P}_1 \qquad \text{for} \qquad p = \tfrac{k_x}{\omega} < \tfrac{1}{V_p}, \tag{7}$$

whereas an "S-sensible receiver" would record

$$\acute{S}_2 = \frac{1}{[\acute{S}\acute{P}]_{2-1}} \acute{P}_1 \qquad \text{for} \qquad p = \tfrac{k_x}{\omega} \geq \tfrac{1}{V_p}, \tag{8}$$

where V_p is the velocity of P waves at the first layer.

DIVIDING THE DATA INTO SNELL ZONES

Although the use of the critical horizontal slowness to perform the separation between P and S waves is effective for shallow reflectors, the finite size of the cable restricts its application to deep reflectors because a converted wave with horizontal slowness larger than the P wave critical slowness will be received at very large offsets.

A possible way to overcome the limitations associated with the finiteness of the field aperture is to use a criterion that somehow takes into account the depth of the reflector. Transforming the data into the τ-p domain provides the necessary flexibility for use of a variable slowness cutoff. Tatham and Goolsbee (1984) showed also that better results can be achieved when a hyperbolic velocity filtering is used during the τ-p transform to limit the range of reasonable stacking velocities.

As we will see, the separation of the data into ranges of *Snell rays* would seem to be another appropriate way to perform the slowness filtering with a variable-slowness cutoff.

A Snell ray (Ottolini, 1982) can be defined for a plane-layered earth as a ray that keeps a constant horizontal slowness (obeys the Snell law) while it propagates through the subsurface, as illustrated in Figure **??**. The reflection points corresponding to a Snell ray with ray parameter (or horizontal slowness) p are defined by

$$\frac{\partial x}{\partial t} = pv^2(z), \qquad \text{where} \qquad v(z) \qquad \text{is the interval velocity at depth } z.$$

For the simple case of a plane-layered earth, the horizontal slowness of a P wave will always be lower than the horizontal slowness of a converted wave recorded at the same position in the *x-t* domain. It is possible then to choose a set of ray parameters and divide the data into regions whose P waves Snell rays are limited by two adjacent values (Figure **??**). Since the converted waves inside each region will have higher horizontal slownesses than the P waves, a different filter can be applied to each region; the cutoff value is given by the ray parameter of the Snell ray that defines the upper limit of that region.

SYNTHETIC MODEL AND REALITY

The method has been applied on a 1D elastic model, generated by a program that uses Haskel-Thompson propagation matrices. Figure **??** shows the original synthetic seismogram and the model, which contains a total of seven layers overlaid by water, each of them with constant density and Poisson's ratio. The group interval is 50 m with 48 channels and a maximum offset of 2650 m.

To facilitate the identification of each of the several superposed events, four other auxiliary synthetics were also generated. Two of them are simple ray-tracing synthetics for PSPP-PPSP primary arrivals (Figure **??***a*) and for PSSP primary arrivals (Figure **??***b*); the sea-bottom reflection is also included in both seismograms. The other two correspond to an acoustical equivalent seismogram using the same P wave velocities as the elastic model (Figure **??***a*), and a seismogram free of **"water-layer multiples"** (Figure **??***b*), where air was replaced by water.

All of the three most basic converted modes (PPSP, PPSP and PSPP) are present on Figure **??***b* (without the interference of multiples), but not on Figure **??***a*; for example, the events that reach the farthest offset of Figure **??***b* at 2.4s (PSPP) and 3.0s (PSSP) or the nearest offset at 0.9s (PSPP), 1.05(PSSP) and 1.8(PSPP) and are all absent on Figure **??***a* and hard to distinguish on Figure **??***b* .

A comparison between the model before and after the application of the multiples-suppression algorithm is showed in Figures **??***a* and **??***b.* It is evident that the method fails to remove the multiples in the near traces. The reasons for that result are discussed along with the description of the method in Appendix A. Whereas the suppression's efficiency at the small offsets is not relevant for the present goals, the algorithm's performance at large offsets is crucial for the isolation of the PSSP wavefield. Several converted-waves arrivals are more evident after the multiple's attenuation, such as the reflections that cross the farthest offset at 3.0s and 4.05s on section *b* of Figures **??** and **??** and are absent on section *a* of both figures. Meanwhile, some further refinement is still required for the multiples-removal process at large offsets, as it can be seen by the presence of a PPSP water-bottom multiple at 2.7s in the last traces of the deconvolved data.

When a constant horizontal-slowness filter is used to perform the S wavefield separation, the result exhibits an "artificial appearance" due to the restricted slowness range accepted by it. In contrast, the splitting method based on the separation of the data into different Snell domains preserves a most suitable range of stepouts in the S wavefield section. Figures ??*a* and ??*b* provide a good comparison between the two methods. All the events are doubtless more clearly discernible in *b* than in *a* . A correlation between Figure ?? (which contains the expected positions of the converted primary reflections) and Figure ??*b* (which corresponds to the separated S field) shows that, although most of the desired events are present, some multiples associated with converted waves are also visible, such as the ones that intercept the last trace at 3.5s and 4.5s. These events correspond to multiples that were not satisfactorily eliminated in the suppression process.

Since multiples and primaries have the same horizontal slownesses, the multiples of P waves will be also eliminated during the filtering process when a constant slowness cutoff is used. However, when the data is divided into different ranges of horizontal slowness so that a variable filter can be applied, the multiples and primaries inside a region will have different horizontal slownesses and the contamination with multiples will became critical.

An overall idea of the differences between the two methods of mode separation and the contribution of the multiples-suppression can be achieved if we contrast the velocity analysis panels regarding each step (Figure ??*a* -*d*). The panel in *a* refers to the original model while the velocity analysis in *b* corresponds to the output of the multiples-suppression process. The differences of interest appear only on the shallow events, like the one with velocity 1350m/s at 0.75s that is present only on *b* or the one with velocity 1300m/s at 2.2s that is stronger in *b* . In addition, the velocity panel corresponding to the constant-slowness splitting is displayed in *c* while the last panel (*d*) refers to the variable-slowness filtering applied over different Snell zones. It is important to notice that whereas the shallow events are easy to identify in *c* and most of the energy associated with multiples has been suppressed, an undesirable trend corresponding to aliased energy strongly contaminates the whole panel. The use of a variable ray parameter filter, however, provides a clean panel with a definite trend of converted-waves' stacking velocities.

The same procedures were applied on data recorded by GECO in Barents Sea (offshore Norway). The data is composed of 48 channels with a group interval of 50 m and maximum offset of 2650 m. The water depth is close to 300 m and the ocean floor is "semi-hard" (P velocity around 1850 m/s). A shot gather of this data is shown in Figure ??*a* and the same gather, after multiples-suppression and separation of the converted wavefield by the variable slowness algorithm, is displayed in *b* . The velocity panels corresponding to the two seismograms are shown in *c* and *d* . Although a nonambiguous interpretation of the velocity trend is not feasible, it is possible to delimit with reasonable confidence the range of acceptable stacking velocities for the converted waves.

The application of the method in a more appropriate dataset is still required, because the poor quality of the present data and the inadequacy of the survey parameters (maximum offset and group interval) strongly restrict the resolution of any prestack procedure and compromise a critical evaluation of its efficiency.

CONCLUSION

The critical point in the attainment of an elastic wavefield in marine environments is the process of separation of the several modes that cohabit in the recorded data. Particularly important is the step regarding the deconvolution of water-bottom multiples and peg-legs, that will share a common range of horizontal slownesses with the converted modes associated with deep reflectors. The use of a constant slowness filter in the ω-k_x domain proves to be ineffective for deep reflectors, for which even the converted modes have a small ray parameter value. The usual solution is to perform the splitting process in the τ-p domain, in which an appropriate filter can be designed for each particular case. In this paper an alternative method was introduced: it makes use of Snell rays to delimit zones of different P wave horizontal slownesses in the data, and applies a different filter to each region. Although a better treatment of the multiples is still required, the method shows to be effective even in a conventional survey. The next step is the application of the whole process in a more appropriate dataset (long cable, small group interval, etc.).

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APPENDIX A

As stated before the conditions most suitable for the generation of converted waves are also appropriate for the generation of water-bottom multiples and peg-legs. This appendix describes the method used to attenuate this kind of multiples before the separation process is applied.

Attenuation of Multiples

The third term on equation (6) corresponds to the multiples and peg-legs associated with the water bottom and must be removed so that equations (7) and (8) can be used to separate the two upcoming waves. The process used is a simplified version of the method developed by Morley (1982) and described by Wiggins (1988) for the case of a nearly horizontal ocean floor.

As illustrated in Figure **??**, the ocean-bottom multiples associated with the upcoming wavefield can be predicted using the following process:

1. Replacement of the air layer by a water layer, above the water surface.
2. Upward continuation of the wavefield by a distance of two water depths.
3. Multiplication by a factor that corresponds to the product of the reflection coefficients of the water bottom and water surface.

Similarly, the downgoing wavefield (or ghost), can have its multiples predicted by the same process, except that the wavefield must be continued downward instead of upward. Since the operators to downward-continue a downgoing wave or upward-continue an upcoming wave in a homogeneous media are identical, the predicted wavefields corresponding to the multiples can be expressed by

$$m(x,t) = \alpha(p)\mathcal{L}(x,t,z)d(x,t),$$

where $\alpha(p)$ is the product of the two mentioned reflection coefficients at horizontal slowness p, and z is the water depth. The determination of $\alpha(p)$ and z is achieved by an optimization algorithm that minimizes the objective function

$$\delta(\alpha_{(p)}, z) = \sum_x \sum_t \parallel d(x,t) - m(\alpha, z, x, t) \parallel^2 .$$

This optimization is easily performed in the ω-k_x domain because the extrapolation operator has the form of a simple phase shift, and the decomposition into different values of slowness is accomplished by simple radial slices within the transform plane. The objective function on this domain assumes the form

$$\delta(\alpha_{(p)}, z) = \sum_k \sum_\omega \parallel d(k,\omega) - m(\alpha, z, k, \omega) \parallel^2 .$$

The comparison between the computed curve $\alpha(p)$ and the theoretical curve for the ocean-floor reflection coefficient (Figure **??**) shows that the computed reflection coefficient diverges from the theoretical at small and large values of slowness. The reason for this result is that no primaries are recorded at small angles (because of the initial offset gap), and no multiples are recorded with large incident angles. The primaries that generated the multiples that are present in the near traces were not recorded, and so, can not be used to predict their multiples. Another way to see the problem is to recall that the upward continuation process corresponds to a propagation of energy from small to large offsets and in the direction of increasing time. Therefore, the near traces in the model will not have enough energy to properly represent the multiples. It is important to notice also that in real data, the source's directiveness will have a major influence on $\alpha(p)$.

Although the efficiency of the method at small offsets is not critical for the present goals of recovering the converted waves where they are sufficiently strong (large offsets), a possible way to improve the performance at near offsets is the use of an extrapolation process to fill in the gap before the upward continuation.