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Cascaded 15-degree normal moveout

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ABSTRACT

The superposition of 15-degree partial normal moveouts is proved to converge to normal moveout. This relation is a basis for cascaded migrations.

INTRODUCTION

Claerbout (1986) examines the expression of normal moveout as the superposition of N^{th} partial normal moveouts. This relation serves him as a basis for cascaded migrations. In this paper an exact proof of the relation is given.

15-DEGREE PARTIAL NORMAL MOVEOUT

Let us repeat basic definitions from Claerbout's paper. A truncated Taylor series of the normal moveout equation is given by the equation:

$$\tau = t - \frac{x^2/v^2}{2t}. \tag{1}$$

The N^{th} partial normal moveout is given by the equation:

$$\tau_N(t) = t - \frac{x^2/v^2}{2tN}. \tag{2}$$

The N^{th} partial normal moveout can be applied N times:

$$T_N(t) = \tau_N \tau_N \cdots \tau_N(t) \tag{3}$$

The equation to be proved is

$$T_\infty = \sqrt{t^2 - x^2/v^2}; \tag{4}$$

i.e., normal moveout can be expressed as a superposition of N N^{th} partial normal moveouts with arbitrary precision as $N \to \infty$.

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PARTIAL NORMAL MOVEOUT

The case of the N^{th} partial normal moveout that is not a truncated Taylor series is easier, so let us examine it first. Define now the N^{th} partial normal moveout by

$$\tau_N(t) = \sqrt{t^2 - \frac{x^2}{Nv^2}}. \tag{5}$$

We can see that the truncated Taylor series of equation (5) is equation (2). Now we will prove that

$$T_N(t) = \tau_N \tau_N \cdots \tau_N(t) = \sqrt{t^2 - \frac{x^2}{v^2}} \tag{6}$$

holds exactly for all N. Let us denote the application of the N^{th} partial normal moveout i times by

$$\mathcal{T}_i(t) = \tau_N \tau_N \cdots \tau_N(t) \tag{7}$$

It holds true that

$$\mathcal{T}_N = T_N. \tag{8}$$

The superposition of $N^t h$ partial normal moveouts may be written

$$\mathcal{T}_i = \sqrt{\mathcal{T}_{i-1}^2 - \frac{x^2}{Nv^2}} \tag{9}$$

for $i = 1, 2, \cdots, N$ and $\mathcal{T}_0 = t$. Let us rewrite this equation into a system of equations:

$$\mathcal{T}_1^2 = \mathcal{T}_0^2 - \frac{x^2}{Nv^2} \tag{10}$$

$$\mathcal{T}_2^2 = \mathcal{T}_1^2 - \frac{x^2}{Nv^2} \tag{11}$$

$$\cdots \tag{12}$$

$$\mathcal{T}_N^2 = \mathcal{T}_{N-1}^2 - \frac{x^2}{Nv^2} \tag{13}$$

By summing up all the equations we get

$$\mathcal{T}_N^2 = t^2 - N\frac{x^2}{Nv^2} = t^2 - \frac{x^2}{v^2}, \tag{14}$$

which is the desired result.

PROOF FOR 15-DEGREE PARTIAL NORMAL MOVEOUTS

Let us write

$$\mathcal{T}_{i+1} = \sqrt{\mathcal{T}_i - \frac{x^2}{Nv^2}} \tag{15}$$

for the nontruncated case and

$$\tilde{\mathcal{T}}_{i+1} = \tilde{\mathcal{T}}_i - \frac{x^2}{2Nv^2\tilde{\mathcal{T}}_i} \tag{16}$$

for the truncated case. We will estimate the difference $(T_N^2 - \tilde{T}_N^2)$ and show that this difference tends to zero as $N \to \infty$. We can write

$$\begin{aligned}
\mathcal{T}_{i+1}^2 - \tilde{\mathcal{T}}_{i+1}^2 &= (\mathcal{T}_i^2 - \frac{x^2}{Nv^2}) - (\tilde{\mathcal{T}}_i - \frac{x^2}{2Nv^2\tilde{\mathcal{T}}_i})^2 && (17)\\
&= (\mathcal{T}_i^2 - \frac{x^2}{Nv^2}) - (\tilde{\mathcal{T}}_i^2 - \frac{x^2}{Nv^2} + \frac{x^4}{4N^2v^4\tilde{\mathcal{T}}_i^2}) && (18)\\
&= (\mathcal{T}_i^2 - \tilde{\mathcal{T}}_i^2) - \frac{x^4}{4N^2v^4}\frac{1}{\tilde{\mathcal{T}}_i^2} && (19)
\end{aligned}$$

As we did in the previous section, we can rewrite this equation into a system of equations:

$$\begin{aligned}
\mathcal{T}_1^2 - \tilde{\mathcal{T}}_1^2 &= (\mathcal{T}_0^2 - \tilde{\mathcal{T}}_0^2) - \frac{x^4}{4N^2v^4}\frac{1}{t^2} && (20)\\
\mathcal{T}_2^2 - \tilde{\mathcal{T}}_2^2 &= (\mathcal{T}_1^2 - \tilde{\mathcal{T}}_1^2) - \frac{x^4}{4N^2v^4}\frac{1}{\tilde{\mathcal{T}}_1^2} && (21)\\
&\cdots && (22)\\
\mathcal{T}_N^2 - \tilde{\mathcal{T}}_N^2 &= (\mathcal{T}_{N-1}^2 - \tilde{\mathcal{T}}_{N-1}^2) - \frac{x^4}{4N^2v^4}\frac{1}{\tilde{\mathcal{T}}_{N-1}^2} && (23)
\end{aligned}$$

By summing up all these equations we get

$$T_N^2 - \tilde{T}_N^2 = -\frac{x^4}{4N^2v^4}\sum_{i=0}^{N-1}\frac{1}{\tilde{\mathcal{T}}_i^2} \tag{24}$$

Now we need to prove that there exists a constant K independent of N such that

$$\frac{1}{\tilde{\mathcal{T}}_i^2} < K \tag{25}$$

for all $i = 0, 1, \cdots, N-1$. From equation (16) we can see that

$$\tilde{\mathcal{T}}_i > \tilde{\mathcal{T}}_{i+1} \tag{26}$$

Similarly from equation (24) we can see that

$$\tilde{T}_N > T_N \tag{27}$$

Putting both equations together we have

$$\tilde{\mathcal{T}}_1 > \tilde{\mathcal{T}}_2 > \cdots > \tilde{T}_N > T_N \tag{28}$$

or

$$\frac{1}{\tilde{T}_i^2} < \frac{1}{T_N^2} = \frac{1}{t^2 - x^2/v^2} \tag{29}$$

where the last equation follows from equation (14). If we define

$$K = \frac{1}{t^2 - x^2/v^2} \tag{30}$$

then from equation (24) we have

$$|T_N^2 - \tilde{T}_N^2| < \frac{x^4}{4N^2v^4} NK = \frac{x^4 K}{4Nv^4} \tag{31}$$

which tends to zero as $N \to \infty$. This proves that the N^{th} partial normal moveout applied N times is equal to the normal moveout for large N.

The convergence of cascaded NMO to a semicircle can be clearly seen on Figure **??**a. The proof of convergence holds for $x/v \neq t$. The behavior for $x/v = t$ can be seen from sequences at the sides of Figure **??**b. The figure convinces us that there is convergence even for $x/v = t$, but that it is extremely slow.

RATE OF CONVERGENCE

The rate of convergence may be estimated from equation (31). It can be rewritten

$$|T_N - \tilde{T}_N| < \frac{x^4}{8Nv^4 T_N^2 (T_N + \tilde{T}_N)}. \tag{32}$$

Because

$$T_N + \tilde{T}_N > 2T_N \tag{33}$$

we can obtain this estimate of the number of iterations:

$$N > \frac{x^4/v^4}{8(t^2 - x^2/v^2)^{3/2}\epsilon}, \tag{34}$$

where ϵ is the required precision.

The number of iterations estimated from equation (34) is higher than the real number of iterations needed to achieve the desired precision. For a low number of iterations (up to about 10), inequality (34) gives a good estimate of the actual number of iterations. Curves of the numbers of iterations required to achieve particular precisions are shown on Figure **??**c. These curves are not computed from equation (34), but they are the numbers of iterations really needed to achieve the required precision.

SUMMARY

The proof that NMO can be expressed as the superposition of N N^{th} 15-degree partial normal moveouts with arbitrary precision as $N \to \infty$, can be summarized in this way:

- The relation holds exactly for partial normal moveouts.
- The 15-degree partial normal moveout approximates a partial normal moveout with an error that tends to zero faster than $1/N$ with increasing N, so that the total error (=sum of N errors) tends to zero with increasing N.

In cascaded migrations, a small number of iterations is used; hence inequality (34) may be used for an estimation of the number of iterations required to achieve angle accuracy.

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REFERENCES

Claerbout, J.F.,1986, Velocity extrapolation by cascaded 15-degree migration: SEP-**48**, 79-84.