Stanford Exploration Project, Report 61, January 13, 1998, pages 1–68

Kinematics of Prestack Partial Migration in a variable velocity medium

Alexander M. Popovici and Biondo Biondi[1]

keywords: *velocity, DMO, prestack, modeling*

ABSTRACT

Prestack partial migration (PSPM) is a well known process for transforming to zero offset the prestack data, when the velocity function is slowly varying. To adapt PSPM to media in which the velocity $\mathbf{v(z)}$ is a rapid function of $\mathbf{z}$ the generalized PSPM impulse response is computed using raytracing and is applied to the data as an integral operator. The impulse response of the PSPM operator can be computed by considering the PSPM process as the combination of full prestack migration followed by zero-offset modeling.
The kinematics of the proposed PSPM operator and the ones of the constant velocity PSPM differ significantly in some realistic cases, such as in presence of a low velocity layer or when a hard sea-bottom causes a jump in velocity.

INTRODUCTION

Prestack data are usually transformed to zero offset by the application of normal moveout (NMO) followed by dip moveout (DMO). Different algorithms have been proposed to perform DMO assuming a constant velocity medium (Deregowski and Rocca, 1981; Hale, 1983; Jakubowitz, 1984; Biondi and Ronen, 1987). These conventional DMO operators are kinematically correct when velocity is constant and they are approximately correct when velocity linearly increases with depth (Hale, 1983), but they cannot be applied when velocity varies rapidly. In this case the prestack partial migration (PSPM) process cannot even be split in NMO followed by DMO, but must be applied as a single process.

A PSPM algorithm that transforms the prestack data to zero offset, even when velocity is a general 2-D function, is an attractive alternative to full prestack migration because it requires less computer resources. In 2-D the spatial aperture of the PSPM operator in constant offset sections is limited by the offset, while the corresponding full prestack migration operator has unlimited aperture. In 3-D the gain in efficiency is even more important; the impulse response of full prestack migration is

[1]**email:** not available

a a 3-D ellipsoid, while the impulse response of the PSPM operator is a simple 2-D ellipse lying in a vertical plane passing through the source and the receiver. Another application where PSPM has potential advantages over full prestack migration is velocity estimation. The velocity function assumed for applying PSPM influences the alignment in time of the reflections over offset but not their absolute position. On the contrary, the choice of the migration velocity influences the absolute position of the reflections (Fowler, 1988).

The determination of the correct amplitudes is important for many applications. In particular when DMO is used for Amplitude Versus Offset (AVO) analysis of the prestack data (Gardner and Forel, 1988; Black and Egan, 1988; Beasley and Mobley, 1988).

We propose in this paper an algorithm for computing the impulse response of the PSPM operator for any $\mathbf{v(z)}$ velocity model. The method can be easily adapted for $\mathbf{v(x,z)}$. We apply the proposed algorithm for computing the kinematics of the impulse response when velocity is constant and for different velocity functions of depth. We show that the impulse response in variable velocity media not only differs from the constant velocity one by a time shift but in certain cases triplications appear in the impulse response. The proposed algorithm can be applied to partially migrate the prestack data using the impulse response as an integral operator.

PRESTACK PARTIAL MIGRATION

Prestack depth migration of reflectors is a two-step process: focusing followed by mapping. In conventional processing the focusing is accomplished by normal moveout (NMO) followed by stacking, while the mapping is provided by poststack depth migration. In presence of dipping reflectors the NMO step must be followed by dipmoveout to properly focus the reflections from dipping beds. Migrating the data in two steps has some advantages with respect to migrating it in only one step. One theoretical advantage is that focusing produces a result in time and not in depth, therefore focusing enhances the reflectors in a zero offset section but does not affect their position. This property of focusing satisfies both the conservatives, who think that velocity cannot be estimated well enough to rely on the result of mapping, and the anisotropic people, who believe that horizontal (focusing) velocities and vertical (mapping) velocities are different. Efficiency is the other important advantage in splitting the migration process. Only the stacked data is mapped to depth and the whole prestack data is focused instead of being fully migrated.

DMO

DMO is much cheaper than full prestack migration because the spatial aperture of the DMO operator in constant offset sections is limited by the offset, while the corresponding full prestack migration operator has unlimited width. The gain in efficiency is important in a 2-D medium, but it is dramatic in a 3-D medium.

DMO is usually applied as a velocity independent processing. A velocity independent DMO is correct when the velocity is constant; it is approximately correct when the velocity varies with depth, but it produces wrong the results in the presence of strong lateral velocity variations. In these cases the standard processing of NMO followed by DMO cannot be used because the moveouts in common midpoint gathers are not any more hyperbolic, but the focusing can still be accomplished in the single step procedure which is the prestack partial migration (Yilmaz and Claerbout, 1980).

THE PSPM OPERATOR IN CONSTANT VELOCITY MEDIA

Prestack partial migration is a mapping of prestack data to zero-offset data. This mapping can be subdivided in two steps:

1. Full prestack migration to a depth model. An input data point in a constant offset section, is migrated into an ellipse. In other words a single impulse on a trace can be generated by a reflection in any point on the reflecting elliptic surface.

2. Zero-offset modeling. Each point on the ellipse can be modeled into a hyperbola, which is obtained by considering the chosen point as a point diffractor.

As a result, an impulse in a constant offset section will be moved in a zero offset section accounting also for the effect of the dipping reflector. The element which fixes the position of the migrated point on the ellipse is the dipping angle α. The coordinates of the points on the ellipse are functions of the dip angle α.

The mapping of the input point (t_h, x_h) from a constant offset section into the point (t_0, x_0) in a zero-offset section can be expressed as

$$(t_h, x_h) = migration(\mathbf{v}, \mathbf{h}, \alpha) \Longrightarrow (\mathbf{z_m}, \mathbf{x_m}; \alpha)$$

$$(z_m, x_m; \alpha) = modeling(\mathbf{v}, \alpha) \Longrightarrow (\mathbf{t_0}, \mathbf{x_0}; \alpha).$$

Given the geometry in Figure 1, the coordinates of the point P can be expressed as a function of the semiaxes of the ellipse (**a** and **b**) and the dip angle α. The semiaxis are $\mathbf{a} = t_h V$ and $\mathbf{b} = \sqrt{t_h^2 V^2 - h^2}$, where V is half the earth velocity. The equation of the ellipse is:

$$\frac{x_m^2}{a^2} + \frac{z_m^2}{b^2} = 1 \tag{1}$$

Which can be rewritten as

$$z_m^2 a^2 = a^2 b^2 - x_m^2 b^2$$

The equation of the tangent to the ellipse is $z_m = \mathbf{m}x_m + \mathbf{c}$ where $\mathbf{m} = \tan\alpha$ and $\mathbf{c} = \sqrt{a^2\tan^2\alpha + b^2}$.

$$z_m = x_m \tan\alpha + \sqrt{a^2\tan^2\alpha + b^2} \tag{2}$$

Squaring equation 2 and multiplying both sides with $\mathbf{a^2}$ we obtain:

$$z_m^2 a^2 = a^2(x_m^2 \tan^2\alpha + 2x_m \tan\alpha\sqrt{a^2\tan^2\alpha + b^2} + a^2\tan^2\alpha + b^2)$$

As the intersection of the tangent to the ellipse should satisfy both equation (1) and (2), by equalizing both sides of $z_m^2 a^2$ we obtain:

$$x_m^2(b^2 + a^2\tan^2\alpha) + 2x_m a^2\tan\alpha\sqrt{a^2\tan^2\alpha + b^2} + a^4\tan^2\alpha + a^2b^2 = a^2b^2 \tag{3}$$

$$(x_m\sqrt{a^2\tan^2\alpha + b^2} + a^2\tan\alpha)^2 = 0 \tag{4}$$

From the previous equation (4) we obtain the value for x_m, and introducing this value in equation 1, we obtain z_m.

$$x_m = -\frac{a^2\tan\alpha}{\sqrt{a^2\tan^2\alpha + b^2}} \tag{5}$$

$$z_m = \frac{b^2}{\sqrt{a^2\tan^2\alpha + b^2}} \tag{6}$$

Each point on the ellipse corresponding to a certain parameter α (dipping angle), can be considered a point diffractor and therefore generate a hyperbola. For a zero-offset section the point will be situated on the modeling hyperbola with the following coordinates:

$$t_0 = \frac{z_m}{V\cos\alpha} \tag{7}$$

$$x_0 = x_m + z_m\tan\alpha \tag{8}$$

Inserting equations (5,6) in (7,8) we obtain:

$$t_0 = \frac{b^2}{V}\sqrt{\frac{1}{a^2\sin^2\alpha + b^2\cos^2\alpha}} \tag{9}$$

$$x_0 = -h^2\sin\alpha\sqrt{\frac{1}{a^2\sin^2\alpha + b^2\cos^2\alpha}} \tag{10}$$

which are the parametric equations (function of the dip angle α) of the well known DMO ellipse.

In the end the DMO operator contoured for only half space will look like in Figure 3. We demonstrate in Appendix A that the operator follows the envelope of the hyperbolas.

PSPM IMPULSE RESPONSE IN MEDIA WITH VARIABLE VELOCITY

The above formulation of the DMO operator allows us to obtain the impulse response of the DMO in a medium with variable velocity in depth. The mappings can be evaluated using ray-tracing for an arbitrary velocity model. The migration depends on the half offset h and thus must be computed for each common-offset section, while the modeling is independent from the offset of the data and must be evaluated only once. The algorithm used to investigate the DMO operator follows the definition of the PSPM method and can be divided in two parts:

1. Construct the full migration depth model. For a constant velocity medium this is equivalent to constructing the migration ellipse for a constant offset section. For a variable velocity medium, the loci of the points with equal travel time from source to receiver is a curve resembling an ellipse or a superposition of several ellipses. See Figures ?? for an example of migration depth models.
2. Zero-offset modeling. Given the depth model (the reflector which generates the same impulse in the constant offset section) we raytrace back at 90 degree from the reflector, simulating the zero-offset case. The intersection of the ray with the surface will give the x coordinate of the DMO operator, while the travel time along the raypath will provide the time coordinate. See Figures ?? for an example of DMO impulse responses.

The raytracing algorithm is based on solving the acoustic wave equation in the high frequency approximation (eikonal equation). The velocity model has only a depth variation, though the algorithm can be easily modified to handle lateral velocity variation as well. The velocity is assumed to be a continuous function of depth, and the smoothness of the variation can be controlled by a B spline interpolation subroutine.

For a medium with linear increase of the velocity with depth $v(z) = v_0 + az$, an impulse will map in a elongated ellipse, the elongation being proportional with the coefficient a. The coefficient a varies from 0.3 to 1.3 according to Telford. The deviation from the constant velocity case is negligible for an coefficient a smaller then 1, when the DMO velocity is chosen correctly.

For media where velocity is not a linear function of depth, the DMO impulse response may have serious variations from the constant velocity case. Triplications can occur in the normally ellipse-shaped DMO impulse response. Figures ?? show an example of atypical DMO operators.

More here on DMO differences!!!

INTEGRAL DMO WITH RAYTRACING

Describe here algorithm for DMO to real data

One common objection to an exact PSPM is the presence of blind zones when lateral velocity gradients are large (Deregowski, 1986; Hale, 1988). A simple remedy of this problem that we would like to investigate is using PSPM for transforming data not to zero-offset, but to a sufficiently large offset without blind zones, or, if needed, to two different offsets.

REFERENCES

Beasley, C. J., and Mobley, E., 1988, Amplitude and antialiasing treatment in (x,t) domain DMO: paper presented at the 58th Annual International SEG Meeting, Anaheim, CA.

Black, J. L., and Egan, M. S., 1988, True amplitude DMO in 3-D: paper presented at the 58th Annual International SEG Meeting, Anaheim, CA.

Dellinger, J., and Muir, F., 1988, Migration = focus + map: SEP – **57**, 307-312.

Deregowski, S. M., and Rocca, F., 1981, Geometrical optics and wave theory of constant-offset sections in layered media: Geophysical Prospecting

Deregowski, S. M., An integral implementation of dip moveout: paper presented at the 47th Annual International EAEG Meeting, Budapest, Hungary.

Deregowski, S. M., 1986, What is DMO?: First Break, **4**, 6-24.

Etgen, J., 1988, Prestack depth migration velocity analysis: linear theory: SEP – **59**, 121-140.

Fowler, P., 1988, Seismic velocity estimation using prestack time migration: Ph. D. dissertation, Stanford University.

Gardner, G. H. F., and Forel, G., 1988, Amplitude preservation equations for DMO: paper presented at the 58th Annual International SEG Meeting, Anaheim, CA.

Gardner, G. H. F., and Uren, N. F., 1988, Dip moveout in anisotropic media: paper presented at the 58th Annual International SEG Meeting, Anaheim, CA.

Hale, I. D., 1988, Dip-moveout processing: Course notes from SEG continuing education course, SEG.

Yilmaz, O., and Claerbout, J. F., 1980, Prestack partial migration: Geophysics, **45**, 1753-1779.

Appendix A

The impulse response of the DMO operator is the envelope to the family of hyperbolas obtained by considering each point on the migration ellipse, a point diffractor. This is a rather peculiar property of the DMO operator and we determine here the equation of the envelope of the hyperbolas, which coincides with the DMO operator. While the equation of the envelope is the equation of an ellipse with the horizontal semiaxis equal with the offset, the impulse response of the DMO is only a part of the total ellipse, the maximum **x** coordinate being

$$\frac{x_{max} = h^2}{a}$$

; where **h** is the offset and **a** is the semiaxis of the full constant offset migration ellipse

$$\frac{a = \mathbf{v} t_h}{2}$$

.

The equation of the family of hyperbolas in Fig.A.1 is:

$$z^2 = z_m^2 + (x_m - x)^2 \tag{11}$$

The points of coordinates (x_m, z_m) are situated on the ellipse, representing a parametrized form of the ellipse equation.

$$x_m = -\frac{a^2 \tan\alpha}{\sqrt{a^2 \tan^2\alpha + b^2}}$$

$$z_m = \frac{b^2}{\sqrt{a^2 \tan^2\alpha + b^2}}$$

In expanded form the function $F(x, z, t)$ where $t = \tan\alpha$ will be:

$$F(x, z, t) = z_m^2 + (x_m - x)^2 - z^2 = \frac{b^4}{a^2t^2 + b^2} + (\frac{-a^2 t}{\sqrt{a^2t^2 + b^2}} - x)^2 - z^2 = 0 \tag{12}$$

The envelope of the family of curves represented in equation ?? is obtained by eliminating the parameter **t** from the two equations:

$$F(x, z, t) = 0$$

$$\frac{\partial F(x, z, t)}{t} = 0$$

We have then the equations:

$$F(x,z,t) = \frac{b^4 + a^4t^2}{a^2t^2 + b^2} + \frac{2a^2tx}{\sqrt{a^2t^2+b^2}} + x^2 - z^2 = 0 \tag{13}$$

$$F'(x,z,t) = \frac{2a^2b^2t(a^2-b^2)}{(a^2t^2+b^2)^2} + \frac{2a^2b^2x}{(a^2t^2+b^2)^{\frac{3}{2}}} \tag{14}$$

$$F'(x,z,t) = \frac{2a^2b^2(t(a^2-b^2) + x\sqrt{a^2t^2+b^2})}{(a^2t^2+b^2)^2} = 0$$

Which gives for **t**:

$$t(b^2-a^2) = x\sqrt{a^2t^2+b^2}$$

$$t^2 = \frac{b^2x^2}{(b^2-a^2)^2 - a^2x^2} \tag{15}$$

Introducing **t** in ?? we obtain:

$$z^2 = b^2 - \frac{x^2b^2}{(b^2-a^2} \tag{16}$$

Because **a** and **b** represent the semiaxis of the ellipse, the offset **h** is given by $h^2 = a^2 - b^2$. The equation ?? can be rewritten as:

$$z^2 = b^2 - \frac{x^2b^2}{h^2} \tag{17}$$

$$\frac{z^2}{b^2} + \frac{x^2}{h^2} = 1$$

The equation of the envelope is the equation of an ellipse, with the vertical semiaxis equal to **b** and the horizontal semiaxis equal to the offset **h**. As the depth **z** is given by

$$z = \mathbf{v}t_dmo$$

and the semiaxis **b** is given by

$$b = \mathbf{v}t_n$$

where

$$t_n$$

is the time impusle after the NMO correction, we can rewrite the equation as

$$\frac{t_dmo^2}{{t_n}^2} + \frac{x^2}{h^2} = 1 \tag{18}$$

which is the well known equation of the full DMO ellipse.