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A modified anisotropic system

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INTRODUCTION

Some years back, Muir & Dellinger (1985), in an attempt to simplify the processing of multi-component seismic data, introduced a system of equations that unified ray and wave theory in an approximate manner. This allowed, for example, time/offset relations to be converted to dispersion relations which could then form the basis for anisotropic imaging schemes.

This paper removes one drawback from that scheme—the use of an anelliptic factor that had no counterpart in normal experience—and broadens it sufficiently that it could find application in the three geometries now in common use, surface-to-surface, borehole-to-surface, and borehole-to-borehole.

WAVE EQUATIONS

The key equation is the dispersion relation, the equation that links the temporal and spatial frequencies of an eigenfield in the Fourier domain. The four variable parameters all have units of velocities squared. W_x and W_z are the actual horizontal and vertical square velocities that apply equally well to waves and rays, and W_{xnmo} and W_{znmo} are the apparent square velocities as determined from move-out measurements. For example, W_{xnmo} is the square NMO velocity that we would use every day in surface-to-surface data processing. If the x subscript seems unusual for a paraxial measurement about the vertical, remember that in an elliptic world it is the horizontal velocity that move-out measures.

$$\omega^2 = \frac{W_x^3 k_x^6 + W_x^2 (2W_z + W_{znmo}) k_x^4 k_z^2 + (2W_x + W_{xnmo}) W_z^2 k_x^2 k_z^4 + W_z^6 k_z^6}{(W_x k_x^2 + W_z k_z^2)^2} \tag{1}$$

With minor modifications that are discussed in Muir & Dellinger (1985), there are precisely equivalent plane wave and partial differential equations.

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RAY EQUATIONS

For the ray equations we need to define four new parameters, all with units of inverse velocity squared. These parameters are defined:

- $M_x = W_x^{-1}$
- $M_z = W_z^{-1}$
- $M_{xnmo} = W_{xnmo}^{-1}$
- $M_{znmo} = W_{znmo}^{-1}$

If we further define:

- ϕ is the angle from horizontal
- c is $\cos(\phi)$
- s is $\sin(\phi)$

then

$$M(\phi) = \frac{M_x^3 c^6 + M_x x^2 (2M_z + M_{znmo}) c^4 s^2 + (2M_x + M_{xnmo}) c^2 s^4 + M_z^3 s^6}{(M_x c^2 + M_z s^2)^2} \qquad (2)$$

Again, in a manner shown in the referenced paper, we can develop move-out equations (about the x or z axes) from this ray equation.

CONCLUSION

Are these wave and ray equations precisely equivalent? No. Equation (1) can be made to approximate dispersion relations that have a horizontal symmetry plane. Approximate it both in the neighbourhood of the z axis for conventional seismic and VSP data, and in the neighbourhood of the x axis for cross-hole data. Likewise Equation (2) will then be a good paraxial approximation in the neighbourhood of both the z and the x axes.

REFERENCES

Muir,F. and Dellinger, J., 1985, A Practical Anisotropic System: SEP–**44**, 55–58.